Rational Bubbles and Default in Growing Economies

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Abstract

Emerging economies display periods of rapid growth and large capital inflows, followed by sudden stops in their borrowing ability and consequent financial crises. Recoveries are characterized by lower growth rates and often reversed balance of payments. I construct a model where capital inflows and high growth rates are initially self-sustaining. Growth is fueled both by capital accumulation and by higher employment of a fixed factor. As the fixed factor becomes scarce, growth slows down and incentives arise for the country to default on its foreign debt. The key assumption that allows for the initial large capital inflow is that the endowment of the fixed factor is not known with certainty, which implies that the timing of default is uncertain. I calibrate the model using data from the Asian crisis and I find that the model is capable of replicating key elements of the pre and post crisis years.

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1 Introduction

The Asian crisis of the late '90 is considered one of the deepest financial crises of the past 20 years. It was not only dramatic in size, but largely unexpected. Figure (1) shows the GDP growth rate for four Asian crises in the years 1987-2007.

PLEASE ENTER FIGURE 1 HERE

The year of the crises is clearly marked by a fall of the GDP of about 10% in most of the countries. Foreign capital inflows, shown in Figure 2, that had helped fueling the growth in the first half of the decade, stopped. The year 1998 marks also a permanent shift in the pattern of growth, which was on average 8.1% in the pre-crises years, and 5% thereafter. A similar pattern is followed by the investment rate shown in Figure 3.

PLEASE ENTER FIGURE 2 HERE

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Such pre- and post- crisis patterns are not specific to the experience of the emerging Asian economies. As Lee and Rhee (2000) and Ranciere, Tornell and Westermann (2005) show, the same regularities are confirmed in a wider sample of countries that have experience financial crises.
The goal of this paper is to explore the relation between growth and default events. I show that periods of rapid growth followed by a sudden financial turmoil can be the natural outcome of a growth process characterized by diminishing marginal returns and limited international enforceability of debt contracts. The initially high economic growth is driven both by capital accumulation and by increasingly higher rates of employment of a non-reproducible fixed factor. As the economy reaches the full factor employment, the growth rate of the economy is expected to slow down, because of the law of decreasing marginal returns. Consequently, with time, the flow of capital into the country becomes less intense and the new debt issued is not enough to roll over the previous debt. At this point, the balance of payment should reverse, since borrowers need to start paying back their debt to the rest of the world. Incentives to default on debt obligations arise whenever the only punishment for a defaulting individual is perpetual exclusion from the financial market, as in Bulow and Rogoff (1989). A backwards induction argument implies that, in the anticipation of this future default scenario, international investors stop immediately their lending activity to

the country, who then suddenly finds itself with a large debt to settle and no ability to issue new debt. The sudden stop in the capital inflow forces the country to default. Since in the stage of slowing growth international debt is not sustainable, the investment rate and the GDP growth rate, which were previously boosted by the capital inflow, drop. The year of the crisis, then, marks exactly this moment of discontinuity.

Once more, it is worth stressing the point that, in this model, a financial crisis is a natural outcome of the growth process, and is not the pathological result of a distorted and inefficient investment activity, nor the effect of exogenous shocks to the productivity of the economy.

2 The Model

There is a small economy inhabited by a unit measure of identical households and a production sector constituted by competitive firms operating a constant return to scale technology. The household can borrow from competitive and risk neutral international lenders, who take the fixed international risk free rate as given.

Firms' output can be used both for consumption or for investment purposes, and is produced using a two-factors technology

$$F(K, H) = A_1 K^{\alpha} H^{1-\alpha} \tag{1}$$

where K denotes the aggregate stock of capital which evolves according to the standard linear accumulation rule

$$K_{t+1} = (1-\delta)K_t + X_t$$

where X_t is the investment.

The second factor of production, H, is in fixed supply $\bar{H}$, which implies that the economy displays fundamental decreasing marginal returns to the reproducible factor K whenever H is employed at its highest level $\bar{H}$. The cost of supplying, up to $\bar{H}$, an amount H_t for the production in period t is $\underline{w}H_t$, with $\underline{w} > 0$.[1]

The presence of decreasing marginal returns is crucial to the model and different interpretations that can be given to H. First, as in Parente and Prescott (1994) and Chari and Hopenhayn (1991), the factor H can be thought of as technology which is slowly adopted through the economy at a

[1]Contrary to K, there is no capacity accumulation problem for H. This is done for simplicity and does not affect the qualitative results of the model.

cost $\underline{w}$ for any efficiency unit of H, or as human capital. The bound $\bar{H}$ is then interpreted as a technological barrier.
Alternatively, similarly to Lewis (1954), H can be a raw input, such as number of hours worked or as natural resources. The idea is that, at the early stages of development, there is an input, such as labor, which is greatly unemployment or employed in an informal sector where it gains a shadow price $\underline{w}$. As the economy grows, more H is drawn from the informal sector at a cost $\underline{w}$. In this case, the rapid initial growth of the economy is driven both by accumulation of K *and* by higher employment rates of H, until the full employment $\bar{H}$ is reached. As an empirical example I plot in Figure 3 [2] and Figure 4, respectively, the employment rate and the participation rate in South Korea. As we can see, both indicators feature a steady growth in the years leading up to the crisis, and then settle down to values that never exceed the pre-crisis levels. A large increase in the labor input indeed took place in the emerging Asian economies due to large regional migration flows (Prema-chandra [2006]). [3]

PLEASE ENTER FIGURE 4 HERE

PLEASE ENTER FIGURE 5 HERE

I introduce into the model an element of uncertainty, with respect to the growth path of the economy, by allowing the aggregate fix endowment $\bar{H}$ to be a random variable. The value of $\bar{H}$ is drawn at the beginning of the times from a distribution with c.d.f. $G(\cdot)$.
Both K and H are provided to the production sector by the housholds. I denote with lowercase letters the variables relative to the household choice. Since $\bar{H}$ is unknown, the household faces a stochastic constraint on the supply of h. As long as $\bar{H}$ doesn't bind because it has not been reached in aggregate, the household supply of h must simply satisfy

$$h \geq 0$$

However, whenever $\bar{H}$ is reached, and h is constrained by

$$h \leq \bar{H}$$

[2] The employment rate equals one minus the unemployment rate.

[3] Similar patterns are followed by Indonesia, Malaysia and Thailand, but the availability and quality of data is limited. *Source*: ILO.

The stochastic nature of the labor supply constraint, and its evolution over time, can be usefully modeled as a sequence of shocks s_t, so that the labor supply constraint can be rewritten as

$$s_t h_t \leq 1$$

where $s_t \in \left\{0, \frac{1}{\bar{H}}\right\}$. Clearly, if $s_t = 0$ the household is not constrained, while if $s_t = 1/\bar{H}$ the constraint becomes $h_t \leq \bar{H}$. If $s_t = 1/\bar{H}$, the maximum amount of labor that can be supplied is then known to all agents at any time $\tau \geq t$. Therefore, a consistent definition of the shocks requires that

$$s_t = \frac{1}{\bar{H}} \quad \Rightarrow \quad s_\tau = \frac{1}{\bar{H}} \quad \forall \tau \geq t$$

I define s^t a history of shocks up to time t. I will denote with s_0^t a history at time t where the bound $\bar{H}$ has not been reached, and with s_H^t a history at t where the bound has been reached yet. The following example illustrates the two types of histories at time $t = 5$

$$s_0^t = (0, 0, 0, 0, 0)$$
$$s_H^t = \left(0, 0, 0, \frac{1}{\bar{H}}, \frac{1}{\bar{H}}\right)$$

In the example above, s_H^t represents a history where $\bar{H}$ is reached at time 4.
The stochastic distribution of the shocks s_t induces a probability measure μ over histories. Even though the representative household takes μ_0 as given, μ_0 will depend, in equilibrium, on the unconditional distribution $G(\cdot)$ and on the the endogenous rate at which H is absorbed into production.
At any history s^t the household is endowed with capital $k(s^{t-1})$ which she rents to the firms, and maturing debt $d(s^{t-1})$ to be repaid to international lenders. The household decides how much factor $h(s^t)$ to supply, the new level of capital $k(s^t)$ and the new amount $d(s^t)$ of international debt to issue. The household can also decide whether to default on its current debt, which is then erased from the household's budget constraint, at the penalty of a permanent exclusion from any future borrowing possibility. The indicator function $i(s^t)$ equals zero if the household is in the default state and one otherwise.
Define $r_k(s^t)$, $w(s^t)$, $q(s^t)$ as, respectively, the rental rate of capital, the price of h and the price of a discount bond at history s^t.

The representative household takes as given the functions $r(s^t)$, $w(s^t)$, $q(s^t)$, μ_0 and solves

$$\max_{\{c(s^t),x(s^t),h(s^t),d(s^t),i(s^t)\}} E_{\mu_0}\left[\sum_{t=0}^{\infty}\sum_{s^t}\beta^t u(c(s^t))\right]$$

$$\text{s.t.}\quad c(s^t)+x(s^t)+d(s^{t-1})[1+i(s^{t-1})-i(s^t)] = r_k(s^t)k(s^{t-1})+[w(s^t)-\underline{w}]l(s^t) + q(s^t)d(s^t) \tag{2}$$

$$k(s^t)=(1-\delta)k(s^{t-1})+x(s^t) \tag{3}$$

$$s_t h(s^t)\le 1 \tag{4}$$

$$i(s^t)\in\{0,i(s^{t-1})\} \tag{5}$$

$$d(s^t)\le[1-i(s^t)]mk(s^t) \tag{6}$$

$$i(s^{-1})=1,\quad k(s^{-1}),d(s^{-1})\quad \text{given} \tag{7}$$

The indicator function $i(s^t)$ determines the default choice by the househod. If $i(s^t)=1$ following a history s^{t-1} where $i(s^{t-1})=0$, then the household defaults at history s^t. It follows that $1+i(s^{t-1})-i(s^t)=0$ so current debt $d(s^{t-1})$ is erased from the budget constraint. However, $i(s^\tau)=1$ at history $s^\tau\ge s^t$ following histoy s^t, and the household can only invest abroad, but not borrow, i.e. $d(s^\tau)\le 0$.

The constant m is an exogenous borrowing limit that is aimed at capturing the maximum amount of debt that the household could ever commit to repay. A more thorough discussion on such a bound is left to Section 2.2. I assume

$$0\le m<1$$

so that the borrowing constraint (6) implies a bound each period on the aggregate debt capital ratio, which can be seen as a limit to leverage. The problem of the firm is a standard static problem that gives the the rental rate of capital and H. Choice variables of the firm are denoted with uppercase letters. At any history s^t aggregate capital $K(s^t)$ and land $L(s^t)$ rent by firms must satisfy

$$F_1(K(s^t),H(s^t))=r_k(s^t) \tag{8}$$

$$F_2(K(s^t),H(s^t))=w(s^t) \tag{9}$$

Finally, we need to define the problem of the risk neutral international investors. As for the firm's problem, the international investor's problem is completely characterized by the pricing function q. By denoting with r_f the constant international risk free rate we obtain

$$q(s^t) = \frac{\mu_0(\{s^{t+1} : i(s^{t+1}) = 0\}|s^t)}{1 + r_f} \tag{10}$$

The key restriction introduced in the model is that G is the c.d.f. of a Pareto distribution

$$Prob\{\bar{H} < H|H_{min}\} = G(H) = 1 - \left(\frac{H_{min}}{H}\right)^{\eta} \tag{11}$$

with $H_{min} > 0$, $0 < \eta < 1$.[4]
In this context, the Pareto distribution has a convenient "memoryless" property. As it will be evident, as H grows at a constant rate, uncertainty about the level $\bar{H}$ does not slowly vanishe. In some sense, as the economy grows, the household does not feel that $\bar{H}$ is getting. The permanence of a non vanishing degree of uncertainty as more H is employed is a crucial property that I impose to the model.

An equilibrium is defined as follows

Definition 1. *An equilibrium is given by aggregate prices* $\{r(s^t), w(s^t), q(s^t)\}$, *a measure* μ_0 *over histories* s^t, *individual policy functions for the household* $\{c(s^t), k(s^t), x(s^t), h(s^t), d(s^t), i(s^t)\}$ *and policy functions for firms* $\{K(s^t), H(s^t)\}$ *such that*

i) the individual policy functions solve the household's problem, given prices and the probability measure μ_0.

ii) the pricing functions q, r, w *solve (8), (9), (10).*

iii) Markets clear.

$$K(s^t) = k(s^t)$$
$$H(s^t) = h(s^t)$$

iv) Feasibility

$$c(s^t) + x(s^t) = F(k(s^t), l(s^t)) + q(s^t)d(s^t) - d(s^{t-1})$$

[4]Notice that for $0 < \eta < 1$ the Pareto distribution has no finite first and second moments. This is of no consequence for the results that I derive, since all the variables will be well defined (finite). The restriction of η implies that the tail of the distribution is sufficiently "fat"

v) labor supply is constrained if and only if the aggregate bound $\bar{H}$ has been reached

$$s_t = \frac{1}{\bar{H}} \Leftrightarrow H(s^t) = \bar{H} \tag{12}$$

vi) for any history s^t with shock s_t, the probability measure μ_0 is defined recursively by the transition probabilities

$$\mu_0(\{s_0^{t+1}\}|s_0^t) = \left(\frac{H(s_0^t)}{H(s_0^{t+1})}\right)^\eta \tag{13}$$

$$\mu_0(\{s_H^{t+1} : s_{H,t+1} = s_{H,t} = 1/\bar{H}\}|s_H^t) = 1 \tag{14}$$

and for $H(s_0^t) \leq \tilde{H} \leq H(s_0^{t+1})$,

$$\mu_0(\{s_H^{t+1} : s_{H,t+1} \geq 1/\tilde{H}\}|s_0^t) = \left[1 - \left(\frac{H(s_0^t)}{\tilde{H}}\right)^\eta\right] \tag{15}$$

The last four equation define the equilibrium properties of the endogenous probability measure μ over the sequence of shocks. Essentially, the measure μ is defined by a fixed point problem. Given,μ_0, agents solve their problems so that an equilibrium sequence H_t of supply of the factor H is determined. The equilibrium conditions requires that the shock $s_t = \frac{1}{\bar{H}}$ is received exactly at time t when $\bar{H}$ becomes binding. Moreover, the transition probabilities for the shocks must be consistent at each time with the ex-ante distribution $G(\cdot)$, the current observed level of $H(s_t)$ and its future unconstrained value $H(s_0^{t+1})$. The fact that the transition probability depends just on the growth rate $\frac{H(s_0^t)}{H(s_0^{t+1})}$ is a property entirely due to the choice of a Pareto form for the c.d.f. G.
To start characterizing an equilibrium I prove the following lemma

Proposition 1. *In any equilibrium, $i(s_H^t) = 1$ for all histories s_H^t. Moreover, if $D(s_0^\tau) > 0$ for some τ, then $i(s_0^t) = 0$ for all s_0^t.*

Proof. Consider any history s_H^t. At such history the aggregate value $\bar{H}$ has been reached and the capital that can be feasibly accumulated by the economy is bounded above by $\bar{K}$, defined implicitly by

$$\delta \bar{K} = A_1 \bar{K}^\alpha \bar{H}^{1-\alpha}$$

Therefore, the aggregate debt for the economy is bounded above by $\bar{D} = m\bar{K}$. Since $s_H^t = 1/\bar{H}$ it follows that $s_H^\tau = 1/\bar{H}$ for all histories $s^\tau \geq s^t$ and

all the uncertainty is resolved from time t. Strictly positive levels of debt can then be sustained only if default never occurs after history s_H^t. In this case, the price of bond must be $q(s_H^t) = \frac{1}{1+r_f}$ for any $s_H^\tau \geq s_H^t$. We know, therefore, that the present value at time t of the debt at an arbitrarily far moment τ goes to zero, that is , for any $\epsilon > 0$ there exists a
tau sufficiently large such that

$$\frac{D(s_H^\tau)}{(1+r_f)^{\tau-t}} \leq \frac{m\bar{K}}{(1+r_f)^{\tau-t}} < \epsilon \tag{16}$$

We can apply the Bulow-Rogoff result (1989) and conclude that the household will default at some history $s_H^{\tilde{\tau}} \geq s_H^t$. But this contradicts the assumption of no default in equilibrium after s_H^t. No borrowing can then be supported from history s_H^t on. It follows that the household is better off by defaulting at history s_H^t and avoid repaying her outstanding debt $D(s^{t-1})$. We conclude that $i(s_H^t) = 1$ at any history $s^\tau \geq s_H^t$.
Applying this result, the proof of the last part of the proposition can be explained using Figure 6. Suppose that at history s_0^{t+1} agents expect $i(s_0^{t+2}) = 1$. Then default takes place with certainty at time $t+2$ and no lending would occur at time $t+1$. It follows that the household is better of by defaulting at $t+1$ instead of waiting $t+2$. A backwards induction argument then implies that a necessary condition for a strictly positive amount of debt to be sustained at some history is that the household never defaults at all histories s_0^t.

PLEASE ENTER FIGURE 6 HERE

□

Proposition 2 tells us that, if we are searching for non-autarkic equilibria we must focus on equilibrium paths where the representative household never defaults at histories s_0^t, i.e. as long as the supply of H is unconstrained. Notice, in particular, that Proposition 1 is completely general, since no assumption about the form of $G(\cdot)$ is needed.
In the rest of the section I give conditions for the existence of equilibria where default never arises along histories s_t^0. In such equilibria, the probability of transitioning from history s_0^t to history s_H^{t+1} is exactly the probability of default at time $t+1$, and the price of the bond charged by the risk neutral international investors at any history s_t^0 satisfies

$$q(s_0^t) = \frac{1}{1+r_f}\mu(\{s_0^{t+1}\}|s_0^t) = \frac{1}{1+r_f}\left[\frac{H(s_0^t)}{H(s_0^{t+1})}\right]^\eta \tag{17}$$

As seen in the proof of Proposition 1, the application of Bulow-Rogoff result implies that default always takes place when $\bar{H}$ is reached. In order to avoid default at unconstrained histories s_0^t the Bulow-Rogoff results must be not valid along those paths. The key lays into breaking the present value condition (16). In fact, if the growth rate of the economy is sufficiently high, the debt can grow at a rate such that new debt issued is at least enough to roll over the maturing debt. If this is true, the household has clearly no incentive to default and exit the financial market. Of course, debt rolling over takes place only if the debt grows at a rate at least as big as the interest rate. I will call such a condition the *bubble condition.*
Let's now characterize the equilibrium at any history s_0^t through its first order conditions. As long as the supply of labor is unconstrained, the first order condition of the household implies

$$w(s_0^t) = \underline{w} \tag{18}$$

and therefore, using the equilibrium condition (9) from the firm's problem,

$$\underline{w} = A_1(1-\alpha)\left[\frac{K(s_0^t)}{H(s_0^t)}\right]^{\alpha}$$

and

$$K(s_0^t) = \kappa H(s_0^t), \quad \kappa \equiv \left(\frac{\underline{w}}{A_1(1-\alpha)}\right)^{\frac{1}{\alpha}} \tag{19}$$

Moreover, since by construction at any history s_H^t the constraint (4) is binding, condition (9) implies

$$w(s_H^t) = A_1(1-\alpha)\left[\frac{K(s_H^t)}{\bar{H}}\right]^{\alpha} \tag{20}$$

It follows that at any history s_0^t output and interest rates are

$$Y(s_0^t) = A_0 K(s_0^t)$$

$$r(s_0^t) = \alpha A_0$$

where

$$A_0 = A_1^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{\underline{w}}\right)^{\frac{1-\alpha}{\alpha}}$$

The AK form of the production function along unconstrained histories will allow us to give conditions for the the existence of a balanced growth path along those histories. On the contrary, at history s_H^t after default,

$$Y(s_H^t) = K(s_H^t)^\alpha \bar{H}^{1-\alpha}$$

$$r(s_H^t) = \alpha K(s_H^t)^{\alpha-1} \bar{H}^{1-\alpha}$$

I turn now to the optimal choice of capital and debt for the household. First, I make the following standard assumptions

$$\beta = \frac{1}{1+r_f}$$

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad \sigma \geq 0, \neq 1$$

The first order conditions for the optimal choice capital $k(s^t)$ and debt $d(s^t)$ are, respectively,

$$\begin{aligned} u'(c(s_0^t) = \beta \left[\frac{H(s_0^t)}{H(s_0^{t+1})}\right]^\eta u'(c(s_0^{t+1})) \left[r_k(s_0^{t+1}) + 1 - \delta\right] + m\lambda_d(s_t^0) \\ + \beta \int_{H(s_0^t)}^{H(s_0^{t+1})} u'(c(s_H^{t+1})) \left[r_k(s_H^{t+1}) + 1 - \delta\right] \eta \left[\frac{H(s_0^t)}{H}\right]^{\eta+1} \frac{1}{H(s_0^t)} dH \end{aligned} \tag{21}$$

$$\begin{aligned} \lambda_d(s_0^t) &= q(s_0^t) u'(c(s_0^t)) - \beta \left(\frac{H(s_0^t)}{H(s_0^{t+1})}\right)^\eta u'(c_0^{t+1}) \\ &= q(s_0^t)[u'(c(s_0^t) - u'(c(s_0^{t+1})] \end{aligned} \tag{22}$$

where $\lambda_d(s_t^0)$ is the Kuhn-Tucker multiplier of the borrowing constraint (6). I derive now some homotheticity properties that prove to be very useful in the simplification of the first order conditions of the household. First of all, consider the situation where the constraint on H becomes binding at time t where capital is $K(s_0^{t-1})$. Given that households default at time t and $\bar{H}$ binds from that moment on, all the uncertainty is resolved, and the equilibrium from time t is the standard solution to a 'one-sector growth model" with inelastic labor supply equal to $\bar{H}$ and initial capital level equal to $K(s_0^{t-1})$.

After default, the problem can be written in recursive form as a the choice of a capital policy function $K'(K;\bar{H})$ that solves

$$V(K;\bar{H}) = \max_{K'} \frac{1}{1-\sigma}[A_1 K^{\alpha}\bar{H}^{1-\alpha} - \underline{w}\bar{H} + (1-\delta)K - K']^{1-\sigma} + \beta V(K;\bar{H})$$

It is an easy task to verify that

$$V(K;\bar{H}) = \bar{H}^{1-\sigma} V\left(\frac{K}{\bar{H}};1\right)$$

The associated consumption policy function $C(K;\bar{H})$ must then satisfy

$$C(K;\bar{H}) = \bar{H} C\left(\frac{K}{\bar{H}};1\right) \equiv \bar{H}\tilde{c}\left(\frac{K}{\bar{H}}\right)$$

for some function $\tilde{c}(\cdot)$. Therefore, at any history s_H^t, where capital is K_t,

$$u'(c(s^t)) = u'(C(K;\bar{H})) = \bar{H}^{-\sigma} u'(\tilde{c}(K/\bar{H}))$$

The second homotheticity property that we use characterizes the price q of the bond at histories s_0^t as dependent only on the growth rate of capital. In fact, because of (19)

$$q(s_0^t) = \left[\frac{H(s_0^t)}{H(s_0^{t+1})}\right]^{\eta} = \left[\frac{K(s_0^t)}{K(s_0^{t+1})}\right]^{\eta}$$

Prior to default, the first order conditions can be then be rewritten just in terms of capital levels

$$\begin{aligned} u'(c(s_0^t) &= \beta\left(\frac{K_t^0}{K_{t+1}^0}\right)^{\eta} u'(c(s_0^{t+1}))\left[\alpha A_0 + 1 - \delta\right] + m\lambda_d(s_t^0)\beta\kappa\eta \\ &\cdot \int_{\kappa^{-1}K_t^0}^{\kappa^{-1}K_{t+1}^0} H^{-\sigma}\left[\tilde{c}\left(\frac{K_{t+1}^0}{H}\right)\right]^{-\sigma}\left[\alpha A_1\left(\frac{K_{t+1}^0}{H}\right)^{\alpha-1} + 1 - \delta\right]\left[\frac{K_t^0}{H}\right]^{\eta+1}\frac{dH}{K_t^0} \end{aligned} \quad (23)$$

Finally, let $g_{t+1}^0 = \frac{K_{t+1}^0}{K_t^0}$ and change the integrating variable to $\bar{\kappa} = \frac{\kappa}{K_t^0}H$ and obtain

$$\begin{aligned} u'(c(s_0^t)) = \beta\left(g_{t+1}^0\right)^{-\eta} u'(c(s_0^{t+1}))\left(\alpha A_0 + 1 - \delta\right) + m\lambda_d(s_t^0) + \beta\eta\kappa^{\sigma}(K_t^0)^{-\sigma} \\ \cdot \int_1^{g_{t+1}^0} \bar{\kappa}^{-\sigma-\eta-1}\tilde{c}\left(\bar{\kappa}^{-1}\kappa g_{t+1}^0\right)^{-\sigma}\left[\alpha A\bar{\kappa}^{1-\alpha}(\kappa g_{t+1}^0)^{\alpha-1} + 1 - \delta\right] d\bar{\kappa} \end{aligned} \quad (24)$$

Proposition 2. *Suppose that the following condition is satisfied*

$$\alpha A_0 + 1 - \delta > (1 + r_f)^{\frac{1+\sigma}{1-\eta}} \tag{25}$$

then there exists an $m > 0$ *such that* $\frac{D(s_t^0)}{K(s_t^0)} = m$ *and* $\frac{K(s_{t+1}^0)}{K(s_t^0)} = \frac{C(s_{t+1}^0)}{C(s_t^0)} = g_0 > 1$ *for all* s_0^t *is an equilibrium. Moreover, if* σ *is small, the equilibrium growth rate* g_0 *is uniquely determined and is strictly increasing in* m.

The proof of Proposition 2 is reported in appendix. Recall that the household prefers not to default along unconstrained histories if and only if the debt is at least rolled over every period. Along a balance growth path the debt grows at the constant rate g_0, hence the rolling over condition is possible whenever the growth rate of the economy is greater than the country's interest rate, i.e.

$$g_0 \geq \frac{1}{q_0} \tag{26}$$

or, using (17)

$$g_0^{1-\eta} = 1 + r_f$$

Since, if (26) holds, the value of an asset (debt) grows at a rate bigger than the interest rate, I refer to (26) as *bubble condition.* Inequality (25) requires that the productivity of the economy, summarized by αA_0, be sufficiently large compared with the risk free rate, for given values of the remaining exogenous parameters. Recalling that we have assumed $\eta \in (0, 1)$[5], if the productivity is high then the balanced growth rate g_0 will satisfy the bubble condition.

The requirement that σ has to be small to obtain uniqueness of the equilibrium is, in my simulations, quite weak, since uniqueness turns out to be guaranteed for reasonably large σ. The theoretical issue, in fact, is that large coefficients of risk aversion generate a precautionary motive for saving, which could lead to multiplicity of equilibria. High growth rates increase the probability of default and this increases the investment rate since borrowers want to issue as much debt as possible before the economy enters autarky. Therefore both high growth-high risk and low growth-low risk equilibria might arise.

Proposition 2 states that growth and sustainability of capital inflows are strictly related. In the next section I emphasize the fact that growth and capital inflows are indeed related in a circular fashion: not only growth allows debt sustainability, but also capital inflows spur growth.

[5] See note 4

2.1 The self-reinforcing effect of growth and capital inflow

At this point we can reassess the essential question that motivates this paper: why do large capital inflows and high growth rates eventually culminate in a financial crisis? The answer that I propose is that growth and capital inflow are self-reinforcing. As long as the emerging economy is steadily growing by accumulating capital *and* using the stock of its underemployed resources, the economy runs a growing, negative, balance of payments. There is no incentive for borrowers to default on their debt, since the punishment of future exclusion from the financial market would deprive them of valuable resources. However, when the fixed factor is finally fully employed, the economy enters the world of decreasing marginal returns, doomed by a slowing growth rate. With the rate of growth slowing down, the capital inflow decreases. There has to be a point where the balance of payment is expected to reverse, since the borrowers will have to start paying back their debt. At this point, the borrowers are better off by defaulting and enter autarky. Notice that a long period may elapse before the reversal in the capital flow takes place. Nonetheless, in the anticipation of this event, whose date is now certain, a backwards induction argument implies that lenders stop providing funds from the very moment when the supply of the fixed factor becomes binding. In other words, there is a sudden stop in the capital inflow and default on existing debt.

As in the proof of Proposition 2, I define $\hat{c}(g_0) = c(s_0^t)/k(s_0^t)$ as the detrended consumption level along the balanced growth path. The growth rate g_0 is the fixed point of the first order condition

$$g_0^{-\eta-\sigma}[\alpha A_0 + 1 - \delta + m(g_0^\sigma - 1)] - \frac{1}{\beta} + \eta \hat{c}(g_0)^\sigma \kappa^\sigma$$
$$\cdot \int_1^{g_0} \bar{\kappa}^{-\sigma-\eta-1} \tilde{c}\left(\bar{\kappa}^{-1}\kappa g_0\right)^{-\sigma} \left[\alpha A \bar{\kappa}^{1-\alpha}(\kappa g_0)^{\alpha-1} + 1 - \delta\right] d\bar{\kappa} = 0$$

Notice that the marginal product of capital in the non-default history is augmented by the term $m(g_0^\sigma - 1)$. This is because the access to the international financial market allows the households to finance part of the capital stock by issuing debt, rather than by forgoing her current consumption. This *leverage effect* increases the effective return on capital and is stronger the wider is the ability m of the household to issue debt. We then conclude that access to the international financial market and capital inflows reinforce growth.

2.2 The borrowing constraint

I conclude the theoretical section of this paper with a discussion on the borrowing constraint. In fact, a crucial assumption of the model is that the household is constrained on the maximum amount of debt that she can issue. The borrowing constraint that I have assumed implies that the household can borrow each period up to a constant fraction of her wealth[6]. This turns out to be exactly the form of borrowing constraint used, for instance, by Barro et al. (1995) and bears the resemblance of a collaterization constraint. This interpretation, however, is not consistent with my framework: no collateral or income are seized after default, and the household is free to revert to state where she is excluded from future borrowing. Notice also that, contrary to Perri and Kehoe (2002) or Kehoe and Levine (2000), the choice of default implies no cost for the household in terms of forgone ability to smooth income fluctuations due to lack of access to the financial market[7]. In fact, in equilibrium, there are simply no states where debt is accumulated and others where it is repaid. In other words, there is no insurance value in the ability access to the financial market, debt is only accumulated and is never repaid.

As noted, the arrangements of the financial market that I assume are those of Bulow and Rogoff (1989). The existence of a bubble breaks the Bulow-Rogoff result and allows strictly positive levels of debt to be sustained. In this terms, my model shares some features present in Hellwig and Lorenzoni (2007). The existence of a "bubble", for which I mean the existence of an asset whose value grows at a rate greater than the interest rate, requires the asset to be in limited supply (see, for instance, the discussion in Jovanovic [2007]). The requirement that the supply of debt asset is limited each period is guaranteed by the borrowing constraint. Without such a constrain, which in equilibrium is always binding whenever the bubble condition holds, there would be no equilibrium in the model, for the amount of debt issued would be undetermined. To justify existence of a borrowing constraint of the form presented here, I refer to the results of Piazza (2008)[8] who, in a related work, shows that the borrowing constraint can be obtained endogenously in a model where exclusion from the financial market can only be

[6] It is straightforward to see that the same results are obtained if the upper bound on borrowing is a constant fraction of the current income or of the present value of future income.

[7] Another feature of my model, which is not present in the literature previously cited, is that I allow for the possibility of saving abroad, after default, at the world's interest rate. This feature, however, has no effect on the results and could be dispensed of.

[8] The paper is available on request

temporary. The results extend also to the case where default implies some kind of output (or collateral) costs. The idea behind these results is that, if exclusion from borrowing is only temporary, too high levels of debt are not sustainable: the household would have incentives to accumulate high stocks of debt, default on it and the re-enter the financial market. In an environment that features the homotheticity properties presented here, such a borrowing limit increases with the income - and hence the capital - of the household.

2.3 Calibration

In this section I perform a calibration exercise based on the evidence from a group of asian countries for the years 1987-2007. The goal of this exercise if to assess whether the model is able to replicate qualitatively and quantitatively the stylized facts presented in section 1.

To perform the calibration, I modify the model in the following way. First of all, let's normalize the cost of supplying H, $\underline{w} = 1$. The cost x_h of supplying a an amount $H \leq \bar{H}$ is then equal to $x_h = H$. Figure (7) shows the "production function" $H(x_h)$. The factor H can be increase with a linear technology up to the point $\bar{H}$, after which any increase in the input x_h is ineffective. This type of production function poses a problem for our calibration. Once $\bar{H}$ is reached, decreasing marginal returns hit very strongly, since H cannot be increased any further. The standard result of a neoclassical growth model says that, as long as α is not too large, the growth rate of the economy would decreases very quickly towards zero. But this is in contrast with the evidence from Figure 1, where the average growth rate preserves a trend-type behavior even after the 1998 crisis. I therefore introduce an new form for the production function, which allows H to be increased even after the stochastic point $\bar{H}$, but at an increasing marginal cost. The modified production function is

$$H = \begin{cases} x_h & x_h \leq \bar{H} \\ \bar{H} + \frac{\bar{H}}{\theta}\left[\left(\frac{x_h}{\bar{H}}\right)^{\theta} - 1\right] & x_h > \bar{H} \end{cases} \tag{27}$$

$0 \leq \theta < 1$. As Figure 8 shows, for $\theta = 0$ the production function is the same as the original one. As θ gets close to 1 decreasing marginal returns get milder. All the results obtained so far carry over to the new production function: after $\bar{H}$ is reached the GDP of the economy is bounded and the present value condition (16) for the validity of the Bulow-Rogoff result holds. To preserve the trend-like behavior of the growth rate after the default period I set an exogenous value of $\theta = 0.99$. This allows me to stress an important point: *even a very mild slow down in the growth rate can trigger a financial crisis, as long as investors perceive such a slow down as the beginning of a stage of slowing growth.*

The calibrated values for α, σ and δ are quite standard in the macro literature. Given the normalization $\underline{w} = 1$, the value of A_1 is uniquely determined by A_0, which in turn is the reciprocal of the capital output ratio along the balanced growth path. Using Nehru and Dhareshwar's (1993) database, I set $A_0 = 2$. The risk free rate is calibrated to the average real return of the US 5-years Bond for the period 1990-2000. I calibrate η using the equilibrium risk premium $\frac{1}{q(g_0)} - (1 + r_f)$ calculated from the EMBI database for the years 1994-1997. Finally, the value of m is chosen using the fact that the Debt/GDP ratio equals m/A_0 along the balanced growth path[9].

Table 1 summarizes the values given to the calibrated parameters, together with the comparison between the equilibrium results and the empirical values. The statistics of interest are the growth rate g_0 and the investment/GDP ratio x_0/y_0 prior to default, the average growth rate g_H and the investment/GDP ratio x_H/y_H over 10 periods after default. Since the model is not meant to capture the output costs due to the financial crisis, I exclude the year 1998 from the computation of the target (empirical) values. In Figures 9-10 I report both the average values for these variables for the four countries analyzed and the simulated results, by assuming that in 1998 the bound $\bar{H}$ is reached. The model captures the three main features presented in Section 1. First of all the year of the crisis marks a structural break in the growth rate of the economy: the lack of the leverage effect from access to the international financial market after default depresses the growth rate of the economy. Second, the financial crisis takes place in a context of high growth both prior and after the crisis: the growth process is then "solid" and the crisis might then appear as "unexpected". Third, the large capital inflow prior to the default moment increases the investment rate, which permanently drops by 5% in the post-crisis periods.

As I have already pointed out, the model does not consider the output costs associated with the disruption of the financial markets, therefore the drop in output realized in 1998 cannot be replicated. Nonetheless we can make some considerations regarding the welfare effect of financial openness by comparing our calibrate example, with $m = 30\%$, with the case $m = 0$, which corresponds to financial autarky. In this latter the economy would

[9]The debt GDP ratio is a volatile statistics, reflecting nominal movement in the exchange rates. Using the IMF World Economic Outlook I choose a Debt/GDP ratio of 60% as an empirically reasonable number.

Table 1: Calibration Parameters and Results

Parameters	Target Values
$\theta = 0.99$	-
$\alpha = 0.4$	-
$\sigma = 2$	-
$\delta = 5\%$	-
$r_f = 3\%$	T-Bond (real)
$m = 30\%$	$\frac{D}{Y} = 60\%$
$\eta = 0.20$	$1/q_0 - 1 - r_f = 1.8\%$
$A_0 = 0.5$	$\frac{K_0}{Y_0} = 2$
$g_0 = 7.8\%$	8.1%
$g_H = 5.7\%$	5%
$X_0/Y_0 = 25\%$	30%
$X_H/Y_H = 20\%$	20.1%

have never experience a crisis event, however, as shown in Figure 9, the growth rate would have been much smaller prior to 1998 when the country would have smoothly entered the world of decreasing marginal returns.[10] Finally, Figure 11 shows the growth path that the country would follow were the debt contracts completely enforceable. In this case, there would be no default event and, since decreasing marginal returns hit very mildly once $\bar{H}$ is reached, the marginal product of capital remains high for a long number of periods, and the country could still take advantage of the leverage effect and sustain a high growth rate.

3 Conclusions

In this paper I constructed a model of economic growth of a small open economy, where a rapid period of growth is followed by a sudden financial

[10] Strictly speaking, *for a given* realization of $\bar{H}$, the autarkic economy would have reached $\bar{H}$ at a later date than the open economy, where the growth rate is bigger and thus convergence to the steady state is faster.

crisis and a persistent fall in the growth rate of output. The crisis arises as a natural consequence of the fact the evolution of the growth rate is uncertain because of the existence of a factor whose fixed endowment is unknown. The rate of employment of such factor increases with the economic expansion but, under suitable assumptions, the degree of uncertainty about the level of the endowment does not simply vanish over time. Uncertainty is resolved only when the factor becomes fully employed. Large capital inflows and high growth rates, associated with the initial periods of growth are self reinforcing. At this stage, the dynamics of the capital inflow resemble those of a bubble. When full employment of the fixed factor is reached decreasing marginal returns start to slow down the growth rate. The bubble bursts and debt becomes unsustainable. The consequent sudden stop in the capital inflow depresses the growth rate of the economy, which can nonetheless remain high for a long number of periods. The model fits quite well some macro facts taken from the Asian crisis. The model shows that growth and debt sustainability are strictly connected, and that "unexpected" financial crisis can be rationalized in a model of growth with limited enforceability on international debt and uncertainty on the timing of appearance of decreasing marginal returns.

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5 Appendix 1

Proof of Proposition 2.

Proof. Suppose that capital grows at a constant gross rate rate g_0. Consider a path along which debt and consumption grow at the rate g_0. By feasibility

$$\begin{aligned}\frac{c_t^0}{k_t^0} &= A_0 + 1 - \delta - g_0 - [A(1-\alpha)]^{\frac{1}{\alpha}} \underline{w}^{\frac{\alpha-1}{\alpha}} + \left[\frac{g_0^{1-\eta}}{1+r_f} - 1\right]\frac{d_t}{k_t} \\ &= \alpha A_0 + 1 - \delta - g_0 + m\left[\frac{g_0^{1-\eta}}{1+r_f} - 1\right] \equiv \hat{c}(g_0)\end{aligned} \tag{28}$$

It follows that

$$u'(c(s_0^{t+1})) = g_0^{-\sigma} u'(c_t^0) = g_0^{-\sigma}(c_t^0)^{-\sigma}$$

The first order condition (24) becomes

$$\begin{aligned}(k_t^0)^{-\sigma}\hat{c}(g_0)^{-\sigma} &= \beta g_0^{-\eta-\sigma}(k_t^0)^{-\sigma}\hat{c}(g_0)^{-\sigma}(\alpha A_0 + 1 - \delta) + m\beta g_0^{-\eta}\hat{c}(g_0)^{-\sigma}(k_t^0)^{-\sigma}(1 - g_0^{-\sigma}) \\ &\quad + \beta\eta(k_t^0)^{-\sigma}\kappa^{\sigma}\int_1^{g_0} \bar{\kappa}^{-\sigma-\eta-1}\tilde{c}\left(\bar{\kappa}^{-1}\kappa g_0\right)^{-\sigma}\left[\alpha A\bar{\kappa}^{1-\alpha}(\kappa g_0)^{\alpha-1} + 1 - \delta\right] d\bar{\kappa}\end{aligned}$$

or

$$\begin{aligned}&g_0^{-\eta-\sigma}(\alpha A_0 + 1 - \delta + m g_0^{\sigma} - m) - \frac{1}{\beta} \\ &\quad + \eta\hat{c}(g_0)^{\sigma}\kappa^{\sigma}\int_1^{g_0} \bar{\kappa}^{-\sigma-\eta-1}\tilde{c}\left(\bar{\kappa}^{-1}\kappa g_0\right)^{-\sigma}\left[\alpha A\bar{\kappa}^{1-\alpha}(\kappa g_0)^{\alpha-1} + 1 - \delta\right] d\bar{\kappa} = 0\end{aligned} \tag{29}$$

Assume that $m = 0$. Define $\bar{g}$ as

$$\bar{g} = \alpha A_0 + 1 - \delta$$

Notice that $\hat{c}(\bar{g}) = 0$ and hence for $g_0 = \bar{g} - \epsilon$ the left hand side of (29) is negative. Now define $\underline{g}$ so that

$$\underline{g}^{1-\eta} = 1 + r_f \tag{30}$$

It is easy to show that if (25) holds we have

$$\underline{g}^{-\eta-\sigma}(\alpha A_0 + 1 - \delta) \geq \frac{1}{\beta}$$

Moreover, $\hat{c}(\underline{g}) > \hat{c}(\bar{g}) = 0$. It follows that for $g_0 = \underline{g}$ the left hand side of (29) is positive, which proves that there exists a g_0 for which (29) holds. By continuity there exists an $m > 0$ and a $g_0(m)$ for which (29) holds. To prove the second part of the proposition it is useful to go back to the original first order condition (21). For $\sigma = 0$ the equation becomes

$$\frac{1}{\beta} = E\left[r_{k,t+1}(g_0) + 1 - \delta \Big| \frac{H(s_0^t)}{H(s_0^{t+1})}\right]$$

and remember that $\frac{H_t^0}{H_{t+1}^0} = \frac{K_t^0}{K_{t+1}^0} = g_0$. A first effect of a higher growth rate g_0 is to decrease the marginal products $r_k(s^{t+1})$ in all all the histories s_H^{t+1} and for al the realizations of $\bar{H}$, since marginal products in those states depend negatively on $\frac{K_{t+1}^0}{\bar{H}}$. On the contrary, the marginal product of capital in history s_{t+1}^0 is constant and equals $r_k(s_0^{t+1}) = \alpha A_0 \geq r_k(s_H^{t+1})$ for all s_H^{t+1}. The first effect is standard and is due to the concavity of the production function. The second effect of a higher growth rate g_0 is to make more likely states of default where, as just mentioned, the marginal product of capital is smaller than that in the non defaulting state. It follows that the expected marginal product of capital is strictly decreasing in g_0 which gives us the uniqueness result stated in the proposition for the case $\sigma = 0$. By compactness, for a given m, of the feasible set of growth rates, and by continuity the equilibrium growth rate g_0 is unique also for σ close to zero. The multiplier λ_d can be written, along a balanced growth path, as

$$\lambda_{d,t}^0 = \beta\hat{c}(g_0)^{-\sigma}(k_t^0)^{-\sigma}g_0^{-\eta}(1 - g_0^{-\sigma}) = \beta c(g_0)^{-\sigma}(k_t^0)^{-\sigma}\hat{\lambda}_d(g_0)$$

For σ small, $\hat{\lambda}_d(g_0)$ is a strictly decreasing function and the first order condition is

$$\frac{1}{\beta} = E\left[\frac{c_{t+1}^{-\sigma}}{(k_t^0)^{-\sigma}\hat{c}(g_0)^{-\sigma}}(r_{k,t+1}(g_0) + 1 - \delta)\Big| g_0\right] + m\hat{\lambda}_{d,t}^0(g_0)$$

By the previous part of the proof and the monotonicity property of $\hat{\lambda}_d$, for every m the right-hand side of the above equation is strictly decreasing in g_0, provided that σ is small. Since an increase in m increases the right-hand side, we conclude that we associate to higher values of m higher growth rates g_0. □

6 Appendix 2

Figure 1: Growth Rates Selected Asian Countries

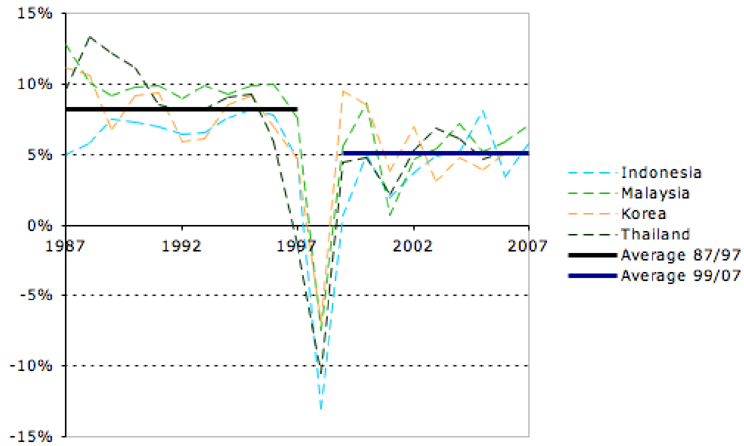

Figure 2: Current Account Selected Asian Countries

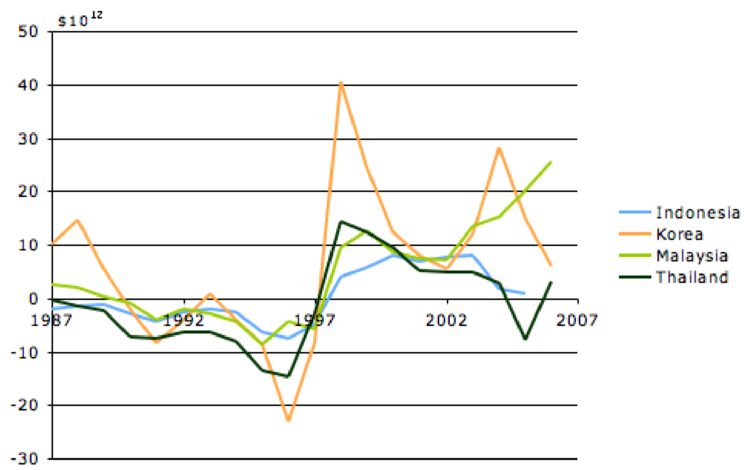

Figure 3: Investment-GDP Ratios Selected Asian Countries

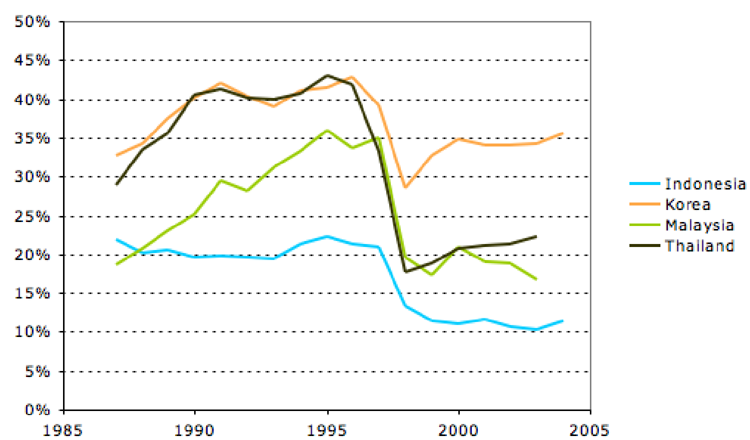

Figure 4: South Korea, Employment Rate

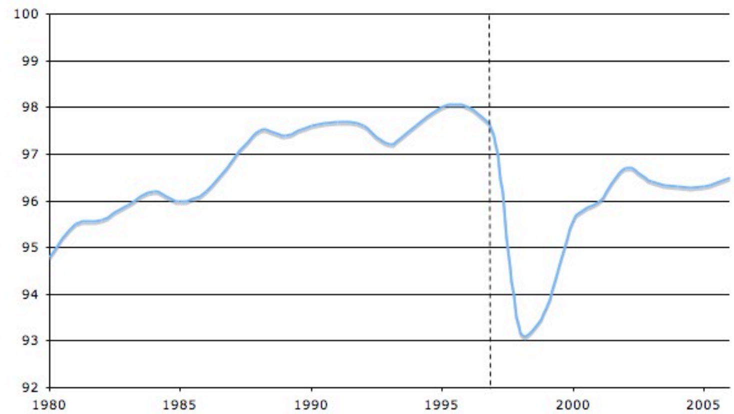

Figure 5: South Korea, Participation Rate

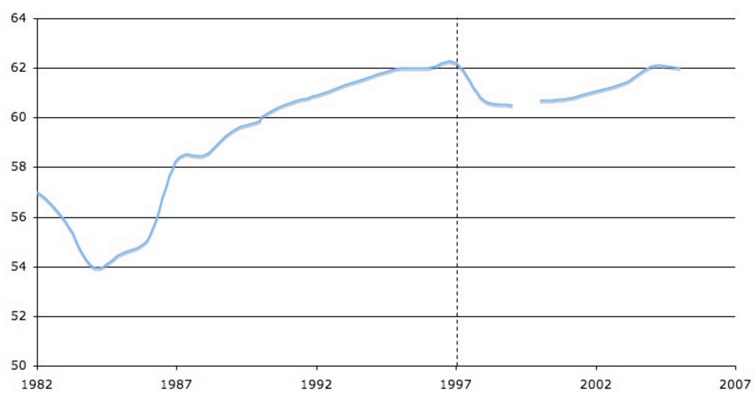

Figure 6: The Sequence of Histories

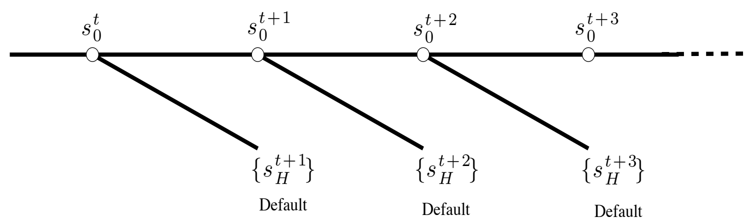

Figure 7: Production function $H(x_H)$

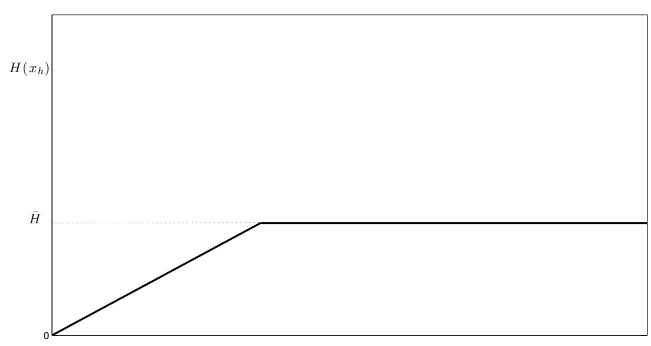

Figure 8: Modified production function $H(x_H)$

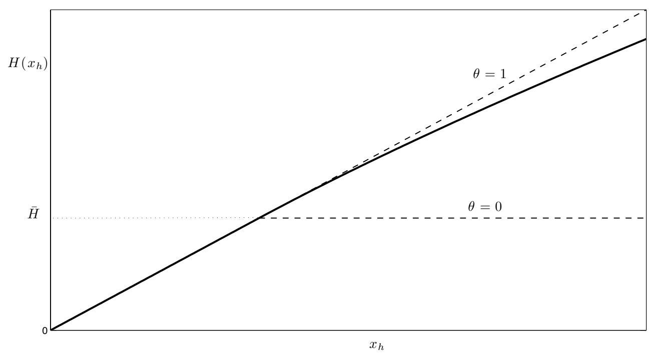

Figure 9: Actual and Simulated Growth Rates

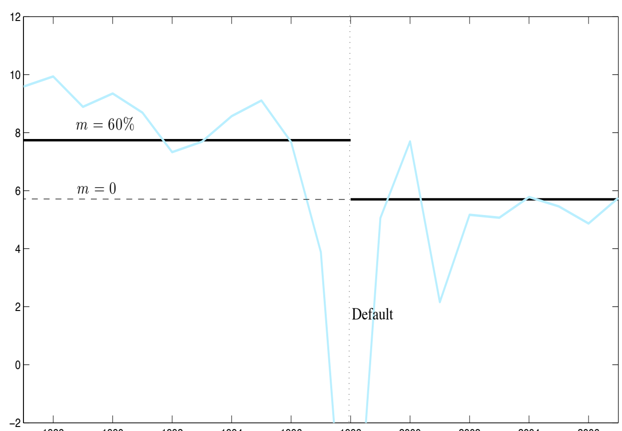

Figure 10: Actual and Simulated Investment/GDP

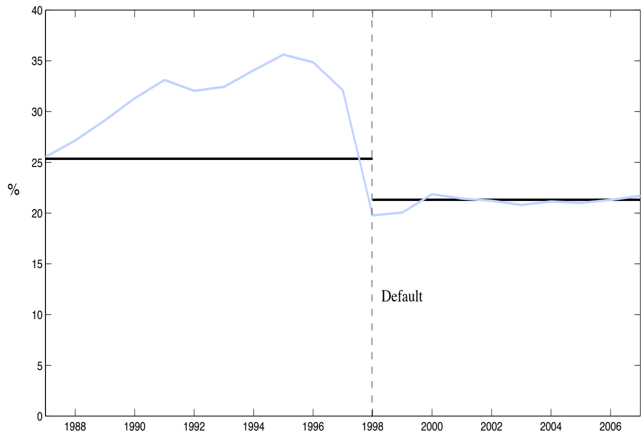

Figure 11: Actual and Simulated Growth Rates