Moral Hazard and Efficiency in General Equilibrium with Anonymous Trading

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Abstract

How efficient is the market system in environments with private information? A "folk theorem," originating among others in the work of Stiglitz, maintains that competitive equilibria are always or "generically" inefficient, unless contracts directly specify consumption levels as in the work by Prescott and Townsend (thus bypassing trading in anonymous markets). This paper critically reevaluates these claims in the context of a general equilibrium economy with moral hazard. We first formalize this folk theorem. Firms offer contracts to workers who choose an effort level that is private information and that affects worker productivity. To clarify the importance of trading in anonymous markets, we introduce a *monitoring partition*, such that employment contracts can specify expenditures over subsets in the partition, but cannot regulate how this expenditure is subdivided among the commodities within a subset. We say that preferences are *nonseparable* when the marginal rate of substitution across commodities within a subset in the partition depends on the effort level. We prove that the equilibrium is always inefficient when a competitive equilibrium allocation involves less than full insurance and preferences are nonseparable.

This result appears to support the conclusion of the above-mentioned folk theorem. Nevertheless, our two main results highlight its limitations. First, most common preference structures do not satisfy the separability condition. We show that when there is *partial separability* in preferences, competitive equilibria with moral hazard are constrained optimal, in the sense that a social planner who can regulate and monitor all consumption levels cannot improve over competitive allocations. Second, we endogenize the monitoring partition by allowing firms to pay a cost in order to observe (specify) expenditures over finer partitions of commodities. We prove that when the costs of monitoring are sufficiently small, the competitive equilibrium is approximately constrained optimal, despite the cost of monitoring that it incurs (relative to the social planner who does not incur them). These results imply that considerable care is necessary in invoking the folk theorem about the inefficiency of competitive equilibria with private information.

Still Preliminary. Comments Welcome.

1 Introduction

A central question for economic theory is the efficiency of competitive markets. In economies with complete markets, this question is conclusively answered by the celebrated First and Second Welfare Theorems, which show that, under some regularity conditions, competitive equilibria are Pareto optimal and every Pareto optimal allocation can be decentralized as a competitive equilibrium. Nevertheless, the complete market benchmark does not cover many empirically-relevant economies where missing markets are ubiquitous. Arguably the most important reason for missing markets in practice is *private information*. Individual agents know more about their preferences, risks and actions than the market can observe. Despite a sizable literature on this topic, efficiency properties of economies with private information are not yet fully understood. In this paper, we investigate the efficiency of competitive equilibria in a subclass of economies with private information, those with *moral hazard*, where individuals take privately-observed actions affecting their endowments (and/or production).

One approach to the study of efficiency in moral hazard economies has been pioneered by Prescott and Townsend (1984a, 1984b). Prescott and Townsend propose the important idea of considering insurance contracts as commodities that should also be priced in equilibrium. Prescott and Townsend show that competitive equilibria with moral hazard are (constrained) Pareto optimal under two key assumptions: *exclusivity* and *full monitoring.* The first implies that individuals can sign exclusive contracts and is a good starting point for the study of employment contracts.[1] We focus on exclusive contracts throughout the paper. The second assumption, full monitoring, is more problematic. Under full monitoring, contracts specify complete consumption bundles for individuals in different states of nature. This essentially implies that firms or some other outside agency can fully monitor individual consumptions. This assumption is not only unrealistic but also goes against the spirit of "competitive markets". Competitive markets should allow *anonymous trading*, so that individuals are able to buy at least a subset of commodities in anonymous markets without a central agency keeping track of their exact transactions.[2]

[1] Exclusivity may be a less satisfactory assumption for insurance contracts, in particular, when informal insurance is also possible; see, e.g., Arnott and Stiglitz (1991) and Bisin and Guaitoli (2003). Real-world insurance contracts or debt contracts often explicitly regulate what other contracts individuals can sign for the same risks or pledging the revenues of the same business.

[2] Prescott and Townsend also introduce contracts that explicitly allow for randomization over consumption bundles. Although this feature of their work is often emphasized, contracts with randomization are important mainly for technical reasons. Whether we allow for randomization has no qualitative effect on our main results. In this paper, we simplify the notation by excluding randomization. Analogous results with randomization to those stated here are provided in Appendix C, which is available upon request.

A systematic analysis of the structure and efficiency of competitive equilibria with anonymous trading is not available, but a series of papers by Stiglitz and coauthors, most notably, Greenwald and Stiglitz (1986), and also Arnott and Stiglitz (1988, 1991), claim that competitive equilibria under these circumstances are always or "generically" Pareto suboptimal. These claims are supported by local analysis of first-order conditions, though without a rigorous proof that this type of local analysis is valid.[3] Hence one may say that the inefficiency of competitive equilibria with anonymous trading has emerged as a *folk theorem*. This folk theorem is not only of theoretical interest but has been very influential in applied work and is often the basis of arguments for government intervention in insurance, labor and credit markets.

In this paper, we consider a general equilibrium environment where the structure and efficiency of competitive equilibria with anonymous trading can be studied. The economy consists of a large number of firms and risk-averse individuals. Individuals accept employment contracts from firms and choose an effort level, which determines the probability distribution over a vector of production. Individual effort is private information. Commodities in this economy are partitioned, such that expenditures over subsets in a given *monitoring partition* of commodities are observable (for example, how much an individual spends on vacation can be determined, but not how this spending is distributed across different activities in the vacation resort). Employment contracts specify payments to workers and expenditure levels over the subsets in the monitoring partitions as a function of the realization of the state of nature. The Prescott-Townsend economy is a special case where each subset in the monitoring partition is a singleton.[4] After all uncertainty is resolved (the underlying states of the world are realized), individuals allocate the contractually-specified expenditures within the subsets in the partition at given market prices.

We establish the existence of a competitive equilibrium and an indirect maximization problem that characterizes equilibrium allocations (Theorem 1 and Proposition 1). We then formalize the above-mentioned folk theorem. Let us say that there is *no full insurance* at an equilibrium if the marginal rate of substitution of some good between the states is not one. Let us say that preferences are *nonseparable*, if there is a subset in the monitoring partition

[3] This is particularly concerning for two reasons. First and more importantly, this local analysis assumes that a range of Lagrange multipliers are strictly positive, though there is no mathematical or economic reason for them to be so. One of our main results will establish the (constrained) optimality of competitive equilibria, thus invalidating this line of analysis. Second, the local analysis makes use of differentiability assumptions and the first-order approach, which do not generally apply in these environments (see Grossman and Hart, 1983, Rogerson, 1985, Jewitt, 1988 on the first-order approach).

[4] Another special case is one in which the partition consists of a number of singleton elements, which correspond to "monitored goods," and the remainder, which comprises "nonmonitored goods".

such that the marginal rate of substitution between the goods in the subset change if the effort level is modified. Conversely, we say that preferences are *partially separable* when there exists no such subset.[5] Our first main result (contained in Theorems 2 and 3) shows that when preferences are nonseparable and there is no full insurance at an equilibrium, then this equilibrium is constrained suboptimal (inefficient), in the sense that a social planner who is constrained by the same moral hazard problems (but who is allowed to monitor expenditures on all goods) can improve over the equilibrium allocation. Note that this theorem is silent on whether a social planner who is also constrained by the same monitoring technology can implement such a Pareto improvement, and we show that this is not necessarily the case.

While Theorems 2 and 3 seem to give some support to the folk theorem, the rest of our results shed doubt on its general validity and applicability. Our second main result, Theorem 4, shows that competitive equilibria are constrained Pareto optimal when preferences are partially separable. This is a remarkable result for two reasons. First, most preferences used in applied work satisfy this partial separability condition. Second, it establishes that the equilibrium is constrained optimal relative to a very strong notion in which the social planner has access to more instruments than the market (she can monitor and specify expenditures for all goods, whereas contracts can only do so for goods within a subset in the partition). This result suggests that, at least in most of environments considered in applied work, the inefficiencies emphasized by the folk theorem do not arise.

There is at least one sense in which Theorem 4 is incomplete. For different monitoring partitions, the same economy may have preferences that are nonseparable or partially separable and thus exhibit different optimality properties. This raises the question of how the monitoring partition is determined in practice. We believe that the most natural approach to this problem is to allow firms to choose the monitoring partition for their employees as part of the employment contract (by incurring a cost for choosing finer partitions). For example, firms can bring certain activities, such as lunch and part of leisure activities, into the boundaries of the firm as a way of monitoring them. Our final main result (contained in Theorems 6 and 7), investigates the conditions under which endogenous monitoring will create another force towards efficiency. It establishes that when monitoring costs are sufficiently small, a competitive equilibrium with endogenous monitoring is approximately constrained optimal. In particular, for any $\varepsilon > 0$, there exists a set of monitoring costs that are positive but sufficiently small such that the gap between welfare under the social planner's allocation and the corresponding competitive

[5] This is much weaker than "separability" since a complex interaction structure between commodities in different subsets of the partition is allowed.

equilibrium is no more than ε. This is despite the fact that the competitive equilibrium incurs the monitoring costs whereas the benchmark socially planned economy still assumes that the planner can monitor expenditures and consumption of all goods at zero costs. Naturally, Theorems 6 and 7 are silent on the optimality of equilibria when monitoring costs are large. In this case, the right comparison would be between a competitive equilibrium with monitoring and a socially planned economy where the planner also has to incur the same monitoring costs. We show by means of an example that in this case the competitive economy may have too much or too little monitoring relative to the social planner's allocation.

Overall, although Theorems 4, 6, and 7 do not imply that competitive equilibria are always efficient in private information economies, they delineate a range of benchmark situations in which equilibria have very strong optimality properties. They also disprove the very strong claims of suboptimality of competitive equilibria and suggest considerable caution in appealing to the above-mentioned folk theorem.

As the above discussion clarifies, our paper is related to a number of literatures. The relationship of our paper to Prescott and Townsend (1984a,1984b) and to Greenwald and Stiglitz (1986) has already been discussed. Another set of closely related papers are by Geanakoplos and Polemarchakis (1986, 2004) and Citanna, Kajii and Villanacci (1998), which establish the generic inefficiency of competitive equilibria in economies with (exogenously-given) incomplete markets. Our work extends the Geanakoplos-Polemarchakis results to environments with *endogenously incomplete markets* (because of moral hazard), but also highlights that in such environments constrained optimality may result if the appropriate subsets of goods are monitored or if monitoring is endogenous and not too costly. The same issues arise in other economies with price externalities due to endogenously incomplete markets,.for example, Kehoe and Levine (1993) provide results similar to ours for an economy with participation constraints. Our paper is also related to Citanna and Villanacci (2000), which establishes generic inefficiency of equilibria for a moral hazard economy with exclusive contractual relationships in which the principal has all the bargaining power. Our work shows that their inefficiency result crucially depends on the assumption that the principal has the bargaining power. Under the separability assumptions they make for the preferences, the equilibrium is efficient in the polar opposite case in which the agent has all the bargaining power, i.e. when insurance contracts are exclusive and the insurance market is competitive.

In addition to these works, the paper most closely related to the first part of our paper is Lisboa (2001), which establishes the Pareto optimality of competitive equilibria in the context of an economy with moral hazard and fully separable utility. A number of key differences between our work and Lisboa are worth emphasizing. First, Lisboa considers a special case of the model studied here, where utility functions are fully separable across all goods and effort and there is no monitoring of consumption in any subset of goods. Second, our analysis endogenizes monitoring and establishes how approximate constrained efficiency emerges in such environments. Finally, Lisboa's analysis relies on the first-order approach applying everywhere, which is restrictive and not used in our analysis.

Our analysis of endogenous monitoring relates to a number of works in the theory of the firm and in general equilibrium theory. First, the idea that firms decide which activities (consumption choices) to bring into their own boundaries and which to leave to anonymous markets is analogous to Coase's (1937) conception of the interaction between firms and markets. Second, our approach also relates to Holmstrom and Milgrom (1994), who construct a partial equilibrium multi-tasking model in which different sets of activities are brought into the firm. Holmstrom and Milgrom neither discuss the general equilibrium interactions, nor the need to bring certain activities into the firm in order to prevent anonymous trading undoing incentives, nor the implications for (constrained) optimality of equilibria. Finally, an interesting recent paper by Zame (2007) develops a flexible framework for the analysis of firms in general equilibrium, but does not focus on issues of anonymous trading and endogenous monitoring.

There is also a large literature on various different aspects of moral hazard in general equilibrium. Bennardo and Chiappori (2003) and Gottardi and Jerez (2006) discuss the problems that arise in general equilibrium economies with moral hazard because of potential non-transferability of utility. They show how Bertrand competition might lead to equilibria with positive profits. This is an issue that also arises in our model and we provide sufficient conditions (that are not very restrictive) for Bertrand competition to lead to zero profits. Bisin, Geanakoplos, Gottardi, Minelli and Polemarchakis (2007) consider an alternative approach to moral hazard in general equilibrium, where, in contrast to our setup, contracts are not necessarily individualized. This introduces natural externalities across the actions of different individuals signing the same type of contract.[6]

Finally, some of the same issues we emphasize in the context of general equilibrium also arise in the public finance and mechanism design literatures. See, for example, Atkeson and Lucas (1992), Hammond (1987), Allen (1985), Guesnerie (1998), Cole and Kocherlakota (2001), Werning (2001), Kocherlakota (2004), Golosov and Tsyvinski (2006), and Doepke and Townsend (2006). None of these studies derive results similar to our main theorems in this

[6]Also related are recent papers considering adverse selection in general equilibrium, for example, Bisin and Gottardi (1999, 2006) and Jerez (2003).

paper.

The rest of the paper is organized as follows. Section 2 presents the environment. Section 3 defines a competitive equilibrium and provides a characterization and an existence result for equilibria. Section 4 contains the main results for the first part of the paper. It introduces the notion of constrained optimality and provides sufficient conditions for constrained suboptimality and constrained optimality. Section 5 introduces the more general environment with endogenous monitoring and establishes that competitive equilibria are approximately optimal when monitoring costs are sufficiently small. Section 6 concludes. Appendix A contains all the proofs and Appendix B provides a number of additional technical results.

2 Environment

2.1 Preferences

We consider a static production economy with a finite set of goods denoted by G and a finite set of (individual-specific) states of nature denoted by S. We use $g \in G$ and $s \in S$ to index goods and states, and use $|G|$ and $|S|$ to denote the cardinality of these sets. There is a continuum of individual workers, denoted by $\mathcal{N}$, with measure normalized to 1. To simplify the analysis and the exposition, we assume that all workers have identical utility and identical production technology. In particular, each worker chooses an effort level $e \in E$, where $E = \left\{e_1, .., e_{|E|}\right\}$ is a finite subset of a Euclidean space. The effort choice of the worker induces a probability distribution over an endowment (production) vector $y \in \mathbb{R}_+^{|G|}$. We represent this probability distribution by the function q, whereby $q_s(e)$ is the probability of state $s \in S$ for the worker in question when she exerts effort e (naturally with $\sum_{s \in S} q_s(e) = 1$ for all $e \in E$), so that $q_s : E \to [0, 1]$ designates the probability of state s as a function of effort $e \in E$. Each state $s \in S$ is, in turn, associated with the production vector $y_s \in \mathbb{R}_+^{|G|}$. We assume throughout that the realization of states in S (conditional on effort) is independent across individuals and thus with a law of large numbers type argument there is no aggregate uncertainty.

We assume that each worker has VNM preferences over consumption of goods and effort choice represented by

$$U(x, e) = \sum_{s \in S} q_s(e)\, u(x_s, e),$$

where $x_s \equiv \left(x_s^1, ..., x_s^{|G|}\right) \in \mathbb{R}_+^{|G|}$ is the vector of consumption in state s and u denotes the state utility function. Throughout, we use the notation $x_s \equiv (x_s^g)|_{g \in G}$ to designate vectors, and $x = (x_s^g)|_{s \in S, g \in G}$ to designate matrices. We make the following standard assumption on the utility functions:

Assumption A1 For any $s \in S$, the utility function u is continuous in e, continuously differentiable in x, strictly increasing in x_s^g for each $g \in G$, strictly concave in x and satisfies $\lim_{x_s^g \to 0} u(x, e) = -\infty$.

Assumption A2 The function $q_s : E \to [0,1]$ is a probability function, i.e. $\sum_{s \in S} q_s(e) = 1$ for each $e \in E$.

The requirements in these assumptions are standard, perhaps with the exception of the Inada-type assumption that $\lim_{x_s^g \to 0} u(x_s, e) = -\infty$ for each $e \in E$ and $g \in G$, which is adopted to simplify the exposition by ruling out corner solutions.

Throughout, the effort choice of the worker is her private information, so there is a *moral hazard problem* and employment contracts cannot be conditioned on effort choices. The realized production vector is publicly observable and employment contracts can condition on these realizations. Motivated by the discussion in the introduction, we consider a partition $\mathcal{G} = \left\{G_1, .., G_{|\mathcal{G}|}\right\}$ of the set of goods G, such that the employment contracts can specify the worker's expenditure $w_s^{\{G_m\}} \in \mathbb{R}_+$ on each set $G_m \in \mathcal{G}$ as a function of her production vector. The goods in G_m are traded in spot markets that operate after all production vectors are realized. We take $\mathcal{G}$ as given in this section and we endogenize it in Section 5. We use $w^{\mathcal{G}} = \left\{w_s^{\{G_m\}}\right\}_{s \in S, G_m \in \mathcal{G}}$ to denote the matrix where each element denotes the individual's expenditure on a subset of commodities in the monitoring partition at a given state. We denote the vector of prices by $p \in \mathbb{R}_+^{|G|}$. We choose good 1 as the numeraire, i.e. $p^1 = 1$. For any subset of commodities $G' \subset G$, we denote by $p^{G'}$ the corresponding price sub-vector, and by $x^{G'}$ the corresponding consumption sub-matrix.

There exists a set of price taking firms that can produce absent the moral hazard problem. By Theorem 5.4 in Acemoglu (2007), this is equivalent to assuming that there is a representative firm that chooses an allocation $z \in Z$ to maximize profits, where $Z \subset \mathbb{R}^{|G|}$ denotes the aggregate production possibilities set. We require Z to satisfy the following standard assumption:

Assumption A3 $Z \subset \mathbb{R}^{|G|}$ is a closed and convex cone with vertex $\{0\}$ that also satisfies $Z \cap \mathbb{R}_+^{|G|} = \{0\}$.

Given prices p, the representative firm solves

$$Z^*(p) \equiv \arg\max_{z \in Z} zp.$$

We denote $\pi^Z(p)$ as the equilibrium profits of the representative firm, and we assume that $\pi^Z(p)$ is distributed equally among the individuals. Two special cases for the production possibilities set Z are worth noting. First, $Z = \{0\}$ captures the case in which all production involves the moral hazard problem. Second,

$$Z = \left\{ x \in \mathbb{R}^{|G|} \mid Ax' = 0 \right\}, \tag{1}$$

where A is a $m \times n$ matrix for some m with row rank $n-1$, captures the case in which the relative prices are fixed through the linear conversion technology of the representative firm, which corresponds to the partial equilibrium analysis of the moral hazard problem. Under (1), the equilibrium prices p are uniquely pinned down (up to a normalization) as the non-zero solution to $Ap = 0$.

2.2 Firms and Employment Contracts

A large finite number of risk neutral firms can sign employment contracts with the workers. We denote the set of firms by $J = \{1, 2, .., |J|\}$ .Throughout we impose *exclusivity* and assume that each worker can only contract with a single firm. A*n employment contract* between a firm and a worker transfers the worker's resources (i.e. her production y_s and profits accruing to her from the supply side, $\pi^Z(p)$) to the firm[7] and specifies worker's expenditures on each subset of commodities $G_m \in \mathcal{G}$ at each state $s \in S$, $w^{\mathcal{G}} = \left\{ w_s^{\{G_m\}} \right\}_{s \in S, G_m \in \mathcal{G}}$, and prescribes consumption levels for goods $x = (x_s^g)|_{s \in S, g \in G}$ and effort choice $e \in E$. We denote a contract by the tuple $c = \left(x, e, w^{\mathcal{G}}\right)$, and we denote the set of contracts by $C(\mathcal{G}) = \mathbb{R}_+^{|S| \times |G|} \times E \times \mathbb{R}_+^{|S| \times |\mathcal{G}|}$. Notice that since E is a subset of a Euclidean space, $C(\mathcal{G})$ is a subset of a finite dimensional space and its elements, denoted by $c \in C(\mathcal{G})$, are simply vectors. An incentive compatible contract is $c \equiv \left(\hat{x}, \hat{e}, \hat{w}^{\mathcal{G}}\right) \in C(\mathcal{G})$ such that the effort choice and the level of consumption of goods are *incentive compatible* given prices p and wage schedule $\hat{w}^{\mathcal{G}}$. More formally, this means that for a given market price vector p, and given $\hat{w}^{\mathcal{G}}$,

$$\begin{aligned} (\hat{x}, \hat{e}) \quad &\in \quad \arg \max_{x \geq 0, e \in E} U(x, e) \\ \text{such that} \quad & x^{G_m} p^{G_m} \leq \hat{w}^{\{G_m\}} \text{ for each } G_m \in \mathcal{G}. \end{aligned} \tag{2}$$

We denote the set of incentive compatible contracts by $C^I(p, \mathcal{G})$.

The incentive compatibility problem (2) can be conceptually divided into two parts. Given an effort level e, the choice of $\hat{x}$ is uniquely determined since U is strictly concave in x. We

[7] The assumption that agent profits $\pi^Z(p)$ get transferred to the firm is made to simplify notation and is without loss of generality.

denote the choice of x by the function $\mathbf{x}\left(\hat{w}^{\mathcal{G}}, p, e\right)$. By Berge's Maximum Theorem and the strict concavity of U in x, this function is continuous in its arguments. The incentive compatible effort choice is then a solution to the following problem:

$$\hat{e} \in \arg\max_{e \in E} U\left(\mathbf{x}\left(\hat{w}^{\mathcal{G}}, p, e\right), e\right). \tag{3}$$

By compactness of E and continuity of U and $\mathbf{x}$, this problem always has a solution. However, this maximization problem is not necessarily concave and the solution need not be unique or continuous. In view of this, the solution is represented by a correspondence $\mathbf{e}\left(\hat{w}^{\mathcal{G}}, p\right)$, which is upper hemicontinuous and has a closed graph by Berge's Maximum Theorem. We also define *the indirect utility function*

$$V(c, p) = \max_{e \in E} U\left(\mathbf{x}\left(\hat{w}^{\mathcal{G}}, p, e\right), e\right) \tag{4}$$

for each $c \in C^I(p, \mathcal{G})$ and $p \in \mathbb{R}_+^{|G|}$. Even though $\mathbf{e}\left(\hat{w}^{\mathcal{G}}, p\right)$ is not necessarily a continuous function, the indirect utility function is continuous in its arguments (here recall that $c \in C^I(p, \mathcal{G})$ is simply a vector).

2.3 Worker's Contract Choice

Each individual worker $\nu \in \mathcal{N}$ faces a menu of incentive compatible contracts, one from each firm, $c(\nu, j)|_{j \in J}$, and she chooses the contract that maximizes her utility. The worker can also reject every contract offer and keep her endowment. In this case, the worker solves

$$\begin{aligned} &\arg \max_{e \in E, x \in \mathbb{R}_+^{|S| \times |G|}} U(x, e) \\ \text{s.t.} \quad & xp \leq yp + \pi^Z(p). \end{aligned}$$

Let (x, e) be a solution to the preceding problem, and define the contract

$$c(\nu, 0 \mid p) = \left(x, e, \left(w^{\{G_m\}} = x^{G_m} p^{G_m}\right)_{G_m \in \mathcal{G}}\right) \tag{5}$$

as *the outside option* of the worker. Rejecting every contract is equivalent for the worker to accepting the contract $c(\nu, 0 \mid p)$. Hence, a *strategy* for the worker ν is a function $\mathbf{J}_\nu : C^I(p, \mathcal{G})^{|J|} \mapsto J \cup \{0\}$ that specifies the index of the firm she chooses or 0 if she chooses her outside option:

$$\mathbf{J}_\nu\left[c(\nu, j)|_{j \in J}\right] \in \arg\max_{J \cup \{0\}} V(c(\nu, j), p). \tag{6}$$

2.4 Firms' Problem

We assume that firm j' offers a continuum of incentive compatible contracts, one for each worker, taking the prices p, the contracts offered by other firms $(c(\nu, j))|_{\nu\in\mathcal{N},\ j\ \in J-\{j'\}}$, and the worker strategies $\mathbf{J}_\nu|_{\nu\in\mathcal{N}}$ as given. To justify the price taking assumption for the firms, we put an exogenous limit on the measure of contracts a firm can sign, which we denote by L. To simplify notation, we impose this assumption by requiring that all but a measure L of contract offers by a firm is equal to the *null contract,* corresponding to an offer $c_{null} = \left(x = 0, e, w^{\mathcal{G}} = 0\right)$. Note that this contract will never be accepted by a worker. We assume that L is a small number and there are enough firms to provide every worker with non-null employment contracts, i.e., $L \times |J| > 1$. Then, firm j' solves the following problem:

$$\max_{[c(\nu,j')\in C^I(p,\mathcal{G})]_{\nu\in\mathcal{N}}} \int_{\left\{v\ \mid\ \mathbf{J}_\nu\left[c(\nu,j)|_{j\in J}\right]=j'\right\}} \pi\left(c\left(\nu, j'\right), p\right) d\nu. \tag{7}$$
$$\text{such that} \qquad c\left(\nu, j'\right) = c_{null} \text{ except for a measure } L \text{ subset of } \mathcal{N}.$$

Here $\pi(c, p)$ denotes the profit of the firm given the contract offer and prices, which is given by

$$\pi\left(\left(x, e, w^{\mathcal{G}}\right), p\right) = \sum_{s\in S} q_s(e)(y_s p - \sum_{G_m\in\mathcal{G}} w_s^{\{G_m\}}) + \pi^Z(p).$$

3 Existence and Characterization of Competitive Equilibria

3.1 Definition of Equilibrium

Let us refer to the economy described in the previous section as economy $\mathcal{E}$. In this section, we define a competitive equilibrium for economy $\mathcal{E}$ and show that such an equilibrium exists.

Definition 1 A **competitive equilibrium** in economy $\mathcal{E}$ is a collection of contract offers $[c(\nu, j)]_{j\in J,\nu\in\mathcal{N}}$ by the firms, a collection of strategies for the workers $(\mathbf{J}_\nu)_{\nu\in\mathcal{N}}$, prices p, allocations $\left[x(\nu), e(\nu), w^{\mathcal{G}}(\nu)\right]_{\nu\in\mathcal{N}}$ and $z \in Z$, such that

E1: Each worker's contract choice is optimal, that is, for each $\nu \in \mathcal{N}$, J_ν satisfies (6) and the equilibrium allocation satisfies $\left[x(\nu), e(\nu), w^{\mathcal{G}}(\nu)\right] = c(\nu, J(\nu))$, where $J(\nu) \equiv J_\nu(c(\nu, j)|_{j\in J})$.

E2: Firms maximize profits, that is, for each $j' \in J$, $[c(\nu, j')]|_{\nu\in\mathcal{N}}$ solves Problem (7).

E3: The representative firm maximizes profits, i.e. $z \in Z^*(p)$.

E4: Commodity markets clear, that is,

$$z^g + \int_{\mathcal{N}} \sum_{s\in S} q_s(e(\nu))(y_s^g - x_s^g(\nu))d\nu \geq 0, \text{ with equality if } p^g > 0.$$

We also impose the following assumption.

Assumption A4 (Limited Transferability) Let $B(c,\epsilon)$ denote the ϵ neighborhood of contract c in space $C(\mathcal{G})$. Then for each incentive compatible contract $c = \left(x, e, w^{\mathcal{G}}\right) \in C^I(p,\mathcal{G})$ with $x > 0, w^{\mathcal{G}} > 0$, there exist contracts $c_+ = \left(x_+, e, w_+^{\mathcal{G}}\right), c_- = \left(x_-, e, w_-^{\mathcal{G}}\right) \in C^I(p,\mathcal{G}) \cap B(c,\epsilon)$ such that

$$w_+^{\mathcal{G}} \geq w^{\mathcal{G}} \geq w_-^{\mathcal{G}},$$

with strict inequality at least for one component.

This assumption asserts that, for each contract offer c in the interior of the contract space, there exist other contracts that pay weakly more [and respectively less] to the worker, while keeping the incentive compatible effort level of the worker the same. The assumption allows for at least a limited amount of utility transfer between the worker and the firms, while respecting the incentive compatibility constraints. In absence of a condition that allows for this type of utility transfer, Bertrand competition may not drive profits to zero (see Bennardo and Chiappori, 2003). Since this problem is already well understood and is orthogonal to our main concerns, Assumption A4 enables us to focus on the questions of interest for us. Assumption A4 is not difficult to satisfy. For example, Proposition 3 in Appendix B shows that it holds when the preferences have a completely separable component in one of the commodities (or in a subset of commodities $G_m \in \mathcal{G}$).

3.2 Bertrand Competition and the Indirect Problem

The following proposition provides an equivalent characterization for the equilibrium. In particular, a collection of accepted contract offers is part of an equilibrium if and only if all but a measure zero of them maximize the utility of the worker subject to the incentive compatibility constraint and a non-negative profit constraint for the firm. As with all the other proofs in this paper, the proof of this Proposition is in Appendix A.

Proposition 1 Suppose that Assumptions A1-A4 hold. Then, the price vector and allocations $\left[p, \left(\left[x(\nu), e(\nu), w^{\mathcal{G}}(\nu)\right]_{\nu\in\mathcal{N}}, z\right)\right]$ are part of an equilibrium if and only if both of the following conditions are satisfied.

1. For all but a measure zero of workers $\nu \in \mathcal{N}$, the accepted contract $c(\nu) = \left(x(\nu), e(\nu), w^{\mathcal{G}}(\nu)\right)$ is a solution to

$$\max_{c\in C^I(p,\mathcal{G})} V(c,p) \tag{8}$$

$$\text{subject to } \pi(c,p) \geq 0. \tag{9}$$

2. The representative firm maximizes profits and the goods markets clear, that is, Equilibrium Conditions E3 and E4 are satisfied.

   Moreover, at the solution to Problem (8), the constraint (9) is binding, so that each firm makes zero profits in equilibrium.

3.3 Existence of Equilibrium

The assumptions we made so far does not guarantee the existence of an equilibrium. If the solution set to Problem (8) is not upper hemicontinuous, the equilibrium may not exist. We impose the following assumption to ensure the upper hemicontinuity of the solution correspondence to Problem (8). The assumption ensures that the set of incentive compatible contracts $C^I(p, \mathcal{G})$, which is one of the constraint sets of Problem (8), is lower hemicontinuous in prices. In Appendix B, we provide simple conditions on the state utility function u and the probability function q which imply this assumption.

Assumption A5 (Lower Hemicontinuity of Incentive Compatible Contracts) For any incentive compatible contract $c \in C^I(p, \mathcal{G})$ and any sequence $p_n \to p$, there exists a sequence of contracts $c_n \in C^I(p_n, \mathcal{G})$ such that $c_n \to c$.

Theorem 1 Suppose that Assumptions A1-A5 hold. Then, a competitive equilibrium exists.

Assumptions A1-A5 are not very restrictive. in Appendix B, we provide several sufficient conditions on the state utility function u and the probability function q that imply these assumptions and validate our analysis.

4 Efficiency of Competitive Equilibria

4.1 Constrained Efficiency

In this section, we characterize the conditions under which the equilibrium we have described is efficient. Our notion of efficiency, which we call constrained optimality, provides the social planner with the same informational constraints as the firms but with better contracting technology. In particular, we suppose that the planner cannot observe the effort choice of the worker, but the planner can shut down the anonymous trading market in goods and specify the consumption of all goods in the employment contract. We next define the notions of incentive feasibility and constrained optimality.

Definition 2 In economy $\mathcal{E}$, an allocation $[(x(\nu), e(\nu))|_{\nu\in\mathcal{N}}, z]$ is **incentive feasible** if it satisfies the following conditions

1. The allocations $(x(\nu), e(\nu))|_{\nu\in\mathcal{N}}$ are incentive compatible, that is, for $\nu \in \mathcal{N}$, we have

$$e(\nu) \in \arg\max_{e\in E} U(x(\nu), e). \tag{10}$$

2. The representative firm's production is feasible, i.e. $z \in Z$, and the resource constraints hold:

$$z + \int_{\nu\in\mathcal{N}} \sum_{s\in S} q_s(e(\nu))(y_s - x_s(\nu))\, d\nu \geq 0. \tag{11}$$

An allocation $[(x(\nu), e(\nu))|_{\nu\in\mathcal{N}}, z]$ is **constrained optimal** if it is incentive feasible, and there does not exist another incentive feasible allocation $[(\hat{x}(\nu), \hat{e}(\nu))|_{\nu\in\mathcal{N}}, \hat{z}]$ such that

$$U(\hat{x}(\nu), \hat{e}(\nu)) \geq U(x(\nu), e(\nu)),$$

for all $\nu \in N$ with strict inequality for a positive measure of $\nu \in \mathcal{N}$.

4.2 Sufficient Conditions for Inefficiency of Equilibrium

In this subsection, we provide sufficient conditions under which the equilibrium is constrained suboptimal. We first provide formal definitions for the notions of full insurance and nonseparability, which we need to state our results.

Definition 3 We say there is **no full insurance** at the allocation (x, e), if there exists a commodity $g \in G$ and two states $s_1, s_2 \in S$ with $q_{s_1}(e) > 0$ and $q_{s_2}(e) > 0$ such that the marginal rate of substitution for good g between states s_1, s_2 is not equal to 1, that is

$$\frac{du(x_{s_1}, e)/dx_{s_1}^g}{du(x_{s_2}, e)/dx_{s_2}^g} \neq 1.$$

Definition 4 We say that the preferences are **nonseparable** at the allocation (x, e), if there is a state s with $q_s(e) > 0$ for all $e \in E$, a subset $G_m \in G$, and two goods $g_1, g_2 \in G_m$ such that the marginal rate of substitution between g_1 and g_2 at state s changes when effort level is modified, that is

$$\frac{du(x_s, e)/dx_s^{g_1}}{du(x_s, e)/dx_s^{g_2}} \neq \frac{du(x_s, e')/dx_s^{g_1}}{du(x_s, e')/dx_s^{g_2}} \text{ for any } e' \in E \setminus e.$$

Our main result in this subsection is the following theorem which shows that the equilibrium is constrained suboptimal whenever preferences are nonseparable and there is no full insurance at the equilibrium allocation for a positive measure of workers. The intuition for the result is closely related to *double deviations by the worker*, that is, deviations in which a worker switches to an effort level and reoptimizes her consumption of non-monitored goods for the new effort level. When the preferences are nonseparable and there is no full insurance at the equilibrium allocation, double deviations bind in the incentive compatibility constraints, that is, they prevent firms from providing more insurance to the workers. A social planner who can also prescribe the consumption of non-monitored goods is not constrained by double deviations and therefore can provide better insurance without violating the incentive compatibility constraints.

Theorem 2 Consider an economy $\mathcal{E}$ that satisfies Assumptions A1-A3. Let $\left[p, \left(\left[x(\nu), e(\nu), w^{\mathcal{G}}(\nu)\right]_{\nu \in \mathcal{N}}, z\right)\right]$ be the prices and the allocations in an equilibrium. Suppose that there is a positive measure set $N \subset \mathcal{N}$ such that for each $\nu \in N$, the preferences are nonseparable and there is no full insurance at the equilibrium allocation $(x(\nu), e(\nu))$. Then, the allocation $(x(\nu), e(\nu))|_{\nu \in \mathcal{N}}$ is **constrained suboptimal**.

We demonstrate this theorem with a simple example.

Example 1 (Nonseparable Preferences) Suppose that there are two states, $s \in S = \{b, g\}$, which respectively correspond to "success" and "failure". There are two goods, i.e. $G = \{1, 2\}$, and $\mathcal{G} = \{G\}$, that is, the firm can only specify wages and otherwise cannot monitor worker's consumption of goods. For simplicity, we consider a partial equilibrium setting in which $p_1 = p_2 = 1$ (more formally, we assume that there is a linear production technology which can convert the two goods to each other). Suppose there are two effort levels, $e \in \{L = 0, H = 1\}$, corresponding to low or high effort. Assume the firm makes zero profits in the failure state and positive profits in the success state, that is, $\pi_b \equiv y_b p = 0$, $\pi_g \equiv y_g p > 0$. Assume $q_g(e = 1) = 1/2$ and $q_g(e = 0) = 0$. Assume that good 2 is relatively more complements with leisure than good 1. More specifically, the worker's utility function is given by

$$U(x, e) = q_g(e) \ln\left(x_g^1 + 10(1-e)x_g^2\right) + (1 - q_g(e)) \ln\left(x_b^1 + 10(1-e)x_b^2\right). \tag{12}$$

The worker enjoys good 2 only when she does not work, where good 1 is equally enjoyable whether or not she works. Consider a social planner that chooses allocation $\left(x_g^1, x_b^1, x_g^2, x_b^2\right)$. It can be seen that, the social planner implements $e = H$ without providing incentives, that is, by providing the worker the same consumption bundle in both good and bad states, while making sure that the worker consumes only good 1. Hence, the worker's consumption is given by

$$x_b^1 = x_g^1 = (\pi_g + \pi_b)/2. \tag{13}$$

Now consider a firm that offers a wage contract (w_g, w_b). It can be seen that the social planner's full insurance solution is not incentive compatible, that is, given the wages just enough to consume the bundle (13), the worker would instead not work and consume a different bundle. Formally, we have

$$\ln\left(\frac{\pi_g + \pi_b}{2}\right) < \ln\left(\frac{10(\pi_g + \pi_b)}{2}\right),$$

hence the worker can increase her utility with a double deviation in which she changes effort choice and reoptimizes her consumption for the new effort decision. It can be seen that the firm will instead offer the wage contract (w_g, w_b) that is the solution to the following equations:

$$\textit{Budget constraint} \rightarrow (w_g + w_b)/2 = (\pi_g + \pi_b)/2.$$

$$\textit{Incentive compatibility} \rightarrow \frac{1}{2}\ln(w_g) + \frac{1}{2}\ln(w_b) \geq \ln 10w_b.$$

The equilibrium wages and consumption is given by

$$\begin{aligned} w_g &= \frac{100}{101}(\pi_g + \pi_b), w_b = \frac{1}{101}(\pi_g + \pi_b) \\ x_g^1 &= w_g, x_g^2 = 0, x_b^1 = w_b, x_b^2 = 0. \end{aligned} \tag{14}$$

In particular, the firm is only partially insuring the worker, and the worker is strictly worse off with the contract offered by the firm compared to the contract offered by the social planner. A social planner who can monitor all consumption is not constrained by worker's double deviations, and hence can provide better insurance.

The following theorem provides a class of economies in which any equilibrium is constrained suboptimal. The result essentially provides conditions on the preferences and the technology such that Theorem 2 applies at any equilibrium allocation. To ensure that there is no full insurance at any equilibrium allocation, we assume that there exists a shirking effort level that yields no revenues to the firm but is preferred by the worker under full insurance. To ensure that every equilibrium has the strong non-separability property, we assume that there exists a subset in the monitoring partition containing goods whose marginal rate of substitution change monotonically in the effort level. The following result then follows from Theorem 2 (proof omitted).

Theorem 3 Consider an economy $\mathcal{E}$ that satisfies Assumptions A1-A3. Let $e_{shirk} \in E$ be such that $u(x, e_{shrk}) > u(x, e)$ for all $e \in E \setminus \{e_{shirk}\}$ and $x \in \mathbb{R}_{++}^{|S| \times |G|}$, and assume that there exists a state $s_{low} \in S$ such that $y_{s_{low}} = 0$ and $q_{s_{low}}(e_{shirk}) = 1$. Let $E = \left\{e_1, ..., e_{|E|}\right\}$ *and* assume that there exists $G_m \in \mathcal{G}$ and two goods $g_1, g_2 \in G_m$ such that $\nabla_{x^{g_1}} u(x, e_i) / \nabla_{x^{g_2}} u(x, e_i)$ is strictly increasing in $i \in \{1, .., |E|\}$ for any $x \in \mathbb{R}_{++}^{|S| \times |G|}$. Then, each equilibrium allocation is **constrained suboptimal**.

Theorems 2 and 3 provide some support for the folk theorem for the inefficiency of the equilibrium. Note, however, that these theorems rely on a strong notion of optimality which essentially provides the social planner with a better monitoring technology than the firms: the planner, unlike the firm, can specify the consumption of all goods in the employment contract. Theorems 2 and 3 are silent on whether a social planner who is also constrained by the same monitoring technology can implement such a Pareto improvement. To address this issue, we introduce a weaker notion of optimality which constrains the social planner with the same monitoring technology as the firms.

Definition 5 An allocation $\left[p, \left[c(\nu) \equiv (x(\nu), e(\nu), w^{\mathcal{G}}(\nu)\right]_{\nu \in \mathcal{N}}, z\right]$ is ***market feasible*** if the contracts are incentive compatible given prices p, i.e. $c(\nu) \in C^I(p, \mathcal{G})$ for each $\nu \in \mathcal{N}$, and allocations $\left((x(\nu), e(\nu))_{\nu \in \mathcal{N}}, z\right)$ satisfy the resource constraints (*11*). The allocation is **weakly constrained optimal** if it is market feasible and there does not exist another market feasible allocation $\left[\hat{p}, \left[\hat{c}(\nu) \equiv (\hat{x}(\nu), \hat{e}(\nu), \hat{w}^{\mathcal{G}}(\nu)\right]_{\nu \in \mathcal{N}}, z\right]$ such that $U(\hat{x}(\nu), \hat{e}(\nu)) \geq U(x(\nu), e(\nu))$ for all $\nu \in N$ with strict inequality for a positive measure of $\nu \in \mathcal{N}$.

We next provide an example with nonseparable utility, which is constrained suboptimal as implied by Theorem 3, but which is weakly constrained optimal. The example shows that the inefficiency of equilibrium established in Theorems 2 and 3 in part stems from the strong notion of optimality which gives the social planner a technological advantage in monitoring. This suggests that care must be taken in invoking these theorems.

Example 2 (Nonseparable Preferences, Weak Optimality) Consider the same setup as in Example 1 but assume that there are two types of workers, t_1, t_2, each with measure 1/2, with state utility functions given by

$$u^{t_1}(x, e) = \ln\left(x_g^1 + 10(1-e)x_g^2\right) \text{ and } u^{t_2}(x, e) = \ln\left(10(1-e)x_g^1 + x_g^2\right).$$

The equilibrium allocation for type t_1 workers is characterized exactly as in Example 1 [cf. Eq. (14)] while the equilibrium allocation for type t_2 workers is the mirror image allocation in which the consumption of goods 1 and 2 are reversed. We claim that this equilibrium allocation is weakly constrained optimal. Suppose the contrary, i.e. that there is price and allocation system $\left(\hat{p}_1 \equiv 1, \hat{p}_2 \equiv p, \left(\hat{c}\left(t\right) \equiv \left(\hat{x}\left(t\right), \hat{e}\left(t\right) = 1, \hat{w}\left(t\right)\right)\right)|_{t \in \{t_1, t_2\}}\right)$ such that $\hat{c}\left(t\right)$ is incentive compatible given $\hat{p}$ for each $t \in \{t_1, t_2\}$, the resource constraints hold, and each type is weakly better off than the equilibrium allocation (while one type is strictly better off). In view of the technology that can convert each good to the other, the resource constraints can be written as

$$\left(\hat{R}\left(t_1\right) + \hat{R}\left(t_2\right)\right)/2 \leq \left(y_g^1 + y_g^2\right)/2 \equiv \pi^g/2, \tag{15}$$

where $\hat{R}\left(t\right) = \left(\hat{w}_g\left(t\right) + \hat{w}_b\left(t\right)\right)/2$ denotes the social planner's average expenditure (in terms of goods) on type t worker. An analysis as in Example 1 shows that the social planner's allocation is given by

$$\hat{x}_g^1\left(t_1\right) = \frac{100\hat{R}\left(t_1\right)}{100 + \hat{p}^2}, \hat{x}_g^1\left(t_1\right) = \frac{\hat{R}\left(t_1\right)}{100 + \hat{p}^2}, \hat{x}_g^2\left(t_2\right) = \frac{100\hat{R}\left(t_2\right)}{100 + 1/\hat{p}^2}, \hat{x}_g^2\left(t_2\right) = \frac{\hat{R}\left(t_2\right)}{100 + 1/\hat{p}^2}.$$

This is a Pareto improvement over the equilibrium allocation only if

$$\frac{10\hat{R}\left(t_1\right)}{\sqrt{100 + \hat{p}^2}} \geq \frac{10\left(\pi^g/2\right)}{\sqrt{101}} \text{ and } \frac{10\hat{R}\left(t_2\right)}{\sqrt{100 + 1/\hat{p}^2}} \geq \frac{10\left(\pi^g/2\right)}{\sqrt{101}}, \tag{16}$$

with strict inequality for at least one type. Combining Eqs. (15) and (16) yields

$$\sqrt{100 + p^2} + \sqrt{100 + 1/p^2} < 2\sqrt{101}.$$

The left hand side is minimized at $p = 1$ and its minimum value is $2\sqrt{101}$, which shows that the previous inequality cannot hold for any p and yields a contradiction. Hence the equilibrium allocation is weakly constrained optimal even though is not constrained optimal.

4.3 Sufficient Conditions for Efficiency of Equilibrium

Next, we provide a partial converse to Theorem 2 and show that when the worker preferences are partially separable, the equilibrium is constrained optimal. The next definition formalizes our notion of partial separability, which is essentially the opposite of nonseparability in Definition 4. The functional form assumption ensures that preferences are not nonseparable at any equilibrium allocation.

Definition 6 The preferences are **partially separable** if there exists continuous functions $u^{\{G_i\}} : \mathbb{R}_+^{|G_i|} \mapsto \mathbb{R}$ strictly increasing in each of its arguments and another continuous function $u^{\mathcal{G}} : \mathbb{R}_+^{|\mathcal{G}|} \times E \mapsto \mathbb{R}$ strictly increasing in its first $|\mathcal{G}|$ arguments such that

$$u\left(x_s, e\right) = u^{\mathcal{G}}\left(u^{\{G_1\}}\left(x_s^{G_1}\right), u^{\{G_2\}}\left(x_s^{G_2}\right), .., u^{\{G_{|\mathcal{G}|}\}}\left(x_s^{G_{|\mathcal{G}|}}\right), e\right). \tag{17}$$

To see how this definition relates to the discussion in the introduction, note that the marginal rate of substitution at state s between goods $g_1, g_2 \in G_m$ is given by

$$\frac{du\left(x_s, e\right)/dx_s^{g_1}}{du\left(x_s, e\right)/dx_s^{g_2}} = \frac{\partial u^{\mathcal{G}}\left(.\right)/\partial m \ \partial u^{\{G_m\}}\left(x_s^{G_m}\right)/\partial g_1}{\partial u^{\mathcal{G}}\left(.\right)/\partial m \ \partial u^{\{G_m\}}\left(x_s^{G_m}\right)/\partial g_2} = \frac{\partial u^{\{G_m\}}\left(x_s^{G_m}\right)/\partial g_1}{\partial u^{\{G_m\}}\left(x_s^{G_m}\right)/\partial g_2},$$

that is, the MRS between two goods g_1, g_2 in a subset $G_m \in \mathcal{G}$ is independent of effort level e.[8]

Our next result shows that partial separability is sufficient for the competitive equilibrium to be constrained Pareto optimal. The intuition is that, under partial separability, the social planner chooses for the worker the consumption bundle which the worker would have chosen by herself in the anonymous trading market. Hence, there is no benefit to additional monitoring, and competition among firms leads to the allocations that the social planner would have chosen. A complementary intuition is that double deviations in which the worker changes effort level and reoptimizes her consumption bundle accordingly are not valuable, which further implies that the relative price changes caused by other contracts in the economy does not change the insurance-incentive trade-off for a worker, rendering pecuniary externalities ineffective.

Theorem 4 Consider economy $\mathcal{E}$ and suppose that Assumptions A1-A4 hold. Assume also that the preferences are partially separable**.** Let $\left[p, \left(\left[c\left(\nu\right) = \left(x\left(\nu\right), e\left(\nu\right), w^{\mathcal{G}}\left(\nu\right)\right)\right]_{\nu \in \mathcal{N}}, z\right)\right]$ be the prices and the allocations in an equilibrium. Then, the allocation $\left[\left(x\left(\nu\right), e\left(\nu\right)\right)|_{\nu \in \mathcal{N}}, z\right]$ is **constrained optimal**.

We demonstrate this theorem with a simple example in which preferences are fully separable.

Example 3 (Separable Preferences) Consider the same setup as in Example 1 but assume that the worker's utility is given by

$$U\left(x, e\right) = q_g\left(e\right)\left[u\left(x_g^1, x_g^2\right) - c\left(e\right)\right] + \left(1 - q_g\left(e\right)\right)\left[u\left(x_b^1, x_b^2\right) - c\left(e\right)\right],$$

which is fully separable. Consider first a planner who determines an incentive compatible consumption vector $x = \left(x_g^1, x_b^1, x_g^2, x_b^2\right)$ for each worker. Suppose, for simplicity, that the planner offers every worker the same employment contract (the result in Theorem 4 is more general and does not rely on this assumption). The planner is maximizing the representative

[8] This representation is not as general as it could be since it does not allow for the MRS between goods g and g' to depend on the consumption level of other goods. All of our results also apply in that more general case, but we adopt the simpler formulation in Definition 6 since it simplifies the notation and the analysis.

worker's utility subject to incentive compatibility and resource constraints, that is:

$$\begin{aligned} & \max_{e,x} U(x,e) \\ \textit{subject to} \quad & \sum_{s\in S} q_s(e)\left(p^1 x_s^1 + x_s^2\right) = \sum_{s\in S} q_s(e)\,\pi_s \quad \textit{[resource constraint]} \\ & U(x,e) \geq U\left(x,e'\right),\ \textit{for all } e' \qquad \textit{[incentive compatibility].} \end{aligned}$$

To solve this problem, the planner computes the value from implementing some $e \in E$. The planner then implements the effort level e^* that yields the highest utility to the worker. Similar to the analysis by Grossman-Hart (1983), we note that, due to the separability of the utility function between effort choice and consumption, the planner's problem is equivalent to first deciding how much utility to provide in each state, U_s, and then deciding the optimal consumption bundle that provides this level of utility. Hence, the optimal utility from implementing $e^* \in E$ is given by

$$\begin{aligned} & V^S(e^*) = \max_{x,U_g,U_b} q_g(e^*)\,U_g + (1-q_g(e^*))\,U_b - c(e^*) \qquad (18) \\ \textit{such that} \quad & \sum_{s\in S} q_s(e^*)\left(p^1 x_s^1 + x_s^2\right) = \sum_{s\in S} q_s(e^*)\,\pi_s \\ & \sum_{s\in S} q_s(e^*)\,U_s - c(e^*) \geq \sum_{s\in S} q_s\left(e'\right)U_s - c\left(e'\right) \ \textit{ for all } e' \end{aligned}$$

and two additional constraints $u\left(x_g^1, x_g^2\right) = U_g$ and $u\left(x_b^1, x_b^2\right) = U_b$. Note that, given U_s, the planner would like to minimize the cost of providing this utility, that is, x_s^G is the solution to

$$\begin{aligned} & C^S(U_s^*) = \min_{x_s} p^1 x_s^1 + x_s^2 \\ & \textit{s.t.}\ \ u\left(x_s^1, x_s^2\right) = U_s^*, \end{aligned}$$

which is a strictly convex minimization problem that provides a one-to-one relationship between $C^S(U_s^*)$ and the optimum choice of vector x_s. By duality, the optimum vector x_s also maximizes $u\left(x_s^1, x_s^2\right)$ subject to the expenditure being not greater than $C^S(U_s^*)$. Using these observations, Problem (18) can be rewritten as

$$V^S(e^*) = \max_{C^S(U_g),C^S(U_b)} \sum_{s\in S} q_s(e^*) \left\{ \max_{p^1 x_s^1 + x_s^2 \le C^S(U_s)} u\left(x_s^1, x_s^2\right) \right\} - c(e^*) \tag{19}$$

subject to

$$\sum_{s\in S} q_s(e^*) C^S(U_s) = \sum_{s} q_s(e^*) \pi_s$$

$$\sum_{s\in S} q_s(e^*) \left\{ \max_{p^1 x_s^1 + x_s^2 \le C^S(U_s)} u\left(x_s^1, x_s^2\right) \right\} - c(e^*)$$

$$\ge \sum_{s\in S} q_s(e') \left\{ \max_{p^1 x_s^1 + x_s^2 \le C^S(U_s)} u\left(x_s^1, x_s^2\right) \right\} - c(e') \;\; \textit{for all } e'.$$

Now consider the firm that offers a wage contract $\{w_g, w_b\}$. Then, to implement effort level $e^* \in E$, the firm will solve

$$V^E(e^*) = \max_{w_g,w_b} \sum_{s\in S} q_s(e^*) \left\{ \max_{p^1 x_s^1 + x_s^2 \le w_s} u\left(x_s^1, x_s^2\right) \right\} - c(e^*) \tag{20}$$

$$\sum_{s\in S} q_s(e^*) w_s = \sum_{s\in S} q_s(e^*) \pi_s$$

$$\sum_{s\in S} q_g(e^*) \left\{ \max_{p^1 x_s^1 + x_s^2 \le w_s} u\left(x_s^1, x_s^2\right) \right\} - c(e^*)$$

$$\ge \sum_{s\in S} q_g(e') \left\{ \max_{p^1 x_s^1 + x_s^2 \le w_s} u\left(x_s^1, x_s^2\right) \right\} - c(e') \;\; \textit{for all } e'.$$

A comparison of Problems in (19) and (20) shows that the problems are equivalent and the worker receives the same utility in either case. Hence, in this example, monitoring is not valuable, and a firm that can only offer a wage contract provides incentives as well as a social planner who can monitor worker's consumption.

Note that most of the applied work on private information in macroeconomics and public finance assume partially separable preferences [see, for example, Atkeson and Lucas (1992), Golosov and Tsyvinski (2006), Golosov, Kocherlakota and Tsyvinski (2003)], hence Theorem 4 shows that the equilibrium is efficient in many economies of applied interest.

5 Efficiency of Equilibrium in a Moral Hazard Economy with Endogenous Monitoring

The results in Section 4 show that the equilibrium is constrained optimal, loosely speaking, if and only if firms monitor consumption of all commodities whose marginal rate of substitution change with effort choice. Since the monitoring decision by the firms is crucial to understanding the efficiency properties of the equilibrium, we next study a more general framework in which the monitoring partition is a choice variable for the firms. We first describe the environment in which the monitoring decision is endogenized.

5.1 Environment

The environment with endogenous monitoring decisions builds upon the environment described in Section 2.1 in which the monitoring partition is exogenously given. The main departure is that we introduce a simple monitoring technology. Let $\mathcal{P}(G)$ be the set of all partitions of the set G. Each contract between a firm and a worker also stipulates a partition $\mathcal{G} = \{G_1, .., G_m\} \subset \mathcal{P}(G)$ such that firms monitor the worker's expenditure on each subset in partition $\mathcal{G}$ by paying some cost according to the cost function $k(\mathcal{G}) : \mathcal{P}(G) \to \mathbb{R}_+^{|G|}$ in terms of a vector of commodities. We assume that monitoring more subsets is costly, i.e. $k(\mathcal{G}') \geq k(\mathcal{G}'')$ whenever $\mathcal{G}'$ is a weakly finer partition of G than $\mathcal{G}''$, which we denote by $\mathcal{G}' \succeq \mathcal{G}''$,[9] and that monitoring the coarsest partition (i.e. monitoring only worker wages) is for free, $k(\{G\}) = (0, 0, .0)$.

An incentive compatible *monitoring contract* between a firm and the worker consists of a pair that denotes the monitoring partition and a corresponding incentive compatible contract, that is $c^M = (\mathcal{G}, c)$ where $\mathcal{G} \subset \mathcal{P}(G)$ and $c \in C^I(p, \mathcal{G})$. We denote the set of incentive compatible monitoring contracts by $C^M(p)$. Let $V^{\mathcal{G}}(c,p)$ denote the worker's indirect utility function from an incentive compatible contract [cf. Eq. (4)] when partition $\mathcal{G}$ is monitored. We define worker's utility over an incentive compatible monitoring contract $c^M = (\mathcal{G}, c)$ as

$$V^M\left(c^M, p\right) = V^{\mathcal{G}}(c, p).$$

As in Section 2.1, each worker $\nu \in \mathcal{N}$ faces a menu of incentive compatible monitoring contracts, one from each firm, $c^M(\nu, j)\,|_{j \in J}$, and she chooses the contract that maximizes her utility. The worker can also reject every contract offer and keep her endowment. Note that the outside option contract as defined in Eq. (5) yields the same utility to the worker regardless of the monitoring partition. Hence, we define

$$c^M(\nu, 0 \mid p) = \left(\{G\}, \left(c^{\{G\}}(\nu, 0) \mid p\right)\right)$$

[9] A partition $\mathcal{G}' = \left\{G'_1, .., G'_{|\mathcal{G}'|}\right\} \in \mathcal{P}(G)$ is finer than another partition $\mathcal{G}'' = \left\{G''_1, .., G''_{|\mathcal{G}''|}\right\} \in \mathcal{P}(G)$ if for each $G''_i \in \mathcal{G}''$, there exist $\mathcal{G}'_i \subset \mathcal{G}'$ such that $\bigcup_{G'_{i_m} \in \mathcal{G}'_i} G'_{i_m} = G''_i$. Note that this notion describes a partial order over $\mathcal{P}(G)$.

without loss of generality as the outside option of the worker in this environment. Hence, a *strategy* for the worker ν is a function $\mathbf{J}^M_\nu\colon \left(C^M(p)\right)^{|J|} \mapsto J \cup \{0\}$ that specifies the index of the firm she chooses or 0 if she chooses her outside option:

$$\mathbf{J}^M_\nu \left[c^M(\nu,j)|_{j\in J}\right] \in \arg\max_{J\cup\{0\}} V^M(c^M(\nu,j),p). \tag{21}$$

Similar to Section 2.1, firm j' offers a continuum of incentive compatible monitoring contracts, one for each worker, taking the prices p, the contracts offered by other firms $c^M(\nu,j)|_{\nu\in\mathcal{N},\ j\in J-\{j'\}}$, and the worker strategies $\mathbf{J}^M_\nu|_{\nu\in\mathcal{N}}$ as given. We again require that all but a measure L of contract offers by a firm is equal to the *null contract,* corresponding to an offer $c^M_{null} = \left(\mathcal{G}, \left(x^G = 0, e', \left(w^{\mathcal{G}}\right)'\right)\right)$. Then, firm j' solves the following problem:

$$\begin{aligned} &\max_{\left[c^M(\nu,j')\in C^M(p)\right]|_{\nu\in\mathcal{N}}} \int_{\left\{v \;|\; \mathbf{J}^M_\nu\left[c^M(\nu,j)|_{j\in J}\right]=j'\right\}} \pi^M\left(c^M\left(\nu,j'\right),p\right)d\nu \qquad (22)\\ &\text{such that} \qquad c^M\left(\nu,j'\right) = c^M_{null} \text{ except for a measure } L \text{ subset of } \mathcal{N}. \end{aligned}$$

Here $\pi^M\left(c^M,p\right)$ denotes the profit of the firm from the monitoring contract, which is given by

$$\pi^M\left(\left(\mathcal{G},\left(x,e,w^{\mathcal{G}}\right)\right),p\right) = \sum_{s\in S} q_s(e)(y_s p - \sum_{G_m\in\mathcal{G}} w_s^{\{G_m\}}) - k\left(\mathcal{G}\right)p + \pi^Z(p).$$

The second to last term in this expression captures the cost of monitoring the partition $\mathcal{G}$.

5.2 Equilibrium with Monitoring

Let us refer to the economy described in the previous subsection as economy $\mathcal{E}^M$. We define a competitive equilibrium for economy $\mathcal{E}^M$ and show that such an equilibrium exists.

Definition 7 A **competitive equilibrium with monitoring** in economy $\mathcal{E}^M$ is a collection of contract offers $\left[c^M(\nu,j)\right]|_{j\in J,\nu\in\mathcal{N}}$ by the firms, a collection of strategies for the workers $\left(\mathbf{J}^M_\nu\right)|_{\nu\in\mathcal{N}}$, prices p, allocations $\left(\mathcal{G}(\nu),\left[x(\nu),e(\nu),w^{\mathcal{G}}(\nu)\right]\right)_{\nu\in\mathcal{N}}$ and $z\in Z$, such that

E1^M: Each worker's contract choice is optimal, that is, for each $\nu\in N$, J^M_ν satisfies (21) and the equilibrium allocation satisfies $\left(\mathcal{G}(\nu),\left[x(\nu),e(\nu),w^{\mathcal{G}}(\nu)\right]\right) = c^M\left(\nu, J^M(\nu)\right)$, where $J^M(\nu) = J^M_\nu\left(c^M(\nu,j)|_{j\in J}\right)$.

E2^M: Firms maximize profits, that is, $\left[c^M(\nu,j')\right]|_{\nu\in\mathcal{N}}$ solves Problem (7).

E3: The representative firm maximizes profits, that is, $z\in Z^*(p)$.

E4^M: Commodity markets clear, that is,

$$z^g + \int_{\mathcal{N}}\left[\sum_{s\in S} q_s(e(\nu))\left(y^g_s - x^g_s(\nu)\right) - k^g\left(\mathcal{G}(\nu)\right)\right]d\nu \geq 0, \text{ with equality for } p^g > 0.$$

The following assumption is the equivalent of Assumption A4 in this setup.

Assumption A4^M (Limited Transferability for Monitoring Contracts) Assumption A4 holds for $\mathcal{G} = \{G\}$.

Note that if contract $c'' = \left(x, e, w^{\mathcal{G}''}\right)$ lies in $C^I\left(p, \mathcal{G}''\right)$, then, for any partition $\mathcal{G}'$ finer than $\mathcal{G}''$, the corresponding contract $c' = \left(x, e, w^{\mathcal{G}'}\right)$ lies in $C^I\left(p, \mathcal{G}'\right)$, i.e. the set of incentive compatible contracts satisfy a monotonicity property in monitoring partitions. Consequently, Assumption A4^M more strongly implies that Assumption 4 holds for all partitions in $\mathcal{P}(G)$. The following proposition, which is the counterpart of Proposition 1 uniquely characterizes the equilibrium under Assumptions A1-A3 and A4^M. The proof closely follows that of Proposition 1 and hence is omitted.

Proposition 2 Suppose that Assumptions A1-A3 and A4^M hold. Then, the price vector p and allocations $\left(\left[\mathcal{G}(\nu), \left(x(\nu), e(\nu), w(\nu)\right)\right]_{\nu \in \mathcal{N}}, z\right)$ are part of an equilibrium if and only if both of the following conditions are satisfied.

1. For all but a measure zero of workers $\nu \in N$, the accepted contract $c^M(\nu)$ is a solution to

$$\max_{c^M \in C^M(p)} V^M\left(c^M, p\right) \tag{23}$$

$$\text{subject to } \pi^M\left(c^M, p\right) \geq 0. \tag{24}$$

2. The representative firm maximizes profits and the commodity markets clear, that is, Equilibrium Conditions E3 and E4^M are satisfied.

Moreover, at the solution to Problem (23), the constraint (24) is binding, so that each firm makes zero profits in equilibrium.

Note that Problem (23) can be also written as

$$\begin{array}{ccc} \max\limits_{\mathcal{G} \in \mathcal{P}(G)} & \max\limits_{c \in C^I(p,\mathcal{G})} V^{\mathcal{G}}(c, p) \\ \textit{subject to} & \pi^{\mathcal{G}}(c, p) \geq k(\mathcal{G})\, p. \end{array} \tag{25}$$

In particular, in this environment the contract maximization problem can be divided into two steps. For a given monitoring partition $\mathcal{G}$, each equilibrium contract solves the inner maximization problem (25), which is very similar to Problem (8) except for the additional cost term $k(\mathcal{G})\,p$. Once this problem is solved for each $\mathcal{G} \in \mathcal{P}(G)$, the equilibrium contract then maximizes over finitely many partitions. Consequently, the structure of Problem (25) is very similar to Problem (8), and we establish existence of equilibrium under similar conditions. The following assumption is the counterpart of Assumption A5 in the economy with endogenous monitoring.

Assumption A5^M (Lower Hemicontinuity of Incentive Compatible Monitoring Contracts) Assumption A5 holds for $\mathcal{G} = \{G\}$.

Once again, due to the monotonicity property of the incentive compatible contracts, Assumption A5^M more strongly implies that Assumption A5 holds for all $\mathcal{G}' \in \mathcal{P}(G)$. The following theorem establishes the existence of equilibrium. In view of Proposition 2, the proof closely follows the proof of Theorem 1 and hence is omitted.

Theorem 5 Suppose that Assumptions A1-A3 and A4^M-A5^M hold. Then, a competitive equilibrium with monitoring exists.

5.3 Sufficient Conditions for Approximate Efficiency of Equilibrium with Monitoring

In this section, we analyze the efficiency of the competitive equilibria with monitoring. Our benchmark for efficiency is constrained optimality introduced in Section 2, that is, we let the social planner monitor consumption of all commodities without any cost. Our main result provides conditions under which the equilibrium with monitoring is approximately efficient under this strong notion of optimality. We introduce $\epsilon-$constrained optimality as our notion of approximate efficiency.

Definition 8 An allocation $[(x(\nu), e(\nu))|_{\nu\in\mathcal{N}}, z]$ is $\epsilon-$**constrained optimal** if it is incentive feasible, and there does not exist another incentive feasible allocation $[(\hat{x}(\nu), \hat{e}(\nu))|_{\nu\in\mathcal{N}}, \hat{z}]$ such that $U(\hat{x}(\nu), \hat{e}(\nu)) \geq U(x(\nu), e(\nu)) + \epsilon$ for all $\nu \in N$.

The following theorem, which is our first main result in this section, shows that the monitoring equilibrium is $\epsilon-$constrained optimal when monitoring costs are sufficiently small. For the intuition, note that any incentive compatible allocation chosen by the social planner could be replicated by an employment contract which monitors all goods. This contract would potentially lead to losses (bounded by the cost of monitoring), and therefore is not necessarily offered by a firm. However, in view of Assumption A4^M, the firm can shift the losses to the worker by offering a similar contract which breaks even and which provides the worker with slightly less utility than the allocation offered by the social planner. It then follows that the equilibrium is approximately optimal when monitoring costs are sufficiently small.

Theorem 6 Consider economy $\mathcal{E}^M$ and suppose that Assumptions A1-A3 and A4^M-A5^M hold. Let $k^r : \mathcal{P}(G) \rightarrow R_+$ be a reference cost function and let the cost function be given by $k(\mathcal{G}) = Kk^r(\mathcal{G})$ where $K > 0$ is a parameter that controls the level of monitoring costs. There exists $\overline{K} > 0$ such that if $K < \overline{K}$, then any equilibrium allocation is $\epsilon-$***constrained optimal***.

Theorem 6 applies very generally and cautions against the use of the folk theorem for the inefficiency of the equilibrium. However, the theorem crucially relies on the cost of monitoring all commodities being sufficiently small. In view of the vast number of commodities available in the market, the cost of monitoring all of these commodities may be substantial, limiting the applicability of Theorem 6. We next present an analogous result which shows that the equilibrium is approximately efficient under less stringent assumptions.

Theorem 7 Consider economy $\mathcal{E}^M$ and suppose that Assumptions A1-A3 and A4^M-A5^M hold. Let $K^{\max} > 0$ and assume that $k(\mathcal{G}) \in [0, K^{\max}]$ for all $\mathcal{G} \in \mathcal{P}(G)$. Consider a representation for the state utility function

$$u(x_s, e) = u^{\mathcal{G}^*}\left(u^{\{G_1\}}\left(x_s^{G_1}\right), u^{\{G_2\}}\left(x_s^{G_2}\right), .., u^{\left\{G_{|\mathcal{G}^*|}\right\}}\left(x_s^{G_{|\mathcal{G}^*|}}\right), e\right), \tag{26}$$

where $\mathcal{G}^* = \left\{G_1, .., G_{|\mathcal{G}^*|}\right\}$ is a partition of G, $\left\{u^{\{G_i\}}\right\}_{G_i \in \mathcal{G}^*}$ are strictly increasing and continuously differentiable functions, and $u^{\mathcal{G}^*}$ is a continuously differentiable function strictly increasing in its first $|\mathcal{G}^*|$ arguments. For any $\epsilon > 0$, there exists $\overline{K}_\epsilon > 0$ such that if $k(\mathcal{G}^*) < \overline{K}_\epsilon$, then any equilibrium allocation is $\epsilon-$***constrained optimal***.

Note that every state utility function has at least one representation of the form in (26) with the finest partition $\mathcal{G}^* = \left\{\{G_1\}, \{G_2\}, .., \left\{G_{|G|}\right\}\right\}$, hence Theorem 7 applies at least as generally as Theorem 6. For preferences that admit a representation of the form in (26) with a coarser partition $\mathcal{G}^*$, Theorem 7 applies more generally than Theorem 6 since it only requires the monitoring cost for partition $\mathcal{G}^*$ to be sufficiently small and allows the monitoring costs for the remaining partitions to be potentially large.

The structure of preferences implied by the representation in (26) has a natural interpretation. Suppose that the worker has preferences, represented by the outer utility function $u^{\mathcal{G}^*}$, over a number of services $H \equiv \left\{H_s^{\{G_1\}}, .., H_s^{\left\{G_{|\mathcal{G}^*|}\right\}}\right\}_{s \in S}$ received at each state, and effort choice e. These services may correspond to broad notions such as health care, vacations, transportation needs, procrastination and so on, or they may more narrowly correspond to types of goods such as hospitals, medical drugs, hotel stays, cars, flights, surfing the net, watching TV etc. The level of abstraction is up to the modeler but it is important to include the services which are relatively complementary or relatively substitutable with leisure choice (or equivalently with effort choice). The service $H^{\{G_m\}}$ is in turn provided by consumption of some or all goods in $G_m = \{g_1, .., g_m\}$, which is captured by the inner utility function u^{G_m}. In other words, the inner utility function u^{G_m} may be viewed as a production function which takes goods $\{g_1, .., g_m\}$ as inputs and outputs the service so that $H_s^{\{G_m\}} = u^{G_m}\left(x_s^{G_m}\right)$. The critical assumption imposed by the representation in (26) is that the marginal rate of substitution between goods that provide a particular service $H^{\{G_m\}}$ does not depend on the effort choice. For example, the effort choice of the worker may affect how long a vacation she takes and hence how much she enjoys a vacation, but it does not have a significant effect on her relative ranking of vacation options or holiday resorts.

In view of this interpretation, Theorem 7 shows that the equilibrium may be close to efficient when there are relatively few services that interfere with the worker's effort choice which are also not too costly to monitor. For example, if the major aspects of worker needs that interfere with effort choice are vacations, procrastination, health care, and if it is not too costly for the firm to monitor (and enforce) how long a vacation the worker takes, how much time she spends in the office, and how good a health care she receives (i.e. whether or not she has health insurance), then the theorem suggests a presumption that equilibrium will be close to efficient. It could be very costly to monitor where the worker spends her vacation, or whether she watches TV or plays computer games in her procrastination time, but these costs do not undo the approximate efficiency of the equilibrium.

Theorems 6 and 7 provide conditions under which the equilibria are near constrained optimal despite the fact that the competitive equilibrium incurs the monitoring costs whereas the benchmark socially planned economy assumes that the planner can monitor expenditures and consumption of all goods at zero costs. Naturally, these theorems are silent on the optimality of equilibria when monitoring costs are large. In this case, the right comparison would be between a competitive equilibrium with monitoring and a socially planned economy where the planner also has to incur the same monitoring costs. We address this issue by introducing a weaker notion of optimality which we call monitoring optimality. We then describe an example economy in which the equilibrium is inefficient with respect to this weaker notion of optimality.

Definition 9 In economy $\mathcal{E}^M$, an allocation $\left[p, \left[c^M(\nu) \equiv \left(\mathcal{G}(\nu), (x(\nu), e(\nu), w^{\mathcal{G}}(\nu))\right)\right]_{\nu \in \mathcal{N}}, z\right]$ is **monitoring feasible** if the contracts are incentive compatible given prices p, i.e. $c^M(\nu) \in C^M(p)$ for each $\nu \in \mathcal{N}$, and allocations $\left(\mathcal{G}(\nu), (x(\nu), e(\nu))_{\nu \in \mathcal{N}}, z\right)$ satisfy the resource constraints

$$z^g + \left[\int_{\mathcal{N}} \left(\sum_{s \in S} q_s(e(\nu))\,(y_s - x_s(\nu))\right) - k^g\left(\mathcal{G}(\nu)\right)\right] d\nu \geq 0. \tag{27}$$

The allocation is **monitoring optimal** if it is monitoring feasible and there does not exist another monitoring feasible allocation $\left[\hat{p}, \left[\hat{c}^M(\nu) \equiv \left(\hat{\mathcal{G}}(\nu), (\hat{x}(\nu), \hat{e}(\nu), \hat{w}^{\mathcal{G}}(\nu))\right)\right]_{\nu \in \mathcal{N}}, z\right]$ *such that* $U(\hat{x}(\nu), \hat{e}(\nu)) \geq U(x(\nu), e(\nu))$ for all $\nu \in \mathcal{N}$ with strict inequality for a positive measure of $\nu \in \mathcal{N}$.

Our next example provides a case with significant monitoring costs in which the equilibrium is not monitoring optimal. The example provides an economy with two equilibria which can be Pareto ranked, therefore the Pareto worse equilibrium is not monitoring optimal.

Example 4 (Monitoring Inefficiency) We consider the setup in Example 1 but depart from it in three respects. First, we drop the partial equilibrium assumption and require the prices $(p_1 = 1, p_2 = p)$ to clear the markets. Second, we assume that preferences are given by

$$u(x_s, e) = \frac{e}{2} \ln x_s^1 + \left(1 - \frac{e}{2}\right) \ln x_s^2 - ce^2.$$

Third, we assume monitoring decisions are endogenous and the cost of monitoring is given by $k(\{\{1\}, \{2\}\}) = \left[k^1, k^2\right] \geq 0$.

In this economy, workers might sign one of the two possible contracts. In the type 1 contract, consumption is monitored and effort is high, whereas in the type 2 contract, consumption is not monitored and effort is high. We calculate the worker utilities from each contract type for given price level p, subject to the constraint that firms break even. Firms which offer the type 1 contracts solve

$$\begin{aligned}
V_1(p) \quad &\equiv \quad \max_{\{x_s^i\}} \frac{1}{4} \left(\sum_{s \in \{g,b\}, i \in \{1,2\}} \ln x_s^i \right) - c \\
s.t. \quad & \quad \frac{x_g + x_b}{2} [1, p]' = \left(\frac{y_g}{2} - k\right) [1, p]', \quad \textit{[resource constraints]} \\
& \quad \frac{1}{4} \left(\sum_{s \in \{g,b\}, i \in \{1,2\}} \ln x_s^i \right) - c \geq \ln x_b^2. \quad \textit{[incentive compatibility (IC)].}
\end{aligned}$$

When $\ln p > 2c$, the value of this problem is given by

$$V_1(p) = \ln\left(\frac{(y_g - 2k)[1,p]'}{4}\right) - \frac{\ln p}{2} - c. \qquad (28)$$

Firms which offer type 2 contracts solve

$$\begin{aligned} V_2(p) \quad &\equiv \quad \max_{\{w_s, x_s^i\}} \frac{1}{4}\left(\sum_{s\in\{g,b\}, i\in\{1,2\}} \ln x_s^i\right) - c \\ s.t. \quad & \quad x_s^1 = w_s/2,\ x_s^2 = w_s/2p \text{ for each } s \in \{g,b\}, \quad \textit{[anonymous market constraints]} \\ & \quad (w_g + w_b)/2 = y_g[1,p]'/2, \quad \textit{[resource constraints]} \\ & \quad \frac{1}{4}\left(\sum_{s\in\{g,b\}, i\in\{1,2\}} \ln x_s^i\right) - c \geq \ln\frac{w_b}{p}, \quad \textit{[incentive compatibility (IC)]}, \end{aligned}$$

where the IC constraint now takes into account the fact that the worker will reoptimize her consumption choice after shirking. When $s(p) \equiv 2c + 2\ln 2 - \ln p > 0$, the value of this problem is given by[10]

$$V_2(p) = \left[\ln\left(\frac{\exp(s(p))^{1/2}}{\exp(s(p)) + 1}\right) - \ln\frac{1}{2}\right] + \ln\left(\frac{(y_g[1,p]')}{4}\right) - \frac{1}{2}\ln p - c. \qquad (29)$$

A comparison of Eqs. (28) and (29) demonstrates that the monitoring contract has the advantage of offering better insurance [since Eq. (28) does not have the negative first term that Eq. (29) has] but it incurs additional monitoring costs.

We next characterize the symmetric equilibria of this economy. If all workers sign type 1 contracts, market clearing in goods implies that the price of good 2 is given by $p_{type1} = \left(y_g^1 - 2k^1\right)/\left(y_g^2 - 2k^2\right)$, and this will be an equilibrium price if $V_1(p_{type1}) > V_2(p_{type1})$. Similarly, if all workers sign type 2 contracts, market clearing implies that $p_{type2} = y_g^1/y_g^2$, and this will be an equilibrium price if $V_2(p_{type2}) > V_1(p_{type2})$. It can be checked that under the parameterization $c = 0, y_g = [16,10], k = [2,0]$, we have $p_{type1} = 1.2 < p_{type2} = 1.6$, and both prices correspond to respective equilibria. Moreover, we have $V_2(p_{type2}) > V_1(p_{type1})$, that is, the equilibria are Pareto ranked and the workers are better off in the non-monitoring equilibrium. Consequently the equilibrium with monitoring is monitoring suboptimal.

In this economy, monitoring uses good 1, hence when firms monitor, good 1 becomes scarcer and the relative price of good 2 decreases. Since good 2 is relatively more complementary with

[10] *The first term in this equation [the term in the brackets] captures the cost of less than full insurance for the agent. Since* $s(p) > 0$ and $s'(p) < 0$, *this term is always negative as expected. Moroever,* it is increasing in p *since its* derivative is given by $s'(p)\left[\frac{1-\exp(s(p))}{1+\exp(s(p))}\right] > 0$, *demonstrating that a higher price of good* 2 *makes the incentive constraints less severe and allows the non-monitoring firms to offer more insurance.*

leisure, this makes the incentive constraints more severe and makes monitoring an attractive option for the remaining firms. This opens the way for multiple equilibria: one in which no firm monitors since the incentive constraints are not very severe, and one in which all firms monitor since the incentive constraints are more severe. The monitoring equilibrium is Pareto worse since it corresponds to the case with relatively more severe incentive constraints.

This example demonstrates that there could be over-monitoring in equilibrium. A similar example (with three goods) can be constructed to show that the equilibrium may also feature under-monitoring.

5.4 Monitoring Decisions in Equilibrium

We end this section with a result that characterizes the equilibrium monitoring decisions when the cost of monitoring is sufficiently small. To present the result, we note the following assumption which puts more structure on the preferences and the cost function and which provides a tractable framework to analyze the trade-off between monitoring costs and better insurance provision. First, we classify the goods with a partition $\mathcal{G}^*$ such that the MRS of goods within each subset in the partition is independent of effort choice and the MRS of goods in different subsets in the partition always change with effort choice. The discussion after Theorem 7 provides an interpretation of the preferences that admit such a partition. Second, we also put more structure on the cost function by assuming that it is only possible to choose a monitoring partition which is comparable to $\mathcal{G}^*$, i.e. a monitoring partition should be either weakly finer or weakly coarser than $\mathcal{G}^*$. This assumption is natural in view of our classification of goods with the partition $\mathcal{G}^*$ and the interpretation we have provided.

Assumption A6^M (More Structure on Preferences and the Cost Function) *Assume that the state utility function has a representation*

$$u(x_s, e) = u^{\mathcal{G}^*}\left(u^{\{G_1\}}\left(x_s^{G_1}\right), u^{\{G_2\}}\left(x_s^{G_2}\right), .., u^{\{G_{|\mathcal{G}^*|}\}}\left(x_s^{G_{|\mathcal{G}^*|}}\right), e\right),$$

where $\mathcal{G}^* = \left\{G_1, .., G_{|\mathcal{G}^*|}\right\}$ *is a partition of* G, $\left\{u^{\{G_i\}}\right\}_{G_i \in \mathcal{G}^*}$ *are strictly increasing and continuously differentiable functions, and* $u^{\mathcal{G}^*}$ *is a continuously differentiable function strictly increasing in its first* $|\mathcal{G}^*|$ *arguments. Moreover, let* $E = \left\{e_1, .., e_{|E|}\right\}$ *and assume that this representation satisfies,*

$$\frac{\partial u^{\mathcal{G}^*}(x_s, e_i)/\partial m}{\partial u^{\mathcal{G}^*}(x_s, e_i)/\partial m'} < \frac{\partial u^{\mathcal{G}^*}(x_s, e_{i'})/\partial m}{\partial u^{\mathcal{G}^*}(x_s, e_{i'})/\partial m'} \text{ for all } m < m',\ i < i' \text{ and } x_s \in R_{++}^{|G|},$$

that is, the MRS between goods in different subsets in the partition $\mathcal{G}^*$ *change with effort choice so that goods in smaller indiced subsets are relatively more attractive when the* *higher indiced effort is chosen. Furthermore, given $K^{\max} > 0$, assume that $k(\mathcal{G}) \in [0, K^{\max}]$ for all $\mathcal{G}$ which is comparable to $\mathcal{G}^*$ (i.e. which satisfies either $\mathcal{G} \succeq \mathcal{G}^*$ or $\mathcal{G} \preceq \mathcal{G}^*$), while $k(\mathcal{G}) = (\infty, \infty, .., \infty)$ for any $\mathcal{G} \in P(G)$ which is not comparable to G^*.*[11]

Our next result shows that, when full insurance is not possible and the cost of monitoring the partition $\mathcal{G}^*$ is sufficiently small, firms monitor exactly the partition $\mathcal{G}^*$ (when monitoring strictly finer partitions is strictly costlier). As Theorem 2 establishes, monitoring nonseparable subsets is valuable since it enables better insurance provision while incentivizing the worker to exert the same level of effort. When monitoring is sufficiently cheap, the benefit of monitoring nonseparable subsets exceeds the cost of doing so, which gives the result that firms monitor a partition that is weakly finer than $\mathcal{G}^*$. Moreover, as Theorem 4 establishes, monitoring partitions that are strictly finer than $\mathcal{G}^*$ yield no additional benefits compared to monitoring just the partition $\mathcal{G}^*$. Hence, if monitoring partitions that are strictly finer than $\mathcal{G}^*$ is strictly costlier than monitoring $\mathcal{G}^*$, then firms monitor exactly $\mathcal{G}^*$.

Theorem 8 Consider economy $\mathcal{E}^M$ and suppose that Assumptions A1-A3 and A4^M-A6^M hold. Assume that there exists $e_{shirk} \in E$ such that $u(x, e_{shirk}) > u(x, e)$ for all $e \in E \setminus \{e_{shirk}\}$ and that there exists a state $s_{low} \in S$ such that $y_{s_{low}} = 0$ and $q_{s_{low}}(e_{shirk}) = 1$ (so that full insurance is not possible). There exists $\overline{K} > 0$ such that if $k(\mathcal{G}^*) < \overline{K}$, then any equilibrium allocation $\left(p, \left[c^M(\nu) = \left[\mathcal{G}(\nu), \left(x(\nu), e(\nu), w^{\mathcal{G}(\nu)}(\nu)\right)\right]\right]_{\nu \in \mathcal{N}}, z\right)$ satisfies $\mathcal{G}(\nu) \succeq \mathcal{G}^*$ [where $\mathcal{G}(\nu) \succ \mathcal{G}^*$ only if $k(\mathcal{G}(\nu)) = k(\mathcal{G}^*)$] for all but a measure zero of workers $\nu \in \mathcal{N}$.

Going back to our earlier interpretation of preferences that satisfy the representation in Assumption A6^M (see the discussion after Theorem 7); if the major aspects of worker needs that interfere with effort choice are vacations, procrastination, health care, and if these services are not too costly for the firm to monitor (e.g. if the firm could monitor how long a vacation the worker takes, how much time she spends in the office, the quality of the health care the worker receives), then Theorem 8 suggests a presumption that firms will monitor these services. Moreover, if the worker's relative ranking of the goods that make up the services does not depend on effort choice, for example if the worker's preference between going to Hawaii or Turkey for a vacation does not depend on how hard she works that year, then the theorem suggests a presumption that firms will not monitor consumption of the particular goods that make up the services.

[11] *We use the notation $k(\mathcal{G}) = (\infty, \infty, .., \infty)$ as a shorthand for arbitrarily large but finite costs of monitoring.*

6 Conclusion

This paper investigated the efficiency of competitive equilibria in environments with private information. Prescott and Townsend (1984a, 1984b) establish the constrained optimality of competitive equilibrium in such environments when (insurance, employment or credit) contracts can fully specify consumption bundles. Though important, these results are not applicable to situations in which individuals are allowed to trade in anonymous markets. We view such anonymous trading to be an essential feature of competitive equilibria. Less is known about the structure and efficiency of competitive equilibria in the presence of such anonymous trading.

A "folk theorem," originating in the work of Stiglitz and coauthors maintains that competitive equilibria are always or "generically" inefficient in such environments. This folk theorem has widespread applicability in both applied models and in policy discussions, though it has not been formally investigated. This paper critically reevaluates this folk theorem in the context of a general equilibrium economy with moral hazard. In our economy, firms offer contracts to workers who choose an effort level that is private information and affects the probability distribution of endowment and production vectors. We establish the existence of a competitive equilibrium and characterize some of its properties.

To investigate the efficiency properties of competitive equilibrium, we introduce a *monitoring partition* such that employment contracts can specify expenditures over subsets in the partition but cannot regulate how this expenditure is subdivided among the commodities within a subset. We say that preferences are *nonseparable* when the marginal rate of substitution across commodities within a subset in the partition depends on the effort level. We prove that the equilibrium is always inefficient when a competitive equilibrium allocation involves less than full insurance and preferences are nonseparable. While this result is consistent with the folk theorem on the inefficiency of competitive equilibrium, our main results show why such inefficiency does not always arise and can be mitigated by endogenous monitoring.

First, we show that most common preference structures does not satisfy the separability condition. In particular, when there is *partial separability* in preferences, competitive equilibrium with moral hazard are constrained optimal, in the sense that a social planner who can regulate and monitor all consumption levels cannot improve over these competitive allocations.

Second, we show how the monitoring partition can be endogenized by allowing firms to pay a cost in order to observe (and contract upon) expenditures over finer partitions of commodities. We prove that when the monitoring partition is endogenized and the costs of monitoring are sufficiently small, the competitive equilibrium is approximately constrained optimal. This result is somewhat surprising, since the competitive equilibrium still incurs monitoring costs, whereas the benchmark social planner allocation can costlessly monitor the consumption of all commodities. Our main results therefore imply that the strong suboptimality claims of the folk theorem for competitive equilibria in private information economies with anonymous trading may be misplaced. At the very least, considerable care is necessary in concluding that competitive equilibria are inefficient and government intervention is necessary without knowing the details of preference and information structure.

One of the contributions of our paper is to develop a framework for the analysis of competitive equilibria in which firms choose which subsets of commodities to monitor. This framework can be used for other purposes. One promising application is to use similar analysis to investigate which activities will be bundled within the boundaries of a single firm, thus linking the theory of the firm with general equilibrium analysis (see Zame, 2007, for a very interesting related approach). Another interesting direction would be to further investigate the efficiency of competitive equilibria with private information and anonymous trading by developing worst-case bounds on the social welfare gap in equilibrium and socially planned allocations. Finally, by putting more restrictions on preferences and technology, one can investigate the conditions under which competitive equilibrium allocations will involve too much or too little monitoring of consumption goods relative to a social planner who also incurs monitoring costs.

Appendix A: Proofs of Main Results

We first note the following Lemma which we use in the subsequent analysis. The result follows from Assumption A4 and the proof is omitted.

Lemma 1 Suppose Assumption A4 holds and let $c = (x, e, w) \in C^I(p, \mathcal{G})$ be an incentive compatible contract such that $x > 0$ and $w^{\mathcal{G}} > 0$. Then, for each $\epsilon > 0$, there exists incentive compatible contracts c_+ and c_- such that

$$V(c_-, p) > V(c) - \epsilon, \quad \pi(c_-, p) > \pi(c, p),$$

and

$$V(c_+, p) > V(c, p), \quad \pi(c_+, p) > \pi(c, p) - \epsilon.$$

Proof of Proposition 1. Let $\left(\left[x(\nu), e(\nu), w^{\mathcal{G}}(\nu)\right]_{\nu \in \mathcal{N}}, z\right)$ and p be an equilibrium allocation and price system. The second claim holds by the equilibrium conditions E3 and E4.

To prove the first claim, first note that firms can always offer null contracts and guarantee themselves 0 profits, hence, by the equilibrium condition E2, $\pi(c(\nu), p) \geq 0$, that is, the equilibrium contract $c(\nu)$ is in the constraint set of Problem (8) for all but a measure zero of workers. We claim that $c(\nu)$ solves Problem (8) for all but a measure zero of workers. Suppose the contrary, that is, there exists a positive measure set $\mathcal{N}^* \subset \mathcal{N}$ and contracts $\hat{c}(\nu)|_{\nu \in \mathcal{N}^*}$ such that $c(\nu)$ and $\hat{c}(\nu)$ are both in the constraint set of Problem (8), yet the latter results in a greater expected utility for the worker ν than $V(c(\nu), p)$. Since $\mathcal{N}^* \subset \mathcal{N}$ is of positive measure and the set of firms is finite, there exists a firm j' such that the set $\mathcal{N}^{j'} = \{\nu \in \mathcal{N}^* \mid J(\nu) = j'\}$ is of positive measure. By Assumption A1, we have that the contract $\hat{c}(\nu)$ satisfies $\hat{x}(\nu) > 0$ and $\hat{w}^{\mathcal{G}}(\nu) > 0$, hence by Lemma 1, there exist an incentive compatible contract $\hat{c}_-(\nu)|_{\nu \in \mathcal{N}^{j'}}$ such that

$$V(\hat{c}_-(\nu), p) > V(\hat{c}(\nu), p) - \epsilon \text{ and } \pi(\hat{c}_-(\nu), p) > \pi(\hat{c}(\nu), p) \geq 0,$$

where $\epsilon \equiv V(\hat{c}(\nu), p) - V(c, p) > 0$. We claim that another firm j'' can strictly increase its expected profits by offering the workers in $\mathcal{N}^{j'}$ the allocation $(\hat{c}_-(\nu))|_{\nu \in \mathcal{N}^{j'}}$. We have $J(\nu) = j'$ and $V(\hat{c}_-(\nu), p) > V(c(\nu), p)$, hence it must be the case that every $\nu \in \mathcal{N}^{j'}$ now accepts firm j'''s offer. Then, after the deviation, the profit accruing to firm j'' from the workers in $\mathcal{N}^{j'}$ satisfies

$$\int_{\mathcal{N}^{j'}} \pi(\hat{c}_-(\nu), p)\, dv > 0,$$

that is, firm j'' makes profits from the new customers that it signs. Hence firm j'' has a profitable deviation, which contradicts the equilibrium condition E2.

Next, we show that the constraint $\pi\left(\left(x,e,w^{\mathcal{G}}\right),p\right)\geq 0$ is binding for any solution $\left(x,e,w^{\mathcal{G}}\right)$ to Problem (8). Suppose the contrary, i.e., there exists a maximizer of Problem (8) $c = \left(x,e,w^{\mathcal{G}}\right)$ such that $\pi\left(c,p\right) > 0$. Then, by Lemma 1, there exists an incentive compatible contract c_{+}, such that $\pi\left(c_{+},p\right)\geq 0$, and $V\left(c_{+},p\right) > V\left(c,p\right)$, which is a contradiction to the fact that c is a solution to Problem (8).

Now consider the converse of the argument; let p and $\left[x\left(\nu\right),e\left(\nu\right),w^{\mathcal{G}}\left(\nu\right)\right]_{\nu\in\mathcal{N}}$ be a price and allocation system that satisfies the claims of the proposition. We conjecture a symmetric equilibrium in which every firm offers the same contract. Let $c\left(\nu,j\right) = \left[x\left(\nu\right),e\left(\nu\right),w^{\mathcal{G}}\left(\nu\right)\right]$ for all $\nu\in\mathcal{N}$ and $j\in J$. Divide the set $\mathcal{N}$ into sets of equal measure, denoted by $\left\{\mathcal{N}^{1},\mathcal{N}^{2},..,\mathcal{N}^{|J|}\right\}$. Let the worker choice $\mathbf{J}_{\nu}$ be equal to

$$\mathbf{J}_{\nu}\left(c\left(\nu,j\right)\ |_{j\in J}\right) = j' \text{ for } \nu\in\mathcal{N}^{j'}$$

for the equilibrium contract set, and equal to an arbitrary selection that satisfies (6) for an off equilibrium contract set. We claim that the firm choices and the worker's strategies that we describe constitute an equilibrium with the price and allocation system p and $\left[x\left(\nu\right),e\left(\nu\right),w^{\mathcal{G}}\left(\nu\right)\right]_{\nu\in\mathcal{N}}$.

The equilibrium condition E1 is satisfied by definition for an off equilibrium contract offer set, and is satisfied for the equilibrium contract set since the workers face symmetric offers. Commodity market clearing condition E3 is satisfied by assumption. To prove that condition E2 holds, suppose the contrary, that is, there exists a firm j' who can deviate by offering a collection of incentive compatible contracts $\left[\hat{c}\left(\nu,j'\right)\right]|_{\nu\in\mathcal{N}}$ and make strictly positive profits. Then, there exists $\mathcal{N}^{*}\subset\mathcal{N}$ with positive measure such that for $\nu\in\mathcal{N}^{*}$,

$$J\left(\nu\right) = j' \text{ after the deviation by firm } j'\text{, and}$$

$$\pi\left(\hat{c}\left(\nu,j'\right),p\right) > \pi\left(c\left(\nu\right),p\right) = 0. \tag{30}$$

Since the worker $\nu\in\mathcal{N}^{*}$ chooses the contract offered by j' over her previous contract, we have

$$V\left(\hat{c}\left(\nu,j'\right),p\right)\geq V\left(c\left(\nu\right),p\right). \tag{31}$$

By Lemma 1 and equations (30) and (31), there exists an incentive compatible allocation $\hat{c}_{+}\left(\nu,j'\right)$ such that

$$\pi\left(\hat{c}_{+}\left(\nu,j'\right),p\right) > 0, \text{ and } V\left(\hat{c}_{+}\left(\nu,j'\right),p\right) > V\left(c\left(\nu\right),p\right).$$

Hence $c\left(\nu\right)$ is not a solution to Problem (8) for $\nu\in\mathcal{N}^{*}$, which is a contradiction to the assumption in the proposition. We conclude that E2 also holds, and the strategies, the price vector, and allocations constructed as above constitute an equilibrium, completing the proof of the proposition. ■

Proof of Theorem 1. The key step in establishing the existence of the equilibrium is to show that the solution correspondence to Problem (8) is upper hemicontinuous in p. Note that, even with Assumption A5, Berge's maximum Theorem does not apply to Problem (8). Since π is a linear function in p, the solution correspondence $\{c \mid \pi(c,p) \geq 0\}$, which is the second constraint to Problem (8), is also continuous in p. However, the intersection of two continuous correspondences is not guaranteed to be lower hemicontinuous, hence Berge's maximum Theorem does not immediately apply. Nevertheless, Assumptions A4 and A5 together put enough structure on the problem to apply the Generalized Maximum Theorem by Ausubel and Deneckere (1990) and show that the solution set to Problem (8) is an upper hemicontinuous correspondence. Since a direct application of their theorem requires us to introduce additional notation, our proof derives this from the first principles.

Let $\left(x_n, e_n, w_n^{\mathcal{G}}\right)|_{n\in\{1,2,..\}}$ be solutions to this problem for $(p_n)|_{n\in\{1,2,..\}}$ such that

$$\lim_{n\to\infty}\left(x_n, e_n, w_n^{\mathcal{G}}\right) = \left(x, e, w^{\mathcal{G}}\right) \text{ and } \lim_{n\to\infty} p_n = p.$$

Note that $\left(x, e, w^{\mathcal{G}}\right)$ satisfies the constraints of Problem (8) for the price vector p. We claim that $\left(x, e, w^{\mathcal{G}}\right)$ is a solution to Problem (8) for the price vector p. Suppose the contrary, then there exists $\left(\hat{x}, \hat{e}, \hat{w}^{\mathcal{G}}\right)$ that satisfies the constraints of Problem (8) and yields a strictly greater utility for the worker. By Lemma 1, there exists another contract $\left(\overline{x}, \overline{e}, \overline{w}^{\mathcal{G}}\right)$ that satisfies the incentive compatibility constraints, that satisfies the non-zero profit constraint strictly, that is,

$$\pi\left(\left(\overline{x}, \overline{e}, \overline{w}^{\mathcal{G}}\right), p\right) > 0, \tag{32}$$

and that yields a strictly greater utility for the worker, that is,

$$U(\overline{x}, \overline{e}) > U(x, e). \tag{33}$$

Since $\left(\overline{x}, \overline{e}, \overline{w}^{\mathcal{G}}\right) \in C^I(p, \mathcal{G})$, by Assumption A5, there exists $\left(\overline{x}_n, \overline{e}_n, \overline{w}_n^{\mathcal{G}}\right) \to \left(\overline{x}, \overline{e}, \overline{w}^{\mathcal{G}}\right)$ such that $\left(\overline{x}_n, \overline{e}_n, \overline{w}_n^{\mathcal{G}}\right) \in C^I(p_n, \mathcal{G})$ for each n. Since π and U are continuous functions, and using equations (32) and (33), there exists n such that

$$\pi\left(\left(\overline{x}_n, \overline{e}_n, \overline{w}_n^{\mathcal{G}}\right), p_n\right) > 0, \text{ and } U(\overline{x}_n, \overline{e}_n) > U(x_n, e_n).$$

But since $\left(\overline{x}_n, \overline{e}_n, \overline{w}_n^{\mathcal{G}}\right)$ is incentive compatible given p_n, the previous inequalities contradict the fact that $\left(x_n, e_n, w_n^{\mathcal{G}}\right)$ is a solution to Problem (8) given the price vector p_n. We conclude that the solution correspondence to Problem (8) is upper hemicontinuous.

The rest of the proof follows standard arguments. For a given price p, define the excess demand correspondence by

$$D(p) = \left\{ \begin{array}{c} \int_{\mathcal{N}} \sum_{s \in S} (x_s(\nu) - y_s) q_s(e(\nu)) d\nu - z \in \mathbb{R}^{|G|} \mid \left(x(\nu), e(\nu), w^{\mathcal{G}}(\nu)\right) \\ \text{solves Problem (8) for each } \nu \in \mathcal{N}\text{; and } z \in Z^*(p). \end{array} \right\} \tag{34}$$

Since there is a continuum of individuals, the demand correspondence $D(p)$ is convex valued. Since the solution set to Problem (8) is an upper hemicontinuous correspondence in p, $D(p)$ is also upper hemicontinuous in p. We have, $\lim_{p^g \to 0} D^g(p) > 0$, since the supply of good g is bounded from above, while the demand of each worker tends to infinity as $p^g \to 0$ by Assumption A1. Similarly, $\lim_{p^g \to \infty} D^g(p) < 0$ since the supply is bounded from below by a positive number by the assumption $y > 0$, while, as $p^g \to \infty$,the demand of each worker tends to zero by the budget constraint of the worker and Assumption A1. Therefore, Kakutani's Fixed Point Theorem applies to this economy and there exists p such that $0 \in D(p)$. By definition of $D(p)$, there exists $\left(x(\nu), e(\nu), w^{\mathcal{G}}(\nu)\right)|_{\nu \in \mathcal{N}}$ such that $\left(x(\nu), e(\nu), w^{\mathcal{G}}(\nu)\right)$ solves Problem (8) for each $\nu \in \mathcal{N}$ and the commodity markets clear. By Proposition 1, the price vector p and the allocations $\left(x(\nu), e(\nu), w^{\mathcal{G}}(\nu)\right)|_{\nu \in \mathcal{N}}$ correspond to an equilibrium, completing the proof of the theorem. ■

Proof of Theorem 2. Consider a worker $\nu \in N$ and denote her allocation by $\bar{c} \equiv \left(\overline{x}, \overline{e}, \overline{w}^{\mathcal{G}}\right) \equiv \left(x(\nu), e(\nu), w^{\mathcal{G}}(\nu)\right)$. Since $(\overline{x}, \overline{e})$ does not feature full insurance, there exists $s_1, s_2 \in S$ and $g \in G$ such that the MRS for good g between states s_1 and s_2 is not equal to 1. We will reallocate the consumption of good g, $\overline{x}^g$, across states so that the worker receives better insurance while her equilibrium effort choice $\overline{e}$ remains incentive compatible. Formally, we will show that there exists $\tilde{x}^g \in \mathbb{R}_+^{|S|}$ such that

$$\sum q_s(\overline{e})\, \tilde{x}_s^g = \sum q_s(\overline{e})\, \overline{x}_s^g, \tag{35}$$

$$\overline{e} \in \arg\max_{e \in E} U\left(\overline{x}^{\mathcal{G}\setminus\{g\}}, \tilde{x}^{\{g\}}, e\right), \tag{36}$$

$$\text{and } U\left(\overline{x}^{\mathcal{G}\setminus\{g\}}, \tilde{x}^{\{g\}}, \overline{e}\right) > U(x, \overline{e}). \tag{37}$$

Once we show the existence of such $\tilde{x}^{\{g(\nu)\}}(\nu)$ for each $\nu \in N$, it follows that the equilibrium allocation is constrained suboptimal since the allocation $\left(\left[(x(\nu), e(\nu))|_{\nu \in \mathcal{N}\setminus N}, \left(x^{\mathcal{G}\setminus\{g(\nu)\}}(\nu), \tilde{x}^{\{g(\nu)\}}(\nu), e(\nu)\right)|_{\nu \in N}\right], z\right)$ is incentive compatible for each $\nu \in \mathcal{N}$, satisfies the resource constraints, and strictly improves the utility of a non-zero measure of individuals while leaving the rest of the individuals indifferent.

Since the MRS for good g between states s_1 and s_2 is not equal to 1 (where $q(s_1) > 0$ and $q(s_2) > 0$), for any $\epsilon > 0$, there exists a vector $v_\epsilon \in \mathbb{R}$ with length in $(0, \epsilon)$, such that $\tilde{x}^{\{g\}} =$

$\overline{x}^{\{g\}} + v_\epsilon$ satisfies the resource constraints in (35) and $U\left(\overline{x}^{\mathcal{G}\backslash\{g\}}, \overline{x}^{\{g\}} + v_\epsilon, \overline{e}\right) > U(\overline{x}, \overline{e})$. Fix a sequence of such vectors $\left(v_{1/n}\right)_{n=1}^{\infty}$. We claim that there exists n such that $\tilde{x}^{\{g\}} = \overline{x}^{\{g\}} + v_{1/n}$ also satisfies the incentive compatibility constraint in (36). Suppose the contrary, then, for each element of the sequence $\left(v_{1/n}\right)_{n=1}^{\infty}$, there exists $e_n \in E\backslash\ \overline{e}$ such that

$$U\left(\overline{x}^{\mathcal{G}\backslash\{g\}}, \overline{x}^{\{g\}} + v_{1/n}, e_n\right) > U(\overline{x}, \overline{e}). \tag{38}$$

Since $E\backslash\overline{e}$ is a finite set, there exists $\tilde{e} \in E\backslash\overline{e}$ such that $e_n = \tilde{e}$ for infinitely many values of n. Consider the sequence of indices $(n_j)_{j=1}^{\infty}$ such that $e_{n_j} = \tilde{e}$. Since $\lim_{j\to\infty} n_j \to \infty$, we have $\lim_{j\to\infty} v_{1/n_j} \to 0$. Then, considering equation (38) for $n \in (n_j)_{j=1}^{\infty}$, and taking the limit as $j \to \infty$, we have

$$U(\overline{x}, \tilde{e}) \geq U(\overline{x}, \overline{e}),$$

for some $\tilde{e} \neq \overline{e}$.

Note that, since preferences are nonseparable at $(\overline{x}, \overline{e})$, there exists $g_1, g_2 \in G_m \in \mathcal{G}$ and $s \in S$ with $q_s(e) > 0$ for all $e \in E$ such that

$$\frac{du(\overline{x}_s, \tilde{e})/dx_s^{g_1}}{du(\overline{x}_s, \tilde{e})/dx_s^{g_2}} \neq \frac{du(\overline{x}_s, \overline{e})/dx_s^{g_1}}{du(\overline{x}_s, \overline{e})/dx_s^{g_2}} = \frac{p^{g_1}}{p^{g_2}},$$

where the last equality follows since $\overline{c} \in C^I(p, \mathcal{G})$. The last equation further implies that $\mathbf{x}_s^{\{g_1,g_2\}}\left(\overline{w}^{\mathcal{G}}, p, \tilde{e}\right) \neq \overline{x}_s^{\{g_1,g_2\}}$, and since $q_s(\tilde{e}) > 0$, this further implies

$$U\left(\mathbf{x}\left(\overline{w}^{\mathcal{G}}, p, \tilde{e}\right), \tilde{e}\right) > U(\overline{x}, \tilde{e}) = U(\overline{x}, \overline{e}),$$

and yields a contradiction in view of the fact that $\left(\overline{x}, \overline{e}, \overline{w}^{\mathcal{G}}\right)$ is an incentive compatible contract. Therefore, there exists a vector $v_{1/n}$ such that $\tilde{x}^{\{g\}} = \overline{x}^{\{g\}} + v_{1/n}$ satisfies equations $(35)-(37)$, which completes the proof of the theorem. ∎

Proof of Theorem 4. By the equilibrium conditions, $((x(\nu), e(\nu))|_{\nu\in\mathcal{N}}, z)$ is incentive feasible. Assume, for sake of contradiction, that $((x(\nu), e(\nu))|_{\nu\in\mathcal{N}}, z)$ is constrained suboptimal. Then, there exists an incentive feasible allocation $((\hat{x}(\nu), \hat{e}(\nu))|_{\nu\in\mathcal{N}}, \hat{z})$ such that

$$U(\hat{x}(\nu), \hat{e}(\nu)) \geq U(x(\nu), e(\nu)) \tag{39}$$

with strict inequality for a positive measure of $\nu \in \mathcal{N}$.

Consider $\nu \in \mathcal{N}$ such that the inequality (39) holds, and drop the ν's from the notation for convenience. We consider the contract $\hat{c} = (\overline{x}, \hat{e}, \overline{w}^{\mathcal{G}})$, where, for each $s \in S$ and $G_m \in \mathcal{G}$, $\overline{w}_s^{\{G_m\}}$ is defined as the minimum value and $\overline{x}_s^{G_m}$ as a solution to the following convex minimization problem:

$$\begin{aligned} & \min_{x_s^{G_m}} \ x_s^{G_m} p^{G_m} \\ \text{such that} \quad & u^{\{G_m\}}(x_s^{G_m}) = u^{\{G_m\}}(\hat{x}_s^{G_m}). \end{aligned} \tag{40}$$

We claim that the contract $\hat{c}$ is incentive compatible given prices p but yields a non-positive profit to the firm, that is,

$$\pi(\hat{c}, p) = \sum_{s \in S} q_s(e)(y_s p - \sum_{G_m \in \mathcal{G}} \overline{w}_s^{\{G_m\}}) + \pi^Z(p) \leq 0. \tag{41}$$

We first show that $\hat{c} \in C^I(p, \mathcal{G})$. For each $G_m \in \mathcal{G}$, let $v^{\{G_m\}}(w_s^{\{G_m\}}, p)$ be given as the optimal value of the following strictly convex optimization problem:

$$\begin{aligned} v^{\{G_m\}}(w_s^{\{\mathcal{G}_m\}}, p) \quad &\equiv \quad \max_{x_s^{G_m} \geq 0} \ u^{\{G_m\}}(x_s^{G_m}) \\ \text{such that} \quad & \quad x_s^{G_m} p^{G_m} \leq w_s^{\{G_m\}}. \end{aligned} \tag{42}$$

The partial separability assumption implies that, $\overline{x} = \mathbf{x}\left(\overline{w}^{\mathcal{G}}, p, \hat{e}\right)$ if and only if $\overline{x}_s^{G_m}$ solves Problem (42) given $(\overline{w}_s^{\{\mathcal{G}_m\}}, p)$ for each $s \in S$ and $G_m \in \mathcal{G}$, and $\hat{e} \in \mathbf{e}\left(\overline{w}^{\mathcal{G}}, p\right)$ if and only if

$$\hat{e} \in \arg\max_{e \in E} \sum_{s \in S} q_s(e) u^{\mathcal{G}}\left(\left\{v^{\{G_m\}}\left(\overline{w}_s^{\{G_m\}}, p\right)\right\}_{G_m \in \mathcal{G}}, e\right). \tag{43}$$

Since Problems (42) and (40) are dual to each other, $\overline{x}_s^{G_m}$ solves (42) given $(\overline{w}_s^{\{G_m\}}, p)$. Moreover, we also have

$$u^{\{G_m\}}(\hat{x}_s^{G_m}) = u^{\{G_m\}}(\overline{x}_s^{G_m}) = v^{\{G_m\}}(\overline{w}_s^{\{G_m\}}, p).$$

This, together with the fact that $(\hat{x}, \hat{e})$ satisfies the incentive compatibility condition in (10), implies that $\hat{e}$ solves Problem (43). Hence $\hat{c}$ is incentive compatible. Moreover, we have

$$\begin{aligned} U(\overline{x}, \hat{e}) \quad &= \quad \sum_{s \in S} q_s(\hat{e}) u^{\mathcal{G}}\left(\left\{u^{\{G_m\}}\left(\overline{x}_s^{G_m}\right)\right\}_{G_m \in \mathcal{G}}, \hat{e}\right) \\ &= \quad \sum_{s \in S} q_s(\hat{e}) u^{\mathcal{G}}\left(\left\{u^{\{G_m\}}\left(\hat{x}_s^{G_m}\right)\right\}_{G_m \in \mathcal{G}}, \hat{e}\right) \\ &= \quad U(\hat{x}, \hat{e}) \geq U(x, e), \end{aligned} \tag{44}$$

where the last inequality follows from (39). The inequality in (44) implies that the incentive compatible contract $\hat{c} = (\overline{x}, \hat{e}, \overline{w}^{\mathcal{G}})$ yields at least as high a utility to workers as the equilibrium contract $c = \left(x, e, w^{\mathcal{G}}\right)$. Now suppose that the inequality in (41) doesn't hold, that is, the contract $\hat{c}$ yields a positive profit to the firm. Then, by Lemma 1 and in view of the inequality in (44), there exists another incentive compatible contract, $\hat{c}_+$ such that

$$\pi\left(\hat{c}_+, p\right) \geq 0, \text{ and } V\left(\hat{c}_+, p\right) > V\left(\hat{c}, p\right) = U\left(\hat{x}, \hat{e}\right) \geq U\left(x, e\right) = V\left(c, p\right),$$

which is a contradiction to the fact that c solves Problem (8), proving that the inequality in (41) holds.

Note that, since $\overline{w}_s^{\{G_m\}}$ is the optimal value of Problem (40) and $\hat{x}_s^{G_m}$ is in the constraint set, we have, for each s,

$$\hat{x}_s^{G_m} p^{G_m} \geq \overline{w}_s^{\{G_m\}},$$

which, together with Eq. (41), implies

$$\sum_{s \in S} q_s\left(\hat{e}\left(\nu\right)\right)\left(y_s - \hat{x}_s\left(\nu\right)\right)p + \pi^Z\left(p\right) \leq 0, \tag{45}$$

where we have put the worker index ν back into the notation. It can be seen that, for $\nu \in \mathcal{N}$ such that the inequality in (39) is strict, the inequality in (45) is also strict. Moreover, since $z \in Z^*\left(p\right)$, we have $\pi^Z\left(p\right) = zp \geq \hat{z}p$. Hence, integrating equation (45) over $\nu \in \mathcal{N}$, we have

$$\hat{z}p + \int_{\mathcal{N}} \sum_{s \in S} q_s\left(\hat{e}\left(\nu\right)\right)\left(y_s - \hat{x}_s(\nu)\right)p d\nu < 0.$$

Finally, the last equation yields a contradiction since $\left(\hat{x}\left(\nu\right), \hat{e}\left(\nu\right)|_{\nu \in \mathcal{N}}, \hat{z}\right)$ satisfies the resource constraints in Definition 2.

We prove the following Lemma which we use in the proofs of Theorems 6 and 7. ■

Lemma 2 *Let Assumptions A1-A3 and A4^M-A5^M hold. Consider the problem*

$$\begin{aligned} P\left(\delta, p, \mathcal{G}\right) : v\left(\delta, p, \mathcal{G}\right) \equiv & \max_{c \in C^I(p, \mathcal{G})} V^{\mathcal{G}}\left(c, p\right) \\ s.t. \quad & \pi^{\mathcal{G}}\left(c, p\right) \geq \delta, \end{aligned} \tag{46}$$

of maximizing worker utility subject to incentive compatibility constraints and the constraint that profits exceed a fixed level δ. Let $\left(\delta \in \mathbb{R}, p \in \mathbb{R}_{++}^{|G|}, \mathcal{G} \in \mathcal{P}\left(G\right)\right)$ be a configuration of parameters such that the constraint set of $\mathcal{P}\left(\delta, p, \mathcal{G}\right)$ is non-empty in a neighborhood of the parameters $\left(\delta, p\right)$. Then, v is a well defined continuous function in a neighborhood of $\left(\delta, p\right)$ which is strictly decreasing in δ.

To see why Problem 46 and Lemma 2 are useful, note that by Eq. (25), given prices p, a contract ($\mathcal{G}$,c) is part of a monitoring equilibrium only if

$$v\left(k\left(\mathcal{G}\right)p, p, \mathcal{G}\right) \geq v\left(k\left(\mathcal{G}'\right)p, p, \mathcal{G}'\right) \text{ for any } \mathcal{G}' \in \mathcal{P}\left(G\right). \tag{47}$$

Note also that the equilibrium utility of a worker is given by $v\left(k\left(\mathcal{G}\right)p, p, \mathcal{G}\right)$.

Proof of Lemma 2. Similar to Problem 8, $P\left(\delta, p, \mathcal{G}\right)$ is maximizing a continuous function over a compact set and hence has a solution whenever the constraint set is non-empty. It follows by Assumption A4^M that v is strictly decreasing in δ. We next show that v is continuous in δ and p with an argument similar to the proof of Theorem 1. Consider a sequence $\{\delta_{n'}, p_{n'}\}_{n'=1}^{\infty} \to (\delta, p)$ and let $c_{n'} \equiv \left(x_{n'}, e_{n'}, w_{n'}^{\mathcal{G}}\right)$ be the solution to Problem $P\left(\delta_{n'}, p_{n'}, \mathcal{G}\right)$ for each n'. Since $\left(x_{n'}, e_{n'}, w_{n'}^{\mathcal{G}}\right)$ lies in a compact set (namely the constraint set), it has a convergent subsequence $\left(x_n, e_n, w_n^{\mathcal{G}}\right) \to \left(x, e, w^{\mathcal{G}}\right)$. We claim that $c \equiv \left(x, e, w^{\mathcal{G}}\right)$ solves Problem $P\left(\delta, p, \mathcal{G}\right)$ which in turn proves that $v\left(\delta_n, p_n, \mathcal{G}\right) \to v\left(\delta, p, \mathcal{G}\right)$, i.e. that v is continuous in δ and p. Suppose the contrary, i.e. suppose that there exists $\hat{c} \equiv \left(\hat{x}, \hat{e}, \hat{w}^{\mathcal{G}}\right)$ which is in the constraint set of Problem $P\left(\delta, p, \mathcal{G}\right)$ and which satisfies $V^{\mathcal{G}}\left(\hat{c}, p\right) > V^{\mathcal{G}}\left(c, p\right)$. Since Assumption A4^M holds, there exists some $\hat{c}_- \equiv \left(\overline{x}, \overline{e}, \overline{w}^{\mathcal{G}}\right)$ such that $\hat{c}_- \in C^I\left(p, \mathcal{G}\right)$ and

$$V^{\mathcal{G}}\left(\hat{c}_-, p\right) > V^{\mathcal{G}}\left(c, p\right), \text{ and } \pi^{\mathcal{G}}\left(\hat{c}_-, p\right) > \delta. \tag{48}$$

Since $p_n \to p$ and since $C^I\left(p, \mathcal{G}\right)$ is lower hemicontinuous by Assumption A5^M, there exists a sequence $\hat{c}_n \to \hat{c}_-$ such that $\hat{c}_n \in C^I\left(p_n, \mathcal{G}\right)$ for each n. Since $p_n \to p$ and V^G is continuous, we have $V^{\mathcal{G}}\left(\hat{c}_n, p_n\right) \to V^{\mathcal{G}}\left(\hat{c}_-, p\right)$ and $V^{\mathcal{G}}\left(c_n, p_n\right) \to V^{\mathcal{G}}\left(c, p\right)$, and hence, in view of the first inequality in (48), there exists sufficiently large $\overline{n}_1$ such that for all $n > \overline{n}_1$ we have $V^{\mathcal{G}}\left(\hat{c}_n, p_n\right) > V^{\mathcal{G}}\left(c_n, p_n\right)$. Moreover,since $\pi^{\mathcal{G}}$ is continuous we have $\pi^{\mathcal{G}}\left(\hat{c}_n, p_n\right) \to \pi^G\left(\hat{c}_-, p\right)$ and hence, in view of the fact that $\delta_n \to \delta$ and the second inequality in (48), there exists sufficiently large $\overline{n}_2$ such that for all $n > \overline{n}_2$ we have $\pi^{\mathcal{G}}\left(\hat{c}_n, p_n\right) > \delta_n$. Consequently, we have

$$V^{\mathcal{G}}\left(\hat{c}_n, p_n\right) > V^{\mathcal{G}}\left(c_n, p_n\right) \text{ and } \pi^{\mathcal{G}}\left(\hat{c}_n, p_n\right) > \delta_n \text{ for all } n > \max\left(\overline{n}_1, \overline{n}_2\right).$$

This yields a contradiction in view of the fact that the contract $\hat{c}_n$ satisfies the constraints of Problem $P\left(\delta_n, p_n, \mathcal{G}\right)$ for all $n > N$ and attains a higher utility than the contract c_n. Hence c solves Problem $P\left(\delta, p, \mathcal{G}\right)$ and $v\left(\delta, p, \mathcal{G}\right)$ is continuous in δ and p as desired. ■

Proof of Theorem 6. As a preliminary step, we show that when the level of monitoring costs are restricted to lie in a compact set, the equilibrium prices also lie in a compact subset P of $\mathbb{R}_{++}^{|G|}$. To see this, restrict the monitoring cost level K to lie in a compact set $[0, K^{\max}]$ for some $K^{\max} > 0$. Recall that the analysis in the proof of Theorem 1 establishes that the excess demand correspondence $D\left(p\right)$ is upper hemicontinuous in p. The same argument establishes that the excess demand correspondence $D\left(p, K\right)$ is upper hemicontinuous in both p and K. Then, the equilibrium price correspondence

$$P^{eq}\left(K\right) \equiv \left\{p \in \mathbb{R}_+^{|G|} \mid p_1 = 1,\ D\left(p, K\right) = 0\right\}$$

is also upper hemicontinuous since it is the intersection of a upper hemicontinuous correspondence with a hyperplane. Fix some $K \in [0, K_{\max}]$ and note that the analysis in the proof of Theorem 1 also establishes that there exists $p_{\min}(K) > 0$ and $p_{\max}(K) > 0$ such that $D^g(p_{\min}(K)) > 0$ and $D^g(p_{\max}(K)) < 0$ for each $g \in G$. Consequently, $P^{eq}(K) \subset [p_{\min}(K), p_{\max}(K)] \subset \mathbb{R}^{|G|}_{++}$, that is the equilibrium price set is bounded for a given parameter K. Moreover, $P^{eq}(K)$ is also closed valued since $D(p, K)$ is upper hemicontinuous in p, which further implies that $P^{eq}(K)$ is compact valued. Since $P^{eq}(K)$ is a compact valued and upper hemicontinuous correspondence, the set $P \equiv \cup_{K \in [0, K^{\max}]} P^{eq}(K)$ is compact. Since $P^{eq}(K)$ lies in $\mathbb{R}^{|G|}_{++}$ for each $K \in [0, K^{\max}]$, so does P. We conclude that whenever $K \in [0, K^{\max}]$, any equilibrium price lies in a compact set $P \subset \mathbb{R}^{|G|}_{++}$, as desired. Assume that $K \in [0, K^{\max}]$ for the rest of the proof.

We next prove the theorem. Let $\mathcal{G}^{\max} = \left\{\{g_1\}, .., \left\{g_{|G|}\right\}\right\}$ denote the finest partition in $\mathcal{P}(G)$. Let v be the function defined in Lemma 2 and let $\eta(\epsilon, p)$ be defined as the unique real number that satisfies

$$v(0, p, \mathcal{G}^{\max}) = v(\eta(\epsilon, p), p, \mathcal{G}^{\max}) + \epsilon.$$

Note that η is well defined since v is continuous and strictly decreasing in its first variable (cf. Lemma 2). Moreover, η is continuous in ϵ and p, is increasing in ϵ and satisfies $\eta(\epsilon, p) \gtreqless 0$ if $\epsilon \gtreqless 0$. Let $\overline{K}_\epsilon$ be given by

$$\overline{K}_\epsilon \equiv \min_{p \in P} \left(\eta(\epsilon, p) / k^r(\mathcal{G}^{\max}) p\right),$$

where $\overline{K}_\epsilon > 0$ since η is continuous and P is a compact subset of $\mathbb{R}^{|G|}_{++}$. Note that whenever $K < \overline{K}_\epsilon$, we have

$$k(\mathcal{G}^{\max}) p = K k^r(\mathcal{G}^{\max}) p < \eta(\epsilon, p) \text{ for all } p \in P. \tag{49}$$

Let $\left(p, \left[\left[\mathcal{G}(\nu), c(\nu) = \left(x(\nu), e(\nu), w^{\mathcal{G}(\nu)}(\nu)\right)\right]_{\nu \in \mathcal{N}}, z\right]\right)$ be an equilibrium allocation. To get a contradiction, suppose that it is not $\epsilon-$constrained optimal, that is, suppose that there is an allocation $\left([\hat{x}(\nu), \hat{e}(\nu)]_{\nu \in \mathcal{N}}, \hat{z}\right)$ which is incentive feasible and which improves the utility of every worker by at least ϵ. Fix some $\nu \in \mathcal{N}$ and let $\left(\mathcal{G}, c = \left(x, e, w^{\mathcal{G}}\right)\right)$ denote the equilibrium allocation and $(\hat{x}, \hat{e})$ denote the ϵ-improving allocation corresponding to $\nu \in \mathcal{N}$. We claim that the allocation $(\hat{x}, \hat{e})$ generates net losses at equilibrium price p, i.e. we claim

$$\sum_{s \in S} q_s(\hat{e})(y_s - \hat{x}_s) p + \pi^Z(p) < 0. \tag{50}$$

To prove the claim in (53), consider the contract $\hat{c} = \left(\mathcal{G}^{\max}, \left(\hat{x}, \hat{e}, \left\{\hat{w}_s^{\{G_m\}} \equiv \hat{x}_s^{G_m} p^{G_m}\right\}_{s \in S, G_m \in \mathcal{G}^{\max}}\right)\right)$. Since $\mathcal{G}^{\max}$ is the finest monitoring partition and $\hat{e}$ is incentive compatible given $\hat{x}$, we have that $\hat{c} \in C^I(p, \mathcal{G}^{\max})$. We also have

$$\begin{aligned} V^{\mathcal{G}^{\max}}(\hat{c}, p) = U(\hat{x}, \hat{e}) &\geq V^{\mathcal{G}}(c, p) + \epsilon \\ &\geq v(k(\mathcal{G}^{\max}) p, p, \mathcal{G}^{\max}) + \epsilon \\ &> v(\eta(\epsilon, p), p, \mathcal{G}^{\max}) + \epsilon \\ &= v(0, p, \mathcal{G}^{\max}), \end{aligned} \tag{51}$$

where the first inequality uses the fact that $(\hat{x}, \hat{e})$ is ϵ-improving over the equilibrium allocation, the second inequality uses the revealed preference type argument that the equilibrium is weakly better than the best utility attained by offering $\mathcal{G}^{\max}$ [cf. Eq. (47)], the third inequality follows from Eq. (49), and the forth line uses the definition of $\eta(\epsilon, p)$. By the inequality $V^{\mathcal{G}^{\max}}(\hat{c}, p) > v(0, p, \mathcal{G}^{\max})$ established in (51), the definition of v as the solution to Problem (46), and the fact that $\hat{c} \in C^I(p, \mathcal{G}^{\max})$, we must have $\pi^{\mathcal{G}^{\max}}(\hat{c}, p) < 0$, which is equivalent to our claim in (53).

Next, we put $\nu \in \mathcal{N}$ back into the notation and add up the inequality in Eq. (53) over all $\nu \in \mathcal{N}$, which yields

$$\int_{\mathcal{N}} \left(\sum_{s \in S} q_s(\hat{e}(\nu)) (y_s - \hat{x}_s(\nu)) p \right) d\nu + \pi^Z(p) < 0.$$

Since $z \in Z^*(p)$, we have $\pi^Z(p) = zp \geq \hat{z}p$. Then, the previous inequality yields a contradiction in view of the fact that the allocation $\left([\hat{x}(\nu), \hat{e}(\nu)]_{\nu \in \mathcal{N}}, \hat{z}\right)$ satisfies the resource constraints in (11). ■

Proof of Theorem 7. As in the proof of Theorem 6, as a preliminary step, we show that the equilibrium prices lie in a compact subset P of $\mathbb{R}^{|G|}_{++}$. Note that we have $k(\mathcal{G}) \in [0, K^{\max}]$ for all $\mathcal{G} \in \mathcal{P}(G)$. Consider the set of valid cost functions

$$\mathcal{K} = \left\{ \{k(\mathcal{G})\}_{\mathcal{G} \in \mathcal{P}(G)} \mid k(\mathcal{G}) \in [0, K^{\max}], k(\{G\}) = 0, \text{ and } k(\mathcal{G}_1) \leq k(\mathcal{G}_2) \text{ for all } \mathcal{G}_1 \leq \mathcal{G}_2 \right\}.$$

Note that $\mathcal{K}$ is a compact subset of the Euclidean space. Moreover, for each $\{k(\mathcal{G})\}_{\mathcal{G} \in \mathcal{P}(G)} \in \mathcal{K}$, the corresponding equilibrium set is compact and the equilibrium correspondence is upper hemicontinuous in the cost vector $\{k(\mathcal{G})\}_{\mathcal{G} \in \mathcal{P}(G)}$, by the same arguments as in Theorem 6. It follows that there exists a compact set $P \subset \mathbb{R}^{|G|}_{++}$, such that equilibrium price lies in P.

We next provide the proof for Theorem 7, which is similar to the proof of Theorem 6. Let $\eta(\epsilon, p)$ be defined as the unique real number that satisfies

$$v(0, p, \mathcal{G}^*) = v(\eta(\epsilon, p), p, \mathcal{G}^*) + \epsilon.$$

Note that η is well defined since v is continuous and strictly increasing in its first variable (cf. Lemma 2). Moreover, η is continuous in ϵ and p, is increasing in ϵ and satisfies $\eta(\epsilon, p) \gtreqless 0$ if $\epsilon \gtreqless 0$. Let $\overline{K}_\epsilon$ be given by

$$\overline{K}_\epsilon \equiv \min_{p \in P} \left(\eta(\epsilon, p) / \left(\sum_{g \in G} p^g \right) \right),$$

where $\overline{K}_\epsilon > 0$ since η is continuous and P is a compact subset of $\mathbb{R}^{|G|}_{++}$. Note that whenever $k(\mathcal{G}^*) < \overline{K}_\epsilon$, we have

$$k(\mathcal{G}^*) p < \eta(\epsilon, p) \text{ for all } p \in P. \tag{52}$$

Let $\left(p, \left[\left[\mathcal{G}(\nu), c(\nu) = \left(x(\nu), e(\nu), w^{\mathcal{G}(\nu)}(\nu)\right)\right]_{\nu \in \mathcal{N}}, z\right]\right)$ be an equilibrium allocation. To get a contradiction, suppose that it is not $\epsilon-$constrained optimal, that is, suppose that there is an allocation $\left([\hat{x}(\nu), \hat{e}(\nu)]_{\nu \in \mathcal{N}}, \hat{z}\right)$ which is incentive feasible and which improves the utility of every worker by at least ϵ. Fix some $\nu \in \mathcal{N}$ and let $\left(\mathcal{G}, c = \left(x, e, w^{\mathcal{G}}\right)\right)$ denote the equilibrium allocation and $(\hat{x}, \hat{e})$ denote the ϵ-improving allocation corresponding to $\nu \in \mathcal{N}$. We claim that the allocation $(\hat{x}, \hat{e})$ generates net losses at equilibrium allocation p, i.e. we claim

$$\sum_{s \in S} q_s(\hat{e})(y_s - \hat{x}_s) p + \pi^Z(p) < 0. \tag{53}$$

To prove the claim in (53), we construct the contract $\overline{c} = \left(\mathcal{G}^*, \left(\overline{x}, \hat{e}, \left\{\overline{w}_s^{\{G_m\}} \equiv \overline{x}_s^{G_m} p^{G_m}\right\}_{s \in S, G_m \in \mathcal{G}^*}\right)\right)$ where, for each $G_m \in \mathcal{G}^*$ and $s \in S$, $\overline{x}_s^{G_m}$ is given as the solution to

$$\begin{aligned} \overline{x}_s^{G_m} &\equiv \arg\min_{\tilde{x}_s^{G_m}} \tilde{x}_s^{G_m} p^{G_m} \\ \text{s.t. } u^{\{G_m\}}\left(\tilde{x}_s^{G_m}\right) &= u^{\{G_m\}}\left(\hat{x}_s^{G_m}\right). \end{aligned} \tag{54}$$

By the representation for preferences in (26) and the fact that $\hat{e}$ is incentive compatible given $\hat{x}$, we have that $\overline{c} \in C^I(\mathcal{G}^*, p)$. By Proposition 2, the equilibrium contract $(\mathcal{G}, c)$ solves the inner problem in 25, hence we have

$$V^{\mathcal{G}}(c, p) = v(k(\mathcal{G}) p, p, \mathcal{G}) \geq v(k(\mathcal{G}^*) p, p, \mathcal{G}^*), \tag{55}$$

where the last inequality follows by a revealed preference type argument [cf. Eq. (47)]. Then, we have

$$\begin{aligned} V^{\mathcal{G}^*}(\overline{c}, p) = U(\hat{x}, \hat{e}) &\geq V^{\mathcal{G}}(c, p) + \epsilon \\ &\geq v(k(\mathcal{G}^*) p, p, \mathcal{G}^*) + \epsilon \\ &> v(\eta(\epsilon, p), p, \mathcal{G}^*) + \epsilon \\ &= v(0, p, \mathcal{G}^*), \end{aligned} \tag{56}$$

where the equality in the first line uses the definition of $\overline{c}$ and the fact that $\overline{c} \in C^I(\mathcal{G}^*, p)$, the inequality in the first line uses the fact that $\hat{c}$ is $\epsilon-$improving over the equilibrium allocation, the second line uses the inequality in (55), the third line uses the inequality in (52) and the forth line uses the definition of $\eta(\epsilon, p)$. Using (56), the definition of v as the solution to (46), and the fact that $\overline{c} \in C^I(\mathcal{G}^*, p)$, we deduct $\pi^{\mathcal{G}^*}(\overline{c}, p) < 0$. This further implies

$$\begin{aligned}\sum_{s\in S} q_s(\hat{e})(y_s - \hat{x}_s)p + \pi^Z(p) &\leq \sum_{s\in S} q_s(\hat{e})\left(y_s p - \sum_{G_m\in\mathcal{G}^*} \hat{x}_s^{G_m} p^{G_m}\right) + \pi^Z(p) \\ &\leq \sum_{s\in S} q_s(\hat{e})\left(y_s p - \sum_{G_m\in\mathcal{G}^*} \overline{x}_s^{G_m} p^{G_m}\right) + \pi^Z(p) \\ &= \pi^{\mathcal{G}^*}(\overline{c}, p) < 0,\end{aligned}$$

where the first line uses simple algebra, the second line uses the definition of $\overline{x}$ in Problem (54), and the last line uses the definition of $\pi^{\mathcal{G}^*}(\overline{c}, p)$. This proves our claim in Eq. (53).

Next, we put $\nu \in \mathcal{N}$ back into the notation and add up the inequality in Eq. (53) over all $\nu \in \mathcal{N}$, which yields

$$\int_{\mathcal{N}} \left(\sum_{s\in S} q_s(\hat{e}(\nu))(y_s - \hat{x}_s(\nu))p\right) d\nu + \pi^Z(p) < 0.$$

Since $z \in Z^*(p)$, we have $\pi^Z(p) = zp \geq \hat{z}p$. Then, the previous inequality yields a contradiction in view of the fact that the allocation $\left([\hat{x}(\nu), \hat{e}(\nu)]_{\nu\in\mathcal{N}}, \hat{z}\right)$ satisfies the resource constraints in (11). ■

Proof of Theorem 8. An argument similar to the proof of Theorem 7 establishes that equilibrium prices lie in a compact subset P of $\mathbb{R}^{|G|}_{++}$. We present two claims which we will use to prove the theorem. Our first claim is that

$$v(\delta, p, \mathcal{G}^*) > v\left(\delta, p, \mathcal{G}'\right), \text{ for all } \mathcal{G}' \prec \mathcal{G}^*, \tag{57}$$

where v is the function introduced in Lemma 2. In words, we claim that, keeping monitoring costs constant, monitoring $\mathcal{G}^*$ generates a higher value than monitoring a partition that is strictly coarser than $\mathcal{G}^*$. We prove this claim with an argument similar to the proof of Theorem 2. Let $\hat{c} = \left(\hat{x}, \hat{e}, \hat{w}^{\mathcal{G}'}\right)$ be a contract that solves Problem $P(\delta, p, \mathcal{G}')$. Note that we also have $\hat{c} \in C^I(p, \mathcal{G}^*)$ since $\mathcal{G}' \prec \mathcal{G}^*$.

First, we will show

$$U(\hat{x}, \hat{e}) > U(\hat{x}, \tilde{e}) \text{ for any } \tilde{e} \in E \setminus \hat{e}. \tag{58}$$

Suppose the contrary, that is suppose $U(\hat{x}, \hat{e}) = U(\hat{x}, \tilde{e})$ for some $\tilde{e} \neq e$. Since $\mathcal{G}' \prec \mathcal{G}^*$, there exists $G'_m \in \mathcal{G}'$ and $G_{m_1}, G_{m2} \in \mathcal{G}^*$ such that $G_{m_1} \cup G_{m2} \subset G'_m$. Let $g_1 \in G_{m_1}$ and$g_2 \in G_{m_2}$ Then, by Assumption A6^M, for any $s \in S$, we have

$$\frac{du(\hat{x}, \tilde{e})}{dx_s^{g_1}} / \frac{du(\hat{x}, \tilde{e})}{dx_s^{g2}} \neq \frac{du(\hat{x}, \hat{e})}{dx_s^{g_1}} / \frac{du(\hat{x}, \hat{e})}{dx_s^{g2}} = p^1/p^2,$$

where the equality follows since $g_1, g_2 \in G'_m$ and $\hat{c} \in C^I(p, \mathcal{G}')$. Then, given prices p, the worker can switch to effort $\tilde{e}$, and change $\hat{x}_s^{g_1}, \hat{x}_s^{g_2}$ to some $\tilde{x}_s^{g_1}, \tilde{x}_s^{g_2}$so that $p^1\tilde{x}_s^{g_1} + p^2\tilde{x}_s^{g_2} = p^1\hat{x}_s^{g_1} + p^2\hat{x}_s^{g_2}$ and that $u(\tilde{x}, \tilde{e})$ is higher than $u(\hat{x}, \tilde{e}) = u(\hat{x}, \hat{e})$. But this yields a contradiction to the fact that $\hat{c} \in C^I(p, \mathcal{G}')$, proving Eq. (58). Second, note that by the assumption of a shirking effort level e_{shirk}, $\hat{c}$ does not entail full insurance, that is, there exists $g \in G$ and $s_1, s_2 \in S$ with $q(s_1), q(s_2) > 0$ such that

$$\frac{du(\hat{x}_{s_1}, e)/dx_{s_1}^g}{du(\hat{x}_{s_2}, e)/dx_{s_2}^g} > 1. \tag{59}$$

We will show that the firm can provide better insurance while still breaking even and incentivizing effort level $\hat{e}$. Note also that we have $du(\hat{x}_s, e)/dx_s^{G_m} = \lambda_s p^{G_m}$ for each $s \in S$ for some $\lambda_s > 0$. Eq. (59) implies $\lambda_{s_1} > \lambda_{s_2}$. Let $\hat{w}_s^{G_m} \equiv \hat{x}_s^{G_m} p^{G_m}$ for each $s \in S$ and G_m. Define the wage contract

$$\hat{w}^{\mathcal{G}^*}(\eta) = \left\{ \left(\hat{w}_s^{\{G_m\}} \right) |_{(G_m,s) \in \mathcal{G}^* \times S \setminus \{G_m\} \times \{s_1,s_2\}}, \hat{w}_{s_1}^{\{G_m\}} - \eta q(s_2), \hat{w}_{s_2}^{\{G_m\}} + \eta q(s_1) \right\}. \tag{60}$$

Define $\hat{x}(\eta) \equiv \mathbf{x}\left(\hat{w}^{\mathcal{G}^*}(\eta), p, \hat{e}\right)$. Note that we have $\lim_{\eta \to 0} \hat{w}^{\mathcal{G}^*}(\eta) = \hat{w}^{\mathcal{G}^*}$, and that $\lim_{\eta \to 0} \hat{x}(\eta) = \hat{x}$ since $\hat{c} \in C^I(p, \mathcal{G}^*)$. Then, by Eq. (58), we have that there exists $\eta_1 > 0$ such that for all $\eta \in [0, \eta_1)$

$$U(\hat{x}(\eta), \hat{e}) > U(\hat{x}(\eta), \tilde{e}) \text{ for any } \tilde{e} \in E \setminus \hat{e}.$$

It follows that $\hat{c}(\eta) \equiv \left(\hat{x}(\eta), \hat{e}, \hat{w}^{\mathcal{G}^*}(\eta)\right) \in C^I(p, \mathcal{G}^*)$ when $\eta < \eta_1$. Moreover, since $\lambda_{s_1} > \lambda_{s_2}$, by Eq. (60), there exists $\eta_2 > 0$ such that for all $\eta \in [0, \eta_2)$, we have $U(\hat{x}(\eta), \hat{e}) > U(\hat{x}, \hat{e})$ since the worker receives better insurance with $\hat{x}(\eta)$ than $\hat{x}$. Finally, again by Eq. (60), we have $\pi^{\mathcal{G}^*}(\hat{c}(n), p) = \pi^{\mathcal{G}'}(\hat{c}, p) \geq \delta$. Combining our findings shows that $\hat{c}(n)$ is in the constraint set of Problem $P(\delta, p, \mathcal{G}^*)$ and yields a higher utility than $\hat{c}$, which in turn implies Eq. (57), proving our first claim.

Our second claim is that

$$v(\delta, p, \mathcal{G}^*) = v\left(\delta, p, \mathcal{G}'\right), \text{ for all } \mathcal{G}' \succeq \mathcal{G}^*, \tag{61}$$

that is, there is no additional value gain from monitoring strictly finer partitions of $\mathcal{G}^*$. To see this, let $c = \left(x, e, w^{\mathcal{G}^*}\right)$ be a solution to $\mathcal{P}(\delta, p, \mathcal{G}^*)$. Since $\mathcal{G}'$ is weakly finer than $\mathcal{G}^*$, we have that $c^{'} = \left(x, e, w^{\mathcal{G}'}\right) \in C^I\left(p, \mathcal{G}^{'}\right)$ and $\pi^{\mathcal{G}'}(c', p) = \pi^{\mathcal{G}^*}(c, p)$, which in turn implies that $v(\delta, p, \mathcal{G}') \geq v(\delta, p, \mathcal{G}^*)$, that is, monitoring more commodities is weakly more valuable (when we keep the cost of monitoring constant). Conversely, let $c = \left(x, e, w^{\mathcal{G}^{'}}\right)$ be a solution to $\mathcal{P}\left(\delta, p, \mathcal{G}^{'}\right)$. Since preferences have the representation in Assumption A6^M, it must be the case that

$$\begin{aligned} x_s^{G_m} &= \arg\max u^{\{G_m\}}\left(\tilde{x}_s^{G_m}\right) \\ \text{s.t.} & \quad \tilde{x}_s^{G_m} p^{G_m} \leq x_s^{G_m} p^{G_m}, \end{aligned} \tag{62}$$

since, otherwise the value of $\mathcal{P}\left(\delta, p, \mathcal{G}^{'}\right)$ could be improved (in view of the fact that $\mathcal{G}' \succeq \mathcal{G}^*$). Then, Eq. (62) and the fact that $c \in C^I\left(p, \mathcal{G}^{'}\right)$ imply that $c = \left(x, e, w^{\mathcal{G}^*}\right) \in C^I(p, \mathcal{G}^*)$. Moreover, we still have $\pi^{\mathcal{G}'}((\mathcal{G}', c'), p) = \pi^{\mathcal{G}^*}((\mathcal{G}^*, c), p)$, which implies $v(\delta, p, \mathcal{G}') \leq v(\delta, p, \mathcal{G}^*)$, i.e. there is no additional value to monitoring partitions finer than $\mathcal{G}^*$. Combining our findings gives Eq. (61) as desired.

We next prove the first part of the theorem. If $\mathcal{G}^* = \{G\}$, the statement trivially holds since any $\mathcal{G} \in \mathcal{P}(G)$ satisfies $\mathcal{G} \succeq \{G\}$. Therefore, suppose $\mathcal{G}^* \succ \{G\}$. For any partition $\mathcal{G}' \prec \mathcal{G}^*$, let $\delta(p, \mathcal{G}')$ be the unique real number that satisfies

$$v\left(\delta\left(\mathcal{G}'\right), p, \mathcal{G}^*\right) = v\left(0, p, \mathcal{G}'\right).$$

Such $\delta(p, \mathcal{G}')$ is uniquely defined by the fact that v is continuous and strictly decreasing in δ (cf. Lemma 2), and is positive in view of the inequality in (57). Moreover, $\delta(p, \mathcal{G}')$ is continuous in p. Let $\overline{K}$ be given by

$$\overline{K} = \min_{\{\mathcal{G}' \in \mathcal{P}(G) \ | \ \mathcal{G}' \prec \mathcal{G}^*\}, p \in P} \left(\delta\left(p, \mathcal{G}^{'}\right) / \left(\sum_{g \in G} p^g\right)\right) > 0,$$

so that, whenever $k(\mathcal{G}^*) < \overline{K}$, we have

$$k(G^*)\, p < \delta\left(p, \mathcal{G}^{'}\right) \text{ for all } \mathcal{G}' \prec \mathcal{G}^* \text{ and any } p \in P. \tag{63}$$

Let $k(\mathcal{G}^*) < \overline{K}$ and consider any equilibrium allocation $\left(p, \left[\left[\mathcal{G}(\nu), c(\nu) = \left(x(\nu), e(\nu), w^{\mathcal{G}(\nu)}(\nu)\right)\right]_{\nu \in \mathcal{N}}, z\right]\right)$. Combining Eq. (61), the fact that v is strictly decreasing in δ, and the fact that monitoring finer partitions is costlier, we have

$$\begin{aligned} v(k(G^*)\, p, p, \mathcal{G}^*) &\geq v\left(k\left(G^{'}\right) p, p, \mathcal{G}^*\right) \\ &= v\left(k\left(G^{'}\right) p, p, \mathcal{G}^{'}\right) \text{ for all } \mathcal{G}' \succ \mathcal{G}^*, \end{aligned} \tag{64}$$

where the equality uses Eq. (61) and the inequality is strict whenever $k(\mathcal{G}') \neq k(G^*)$. Similarly, for the partitions that are strictly coarser than $\mathcal{G}^*$, combining Eq. (57), the definition of $\delta(p, \mathcal{G}')$, Eq. (63) and the fact that c is strictly decreasing in δ, we have

$$\begin{aligned} v(k(G^*)p, p, \mathcal{G}^*) &> v(\delta(p, \mathcal{G}'), p, \mathcal{G}^*) \\ &= v\left(0, p, \mathcal{G}'\right) \geq v\left(k\left(G'\right)p, p, \mathcal{G}'\right), \text{ for all } \mathcal{G}' \prec \mathcal{G}^*. \end{aligned} \tag{65}$$

In view of Eq. (47), the inequalities in (64) and (65) imply that for any solution $c = \left(\mathcal{G}, \left(x, e, w^{\mathcal{G}}\right)\right)$ to Problem 25, we have $\mathcal{G} \succeq \mathcal{G}^*$ where strict inequality is possible only if $k(\mathcal{G}) = k(G^*)$. Hence, by Proposition 2, we conclude that $\mathcal{G}(\nu) \succeq \mathcal{G}^*$ for all but a measure zero of workers $\nu \in \mathcal{N}$ and that $G(\nu) \succ G^*$ only if $k(\mathcal{G}(\nu)) = k(\mathcal{G}^*)$, completing the proof. ■

Appendix B: Class of Economies that Satisfy Assumptions A1-A5

We first introduce an additional notion of separability for the utility function, which we use to establish conditions that imply Assumptions A1-A5.

Definition 10 The preferences have a **completely separable** component if there exists $G_{sep} \in G$ and continuous functions $u^{G_{sep}}$ and $u^{G \backslash G_{sep}}$ increasing in respectively $x_s^{G_{sep}}$ and $x_s^{G \backslash G_{sep}}$, such that

$$u(x_s, e) = u^{G_{sep}}\left(x_s^{G_{sep}}\right) + u^{G \backslash G_{sep}}\left(x_s^{G \backslash G_{sep}}, e\right). \tag{66}$$

We introduce this condition as a convenient way to satisfy the limited transferability Assumption A4. Note that this assumption is not hard to satisfy: it will hold for example if there is one good, which is completely separable from effort choice and consumption of which can be monitored by the firm. Next, we introduce the "effort targeting" condition for the probability function, which loosely corresponds to the requirement that firms should be able to induce any level of effort if they wish to do so.

Definition 11 The probability function $q(e)$ satisfies **effort targeting** if for each $e \in E$, there exists a vector of utility transfers $t \in \mathbb{R}^{|S|}$ such that

$$e = \arg\max_{\tilde{e} \in E} \sum_{s \in S} q_s(\tilde{e}) t_s, \tag{67}$$

that is, every effort level $e \in E$ can be targeted with a vector of utility transfers over the states.

Note that when E is a finite set, $\{e_1, e_{2,\ldots,}e_r\}$, a sufficient condition for this assumption to hold is that the matrix $[q_s = q_s(e_j)]_{s\in S, j\in E}$ has full row rank. In particular, in the commonly studied case of two effort levels, this assumption holds when there are at least two states and the efforts do not have identical success probabilities for all of the states.

The following lemma provides several conditions for the state utility function u and the probability function q which imply Assumptions A4 and A5.

Lemma 3 *In each of the following cases, the economy described so far satisfies Assumptions A4 and A5:*

1. *The preferences have a completely separable component and the probability function q satisfies effort targeting.*

2. *The preferences have a completely separable component and are partially separable.*

Proof of Lemma 3. We first show that Assumption A4 holds when the preferences have a completely separable component with $G_{sep} \in \mathcal{G}$. Consider $c = \left(x^G, w^{\mathcal{G}}\right) \in \mathbb{R}_{++}^{|S|\times|G|} \times \mathbb{R}_{++}^{|S|\times|\mathcal{G}|} \in C^I(p, \mathcal{G})$ and $\epsilon > 0$. For each $\delta \in \mathbb{R}_+$ and $G_m \in \mathcal{G}$, define $w_s^{\{G_{sep}\}}(\delta) \in \mathbb{R}_+$ as the solution and $x_s^{G_{sep}}(\delta)$ as the optimum to the following convex optimization problem:

$$w_s^{\{G_{sep}\}}(\delta) = \min_{\hat{x}_s^{G_{sep}} \geq 0} \hat{x}_s^{G_{sep}} p^{G_{sep}} \tag{68}$$
$$\text{such that} \quad u^{G_{sep}}(\hat{x}_s^{G_{sep}}) \geq u^{G_{sep}}\left(\hat{x}_s^{G_{sep}}\right) + \delta.$$

For $\delta = 0$, we have $w_s^{\{G_{sep}\}}(0) = w_s^{\{G_{sep}\}}$ since the contract $c \in C^I(p, \mathcal{G})$. We also have that $w_s(\delta)$ is continuous and strictly increasing. Let $\delta_+ > 0$ be such that $w_s^{\{G_{sep}\}}(\delta_+) \in \left(w_s^{\{G_{sep}\}}, w_s^{\{G_{sep}\}} + \epsilon\right)$ for each s. Then, we have

$$u^{G_{sep}}\left(x_s^{G_{sep}}(\delta_+)\right) = u^{G_{sep}}\left(x_s^{G_{sep}}\right) + \delta_+, \text{ for each } s \in S,$$

which, by incentive compatibility of c and the fact that u has a completely separable component implies $\mathbf{e}\left(w^{\{G_{sep}\}}(\delta_+), w^{\mathcal{G}\backslash\{G_{sep}\}}, p\right) = \mathbf{e}\left(w^{\mathcal{G}}, p\right)$. Then, $c_+ = \left(x^{G_{sep}}(\delta_+), x^{\mathcal{G}\backslash G_{sep}}, w^{\{G_{sep}\}}(\delta_+), w^{\mathcal{G}\backslash\{G_{sep}\}}, e\right)$ constitutes a limited incentive compatible transfer to the worker and satisfies the conditions of Assumption A4. A similar argument can be used to construct c_- hence Assumption A4 holds.

We next show that, if, in addition to the state utility function having a completely separable component, q satisfies the effort targeting condition, then Assumption A5 holds. Consider the price vector p and an incentive compatible contract $c \equiv \left(x, e, w^{\mathcal{G}}\right)$. Consider a sequence $(p_n)_{n=1}^{\infty} \to p$ and any $\epsilon > 0$. We will show that there exists N such that for all $n > N$, there exist incentive compatible contracts $c_n \equiv \left(x_n, e, w_n^{\mathcal{G}}\right)$ for p_n such that

$$e_n \in B\left(e, \epsilon\right), \text{ and } w_n \in B\left(w, \epsilon\right), \tag{69}$$

which in turn implies Assumption A5, since $\lim_{n\to\infty} c_n = c$. Consider the vector t that identifies effort level e, i.e., the transfer vector such that equation (67) holds for e. Note that equation (67) also holds for a transfer vector rt, where $r > 0$ is an arbitrary positive scalar. Let $r > 0$ be such that $w_s^{\{G_{sep}\}}\left(rt_s\right) \in B\left(w_s^{\{G_{sep}\}}, \epsilon\right)$ for each s. Let

$$\bar{\epsilon} = \sum_{s\in S} rt_s q_s\left(e\right) - \max_{e'\in E\setminus B(e,\epsilon)} \sum_{s\in S} rt_s q_s\left(e'\right), \tag{70}$$

which is a positive scalar by equation (67). Since the indirect utility function $V\left(c, p\right)$ and the function $U\left(\mathbf{x}\left(w^{\mathcal{G}}, p, e\right), e\right)$ are continuous in p, there exists N such that for all $n > N$,

$$V\left(c, p_n\right) < V\left(c, p\right) + \bar{\epsilon}/2$$

and

$$U\left(\mathbf{x}\left(w^{\mathcal{G}}, p_n, e\right), e\right) > U\left(\mathbf{x}\left(w^{\mathcal{G}}, p, e\right), e\right) - \bar{\epsilon}/2.$$

Since effort level e attains $V\left(c, p\right)$, these two inequalities jointly imply

$$U\left(\mathbf{x}\left(w^{\mathcal{G}}, p_n, e'\right), e'\right) < U\left(\mathbf{x}\left(w^{\mathcal{G}}, p_n, e\right), e\right) + \bar{\epsilon} \tag{71}$$

for all $n > N$ and $e' \in E$. Consider p_n where $n > N$, and define $\left(x_n, w_n^{\mathcal{G}}\right)$ by $(x_n)_s^{G_{sep}} = x_s^{G_{sep}}\left(rt_s\right)$, $(w_n)_s^{\{G_{sep}\}} = (w_n)_s^{\{G_{sep}\}}\left(rt_s\right)$ for each $s \in S$ [cf. Eq. (68)] and $(x_n)_s^{G\setminus G_{sep}} = x_s^{G\setminus G_{sep}}$, $(w_n)_s^{\mathcal{G}\setminus\{G_{sep}\}} = (w_n)_s^{\mathcal{G}\setminus\{G_{sep}\}}$ for each $s \in S$. We will show that the allocation $\left(x_n, w_n^{\mathcal{G}}\right)$ and any $e_n \in \mathbf{e}\left(w_n^{\mathcal{G}}, p_n\right)$ satisfy the claims in (69). By construction of x_n and $w_n^{\mathcal{G}}$, we only need to show that the incentive compatible effort level e_n lies in $B\left(e, \epsilon\right)$. By incentive compatibility, e_n satisfies

$$U\left(\mathbf{x}\left(w_n^{\mathcal{G}}, p_n, e_n\right), e_n\right) \geq U\left(\mathbf{x}\left(w_n^{\mathcal{G}}, p_n, e\right), e\right).$$

Using the fact that preferences have a completely separable component [cf. Eq. (66)] and the construction of the contract $\left(x_n, w_n^{\mathcal{G}}\right)$, this inequality can be rewritten as

$$\sum_{s\in S} rt_s q_s\left(e_n\right) + U\left(\mathbf{x}\left(w^{\mathcal{G}}, p_n, e_n\right), e_n\right) \geq \sum_{s\in S} rt_s q_s\left(e\right) + U\left(\mathbf{x}\left(w^{\mathcal{G}}, p_n, e\right), e\right).$$

Combining the previous inequality with (71) implies

$$\sum_{s\in S} rt_s q_s(e_n) > \sum_{s\in S} rt_s q_s(e) - \bar{\epsilon},$$

which, by definition of $\bar{\epsilon}$ in (70), implies $e_n \in B(e,\epsilon)$. Hence (69) holds for $\left(x_n, e_n, w_n^{\mathcal{G}}\right)$, which implies that Assumption A5 holds. This completes the proof of the first part of the lemma.

We next show that Assumption A5 also holds when the preferences are partially separable and have a completely separable component. For each $G_m \in \mathcal{G}$, let $v^{\{G_m\}}(w_s^{\{G_m\}}, p)$ be the optimum and $\tilde{x}^{G_m}\left(w_s^{\{G_m\}}, p\right)$ be the solution to the problem defined in (42). The partial separability assumption implies that, $x = \mathbf{x}\left(w^{\mathcal{G}}, p, e\right)$ if and only if $x_s^{G_m} = \tilde{x}^{G_m}\left(w_s^{\{G_m\}}, p\right)$ for each $s \in S$ and $G_m \in \mathcal{G}$, and $e \in \mathbf{e}\left(w^{\mathcal{G}}, p\right)$ if and only if

$$e \in \arg\max_{\hat{e}\in E} \sum_{s\in S} q_s(\hat{e}) u^{\mathcal{G}}\left(\left\{v^{\{G_m\}}\left(w_s^{\{G_m\}}, p\right)\right\}_{G_m\in\mathcal{G}}, \hat{e}\right). \tag{72}$$

Consider the price vector p and an incentive compatible allocation $\left(x, e, w^{\mathcal{G}}\right)$. Consider a sequence $(p_n)_{n=1}^{\infty} \to p$. Note that $v^{\{G_m\}}\left(w_s^{\{G_m\}}, p\right)$ is continuous in $w_s^{\{G_m\}}$ and p, and is strictly increasing in $w_s^{\{G_m\}}$. Moreover, from Assumption A1, $\lim_{w_s\to 0} v^{\{G_m\}}(w_s^{\{G_m\}}, p) = -\infty$, and $\lim_{w_s\to\infty} v^{\{G_m\}}(w_s^{\{G_m\}}, p) = \infty$. Then, for each p_n and $s \in S$, $G_m \in \mathcal{G}$, there exists a unique $w_s^{\{G_m\}}(p_n)$ such that

$$v^{\{G_m\}}(w_s^{\{G_m\}}(p_n), p_n) = v^{\{G_m\}}(w_s^{\{G_m\}}, p), \tag{73}$$

moreover, by continuity of $v^{\{G_m\}}$, $\lim_{n\to\infty} w_s(p_n) \to w_s$. Let $x(p_n) \equiv \left\{\tilde{x}^{G_m}\left(w_s^{\{G_m\}}(p_n), p_n\right)\right\}_{s\in S, G_m\in\mathcal{G}}$, and note, by the continuity of $\tilde{x}^{G_m}$, that $\lim_{n\to\infty} x(p_n) \to x$. Moreover, equations (73) and (72) imply that the allocation $\left(x(p_n), e, w^{\mathcal{G}}(p_n)\right)$ is incentive compatible given the price vector p_n, for each n. Since the sequence $\left(x(p_n), e, w^{\mathcal{G}}(p_n)\right)_{n=1}^{\infty}$ converges to $\left(x, e, w^{\mathcal{G}}\right)$, we establish that the incentive compatible allocation correspondence is lower hemicontinuous, as desired. Putting our findings together implies part 2 of the lemma. ■

References (very incomplete)

Allen, Franklin (1985) “Repeated Principal-Agent Relationships with Lending and Borrowing,” *Economic Letters*, 17, 27-31.

Arnott, Richard and Joseph Stiglitz (1988) “Basic Analytics of Moral Hazard,” *Scandinavian Journal of Economics*, 90, 383-413.

Arnott, Richard and Joseph Stiglitz (1991) “Moral Hazard and Nonmarket Institutions: Dysfunctional Crowding Out of Peer Monitoring,” *American Economic Review*, 81, 179-190.

Atkeson, Andy and Robert Lucas (1992) “On Efficient Distribution with Private Information,” *Review of Economic Studies*, 59,.427-453.

Bennardo, Alberto and Pierre-Andre Chiappori (2003) “Bertrand and Walras Equilibria under Moral Hazard,” *Journal of Political Economy*, 111, 785-817.

Bisin, Alberto, John Geanakoplos, Piero Gottardi, Enrico Minelli and Herakles Polemarchakis (2007) “Markets and Contracts,” forthcoming *Journal of Mathematical Economics.*

Bisin, Alberto and Piero Gottardi (1999) “Competitive Equilibria with Asymmetric Information,” *Journal of Economic Theory*, 87, 1-48.

Bisin, Alberto and Piero Gottardi (2006) “Efficient Competitive Equilibria with Adverse Selecion,” *Journal of Political Economy*, 114, 485-516.

Bisin, Alberto and Danilo Guaitoli (2004) “Moral Hazard and Non-Exclusive Contracts,” *Rand Journal of Economics*, 35, 306-328.

Citanna, Alessandro and Antonio Villanacci (2000) “Competitive Equilibrium with Moral Hazard in Economies with Multiple Commodities,” *Journal of Mathematical Economics*, 38, 117-147.

Cole, Harald and Narayana Kocherlakota (2001) “Efficient Allocations with Hidden Income and Hidden Storage,” *Review of Economic Studies*, 68, 523-542.

Citanna, Alessandro, Atsusi Kajii and Antonio Villanacci (1998) “Constraints Suboptimality in Incomplete Markets: A General Approach and Two Applications,” *Economic Theory*, 11, 495-521.

Doepke, Matthias and Robert Townsend (2006) “Dynamic Mechanism Design with Hidden Income and Hidden Actions,” *Journal of Economic Theory*, 126, 235-285.

Geanakoplos, John and Herakles Polemarchakis (1986) “Existence, Regularity, and Constrained Suboptimality of Competitive Allocations When the Asset Market Is Incomplete” in W. Heller, R. Starr and D. Starrett (editors) *Uncertainty, Information and Communication: Essays in Honor of Kenneth Arrow*, Cambridge University Press, volume 3.

Geanakoplos, John and Herakles Polemarchakis (2000) “Pareto Improving Taxes,” Yield mimeo.

Golosov, Michael, Narayana Kocherlakota and Aleh Tsyvinski (2003) ”Optimal Indirect and Capital Taxation,” *Review of Economic Studies,* 70, 569–587.

Golosov, Michael and Aleh Tsyvinski (2006) “Optimal Taxation with Endogenous Insurance Markets” *Quarterly Journal of Economics*, 122, 487-532.

Greenwald, Bruce and Joseph Stiglitz (1986) “Externalities in Economies with Imperfect Information and Incomplete Markets,” *Quarterly Journal of Economics*, 101, 229-264.

Gottardi, Piero and Belen Jerez (2006) “A Note on Walrasian Equlibria with Moral Hazard and Aggregate Uncertainty,” mimeo.

Guesnerie, Roger (1998) *A Contribution to the Pure Theory of Taxation*, Econometric Society Monographs, Cambridge University Press, New York, NY.

Hammond, Peter (1987) “Markets As Constraints: Multilateral Incentive Compatibility in Continuum Economies,” *Review of Economic Studies*, 54, 399-412.

Jerez, Belen (2003) “A Dual Characterization of Economic Efficiency,” *Journal of Economic Theory,* 112, 1-34.

Jewitt, Ian (1988) “Justifying the First Order Approach to Principal-Agent Problems,” *Econometrica*, 56, 1177-1180.

Kehoe, Timothy and David Levine (1993) ”Debt-Constrained Asset Markets,” *Review of Economic Studies*, 60, 865-888.

Kocherlakota, Narayana (2004) “Figuring out the Impact of Hidden Savings on Optimal Unemployment Insurance,” *Review of Economic Dynamics*, 7, 541-554.

Lisboa, Marcos (2001) “Moral Hazard and General Equilibrium in Large Economies,” *Economic Theory*, 18, 555-585.

Prescott, Edward C. and Robert Townsend (1984a) “General Competitive Analysis in an Economy with Private Information”, *International Economic Review*, 25, 1-20.

Prescott, Edward C. and Robert Townsend (1984b) “Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard”, *Econometrica* 52, 21-45.

Rogerson, William (1985) “The First-Order Approach to Principal-Agent Problems,” *Econometrica*, 53, 1357-367.

Werning, Ivan (2001) “Optimal Unemployment Insurance With Unobservable Savings,” mimeo.