Capital Requirements and Bank Failure

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June 2009

Abstract

This paper studies the effect of capital requirements on bank’s probability of failure and entrepreneurs’ risk. Higher capital requirements reduce banks’ leverage and, for given asset risk, reduce the probability of bank failure. But higher capital requirements increase the cost of funding, which leads to higher loan rates and, possibly, riskier loans. Although the net effect is ambiguous, numerical results for a competitive banking system with imperfectly correlated loan defaults, show that a non monothonic relationship between capital requirements and the risk of bank failure generally obtains. Results for a monopolistic setup and a general correlation function among loan defaults are discussed.

Keywords: Bank Failure, Buffers, Capital requirements, Loans’ correlation, Loan rates.

JEL Classification: G21, G28, E43

*I would like to thank Douglas Gale, Michal Kowalik, Gerard Llobet, Rafael Repullo, Javier Suarez and Ernst-Ludwig von Thaden for their comments. Financial support from the Spanish Ministry of Education (Grant BES-2006-13469) is gratefully acknowledged. Address for correspondence: CEMFI, Casado del Alisal 5, 28014 Madrid, Spain. Phone: 34-914290551. E-mail: dmartinez@cemfi.es.
1 Introduction

The view that capital requirements enhance the safeness of the banking system is widespread throughout the banking literature. The main rationale underlying this statement focusses on the implications for banks risk taking decisions of an increase in the exposure of their own funding. Due to limited liability and the access to secured deposits, banks have an incentive to choose risky projects which ultimately increase the probability of bank failure. Increasing the percentage of the investment funded by banks inside resources, from now on capital, ameliorates this risk taking behavior of banks. Numerous studies have shown the importance of capital requirements in order to limit the risk taking behavior of banks. Examples of this literature are the studies of Stiglitz et al. (2000) and Repullo (2004).\textsuperscript{1}

The literature on capital requirements has had an important role in establishing frameworks for the development of the Basel I Capital Accord (1988) and more recently the Basel II Capital Accord which establish the capital requirements that banks must adopt in order to undertake their investments.

Previous studies analyzing the effect of capital requirements in banks’ probability of failure assume that banks invest in exogenous assets with fixed return distributions. This overlooks the effects that banks actions have on their investment. This issue is particularly important when studying the banking sector as more than half of the assets in an average bank’s portfolio is constituted by loans.\textsuperscript{2} As Stiglitz and Weiss (1981) argued on their seminal paper on credit rationing, the optimal response of loans’ riskiness varies with the loan rate. More precisely, higher loan rates lead to higher risk taking from the part of the entrepreneurs.

This paper departs from the previous literature and analyses the effects of capital requirements on bank’s probability of failure taking into account the optimal response of entrepreneurs to different loan rates in a setup of moral hazard between the bank and en-

\textsuperscript{1}It must be noted that some studies, for example Koehn and Santomero (1980), in a mean-variance frontier setup, and Blum (1999), in a dynamic setup, have found ambiguous results on the effects of capital requirements on the probability of bank failure.

\textsuperscript{2}Source: Federal Deposit Insurance Comission.
entrepreneurs. The basic model analyzes the effects of capital requirements on the probability of bank failure in a setup of perfect competition among banks with imperfect correlation among loan defaults. In order to model loans’ default we implement the single risk factor model, which is the baseline model used in Basel II Capital Accord.\(^3\)

In our setup increasing capital requirements increases the fraction of defaulting loans that a bank can absorb without undergoing failure, *capital buffer effect*. But increasing capital requirements also increases the equilibrium loan rate charged to entrepreneurs as the cost of funding of the bank increases. By increasing the loan rate charged to entrepreneurs, banks have a higher probability of a given loan defaulting, as entrepreneurs choose to have riskier loans, which increases their probability of failure, *risk shifting effect*.\(^4\) But on the other hand when the loan rate increases revenues of a non defaulting loan increase, which increases the fraction of loans that can default before the bank undergoes failure, *margin effect*.

This model delivers no closed form solution concerning the overall effect of capital requirements on the probability of bank failure. Hence, we undergo a numerical analysis of the model in order to give intuitive results of the underlying forces present in our model. Generally, a U-shaped relationship between capital requirements and banks’ probability of failure is obtained. High and low levels of capital requirements result in high probability of bank failure and intermediate levels of capital requirements result in low probability of bank failure.

In order to analyze different competitive structures among the banking system we analyze the effects of capital requirements in a setup of a monopolistic bank. In this setup we show how, contrary to the basic setup, the equilibrium loan rate is not monotonically increasing in capital requirements. Banks internalize the effects of an increase in the loan rates in the risk of losing their investment when maximizing their profits. This results in non monotonic responses of the loan rate to an increase in the capital requirements. When the loan rate has large effects on the optimal choice of risk of entrepreneurs, high risk shifting, the bank

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\(^3\) This model of imperfect correlation among loan default has the advantage of allowing for different degrees of correlation with only one parameter.  
\(^4\) This effect was first explained by Stiglitz and Weiss (1981).
lowers the equilibrium loan rate charged to entrepreneurs. The bank internalizes that by decreasing the loan rate the revenues it obtains from a non defaulting entrepreneur decline, but this effect is dominated by the fact that the entrepreneurs choose safer strategies and default with lower probability. The opposite is true when the risk shifting effect is low. Once the optimal response of the loan rate is solved we show how in the case of a monopolistic bank there is no monotonic relationship between capital requirements and the probability of bank failure.

The first two setups deliver an equilibrium capital equal to the capital required by the regulator. We analyze a setup with a monopolistic infinitely lived bank and show how the equilibrium capital chosen by the bank can be higher than that imposed by the regulator. However, it is shown how the capital that the bank chooses is going to differ from the capital which minimizes the probability of bank failure. Hence, even in a dynamic setup with non-binding capital requirements, there is scope for capital regulation in order to minimize the probability of bank failure.

Although we use the single risk factor model as the underlying correlation structure among loan defaults, many models have studied the default structure of loans as different to the single risk factor model. Hence, we generalize the correlation structure among loan defaults and show how the qualitative results of the main section remain.

Finally we analyze the effects of credit cycles in the probability of banking failure and how capital requirements should react. It is shown how when the economy enters a high default cycle capital requirements should adapt. If capital requirements should increase or decrease when an economy enters a high default state, depends on the correlation among loan defaults. When loans have low correlation, capital requirements should decrease when the economy enters a high default state, being the opposite true when the correlation among loan defaults is high. Hence the qualitative response of capital requirements to different credit cycles depends on the correlation among loan defaults.

The rest of the paper is structured as follows: Section 2 presents the entrepreneur setup. In section 3 we analyze the Bertrand equilibrium to the problem. Section 4 studies the effects of capital requirements in the probability of failure the banking sector. Section 5 shows
numerical solutions for the basic setup. Section 6 studies the monopolistic equilibrium both in a static and dynamic setup. Section 7 generalizes the correlation structure. Section 8 studies the effects of credit cycles and finally section 9 concludes.

2 The Model

Consider an economy with three types of risk neutral agents: entrepreneurs, indexed by $i$, banks, indexed by $j$, and depositors.

The timing of the model is as follows: At date 0 banks raise their funding in order to grant loans to entrepreneurs and charge a (net) loan rate $r$. Once the loan is set entrepreneurs choose the risk of the project. At date 1 the realization of the project occurs and entrepreneurs either pay back the loan or default. Once banks receive all the payments from the entrepreneurs, they are able to pay back their depositors or fail.

2.1 Entrepreneurs

There is a continuum of penniless entrepreneurs characterized by a continuous distribution of reservation utilities with support $\mathbb{R}_+$. Let $G(u)$ denote the measure of entrepreneurs that have reservation utility less than or equal to $u$.

Each entrepreneur has the opportunity of undertaking a project which has the following stochastic return structure:

$$R(p_i) = \begin{cases} 
1 + \alpha(p_i), & \text{with probability } 1 - p_i \\
1 - \lambda, & \text{with probability } p_i
\end{cases}$$

Where the probability of default, $p_i \in [0, 1]$, is the unverifiable choice variable of the entrepreneurs. Parameter $\lambda \in [0, 1]$ defines the loss given default of the project and is independent of $p_i$. The net return of the project $\alpha(p_i)$ satisfies $\alpha'(p_i) > 0$ and $\alpha''(p_i) \leq 0$. Hence, riskier projects yield higher revenues in the case of no default. To guaranty an interior optimum we assume $\alpha(0) < \alpha'(0)$.

\footnote{This return structure mimicks that of Allen and Gale (2004).}
In order to determine when a project defaults we use the single risk factor model of Vasicek (2002), according to which the default of the project of entrepreneur $i$ is driven by the realization of project $i$’s latent variable $y_i$. Entrepreneurs $i$’s project fails when $y_i < 0$. where

$$y_i = -\Phi^{-1}(p_i) + \sqrt{\rho} \ z + \sqrt{1 - \rho} \ \varepsilon_i$$

The random variable $y_i$ is the sum of three terms: $-\Phi^{-1}(p_i)$ is a deterministic term that is decreasing in the probability of failure $p_i$ chosen by the entrepreneur, $z$ is a systematic risk factor that affects all projects in the same way, and $\varepsilon_i$ is an idiosyncratic risk factor that only affects the project of entrepreneur $i$. It is assumed that $z$ and $\varepsilon_i$ are standard normal random variables, independently distributed from each other as well as, in the case of $\varepsilon_i$, across projects. $\Phi(\cdot)$ denotes the cumulative density function of a standard normal random variable, and $\Phi^{-1}(\cdot)$ its inverse. Parameter $\rho \in [0, 1]$ determines the extent of correlation in project failures. Note that if $\rho = 0$ the systematic risk factor does not play any role and we have statistically independent failures. On the other hand if $\rho = 1$ the idiosyncratic risk factor does not play any role and we have perfectly correlated failures.\(^6\)

As entrepreneurs are penniless they need a unit loan from the bank in order to undergo their project. Banks charge a net interest rate, $r$, for the loan.\(^7\) Due to limited liability entrepreneurs only care about the net return of the project $\alpha(p_i) - r$ when the project does not default $1 - p_i$. Therefore, entrepreneur’s problem is to maximize the expected revenue of the project in the case of no default, $u(r)$.

$$u(r) = \max_{p_i} (1 - p_i)(\alpha(p_i) - r)$$

As entrepreneurs only differ in their reservation utility, the optimum choice of default, $p_i$, is the same for all entrepreneurs, from now on $p$. The optimal choice of default by

\(^6\)Note that $\sqrt{\rho} \ z + \sqrt{1 - \rho} \ \varepsilon_i \sim N(0, 1)$ implies

$$\Pr(y_i < 0) = \Pr[\sqrt{\rho} \ z + \sqrt{1 - \rho} \ \varepsilon_i < \Phi^{-1}(p_i)] = \Phi[\Phi^{-1}(p_i)] = p_i$$

This is required in order to have that the probability of default of a project is equal to the choice of default of entrepreneurs.

\(^7\)The pricing of the loan will be analyzed in section 3.
entrepreneurs is implicitly characterized by the following first order condition:

\[(1 - p)\alpha'(p) - (\alpha(p) - r) = 0.\]  

(1)

**Lemma 1** When the loan rate is increased entrepreneurs optimally decide to invest in riskier projects.

**Proof** Differentiating equation (1) and taking into account that \(\alpha'(p) > 0\) and \(\alpha''(p) \leq 0\) we obtain that the optimal expected default varies positively with the loan rate.

\[
p_r = \frac{dp}{dr} = -\frac{1}{-2\alpha'(p) + (1 - p)\alpha''(p)} > 0. \square
\]

When the loan rate is increased the revenues that entrepreneurs obtain in the case of survival diminish. Entrepreneurs react to an increase in the loan rate by choosing riskier strategies that, although make default more probable, increase the return of the project in case of survival. This will be noted as the risk-shifting effect.\(^8\)

Entrepreneur \(i\) asks for a loan to undertake the project only if his expected revenue of undertaking the project \(u(r)\) is higher than his reservation utility \(u_i\). As each entrepreneur needs for a unit loan, the aggregate demand for loans for a given loan rate \(L(r)\) is given by \(G(u(r))\). Using the envelope condition it is direct to show that \(L'(r) < 0.\(^9\) When the loan rate is increased the expected utility of the project is reduced and fewer entrepreneurs find it optimal to undergo the project.

Once the optimal decision of default probability for each entrepreneur has been obtained, we now derive the distribution of defaults in the economy. With a continuum of projects idiosyncratic risk is diversified away, so the aggregate failure rate \(x\) (the fraction of projects that default in the economy) is only a function of the realization of the systematic risk factor \(z\). By the law of large numbers the failure rate \(x\), conditional on the realization of the macroeconomic shock is equal to:

\[
\gamma(z) = \Pr \left[ -\Phi^{-1}(p) + \sqrt{\rho} z + \sqrt{1 - \rho} \varepsilon_i < 0 \mid z \right] = \Phi \left( \frac{\Phi^{-1}(p) - \sqrt{\rho} z}{\sqrt{1 - \rho}} \right)
\]

\(^8\)It is also direct to show how increasing the loan rate decreases the efficiency of the projects that the entrepreneurs undertake. By limited liability entrepreneurs choose to have riskier projects than the efficient ones. As the loan rate increases the risk increases and hence, the efficiency of the projects is reduced.

\(^9\)It is direct to show that the inverse demand for loans \(r(L)\) satisfies \(r'(L) < 0\).
Using the fact that $z \sim N(0,1)$, the unconditional cumulative density function of the aggregate failure rate can be expressed as

$$F(x) = \Pr \{ \gamma(z) \leq x \} = \Pr \{ z \leq \gamma^{-1}(x) \} = \Phi \left( \frac{\sqrt{1-\rho} \Phi^{-1}(x) - \Phi^{-1}(\rho)}{\sqrt{\rho}} \right)$$ (2)

For $\rho \in (0,1)$ the c.d.f. $F(x)$ is continuous and increasing, with $\lim_{x \to 0} F(x) = 0$ and $\lim_{x \to 1} F(x) = 1$. It can also be shown that $E(x) = \int_0^1 x \, dF(x) = p$. Note that $\partial F/\partial p < 0$, so changes in the probability of failure $p$ lead to a first-order stochastic dominance shift in the distribution of the failure rate $x$. When entrepreneurs choose to have riskier projects the distribution of defaults is shifted to the right. On the other hand we can see that $\partial F/\partial \rho \geq 0$ if and only if $x \leq \Phi(\sqrt{1-\rho} \Phi^{-1}(p))$, so changes in the correlation parameter $\rho$ lead to a mean-preserving spread in the distribution of the failure rate $x$.

When $\rho \to 0$, independent failures, the distribution of the failure rate approaches the limit $F(x) = 0$ for $x < p$ and $F(x) = 1$ for $x \geq p$. The single mass point at $x = p$ implies that a fraction $p$ of the projects fail with probability 1. Hence, with independent defaults as the idiosyncratic shock is diversified, the fraction of loans defaulting is deterministic and equal to $p$.

When $\rho \to 1$, perfectly correlated defaults, the distribution of the failure rate approaches the limit $F(x) = \Phi(-\Phi^{-1}(p)) = 1 - p$, for $0 < x < 1$. The mass point at $x = 0$ implies that with probability $1 - p$ no project fails, and the mass point at $x = 1$ implies that with probability $p$ all projects fail. Hence in this situation a portfolio of loans replicates exactly the same return structure as an individual loan. Note that when loans are perfectly correlated when one loan defaults all of them default and viceversa.

## 3 Bertrand competition among Banks

This section analyses the equilibrium loan rate and capital holdings in the economy assuming that banks compete à la Bertrand for loans. I focus on symmetric Nash equilibria.
3.1 Bank’s problem

At period 0 bank \( j \) chooses the loan rate, \( r_j \), and capital, \( k_j \), in order to maximize its expected profits, \( \pi(r_j, k_j) \).

Banks finance themselves from deposits and from own resources, from now on capital. We assume a perfectly competitive, infinite supply of deposits which are fully insured and normalize the deposit rate to 0. Banks’ capital is costly, being the cost of capital \((1+\delta)\), with \( \delta > 0 \). The assumption of costly capital is a classical assumption when taking into account the scarcity of bankers’ wealth or the existence of a premium for the agency problems faced by bankers.\(^{10}\) We assume the existence of capital regulation which establishes a minimum fraction of capital per unit of loans, \( \hat{k} \), that banks have to hold in order to grant loans. Hence, for every unit of loans a bank grants it will have \((1-k_j)\) amount of deposits and \( k_j \) amount of capital.

When deciding the loan rate they charge, banks take into account that their amount of loans, \( l_j \), varies with the loan rate they charge. Being

\[
 l_j(r_j, r_{-j}) = \begin{cases} 
 0 & \text{if } r_j > \min(r_{-j}) \\
 \frac{L(r_j)}{n} & \text{if } r_j = \min(r_{-j}) \\
 \frac{L(r_j)}{n} & \text{if } r_j < \min(r_{-j}) 
\end{cases}
\]

where \( r_{-j} \) stands for the loan rates the other banks set and \( n \) stands for all banks that set the minimum loan rate.

By limited liability, a bank, at period 1 obtains the revenues of their loan portfolio, to which they have to subtract the repayment to depositors, in case of no bank failure or, in case of failure, obtains no revenues. At period 0 banks undergo the cost of capital. Hence, bank’s problem can be written as:\(^{11}\)

\[
\max_{r_j, k_j} \pi(r_j, k_j, r_{-j}) \\
\text{s.t. } k_j \geq \hat{k}
\]

Where \( \pi(r_j, k_j, r_{-j}) = l_j E \{ \max[(1-x)(1+r_j) + x(1-\lambda) - (1-k_j), 0] - k_j(1+\delta) \} \) and \( x \) stands for the fraction of loans defaulting in bank’s \( j \) portfolio, which is a random variable

\(^{10}\)Holmström and Tirole (1997) and Diamond and Rajan (2000).
\(^{11}\)No participation constraint is needed as banks can always obtain 0 profits by setting an interest rate higher than its competitors.
following the cumulative distribution $F(x)$, which has been derived in the previous section. As I am focussing in symmetric Nash equilibrium I denote $k_j = k_{-j} = k$ and $r_j = r_{-j} = r$.

### 3.2 Equilibrium

In equilibrium capital requirements are binding. Capital is costly for the bank and has only the benefit of saving on depositors, which have been normalized to be costless.\(^{12}\) Hence, by choosing the lowest possible capital banks maximize their profits independent of the loan rate they charge.\(^{13}\)

The equilibrium deposit rate is equal to 0, which is the normalized rate at which depositors are indifferent between depositing in the bank or not. Note that as there is an infinite supply of deposits models a la Yanelle (1997) do not apply and banks do not find it profitable to pay more than the cost of deposit to depositors as they can not corner the deposit market.\(^{14}\).

The equilibrium loan rate satisfies that it is the minimum loan rate which allows banks to have non negative expected rents. If banks could charge a lower loan rate which allows them to have non negative expected returns they would lower the loan rate by a minimum amount and would achieve the whole demand of loans.

**Proposition 1** Equilibrium $k$ and $r$ are $k = \hat{k}$ and $r = \hat{r}$ such that $\exists r' < \hat{r} \mid \pi(r', \hat{k}) > 0$

**Proof** If $k > \hat{k}$ then $\pi(r, k) < \pi(r, \hat{k})$ for all $r$. So banks maximize their profits by choosing $k = \hat{k}$.

If there existed a loan rate $r' < \hat{r}$ such that $\pi(r', \hat{k}) \geq 0$, then the usual undercutting would occur and $\hat{r}$ would not be the equilibrium loan rate.

In this situation $\hat{k}$ is unique but there may be multiple equilibria in the loan rate. This multiple equilibria are characterized by $\hat{r} \subset R$ where $R = r \mid \exists r' < r \mid \pi(r', \hat{k}) > 0$.\(\blacksquare\)

\(^{12}\)This result holds in a setup of bertrand competition as long as the cost of deposits is lower than the cost of capital.

\(^{13}\)In Section 5 we analyze a dynamic monopolistic setup in which capital requirements may not be binding.

\(^{14}\)Please see the appendix for other possible equilibria. It is discussed how in this setup the equilibrium with deposit rates being equal to 0 is the only equilibrium with possitive lending strategies and probabilities of bank failure different from one. It is proved how the equilibrium we analyze in this main section is the unique Nash equilibrium when a non monetary cost of running the bank is assumed.
The level of capital requirements also determine the existence of the banking industry. It can be shown how there would be no banking industry if $\tilde{r} \mid \pi(\tilde{r}, \tilde{k}) \geq 0$ which is directly related to the level of capital requirements. When capital requirements are too high it might occur that the cost of raising the funds does not make it profitable for the banks to lend.

Under the differentiability conditions on the optimal choice of default of entrepreneurs, and given that the distribution function of the fraction of loan defaulting follows a normal distribution, the equilibrium expected profits of the bank would be equal to 0. This is direct to show as given the previous assumptions the expected profit function $\pi(r_j, k_j, r_{-j})$ is continuos and differentiable. As in any general perfect competition model, if expected profits where positive banks would undercut the loan rate until expected profits where equal to 0, hence in equilibrium $\pi(\tilde{r}, \tilde{k}) = 0$.

### 3.3 Increasing capital requirements

Once the equilibrium for a given capital requirement, $\tilde{k}$, has been derived, we analyze the effects of an increase in capital requirements. We assume that the regulator increases the capital requirements and sets a new capital requirement $\bar{k} > \tilde{k}$.

When capital requirements are increased the financing cost of the banks increase, recall that capital is more costly than deposits. As their financing cost has been increased, banks are forced to increase the loan rates they charge to entrepreneurs.

**Proposition 2** When capital requirements are increased, the equilibrium loan rate, if it exists, will be increased, $\bar{r} > \tilde{r}$.

**Proof.** We know that $\pi(r, \bar{k}) > \pi(r, \tilde{k})$ and $\pi(\tilde{r}, \bar{k}) < 0$. So $\tilde{r}$ can not be the equilibrium loan rate. Proposition 1 shows that $\exists r' \leq \tilde{r} \mid \pi(r', \bar{k}) \geq 0$ then $\exists r' < \tilde{r} \mid \pi(r', \bar{k}) \geq 0$. So $\exists r' \leq \tilde{r} \mid \pi(r', \bar{k}) \geq 0$. Then it must be $\bar{r} > \tilde{r}$. ■

Hence we can conclude that increasing capital requirements increases the equilibrium loan rate that banks charge. This increase in the loan rate is a cost for the entrepreneurs which will lead them to take higher inefficient risk, recall lemma 1. These higher inefficient defaults in the non financial sector, as well as the lower amount of projects that are undergone, is the
first effect that the regulator should take into account when establishing capital requirements. Increasing capital requirements may have a beneficial effect for the banking sector,\textsuperscript{15} but the regulator should not forget the negative effects it will have in the non-financial sector, in this case the entrepreneurs.

4 Banks’ probability of failure analysis

This section studies the effects that capital regulation has on banks’ probability of failure, \( q(r, k) \).

We define the probability of bank’s failure as the probability of a bank not having enough money to pay back its depositors at \( t = 1 \). This definition assumes that regulators have a flexible closing rule, i.e. that banks which have enough money to pay depositors but less money than capital requirements do not undergo failure.\textsuperscript{16} Hence, a bank undergoes failure when the fraction of loans defaulting in his portfolio, \( x \), is higher than a threshold, \( \hat{x} \):

\[
q(r, k) = \Pr((1 - x)(1 + r) + x(1 - \lambda) - (1 - k) < 0) = \Pr (x > \hat{x})
\]

where \( \hat{x} = \frac{r+k}{r+\lambda} \) is the threshold fraction of loans that can default in a bank’s portfolio in order for a bank not to fail.\textsuperscript{17}

Equation (4) shows how the probability of banks’ failure is determined by the distribution of loans’ default. Recall equation (2) which establishes the cumulative distribution of loans’ default rate as

\[
F(x) = \Phi \left( \frac{\sqrt{1 - \rho} \ \Phi^{-1}(x) - \Phi^{-1}(\rho)}{\sqrt{\rho}} \right)
\]

Using equation (2) and the symmetry of the standard normal distribution, banks probability of failure can be written as:

\[
q(r, k) = \Pr (x > \hat{x}) = \Phi \left( \frac{\Phi^{-1}(p) - \sqrt{1 - \rho} \ \Phi^{-1}(\hat{x})}{\sqrt{\rho}} \right)
\]

\textsuperscript{15}Which as I show in the following section is not always true.

\textsuperscript{16}For a discussion of different types of closing rules please see Elizalde and Repullo (2007).

\textsuperscript{17}It has to be assumed that \( \lambda > k \) in order to have risky banks. This implies that when all loans of a bank default the bank can not pay its depositors.
The evolution of banks’ probability of failure is determined by the derivative of the equation (5) with respect to capital. Three effects determine how banks’ probability of failure evolves with capital requirements: the capital buffer effect, the risk-shifting effect and the margin effect.

The first effect is the **capital buffer effect**. When projects’ failures are not perfectly correlated the higher the capital requirement the higher the fraction of defaulting loans a bank can absorb. This is because the higher the capital requirements the lower the relative amount of deposits a bank has to repay. Hence, the threshold number of defaulting loans a bank can absorb, \( \hat{x} \), increases with the capital requirements.

This effect is negative, in the sense that higher capital requirements, caeteris patribus, always lower the probability of bank failure. Partially deriving equation (5) with respect to capital and using the properties of the standard normal distribution function, it is shown that

\[
g_k(r, k) = -\frac{\sqrt{1-\rho}}{\sqrt{\rho}} \phi \left( \frac{\Phi^{-1}(\rho) - \sqrt{1-\rho} \Phi^{-1}(\hat{x})}{\sqrt{\rho}} \right) \frac{d\Phi^{-1}(\hat{x})}{d\hat{x}} \frac{r}{r + \lambda} < 0
\]

where \( \phi \) is the probability density function of a \( N(0, 1) \) variable.

Equation (6) highlights that higher correlations, caeteris patribus, leads to a smaller capital buffer. When projects are highly correlated, defaults came in very high proportion, hence the effectiveness of any buffer is reduced. In the case of perfect correlation, \( \rho = 1 \), no capital buffer exists, as when projects default they all default at once and bank has not enough money to repay depositors.\(^{18}\)

At this stage is useful to recall proposition 2 which claims that increasing the capital requirements increases the loan rate. This is important as the risk-shifting and the margin effect arise because of the change in the equilibrium loan rate when capital requirements are increased.

**The risk-shifting effect.** When loan rates are increased, entrepreneurs optimally choose to have riskier projects which increases the probability of one given loan defaulting. Hence,

\(^{18}\)Recall the implicit assumption \( \lambda > \hat{k} \).
when loan rates are increased $p$ is increased and this affects banks probability of failure positively (increasing banks probability of failure).

\[ q_p(r, k) = \frac{1}{\sqrt{\rho}} \phi \left( \frac{\Phi^{-1}(p) - \sqrt{1 - \rho} \Phi^{-1}(\hat{x})}{\sqrt{\rho}} \right) \frac{d\Phi^{-1}(p)}{dp} > 0 \]  

Equation (7)

The underlying intuition is that when entrepreneurs choose higher default probabilities a bank will be riskier as a higher fraction of its loans will default.

The **margin effect**. The intuition of this effect is that the higher the loan rate the higher the rents a bank earns from the non defaulting loans. This higher rents serve as a buffer to absorb defaults. Formally it can be seen how the threshold that determines banks’ probability of failure, $\hat{x}$, varies positively with the loan rate. Hence, due to this effect increasing the loan rates affects banks probability of failure negatively.

\[ q_x(r, k) = -\sqrt{1 - \rho} \phi \left( \frac{\Phi^{-1}(p) - \sqrt{1 - \rho} \Phi^{-1}(\hat{x})}{\sqrt{\rho}} \right) \frac{d\Phi^{-1}(\hat{x})}{d\hat{x}} \frac{\lambda - k}{(r + \lambda)^2} < 0 \]  

Equations (7) and (8) show how the sign of the margin effect is the opposite of that of the risk-shifting effect. What matters for the analysis is the overall effect that loan rates have on the probability of failure of a bank, which is the joint effect of the margin and risk shifting effect. However, the effect of loan rates in banks’ probability of failure has not a definite sign. Differentiating equation (5) with respect to loan rates and defining $\phi$ as the density function of a $N(0,1)$, it is obtained that

\[ q_r(r, k) = \frac{1}{\sqrt{\rho}} \phi \left( \frac{\Phi^{-1}(p) - \sqrt{1 - \rho} \Phi^{-1}(\hat{x})}{\sqrt{\rho}} \right) \left( \frac{d\Phi^{-1}(p)}{dp} \frac{dp}{dr} - \frac{d\Phi^{-1}(\hat{x})}{d\hat{x}} \sqrt{1 - \rho} \lambda \right) \leq 0 \]  

Equation (9)

This functional form has already been studied by Martinez-Miera and Repullo (2008). The main claim of their study is that for a wide set of parameters the relationship between loan rate (competition) and banks’ probability of failure is non monotonic.\(^{19}\)

Equation (9) highlights the important role that the correlation parameter has on the total effect of loan rates in the probability of banks’ failure. When $\rho \to 1$ it can be seen

\(^{19}\)For a proof of the existence of this type of non monotonic relationship please see MMR
that the margin effect disappears and the only effect that is maintained is the risk shifting effect. When loan defaults are highly correlated it is less probable that the revenues a bank gains from non defaulting loans may serve as a buffer as it is very probable that one default is followed by a large fraction of defaults. In such case the buffer banks obtain from non defaulting loans will not be sufficient in order not to undergo failure.

Hence, the higher the correlation the more probable that capital requirements lead to a riskier banking industry as the two beneficial effects that capital requirements have on banks’ failure, margin effect and capital effect, are reduced.

Taking into account the previous exposition we can derive the following propositions.

**Proposition 3** When loan defaults are perfectly correlated, \( \rho = 1 \), increasing the capital requirements increases the probability of bank failure

**Proof.** When \( \rho = 1 \) the margin effect and the capital buffer effect dissapear, hence, the only effect remaining is the risk shifting effect. In such case it is direct to show that

\[
\frac{dq(r, \hat{k})}{dk} = \frac{1}{\sqrt{\rho}} \phi \left( \frac{\Phi^{-1}(p) - \sqrt{1 - \rho} \Phi^{-1}(\hat{x})}{\sqrt{\rho}} \right) \frac{d\Phi^{-1}(p)}{dp} \frac{dp}{dk} > 0
\]

\[\blacksquare\]

**Proposition 4** With imperfect loan defaults, \( \rho \in (0,1) \), the effect of increasing capital requirements in the probability of banks’ failure is generally ambiguous.

**Proof.** The total effect of capital requirements on banks’ probability of failure can be compactly written as:

\[
\frac{dq(r, k, \hat{k})}{dk} = -\sqrt{1 - \rho} \phi \left( \frac{\Phi^{-1}(p) - \sqrt{1 - \rho} \Phi^{-1}(\hat{x})}{\sqrt{\rho}} \right) \frac{d\Phi^{-1}(\hat{x})}{d\hat{x}} \frac{r}{r + \lambda} + \]

\[\underbrace{\text{Capital buffer effect \text{ (<0)}}_{(10)} \right)

\[+ \frac{1}{\sqrt{\rho}} \phi \left( \frac{\Phi^{-1}(p) - \sqrt{1 - \rho} \Phi^{-1}(\hat{x})}{\sqrt{\rho}} \right) \left( \frac{d\Phi^{-1}(p)}{dp} \frac{dp}{dr} - \frac{d\Phi^{-1}(\hat{x})}{d\hat{x}} \frac{\sqrt{1 - \rho} \left( \lambda - \hat{k} \right)}{(r + \lambda)^2} \right) \frac{dr}{dk} \]

\[\underbrace{\text{Risk shifting + Margin effects \text{ (\leq0)}}}_{(10)} \right)

\[\blacksquare\]
in which all three forces are clearly identified. Equation (10) generally has no definite sign.

The importance of the capital buffer relative to the overall effect of loan rates in banks’ probability of default depends on the intensity of the shift that capital requirements make on the equilibrium loan rate, $\frac{dr}{dk}$, and on the intensity of the risk shifting effect. If this effects are negligible then, the overall effect will be negative as the capital buffer effect prevails. But if capital requirements have a substantial effect in the loan rate and the risk shifting effect dominates the margin effect, increasing capital requirements will increase banks’ probability of default. As no closed form solution can be obtained the next section undergoes a numerical analysis of the model.

5 Numerical analysis

This section analyzes the numerical solution of the problem. Some additional assumptions concerning parameters and functional forms have been made. The main purpose of this section is to clarify the basic results of the theoretical model without any intention of calibrating the inputs or outputs of the model.

5.1 Parameterization

Due to the differentiability conditions imposed on $\alpha(p)$, in equilibrium banks benefits are equal to zero. As $\pi(r_j, k_j) = l_jE[\max[(1 - x)(1 + r) + x(1 - \lambda) - (1 - \lambda_k), 0] - k_j(1 + \delta)] = 0$, and we analyze the effect of capital requirements in banks that have a share of the market $l_j > 0$, then it must be satisfied that the expected benefit of any loan is equal to cero,

$$E[\max[(1 - x)(1 + r) + x(1 - \lambda) - (1 - k_j), 0] - k_j(1 + \delta)] = 0$$

We assume that entrepreneurs optimal response function takes the linear form $p = a + br$. Parameter $a$ can be understood as the probability of default that a given entrepreneur

---

20 A matlab code that resembles the economy has been programmed. The code may be delivered upon request.
21 This solution comes from assuming $\alpha(p) = \frac{1-2a+p}{2b}$
Figure 1 shows the relationship between capital requirements and the probability of bank failure for different values of the correlation parameter. Has independently of the interest rate, and \( b \) is the risk-shifting parameter. In our numerical exercise, we assume that the exogenous probability of default, \( a \), is equal to 1\% and the risk-shifting parameter, if nothing is explicitly mentioned, is equal to 1.5.

Following Basel II specifications, we assume that project’s loss given default, \( \lambda \), is equal to 45\%.

When nothing is explicitly noted, the cost of capital has been set to be equal to 10\%. As in our theoretical setup, the deposit rate is normalized to 0.

### 5.2 Numerical results

This subsection provides numerical results for our model. Figure 1 highlights the importance of correlation in the overall effect of capital requirements on bank’s failure probability. When perfect correlation is assumed, \( \rho = 1 \), the relationship between capital requirements and probability of bank’s failure is monotonically increasing, as it had been shown in the theoretical setup. In such case the positive effect of capital requirements, capital buffer and margin effect disappear and the only effect that exists is the negative risk shifting effect.

When low correlation is assumed, \( \rho = 0.1 \), the relationship is mainly decreasing with
the level of capital requirements that minimize the probability of bank failure being 10%. As the theoretical setup points out when correlation is low the capital buffer and margin effect become more relevant. For intermediate level of correlation, $\rho = 0.5$, a U-shaped relationship between capital requirements and probability of bank failure is clearly obtained. In this latter situation banks probability of failure is minimized for a capital requirement of 12%.

Figure 1 also shows how the higher the correlation the higher the capital requirements banks can absorb. This is due to the fact that banks’ expected profits for a given loan rate are higher the higher the correlation. Higher correlation makes the existence of limited liability more valuable for the bank. Recall from equation (3) that banks’ profits can be seen as a call function and hence, banks value increases with the volatility of their returns.\footnote{Recall how increasing the correlation has a SOSD effect on the distribution of banks’ returns. Hence banks would always choose to invest in highly correlated markets.}

Another important issue concerning Figure 1 is the consequences of the diversification. It can be seen that investing in industries with less correlated outcomes is not always better for banks’ probability of failure. How the level of correlation affects the probability of banking soundness is highly related to the level of capital requirements imposed by the
regulator. Investing in industries with low correlation is safer when the capital requirement is high enough. On the other hand in cases with low capital requirements banks have lower probability of failure when the industry they invest in is highly correlated.\footnote{For empirical evidence supporting this claim please see Acharya et al. (2006).}

In the theoretical setup we have analyzed the importance that the equilibrium loan rate has on determining the final effect of capital requirements in the probability of bank failure. When changes in capital requirements have little effect on the equilibrium loan rate that bank set it is more probable that the capital buffer effect prevails and hence increasing capital requirements in the end reduce the probability of bank failure. In our model the cost of capital is a good parameter to focus on this effect. Figure 2 shows how the larger the cost of funding the bigger the changes in the equilibrium loan rate that higher capital requirements have.

Figure 3 highlights the importance of the cost of capital in the determining the effect of capital requirements on the probability of bank failure. Figure 3 shows how, as predicted in the theoretical setup, the higher the cost of capital the more probable that increasing the capital requirements increases the probability of banks’ failure. As previously explained this
is because the lower the cost of capital the lower the change in the loan rate and hence the higher the relative importance of the capital buffer effect which decreases the probability of bank failure.

6 Model with Monopolistic bank

In the previous section we have assumed perfect competition among banks, Bertrand model, and showed how capital requirements unambiguously increase the equilibrium loan rate. In contrast this section analyzes a model with a monopolistic bank which faces a continuum of entrepreneurs resembling those in section 2. We show how when the nature of competition is changed, under certain circumstances, higher capital requirements reduce the equilibrium loan rate. We also show how the relationship between capital requirements and bank failure is non monotonic. After results for a static monopolistic setup are derived we extend the model to a dynamic monopolistic setup in which we show how capital requirements may not be binding.

6.1 Bank’s problem

We now analyze a setup in which a monopolistic bank decides the loan rate it sets, \( r \), and also decides the fraction of capital, \( k \), it holds in order to maximize its expected profits, \( \pi(r, k) \).

When deciding the loan rate it sets, a bank takes into account that increasing the loan rate is going to decrease the amount of loans it can grant, recall that \( L'(r) < 0 \). As explained in section 2, reducing the loan rate also decreases the revenues that the bank obtains from a non-defaulting entrepreneur, but increases the probability of a given entrepreneur repaying.

As in the previous setup, the bank obtains funding from a infinitely elastic supply of depositors, whose deposit rate is normalized to 0, and from capital, which is costly.\(^{24} \) For every unit of loans a bank grants it will have \( (1 - k) \) amount of deposits and \( k \) amount of

\(^{24}\)In this section it is not crucial to assume that there is an infinite supply of deposits. As the bank is a monopolistic models like Yanelle (97) do not apply. However for simplicity we mantain that assumption.
capital. Recall the existence of capital regulation which establishes that bank capital can not be lower than a threshold, \( \hat{k} \).

By limited liability the bank at period 1 can obtain the gross return of its portfolio of loans, to which it has to subtract the repayment to depositors, in case of no failure, or, in case of failure, obtain 0 revenues. In both cases the bank has to undergo the cost of capital at period 0.

The structure of the problem allows bank’s objective function to be written as:

\[
\max_{r, k} \pi(r, k) \\
\text{s.t } k \geq \hat{k}
\]

Being \( \pi(r, k) = l(r)E \{ \max [(1 - x)(1 + r) + x(1 - \lambda) - (1 - k), 0] - k(1 + \delta) \} \). In this case \( l(r) = L(r) \) as the monopolistic bank is the only bank granting loans.

As in the Bertrand setup bank’s capital is binding, \( k = \hat{k} \). The intuition behind binding capital requirements in this setup is that banks recuperate \( k \) only when the bank does not fail, \( 1 - q(r, k) \leq 1 \), and have to pay for capital a cost that exceeds 1. Formally it can be derived how the first order condition for optimal capital is always negative.

When choosing the loan rate, a bank takes into account not only the amount of loans it grants \( l(r) \), but also the effect that the loan rate has on the probability of default of loans. \( p(r(l)) \) and hence on the distribution of the failure rate \( F(x) \).

Note that for banks problem to have an interior maximum it must be satisfied that the second order condition with respect to the loan rate is negative:

\[
\frac{\partial^2 \pi(r, k)}{\partial r^2} < 0.
\]

6.1.1 Bank’s response to an increase in capital requirements

In the Bertrand setup it has been shown how an increase in capital requirements unambiguously increased the loan rate charged to entrepreneurs. This subsection shows this result is not robust to the introduction of a monopolistic bank.

Using the implicit function theorem and the first order condition with respect to loans, we can obtain how the optimal loan rate, \( r \), that a bank sets varies with capital requirements.
Equation (14) establishes that banks’ optimal response to an increase in capital requirements depends on the sign of the cross derivative of the objective function.

\[
\frac{dr}{dk} = -\frac{\partial^2 \pi(r,k)}{\partial r \partial k} \tag{14}
\]

Hence in the setup of a monopolistic bank, an increase in capital requirements does not necessarily lead to an increase in the loan rate. The reason being that in this setup banks take into account that increasing or decreasing the loan rate has an effect on the exposure to risk they face and, hence, to losing their investment in capital when they maximize their profits.

Given the notation complexity of the cross derivative of the objective function for the general model, we focus on the case of perfectly correlated loans’ default in order to obtain the main intuition of why results in the case of the monopolistic bank can be reversed. Recall that in that case the Bertrand equilibrium delivered that higher capital requirements unambiguously result in higher loan rates and higher probability of bank failure.

6.1.2 Perfect correlation setup

In order to have intuitive and tractable analytic results we assume that banks face perfectly correlated loans, \( \rho = 1 \). This subsection shows how in the monopolistic setup even in the case of perfectly correlated loan defaults higher capital requirements do not always lead to higher loan rates and higher probability of loan failure.

In the setup of perfect correlation among loan defaults, the cross derivative of loans and capital, is equal to \( l'(r) [1 - p(r) - (1 + \delta)] - l p'(r) \). This gives a clear insight of the forces that determine how the loan rate, and hence the probability of bank failure, respond to an increase in bank’s capital requirement.
Proposition 5 In a monopolistic setup, the equilibrium loan rate is not monotonically increasing in capital requirements. Hence, even when $\rho = 1$, the probability of bank’s failure is not monotonic in capital requirements. It depends in the sign of the cross derivative of the objective function $\frac{\partial^2\pi(r,k)}{\partial r \partial k}$.

Proof. Noting $q$ as banks probability of default, from the assumption that entrepreneurs’ default are perfectly correlated, it can be obtained that $q = p$. So the response of banks’ probability of default to an increase on the capital requirements depends on the sign of $\frac{\partial^2\pi(r,k)}{\partial r \partial k} < 0$. It can be obtained that

$$\begin{align*}
I f \frac{\partial^2\pi(r,k)}{\partial r \partial k} < 0 \rightarrow \frac{dq}{dk} < 0 \\
I f \frac{\partial^2\pi(r,k)}{\partial r \partial k} > 0 \rightarrow \frac{dq}{dk} > 0
\end{align*}$$

Proposition 6 Increasing banks’ capital requirements makes a bank riskier if the risk-shifting effect is low enough.

Proof. In order for capital requirements to increase the probability of bank failure, it is needed that $\frac{\partial^2\pi(r,k)}{\partial r \partial k} < 0$ which can be written in terms of the risk shifting effect as $p'(r) < \frac{-r(r)(p(r)+\delta)}{l(r)}$. Hence, a low risk-shifting effect in loans means that banks will pass the cost and so increasing capital requirements is not a good idea in terms of the safeness of the banking industry.

It can be seen how the risk shifting effect plays a crucial role in determining the outcome of the model. With high risk shifting banks respond to an increase in their capital requirement reducing the loan rate which in the setup of perfect correlation unambiguously reduces the probability of bank failure. When the risk-shifting effect is sufficiently low banks will pass the cost and higher the loan rate as this has little effect on the default rate of the entrepreneurs, but when the risk-shifting is sufficiently high the traditional hypothesis by which increasing capital requirements makes a bank behave more prudently holds. In this model the way a bank behaves more prudently is charging lower loan rates as by doing so the default rate of entrepreneurs diminish..
6.2 Dynamic model

This subsection analyzes a discrete time, infinite horizon model in which at the end of each period if the bank does not fail it lends to a new set of entrepreneurs resembling those in section 2.

At each period banks grant loans to entrepreneurs, finance themselves from capital, which are costly, and insured deposits, which have been normalized to be costless. At the beginning of each period a bank has to decide the fraction of capital $k$ and the loan rate $r$ it offers. The bellman equation for this model can be written as

$$V(r, k) = \max_{r,k} \pi(r, k) + \beta(1 - q(r, k))V$$

where $\pi(r, k)$ are one period's expected profits, $q(L, k)$ is the probability of default of the bank, $\beta$ is banks discount factor and $V$ is the value of the bank. Being

$$V(r, k) = \frac{\pi(r^*, k^*)}{1 - \beta(1 - q(r^*, k^*))},$$

where $r^*$ and $k^*$ are the optimal choice of loan rate and capital by the bank.

As previously explained increasing the capital increases the cost of funds of the bank and hence, reduces the one period expected profits of a bank $\frac{d\pi(r, k)}{dk} < 0$. Although this effect is still present in the dynamic framework, capital has a positive effect in this dynamic framework. For a given loan rate, increasing capital makes the probability of bank failure decrease as the buffer of the bank increases. The banker in the dynamic setup acknowledges that by having capital it decreases the probability of bank failure and increases the probability of obtaining the charter value of the bank.

6.2.1 Non binding capital requirements

This setup can show optimal non binding capital requirements. Capital requirements are going to be binding whenever the banks’ optimal holding of capital is smaller than capital requirements. Optimal holding of capital by banks is given by the following expression

\[ V(r, k) = \frac{\pi(r^*, k^*)}{1 - \beta(1 - q(r^*, k^*))}, \]
Expression (16) shows that in this context capital requirements may not be binding. Whenever the previous expression evaluated at the capital requirements and the optimal loan rate is positive capital is not going to be binding. For a given loan rate, increasing the capital holding has a cost of decreasing the one period profits, but increasing capital makes more probable obtaining future rents. Recall that if the loan rate is constant the only effect of having capital is the capital buffer effect. One interesting feature is that the optimal capital is (generally) not going to be equal to the one that minimizes the probability of bank failure. It can be seen that in general the bank would choose a capital holding different to the one that minimizes the probability of bank failure. Hence, even in the context of this dynamic model there is scope for capital regulation.

We can also observe how the importance of the continuation value in determining the amount of capital that a bank holds. Although we have not derived a complete model of imperfect competition we can argue that high competitive markets have low future rents, hence, banks prefer to increase current profits at the expense of reducing the probability of obtaining the low future rents. This claim is supported by section 3 where capital requirements where always binding in the extreme case of perfect competition. Note that in a perfect competition model such as the Bertrand model presented in section 3 Banks have 0 expected profits. Hence their future value is zero and they lose no rents if they default.

7 General correlation function

Although the previous analysis has been done for a specification of the probability of loan’s default resembling that of Basel II, it is probable that the probability of loan’s default is determined by another functional form different than that of the Vasiceck model. Numerous studies concerning loan’s default have modeled this probability as a combined result of:
various aggregate factors,\textsuperscript{25} different density distributions,\textsuperscript{26} etc. In this section I show how the results from the previous sections hold for a general specification of loan’s default with binding capital requirements.

Generally entrepreneurs default can be specified as \( \Pr(y_i < 0) \), which means that entrepreneurs default when the project’s latent variable is lower than a threshold. In a general case the latent variable of the project can be defined as \( y_i = \mu + g(\eta, \theta) \).\textsuperscript{27} Where \( \mu \) is a monotonic transformation of the choice variable from the entrepreneur, \( \eta \) are the random variables that determine the stochastic structure of \( y_i \), typically some of them will be idiosyncratic and some of them aggregate factors, and \( \theta \) are exogenous parameters that determine the relationship between \( \eta \). The function \( g(\eta, \theta) \) can be understood as a random variable \( \beta \) following a cumulative distribution function \( F_{\eta, \theta}(\cdot) \).

**Assumption** Entrepreneur’s choices do not affect the unconditional distribution function of \( \beta \).

This assumption establishes that a given entrepreneur can not affect the distribution of the shocks in the economy. Increasing or decreasing the probability of loans failure does not affect the correlation structure among loans. More specifically \( F_{z, \theta}(\cdot | \mu) = F_{z, \theta}(\cdot) \gamma \mu \).

Under this assumption entrepreneurs probability of default \( p = \Pr(y_i < 0) = F_{z, \theta}(-\mu) \). Hence, it can be seen how \( \mu \) implicitly determines the choice of risk by entrepreneurs \( p \), without any secondary effects on the distribution function \( F_{z, \theta}(\cdot) \). More specifically \( \mu = F_{z, \theta}^{-1}(p) \), being \( F_{z, \theta}^{-1}(p) \) the inverse of the cumulative density function.

Following the same analysis as in section 2 higher loan rates would lead to higher probabilities of default from entrepreneurs. The intuition remains unchanged, when entrepreneurs face higher loan rates they need to increase the risk of their project in order to obtain higher rents in the case of non default.

Proposition 2 which established that with perfect competition higher capital requirement

\textsuperscript{25}For example multivariate risk factor models have been used previously in the literature. This models assume that there is more than one source of aggregate risk.

\textsuperscript{26}Copulas, gamma distributions, etc.

\textsuperscript{27}\( y_i \) can also be modeled as \( h(\mu, g(z, \theta)) \) with the assumption that \( F_{z, \theta}(\cdot | \mu) = F_{z, \theta}(\cdot) \gamma \mu \). In such case all the analysis holds but notation gets difficult to follow.
lead to higher loan rates is obtained without any functional form for loans correlation so no additional proof is needed.

The fraction of loans defaulting in a given bank portfolio, \( x \), will now be driven by a general form of correlation, \( F(x | \mu(r), \theta) \). Note that \( F(x | \mu(r), \theta) \neq F_z(\cdot) \) as \( F(x | \mu, \theta) \) takes into account both the optimal choice of risk of entrepreneurs and also diversification of idiosyncratic risks.\(^{28}\)

As in the basic framework we define the probability of bank failure as the probability of a bank not having enough money to pay their depositors.

\[
q(r, k, \theta) = \Pr((1 - x)(1 + r) + x(1 - \lambda) - (1 - k) < 0) = \Pr(x > \hat{x}) = 1 - F(\hat{x} | \mu(r), \theta) \tag{17}
\]

Where \( \hat{x} = \frac{r + k}{r + \lambda} \).

This generalized setup also exhibits the capital buffer effect, the risk-shifting effect and the margin effect.

The first effect is the **capital buffer effect.** When projects’ failures are not perfectly correlated the higher the capital requirement the higher the fraction of defaulting loans a bank can absorb, as it has to repay less money to its depositors. This effect is always negative, higher capital requirements, \( \text{caeteris paribus} \), will always make a bank safer. This effect can be formally stated taking into account how the threshold fraction of defaulting loans varies with capital requirements.

\[
\frac{d\hat{x}}{dk} = \frac{r}{(r + \lambda)} > 0 \rightarrow
\]

\[
\frac{\partial q(r, k, \theta)}{\partial k} = q_k(r, k, \theta) = -f(\hat{x} | \mu(r), \theta) \frac{r}{(r + \lambda)} < 0. \tag{19}
\]

where \( f(\hat{x} | \mu(r), \theta) \) is the density function

**The risk-shifting effect.** When loan rates are increased, entrepreneurs will optimally choose to have riskier projects. The fraction of loans defaulting in any given economy satisfies that, independently of the exact functional forms assumed, the expected fraction of entrepreneurs defaults is equal to the optimal choice of risk of the entrepreneurs, \( E(x) = p \). When

\(^{28}\)In the previous setup \( F_z(\cdot) = N(0,1) \) and \( F(x | \mu, \theta) = \Phi \left( \frac{\sqrt{1-p} \cdot \Phi^{-1}(x) - \Phi^{-1}(\mu)}{\sqrt{\sigma^2}} \right) \)
loan rates are increased \( p \) increases and so it must be satisfied that \( E(x) \) is increased. In other words increasing the loan rate has a first order statistical dominance effect on the distribution function of entrepreneurs defaults. This means that \( F(x|\mu(r'), \theta) > F(x|\mu(r), \theta) \). where \( r > r' \). Relating this to the probability of bank failure it is direct to show that

\[
q(r, k, \theta) = 1 - F(\hat{x}|\mu(r), \theta) > 1 - F(\hat{x}|\mu(r), \theta) = q(r', k, \theta)
\]

**The margin effect.** The basic intuition of this effect is that, caeteris patribus, the higher the loan rate the more a bank earns from the non defaulting firms, serving these higher revenues as a buffer to absorb possible defaults. Mathematically it can be shown how the threshold of defaulting loans, \( \hat{x} \), varies positively with the loan rate.\(^{29}\)

\[
\frac{d\hat{x}}{dr} = \frac{\lambda - \hat{k}}{(r + \lambda)^2} > 0
\]

Using equation (17) it is direct to see that \( q(r, k, \theta) \) decreases when \( \hat{x} \) is increased. Hence, due to the margin effect higher loan rates make bank more secure.

Not surprisingly the risk shifting and margin effects continue to have different signs, so no global sign for the effect of loan rates on banks probability of failure can be achieved.

\[
\frac{dq(r, k, \theta)}{dr} = q_r(r, k, \theta) \geq 0
\]

For a general functional form, the overall effect of capital requirements on bank’s probability of failure is stated by the following equation.

\[
\frac{dq}{dk} = q_k(r, k, \theta) + q_r(r, k, \theta) \frac{dr}{dk} \geq 0
\]

Equation (22) determines how the importance of the capital buffer, \( q_k(r, k, \theta) \), relative to the overall effect of loan rates in banks’ probability of default, \( q_r(r, k, \theta) \), depends on the intensity of the shift that capital requirements make on the equilibrium loan rate, \( \frac{dr}{dk} \).

As explained in the basic model, when this effect is negligible then, the overall effect of capital requirements on the the probability of bank failure will be negative. But if \( \frac{dr}{dk} \) is

\(^{29}\)Recall that in order to have risky banks \( \lambda > \hat{k} \).
strongly positive and the risk shifting effect dominates the margin effect then the overall effect of increasing capital requirements will be an increase in banks’ probability of default.\textsuperscript{30} Assuming perfect correlation has important consequences in cancelling out the capital buffer effect and the margin effect, hence as the possible positive effects of bank capital are reduced, it is more probable to a claim that capital requirements are prejudicial for banks’ probability of default.

By the previous arguments equation (22) has no definite sign. Hence, until the literature converges to some correlation function that determines loan defaults and the relevant range of parameters that drive this function, the discussion of capital requirements and bank safeness is a challenging issue.

8 Capital requirements and the credit cycle

This section analyzes how the capital requirements that minimize the probability of bank failure change when the economic conditions change. More specifically we analyze how changes in the exogenous probability of default of entrepreneurs affect the capital requirements that minimize the probability of failure of a bank. We show how increasing or decreasing the capital requirements in order to minimize the probability of bank’s failure, depends on the level of correlation we assume. Low levels of correlation lead to a decrease in capital requirements when the exogenous default rate of entrepreneurs increase. On the other hand High levels of correlation lead to an increase in the capital requirements when the exogenous probability of default of the loans increase.

Therefore, it can be concluded that the level of correlation in bank loans’ portfolio is important not only to determine the level of capital requirements that minimize the probability of bank failure but also to implement policies which adapt to different credit cycles.

\textsuperscript{30}Recall that \( \frac{dc}{dk} > 0 \) in the Bertrand model but \( \frac{dc}{dk} \) has not a definite sign in the Cournot setup.
8.1 The model

We analyze how the capital requirements that minimize the probability of bank failure evolve with the credit cycle. We follow the bertrand competition setup and assume additionally that the economy has two states: the high default state and the low default state. In high default states the probability of default of an entrepreneur, conditional on the loan rate charged, is higher than in low default states. This can be modelled by increasing the exogenous probability of default of an entrepreneur, \( a \).

Due to the resemblance of the theoretical setup with the model explained in the main section of the paper, and due to the inexistence of closed form solutions for this problem, we resort to the numerical solution of the model.

First the model is solved assuming low correlation among loan defaults, \( \rho = 0.2 \), and afterwards the model is solved for high correlation among loan defaults, \( \rho = 0.8 \). We assume that the unconditional default probability, \( a \), in low default states is equal to 0.5% and in high default states is equal to 1.5%, keeping all of the other parameters in the values of the benchmark case.

Figure 4 shows how in high default states the probability of bank failure increases, independently of the capital requirements and the correlation among loan defaults. This is because high default states, caeteris patribus, increase the probability of one loan defaulting which makes the bank unambiguously riskier.

Moreover Figure 4 also shows how capital requirements should adapt to the credit cycle. When the credit cycle shifts from a low to a high default state and the correlation among loan defaults is low, capital requirements should be lowered in order to minimize the probability of bank failure. In high default states the effective cost of capital is higher, as with higher probability the bank will default and not recover its capital. Hence, the bank increases the capital requirements.

\[ 31 \text{In the setup of this model we can not model the business cycle as different expected realization of the systemic factor as the entrepreneur will undo the changes of this different spectations by changing his choice of project and hence would leave the probability of default unchanged.} \]

\[ \text{In order to introduce different expectations of the aggregat shock we have to introduce a choice variable in the determination of } y, \text{ being that choice variable the one that affects alpha, not the probability of default (for example effort). In such case we could introduce the credit cycle as different expectations in the aggregate shock.} \]
loan rate it charges to entrepreneurs, which increases the default rate of loans due to the risk shifting effect. By lowering the capital requirements the risk shifting effect is lowered making loans safer. On the other hand by lowering the capital requirements the bank looses some of its capital buffer which makes the bank riskier. Figure 4 shows how when low correlation is assumed capital requirements should decrease when the economy enters a high default state. Hence, capital requirements should be lowered in order to lower the equilibrium loan rate, which in turn makes loans safer and in equilibrium reduces the banks’ probability of failure.

However, the result concerning the evolution of capital requirements with the credit cycle is reversed when the correlation among loan defaults is high. The reason for this different result is that the equilibrium loan rate charged to entrepreneurs depends on the correlation among loan defaults. When correlation is high, increasing the capital requirements has a lower effect in the equilibrium loan rate. Hence in order to ameliorate the higher default rate of entrepreneurs capital requirements should increase when the cycle enters a high default state.

The reason for the different effect of capital requirements in the equilibrium loan rate derives from banks’ limited liability. With limited liability the profit function is a convex payoff function, and, as explained in the main section, correlation has a mean preserving effect on the distribution of loan loses. Hence, a bank with higher correlation among loans
default, has higher expected profits, which in the case of Bertrand competition makes the equilibrium loan rate be lower.

As Figure 5 shows when the correlation in banks’ loan portfolio is high the change on the equilibrium loan rate is smaller than when the correlation is low. Hence, when the correlation is high an increase in capital requirements has little effect in the loan rate, which in turn makes the capital buffer effect prevail over the negative risk shifting effect. In the case of low correlation when the capital requirements is increased the loan rate is increased more than in the high correlation setup. Hence in low correlation setup the risk shifting effect prevails over the capital buffer effect and it is better to decrease the capital requirements in order to minimize the probability of bank failure when the cycle enters a high default state.

As we have just exposed it is crucial for the policy maker to take into account the level of correlation in bank’s loan portfolio in order to increase or decrease the capital requirement when the economy enters a high default state.

9 Conclusions

This paper undergoes a theoretical analysis of the relationship between capital requirements and the probability of bank failure. We study the effects of capital requirements on the probability of bank failure in a setup in which banks invest in loans and face a moral hazard
problem from the part of the loan takers, and in which loan defaults are imperfectly correlated

We show how, on the one hand, higher capital requirements make banks safer as the fraction of loans that a bank can absorb without undergoing failure increases, capital buffer effect. On the other hand, in a setup of perfect competition among banks, increasing capital requirements increases the funding cost of the bank and banks have to increase the loan rates they charge to entrepreneurs. The increase in the loan rate that follows an increase in capital requirements has two effects: (i) it makes entrepreneurs choose riskier projects which in turn result in a higher fraction of loans defaulting in the bank’s portfolio, risk shifting effect and (ii) it increases the revenues that banks obtain from those loans that do not default, margin effect. Overall the increase in loan defaults that banks experience when capital requirements are increased may prevail over the beneficial effects of capital requirements.

As no closed solution for the problem may be obtain we undergo a numerical analysis of our model. We generally obtain a U-shaped relationship between capital requirements and the probability of bank failure. Hence, we can conclude that in our general setup increasing capital requirements over a threshold increases bank’s probability of failure.

The non monotonic relationship between capital requirements and bank failure is maintained in a setup of a monopolistic bank. In this situation capital requirements does not necessarily increase the loan rate the bank charges. It must be noted that contrary to the common view higher capital requirements might lead to higher loan rates from the part of a monopolistic bank, increasing the risk of their portfolio. This occurs when the moral hazard problem between the bank and the entrepreneur is low.

Finally the paper analyzes how capital requirements should adapt to the different economic conditions. It is shown how depending on the correlation among loan defaults, capital requirements should be increased or decreased when the economy enters a high default state, in order to minimize the probability of bank failure.

In conclusion this paper shows how the effect of capital requirements on the probability of banking failure is not a clear-cut issue. The existence of different levels of correlation among loan defaults determines the optimal level of capital requirements as well as the qualitative changes in such level when the economy enters a state of high default among entrepreneurs.
References


Appendix

Appendix A - On the existence of multiple equilibria

This appendix analyses the existence of multiple equilibria which differ from the equilibrium analyzed in section 3 of the model. These equilibria, all in the Bertrand setup, are characterized by deposit rates being higher than the normalized cost of deposit, 0 and with probability of bank failure equal to 1. Nevertheless, as no profitable deviation from the equilibrium characterized in the main section exists, the equilibrium characterized in the main section of the paper is indeed an equilibrium. Importantly the equilibrium analyzed in the main section is the only equilibrium which does not rely on the existence of limited liability of banks and provides probabilities of bank failure different to 1.

The main difference between the main setup of this paper and a setup with simultaneous price competition in the loan and deposit market, as the one analyzed in Yanelle (1997), is the existence of infinite (sufficiently large) inelastic supply of deposits. This assumption makes it impossible for a bank to corner the deposit market. To effectively corner the deposit market a bank has to absorb an infinite supply of deposits. As the demand for loans is bounded then the revenues at period 1 are always negative as banks have to repay $\infty(1 + r_d)$ and the maximum income they obtain are bounded by the monopolistic income. Hence, the overall net revenues of a bank which effectively corners the deposit markets are going to be negative. Due to limited liability the profits of a bank with negative revenues are going to be equal to 0. The fact that limited liability limits banks losses to 0 gives rise to the following equilibria.

No capital equilibrium.

When no capital requirements are established there are infinite equilibria with $r_d > 0$ and $r_l \in [-\infty, \infty]$.

In such situation a bank obtains cero profits and have a probability of failure equal to one. These equilibria exist because of the role of limited liability in my model as well as the deposit insurance, which allows depositors to be indifferent in the case of bank failure. When there are no capital requirements revenues are negative but due to limited liability and no capital cost profits are equal to 0.
No lending equilibrium

When capital requirements are positive $k > 0$, there are infinite equilibria with $r_d > 0$ and $l = 0$. As capital requirements are positive, and the revenues in period 1 are negative, banks have always negative profits when they undergo lending, because of the cost of raising capital. Hence equilibria only exist with no lending, which result in negative net revenues in period 1 and probability of failure equal to 1.

Uniqueness of equilibrium

With a positive and negligible cost of running the bank, $c > 0$, the unique equilibrium is the one characterized in the main section of the model. An important issue is the assumption that the cost of running the bank is non monetary.\(^{32}\) The cost $c$ can be seen as the effort of finding the depositors or the opportunity cost for the banker Assuming an arbitrarily small cost allows for the previous degenerated equilibria explained in the appendix not to exist as limited liability allows for 0 revenues but it does not repay $c$ so profits are negative.

We can conclude that equilibria based on limited liability and probability of failure equal to 1 do not exist with the assumption of positive capital requirements and an arbitrarily small non monetary cost of running the bank (only undergone if the bank has positive loans or deposits). Readers searching for a unique equilibrium should understand the model in the main section as having the cost of running the bank $c$ tending to 0. This assumption is not included in the main section in order to lighten the exposition of the main effects of capital in the probability of bank failure. All the qualitative results remain unchanged when an arbitrary small non monetary cost is assumed.

\(^{32}\)If it was pecunary the banker would be able to raise deposits in order to pay $c$ and by limited liability obtain 0 profits.