Systemic Shocks, Banking Spreads and the External Finance Premium

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Current Draft: June 10, 2009

Abstract

In this paper, we study the dynamics of the external finance premium in an accelerator model of the economy. To better capture the dynamics in financial crises, we use features of a regulated banking sector as well as a notion of systemic risk. The model produces an external finance premium that is an increasing function of systemic risk. As well, it finds that bank leverage is also positively related to increased spreads, but only when the economy is in stress. That is, changes in bank regulatory requirements have little impact in good times, but act as a multiplier when the economy is in a "bad" state.

JEL Classification: E52, E58, G18, G28

KEY WORDS: Financial Accelerator, Capital Adequacy Requirements, Optimal Monetary Policy

*We would like to thank Claudio Borio, Simon Glichrist, Christian Upper, John Taylor, Oreste Tristani, David Vestin, Mark Wynne and conference participants at the 12th annual BIS/CEPR central banking conference and the 12th annual conference of the central bank of Chile for helpful comments and discussions. We also acknowledge the excellent research assistance provided by Jonathan Morse and the support of the Federal Reserve Bank of Dallas and the Federal Reserve Bank of Boston. All remaining errors are ours alone. The views expressed in this paper do not necessarily reflect those of the Federal Reserve Banks of Dallas, the Federal Reserve Bank of Boston, or the Federal Reserve System.

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1 Introduction

The ongoing and now long-lasting crisis has made clear that a generation of new-Keynesian models has yet to fully meet the challenge of evaluating the role of monetary policy under financial stress. These models relied on a range of market imperfections, such as nominal rigidities and monopolistic competition, to allow for non-trivial market power and non-simultaneous and/or non-synchronous price setting behavior. These features allowed an understanding of how relative price misalignments induce demand shifts that can significantly impact output. Thus, monetary policy in as much as inflation can amplify or dampen those relative price misalignments, could also have real effects through an impact on the demand.

These insights generated a wide research agenda that studied the basic role of monetary policy in models with nominal rigidities. As is now widely accepted, most of these models omitted details of financial market imperfections. This omission formed the impetus for a new round of (second-generation) new-Keynesian models. A number of papers turned their attention to the role of financial and credit market imperfections (see, e.g., Bernanke et al. (BGG), 1999, and Carlstrom and Fuerst, 2001). A core component of the insight in the BGG (1999) model was that asymmetric information in lending, due to costly state verification, leads to a premium on external finance. This premium, in turn, is crucial to understanding the role of monetary policy in the economy. This premium provides a link between the financial and real sectors, and makes it possible for monetary policy to operate also through balance sheet effects on the borrowers. The Modigliani-Miller theorem no longer holds.

This insight has motivated a range of research, including our own. The current crisis, however, has revealed that our understanding of the link between the financial and real sectors may not be fully characterized by the external finance premium (EFP), as formulated by BGG (1999). Our contribution in this paper is to illustrate the dynamics of the EFP in an environment that is close to the framework developed by BGG (1999), but with two important departures that lead to distinct dynamic responses: systemic risk and bank leverage. Indeed our model is consistent with current new-Keynesian models of the BGG (1999) type in ‘good’ times. In ‘bad’ times (crisis periods), when losses are potentially large in the secondary markets for capital goods, the model can generate sharp changes in the EFP that are linked to changes in the state of the economy. Indeed, we show that basic features of banking regulation like deposit reserve requirements or capital adequacy requirements add to the costs that entrepreneurs have to pay to borrow from the financial system - and as a consequence can amplify the cycle beyond what the traditional BGG (1999) framework would predict.\footnote{The literature on this is wide ranging, from Bernanke and Lown (1991) argument that the 1992 Basel I deadline contributed to the early 1990s credit crunch to a range of arguments that capital regulation generates magnified business cycles. Some relevant papers include (Berger and Udell (1994), Blum and Hellwig (1995), Brinkmann and Horvitz (1995), Thakor (1996); recent papers include Goodhart et al. (2004), Estrella (2004), Kashyap and Stein (2004), Gordy and Howells (2006). As well Borio (2007) provides a comprehensive literature review.}

To illustrate these phenomenon, we use the model developed in Cohen-Cole and Martinez-Garcia (2009) - a summary of which is provided below. This synthesis model includes leveraged and regulated financial intermediaries as well as systemic risk shocks to the resale value of invested capital (to capture the fact that secondary markets become impaired during financial distress periods). We view this risk as a good proxy for the asset value shocks that has been witnessed in 2007 and 2008.

Our approach is as follows. We start by maintaining the intuition from BGG (1999) that costly state
verification (CSV) and the EFP are core mechanisms for the transmission of shocks. From this starting point, we look to integrate aggregate systemics shocks and the bank balance sheet roles into the financing premium. This enables us to generate a parsimonious characterization of the finance premium that takes the following stylized form,

\[ EFP = AgShock \times AgencyCostChannel \times BalanceSheetChannel. \]

Why do risk and banking integrate nicely in this equation? Risk to capital resale value magnifies the EFP as it generates an additional wedge between the resale value of capital from prior projects and the acquisition cost of new capital. Indeed, our specification of the risk will allow the EFP to have no wedge in 'good’ times and a large wedge in crisis times. Bank balance sheets enter the equation here as the imposition of regulatory requirements add to the costs that entrepreneurs have to pay to borrow. This wedge is a function of the leverage in the system.

Importantly, our approach differs from existing work in a few ways. In one sense, it provides a method via which regulation matters in a tractable way. In comparison to models such as Curdia and Woodford (2008), which generate financial channel effects through exogenous spread changes, our model produces an important role for a disruption of intermediation precisely because of the trade-offs present in regulation. In another sense, it differs because it provides a simple way to think about financial intermediation via leverage and constraint.

We find two general sets of results. First, we find that the external finance premium is an increasing function of both systemic and idiosyncratic risk. Increases in either lead to increases in spreads to external finance. Second, we find that bank regulation impacts spreads as well, but only when the economy is in stress. That is, changes in bank regulatory requirements has little impact in good times, but acts as a multiplier when the economy is in a 'bad’ state.

The remainder of the paper is structured as follows. We fully describe the foundations of our model in section 2 and present our characterization of the external finance premium in section 3. Section 4 outlines a basic parameterization and shows our results. Section 5 concludes.

## 2 Baseline Model

In this section, we describe the stochastic general equilibrium model from Cohen-Cole and Martinez-Garcia (2009). The financial system is hampered by asymmetries of information in the borrower-lender relationship and costly state verification on one hand, and constrained by regulatory features like capital adequacy and deposit reserve requirements on the other hand. The economy is populated by a continuum of households and entrepreneurs, each with unit mass. In addition, the economy includes three types of non-financial firms – capital goods producers, wholesale producers, and retailers – and one type of financial institution: the banks. All firms, whether financial or non-financial, operate under perfect competition, except for the retailers that exploit a monopoly power in their own varieties. Ownership of all the firms is given to the households, except for wholesale producers who are owned and operated by the entrepreneurs.

The financial system is characterized by banks, who originate the loans and channel funds from the households to the entrepreneurs-borrowers, and a central bank with powers to set both banking regulation as well as monetary policy. Monetary policy is characterized by an interest rate feedback rule in the tradition
Taylor (1993). Banking regulation is summarized in a compulsory reserve requirement ratio on deposits and a capital adequacy requirement on bank capital (or bank equity). The fiscal authority plays a passive role purely acting as a device to eliminate the long-run inefficiency due to monopolistic competition in the retail sector.

In the financial accelerator model of Bernanke et al. (BGG, 1999) entrepreneurs are inherently different from households, hence borrowing and lending is always possible in equilibrium. Financial frictions arise from asymmetric information between entrepreneurs-borrowers and the lenders (i.e., the banking system). Monitoring costs make external financing costly for entrepreneurs and, therefore, the borrowers’ balance sheet conditions play out an important role over the business cycle. Lenders act as a third party inserted between the households and the entrepreneurs-borrowers whose existence and characteristics are all assumed.

Hence, the balance sheet of the lenders that originate the loans becomes passive because ultimately loan supply must be equal in amount to the deposits demanded by the households. Our benchmark extends the BGG (1999) model to enhance the role of the banking balance sheet. In particular, we explore the role that banking regulation has on the ‘bank’s balance sheet’ channel and its relevance for monetary policy. Subsequently, we also explore the interaction between banking regulation and monetary policy. We fit, nonetheless, in the BGG (1999) tradition since the basic structure of banking relationships, intermediation, and contract loans is taken as given, rather than arising endogenously, and since we also maintain the illusion of a perfectly competitive banking system. Our model also shares an important characteristic with the framework of Kyotaki and Moore (1997) in that asset price movements serve to reinforce credit market imperfections unlike the Carlstrom and Fuerst (1997) framework, as noted by Gomes et al. (2003). We depart from BGG (1999) because we note that banking regulation affects the behavior of banks and, therefore, alters the transmission mechanism in the financial accelerator model. We also depart from them because we introduce systemic (or aggregate) risk on capital income to help us analyze the interest rate spreads, the borrower-lender relationship and the business cycle dynamics in response to ‘rare events’ of large capital income losses.

**Timing in the model** At time $t$, in the morning, the monetary and productivity shocks are realized and observed. Then, entrepreneurs rent the capital acquired at $t-1$ to the wholesale producers. Entrepreneurs and households also supply managerial and non-managerial labor, respectively. In turn, wholesale producers manufacture wholesale goods and sell them to the retailers. Households and entrepreneurs get compensated with competitive wages on their labor, and entrepreneurs receive a rental rate on capital as well as the resale value of the depreciated capital. However, this capital income is subject to both idiosyncratic and systematic risk shocks which change the disposable capital income available to each individual entrepreneur. The systematic risk is modelled as a potential loss on the average size of the idiosyncratic risk, which can become very large on the left tail of the distribution. The systemic and idiosyncratic shocks are realized at this point.

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3Often, the literature has focused on the role of financial intermediation to finance the wage bill instead of the investment bill (see, e.g., Carlstrom and Fuerst, 2001). We look at the financing model instead because it emphasizes the impact of financial frictions. The idea is that investment - unlike labor - is an intertemporal decision. Therefore, the financial accelerator model not only has the potential to amplify the effects of a shock, but by constraining capital accumulation it also propagates the effects of the shock over time.
In the afternoon, the government raises taxes from households to subsidize the retailers and to allow the central bank flexibility to accommodate changes in deposit reserves from the banks (which occur during the evening). Retailers produce their differentiated varieties, and sell them to households and entrepreneurs for consumption and to capital goods producers for investment purposes. All of them bundle up the retail varieties in the same fashion whether it is then used for investment or consumption. Retailers also distribute their profits back to their shareholders, the households. Capital goods producers combine depreciated capital with investment goods to produce new capital, which will be used at time $t + 1$.

In the evening, entrepreneurs repay the outstanding one-period loans made at time $t - 1$, make consumption decisions and obtain loans needed to finance the acquisition of new capital in this period. To finance the capital bill in advance, entrepreneurs use internal funds (disposable income net of consumption) and external funds (financial loans) from the banking system. Banks do not observe idiosyncratic shocks on capital income, and therefore originate the loans after accounting for costly monitoring. Simultaneously, the banking system establishes relationships with households to obtain deposits and bank capital both to meet the regulatory requirements (on reserves and capital adequacy) and to maximize profits. Using their new loans, entrepreneurs loans buy new capital from the capital producers. Entrepreneurs need external financing because they must acquire assets today that will not yield returns until tomorrow, and they do not dispose of enough funds to finance the expense directly.

**Idiosyncratic and Systematic Risk** We capture risk in this model as the difference between capital resale price, denoted $\overline{Q}_t$, and new capital value, denoted $Q_t$. Thus, when entrepreneurs buy new capital for risky investment, they do so under conditions that are a functional of the state of the economy. To capture the nature of liquidity-constrained markets, we assume that frictions in the secondary market for used capital prevent arbitrage between the resale value of old capital and the sale value of new capital, i.e. $\overline{Q}_t \neq Q_t$. The resale value will be proportional to Tobin’s $q$, i.e. $\overline{Q}_t = \alpha_t Q_t$. However, we assume that the parties involved in the secondary market (entrepreneurs and capital goods producers) view these frictions as entirely out of their control and, hence, they treat the wedge $\alpha_t$ as an exogenous and random shock. Moreover, there is no centralized market that will ensure a uniform pricing for used capital, so each individual entrepreneur and capital producer matched in the secondary market get a different draw of this random wedge. In other words, $\alpha_t$ enters the model initially as an idiosyncratic shock, not an aggregate one. Aggregation will occur below.

Below, we map this re-sale shock into the idiosyncratic shock to returns on capital used in BGG (1999). That keeps our departure from the original model to a minimum, but requires a comment. We must assume that individual capital goods producers and entrepreneurs are small enough to take them as purely exogenous and out of their control. Otherwise, we could not model the wedge as we do here.

At time $t$, the entrepreneurs-borrowers and the lenders must agree on a contract that facilitates the acquisition of new capital, $K_{t+1}$. The entrepreneurs operate in a legal environment that ensures them

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4Carlstrom and Fuerst (1997) assume that capital goods producers are the ones facing the financing constraints, rather than the producers of final (or wholesale) goods. Alternatively, Carlstrom and Fuerst (2001) focus on the role of financing frictions where the cash-in-advance constraint is placed on the wage bill, rather than on the capital bill. The BGG (1999) framework adopted here is clearly distinct.

5We assume that relationships between banks and households do break up after one period. Hence, every period a new relationship has to be initiated. This implies that bank equity, deposits and loans have the same maturity. Therefore, we entirely abstract from the important problem of maturity mismatch in the bank’s balance sheet.
limited liability. Hence, in case of default at time $t+1$, the banks can only appropriate the total capital income of the entrepreneurs. Loan are restricted to take the standard form of a one-period risky debt contract as in Townsend (1979), Gale and Hellwig (1985) and BGG (1999). The capital income of an individual entrepreneur at time $t+1$ on capital acquired at time $t$ is greatly affected by the realization of the idiosyncratic shock.

We interpret this shock $\omega_{t+1}$ as a reduced form representation for the exogenous losses on the resale value of depreciated or used capital due to frictions in the secondary market that are left unmodelled. Those frictions imply a wedge between the resale value of capital and the acquisition cost of new capital (or Tobin’s $q$) within the period. It is assumed that the shock is not known at time $t$ when the contract is signed, and can only be observed privately by the entrepreneur itself at time $t+1$. Banks, however, have access to a costly monitoring technology that permits them to uncover the true realization at a cost.

We denote $(\omega_{t+1} | s_{t+1})$ the density and $\Phi (\omega_{t+1} | s_{t+1})$ the cumulative distribution of $\omega_{t+1}$ conditional on a given realization of the aggregate shock $s_{t+1}$. The mean return of each entrepreneur on its capital acquisition is a function of the aggregate shock $s_{t+1}$ (e.g., Faia and Monacelli, 2007). The aggregate shock $s_{t+1}$ captures our notion of systemic risk on the resale value of depreciated capital, which has the effect of shifting the distribution of the risky capital income. The systemic risk shock, $s_{t+1}$, follows an AR(1) process of the following form,

$$s_t = (1 - \rho_s) s + \rho_s s_{t-1} + \varepsilon_t^s,$$

where $\varepsilon_t^s$ is a zero mean, uncorrelated and normally-distributed innovation. The parameter $-1 < \rho_s < 1$ determines the persistence of the productivity shock, and $\sigma_s^2 > 0$ the volatility of its innovation. We assume that the unconditional mean, i.e. $-\infty < s < +\infty$ ($s \neq 0$), is fixed at one for simplicity, and also that the innovations of the systemic risk are potentially correlated with the innovations of the productivity shocks, i.e. $-\infty < \sigma_{s,s} \equiv \text{cov} (\varepsilon_t^s, \varepsilon_t^s) < +\infty$.

The expected idiosyncratic shock on capital income, $\omega_{t+1}$, conditional on the realization of the aggregate shock, $s_{t+1}$, is given by,

$$\mathbb{E} [\omega_{t+1} | s_{t+1}] = 1 - J (s_{t+1}),$$

where $0 \leq \lambda \equiv J (s) < 1$ determines the level of the expected losses in steady state, and $-\infty < \xi \equiv J' (s) < +\infty$ characterizes the sensitivity of the expected losses to the systemic risk shock in the secondary market. We believe this specification is flexible enough to allow for catastrophic losses due to the systemic risk shock, $s_{t+1}$.

By choosing $\lambda$ sufficiently close to zero, we ensure that the expected idiosyncratic shock mean is relatively close to one, i.e. $\mathbb{E} [\omega_{t+1} | s_{t+1}] \approx 1$. This is the assumption in BGG (1999), and means in our interpretation that on average entrepreneurs get a resale value on their depreciated capital that is approximately equal to the acquisition cost of new capital. By choosing $\xi$ sufficiently high, we ensure that the transition towards catastrophic losses would be abrupt, rather than gradual. That gives us a practical approximation to a world with two regimes: in good times, the economy is close to the BGG (1999) framework; in bad times (when the systemic risk shock hits), the economy confronts catastrophic losses on the secondary capital markets.

In any event, there is a non-negligible probability that markets will fall into disarray due to catastrophic losses in the secondary market. We conjecture that the systemic risk shock is positively correlated with systemic risk in the secondary market. We believe this specification is flexible enough to allow for catastrophic losses due to the systemic risk shock, $s_{t+1}$.

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6For a discussion of optimal contracts in a dynamic costly state verification framework, see Monnet and Quintin (2005).
the productivity shock, i.e. $\sigma_{a,a} > 0$, implying that periods of catastrophic losses are more likely whenever productivity is also unusually low. We would speculate that following a different monetary policy rule depending on whether the economy is in bad times and good times could make sense in this context. We keep the same structure as BGG (1999), which implies that when systemic losses occur in ‘bad times’ they hit the income generated by the capital acquired by the entrepreneurs, but do not affect the stock of physical capital directly.

2.1 Loan Origination

In this section, we walk briefly through loan contracting to illustrate how systemic risk will link to the costly state verification framework.

At time $t$, the entrepreneur-borrower and the lender must agree on the terms of the loan to be repaid at time $t+1$. Default on a loan signed at time $t$ occurs when the income from capital received at time $t+1$ after the idiosyncratic shock $\omega_{t+1}$ and the aggregate shock $s_{t+1}$ are realized, falls short of the amount that needs to be repaid. Hence the default space is characterized by,

$$\omega_{t+1} R^e_{t+1} P_t Q_t K_{t+1} \leq I^l_{t+1} L_{t+1}, \quad (3)$$

where $I^l_{t+1}$ is short-hand notation for the amount of repayment agreed at time $t$ per unit of loan, $L_{t+1}$ defines the loan size, and $P_t$ is a consumption price index. A risky one-period loan contract at time $t$ can be defined in terms of a threshold on the capital losses, $\varpi_t$, and $R^e_{t+1} P_t Q_t K_{t+1}$, such that the repayment function is given by,

$$I^l_{t+1} L_{t+1} = \varpi_{t+1} R^e_{t+1} P_t Q_t K_{t+1}. \quad (4)$$

Another way to interpret the implication of equations (3) and (4) is that loans can at most appropriate the capital income of the entrepreneur, so naturally loan repayment must be proportional to a measure of capital income.

When default occurs, i.e. when $\omega_t < \varpi_t$, the entrepreneur cannot repay the amount it owns based on the received capital income. To avoid misreporting on the part of the defaulting entrepreneur, the lender must verify the individual’s capital income. That requires the lender to expend in monitoring costs the nominal amount of $\mu \omega_{t+1} R^e_{t+1} P_t Q_t K_{t+1}$. In that case, the entrepreneur gets nothing, while the bank gets $(1 - \mu) \omega_{t+1} R^e_{t+1} P_t Q_t K_{t+1}$. If the entrepreneur does not default, i.e. if $\omega_t \geq \varpi_t$, then he pays $\varpi_{t+1} R^e_{t+1} P_t Q_t K_{t+1}$ back to the lender and keeps the rest for himself, i.e. the entrepreneur gets to keep $(\omega_{t+1} - \varpi_{t+1}) R^e_{t+1} P_t Q_t K_{t+1}$.

We take this defaulting rule and the implied distribution of capital income between the entrepreneur-borrower and the lender as given. At time $t+1$, the capital income expected by the entrepreneur after observing all aggregate shocks, but before the realization of its own idiosyncratic shock $\omega_{t+1}$, can be computed.
as,

\[
\int_{\omega_{t+1}}^{+\infty} \left[ \omega_{t+1} R_t^e P_t Q_t K_{t+1} - I_t^1 L_t^1 \right] \phi \left( \omega_{t+1} \mid s_{t+1} \right) d\omega_{t+1}
\]

\[
= R_t^e P_t Q_t K_{t+1} \left[ \int_{\omega_{t+1}}^{+\infty} \left( \omega_{t+1} - \omega_{t+1} \right) \phi \left( \omega_{t+1} \mid s_{t+1} \right) d\omega_{t+1} \right]
\]

\[
= R_t^e P_t Q_t K_{t+1} f \left( \omega_{t+1}, s_{t+1} \right),
\]

where,

\[
f \left( \omega_{t+1}, s_{t+1} \right) \equiv \int_{\omega_{t+1}}^{+\infty} \omega_{t+1} \phi \left( \omega_{t+1} \mid s_{t+1} \right) d\omega_{t+1} - \omega_{t+1} \left( 1 - \Phi \left( \omega_{t+1} \mid s_{t+1} \right) \right)
\]

which, by the law of large numbers, can be interpreted also as the fraction of the expected capital income that accrues to the entrepreneurs. With the information available at time \( t \), entrepreneurs expected capital income should therefore be equal to,

\[
P_t Q_t K_{t+1} E_t ^{R_t^e} E_t \left[ U_t^e f \left( \omega_{t+1}, s_{t+1} \right) \right],
\]

where

\[
U_t^e \equiv \frac{R_t^e}{E_t \left[ R_t^e \right]} \]
determines the ratio of returns between the realized and the expected capital income, and \( E_t \left[ U_t^e \right] = 1 \). The amount of physical capital itself, \( K_{t+1} \), is known because it has been determined at time \( t \). The information up to time \( t \) includes the realization of the aggregate shock \( s_t \) which has a direct impact on the amount of capital income that would be going to the entrepreneurs in the next period.

Defining

\[
g \left( \omega_{t+1}, s_{t+1} \right) \equiv \left( 1 - \mu \right) \int_{0}^{\omega_{t+1}} \omega_{t+1} \phi \left( \omega_{t+1} \mid s_{t+1} \right) d\omega_{t+1} + \omega_{t+1} \left( 1 - \Phi \left( \omega_{t+1} \mid s_{t+1} \right) \right)
\]

and after some math, we can see that lenders expected capital income will equal,

\[
P_t Q_t K_{t+1} E_t ^{R_t^e} E_t \left[ U_t^e g \left( \omega_{t+1}, s_{t+1} \right) \right],
\]

where \( U_t^e \equiv \frac{R_t^e}{S_t \left[ R_t^e \right]} \) again determines the ratio of returns between the realized and it should be apparent that expected income is a function of \( s_{t+1} \).

**Capital Income Sharing and Dead Weight Loss** In this section, we characterize the sharing of capital income in the context of aggregate risk. A fraction of the capital income, \( J \left( s_{t+1} \right) \), will be transferred to the capital goods producers due to inefficiencies in the secondary market for used capital, while another fraction, \( G \left( \omega_{t+1}, s_{t+1} \right) \), will be lost due to the burden of monitoring. It is worth pointing out that only monitoring costs result in a direct loss of capital income that detracts resources, as we will show later with the resource constraint. But the fact that resources are siphoned out of the hands of borrowers and lenders due to market imperfections somewhere else still has the potential to substantially distort the incentives of both parties involved in the loans and, therefore, to affect the funding of new capital.
We define the following two variables for simplicity of notation:

\[
\Gamma (\omega_{t+1}, s_{t+1}) \equiv \int_0^{\omega_{t+1}} \omega_{t+1} \phi (\omega_{t+1} \mid s_{t+1}) \, d\omega_{t+1} + \omega_{t+1} (1 - \Phi (\omega_{t+1} \mid s_{t+1})),
\]

(10)

\[
\mu G (\omega_{t+1}, s_{t+1}) \equiv \mu \int_0^{\omega_{t+1}} \omega_{t+1} \phi (\omega_{t+1} \mid s_{t+1}) \, d\omega_{t+1}.
\]

(11)

Then, we can rewrite the share on capital income going to the lenders more compactly as,

\[
g (\omega_{t+1}, s_{t+1}) = \Gamma (\omega_{t+1}, s_{t+1}) - \mu G (\omega_{t+1}, s_{t+1}).
\]

(12)

Given the definition of the capital income share going to the entrepreneurs, i.e. \( f (\omega_{t+1}, s_{t+1}) \), it also follows that,

\[
f (\omega_{t+1}, s_{t+1}) = \int_0^{\omega_{t+1}} \omega_{t+1} \phi (\omega_{t+1} \mid s_{t+1}) \, d\omega_{t+1} - \Gamma (\omega_{t+1}, s_{t+1})
= 1 - J (s_{t+1}) - \Gamma (\omega_{t+1}, s_{t+1}),
\]

(13)

where the second equality follows from our characterization of the expectation of the idiosyncratic shock. Based on these definitions, we can infer that the capital income sharing rule resulting from this financial contract satisfies:

\[
f (\omega_{t+1}, s_{t+1}) + g (\omega_{t+1}, s_{t+1}) = 1 - J (s_{t+1}) - \mu G (\omega_{t+1}, s_{t+1}),
\]

(14)

where \( J (s_{t+1}) \equiv 1 - \mathbb{E} [\omega_{t+1} \mid s_{t+1}] \) accounts for the expected systemic losses on the resale value of capital and \( \mu G (\omega_{t+1}, s_{t+1}) \) characterizes the conventional monitoring costs associated with the costly-state verification framework.

The functions \( f (\omega_t, s_t) \) and \( g (\omega_t, s_t) \) represent the sharing rule between entrepreneurs-borrowers and lenders on the capital income implied by the risky one-period loan. Both of them depend on the realization of the systemic risk shock, \( s_{t+1} \). However, as can be inferred, equations (12) and (13) do not add to one. This is due to the fact that there are costs of monitoring to be accounted for, \( \mu G (\omega_{t+1}, s_{t+1}) \), and there are also losses due to systemic risk, \( J (s_{t+1}) \). The losses due to systemic risk ought to be interpreted as transfers of resources from the relationship between borrowers and lenders to a third party outside of it, the capital goods producers.

**The Optimization Problem**  We walk through optimization problem in order to reach our first simplified version of the external finance premium.

The formal contracting problem reduces to choosing the quantity of physical capital, \( K_{t+1} \), and the state-contingent threshold, \( \omega_{t+1} \), that maximize the entrepreneurs’ expected nominal return on capital income net of the loan costs (see equation (7)), i.e.

\[
P_t Q_t K_{t+1} \text{E}_t \left[ R^e_t \right] \text{E}_t \left[ U^e_t (1 - J (s_{t+1}) - \Gamma (\omega_{t+1}, s_{t+1})) \right],
\]

(15)

subject to the state-contingent participation constraint for the lenders (see equation (9)), i.e.

\[
P_t Q_t K_{t+1} U^e_t \text{E}_t \left[ R^e_t \right] \left( \Gamma (\omega_{t+1}, s_{t+1}) - \mu G (\omega_{t+1}, s_{t+1}) \right) \geq I^b_{t+1} L_{t+1} = I^b_{t+1} \left[ P_t Q_t K_{t+1} - N_{t+1} \right],
\]

(16)
where it is implicitly agreed that if lenders participate in this contract, they always supply enough loans, $L_{t+1}$, as long as an *ex ante* participation rate, $I^p_{t+1}$, is guaranteed to them after accounting for all potential losses. In other words, neither we nor BGG (1999) explicitly considers the possibility of credit rationing. All banks share equally on the aggregate size of the loan.

Notice also that, given the definition of $U_{t+1}$, the participation constraint is written to ensure a certain return $I^p_{t+1}$ on the loans for any possible realization of all the aggregate shocks. This implies that all risk arising from either differences between the expected capital income and the realized one or from losses due to systemic valuation risk is taken by the risk-neutral entrepreneur.

The first-order condition with respect to $\lambda (\omega_{t+1}, s_{t+1})$ allows us to derive the shadow cost of enticing the participation of the lenders:

$$\lambda (\omega_{t+1}, s_{t+1}) = \frac{\Gamma_1 (\omega_{t+1}, s_{t+1})}{\Gamma_1 (\omega_{t+1}, s_{t+1}) - \mu G_1 (\omega_{t+1}, s_{t+1})},$$  \hspace{1cm} (17)$$

Efficiency on the participation constraint requires that:

$$\frac{P_t Q_t K_{t+1}}{N_{t+1}^e} = \frac{1}{1 - \left( \frac{R^e_{t+1}}{I^p_{t+1}} \right) \left( \Psi (\omega_{t+1}, s_{t+1}) + J (s_{t+1}) + \Gamma (\omega_{t+1}, s_{t+1}) - 1 \right)},$$  \hspace{1cm} (18)$$

where we define $\Psi (\omega_{t+1}, s_{t+1}) \equiv 1 - J (s_{t+1}) - \Gamma (\omega_{t+1}, s_{t+1}) + \lambda (\omega_{t+1}, s_{t+1}) (\Gamma (\omega_{t+1}, s_{t+1}) - \mu G (\omega_{t+1}, s_{t+1})).$

The optimization also requires the following first-order condition with respect to capital, $K_{t+1}$, to hold,

$$\mathbb{E}_t \left\{ \frac{R^e_{t+1}}{I^p_{t+1}} \Psi (\omega_{t+1}, s_{t+1}) - \lambda (\omega_{t+1}, s_{t+1}) \right\} = 0.$$  \hspace{1cm} (19)$$

Simply re-arranging and allowing for the fact that $U^e_{t+1}$ drops out up to a first order approximation gives us a formulation for the external financing premium:

$$\mathbb{E}_t [R^e_{t+1}] = s \left( \frac{P_t Q_t K_{t+1}}{N_{t+1}^e}, \mathbb{E}_t (s_{t+1}) \right) I^p_{t+1}. $$  \hspace{1cm} (20)$$

This characterization of the external financing premium expands the BGG (1999) framework by adding the explicit possibility that the spread itself be affected by the impact of an aggregate shock, $s_t$.

### 2.2 Banking System

With this initial description of the external finance premium that takes into account systemic risk, we move to a discussion of the role of banking. We assume a continuum of banks of unit mass. In every period, households and banks have to create banking relationships anew. Both households and banks are symmetric and perfectly competitive, so they take all prices as given. The bank offers the households two types of assets for investment purposes: bank equity shares and one-period deposits. At end of period liquidation, all gains are rebated to the shareholders, but there is no resale value on the bank shares. Banks, in turn, use the resources they attract to offer one-period loans to entrepreneurs.

Profits generate a yield, $R^b_{t+1}$, over the value of the banks equity, $B_{t+1}$. Ruling out the possibility of long-lasting relationships between the financial intermediaries and the households implies that bank equity and deposits are indistinguishable for the household. Household deposits are perfectly insured, and pay a risk-free rate, $I_{t+1}$. A pre-condition for an equilibrium to exist in which bank equity and deposits are held...
simultaneously is that households be indifferent between the bank yield, $R_{t+1}^b$, and the risk-free rate, $I_{t+1}$. Otherwise, households would want to save exclusively on either deposits or bank equity, but not both.

At the end of period $t$, the balance sheet of the banking system can be summarized as follows,

$$L_{t+1} + \varpi D_{t+1} = B_{t+1} + D_{t+1},$$

where the right-hand side describes the liabilities, which includes deposits taken at time $t$, $D_{t+1}$, and equity offered at the same time, $B_{t+1}$. The left-hand side shows the assets, $L_{t+1} + \varpi D_{t+1}$. Among the assets, we count the reserves on deposits maintained at the central bank, i.e. $\varpi D_{t+1}$, where $0 \leq \varpi < 1$ represents the compulsory reserve requirement on nominal deposits set by the regulator, and the loans offered at time $t$, $L_{t+1}$. As a matter of convention, $D_{t+1}$ denotes nominal deposits and $L_{t+1}$ nominal loans held from time $t$ to $t+1$. For the same token, $B_{t+1}$ is the bank capital issued at time $t$ to be liquidated at time $t+1$.

We can rewrite more conveniently the balance sheet as,

$$L_{t+1} = (1 + \varpi) D_{t+1},$$

where we define the leverage ratio on bank capital as $u_{t+1} = \frac{B_{t+1}}{L_{t+1}}$. In other words, the rate of transformation from deposits into loans is affected by the compulsory reserve requirement as well as by the bank’s capital leverage policy. In BGG (1999), with $\varpi = 0$ and no bank equity, the transformation rate is one-to-one, i.e. $L_{t+1} = D_{t+1}$. Although the model preserves the basic underlying structure of the bank’s balance sheet in BGG (1999), equation (22) already points out that regulatory features should play a significant role on loan supply.

The banks profits would be realized at time $t+1$, and afterwards the bank should be liquidated. We can express the profits of the banking system as:

$$\Pi_{t+1}^b = I_{t+1}^b L_{t+1} + \varpi T_{t+1} D_{t+1} - R_{t+1}^b B_{t+1} - I_{t+1}^b D_{t+1},$$

The required nominal returns on loans, $I_{t+1}^b$, are determined at time $t$ when the loans are signed with the entrepreneurs-borrowers (see the participation constraint in (16)). Deposits held at the central bank in the form of reserves are also returned to the banks. We assume that they earn an interest on reserves fixed at time $t$, $T_{t+1}$, and designed as a two-part tariff, i.e.

$$T_{t+1} = (1 - c) + \zeta (I_{t+1} - 1),$$

whereby banks pay a fixed fee as a management cost per unit of reserve held at the central bank, $0 < c < 1$, and get back the principal (minus the management fee) and a net rate of return that is proportional to the net risk free rate, $0 < \zeta < 1$. The parameter $\zeta$ denotes the discount rate relative to the monetary policy instrument net rate at which reserves are compensated. Although in most instances the practice is to set this rate of return to zero (i.e., $c = \zeta = 0$), there are precedents for paying interest on reserves.\footnote{Until very recently reserve requirements held at the Federal Reserve did not pay interest. The Federal Reserve announced changes to reserve management after winning the power to pay interest on required and excess reserves on October 3, 2008. The Federal Reserve has argued that paying interest would deter banks from lending out excess reserves and as such would make it easier for the Fed to attain its target rate. We do not model this feature explicitly.} We also
make the simplifying assumption that there is full deposit insurance, as a consequence deposits are riskless and the gross interest rate paid on deposits is equal to the risk-free nominal rate, $I_{t+1}$, which is known at time $t$.

Bank capital shareholders, the households, have to be compensated with a certain nominal yield determined at time $t$, $R_{t+1}^b$. Since each piece of the profit function is decided at time $t$ and is known by the banks and the households, competitive banks end up offering a yield to the shareholders that is also known at time $t$. By arbitrage among the assets available to households, then it must be the case that,

$$ (1 - \epsilon) R_{t+1}^b = I_{t+1}, \quad (25) $$

which insures that households remain indifferent between holding bank capital or deposits. For a competitive banking sector, the profit function in (23) can be re-written as a zero-profit condition (i.e., $\Pi_{t+1}^b = 0$) in the following terms:

$$ \Pi_{t+1}^b \equiv \left[ I_{t+1}^b - v_{t+1} R_{t+1}^b - (1 - v_{t+1}) \left( \frac{I_{t+1} - \omega T_{t+1}}{1 - \omega} \right) \right] L_{t+1} = 0, \quad (26) $$

by using the constraint of the balance sheet in (22). The problem of the banks is to optimize their capital structure, their trade-off between bank equity and deposits, subject to the constraint that banks must offer a yield on bank capital that would make households indifferent given the risk-free rate paid on deposits as given by equation (25). Of course, this problem is also subject to the returns on reserves paid by the central bank as given by equation (24) and to a regulatory constraint on capital adequacy that implies banks must satisfy:

$$ 1 \geq v_{t+1} \equiv \frac{B_{t+1}}{L_{t+1}} \geq \omega, \quad (27) $$

where $0 \leq v < 1$ is equal either to the minimum mandatory capital adequacy requirement set by the regulator, or could be a lower bound that reflects a buffer above the minimum requirement implied by the statutory requirements of the banks.

We shall make two key parametric assumptions to simplify the problem of the banks, and we leave the exploration of more complex banking cost structures for future research. Our goal, at this stage, is to make only the smallest possible departure from the original BGG (1999). We assume that $\zeta = 1 - c$ and, furthermore, that taxes on bank equity are bounded by $0 < 1 - \epsilon^b < \frac{1 - \omega}{1 - \zeta \omega}$. Whenever $\xi = 0$, this bound implies that $\epsilon^b > \omega$; whenever $\xi = 1$, it merely requires that $\epsilon^b > 0$. Given the fact that tax rates are quite often much higher than the minimum reserve ratios, these bounds are likely not excessively restrictive. Both assumptions put together imply that,

$$ R_{t+1}^b > \left( \frac{I_{t+1} - \omega T_{t+1}}{1 - \omega} \right). \quad (28) $$

In other words, it is costlier for banks to finance themselves with bank equity than with deposits and, therefore, the lower bound on the leverage ratio must be binding at all times.

In turn, these assumptions imply that the returns on the portfolio loans that the banks require to
participate in funding the entrepreneurs are fully determined by the cost structure of the banks as follows,

\[ I_{t+1}^b = v P_{t+1}^b + (1 - v) \left( \frac{I_{t+1} - \varpi J_{t+1}}{1 - \varpi} \right) \]

\[ = \left[ v \left( \frac{1}{1 - \nu} \right) + (1 - v) \left( \frac{1 - \varpi \zeta}{1 - \varpi} \right) \right] I_{t+1}. \]  

(29)

This is what we call the balance sheet channel of banking regulation. It can be easily seen that without capital adequacy requirements, i.e. \( v = 0 \), and without reserve requirements, i.e. \( \varpi = 0 \), we would be back in the original world of BGG (1999). Our equation (29) is a heavily parameterized version of the following expression for returns on the loan of portfolio under constant returns to scale,

\[ \frac{I_{t+1}^b}{I_{t+1}} \equiv v_{t+1} \times \frac{\text{cost(bank equity}_{t+1})}{I_{t+1}} + (1 - v_{t+1}) \times \frac{\text{cost(deposits}_{t+1})}{I_{t+1}}, \]  

(30)

where \( v_{t+1} \) represents the leverage ratio as before.

### 3 The External Finance Premium

The relationship in (29) ties down the return on the portfolio of loans to the risk-free rate, which happens to be also the relevant instrument for monetary policy. The regulatory restriction on capital adequacy in (27) does not have the purpose of protecting the financial system from bad outcomes, since that is already taken care off by the contracting problem with the entrepreneurs-borrowers. Instead, the purpose of this regulatory constraint is to effectively give the monetary authority a way to ‘regulate’ the supply of loans without having to manipulate the interest rate directly. Then, we can visualize the banks’ ‘balance sheet’ channel in the framework of BGG (1999) by combining (20) and (29) as follows,

\[ \mathbb{E}_t [R_{t+1}^e] = s \left( \frac{P_t Q_t K_{t+1}}{N_{t+1}} \right) \left[ v \left( \frac{1}{1 - \nu} \right) + (1 - v) \left( \frac{1 - \varpi \zeta}{1 - \varpi} \right) \right] I_{t+1}. \]

(31)

This equation shows how both systemic risk and bank regulatory mechanisms can be integrated into the external financing premium.

#### 3.1 Model Parameterization

In this section, we describe the choice of the parameter values. Our calibration is summarized in Table 1. We follow the literature as closely as possible in our calibration efforts, with a special emphasis to keep the model comparable to the framework of BGG (1999). Some baseline values are calibrated to match existing data. We assume that the discount factor, \( \beta \), equals 0.99 which is consistent with an annual real return around 4%. The Calvo price stickiness parameter, \( \alpha \), is assumed to be 0.75 which implies that the average price duration in our model is 4 quarters. We set the household labor share, \( \psi \), equal to 0.62, the entrepreneurs’ labor share, \( \varphi \), equal to 0.03, and the depreciation rate, \( \delta \), equal to 0.025. The latter implies an annual depreciation rate around 10% which is typical for the U.S. data.

All of this parameter choices are identical to BGG (1999) except for the labor shares. While the combine
labor share, $\psi + \varphi$, is still equal to 0.65 as in BGG (1999), we choose a slightly larger entrepreneurial labor share up from the 0.01 value preferred by BGG (1999). We think this is only a minor departure from the calibration used by these authors. Our steady state calibration has the property that it bounds the consumption share of entrepreneurs, $\gamma_{c,e}$, to be below 0.03 of total resources. Therefore, our choice of the labor share is meant to give entrepreneurs a share of wholesale output (based on labor income) at least as large as their consumption share in order to ensure their ability to save and keep funding the acquisition of new capital.

We set the intertemporal elasticity of substitution, $\sigma$, to 0.5 as argued by Lucas (1990), instead of 1 as in BGG (1999). Given that preferences are additively separable in consumption and labor, we fix the Frisch elasticity of labor supply, $\varphi$, to be equal to the intertemporal elasticity of substitution. This assumption ensures that the economy would be consistent with a balanced growth path. The elasticity of substitution across varieties, $\theta$, is set to 10. This is consistent with a price mark-up of 11% as documented in the U.S. data by Basu (1996).8

The parameterization of the monetary policy rule also differs from BGG (1999). The interest rate inertia parameter, $\rho$, equals 0.9 as in BGG (1999). However, the weight on the inflation target, $\psi_{\pi}$, equals 1.5, and the weight on the output target, $\psi_{y}$, is 0.5. This weighting scheme follows from the trade-off between inflation and output proposed by Taylor (1993). In principle, we set the weight on the asset price of capital (or Tobin's q), $\psi_{q}$, to 0 as in Taylor (1993), but we nonetheless explore the sensitivity of the model to the reaction of the central bank to fluctuations in asset prices.

The regulatory parameters on capital adequacy, $v$, and minimum reserves on deposits, $z$, are fixed according to the current state of regulation itself. We set the leverage maximum to 25, which implies a minimum capital adequacy requirement, $v$, of 0.04. Basel I requirements stipulate that a Tier 19 capital to assets ratio of below 0.04 implies that the institution is 'undercapitalized.' New Basel II requirements allow a reduction of this capital by up to 15%, thus down to just over 0.03, over a long period of time. Being 'well-capitalized' requires a Tier 1 capital to assets ratio of above 0.06. We use the 0.04 level for the banking system as this reflects the point at which financial institutions can be considered in 'distress' from the point of view of the regulator.10 The minimum reserve requirement on deposits, $z$, is set to 0.09 based on current

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8 Up to a first-order approximation, the elasticity of substitution across varieties only enters our considerations because the markup affects the pricing rule for the retailers in steady state. However, we are making the implicit assumption that an optimal tax, $\tau^* = \frac{1}{2}$, subsidy would be implemented implying that retail prices would be equal to marginal costs for the retailers in steady state. In this environment, the parameterization of $\theta$ becomes immaterial because it no longer determines the steady state (unlike in BGG, 1999).

9 Tier 1 capital is defined as common equity, non-cumulative perpetual preferred stock and minority interests in equity accounts of minority shareholders.

10 For chartered national banks (most of the big ones), the minimum requirement is set at 3% tier 1/assets. "However: An institution operating at or near the level in paragraph (b) of this section should have well-diversified risks, including no undue interest rate risk exposure; excellent control systems; good earnings; high asset quality; high liquidity; and well managed on- and off-balance sheet activities; and in general be considered a strong banking organization, rated composite 1 under the Uniform Financial Institutions Rating System (CAMELS) rating system of banks. For all but the most highly-rated banks meeting the conditions set forth in this paragraph (c), the minimum Tier 1 leverage ratio is 4 percent. In all cases, banking institutions should hold capital commensurate with the level and nature of all risks." Thus, 4% tier 1/ assets is really the level of relevance, and the one that we use to calibrate the model. The full rules are here: http://www.federalreserve.gov/ newsevents/press/bcreg/20071102a.htm, and http://www.federalreserve.gov/ newsevents/press/bcreg/2007bcreg.htm. However, regulation depends on the charter. For the national banks charter, see here: http://www.occ.treas.gov/fr/cfrparts/12cfr03.htm#%C2%A7%203.006%20Minimum%20capital%20ratios.
Federal Reserve Board rules.\textsuperscript{11}

Other regulatory features include the management cost per unit of reserves held at the central bank, \( c \), and the discount rate relative to the monetary policy instrument at which reserves are compensated, \( \zeta \). FDIC prevailing rules imply that institutions that are adequately capitalized or well-capitalized pay between 5 – 7 and 28 basis points on deposit insurance which would influence our characterization of \( c \).\textsuperscript{12} The interest rate being offered on reserves is the federal funds rate minus a spread that has been set at 10 basis points since the program was started in October 2008 which should affect our choice of \( \zeta \).\textsuperscript{13} Based on these regulatory facts, the simplifying assumption of \( \zeta = 1 - c \) does not appear to be fundamentally at odds with the data if we calibrate \( \zeta \) to imply a spread between the monetary instrument net rate and the net rate of deposits (excluding also management costs of deposit insurance) of around 10 basis points. Hence, we set \( \zeta \) to 0.999.

The tax rate on the bank capital yield, \( \nu^h \), is set to 0.05 to match the current dividend tax rate in the U.S.\textsuperscript{14} However, we interpret this parameter as more than a simple taxation wedge. It may reflect other non-tax related costs associated with holding bank capital rather than deposit that are left in the background. Deposits offer some insurance protection, while bank capital does not; hence, although there are no defaulting banks in our model there are still reasons why the shareholders may want to ask for a yield above the rate on deposits. We subsume all of that into our choice of the tax rate on bank dividends. Notice also that this tax rate satisfies the restriction \( \nu^h > \frac{\sigma^2}{1-\zeta} \) which ensures that the capital structure of the banking system is well-characterized in our model.

There are a number of parameters that characterize the entrepreneur-borrower relationship with the lenders, which we will calibrate jointly. Those parameters include the relative impatience of the entrepreneurs, \( \eta \), the fraction of losses due to monitoring costs, \( \mu \), the slope of expected systemic losses, \( \lambda \), the depth of the expected systemic losses, \( \xi \), and the volatility of the idiosyncratic shock itself, \( \sigma^2_N \). For that we target a number of key moments partially implied by the data.

Using the approach of BGG (1999), we choose the parameters to imply that in steady state: (a) the annualized risk spread, \( R^e - I \), is equal to two hundred basis points which corresponds approximately to the historical average spread between the prime-lending rate and the six-month Treasury bill rate as in BGG (1999), (b) the real leverage ratio for entrepreneurs, \( \gamma_n \equiv \frac{N}{PK} \), is equal to 0.5 as in BGG (1999), and (c) the quarterly failure rate is of 3\% as in BGG (1999).\textsuperscript{15}

\textsuperscript{11}The Board of Governors of the Federal Reserve System currently requires 0\% fractional reserves from depository institutions having net transactions accounts of up to $9.3 million, requires 3\% from depository institutions having over $9.3 million and up to $43.9 million in net transaction accounts, and requires 10\% for depository institutions having over $43.9 million in net transaction accounts. Most depository institutions fall in highest reserve requirement range, hence our choice of the reserve ratio at 9\%. For more details, see: http://www.federalreserve.gov/monetarypolicy/reservereq.htm

\textsuperscript{12}For more details, see: http://www.fdic.gov/deposit/insurance/assessments/risk.html

\textsuperscript{13}The interest rate paid on required reserve balances is determined by a formula set by the Board. The formula is the average targeted federal funds rate of the Federal Open Market Committee during the institution’s reserve maintenance period less a spread. This spread was set to reflect the risk-free nature of a deposit at a Federal Reserve Bank. For more details, see: http://www.federalreserve.gov/monetarypolicy/reqresbalances.htm

\textsuperscript{14}Congress passed the Jobs and Growth Tax Relief Reconciliation Act of 2003, which included some of the cuts Bush requested and which he signed into law on May 28, 2003. Under the new law, qualified dividends are taxed at a 15\% rate for most individual taxpayers. Qualified Dividends received by low income individuals are taxed at a five percent rate until December 31, 2007 and become fully untaxed in 2008. The 15\% tax rate was set to expire December 31, 2008. However, with the Tax Increase Prevention and Reconciliation Act of 2005, the lower tax rate was extended until the end of 2010.

\textsuperscript{15}Watson and Everett (1996) studied 5,196 startups in 51 managed shopping centers across Australia between 1961 and 1990 to try to determine true failure rates. Their data shows that annual failure rates were greater than 9\% when failure was defined
In order to complete the calibration, we also assume that the relative scarcity of household labor in steady state measured by $\frac{H_e}{H}$ should be consistent with an investment share around, $\gamma_c$, around 20% and a household consumption share, $\gamma_c$, around 69%. These shares reflect the average patterns observed in the U.S. data for private consumption and investment over the period 1992-2008. Finally, we also arbitrarily bound the maximum size of the systemic losses for the entrepreneur on the secondary market for used capital to 30% of the cost of new capital, which implies that the depth of the expected systemic losses, $\xi$, is equal to 5.67. For more details on the calibration of these parameters, see the appendix.

We assume identical AR(1) exogenous processes for the productivity, the monetary, and the systemic risk shocks. We assume all these shocks to be highly persistent. As in Cooley and Prescott (1995), we choose the persistence of the productivity shock, $\rho_a$, to be equal to 0.95. The persistence of the monetary and systemic risk shocks, $\rho_m$ and $\rho_s$, is set at an arbitrarily lower level of 0.9. Cooley and Prescott (1995) propose a volatility of the productivity shock, $\sigma_a^2$, of $(0.007)^2$. However, in line with most of the current literature, we calibrate the volatility of each shock, $\sigma_a^2$, $\sigma_m^2$, and $\sigma_s^2$, to match output volatility in the U.S. data (where the U.S. standard deviation is 1.54%). We arbitrarily set the correlation of the systemic risk shock with the productivity shock, $\rho_{a,s}$, at 0.75 to emphasize the strong linkage between the losses in the secondary market and the productivity of new capital. This parameter, in particular, is subject to extensive sensitivity analysis.

Finally, we select the appropriate capital or investment production constraint parameter, either $\chi$ or $\kappa$, to ensure that investment volatility in the model is as volatile as in the U.S. data (i.e., 3.38 times as volatile as U.S. GDP in real terms).

### 3.2 Results

In this section we describe some of the features of the external finance premium under the structure of the model presented and with the specified calibration. Broadly speaking, we find that the external finance premium is increasing in systemic risk. As systemic risk increases, the external finance premium increases markedly vis-a-vis the benchmark. When we add bank capital requirements, the finance premium increases further. The sensitivity of the premium to the monetary shocks is also a function of the risk level and leverage in each case. The interaction between these forces can be best described by referring back to equation (31).

We show a few examples in the figures reported below to illustrate the power of the mechanism. First, in figures 1a and 1b, we show the impact of a systemic shock (an increase to $S$) on economies with higher macroeconomic and idiosyncratic volatility respectively. As should be apparent, both lead to an increase in the external finance premium. Systemic risk increases the spread through an increase in the cost of lending to cover reduced capital resale values. It is worth noting that changes to the systemic shock lead to larger initial impacts and persist much longer than a corresponding idiosyncratic shock.

simply as "discontinuance of ownership." When failure was defined as bankruptcy, however, the number dropped to less than 1% annually. Another 2% of the owners disposed of their businesses annually to prevent further loss. The authors concluded that cumulatively 65.2% of the businesses failed in a 10-year period – if failure was measured as discontinuance of ownership – but only 5.3% actually filed for bankruptcy during a decade. The upper bound numbers in Watson and Everett (1996) are consistent with the failure rate in BGG (1999), which we also maintain in our model. Our interpretation of failure, hence, is quite broad, not restricted exclusively to bankruptcies.

---

16 The parameter $\xi$ is of special relevance in our framework. In the limit, whenever $\xi \to +\infty$, the losses in the secondary market tend to zero and, therefore, that brings us closer to the standard BGG (1999) model.
Second, to highlight the difference between the two shocks, figures 2a and 2b show the impulse response for a monetary shock under each case (high/low volatility). In the macroeconomic case, a monetary shock leads, as expected, to a decrease in the external finance premium. However, notice that monetary shocks have no differential impact on the external finance premium as idiosyncratic volatility changes. Similar factors are observed in the responses to a productivity shock. Increased productivity leads to a decrease in the external finance premium. This can be seen in figures 3a and 3b. Notice, though, that increased systemic risk decreases this effect. Again, changes in idiosyncratic volatility have no relative impact.

We interpret these results as evidence that the "agency cost" channel in BGG (1999) is less sensitive to macroeconomic shocks. Changes in systemic risk have a direct effect on the spread, unlike shocks to productivity or monetary shocks which operate only through the leverage ratio of the borrowers. Therefore, systemic risk shocks can have a potentially larger role leading to a magnification of their effects under high volatility that is more noticeable.

Capital adequacy requirements appear to impact the external finance premium as well, but only in the context of risk changes. When changes independently of the level of the risk in the economy, they have nearly no effect on the path of the external finance premium (not shown). Indeed, this is largely to be expected. In ‘good’ times, small changes in adequacy requirements are unlikely to change lending criteria. Even in a model such as this, where bank leverage is directly a function of the adequacy requirement, without changes to expected payoffs, lending contracts will not change dramatically. However, when combined with systemic shocks, requirements have a much larger impact. This again is reasonable. In the context of the 2007-2008 financial crisis, one can see that once a systemic shock hit, capital requirements became an important determinant of bank decision-making. Risk spreads increased dramatically. This can be illustrated with figure 4.

4 Concluding Remarks

This paper has shown how to embed risk and bank regulation into a model of the financial accelerator. After deriving a simple characterization of the external finance premium in this context, the paper shows some stylized dynamics that match current intuition on the reaction of spreads to a crisis environment and to bank balance sheet effects.

The model produces an external finance premium that is an increasing function of systemic risk. As well, it finds that bank leverage is also positively related to increased spreads, but only when the economy is in stress. That is, changes in bank regulatory requirements has little impact in good times, but acts as a multiplier when the economy is an a ‘bad’ state.

References


Appendix
US Monetary Policy target rates and Taylor Rule residuals

see Rudebusch (2006) for full discussion. These residuals calculated based on the rule:

\[ i_t = 2.04 + 1.39\pi_t + .92y_t. \]
## A Model - Summary of Equations

### First Order Conditions: Households, Entrepreneurs and Resource Constraint

#### Households

\[
P_tC_t + T_t + D_{t+1} + B_{t+1} = W_t H_t + I_t D_t + (1 - i^h) R^b_t B_t + \Pi_t^e + \Pi_t^f,
\]

\[
\frac{1}{T_{t+1}^e} = \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}^e}{C_t^e} \right)^{-\sigma - 1} \frac{P_t}{P_{t+1}^e} \right],
\]

\[
1 = \beta \mathbb{E}_t \left[ (1 - i^h) R^b_{t+1} \left( \frac{C_{t+1}^e}{C_t^e} \right)^{-\sigma - 1} \frac{P_t}{P_{t+1}^e} \right],
\]

\[
\frac{W_t}{P_t} = \left( \frac{C_t}{C_t^e} \right)^{\sigma - 1} (H_t)^{\sigma - 1},
\]

#### Entrepreneurs

\[
H_t^e = 1,
\]

\[
P_t Q_t K_{t+1} = N_{t+1} + L_{t+1},
\]

\[
P_t C_t^e + P_t Q_t K_{t+1} = W_t^e H_t^e + [1 - J (s_t) - \Gamma (\omega_t, s_t)] R_t^e P_{t-1} Q_{t-1} K_t + L_{t+1},
\]

\[
(\beta\eta) \mathbb{E}_t \left[ (1 - J (s_{t+1}) - \Gamma (\omega_{t+1}, s_{t+1})) \left( R_{t+1}^e \frac{P_{t+1}^e}{P_{t+1}^e} \right) \right] = 1,
\]

#### Resource Constraint

\[
Y_t^w = C_t + C_t^e + X_t + \left( 1 - \left( \frac{P_t^e}{P_t} \right)^\theta \right) Y_t^w + \mu G (\omega_t, s_t) R_t^e \frac{P_{t-1}^e}{P_t^e} Q_{t-1} K_t,
\]

\[
Y_t = \left( \frac{P_t^e}{P_t} \right)^\theta Y_t^w.
\]
First Order Conditions: Non-Financial Firms

Retailers

\[ \sum_{\tau=0}^{\infty} \mathbb{E}_t \left[ (\alpha\beta)^{\tau} \left( \frac{C_{t+\tau}}{C_t} \right)^{-\sigma' X_{t+\tau}} \tilde{Y}_{t,t+\tau}\left( \frac{\tilde{P}_t(z)}{P_{t+\tau}} - \frac{\theta(1-\gamma')}{\theta-1} \frac{P^w_{t+\tau}}{P_{t+\tau}} \right) \right] = 0, \]

\[ \tilde{Y}_{t,t+\tau}(z) = \left( \frac{\tilde{P}_t(z)}{P_{t+\tau}} \right)^{-\theta} Y_{t+\tau}, \]

\[ P_t = \left[ \alpha P_{t-1}^{-\theta} + (1 - \alpha) \tilde{P}_t(z)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \]

\[ \Pi_t^0 = P_t \left( \frac{P_t^w}{P_t^e} \right)^\theta Y_t^w - (1 - \gamma') P_t^w Y_t^w, \]

\[ P_t^* = \left[ \alpha (P_{t-1}^*)^{-\theta} + (1 - \alpha) \tilde{P}_t(z)^{-\theta} \right]^{-\frac{1}{1-\theta}}. \]

Capital Goods Producers

\[ K_{t+1} = (1 - \delta) K_t + \Phi (X_t, X_{t-1}, K_t) X_t, \]

\[ Q_t \left[ \Phi (X_t, X_{t-1}, K_t) + \frac{\partial \Phi (X_t, X_{t-1}, K_t)}{\partial X_t} X_t \right] + \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} Q_{t+1} \frac{\partial \Phi (X_{t+1}, X_{t+1}, K_{t+1})}{\partial X_t} X_{t+1} \right] = 1, \]

\[ \Pi_t^k = P_t Q_t \Phi (X_t, X_{t-1}, K_t) X_t - (\omega_t - 1) (1 - \delta) P_t Q_t K_t - P_t X_t, \]

\[ o_t = \frac{\Pi_t^k}{Q_t}, \omega_t = \frac{(\omega_t-1)K_t^K + \omega_t(1-\delta)P_tQ_tK_t}{(1-\delta)P_tQ_tK_t}. \]

Wholesale Producers

\[ Y_t^w = e^{\omega_t} (K_t)^{1-\psi-\theta} (H_t^e)^{\psi} (H_t^w)^{\theta}, \]

\[ R_t^w = (1 - \psi - \omega) \frac{P_t^w Y_t^w}{K_t}, \]

\[ W_t = \psi \frac{P_t^w Y_t^w}{K_t}, \]

\[ W_t^* = \omega \frac{P_t^w Y_t^w}{K_t}. \]
# First Order Conditions: Financial Institutions and CSV Loans

## Banks

\[
\frac{P_t}{Q_t} = u, \\
I_t = \left[ v \left( \frac{1}{1-u} \right) + (1-v) \left( \frac{1-\xi}{1-\delta} \right) \right] I_{t+1},
\]

## Financial Contracts

\[
R_t = \left( \frac{R_t + (1-\delta)Q_t}{Q_{t-1}} \right) P_t, \\
\Gamma_1 (\omega_{t+1}, s_{t+1}) - \lambda (\omega_{t+1}, s_{t+1}) \left[ \Gamma_1 (\omega_{t+1}, s_{t+1}) - \mu G_1 (\omega_{t+1}, s_{t+1}) \right] = 0, \\
\frac{P_t Q_t K_{t+1}}{N_{t+1}} \left( \frac{R_{t+1} - R_t}{Q_{t+1}} \right) (\Gamma (\omega_{t+1}, s_{t+1}) - \mu G (\omega_{t+1}, s_{t+1})) - \left[ \frac{P_t Q_t K_{t+1}}{N_{t+1}} - 1 \right] = 0, \\
\mathbb{E}_t \left\{ \frac{R_{t+1}}{Q_{t+1}} (1 - J (s_{t+1}) - \Gamma (\omega_{t+1}, s_{t+1}) + \lambda (\omega_{t+1}, s_{t+1}) (\Gamma (\omega_{t+1}, s_{t+1}) - \mu G (\omega_{t+1}, s_{t+1})) - \lambda (\omega_{t+1}, s_{t+1})) \right\} = 0.
\]

## Central Bank

\[
T_t + \delta^\ell R_t^\delta B_t + M_{t+1} = \nu_t P_t^w Y_t^w + \tilde{M}_t^\ell M_{t}, \quad \ell = \frac{1}{\delta}, \\
M_{t+1} = \omega D_{t+1}, \\
\ln (I_{t+1}) = \rho_t \ln (I_t) + (1 - \rho_t) \left[ \psi_\nu \ln \left( \frac{P_t}{T_{t-1}} \right) + \psi_q \ln (Q_t) + \psi_y \ln (Y_t) \right] + m_t.
\]
<table>
<thead>
<tr>
<th><strong>Exogenous Shocks</strong></th>
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</thead>
<tbody>
<tr>
<td><strong>Productivity Shocks</strong></td>
</tr>
<tr>
<td>$a_t = \rho_a a_{t-1} + \varepsilon_t^a$,</td>
</tr>
<tr>
<td><strong>Monetary Shocks</strong></td>
</tr>
<tr>
<td>$m_t = \rho_m m_{t-1} + \varepsilon_t^m$,</td>
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<tr>
<td><strong>Systematic Risk Shocks</strong></td>
</tr>
<tr>
<td>$s_t = (1 - \rho_s) + \rho_s s_{t-1} + \varepsilon_t^s$,</td>
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<tr>
<td>$\mathbb{E}[\omega_t \mid s_t] = \frac{1}{1+\tau} [\xi + \tanh(\lambda \gamma + \gamma s_t)]$.</td>
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</table>
### Tables and Figures

#### Table 1: Benchmark Calibration

<table>
<thead>
<tr>
<th>Structural Parameters:</th>
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</thead>
<tbody>
<tr>
<td>\text{Discount Factor for Households} &amp; \text{0 &lt; } \beta &lt; 1 &amp; 0.99 \text{ BGG (1999)}</td>
<td></td>
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<tr>
<td>\text{Elasticity of Intertemporal Substitution} &amp; \sigma &gt; 0 (\sigma \neq 1) &amp; 0.5 \text{ Lucas (1990)}</td>
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<tr>
<td>\text{Frisch Elasticity of Labor Supply} &amp; \varphi &gt; 0 &amp; 0.5 \sigma = \varphi</td>
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<tr>
<td>\text{Elasticity of Substitution across Varieties} &amp; \theta &gt; 1 &amp; 10 \text{ Basu (1996)}</td>
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<tr>
<td>\text{Calvo Price Stickiness Parameter} &amp; \text{0 &lt; } \alpha &lt; 1 &amp; 0.75 \text{ BGG (1999)}</td>
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<tr>
<td>\text{Depreciation Rate} &amp; \text{0 &lt; } \delta &lt; 1 &amp; 0.025 \text{ BGG (1999)}</td>
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<tr>
<td>\text{Capital Adjustment / Investment Adjustment} &amp; \chi &gt; 0 / \kappa &gt; 0 &amp; \text{ - Match Investment volatility}</td>
<td></td>
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</tr>
<tr>
<td>\text{Non-managerial Labor Share} &amp; \text{0 &lt; } \psi &lt; 1 &amp; 0.62 \text{ BGG (1999)}</td>
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</tr>
<tr>
<td>\text{Managerial Labor Share} &amp; \text{0 &lt; } \mu &lt; 1 &amp; 0.03 \text{ BGG (1999)}</td>
<td></td>
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<tr>
<td>\text{Relative Impatience of Entrepreneurs} &amp; \text{0 &lt; } \eta &lt; 1 &amp; \text{ - Calibrated}</td>
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<tr>
<td>\text{Monitoring Costs} &amp; \text{0 &lt; } \mu &lt; 1 &amp; \text{ - Function of Calibrated Parameters}</td>
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<tr>
<th>Monetary Policy and Regulatory Parameters:</th>
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<tbody>
<tr>
<td>\text{Interest Rate Inertia} &amp; \text{0 \leq } \rho_i \leq 1 &amp; 0.9 \text{ BGG (1999)}</td>
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<tr>
<td>\text{Weight on Inflation Target} &amp; \psi_{\pi} \geq 1 &amp; 1.5 \text{ Taylor (1993)}</td>
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<tr>
<td>\text{Weight on Asset Value (Tobin’s q) Target} &amp; -\infty &lt; \psi_q &lt; +\infty &amp; 0 \text{ Taylor (1993)}</td>
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<tr>
<td>\text{Weight on Output Target} &amp; \psi_y \geq 0 &amp; 0.5 \text{ Taylor (1993)}</td>
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<tr>
<td>\text{Reserve Requirement on Deposits} &amp; 0 \leq \omega &lt; 1 &amp; 0.09 \text{ FRB}</td>
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<tr>
<td>\text{Capital Adequacy Requirement} &amp; 0 \leq \varrho &lt; 1 &amp; 0.04 \text{ Basel II Accord}</td>
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<tr>
<td>\text{Discount Offered on Reserved Rates} &amp; 0 &lt; \zeta &lt; 1 &amp; 0.999 \text{ FDIC, FRB}</td>
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<tr>
<td>\text{Tax Rate on Bank Dividends} &amp; \frac{\pi - \zeta \pi}{1 - \zeta \pi} &lt; l^h &lt; 1 &amp; 0.05 \text{ TIPRA 2005}</td>
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<tr>
<th>Exogenous Idiosyncratic Shock Parameters:</th>
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<tr>
<td>\text{Level of Expected Systemic Losses} &amp; \text{0 \leq } \lambda &lt; 1 &amp; \text{ - Calibrated}</td>
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<tr>
<td>\text{Sensitivity to Expected Systemic Losses} &amp; -\infty &lt; \xi &lt; +\infty &amp; \text{ - Chosen to Regulate } \Theta, \Sigma</td>
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</tr>
<tr>
<td>\text{Volatility of Idiosyncratic Shock} &amp; \sigma_\xi^2 &amp; \text{ - Function of Calibrated Parameters}</td>
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<tr>
<th>Exogenous Aggregate Shock Parameters:</th>
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<tbody>
<tr>
<td>\text{Persistence of Productivity Shock} &amp; -1 &lt; \rho_p &lt; 1 &amp; 0.95 \text{ Cooley &amp; Prescott (1995)}</td>
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<tr>
<td>\text{Volatility of Productivity Shock} &amp; \sigma_p^2 &gt; 0 &amp; \text{ - Match GDP volatility}</td>
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<tr>
<td>\text{Persistence of Monetary Shock} &amp; -1 &lt; \rho_m &lt; 1 &amp; 0.9 \text{ Chosen}</td>
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<tr>
<td>\text{Volatility of Monetary Shock} &amp; \sigma_m^2 &gt; 0 &amp; \text{ - Match GDP volatility}</td>
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<tr>
<td>\text{Persistence of Systemic Risk Shock} &amp; -1 &lt; \rho_s &lt; 1 &amp; 0.9 \text{ Chosen}</td>
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<tr>
<td>\text{Volatility of Systemic Risk Shock} &amp; \sigma_s^2 &gt; 0 &amp; \text{ - Match GDP volatility}</td>
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<tr>
<td>\text{Covariance Productivity, Systemic Shocks} &amp; -\infty &lt; \sigma_{a,s} &lt; +\infty &amp; \frac{\sigma_{a,s}}{\sigma_a \sigma_s} = 0.75 \text{ Chosen}</td>
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<tr>
<td>\text{Unconditional mean of Systemic Risk Shock} &amp; \text{0 &lt; } s &lt; +\infty (s \neq 0) &amp; 1 \text{ For Simplicity}</td>
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This table defines the benchmark parameterization of the structural parameters. The results of the sensitivity analysis for a given parameter are discussed in the paper, but not always reported. A GMM process was used to calibrate the above noted parameters. The simulation entailed defining a range of plausible values for each parameter, as determined by the literature and deemed appropriate by the authors, then a set of these parameters was evaluated and determined best fit in terms of matching the volatility of GDP and the correlation of GDP and other pertinent variables to the data. The detailed methodology and specific values can be obtained directly from the authors upon request.
This figure demonstrates the response of the External Finance Premium to a Systemic Shock. The solid line is the simulation from the baseline parameter specification. The dotted line increases the standard deviation of the systemic risk shock by an order of magnitude.
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