The home bias of the poor: terms of trade effects and portfolios across the wealth distribution

Tobias Broer *

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Abstract

Wealthier people hold a larger part of their savings in risky assets. Using the US Survey of Consumer Finances, I show that wealthier households also have a higher portfolio share of foreign assets than less wealthy people. The empirical evidence suggests that this relative home bias of the poor cannot be explained by fixed participation costs alone, as the portfolio share of foreign assets increases with financial wealth even among participants in foreign asset markets. This paper shows how both biases can arise in a simple 2 country economy with income and portfolio heterogeneity. Poor investors are naturally biased against domestic equity when wages and capital returns are positively correlated, making equity a bad hedge against fluctuations in labour income relative to bonds. Moreover poor investors prefer home to foreign bonds if equilibrium terms of trade movements systematically lead to a fall in the purchasing power of domestic assets in periods of high wages. I show that this is likely to be the case if aggregate shocks at home are more important than abroad. Finally, the model provides a tight link between home bias in the aggregate country portfolio, and the relative home bias of portfolios held by poorer individuals.

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[0]European University Institute, Florence, Italy. Contact: tobias.broer@eui.eu.

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1 Introduction

It is well-documented that household portfolios become more diversified as wealth increases. Campbell (2006) and Guiso et al (2003), for example, show that poor households are less likely to invest in risky assets. A large literature has also documented that aggregate country-portfolios display "home bias" (see Lewis, 1997, for a summary of this literature). But little attention has been devoted to the composition of individual household portfolios between domestic and foreign assets, and the relationship between individual home bias and wealth. I study the US survey of consumer finances (SCF) and show that wealthier investors also seem to invest on average a higher share of their portfolio in foreign assets than investors with lower net wealth.

There are several possible explanations for this bias of poorer investors towards safe and home assets, such as fixed costs of participating in the markets for risky and international assets. I argue that the phenomenon may also result from optimal investment strategies without non-convex costs: equilibrium movements in the relative price of endowments can give poor investors stronger incentives to buy home bonds rather than home equity or foreign bonds. This is because in equilibrium, a positive shock to aggregate domestic output increases the relative price of foreign goods, and thus the price of the domestic consumption basket in terms of domestic goods. When bonds pay off in domestic goods, their real returns are thus inversely related to movements in aggregate output. Therefore, if individual and aggregate endowments are positively correlated, bonds provide a hedge against fluctuations in non-diversifiable individual income risk. Wealthy investors, whose future income is less dependent on endowments, care less about this hedging property than poor investors. Therefore, equilibrium portfolios vary across the wealth distribution and poorer investors tend to have a stronger home bias than rich investors.

The intuition for the results has similarities to Baxter and Jermann (1997) who show that with income from non-marketable human capital, the optimal portfolio of assets consists of two sub-portfolios, one completely diversified, the other designed to hedge against volatility of human capital returns. I show that the hedging portfolio can be dominated by safe domestic assets. And its importance relative to the diversified part of the portfolio declines with increases in total wealth. I consider a two country model with heterogeneous consumers that receive an uncertain amount of a country-specific endowment good every period. I derive analytical portfolio shares by assuming (as Cole and Obstfeld, 1991) that preferences over domestic and foreign goods are unit-elastic and identical across countries and agents. This implies that terms of trade depend only on aggregate country endowments each period.

My paper combines three strands of literature. First, from studies of household finances I take the stylised fact that wealthier individuals have riskier and more diversified portfolios. Using the 2004 wave of the survey of consumer finances, I illustrate how the portfolio share of foreign assets is increasing in investor wealth. Second, I adopt the idea that general equilibrium terms of trade movements can significantly change the optimal portfolios obtained in partial equilibrium (cf. Lewis, 1997, for a survey, or more recently Obstfeld and Rogoff, 2000, van Wincoop and Warnock, 2006, Heathcote and Perri, 2006). Like Baxter and Jermann (1997) I also include non-marketable human capital, but add an idiosyncratic component to its returns. I show that the implications for the composition of individual portfolios across the wealth distribution are consistent with the observed facts. Third, I extend heterogeneous agents models to the open economy.

2 Asset portfolios across the wealth distribution

Wealthier and more educated people are more likely to invest in risky assets. This is well-documented for the US (see for example Campbell, 2006, for a review and an illustration using the 2001 SCF data) and a number of European countries (see Guiso et al, 2003, and Carroll, 2002).

Compared with risky assets in general, or domestic equity, the portfolio shares of non-domestic assets and their determinants are much less documented. It is well-known that average country portfolios have surprisingly low shares of foreign assets - the "home bias in portfolios puzzle". This has been interpreted as a consequence of a more general "local bias" of household portfolios, which overweigh local, regional, and national assets (see e.g. Campbell 2006). But there is very little evidence on the home bias of individual households and its determinants. Campbell et al (2006) conclude for the case of Sweden that international diversification possibilities exist, but are usually exploited only by wealthier individuals, who have a higher share of investments in mutual funds (with an average portfolio share of 25 percent for foreign assets). However, they provide no evidence on direct holdings of foreign assets.

I examine the 2004 wave of the US survey of consumer finances. This survey includes information on the US Dollar value of households' holdings of

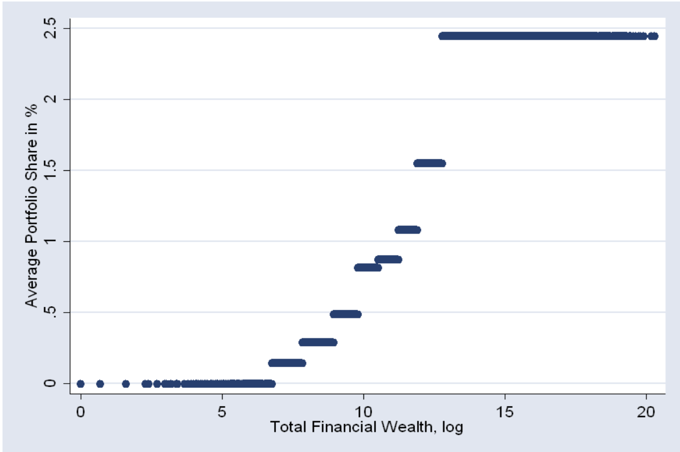

Figure 1: Average portfolio share of total foreign assets for weight-adjusted deciles of the financial wealth distribution

"bonds issued by foreign governments or companies" and "stock in a company headquartered outside of the United States".[1] In order to control for indirect holdings of foreign assets, I include a measure of foreign assets held via mutual funds.[2] The SCF includes data on households' investments in several broad categories of mutual funds. I derive a measure of total foreign asset holdings by summing to individuals' direct investments in foreign equity and bonds the value of their mutual fund shares multiplied by the average portfolio weight of foreign assets in US equity, bond and combination funds.[3] Figure 1 shows the

[1] Question codes x7638 and x7641. An obvious problem of this measure is that it does not refer to non-dollar assets, but to assets issued by foreign issuers, in foreign currency and US dollars.

[2] In other words, I do not look at pension funds here. One reason for this is that individuals' decisions on pension fund investment are taken under a very different set of constraints than other investment decisions. Also, most shares in pension funds are not actively managed as a part of regular portfolio decisions. However, both these arguments do not apply to individual mutual fund investments.

[3] To my knowledge, these average portfolio shares of mutual funds are not readily available from published sources. But Morningstar kindly provided data on portfolio shares of non-US assets for more than 4700 US mutual funds, not including funds of funds. From this I calculated weighted averages for portfolio shares of foreign bonds and equity for the three categories of funds for the year 2003. Since equity (bond) funds seem to often not report zero foreign bond (equity) holdings, I made an adjustment by setting missing observations to zero for all funds that reported portfolio shares summing to at least 99.5 percent. The resulting sample included around 2800 observations for shares of international equity and slightly less for bonds. Using this sample, the average equity mutual fund invested 17.1 percent in foreign shares, while the average bond fund (disregarding funds of government / municipal bonds) invested 3.6 percent abroad. Combination funds invested on average 10.7 percent in non-US

resulting total foreign assets in portfolios of respondents, as a share of total financial wealth, averaged within different deciles of financial wealth. Both the deciles and the averages take account of the fact that the SCF oversamples parts of the population, by applying the weights suggested by Kennickell (1999), and the multiple imputation procedure used for the SCF.[4] The figure shows that the portfolio shares of foreign assets are monotonically increasing in financial wealth.[5]

The measure of total foreign asset holdings used to construct Figure 1 potentially suffers from two kinds of measurement error. First, the responses of households to questions on their asset holdings are accurate only insofar individuals both know the accurate Dollar value of their assets, and truthfully report it. Since I only look at portfolio investments (in other words I disregard directly owned foreign companies), market values of investments are in principle available, and individuals should report their Dollar values at current exchange rates. This may be a strong assumption not only as individuals might not be aware of up-to-date market values for long-term investments or exchange rates, but also, for example, if some of them underreport systematically off-shore investments used to evade tax payments. In the latter case, however, the resulting measurement error would tend to dilute the correlation between wealth and the foreign asset share of the portfolio. So a rejection of the Null hypothesis of no relation would be less likely in the presence of this kind of measurement error.[6]

assets.

[4]To eliminate inconsistencies and missing values, the SCF imputes some values from the other information provided by a Household. However, rather than simply reporting ones best guess for the imputed values, the SCF provides 5 draws per observation from the distribution of the missing values conditional on observables.

[5]The portfolio shares of foreign assets are low relative to those calculated from aggregate US data. But keep in mind that the SCF measure of financial wealth, the denominator or the ratio, includes a large range of assets such as insurance contracts, liquid retirement funds, etc., while the numerator only considers bonds and stocks held directly and via mutual funds. Also, the aggregate shares of foreign assets in the country portfolio cannot directly be read from the graph. The ratios of foreign to total assets of the implied weighted aggregate portfolio are 2.75, 4.08, 3.99 percent for bonds, equities and their total respectively.

[6]To see this, suppose all individuals were to invest x percent of their foreign asset holdings in unreported offshore vehicles. In this case, all portfolio shares would be biased downwards by a factor of $(1-x)/(1-x*SHARE_{true})$, where $SHARE_{true}$ is the true portfolio share of foreign assets. So the ratio of true to reported portfolio shares is most distorted for small portfolios (going to 1-x as $SHARE_{true}$ goes to 0), and declines to 0 as $SHARE_{true} \longrightarrow 1$. The absolute difference between reported and true portfolio shares,however, will be greatest for intermediate portfolios. Thus, as we see foreign asset shares rising from zero to single-digit

A second source of measurement error results from the use of average portfolio shares in the imputation of households' indirect foreign asset holdings via mutual funds. If rich individuals systematically invest in funds with different exposures to foreign assets, this migh distort the observed wealth effect on total foreign assets. But again, this error is likely to dampen the observed relationship between wealth and the portfolio share of foreign assets.[7]

The evidence presented in Figure 1 raises two questions. First, could the rise in average portfolio shares across the wealth distribution merely be due to a higher participation rate of wealthy individuals in the foreign asset market, rather than a rise in individual portfolio shares of participants as they become richer? One factor that could cause such a pattern are fixed costs of entering sophisticated financial markets. These would lead to a non-linear relationship between financial wealth and participation, in the form of a threshold value of assets below which individuals do not hold any foreign assets. Optimal portfolios above the threshold value, however, would not be affected by sunk fixed costs. Thus, any variation in portfolio shares above the threshold value has to be attributed to other factors. A second question is wether financial wealth could simply capture the effects of other important variables, such as education, age, or income. In this case we would expect an analysis that controls for these variables to yield significantly different results.
To answer both of these questions, I perform a more formal econometric analysis. I estimate jointly the probability of participation and the optimal portfolio share of participants with the Heckman (1979) method, conditioning on other variables that were found to be important for portfolio decisions of individuals in previous research. To be precise, I estimate the parameters of the following

percentages in Figure 1, the bias will increase along the wealth distribution.

[7]To see this, suppose all individuals have the same portfolio share of mutual funds, but richer individuals choose funds with a higher (lower) share of foreign assets. Using average mutual fund portfolios introduces measurement error that is negative for rich (poor) individuals, positive for poor (rich) individuals. This biases the wealth effect estimated from observed data towards zero. The bias will be even stronger when richer individuals also have a higher portfolio share of mutual funds. So again, we would be less likely to reject the null of no wealth effect on portfolios in the presence of measurement error, than we would be without it.

2 equation system

$$SHARE = \begin{cases} \alpha + \beta_1 ln(FIN) + \beta_2 ln(INCOME) + \epsilon_1 & \text{if } H > 0 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

with

$$H = a + b_1 AGE + b_2 COL + b_3 FIN_2 + b_4 FIN_3 + b_5 FIN_4 + \epsilon_2 \quad (2)$$

Here H is an indicator variable that captures the probability of participation in foreign asset markets. This probability is a function of age, a dummy variable "COL" that equals 1 when the household head holds a college degree, and a set of dummies FIN_x that capture financial wealth, taking the value 1 when total financial assets of the household fall in the (weight-adjusted) xth quartile. Only when H is above a threshold, normalised to 0, agents participate in foreign asset markets and we observe the variable SHARE, their portfolio share of foreign assets. Conditional on participation the portfolio share is a function of income and financial wealth (FIN). The errors ϵ_1 and ϵ_2 are assumed to follow a joint normal distribution. The equations are estimated jointly with full maximum-likelihood adjusted for sampling weights. Identification is achieved by restricting the effects of financial wealth to be linear in logs in (1), and constant within quartiles in (2), which I take to be a proxy for different possible participation thresholds.[8] Results are reported in table 1, where numbers in italics are standard errors.[9]

The effect of financial wealth is significant (at the 1 percent level) in both equations. Ceteris paribus, individuals in the bottom quartile of the financial wealth distribution are least likely to invest in foreign assets. But after a jump in the likelihood of participation between the first and second quartile, moving further up the wealth distribution has much smaller, and non-monotonous effects. This is in line with a threshold value of assets beyond which a rise in

[8] I also estimated an alternative specification that included income quartiles in the participation equation. While,in the presence of fixed costs of entering foreign asset markets, we would expect financial wealth to determine the participation threshold and not income, current income could act as a proxy for future financial wealth. However, the income quartile dummies turned out to be insignificant, so I excluded them from the final specification.

[9] Again, an additional complication is the use in the SCF of multiple imputations for missing values. To account for this, I estimate the same model for each of the 5 implicates separately and then aggregate the estimation results. For the coefficients and standard errors reported in Table 1, I use the formulae suggested in the SCF codebook (http://www.federalreserve.gov/PUBS/oss/oss2/2004/codebk2004.txt). For the chi^2 value I report a simple average of the following individual values: 73.33, 69.85, 76.36, 71.31, 45.00.

Table 1: Heckman model for participation and portfolio share of foreign assets

Equation (1)					
const	ln(FIN)	ln(INCOME)			
-10.62	1.23	-0.84			
2.82	*0.19*	*0.29*			
Equation (2)					
const	AGE	COL	FIN_2	FIN_3	FIN_4
-2.70	0.000	0.097	1.76	1.22	2.20
0.25	*0.0017*	*0.046*	*0.27*	*0.28*	*0.26*
No of obs	4517	Censored:	3388		
$\chi^2(2)$	67.02				

FIN is the SCF measure of total financial wealth, AGE the age of the household head in years, FIN_x a dummy variable that takes the value 1 when financial wealth falls in the (weight-adjusted) xth quartile of the cumulative distribution, and COL a dummy variable that equals 1 if the head of the household has a college degree. Numbers in italics are standard errors.

wealth does not systematically raise the probability of participation. However, higher financial wealth increases significantly the portfolio share of participants in equation (1), which cannot be attributed to fixed costs. The effect of age on the probability of participation is insignificant, but college graduates have on average an 8 percent higher probability of investing in foreign assets. Finally, for participants the effect of rising income on the portfolio share of foreign assets is negative.

This section has shown that individual portfolio shares of foreign assets increase with financial wealth. There is a significant jump in the probability of participation in foreign asset markets between the first and second financial wealth quartiles, consistent with fixed participation costs. But fixed costs cannot explain the significant positive relationship between portfolio shares and financial wealth for participants. The next section presents a simple model of the international economy, where general equilibrium movements in the relative price of home and foreign goods can lead to the observed pattern of portfolios: poor individuals have a stronger taste for home bonds as in general equilibrium their real payoffs hedge against volatile endowments, which are their dominant source of income.

3 A 2 country heterogeneous agents endowment economy

I consider an economy with 2 countries, home (H) and foreign (F). In each country there is a large number of agents with unit mass. Individual agents are indexed by h, f at home and abroad respectively. They live for two periods, and receive endowments of a country-specific perishable good H or F.
Agents' preferences are described by a von Neumann-Morgenstern utility function that is homogeneous across agents and countries, with constant relative risk aversion over sequences of a Cobb-Douglas aggregate of country-specific goods as for example in Cole and Obstfeld (1991)

$$\mathbb{U}_k = U(c_k)) + \beta E[U(c_k')] \tag{3}$$

$$U(c_k) = \frac{c_k^{1-\gamma} - 1}{1-\gamma} \tag{4}$$

$$c_k = c_{k,H}^{\theta} c_{k,F}^{1-\theta} \tag{5}$$

where $c_{k,I}$ denotes consumption by agent k of good I and $k \in \{h, f\}$. More generally, notation is as follows: Capital letters H,F denote country wide variables or goods, small letters h,f denote individual variables that can vary across agents of country H,F. First subscripts denote agents or countries, second subscripts goods. Second period variables are denoted with a " ' ".

Heterogeneity and uncertainty

Heterogeneity in our economy comes from differences in endowments. More precisely, agents in country K receive individual endowments ϵ_k, ϵ_k' of their specific good in period 1 and 2 respectively. Initial endowments ϵ_k are known at beginning of period 1 before agents choose consumption and portfolios. Income inequality in country K is summarised by the distribution of period 1 endowments across agents Ψ_K^{ϵ}.
ϵ_k', the endowment of individual k in period 2, is the product of two terms: an "individual endowment share" e_k', and a country-wide "aggregate endowment" Y_K'

$$\epsilon_k' = e_k' * Y_K' \tag{6}$$

"Idiosyncratic risk" is given by the probability distribution of e_k', the period 2 endowment shares of individual k, which I denote $\Psi_k^{e'}$. For simplicity I assume that second period endowments are i.i.d. across agents and independent of all aggregate variables. Also I normalise $\int\int e_k' \Psi_k^{e'} \Psi_K^{\epsilon} = 1$ - the sum across all

agents of expected period 2 individual endowments is 1. By independence and the law of large numbers this means the sum of realised endowment shares is always 1 and aggregate period 2 output in country K simply equals Y'_K.[10]
"Aggregate risk" is summarized by the probability distribution of Y'_H and Y'_F, the period 2 aggregate endowments in both countries, which I denote $\Psi^{Y'}_H, \Psi^{Y'}_F$.
Y'_H and Y'_F are independent of each other.
I assume that all period 2 random variables are log-normally distributed: $(\widehat{e'}_h, \widehat{e'}_f, \widehat{Y'}_H, \widehat{Y'}_F)' \sim N(\ (\overline{e'_h}, \overline{e'_f}, \overline{Y'_H}, \overline{Y'_F})',\ \Sigma)$, where a hat denotes natural logarithms $\widehat{z} = ln(z)$ and Σ is a diagonal matrix with diagonal entries $V_{e_h}, V_{e_f}, V_{Y_H}, V_{Y_F}$.

Incomplete asset markets and borrowing constraints
I impose the simplest structure of asset markets that allows me to analyse two kinds of trade-offs in optimal portfolios: the choice between safe and risky assets on the one hand, and between home and foreign assets on the other.
Like Hugget (1993), agents trade domestically in individual "IOUs" that are in zero net supply and denominated in domestic goods. These are "safe" assets in the sense that for 1 unit of H goods invested today, IOUs in H always pay R^b_H units of good H next period (where "b" stands for "bonds"). Equivalently, foreign IOUs pay R^b_F units of F goods.
I look at two cases of 2-asset portfolios, where agents can invest in domestic IOUs, plus either risky shares in a national mutual fund (Case 1), or foreign IOUs (Case 2). Shares are risky in the sense that their payoffs are proportional to the stochastic aggregate endowment. Thus the return on home shares is $R^s_H Y'_H$ per unit of H goods invested, equivalently for F. One obvious implication of the exogenous incompleteness of asset markets is that individual claims to future endowments are non-tradable, and that the resulting risk thus is non-diversifiable.
I denote h's holdings of home and foreign bonds by $a^b_{h,H}$ and $a^b_{h,F}$ respectively, and her holdings of shares by $a^s_{h,H}$ and $a^s_{h,F}$. Asset quantities are denoted in endowment goods of the owner. So if h holds a portfolio $a^b_{h,H}, a^b_{h,F}$, she owns $a^b_{h,H}$ units of H IOUs and $\frac{a^b_{h,F}}{p}$ units of F IOUs. I denote the vector of returns as $\overline{R}$, the vectors of assets held by individuals in H, F as $\overline{a_h}, \overline{a_f}$, and the total value, in terms of their domestic good, of their assets at the end of period 1 as a_h, a_f.
I assume both IOUs and shares have zero default probability. Consistent with this, agents can credibly promise to repay only in units of their income - so

[10] For the derivation of a law of large numbers for continuum economies, see Uhlig (1996).

borrowing contracts are always written in the endowment good of the issuer. This means agents can issue only domestic assets, but invest both at home and abroad. One consequence of the no-default assumption are individual borrowing constraints: agents in country K can only issue IOUs and mutual fund shares up to maximum amounts B^b_K, B^s_K. The borrowing limits play no further role in the discussion as I concentrate on interior portfolios.[11]

Household's problem

A typical home Household h maximises expected lifetime utility by choosing in period 1 consumption and a vector of assets $\overline{a_h}$ subject to her budget constraint, borrowing constraints for domestic assets and the non-negativity of foreign asset holdings, taking as given the relative price of goods this period and the vector of returns $\overline{R}$. h's problem is thus given as:

$$max_{c_h,c'_h,\overline{a_h}} \frac{c_h^{1-\sigma}-1}{1-\sigma} + \beta E_{\lambda'}\{\frac{c_h'^{1-\sigma}-1}{1-\sigma}\} \tag{7}$$

Subject to the constraints

$$\begin{aligned} c_h &= \frac{\epsilon_h - \sum_{i\in\{b,s\}} a^i_{h,H} - \sum_{j\in\{b,s\}} a^j_{h,F}}{p_H} \\ c'_h &= \frac{\epsilon'_h + R^b_H a^b_{h,H} + R^s_H Y'_H a^s_{h,H} + (R^b_F a^b_{h,F} + R^s_F Y'_F a^s_{h,F})\frac{p'}{p}}{p'_H} \\ a^i_{h,H} &\geq B^i_H,\ for\ i \in \{b,s\} \\ a^j_{h,F} &\geq 0,\ for\ j \in \{b,s\} \\ \epsilon'_h &= e'Y'_H \end{aligned}$$

where $p_H = \theta^{-\theta}(1-\theta)^{-(1-\theta)}p^{1-\theta}$ is the home consumption price index. The problem of a typical foreign household is symmetric.

Note that the expectation in (7) is taken across the joint distribution of the random variables $e'_h, e'_f, Y'_H, Y'_F, p'$. Thus, to solve her problem, h needs to know the distribution of the equilibrium relative price next period, as this determines the real value of her assets and endowments. But in equilibrium, p' potentially

[11] The "natural" limit to total borrowing in riskless assets would equal the present discounted value of minimum future income $B_K = b_K \frac{\epsilon'_{K,min}}{R}$, which is the highest amount agents can repay for sure. But with log-normal endowments there is a positive probability of having endowment realisations arbitrarily close to 0, such that this formulation does not lead to a non-zero borrowing limit. The problem can be avoided by introducing a positive non-stochastic minimum endowment level for all agents in a country. This can be chosen such that the resulting natural borrowing limit equals the sum of B^b_K and B^s_K above. As I concentrate on interior portfolios, I prefer the simpler formulation in terms of B^b_K and B^s_K directly.

depends on the demand functions of all individual agents, and thus on the joint distribution of endowments and the distribution of equilibrium asset holdings at the end of period 1.

Note also that in (7), households can be constrained by any combination of the borrowing limits on assets, and in most equilibria there will be some constrained households. In the analysis of portfolios, I concentrate on unconstrained households with interior portfolios. For simplicity I specify the vector of assets in two different ways, allowing either cross-border trade in equity or in IOUs.

4 Competitive equilibrium

This section defines a competitive equilibrium and discusses the properties of the equilibrium terms of trade, before the following section looks at portfolios. An appendix discusses existence and uniqueness.

4.1 Definition of Competitive equilibrium

A competitive equilibrium is

1. A **Consumption allocation**:
   For every agent k, a consumption sequence of both goods for both periods: $c_{k,H}, c_{k,F}, c'_{k,H}, c'_{k,F}$, where $c'_{k,J}$ is a random variable depending on the realisation of period 2 uncertainty.

2. A set of **Portfolios**:
   For every agent k, a vector $\overline{a_k}$ specifying holdings of all assets in the economy at the end of period 1.[12]

3. A **Price system**, consisting of

   - p, p', the relative prices of F goods in terms of H goods in period 1 and 2, where p' is a random variable with distribution $\Psi^{p'}$.
   - $\overline{R}$, the vector of asset returns

such that

1. Agents allocate their funds optimally across goods in period 2 given a particular realisation p'.

[12]Summed across all agents individual quantities imply an **Aggregate consumption allocation** for consumption of good K in country J $C_{J,K} = \int c_{j,K} d\Psi^e_J, C'_{J,K} = \int c'_{j,K} d\Psi^{e'}_J$, as well as a **Country portfolio** of gross and net asset holdings, and a **Net asset position** once net holdings of all assets in a country are summed at period 1 prices.

2. the allocation solves every household's problem (7) in period 1 given a relative price p, a distribution $\Psi^{p'}$, and rates of return $\overline{R}$.

3. markets clear:

   - for goods: $\int c_{h,H} d\Psi^e_H + c_{f,H} d\Psi^e_F = Y_H, \ \int c_{h,F} d\Psi^e_H + c_{f,F} d\Psi^e_F = Y_F$ in both periods
   - and assets: $\int a^i_{h,J} d\Psi^e_H + \int p a^i_{f,J} d\Psi^e_F = 0, \ \forall \ i \in \{b, s\}, \ J \in \{H, F\}$ (each asset is in zero net supply)
   - The distribution of the future relative price $\Psi^{p'}$ is consistent with the joint distribution of random variables e'_h, e'_f, Y'_H, Y'_F, and individual asset holdings at the end of period 1.

It is useful to compare this equilibrium definition to the well-known setup in Krusell and Smith (1998). In their recursive framework with capital accumulation, agents need to know the law of motion for the joint distribution of individual asset holdings and (aggregate and idiosyncratic) shocks, as this determines aggregate savings and thus the returns to capital. Here, despite the endowment setup, the problem is similar: agents real resources next period are a function of relative price p'. But in equilibrium p' depends on individual demands for goods, which are a function of endowments and savings. So to determine the distribution of relative prices $\Psi^{p'}$, agents need to map today's distribution of endowments into a distribution of tomorrow's individual resources, using the joint law of motion of aggregate endowments and individual endowment shares, and the optimal savings of each agent today. As the next section shows the assumption of identical homothetic preferences for all agents in a particular country implies that a country's demand for goods only depends on the relative price and aggregate savings and endowment, not individual savings or endowments. But individual uncertainty and heterogeneity still matter for aggregate savings and net asset positions. Identical preferences across countries ensure that even the aggregate net asset positions do not matter for excess demands, so aggregate endowments tomorrow completely determine aggregate demand for goods and thus market-clearing prices.

4.2 Equilibrium terms of trade movements

An important consequence of the identical homothetic preferences across goods is that the optimal expenditure shares are identical for all agents independent of endowment income. Since assets are in zero net supply, this implies that excess demands are independent of the heterogeneity in the economy. It follows from this that aggregate endowments Y_h, Y_f map directly into a market clearing relative price p independent of the heterogeneity in the economy.[13]

$$p = \frac{1-\theta}{\theta}\frac{Y_H}{Y_F} \quad \forall \Psi_F^{e\prime}, \Psi_H^{e\prime}, \Psi_F^{\epsilon}, \Psi_H^{\epsilon} \tag{13}$$

As a consequence of the unit-elasticity of the consumption basket, for consumers with constant relative risk aversion, these equilibrium price movements yield complete insurance against country-endowment shocks: prices move against endowments to optimally spread risk across countries. So with representative agents, there is no incentive for asset trade (see Cole and Obstfeld (1991)). However, in an environment with heterogeneity in incomes, agents do have incentives to trade assets to smooth consumption and engage in precautionary savings. Moreover, their asset portfolios will generally depend on their period 1 income, which, given the assumption of i.i.d. period 2 endowments, maps directly into lifetime wealth.

A feature of unit-elastic demand for goods closely related to the perfect insurance result is that claims to country-endowments, or national mutual fund shares, must have equal stochastic consumption payoffs in equilibrium

$$\frac{R_H^s Y_H p_H}{p_H'} = R_H^s Y_H'^{\theta} Y_F'^{\theta} \tag{14}$$

$$\frac{R_F^s Y_F p_F}{p_F'} = R_F^s Y_H'^{\theta} Y_F'^{\theta} \tag{15}$$

[13]To see this, denote by $s_h(s_f)$ agents' total expenditure in terms of H (F) goods in any period. Cobb-Douglas preferences imply constant budget shares, so optimality requires

$$c_{h,H} = \theta s_h, \; c_{h,F} = (1-\theta)\frac{1}{p}s_h \tag{8}$$

$$c_{f,H} = \theta p s_f, \; c_{f,F} = (1-\theta)s_f \tag{9}$$

where s_k denotes expenditure of agent k (endowments minus net asset investment) in domestic goods. When agents spend a_h, a_f units of their endowment good on assets, the **excess demand for H goods** in the first period is:

$$\int \theta(\epsilon_h - a_h) d\Psi_H^e + \int \theta p(\epsilon_f - a_f) d\Psi_F^e = Y_H \tag{10}$$

Zero net assets implies that individual asset holdings sum to 0 across countries once expressed in the same currency:

$$\int_{\Omega_H^e} a_h d\Psi_H^e + \int p a_f d\Psi_F^e = 0 \tag{11}$$

So the excess demand function becomes

$$\int \theta\epsilon_h d\Psi_H^e + \int \theta p\epsilon_f d\Psi_F^e = \theta Y_H + \theta p Y_F = Y_H \tag{12}$$

where I set the period 1 relative price to 1 for simplicity. So with international trade in shares agents are always indifferent between home and foreign mutual fund shares. In this sense, the equilibrium portfolio is never unique with international trade in shares. Also, there may be two equilibria with high and low rates of interest in some cases. An appendix discusses conditions for uniqueness and existence of equilibrium.

5 Optimal portfolios

I now derive the optimal asset allocation when agents can trade more than just domestic IOUs. I first look at optimal portfolios when agents can also invest in risky mutual fund shares (domestic or foreign) to see how the portfolio weights of shares evolve across the wealth distribution. Then I perform the same exercise for trade in domestic and foreign IOUs.

Since real payoffs to home and foreign mutual fund shares are always equal, I drop superscripts on returns and asset quantities in this section: returns on home and foreign IOUs are from now on simply denoted as R_H, R_F, those on shares as R_S, and h's corresponding asset holdings as $a_{h,H}, a_{h,F}, a_{h,S}$.

5.1 Portfolio shares of risky assets

Note that agents with low wealth have future income dominated by endowments. Thus they have a stronger preference than rich investors for assets that hedge against the volatility of real endowment income, which after substituting equilibrium price movements is $e'_h Y_H'^{\theta} Y_F'^{1-\theta}$. But share returns have real payoffs of $r_s Y_H'^{\theta} Y_F'^{1-\theta}$ - perfectly correlated with aggregate endowments. Bonds on the other hand have real payoffs of $\frac{R_H}{p'_H} R_H Y_H'^{\theta-1} Y_F'^{1-\theta}$. These decline in aggregate H endowment due to movements in the equilibrium price index p_H and thus have better hedging properties as long as there is volatility in aggregate home endowments. Therefore, a poor investor will typically short equity to invest in bonds. "Wealthy" investors with high asset income on the other hand will always have a diversified portfolio. This is stated in proposition 1.

Proposition 1
When agents can invest in domestic assets and mutual fund shares at home and abroad, the portfolio weight of shares is increasing in total wealth, and strictly so for agents with a diversified portfolio.

Proof of proposition 1

The fact that terms of trade movements equalise the stochastic payoffs of home and foreign mutual fund shares allows me to concentrate on the allocation of assets between home IOUs $a_{h,H}$ and total shares $a_{h,S}$, noting that the portfolio is not unique over home and foreign shares. Agents thus solve their problem (7) with $\overline{a_h} = (a_{h,H}, a_{h,S})$ subject to $a_{h,H} \geq B^b_H, a_{h,S} \geq B^s_H$.

The first order conditions for unconstrained holders of bonds and shares are, respectively

$$c_h^{-\sigma} = \beta R_B E[\frac{c_h'^{-\sigma}}{p'_H}] \quad (16)$$

$$c_h^{-\sigma} = \beta R_S E[Y_H \frac{c_h'^{-\sigma}}{p'_H}] \quad (17)$$

Together they imply an arbitrage condition for interior portfolios

$$E[\ c_h^{-\sigma} \frac{(R_B - R_S Y_H)}{p'_H}\ ] = 0 \quad (18)$$

Given log-normality of the returns, we use the approximation that the sum of log-normally distributed variables is itself log-normally distributed (see for example Van Wincoop and Warnock (2006)), and apply it to <u>real</u> returns in (18). This yields the following approximation of second period marginal utility of consumption

$$log(c_h'^{-\sigma}) = log[(\overline{e_h'}Y_H + a_h)(\frac{\widetilde{\widetilde{e_h'}}Y_H'}{p_H'} + \frac{R_H \widetilde{\widetilde{a_{h,H}}}}{p_H'} + \frac{R_S(\widetilde{\widetilde{a_{h,S}}})Y_H}{p_H'})^{-\sigma}]$$

$$\approx -\sigma log(\overline{e_h'}Y_H + a_h) - \sigma[\widetilde{\widetilde{e_h'}}[\varepsilon_h' + \theta y_H + (1-\theta)y_F] + \widetilde{\widetilde{a_{h,H}}}[r_H - (1-\theta)y_H + (1-\theta)y_F] + (1 - \widetilde{\widetilde{a_{h,H}}})[r_S + \theta y_H + (1-\theta)y_F]$$

where a double tilde denotes ratios with respect to $\overline{e_h'}Y_H + a_h$, total future income evaluated at expected endowment shares, zero growth and zero net returns $r_b, r_s = 0$. r_s is the net return on shares excluding output growth, defined by $R_S Y_H' = (1 + r_S)\frac{Y_H'}{Y_H}$, and $y_H = \widehat{Y_H'} - \widehat{Y_H}, y_F = \widehat{Y_F'} - \widehat{Y_F}$ denote growth rates. ε_h' is the log-deviation from the expected endowment share. Using $E[X] = e^{E[\widetilde{X}] + \frac{1}{2}Var(\widetilde{X})}$, we can write both sides of our arbitrage equation (18) as (exponentials of) linear functions of expectations and variances of three normal random-variables $\widehat{e'}, \widehat{Y_H'}, \widehat{Y_F'}$. Denoting ratios with respect to total assets with a simple tilde, we can solve the resulting expression for $\widetilde{a_{h,S}} = \frac{a_{h,S}}{a_h}$, the share of equities in the portfolio:

$$\begin{aligned} \widetilde{a_{h,S}} &= [\frac{r_S + y_H - r_B}{\sigma V_{Y_H}} + \frac{\frac{1}{2} + (1-\theta)(\sigma - 1)}{\sigma}](1 + \widetilde{e_h'}) - \widetilde{e_h'} \\ &= \widetilde{a_{h,S}}|_{\overline{e_h}=0} + (\widetilde{a_{h,S}}|_{\overline{e_h}=0} - 1)\widetilde{e_h'} \quad (19) \end{aligned}$$

Thus, we can think about the equity portfolio share as the sum of two sub-portfolios: first, a "limiting portfolio" $\widetilde{a_{h,S}}|_{\overline{e_h}=0}$ that would be optimal in the absence of endowment income (or as assets go to infinity and endowment income becomes relatively irrelevant); and a subportfolio that hedges against endowment risk. As the limiting portfolio share is bounded by 1, the term multiplying the $\widetilde{e'_h}$, the ratio of expected endowments to total asset holdings, is negative. Given that expected endowments are equal for all agents, the portfolio share of mutual fund shares is thus increasing in total assets.

To understand the intuition behind Proposition 1, it is worth looking at the determinants of the two sub-portfolio shares in (19) in more detail. The limiting portfolio $\widetilde{a_{h,S}}|_{\overline{e_h}=0}$ for a pure financial investor is standard: it equals a constant, plus the return differential scaled by the risk aversion-weighted variance in home endowments. Portfolios are thus more sensitive to return differentials, the lower risk aversion or the lower the aggregate home volatility that drives a wedge between the risk attributes of home IOUs and risky shares.
The sub-portfolio hedging against endowment risk is proportional to relative expected endowments $\widetilde{e'_h}$. The factor of proportionality $(\widetilde{a_{h,S}}|_{\overline{e_h}=0} - 1)$ is itself a function of the limiting portfolio share of risky assets. This can be interpreted as a "portfolio balance effect": optimal portfolios specify the relative size of different claims to total income. To keep the ratio of an asset to total income constant when expected endowment income rises, its share in the financial portfolio has to increase the stronger the more important its share in the optimal portfolio. The second term in the hedging portfolio $-\widetilde{e'_h}$ can be interpreted as a pure "hedging effect": endowment income and shares are perfect substitutes in the portfolio as they have the same stochastic payoffs.
This result is similar to that obtained by Baxter and Jermann(1997), who show that the optimal portfolio of a representative agent can be split in the sum of a diversified portfolio and one that hedges against endowment risk. In my framework with heterogeneous endowments, given that expected endowment is the same across agents, rich investors, i.e. those with more financial wealth or assets, hold a larger fraction of shares. The monotonicity of the portfolio weight of risky assets also ensures that there is a cut-off value of total assets a_h^* below which agents optimally short the mutual fund shares as much as they can. Thus, the portfolio weight of shares is increasing across the whole support of the wealth distribution, and strictly so for unconstrained agents.

5.2 Portfolio shares of foreign IOUs (Preliminary)

I now look at optimal portfolios in the second case, when agents can, in addition to home IOUs, also invest in foreign IOUs, but not in mutual fund shares. As before, poor agents have stronger incentives to invest in good hedges against the volatility of endowments. And unlike risky mutual fund shares, home and foreign IOUs do not have the same hedging properties. Real payoffs of home IOUs ($R_H Y_H'^{(\theta-1)} Y_F'^{1-\theta}$) are low when home endowment is high relative to foreign endowment (yielding a high home consumption price). And the inverse holds for real returns to foreign IOUs ($R_F Y_H'^{\theta} Y_F'^{-\theta}$). Endowment income ($e_h' Y_H'^{\theta} Y_F'^{1-\theta}$) on the other hand comoves positively with both home endowment (due to the comovement of wages and aggregate output) and foreign endowment (due to lower home consumption prices when aggregate foreign endowment is high). If the volatility of real labour income is dominated by the volatility of home endowments, for example because there is home bias in consumption or because the home endowment is simply more volatile, agents with a higher share of labour income have a stronger preference for home IOUs that hedge against this volatility. "Wealthy" individuals on the other hand, whose future income is dominated by asset returns, have a diversified portfolio. This is stated in proposition 2. The relative properties of home and foreign assets as a hedge against labour income risk turn out to be closely linked to their relative share in the optimal diversified portfolio of an investor with negligible endowment income, and the relative volatility of aggregate endowments.

Proposition 2
With international trade in IOUs and no trade in shares, home bias generally changes across the wealth distribution. Moreover, poorer domestic agents have stronger home bias if any one of the following conditions hold
i) the portfolio share of home bonds in the limiting diversified portfolio is bigger than the ratio of foreign to total aggregate endowment volatility.
ii) the limiting portfolio is perfectly balanced and the volatility of the home aggregate endowment is larger than abroad.
iii) Countries are symmetric, $\sigma > 1$, and there is home bias in consumption.

Proof of proposition 2
Agents now choose their holdings of home and foreign IOUs, i.e. $\overline{a_h} = a_{h,H}, a_{h,F}$, to solve their problem (7) subject to $a_{h,H} \geq B_H^b, a_{h,F} \geq 0$. Again, solutions to h's problem may be interior for none, one or both assets. For interior portfolios,

the first order condition for H IOUs is unchanged

$$c_h^{-\sigma} = \beta R_H E\{\frac{c_h^{-\sigma}}{p'_H}\} \tag{20}$$

For F assets, the first order condition becomes

$$c_h^{-\sigma} = \beta R_F E\{[\frac{p'}{p}]^\theta \frac{c_h'^{-\sigma}}{p'_H}\} \tag{21}$$

Given log-normality of the three random variables e'_h, Y'_H, Y'_F, we can again derive analytically the share of foreign and home bonds $\widetilde{a_{h,F}}, \widetilde{a_{h,F}}$ in the portfolio as a function of $\widetilde{e'_h}$, the ratio of expected endowment to total individual asset holdings. This yields the following expressions for the portfolio share of home and foreign assets

$$\begin{aligned}
\widetilde{a_{h,F}} &= \frac{r_F + y_f - r_H}{\sigma V_{tot}} + \frac{\frac{1}{2} + (1-\theta)(\sigma - 1)}{\sigma}(1 + \widetilde{e'_h}) - \widetilde{e'_h}\frac{V_{y_H}}{V_{tot}} \\
&= \widetilde{a_{h,F}}|_{\overline{e_h}=0} + (\widetilde{a_{h,F}}|_{\overline{e_h}} - \frac{V_{y_H}}{V_{tot}})\widetilde{e'_h} \qquad (22)\\
\widetilde{a_{h,H}} &= \frac{r_H - y_h - r_F}{\sigma V_{tot}} + \frac{\frac{1}{2} + \theta(\sigma - 1)}{\sigma}(1 + \widetilde{e'_h}) - \widetilde{e'_h}\frac{V_{y_F}}{V_{tot}} \\
&= \widetilde{a_{h,H}}|_{\overline{e_h}=0} + (\widetilde{a_{h,H}}|_{\overline{e_h}=0} - \frac{V_{y_F}}{V_{tot}})\widetilde{e'_h} \qquad (23)
\end{aligned}$$

Here V_{tot} is simply the sum of endowment variances at home and abroad. Again, the portfolio shares are the sum of a term reflecting the limiting portfolio absent endowment risk, and a hedging term reflecting the importance of expected endowments relative to asset holdings. The latter is generally non-zero, so portfolios differ across the wealth distribution, as $\widetilde{e'_h}$ falls with agents' total assets. More particularly, there is relative home bias of the poor whenever the sign of the hedging term $(\widetilde{a_{h,H}}|_{\overline{e_h}=0} - \frac{V_{y_F}}{V_{tot}})$ in (23) is negative, which is equivalent to condition i) in Proposition 2. As in Case 1, the hedging term is the sum of the limiting portfolio share and a function of the relative volatility of home and foreign aggregate endowments. There is relative home bias of poorer agents (with a larger $\widetilde{e'_h}$) whenever either home volatility is relatively important, giving home agents incentives to hold home assets for hedging purposes, and / or when the weight of home assets in the limiting portfolio is large, leading to an important "portfolio balance" effect.[14] This effect is likely to support

[14] As in Case 1, this arises because the optimal portfolio specifies the relative importance of all claims to income. To keep relative claims to income from different sources at their optimal levels as expected endowment income rises, the asset shares in the financial portfolio need to rise proportional to their weights in the limiting portfolio.

relative home bias whenever the limiting portfolio share of home bonds is large, i.e. when there is stronger home bias in consumption or a positive return differential between home and foreign assets. If countries are symmetric in the sense of equal expected real returns and volatilities, $\theta > \frac{1}{2}$ (condition iii)) is a sufficient condition for relative home bias. And if the limiting portfolio is balanced ($\widetilde{a_{h,F}} = \widetilde{a_{h,H}} = \frac{1}{2}$), only the relative aggregate volatilities matter, and the condition reduces to $V_{y_H} > V_{y_F}$ (condition ii)).

This section has shown the main results of this paper. In my model, the portfolio share of risky assets declines with wealth, in line with the empirical evidence. Also, we saw that if wealthy investors have a taste for home assets, in the sense of a larger portfolio share in their diversified portfolio, and / or the variance of aggregate endowments at home is larger than abroad, then poor home investors have an even stronger taste for home assets, so home bias declines with wealth.

6 Aggregate and individual Home Bias

So far, this paper has looked at **relative** home bias, or the portfolio share of domestic assets across the wealth distribution. But it turns out that the model creates an immediate link between **relative** home bias of individuals' portfolios on the one hand and **aggregate** home bias of country portfolios on the other. For this, we first need a definition of home "bias", relative to some "unbiased" aggregate portfolio. The literature offers 2 such benchmarks: in Finance, actual portfolios are often compared to an optimal hedge portfolio given the observed payoff structure of assets (see e.g. Lewis 1999). The macroeconomic literature on the other hand is often concerned with showing that home bias is an optimal outcome in general equilibrium. Thus, this literature tends to define home bias relative to a more ad hoc benchmark, for example the portfolio consistent with completely diversified portfolios in all countries, implying portfolio shares equal to the share of a country's assets in world supply (see e.g. Van Wincoop and Warnock (2006)). As I am concerned with the impact of non-diversifiable income risk, I take as a benchmark the optimal portfolio of a financial investor without future endowment income, which I called the limiting portfolio above. Thus I define aggregate home bias of a country portfolio as a portfolio share of home assets that exceeds that in the limiting portfolio of a financial investor.

The following proposition makes a link between aggregate and relative home bias. It uses the difference of portfolio shares in the limiting portfolio $\Delta\widetilde{a_{h,H-F}}|_{\overline{e_h}=0} = \widetilde{a_{h,H}}|_{\overline{e_h}=0} - \widetilde{a_{h,F}}|_{\overline{e_h}=0}$ and the difference in aggregate endowment volatilities $\Delta V_{H-F} = V_{y_H} - V_{y_F}$.

Proposition 3

With international trade in IOUs and no trade in shares, as long as the portfolio balance effect does not more than offset the hedging effect ($sign(\Delta\widetilde{a_{h,H-F}}|_{\overline{e_h}=0}) = sign(\Delta\widetilde{a_{h,H-F}}|_{\overline{e_h}=0} + \Delta V_{H-F})$), there is relative home bias of poorer households if and only if the country portfolio has aggregate home bias.

Proof of proposition 3

The intuition for proposition 3 comes from equation (23): the limiting portfolio share of home assets and the relative variance of aggregate endowments together determine the relative home bias of individuals. Suppose first that the limiting portfolio is completely diversified ($\Delta\widetilde{a_{h,H-F}}|_{\overline{e_h}=0} = 0$). Then a higher variance of home aggregate endowments ($\Delta V_{H-F} > 0$) implies an increasing share of home assets as $\widetilde{e}$ gets larger, i.e. as we move down the wealth distribution and endowment income becomes more important. But an increasing individual share of home assets for poorer agents relative to the limiting portfolio implies an aggregate share of home assets that exceeds that of the limiting portfolio. To conclude the proof, we need to show that the portfolio decisions of constrained agents do not change this intuition, as (23) only holds for unconstrained agents with an interior portfolio. Also, when we allow countries to be asymmetric, this intuition only holds as long as the portfolio balance effect of variation in relative endowment income does not more than offset the hedging effect, as we assume in the condition of Proposition 3.

Only if

According to equation (23), if $sign(\widetilde{a_{h,H}} - \widetilde{a_{h,F}}) = sign(\widetilde{a_{h,H}} - \widetilde{a_{h,F}} + \Delta V)$, the difference in portfolio shares increases as we move down the wealth distribution and the hedging terms in (23) and (22) become more important. In other words, all agents with interior portfolios have less balanced portfolios than the limiting portfolio, although there may be relative home or foreign bias. Now suppose there is relative home bias. According to the condition this requires a larger share of home bonds in the limiting portfolio. So as we move down the wealth distribution the share of foreign assets drops, starting from a value strictly below $\frac{1}{2}$. Thus, there is an interior portfolio corresponding to a value of current wealth a_h*, where the optimal level of foreign assets is zero and that for home assets positive. For all $0 < a_h < a_h*$ the home asset share is 1: the marginal utility from investing in home assets is greater than that for foreign assets. Thus both interior and constraint portfolios have a share of home assets that is bounded below by the share in the limiting portfolio, which implies home bias in the country portfolio on aggregate. So under the condition given in proposition 3, relative home bias implies aggregate home bias.

If

I prove this by showing the contrapositive. Assume that there is relative foreign bias at home, and that the condition in proposition 3 holds. From a similar argument as in the previous paragraph, all agents that are constrained will have non-positive holdings of home assets, as home asset shares for interior portfolios drop faster to zero with declining wealth than foreign asset shares, starting from a value below $\frac{1}{2}$. Thus, all portfolios have foreign bias bounded below by the portfolio share of foreign assets in the limiting portfolio, which is greater then 1/2. So relative foreign bias implies no aggregate home bias. Noting that aggregate home bias is defined with respect to the limiting portfolio, and thus requires at least some variation of portfolios across the wealth distribution, excludes the knife-edge case where the portfolio balance and hedging effect exactly cancel. So aggregate home bias implies relative home bias.

7 Robustness of the results

7.1 International trade in both shares and IOUs

An interesting extension of the above analysis is to look at international trade of both IOUs and shares. In this case, h's problem is to solve (7) with $\overline{a_h} = a_{h,H}, a_{h,F}, a_{h,S}$, subject to $a_{h,H} \geq B_H^b, a_{h,F} \geq 0, a_{h,S} \geq B_H^s$. This case is more complicated as shares partly share the hedging properties of both IOUs: their hedging properties against aggregate volatility at home are equal to those of foreign IOUs, while they hedge in the same way against aggregate foreign volatility as home IOUs. Also, shares have the exact same hedging properties against aggregate volatility as individual endowments. So shares are always substitutes for endowment income, while their substitution properties with home or foreign IOUs in h's portfolio depend on the relative aggregate volatility in the home and foreign country.

Using the same approximation as above, the optimal interior portfolio is now described by the following system of equations

$$\sigma(V_{y_H}+V_{y_F})\widetilde{a_{h,F}}+\sigma V_{y_H}\widetilde{a_{h,S}}=$$
$$\{r_F+y_H-y_F-r_H+(\frac{1}{2}+(1-\theta)(\sigma-1))(V_{y_H}+V_{y_F})\}(1+\widetilde{e'_h})-\widetilde{e'_h}V_{y_H} \quad (24)$$
$$\sigma V_{y_H}\widetilde{a_{h,F}}+\sigma V_{y_H}\widetilde{a_{h,S}}=$$
$$\{r_S+y_H-r_H+(\frac{1}{2}+(1-\theta)(\sigma-1))V_{y_H}\}-\widetilde{e'_h}V_{y_H} \quad (25)$$

where the first equation is derived from the arbitrage condition for foreign vs. home IOUs, the second from that for shares vs. home IOUs. Note that the right-hand sides are comparable expressions of excess returns over home IOUs, parameters and the ratio of expected income to total assets as before. But they now equal a weighted sum of the portfolio shares of foreign bonds and shares - reflecting the substitution relationships among different assets. The system can easily be solved to get

$$\widetilde{a_{h,F}}=[\frac{r_F+y_f-r_S}{\sigma V_{y_f}}+\frac{\frac{1}{2}+(1-\theta)(\sigma-1)}{\sigma}](1+\widetilde{e'_h}) \quad (26)$$
$$\widetilde{a_{h,S}}=(\frac{r_S+y_h-r_H}{\sigma V_{y_h}}+\frac{r_S+y_f-r_F}{\sigma V_{y_f}})(1+\widetilde{e'_h})-\widetilde{e'_h} \quad (27)$$
$$\widetilde{a_{h,H}}=[\frac{r_H-y_h-r_S}{\sigma V_{y_h}}+\frac{\frac{1}{2}+\theta(\sigma-1)}{\sigma}](1+\widetilde{e'_h}) \quad (28)$$

The expressions for $\widetilde{a_{h,H}}$, $\widetilde{a_{h,F}}$ are symmetric. Both the portfolio shares of home and foreign IOUs are increasing in the ratio of expected endowment income to total assets $\widetilde{e'_h}$, i.e. decreasing in total asset holdings or wealth. The share of home and foreign mutual fund shares $\widetilde{a_{h,S}}$ is decreasing with $\widetilde{e'_h}$ and thus increases with wealth under similar conditions as before. In fact, when return differentials are zero, there is perfect substitution between share holdings and claims to endowments.

To determine how the share of total foreign assets evolves with individual wealth, we have to bear in mine that $\widetilde{a_{h,S}}$ comprises both home and foreign equity. In my model their individual portfolio weights $\widetilde{a_{h,H,S}}, \widetilde{a_{h,F,S}}$ are not unique, as their payoff distribution is exactly equal. Introducing the ratio of foreign shares to total shares as a parameter, $\frac{a_{h,F,S}}{a_{h,S}}=q$, the total portfolio share of foreign assets is $\widetilde{a_{h,F}}+q\widetilde{a_{h,S}}$. For the completely symmetric case with $q\geq 1/2$, the result of proposition 2 goes through unchanged, so home bias in consumption, $\theta>1/2$, is a sufficient condition for declining relative home bias in portfolios.

7.2 Structure of uncertainty

7.2.1 Idiosyncratic risk

Above I assumed that idiosyncratic risk is i.i.d. across agents, implying that having high income today does not have any implications for my expected future income. It is interesting to examine the consequences of relaxing this assumption. Note first that the variance of future endowments is not a direct argument of the arbitrage equations. Nevertheless, it determines precautionary savings and thus the size of the portfolio. Second, with i.i.d. endowments, future expected income is the same for all agents. So higher current income translates to higher assets and a lower ratio of expected future income to the sum of total claims $\widetilde{e'_h}$. This yielded the link between current wealth and portfolio shares of propositions 1 and 2. But of course, if we distinguish agents directly by their $\widetilde{e'_h}$, all results of the previous section still hold true. Thus, independently of the structure of uncertainty, agents with a higher ratio of expected endowment income to total claims will have a higher share of domestic and safe assets.

7.2.2 Relaxing log-normality

Without assuming log-normality, there is no closed-form solution for portfolio shares. But for the case of trade in shares, one can still characterise portfolios by totally differentiating (18) and solving for the slope of the IOU portfolio-share as a function of total assets $\frac{\delta \widetilde{a_{h,H}}}{\delta a_h}$. The slope is zero only asymptotically for high total assets. Thus portfolios differ across the wealth distribution and an asymptotic portfolio exists. Also, for most equilibria the share of bonds decreases with total assets.[15]

For international trade in IOUs the general characterisation of portfolios is more difficult, although it is easy to show that when $\theta \neq \frac{1}{2}$, portfolios differ across

[15]The expression is equal to

$$\frac{\delta \widetilde{a_{h,H}}}{\delta a_h} = -\frac{cov(c_h^{-\sigma} \frac{(R_B - R_S Y_H)}{p'_H}, \frac{\widetilde{a_{h,H}} R_B + (1-\widetilde{a_{h,H}}) R_S Y_H}{\epsilon_h + a_h [\widetilde{a_{h,H}} R_B + (1-\widetilde{a_{h,H}}) R_S Y_H]})}{cov(c_h^{-\sigma} \frac{(R_B - R_S Y_H)}{p'_H}, a_h (R_B - R_S Y_H))} = \tag{29}$$

$$-\frac{cov(MU\ fr\ B\ -\ MU\ fr.\ Sh,\ asset\ share\ of\ future\ income)}{cov(MU\ fr\ B\ -\ MU\ fr\ Sh,\ return\ fr\ B\ -\ return\ fr\ Sh\ (in\ H\ goods))} \tag{30}$$

individual wealth levels. [16]

8 Conclusion

In this paper I have shown that, according to the Survey of Consumer Finances, wealthier US Households invest a higher share of their portfolio both in risky and international assets. This result continues to hold when I take account of the fact that poorer households are less likely to participate in more sophisticated financial markets.

I constructed a 2 country model that can account for this finding. Agents in the model receive stochastic endowments of a country-specific tradable good and I assume that there is idiosyncratic and country-specific endowment shocks. Agents are prevented from access to a complete set of asset markets but can trade in riskless assets and/or in equity. Assuming log-normal returns, I derived asset portfolios under alternative assumptions regarding the structure of asset markets but maintaining the assumption of no insurance against idiosyncratic risk.

In this model, terms of trade movements imply that poorer households can partly insure against income volatility by holding domestic or foreign bonds. Wealthier investors, whose income share of endowments is less important, care less about this hedging property than poor investors and therefore hold a more diversified portfolio. I showed that the condition for home bias in individual portfolios to fall with rising wealth is closely linked to the relative shares of assets in an optimal diversified portfolio for an investor with negligible endowment income. In this sense, if wealthy investors have a preference for home assets, poor investors have an even stronger taste for these, so home bias falls with wealth.

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[16] The expression of the slope of $\widetilde{a_H}$, the share of home bonds in a diversified portfolio is equal to

$$\frac{\delta \widetilde{a_H}}{\delta a_h} = -\frac{cov(c_h^{-\sigma}\frac{(R_H - \frac{p'}{p}R_F)}{p'_H}, \frac{\widetilde{a_H}R_H + (1-\widetilde{a_H})\frac{p'}{p}R_F}{\epsilon_h + a_h[\widetilde{a_H}R_H + (1-\widetilde{a_H})\frac{p'}{p}R_F]})}{cov(c_h^{-\sigma}\frac{(R_H - \frac{p'}{p}R_F)}{p'_H}, \frac{a_h(R_H - \frac{p'}{p}R_F)}{\epsilon_h + a_h[\widetilde{a_H}R_H + (1-\widetilde{a_H})\frac{p'}{p}R_F]})} = \tag{31}$$

$$-\frac{cov(MU\ fr\ H\ bond\ -\ MU\ fr.\ F\ bond,\ asset\ share\ of\ future\ income)}{cov(MU\ fr\ H\ bond\ -\ MU\ fr.\ F\ bond,\ return\ differential\ in\ H\ goods)} \tag{32}$$

Again, the evolution of the share of home bonds in the portfolio depends on the covariance of three functions of the random variables e'_h, Y'_H, Y'_F. The share of asset income in total income is again non-stochastic for large asset income. So there is a limit portfolio for $a_h \longrightarrow \infty$. However, the intuition on when this expression is positive / negative is more difficult.

With regards to policy this study implies that the welfare loss from poorer households' non-participation in sophisticated financial markets may be less important than thought. In future research it would be interesting if this result also holds in different environments. Particularly, one could try to relax the assumptions of unit-elastic preferences, or the independence of aggregate endowments at home and abroad. And one could explore how the model deals with shocks to demand, rather than the supply shocks to endowments this study has looked at.

9 References

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10 Appendix: Existence and uniqueness of equilibrium

In section 4, I showed that the equilibrium relative price of goods is independent of heterogeneity and the allocation of assets. As long as agents have some preference for both goods ($0 < \theta < 1$), (13) thus describes a non-empty, single-valued mapping from the two-dimensional space of aggregate endowments into a market-clearing price. In other words a market-clearing price of goods always exists and is unique for any combination of Y_H, Y_F.

The excess demands for assets are the sum of the quantities solving (7), integrated across the distribution of unconstrained agents in both countries, plus maximum borrowing multiplied by the measure of constrained agents. For example, for Home IOUs, remembering that these can only be issued by Home agents and that asset quantities are denoted in terms of domestic goods for home and foreign agents, we get

$$a_H = \int a_{h,H} d\Psi_H^{\epsilon} + p \int a_{f,H} d\Psi_F^{\epsilon} \tag{33}$$

$$= B_H^b \Psi_H^{\epsilon}(-\infty, \epsilon_h *) + \int_{e_h *}^{\infty} a_{h,H} d\Psi_H^{\epsilon} + p \int_0^{\infty} a_{f,H} d\Psi_F^{\epsilon} \tag{34}$$

where $e*$ denotes the level of current home endowment share that solves the first order condition for borrowing at the maximum level B_H^b.

Under financial autarky, existence of an equilibrium price vector $R = (R_H^b, R_H^s)$ is easy to prove by a fixed point argument. Local uniqueness of both consumption allocation and portfolios can also be shown.

However, global uniqueness is more difficult to prove as individual asset demands are not necessarily monotone in relative returns. Two special cases where the equilibrium can be shown to be globally unique are when $b_i = 0$, $i \in \{b, s\}$ (only domestic trade in either bonds, or shares), and either $\sigma \leq 1$ (substitution effects dominate income effects) or $b_j = \infty$ (unconstrained issuance of assets). This is because with one asset only, total excess demand shows no inter-asset substitution effects. Then, for $\sigma < 1$, all individual asset demands, and therefore total excess demand for assets, are monotone in returns as the substitution effect dominates. For $\sigma > 1$ savers may have decreasing asset demand (as the income effect dominates). But borrowers' asset demand is always increasing in returns, with an elasticity higher than that of savers at optimal borrowing levels as long as everybody faces the same period 2 uncertainty. So if all borrowers are unconstrained the total excess demand is again upward sloping in returns, and the equilibrium globally unique. However, even with only one asset, when

a lot of borrowers are constrained, there may be multiple equilibria, as the non-monotonous asset demands of savers can dominate total excess demand.
With more than 1 asset, possibly traded across countries, the equilibrium is not generally globally unique. But conditions for global uniqueness can be derived for example by imposing the gross substitution property on the system of individuals' arbitrage equations. For the analysis here this is not a problem, however, as I only look at interior portfolios, given an equilibrium vector of returns $\overline{R}$. I do not solve for the equilibrium explicitly, which will be a function of the particular specification of distributions and borrowing constraints in both countries.