Importance Sampling and the Spanish Financial System

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October 18, 2013

Abstract

This paper quantifies the credit risk loss distribution of the Spanish financial system by introducing a general Monte Carlo importance sampling (IS) approach. We start obtaining all the required information for the Vasicek (1987) model. Then we quantify the loss distribution under the standard IS method introduced by Glasserman and Li (2005) and allocate the total risk over the different institutions in the Spanish financial system. We also extend the current IS framework to deal with more general assumptions like random recoveries and market valuation. Our results show that this approach can be very useful for banking supervisors from a macroprudential point of view and that the risk allocation can vary considerably depending on the valuation model under analysis.

Keywords: Monte Carlo, importance sampling, credit risk, macroprudential supervision, risk allocation, VaR, expected shortfall.

JEL classification: C15, C63, G21

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1 Introduction

This paper quantifies the credit risk loss distribution of the Spanish financial system under a general Monte Carlo importance sampling (IS) model. One of the main activities in financial institutions consists on financing investors and paying depositors. Under the Basel regulation the financial institutions are required to have a minimum level of own resources so that they will not go bankruptcy in the case that investors do not pay back their loans.

Micro-prudential financial regulation focuses on a one by one supervision of the financial institutions in order to ensure a maximum default probability of each institution, however a macro-prudential financial regulation focuses on the whole loss distribution of the financial system. In the past regulators did just a micro-prudential supervision (see Basel (2006)) however they have recently switched to a macro-prudential supervision (see Basel (2011) and Basel (2012b)) that tries to capture the interconnectedness between the financial institutions, their size and the magnitude of the possible negative effects in the economy.\footnote{In Basel (2012a) the Basel committee enforces the use of risk measures that do not only consider the probability of high losses but the magnitude of those losses.} Over the current economic crisis many financial institutions had to be rescued by the governments due to their size and potential negative effects in the economy, among others we have Fannie Mae, Freddie Mac, AIG, Northern Rock, RBS, Lloyds, Nordea, Dexia, ING, Fortis, IKB, Commerzbank, Hypo Real Estate or Bankia, CAM, Catalunya-Caixa, Novacaixagalicia (NCG), and Unnim in Spain and some have merged or been absorbed by others financial institutions. Therefore knowing the loss distribution of a whole financial system and being able to correctly allocate the risk of each institution is crucial for a good banking supervision and the financial system stability.

This paper estimates the loss distribution of the Spanish financial system under the model introduced in Vasicek (1987). This model is widely used in practice and is the starting point for the Basel Internal Rating Based capital charges (see Basel (2006)). As far as we know, Campos et al. (2007) is the only previous study that tried to measure the risk of the Spanish financial system. However, these authors a) did not take into account the diversification effect of the institutions that are not only based in Spain, b) used a base recovery value of 60% which, according to USA default data, is too high and c) did not allocate the risk over the different financial institutions. Bennet (2002), Kuritzkes et al. (2002), and Cariboni et al. (2011) used a similar approach to that in Campos et al. (2007) to define an optimum deposits insurance fund.

As we have said, Campos et al. (2007) considered a unique macroeconomic factor that links all the institutions in the economy. Our paper goes one step forward as we define as many factors as countries. We propose to use the public information of consolidated net interest income generated by the banking groups in the different countries (see BBVA (2009) and Santander (2009)) as a way...
to capture the risk exposure of the institutions to the different countries.

We use the Monte Carlo Importance Sampling (IS) technique introduced in Glasserman (2005) and Glasserman and Li (2005) to measure and allocate the total risk of a certain portfolio. One of the main advantages of the technique is that it can generate very accurate loss distributions and risk allocation at a low computational cost compared with that of the standard Monte Carlo method. In addition, compared with other approximate methods to obtain loss distributions like those in Pykhtin (2004) and Huang et al. (2007), its accuracy can be improved by increasing the number of simulations.

To address some criticism raised from the constant recoveries assumption we have used the FDIC data to extend the IS model to deal with random recoveries. After testing several random recoveries models, our results show that the random recoveries impact on the risk allocation over the different institutions but not on the portfolio 99.9% probability loss. We have also extended the IS framework in Glasserman and Li (2005) to obtain the market valuation of the portfolio by using a model similar to that in Grundke (2009). The impact of this valuation on the loss distribution can double that of the random recovery model.

This paper introduces two major contributions. First, we measure and allocate the risk of the Spanish financial system under the IS method. Second, we extend the IS method to deal with more realistic assumptions such as random recoveries and market valuation. Our results show that different financial systems can satisfy the micro-prudential regulation but have completely different loss distributions. We also highlight that a simple default mode model can seriously underestimate the possible losses and the risk allocation compared with a market mode model. We suggest not to focus only in one model but to test the impact of the different models to assess the solvency of a financial system and the impact of each financial institution. We believe that our approach goes one step forward in the current risk measurement methods applied by financial system supervisors and it can be a basic tool to identify Systemically Important Financial Institutions (SIFI) and to quantify the required capital surcharge for these institutions (see Basel (2011)).

This paper is organized as follows. Section 2 reviews the main ideas regarding credit risk and the Vasicek (1987) model. Section 3 introduces the IS model proposed in Glasserman and Li (2005) and explains the optimal changes in the sampling distributions. Section 4 describes the main features of the Spanish financial system portfolio. Section 5 presents the IS results, loss distribution, and risk allocation for this financial system. Section 6 develops some extensions of the IS method such as random recoveries and market mode valuation. Section 7 summarizes our main results and concludes.
2 The Vasicek (1987) Model

Vasicek (1987) introduced one of the most extended credit risk models assuming that the default behavior of a given client \( j \) (or counterparty) is driven by a set of macroeconomic factors \( Z = \{z_1, z_2, \ldots, z_k\} \) and an idiosyncratic (client-specific) term \( \varepsilon_j \). The factors \( \{z_i\}_{i=1}^{k} \) and \( \varepsilon_j \) are independent and distributed as standard normal random variables.\(^2\) Under these assumptions, default is modeled through the so called asset value of the client \( j \), defined as

\[
V_j = \sum_{f=1}^{k} \alpha_{f,j} z_f + \varepsilon_j \sqrt{1 - \sum_{f=1}^{k} \alpha_{f,j}^2}
\]

This client defaults in her obligations if \( V_j \) falls below a given default threshold level \( k \). As \( V_j \sim N(0, 1) \), we have that \( k = \Phi^{-1}(PD_{j,C}) \), where \( \Phi(\cdot) \) denotes the normal distribution function and \( PD_{j,C} \) denotes the historical average default rate of the client \( j \) over long enough periods.\(^3\)

Given the specification (1) and conditional to the macroeconomic factors \( Z \), the default probability of the client \( j \) is

\[
Prob(D_j = 1|Z) = Prob(V_j \leq k|Z) = \Phi\left( \frac{\Phi^{-1}(PD_{j,C}) - \sum_{f=1}^{k} \alpha_{f,j} z_f}{\sqrt{1 - \sum_{f=1}^{k} \alpha_{f,j}^2}} \right)
\]

Bank portfolios are composed of this kind of contracts. The total loss of a portfolio including \( M \) contracts or clients is given as \( L = \sum_{j=1}^{M} x_j \), being \( x_j \) the individual loss of the client or contract \( j \). Under homogeneous granular portfolios,\(^4\) the probability of default \( Prob(D_j = 1|Z) \) and the observed default rates \( DR_z \) tend to be equal. This means that all the idiosyncratic risks of the different clients disappear and there is no uncertainty on the loss conditional to the macroeconomic scenario.

Under granular homogeneous single factor portfolios, the unconditional default rate distribution function is given as

\[
Prob(DR_z \leq L) = Prob\left( \Phi\left( \frac{\Phi^{-1}(PD_C) - \alpha z}{\sqrt{1 - \alpha^2}} \right) \leq L \right) = \Phi\left( \frac{\Phi^{-1}(L)\sqrt{1 - \alpha^2} - \Phi^{-1}(PD_C)}{\alpha} \right)
\]

Since the Basel II accord, the banking regulation uses the Vasicek (1987) asymptotic single factor model and forces the financial institutions to have an amount of own resources, equity, and other assets with similar behavior to the equity equal to the worst loss with a 99.9% probability.

\(^2\)Dependent factors can always be orthogonalized.

\(^3\)It might be more useful to think on the historical average default rates of clients similar to \( j \) rather than on the historical average default rates of the client \( j \).

\(^4\)This type of portfolios is made up of many identical contracts, with the same risk parameters.
The estimation of the portfolio loss distribution requires estimating $PD_C$ for the different portfolios. This can be done by using the historical default rates of the portfolios but another components are also needed:

1. **EAD**: Exposure at default, the amount of money owed by the investor when he defaults.
2. **LGD**: Loss given default, the final loss after all the recovery processes.\(^5\)
3. $\alpha$: Sensitivity of the asset value to the macroeconomic factors. The Basel accord provides standard $\alpha$ values for the different portfolios of a bank.

Then, the portfolio loss can be expressed as

$$L = \sum_{j=1}^{M} x_j = \sum_{j=1}^{M} EAD_j LGD_j 1(V_j \leq \Phi^{-1}(PD_{j,C}))$$

In the general case of non-granular, non-homogeneous and multi-factor portfolios, the loss distribution of a loan portfolio can be obtained by Monte Carlo methods or by approximated ones.

It should be noted that our objective is to know just some statistical measures of the accumulated loss distribution $F(L)$, being the most important the following ones:

1. Value at Risk: $VaR(q) = F^{-1}(q)$.\(^6\)
2. Expected Shortfall or Tail VaR, that is, the expected loss given that a minimum loss level has been reached: $ES(q) = E(L|L \geq VaR(q))$.
3. Risk contributions of the client $j$. We can consider two alternatives:
   
   (a) Value at Risk contribution, $CVaR_j(q) = E(x_j|L = VaR(q))$.
   
   (b) Expected Shortfall contribution, $CES_j(q) = E(x_j|L \geq VaR(q))$.

### 3 Importance sampling for credit risk

The importance sampling (IS) is a Monte Carlo simulation method that helps to estimate expectations of random variables through an smart change of the sampling distribution. It has been widely used to valuate market derivatives such as options, see Glasserman (2003) and Bolia and Juneja (2005). As explained previously, the most general measure in credit risk is $Prob(L \geq l)$, directly

\(^5\)For a certain client $j$, both $EAD_j$ and $LGD_j$ are random variables although they are commonly assumed to be constant. Along the paper, we will indicate whether the LGD is in percentage terms of the EAD or in euros.

\(^6\)The Basel regulation requires a bank to have an amount of own resources equal to the $VaR(99.9\%)$. 


related to the VaR at a given confidence level, or the maximum loss with a given probability. Then, to apply the IS method, we start transforming this probability into an expectation as follows

\[ Prob(L \geq l) = E(1(L \geq l)) = \int_{-\infty}^{\infty} 1(L \geq l) f(L) dL = \int_{-\infty}^{\infty} 1(L \geq l) \frac{f(L)}{g(L)} g(L) dL \]

One estimator of \( Prob(L \geq l) \) is then given as

\[ \hat{Prob}(L \geq l) = \frac{1}{N} \sum_{i=1}^{N} 1(L_i \geq l) \frac{f(L_i)}{g(L_i)} \]

where \( L_i \) is sampled from \( g(L) \). As the simulated random variables are independent, the variance of this estimator is\(^7\)

\[ Var(\hat{Prob}(L \geq l)) = \frac{1}{N^2} \sum_{i=1}^{N} \text{Var} \left( 1(L_i \geq l) \frac{f(L_i)}{g(L_i)} \right) = \frac{1}{N} \text{Var} \left( 1(L_i \geq l) \frac{f(L_i)}{g(L_i)} \right) \approx \frac{1}{N} \left[ \frac{1}{N} \sum_{i=1}^{N} 1(L_i \geq l) \frac{f^2(L_i)}{g^2(L_i)} - \left( \frac{1}{N} \sum_{i=1}^{N} 1(L_i \geq l) \frac{f(L_i)}{g(L_i)} \right)^2 \right] \]

where we have used sample statistics. Using this variance estimate and the central limit theorem we can get the confidence intervals of the probability estimates.

The expected shortfall (ES) is defined as

\[ ES = E(L|L \geq l) = \int_{-\infty}^{\infty} L f(L|L \geq l) dL = \frac{\int_{l}^{\infty} L f(L) dL}{\int_{l}^{\infty} f(L) dL} \]

and can be estimated using the IS method as

\[ \hat{ES} = \frac{\sum_{i=1}^{N} L_i 1(L_i \geq l) \frac{f(L_i)}{g(L_i)}}{\sum_{i=1}^{N} 1(L_i \geq l) \frac{f(L_i)}{g(L_i)}} \]

The estimators for the VaR and ES risk contributions of the client \( j \) are respectively

\[ CVaR_j = \frac{\sum_{i=1}^{N} x_{j,i} 1(L_i = l) \frac{f(L_i)}{g(L_i)}}{\sum_{i=1}^{N} 1(L_i = l) \frac{f(L_i)}{g(L_i)}} \]

\[ CES_j = \frac{\sum_{i=1}^{N} x_{j,i} 1(L_i \geq l) \frac{f(L_i)}{g(L_i)}}{\sum_{i=1}^{N} 1(L_i \geq l) \frac{f(L_i)}{g(L_i)}} \]

As \( CVaR_j \) cannot be implemented computationally, the following modification is required:

\[ \hat{CVaR}_j = \frac{\sum_{i=1}^{N} x_{j,i} 1(l(1 - R) \leq L_i \leq l(1 + R)) \frac{f(L_i)}{g(L_i)}}{\sum_{i=1}^{N} 1(l(1 - R) \leq L_i \leq l(1 + R)) \frac{f(L_i)}{g(L_i)}} \]

\(^7\)It can be noted that the variance of this estimator vanishes for the sampling distribution \( g(L_i) \propto 1(L_i \geq l) f(L_i) \).
where $R$ is an interval defining parameter. From now on we will employ $R = 1\%$.

The confidence intervals of the expected shortfall and the risk contributions can be derived using Serfling (1980) to obtain that

$$Var(\hat{ES}) = N \sum_{i=1}^{N} (L_i - \hat{ES})^2 \mathbf{1}(L_i \geq l) \left( \frac{f(L_i)}{g(L_i)} \right)^2$$

This equation can be extended to provide estimators of the variance of the empirical estimates of the ES and VaR risk contributions just replacing $(L_i - \hat{ES})$ by $(x_{j,i} - \hat{ES})$ or $(x_{j,i} - \hat{VaR})$ and $\mathbf{1}(L_i \geq l)$ by $\mathbf{1}(l(1 - R) \leq L_i \leq l(1 + R))$.

So far no functional form for the function $g(L)$ has been suggested. Glasserman and Li (2005) suggested to obtain $g(L)$ in two steps, changing a) the default probabilities conditional on the macroeconomic factors and b) the macroeconomics factors distribution, respectively.

### 3.1 Optimal conditional distribution

Conditional to the macroeconomic factors realization, the default probability of the client $j$ is

$$PD_{j,Z} = \Phi \left( \frac{\Phi^{-1}(PD_{j,C}) - \sum_{f=1}^{k} \alpha_{f,j} z_{f}}{\sqrt{1 - \sum_{f=1}^{k} \alpha_{f,j}^2}} \right)$$

Glasserman and Li (2005) suggested to change the default probability by a new one using an exponential twist

$$PD_{j,Z,\theta} = \frac{PD_{j,Z} e^{LGD_j EAD_j \theta}}{1 + PD_{j,Z} (e^{LGD_j EAD_j \theta} - 1)}$$

The change in the default probability of a client depends only on his specific default parameters plus a parameter $\theta$, common for all the clients. Under this twist, the weight to be assigned to every loss simulation $i$ of the total portfolio is

$$W_{1,i} = \frac{f(D_{i,1}, \cdots, D_{i,M})}{g(D_{i,1}, \cdots, D_{i,M})} = \prod_{j=1}^{M} \left( \frac{PD_{j,Z}}{PD_{j,Z,\theta}} \right)^{D_{j,i}} \left( \frac{1 - PD_{i,Z}}{1 - PD_{j,Z,\theta}} \right)^{1-D_{j,i}}$$

where $D_{j,i}$ is the default indicator of the client $j$ in the simulation $i$. A little algebra leads to

$$W_{1,i} = e^{-L_{\theta} + \psi(\theta)}$$

where

$$\psi(\theta) = \sum_{j=1}^{M} \ln \left( 1 + P_{j,Z} (e^{LGD_j EAD_j \theta} - 1) \right)$$

(2)
Note that, conditional to the macroeconomic state $Z$, the losses of every client $j$ are independent. Then, (2) implies that $\psi(\theta)$ is the cumulant generating function of the random variable $L(Z)$, with an important role in the saddlepoint approximation method.

Now the problem is to estimate the optimal value of $\theta$ that minimizes the variance of the estimator under the new distribution $g(L, \theta)$. Glasserman and Li (2005) proved that

$$\text{Var}_{g(L, \theta)} \left( \frac{1}{N} \sum_{i=1}^{N} 1(L_i \geq l) W_{1,i} \right) \leq e^{-2\theta L + 2\psi(\theta)}$$

Differentiating this upper bound and using the convexity of $\psi(\theta)$, the optimum shift $\theta$ satisfies $\psi'(\theta_l) = l$ if $l > \psi'(0)$ being null otherwise. Straightforward calculations lead to $\psi'(\theta) = \sum_{j=1}^{M} \text{LGD}_j \text{EAD}_j \text{PD}_{j,Z,\theta} = E_{g(L, \theta)}(L)$.

The intuition behind this result is that we aim to obtain high enough losses close to the loss value $l$. Under the current macroeconomic factor simulations, expected losses can be much lower than $l$ and, then, the default probabilities are changed so that the new expected losses equate the desired loss level, this is done by using $\theta_l \geq 0$. However, if the actual expected losses are higher than the desired one ($l$), default probabilities are not changed at all. In this case $\theta_l$ should be negative to get an expected loss of $l$.

If the VaR based loss contributions (CVaR) are calculated, the default probabilities will always be shifted to the desired loss level $l$, so that many simulations will lay inside the interval $l(1 \pm R)$. According to our experience, the number of simulations in the VaR interval can be doubled from that obtained when forcing $\theta_l \geq 0$.

Another interesting property of the Glasserman and Li (2005) approach is that, as $\psi(\theta)$ equates the cumulant generating function, the optimization problem $\psi'(\theta) = l$ to be solved under the IS method coincides with that solved under a saddlepoint approach. The value $\theta_l$ is computed through a non-linear iterative process that departs from an initial estimate obtained by applying a third-order Taylor expansion to $\psi(\theta)$ around $\theta = 0$.\(^8\)

### 3.2 Optimal macroeconomic distribution

As with the default probability it is possible to change the distribution of the macroeconomic factors to a new one that reduces the variance of the estimates. The probability we are interested in is

$$\text{Prob}(L \geq l) = \int_{-\infty}^{\infty} \text{Prob}(L \geq l|Z)f(Z)dZ \propto \int_{-\infty}^{\infty} \text{Prob}(L \geq l|Z)e^{-\frac{Z^2}{2}} dZ$$

The optimal sampling distribution $g(Z)$ satisfies $g(Z) \propto \text{Prob}(L \geq l|Z)e^{-(Z/Z)^2/2}$. Sampling from this distribution is complex but feasible through the Markov chain Monte Carlo technique using

\(^8\)We used this expansion to approximate the non-linear problem that has to be solved and defined a rule to choose among the three possible solutions. This approach generated initial estimates very close to the real value of $\theta_l$.  

the Metropolis-Hasting algorithm. However, Glasserman and Li (2005) suggested sampling from a normal distribution with the same mode as the optimum distribution, that is, \( g(Z) \sim N(\mu, I) \), where \( \mu = \max_Z \{ \text{Prob}(L \geq l | Z) e^{-(Z^T Z)/2} \} \). According to this, a new weight \( W_{2,i} = e^{-\mu^T Z + \mu^T \mu/2} \) has to be applied and the IS estimators will be given by

\[
\hat{\text{Prob}}(L \geq l) = \frac{1}{N} \sum_{i=1}^{N} 1(L_i \geq l) W_{1,i} W_{2,i}
\]

\[
\hat{E}(L|L \geq l) = \frac{\frac{1}{N} \sum_{i=1}^{N} L_i 1(L_i \geq l) W_{1,i} W_{2,i}}{\text{Prob}(L \geq l)}
\]

It still remains to estimate \( \text{Prob}(L \geq l | Z) \). To this aim, we decided to use a simple approach assuming that \( L|Z \sim N(a, b^2) \) where\(^9\)

\[
a = E(L|Z) = \sum_{j=1}^{M} PD_{j,Z} LGD_j EAD_j
\]

\[
b^2 = \text{Var}(L|Z) = \sum_{j=1}^{M} \text{Var}(x_j|z) = \sum_{j=1}^{M} PD_{j,Z} (1 - PD_{j,Z}) LGD_j^2 EAD_j^2
\]

4 Portfolio data

We evaluate alternative credit risk measures (loss distribution and risk contributions) considering the 157 financial entities covered by the Spanish deposit guaranty fund (FGD) at December, 2010.\(^10\) This fund was analyzed in Campos et al. (2007) by using a simple single factor model and Monte Carlo simulations. These authors just tested a range of constant LGDs not directly linked to historical recovery rates and did not estimate any risk contribution measure. We will try to overcome these limitations and will assume that the two biggest institutions (BBVA and Santander) are exposed to other economies and, hence, to other macroeconomic factors.

4.1 Probability of default (PD)

We use the credit ratings available at December, 2010 for the Spanish financial institutions and the historical observed default rates reported by the rating agencies\(^11\) to infer a probability of default. This probability is obtained adjusting an exponential function to the default rates of the ratings up to B- and imposing a value of 0.3% for a rating AA, a commonly accepted feature. Entities with

\(^9\) Other alternatives such as the constant approach or the tail bound approach can be found in Glasserman and Li (2005).

\(^10\) The FGD is built up to help the financial system stability and includes the three previously existing funds (for banks, saving banks, and cooperative banks) that were merged in October, 14th, 2011 under the Real Decreto 16/2011.

no external rating are assigned one notch less than the average rating of the portfolio with external rating. This implies that banks without external rating are assigned a A- rating and the remaining institutions a BB+ rating, values that are consistent with Campos et al. (2007). Once a rating is recovered, a long-term default rate is assigned to each institution.

We obtain that the S&P and Moody’s ratings have very similar historical default rates for the different rating letters while Fitch rating is very different from the other two. Even though Fitch and S&P use the same letters to measure credit risk, the underlying default risk is different, specially for the very bad ratings. Luckily, no institution had this rating at the date of analysis and, then, we can still use the calibrated probabilities of default.

4.2 Exposure at default (EAD)

Details on assets, liabilities, and deposits for the FGD institutions are available in the AEB, CECA, and AECR webpages. The FGD covers not only depositors but also any loss due to a Governmental intervention of a financial institution. Hence, our analysis focuses on total assets losses and not only on losses to depositors.

Balance information at December 2010 was used for the analysis. As many mergers took place during 2010 (see Table 1), we have summed all the information from the different institutions that belong to the same group.

4.3 Loss given default (LGD)

Bennet (2002) computed the losses due to financial institutions default in the deposits guarantee fund in United States (FDIC) and showed that the average losses are bigger in the smallest banks

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12This average is computed weighting by assets and distinguishing between banks and saving banks.
13For the sake of brevity, these results are not reported here and are available upon request.
14AEB is the Spanish Bank Association, CECA is the Spanish Saving Bank Association, and AECR is the Spanish Credit Cooperatives Association. Other sources as Bankscope were tested, however the set of available institutions was smaller.
for the period 1986-1998. We update this analysis for the period 1986-2009 using FDIC public data and the banks assets are updated using the USA CPI series aiming to have comparable asset sizes. We obtain an average LGD for deposits of 20.73% but this value may be biased as there are many observations in the initial and final years of the database. Hence, we decide to use $E(E(LGD_{j,t}|t))$ as an estimate of the real average LGD and obtain 18.35%, that is, 88.56% of the initial average LGD. Then, we estimate $E(LGD_{j,t}|\text{Asset Bucket})$ and multiply it by the 88.56% adjustment factor. Finally, these LGDs on deposits are transformed into the LGDs on assets using a multiplicative factor of 1.378.\footnote{This factor is based on the numbers obtained in Bennet and Unal (2011) that used FDIC data for 1986-2007 and estimated an average depositors LGD of 24.4%, equivalent to a 29.95% total LGD over assets before the time effect and a 33.61% after the discount effect.}

Table 2 provides the LGDs obtained in this way.

4.4 Factor correlation ($\alpha$)

We use the total factor sensitivities ($\alpha_j$) stated in the Basel accord. These values range between $\sqrt{0.12}$ and $\sqrt{0.24}$ according to the formula $\sqrt{0.12\omega + 0.24(1-\omega)}$ where $\omega = \frac{1-e^{-50PD}}{1-e^{-50}}$.\footnote{However, in practice, most of the entities show sensitivities closer to $\sqrt{0.24}$.} Then, our values are between those used in Kuritzkes et al. (2002) and Campos et al. (2007) ($\sqrt{0.15}$ and $\sqrt{0.30}$, respectively). Recently, the Basel III accord has increased the previous Basel II correlations by a factor of $\sqrt{1.25}$. In this way, we would generate correlations in the range of those used in Campos et al. (2007). In the following analysis we use the Basel III correlations.

We assume geographic macroeconomic factors and that all the financial institutions are exposed only to the Spanish factor except for BBVA and Santander that are exposed to additional geographies. This assumption seems reasonable and its motivation can be seen in Figure 1 which shows that, among the 25 biggest financial institutions, apart from these two entities, only Barclays is not a fully Spain based bank and its share is very small.

The exposure of BBVA and Santander to the macroeconomic factors is computed using the reported net interest income by geography obtained from the public 2010 annual reports. We think that this variable can be a good proxy of the risk faced by a financial institution and, then, it can indicate appropriately its exposure to the different countries in which the institution operates. Hence, an income based allocation method can be better than a method only based on exposures that would assign small weights to the non-Spanish geographies.

Finally, we assume that the correlation between the macroeconomic factors for different countries is equal to that between the GDP of the countries.\footnote{These correlations are available upon request.}
Table 3 shows the exposure of BBVA and Santander to the different countries according to their net interest incomes. As these country factors are correlated, those exposures have to be standardized so that the total variance of the sum of each client’s macroeconomic factors equates one.

[INSERT TABLE 3 AROUND HERE]

4.5 Portfolio expected loss and Basel loss distribution

The total assets, expected loss, and BIS 99.9% probability loss for the Spanish financial institutions are 2,921,504 MM €, 453 MM €, and 13,733 MM €, respectively.

The left graph in Figure 2 shows the share of these variables for the biggest (ordered by assets) 25 financial institutions.

[INSERT FIGURE 2 AROUND HERE]

Two conclusions can be extracted from this Figure:

1. Expected loss and Basel 99.9% probability loss generate a very similar ordering.

2. The ordering according to the assets amount is very different from that based on expected or Basel losses.

The right graph in Figure 2 shows the expected loss and Basel 99.9% loss divided by the size of each institution. We find that the two biggest institutions (BBVA and Santander) share very low risk parameters.

We will introduce now the results obtained with the IS method as a way to deal with non-granular and multifactorial portfolios. The main ideas behind this modification of the asymptotic single factor model are a) BBVA and Santander have some diversification effects as they are exposed to more than one macroeconomic factor that reduces their risk and b) having non-granular portfolios increases the risk.

5 Importance sampling results

We start orthogonalizing the country factors by applying principal components analysis. As the correlation between the different economies is very high we end up having a very important common factor across all the financial institutions. When we obtain the optimum change in the factor mean for a target loss of 10 times the expected loss we get a 1.62 value in the main common factor and zero otherwise.
Figure 3 shows the loss distribution under IS and Monte Carlo simulations. According to the Basel model the loss level with 99.9% probability is 13,733 MM€. While under multifactorial non-granular portfolios this loss level is 32,102 MM€, 2.3 times more!\(^\text{18}\)

![INSERT FIGURE 3 AROUND HERE]

Figure 4 shows the results for the expected shortfall. The VaR contributions are usually less stable as few simulations fall inside the interval. That is why it is quite common using the expected shortfall contributions at a loss level whose tail expectation equals the \(\text{VaR}(99.9\%) = 32,102\) MM€. In this case this loss level is 16,274 MM€.

![INSERT FIGURE 4 AROUND HERE]

Figure 5 shows the risk allocation rule according to the ES and VaR contributions and the confidence intervals for the IS technique.\(^\text{19}\) These intervals are quite thin after only 10,000 simulations, one of the main advantages of the IS method over the Monte Carlo simulations. Moreover, the IS method can generate many high loss simulations from a thin loss interval and, then, more accurate estimates at a lower computation time. The risk picture is completely different from that obtained using the simple expected loss or the Basel loss model. The main reason for this is that the non-granularity effect increases (decreases) the risk allocated to the biggest (smallest) institutions.

![INSERT FIGURE 5 AROUND HERE]

The main ideas that can be extracted from this Figure are the following:

1. The LGDs (in euros) for BBVA and Santander are higher than the \(\text{VaR}(99.9\%)\). Then their VaR contributions are zero.

2. The LGD of Bankia is 28,948 MM€, close to the \(\text{VaR}(99.9\%)\) value. Then, this firm copes most of the risk under the VaR contribution allocation method.

3. The risk allocations of Caixabank and Unnim have big confidence intervals. This is due to the fact that the LGD of both entities together is close to the \(\text{VaR}(99.9\%)\) and there are few simulations in which Caixabank and Unnim default.

4. The confidence intervals of the 99.9% probability loss ratio are bigger as the risk is adjusted by the institution size and Unnim has the biggest confidence intervals for the risk allocation.

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\(^{18}\) All the figures in the paper are based on the IS results rather than on the Monte Carlo method.

\(^{19}\) For the VaR contributions we have used a ±1% interval around the desired loss level.
6 Importance sampling modifications and extensions

This Section extends the classical IS framework to deal with random recoveries and market valuation. Other extensions were performed:20

1. We found that using the mode for the macroeconomic factor shifts may introduce a low sampled region problem and we developed a method based on the mean of the optimal distribution to overcome this problem.

2. For granular portfolios, we found that the 99.9% probability losses of the Spanish financial would be 13,478 MM €.

3. We also evaluated the suitability of the simulation loop decoupling, based on simulating $N_{Macro}$ macroeconomic scenarios and $N_{Default}$ default scenarios for each (simulated) macroeconomic scenario. This analysis is very interesting in terms of speed and accuracy for portfolios with few counterparties that are exposed to the same macroeconomic factor, as it is our case. The following IS results are based on this extension.

6.1 Random loss given default

So far the LGD has been considered as constant but it is a random variable with the same span as the default rates. Then, it seems natural to assume that the LGD follows a similar distribution to that of the default rate. Considering this, the simplest case assumes that the whole recovery risk comes from macroeconomic factors, for example, a single factor called $z_{LGD}$:

$$LGD_{j,Z} = \Phi \left( \frac{\Phi^{-1}(LGD_{j,C}) - \alpha_j z_{LGD}}{\sqrt{1 - \alpha_j^2}} \right)$$

Under this specification the only parameters to be estimated are $\alpha_j$ and the correlation between $z_{LGD}$ and the rest of the macroeconomic factors. This model also allows to have more macroeconomic factors but the idea is that no idiosyncratic risk is considered.

The previous formula has been widely studied21 and some of their moments have a closed-form expression, for example

$$E(LGD_{j,Z}) = LGD_{j,C}$$
$$E(LGD^2_{j,Z}) = \Phi_2(\Phi^{-1}(LGD_{j,C}), \Phi^{-1}(LGD_{j,C}), \alpha_j^2)$$

We have shown previously that the LGD depends on the institution size and that most of the defaults in our sample correspond to institutions with less than 1,000 MM € in assets. To keep the

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20For the sake of brevity, we just enumerate these extensions whose detailed findings are available upon request.
21See Dullmann et al. (2008) or Gordy (2000).
database as clean as possible we will estimate the parameters using just the institutions with this assets size.

The above formulas and the historical recovery rates from the FDIC data imply $LGD_{j,c} = 19.13\%$, $E(LGD_{j,z}^2) = 4.3178\%$ and, therefore, $\alpha_j = 29.26\%$. Using these estimates we recover the $z_{PD}$ and $z_{LGD}$ factors from the historical default series of the FDIC and obtain that the correlation between the default and recovery factors is 22.63%. The random LGD is introduced replicating the factor correlation of the PD for the LGD as follows:

$$G = \begin{bmatrix}
M_{PD} & 22.63\% & 0\% & \cdots \\
22.63\% & 0\% & \cdots \\
0\% & \cdots & 0\% \\
\cdots & 22.63\%
\end{bmatrix}
\begin{bmatrix}
M_{LGD}
\end{bmatrix}$$

where $M_{PD} = M_{LGD}$ equates the GDP correlation matrix of the different countries.\(^{22}\) Now not only $PD_{j,z}$ has to be estimated but also $LGD_{j,z}$ in every simulation step. The optimal exponential twist and the optimal change in the mean of the macroeconomic factors are obtained using $PD_{j,z}$ and $LGD_{j,z}$.

Figure 6 shows the comparison between the loss distributions of the portfolio under random and constant LGDs. The 99.9% probability loss is 36,970 MM €, that is, 1.15 times the loss level under constant LGD. The equivalent expected shortfall level is 19,326 MM €. Figure 7 shows the risk allocation under VaR and ES for the new 99.9% probability loss level. Comparing with Figure 5 we can see that this model assigns risk to all the institutions, even to Santander whose initial LGD was 53,146 MM €, much higher than the 99.9% probability loss. However, as now the LGD is random, there are some scenarios where Santander defaults and the total loss is close to 36,970 MM €.

Compared with the constant LGD case, the random LGD provides the following facts:

1. The confidence intervals in the risk allocation are wider. Now, in the event of default, the losses have a bigger variability and, hence, the estimation of $E(X_i|L = VaR)$ is also more volatile.

2. The risk allocations based on the VaR and the ES are relatively “similar” and the risk is not concentrated in some institutions as in the case of constant LGD.

\(^{22}\)For BBVA and Santander the weights of the LGD to the different LGD factors are the same as those defined before according to their net interest incomes.
Under the Basel accord, the random LGD is considered under a very broad definition of a downturn LGD, defined as the LGD under a stress scenario. This constant downturn LGD tries to capture somehow the effect of the random LGD.

Under the previous setup, two clients with the same $LGD_{j,C}$ and the same sensitivity to the macroeconomic variables will have the same $LGD_{j,Z}$. To avoid this possibility, an idiosyncratic term $\gamma_j \sim N(0, 1)$ can be included in the previous formula:

$$LGD_{j,z,\gamma_j} = \Phi \left( \frac{\Phi^{-1}(LGD_{j,C}) - \alpha_j (rz_{LGD} + s\gamma_j)}{\sqrt{1 - \alpha_j^2}} \right)$$

with $r^2 + s^2 = 1$. This second specification reduces the correlation between the LGD and the defaults as a new independent term is considered but it can increase the variability of the recoveries.

The parameter $r$ controls the variability in $LGD_{j,Z}$ over the business cycle. According to our data, we obtain $E(Var(LGD_{j,z}|z)) = 1.338\%$; implying a variability that is higher than the average LGDs of the big financial institutions. Intuitively, now, more institutions can generate high and low loss levels compared to the constant LGD case and the confidence intervals will be wider than under constant LGD and under fully macroeconomic random LGD. Calibration of the LGD data provides $r = 59.39\%$

Using these data, for every default and recovery observation in the FDIC database, we recover the value $rz_{LGD} + s\gamma_j$ using the previous formula. Then, for every year, we obtain empirically $E(rz_{LGD} + s\gamma_j)$ that equates $rz_{LGD}$. In this way we estimate $z_{LGD}$ for every year and obtain that the correlation between $z_{LGD}$ and the default driving macroeconomic factor $z_{PD}$ is 19.02%.

This new specification causes some changes in the IS framework. For instance, the exponential twist of the default probabilities conditional to a given set of macroeconomic factors was defined as that generating an expected loss equal to the target loss level. Now, conditional to these factors, $LGD_{j,z}$ is not constant and we have two alternatives to find the optimum exponential twist:

1. To keep using the average loss given default $LGD_{j,C}$ regardless of the macroeconomic factors.
2. To estimate $E(LGD_{j,z} | z)$ and $E\left(LGD_{j,z}^2 | z\right)$ for every macroeconomic factor simulation.

We use the second method given that $E(LGD_{j,z})$ has a closed-form expression given as

$$E(LGD_{j,z}) = Prob(V_{j,z} < \Phi^{-1}(LGD_{j,C})) = \Phi \left( \frac{\Phi^{-1}(LGD_{j,C}) - \alpha rz_{LGD}}{\sqrt{\alpha^2(s^2 - 1) + 1}} \right)$$

Computing the optimal change in the mean of the factors is a bit more complex as it requires $2^{24}$ Then, the LGD can change $\pm 11.56\%$ with respect to its mean.
estimating $\text{Var}(\text{LGD}_{j,z}|z)$ or, equivalently, $E(\text{LGD}_{j,z}^2|z)$, this is,

$$E(\text{LGD}_{j,z}^2|z) = \Phi_2\left(\begin{pmatrix} \Phi^{-1}(\text{LGD}_{j,C}) \\ \Phi^{-1}(\text{LGD}_{j,C}) \end{pmatrix}, \begin{pmatrix} \alpha r z_{\text{LGD}} & \alpha^2 s^2 \\ \alpha^2 s^2 & \alpha^2 s^2 + (1 - \alpha^2) \end{pmatrix}\right)$$

It is worthy to note that the optimal exponential twist is generated using $E(\text{LGD}_{j,Z,\gamma_j}|Z)$ rather than the simulated $\text{LGD}_{j,Z,\gamma_j}$. Then the weight $W_{1,i}$ must be estimated using $E(\text{LGD}_{j,Z,\gamma_j}|Z)$ rather than the realized $\text{LGD}_{j,Z,\gamma_j}$, that is, using $L^*_i = \sum_{j=1}^M D_j E AD_j E (\text{LGD}_j|Z)$ instead of $L_i$.

Figure 8 provides the loss distributions under the three possible specifications: constant LGD, macroeconomic random LGD ($\text{LGD}_1$), and macroeconomic plus idiosyncratic random LGD ($\text{LGD}_2$). It can be seen that considering the idiosyncratic term adds some more risk to the 99.9% loss level.

The effect of the idiosyncratic risk is quite small in the loss distribution. Using the IS results, the 99.9% loss level under the idiosyncratic risk is 37,934 MM €, only 964 MM € more than that under the macroeconomic LGD model. Hence, the impact of the idiosyncratic LGD on the loss distribution is small compared with that of the macroeconomic LGD. It can also be noted that, for small (large) loss levels, the idiosyncratic risk term reduces (increases) the chance of those losses.

Regarding the risk allocation, Figure 9 shows that, in this case, the (absolute and relative) risk allocation has even bigger confidence intervals than in the previous models. The reason is that previously highlighted: given default, the variability of the losses of the client $j$ are wider under the idiosyncratic LGD model than under the pure macroeconomic LGD.

Other LGD distributions have been tested for the pure macroeconomic LGD model ($\text{LGD}_1$) and the mixed macroeconomic and idiosyncratic LGD model ($\text{LGD}_2$). Table 4 includes the resulting loss distributions using the IS method and shows that the results of the different random LGD models for the 99.9% loss level are quite similar in all the cases except for the Log-Normal one.

6.2 Market mode

This Section evaluates the portfolio risk under a market value model instead of a default mode one. Under this model the rating of the companies may change over the time and these changes affect the

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24 The ES equivalent loss is 19,473 MM €.

25 Detailed results are not reported here and are available upon request.
firm valuation. Then, it is more intuitive to talk about the portfolio value for a given scenario rather than about portfolio losses. To calibrate a discount factor we obtain the median CDS spread for a sample of European financial institutions ordered by ratings.\footnote{These data correspond to 5-year senior CDS since 2008 and were obtained from Markit.} Figure 10 illustrates that the worse the ratings the higher the CDS spread and that the spread required by the market has increased considerably since 2008.

We have extended linearly the CDS values to the remaining ratings according to their average default probability and obtained the daily series of the median CDS spread level for each rating grade for the period 2008-2011. We assume that this is a representative spread to obtain a discount factor for the different ratings. However this spread usually assumes a LGD of 60% for bonds while we have an average LGD value of 18.35% x 1.378 = 25.28% over assets.\footnote{As the financial institutions with available data in Markit have a high level of assets, it is quite possible that the LGDs of these entities will be smaller than 25.28% but this is a conservative assumption.} Hence we adjust linearly the spread. We assume an average maturity of 3 years for the assets in the portfolio; this is a mixture of the retail banking assets with longer maturity (like mortgages) and the corporate banking assets with shorter maturity. The average maturity of the assets is a key assumption in the model, the greater the maturity the higher the chance of high losses. Unluckily this information is not public for banks. Table 5 reports the 3-year discount factors obtained for each rating in this way.

To simulate the rating transitions, we use an average rating transitions matrix over the business cycle. We adjust the S&P public data in S&P (2010) to take into account the non-rated companies and we do impose the average probability of default previously adjusted. Table 6 includes the rating transition matrix employed.

### 6.2.1 Migration rule

For a default mode model, the default probability of the client \( j \) conditional to a given macroeconomic scenario is

\[
PD_{j,Z} = \Phi \left( \frac{\Phi^{-1}(PD_{j,C}) - \sum_{f=1}^{k} \alpha_{f,j} z_f}{\sqrt{1 - \sum_{f=1}^{k} \alpha_{f,j}^2}} \right)
\]

This means that, to simulate the defaults, we can generate a random number \( U_j \sim U(0,1) \) and the client defaults if

\[
U_j \leq PD_{j,Z}.
\]
In the case of a market mode model a client can move from an initial rating to a new one. Let $MP_{j,C,IR,FR}$ denote the average probability (over the cycle) for the client $j$ of migrating from an initial rating $IR$ to $FR$, a final one. We can construct the accumulated probabilities $AccumMP_{j,Z,IR,FR}$, $FR = AAA, AA+, \cdots, CCC, D$.\(^{28}\)

Then, for a given macroeconomic state, we can calculate the point in time accumulated probability of migration between ratings, $AccumMP_{j,Z,IR,FR}$, as

$$AccumMP_{j,Z,IR,FR} = \Phi\left(\Phi^{-1}(AccumMP_{j,C,IR,FR}) - \sum_{f=1}^{k} \alpha_{f,j} z_f\right) \sqrt{1 - \sum_{f=1}^{k} \alpha_{f,j}^2}$$

We generate a random number $U_j \sim U(0,1)$. Now, if $U_j \leq AccumMP_{j,Z,IR,D}$, the new rating of the client would be D. If $AccumMP_{j,Z,IR,CCC} \leq U_j \leq MP_{j,Z,IR,D}$, the new rating would be CCC and so on. For each possible rating state the whole portfolio is evaluated.

### 6.2.2 Importance sampling

The IS framework must be modified in two ways: a) the exponential twisting rule should be extended to deal with more than two possible states and b) the conditional portfolio value must be approximated to estimate the macroeconomic factor mean shift.

Given a macroeconomic scenario $Z$, the exponential twist of the migration probabilities $MP$ of the client $j$ from the rating state $IR$ to $FR$ can be extended as follows:

$$MP_{j,Z,IR,FR,\theta} = \frac{MP_{j,Z,IR,FR} e^{V_{j,FR} \theta}}{\sum_{i=1}^{k} MP_{j,Z,IR,i} e^{V_{j,i} \theta}}$$

where $V_{j,i}$ is the loan value to the counterparty $j$ given the rating state $i$, that is, $EAD_j \times DF_i$ where $DF_i$ is the discount factor in the state $i$. Now, the natural extension of the default mode twist to the case of the mark to market valuation is

$$V_{l} = \sum_{j=1}^{M} \sum_{h=1}^{k} \frac{MP_{j,Z,IR,h} e^{V_{j,h} \theta}}{\sum_{i=1}^{k} MP_{j,Z,IR,i} e^{V_{j,i} \theta}}$$

that is, the expected value of the portfolio equates the target value.

We use the normal approximation to change the mean of the factors. Under this approximation, conditional to the macroeconomic state $Z$, the portfolio value is distributed as $N(\mu_Z, \sigma_Z)$ with

$$\mu_Z = \sum_{j=1}^{M} \sum_{h=1}^{k} V_{j,h} MP_{j,Z,IR,h}, \quad \sigma_Z = \sqrt{\sum_{j=1}^{M} \sum_{h=1}^{k} V_{j,h}^2 MP_{j,Z,IR,h} - (\mu_Z)^2}$$

\(^{28}\)For example, $AccumMP_{j,Z,IR,B-} = MP_{j,Z,IR,B-} + MP_{j,Z,IR,CCC} + MP_{j,Z,IR,D}$. 

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According to the current ratings, the market value of the Spanish financial system is 2,842,499 MM €, representing a 2.7% discount with respect to the total assets. Applying the discounting factors to the migration probabilities, we get that the expected value of the portfolio is 2,839,535 MM €. Under a default mode model we focused on 4,528 MM € losses (ten times the expected loss), in this case focusing on the current market value this is equivalent to 2,842,499 - 4,528 = 2,837,971 MM € market value. We will use this number as the target value for the IS method.

We will focus on value losses compared with the current market value rather than with total assets. The idea is that the difference between total assets and the current market value has been previously recognized through profit and losses statement and, hence, it does not represent a possible future loss. It means that debt holders and depositors should be concerned about the possible losses over the current market value and the amount of own resources that the institution has at any time.

Figure 11 shows the loss distribution of the portfolio. For each simulation, losses are obtained as the market value minus the starting market value, 2,842,499 MM €. The 99.9% probability loss is 68,852 MM €, additional to the current market value loss, equal to 79,006 MM €. As the simulation speed is very sensitive to the number of possible states, it is very important to use only clearly different ratings.29

Regarding the VaR and ES based contributions we will allocate the 68,852 MM € loss over the current market value. Figure 12 provides the results and shows that the top contributor is Santander.

7 Conclusions

This paper has successfully extended the IS framework introduced by Glasserman and Li (2005) to the case of random recoveries and market mode models. We also tested the effect of granular portfolios, simulation loop decoupling and mean based macroeconomic factors shift.

All these extensions allow to use this method inside financial institutions or for regulatory purposes. The extensions and modifications have been tested on a portfolio including Spanish financial institutions using Monte Carlo simulations as benchmark. Based on Bennet (2002), the LGD of the different institutions has been obtained and used to estimate the loss distribution of this financial system.

29 The analysis has been performed using the rating scale considering modifiers but it could be done without these modifiers.
According to our results the 99.9% probability losses can range between 30,000 and 70,000 MM € depending on the LGD model and the valuation method employed. However, under a granular portfolio with constant LGD, the 99.9% probability losses would be only 13,478 MM €. The confidence intervals of the loss distribution obtained using the IS approach are very thin regardless of the LGD model or the valuation method used.

The confidence intervals of the risk allocation obtained using IS are much thinner than those obtained with the Monte Carlo method, specially for the VaR based risk allocation. In general, the risk allocation based on VaR has wider confidence intervals than that based on ES. More precisely, under constant LGD, the VaR based risk allocation has thin confidence intervals and requires a low number of simulations. However, as we move to a random LGD framework, the number of simulations required to obtain small confidence intervals in the risk allocation increases considerably. Hence, one possible way to deal with this issue is to use the IS method to estimate the risk allocation in the case of constant LGD and try to extend other methods such as those in Huang et al. (2007), Pykhtin (2004) and Voropaev (2011) to deal with the random LGD risk allocation.

Analyzing the suitability of the allocation methods, we have found that the results can vary considerably. Probably the best approach is to obtain all the possible results and compare them. For example, under the CVaR, a given deal may have a null risk allocation (as happened with BBVA and Santander in the constant LGD model) and, hence, provide a infinite risk adjusted return, but this would lead to a higher concentration.

This kind of analysis can provide a basic tool for regulators when analyzing the solvency of the financial system and when studying the relevance of the financial institutions in the economy. This last issue is specially interesting when establishing the so called systematically important financial institutions surcharge in BIS III.
References


### Appendix of Tables

**Table 1:** Spanish financial institutions involved in a merger / acquisition process or belonging to the same corporation at December, 2010.

<table>
<thead>
<tr>
<th>New Entity</th>
<th>Original Institutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Banca Civica</td>
<td>Caja Municipal de Burgos, Caja Navarra, Caja Canarias, CajaSol, Caja Guadalajara</td>
</tr>
<tr>
<td>Banco Base</td>
<td>Caja Asturias, Banco de Castilla La Mancha, Caja Cantabria, Caja Extremadura</td>
</tr>
<tr>
<td>Banco Mare Nostrum</td>
<td>Caja Murcia, Caixa Penedés, Caja Granada, Caja Sa Nostra</td>
</tr>
<tr>
<td>Banco Popular</td>
<td>Banco Popular, Banco Popular Hipotecario, Banco Popular-e, Popular banca privada</td>
</tr>
<tr>
<td>Bankia</td>
<td>Caja Madrid, Bancaja, Caixa Laietana, Caja Avila, Caja Segovia, Caja Rioja, Caja Insular</td>
</tr>
<tr>
<td>BBK</td>
<td>BBK, Cajasur</td>
</tr>
<tr>
<td>BBVA</td>
<td>BBVA, Finanzia, Banco Depositario BBVA, UNO-E Bank</td>
</tr>
<tr>
<td>Caixabank</td>
<td>La Caixa, Caixa Girona, Microbank</td>
</tr>
<tr>
<td>Caja 3</td>
<td>Caja Inmaculada, Caja Burgos CCO, Caja Badajoz</td>
</tr>
<tr>
<td>Caja España de Inversiones</td>
<td>Caja España, Caja Duero</td>
</tr>
<tr>
<td>CatalunyaCaixa</td>
<td>Caixa Cataluña, Caixa Tarragona, Caixa Manresa</td>
</tr>
<tr>
<td>Novacaixagalicia</td>
<td>Caja Galicia, Caixanova</td>
</tr>
<tr>
<td>Santander</td>
<td>Banco Santander, Banesto, Santander Investment, Openbank, Banif, Santander Consumer Finance</td>
</tr>
<tr>
<td>Unicaja</td>
<td>Unicaja, Caja Jaén</td>
</tr>
<tr>
<td>Unnim</td>
<td>Caixa Sabadell, Caixa Terrassa, Caixa Manlleu</td>
</tr>
</tbody>
</table>
Table 2: LGD estimates for losses on deposits and losses on assets for the period 1986-2009 obtained from the FDIC public data by institution size.

<table>
<thead>
<tr>
<th>Assets</th>
<th>Count</th>
<th>Mean (deposits)</th>
<th>Mean (assets)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; $1bn</td>
<td>1148</td>
<td>18.61%</td>
<td>25.65%</td>
</tr>
<tr>
<td>$1bn - $5bn</td>
<td>49</td>
<td>15.50%</td>
<td>21.37%</td>
</tr>
<tr>
<td>$5bn - $15bn</td>
<td>7</td>
<td>9.95%</td>
<td>13.72%</td>
</tr>
<tr>
<td>&gt; $15bn</td>
<td>8</td>
<td>6.39%</td>
<td>8.82%</td>
</tr>
</tbody>
</table>

Table 3: BBVA and Santander country exposures obtained according to the net interest income data published in their 2010 Annual Reports.

<table>
<thead>
<tr>
<th>Country</th>
<th>BBVA %</th>
<th>Santander %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spain</td>
<td>37.7%</td>
<td>18.8%</td>
</tr>
<tr>
<td>Mexico</td>
<td>33.5%</td>
<td>5.9%</td>
</tr>
<tr>
<td>United States</td>
<td>9.6%</td>
<td>6.8%</td>
</tr>
<tr>
<td>Argentina</td>
<td>2.8%</td>
<td>0%</td>
</tr>
<tr>
<td>Chile</td>
<td>4.0%</td>
<td>5.3%</td>
</tr>
<tr>
<td>Colombia</td>
<td>4.0%</td>
<td>0%</td>
</tr>
<tr>
<td>Peru</td>
<td>4.8%</td>
<td>0%</td>
</tr>
<tr>
<td>Venezuela, RB</td>
<td>3.7%</td>
<td>0%</td>
</tr>
<tr>
<td>Portugal</td>
<td>0%</td>
<td>2.6%</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0%</td>
<td>14.5%</td>
</tr>
<tr>
<td>Brazil</td>
<td>0%</td>
<td>36.8%</td>
</tr>
<tr>
<td>Italy</td>
<td>0%</td>
<td>0.7%</td>
</tr>
<tr>
<td>Finland</td>
<td>0%</td>
<td>0.8%</td>
</tr>
<tr>
<td>Germany</td>
<td>0%</td>
<td>7.5%</td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>
Table 4: Comparison of the 99.9% probability loss levels under different random LGD models. We consider a pure macroeconomic LGD (LGD$_1$), based on transformations of a random normal macroeconomic variable $z_{LGD}$, the random LGD conditional to the macroeconomic variable $z_{LGD}$ (LGD$_2$), and the case of $LGD|z_{LGD}$ with Beta and Gamma distributions.

<table>
<thead>
<tr>
<th>Model</th>
<th>Loss (MM €)</th>
<th>Model</th>
<th>Loss (MM €)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal LGD$_1$</td>
<td>37,160</td>
<td>Probit Normal LGD$_1$</td>
<td>35,999</td>
</tr>
<tr>
<td>Normal LGD$_2$</td>
<td>38,131</td>
<td>Probit Normal LGD$_2$</td>
<td>35,318</td>
</tr>
<tr>
<td>Log-Normal LGD$_1$</td>
<td>29,309</td>
<td>Normal$^2$ LGD$_1$</td>
<td>36,826</td>
</tr>
<tr>
<td>Log-Normal LGD$_2$</td>
<td>36,139</td>
<td>Normal$^2$ LGD$_2$</td>
<td>36,587</td>
</tr>
<tr>
<td>Logit Normal LGD$_1$</td>
<td>35,909</td>
<td>Beta LGD$_2$</td>
<td>37,616</td>
</tr>
<tr>
<td>Logit Normal LGD$_2$</td>
<td>34,997</td>
<td>Gamma LGD$_2$</td>
<td>37,578</td>
</tr>
</tbody>
</table>

Table 5: Discount factor by rating grade based on the average CDS spread and 3 years average maturity.

<table>
<thead>
<tr>
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Table 6: Average 1-year rating migration matrix from S&P (2010).

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<th>AA+</th>
<th>AA</th>
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<th>BBB</th>
<th>BBB-</th>
<th>BB+</th>
<th>BB-</th>
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<th>CCC/C</th>
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</tbody>
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Appendix of Figures

Figure 1: Assets and deposits share of the top twenty-five Spanish financial institutions.

Figure 2: Assets, Expected Loss, and Basel 99.9% loss share of the top 25 Spanish financial institutions. Left and right graphs show, respectively, the amount allocation and the allocated amount relative to the institution size.
Figure 3: Loss distribution using 10,000 importance sampling (IS) and 1,000,000 Monte Carlo (MC) simulations. The black and red lines show, respectively, the Monte Carlo and IS results while the blue lines indicate the 5%-95% confidence interval of the IS estimates. Left and right graphs show, respectively, the tail distribution and the detail of the distribution in the neighborhood of the 99.9% probability loss level.

Figure 4: Expected Shortfall using 10,000 importance sampling (IS) simulations. The red and blue lines show, respectively, the IS results for the expected shortfall estimate and its 5%-95% confidence intervals.
Figure 5: Risk allocation under constant LGD based on expected loss (EL), Basel loss 99.9% (BIS), contributions to VaR (CVaR) and ES (CES) both using importance sampling (IS) and Monte Carlo (MC) criteria. Left and right graphs show, respectively, the total risk allocation and the allocated risk relative to the institution size.

Figure 6: Comparison of the random LGD (Rnd LGD) and constant LGD (Const LGD) loss distributions. Black lines show the results of the Monte Carlo (MC) method using 1,000,000 simulations. The red and blue lines show, respectively, the importance sampling (IS) estimates and their 5%-95% confidence intervals using 10,000 macroeconomic scenarios and 100 default simulations on each macroeconomic scenario. Left and right graphs show, respectively, the tail distribution and the detail of the distribution in the neighborhood of the 99.9% probability loss level.
Figure 7: Risk allocation under macroeconomic random LGD ($LGD_1$) for the VaR (CVaR) and the ES (CES) criteria. Continuous and dashed lines represent, respectively, the IS estimates and the 5%-95% confidence intervals. Left and right graphs show, respectively, the total risk allocation and the allocated risk relative to the institution size.

Figure 8: Comparison of the two random LGD models ($Rnd \ LGD_1$ / $Rnd \ LGD_2$) and constant LGD (Const LGD) loss distributions. Black lines show the results of the Monte Carlo (MC) method using 1,000,000 simulations. The red and blue lines show, respectively, the importance sampling (IS) estimates and their 5%-95% confidence intervals using 10,000 macroeconomic scenarios and 100 default simulations on each macroeconomic scenario. Left and right graphs show, respectively, the tail distribution and the detail of the distribution in the neighborhood of the 99.9% probability loss level.
Figure 9: Risk allocation under mixed macroeconomic and idiosyncratic random LGD ($LGD_2$) for the VaR (CVaR) and ES (CES) criteria. Continuous and dashed lines represent, respectively, the IS estimates and the 5%-95% confidence intervals. Left and right graphs show, respectively, the total risk allocation and the allocated risk relative to the institution size.

Figure 10: Median 5Y CDS spread evolution for a set of European financial institutions ordered by rating grades over the period 2007-2010.
Figure 11: Market mode loss distribution. The black line shows the results of the Monte Carlo (MC) method using 1,000,000 simulations. The red and blue lines show, respectively, the importance sampling (IS) estimates and their 5%-95% confidence intervals using 10,000 macroeconomic scenarios and 100 default simulations on each macroeconomic scenario.

Figure 12: Risk allocation under market valuation for the VaR (CVaR) and ES (CES) criteria. Continuous and dashed lines represent, respectively, the IS estimates and the 5%-95% confidence intervals. Left and right graphs show, respectively, the total risk allocation and the allocated risk relative to the institution size.