Robust Consumption and Portfolio Policies
When Asset Prices Can Jump*

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Abstract

We study the consumption-portfolio allocation problem in continuous time when asset prices follow Lévy processes and the investor is concerned about potential model misspecification. We derive in closed form optimal consumption and portfolio policies that are robust to uncertainty about the hard-to-estimate drift rate and jump intensity parameters. We also compute the detection-error probability and compare various portfolio holding strategies, including robust and non-robust policies.

Keywords: Optimal consumption and portfolio selection; jumps; Lévy processes; robust control; closed form solution.

Subject Classification: 91G10; 60J75; 93E20

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1 Introduction

The study of dynamic intertemporal portfolio choice problems in continuous time has a long history, dating back to Merton (1969) and Merton (1971). In Merton’s model, the investor’s optimization problem consists of optimizing his consumption and portfolio allocation in a riskfree and risky assets. The sources of risk in this framework are all diffusive so that sudden large changes in the risky assets are unlikely to occur. Since then, the framework has been extended to allow for asset price discontinuities, driven by jump processes, including Poisson, stable or more general Lévy processes, and Hawkes processes.¹

In these models, however, the parameters of the asset returns distribution, such as the expected return, jump intensity and jump size distribution are treated as if they were known to the investor. In practice, these parameters are in fact largely unknown and difficult to estimate, so the investor faces a considerable amount of model uncertainty. It is well known that neglecting the uncertainty surrounding the parameters may lead to poor portfolio performance: for instance, mean-variance optimized portfolios are very sensitive to small changes in the input parameters.² As a result, DeMiguel et al. (2009) advocate in favor of “1/n” or equally weighted portfolios over portfolios that are mean-variance optimized, owing in part to the large uncertainty surrounding expected returns. If anything, the presence of jumps should make this problem worse: jumps are rare events, and pinning down their statistical characteristics, even simply their arrival rate, is difficult on the basis of historical returns data.

The literature suggests different methods to approach this problem. In Bayesian decision analysis, the investor forms a prior over models and maximizes his expected utility. Model uncertainty involves averaging over models instead of integrating over shocks. Knightian, ambiguity, and uncertainty aversion are closely related concepts. One way of introducing ambiguity aversion is through the formulation of multiple priors preferences as in Gilboa and Schmeidler (1989). Given such preferences, optimal decisions are taken under the premise that state variables are governed by the worst-case probability model among a set of candidate models. Chen and Epstein (2002) formulate an intertemporal recursive multiple-priors utility problem that incorporates Knightian ambiguity aversion. An extension of this formulation of ambiguity aversion in continuous time is given in Leippold et al. (2007) which combines learning based on optimal Bayesian updating and ambiguity aversion. Uncertainty aversion has been employed in the literature to study consumption and asset allocation problems, although without jumps: see Uppal and Wang (2003).


The robust control method pioneered by Hansen and Sargent focuses on minimizing the worst case loss over the set of possible models, rather than Bayesian averaging over models.\(^3\) The investor has a specific reference model in mind but also considers a set of alternative models (or parameters) when optimizing his decisions. Recognizing that he is unable to know exactly the true underlying model, the investor seeks instead to develop portfolio policies that should perform reasonably well across the set of alternative models which are statistically close to the reference model in the sense of entropy minimization. Trojani and Vanini (2000) and Trojani and Vanini (2004) solve two versions of a robust control problem and examine its impact on the resulting asset allocation. Robustness with respect to model misspecification has also been applied to models of the term structure of nominal interest rates. Ulrich (2013) employs a robust decision making framework to analyze how model uncertainty with respect to monetary policy affects the term premium on nominal bond yields. Kleshchelski and Vincent (2007) present an equilibrium model of the term structure in a robust control setting where consumption growth exhibits stochastic volatility. They show that policies robust to model misspecification amplify the effect of conditional heteroskedasticity in consumption growth.

In studies of optimal intertemporal consumption and investment decisions, the representative agent typically has expected utility with constant relative risk aversion (CRRA), which ties the elasticity of intertemporal substitution of consumption (EIS) to the inverse of the coefficient of relative risk aversion. CRRA preferences make the agent essentially indifferent about the timing of the resolution of uncertainty. The recursive preferences of Duffie and Epstein (1992) disentangle the agent’s risk aversion from his elasticity of intertemporal substitution and therefore make it possible to investigate how the investor’s optimal robust consumption policy changes depending on his preference for the timing of resolution. Intuitively, early as opposed to late resolution of uncertainty seems to be preferred, as when uncertainty about the investor’s future wealth is resolved earlier, he can optimally smooth his consumption across time. For instance, if the investor knows that he is going to receive a positive income shock sometime in the future, for example higher dividends on his stock investments, he may choose to increase his current consumption even though the dividend payments will not be received until a later date. Portfolio and consumption decisions using recursive preferences have been solved, without robustness, by Kraft et al. (2013) and Kraft et al. (2017).

In a model without jumps, Maenhout (2004) studies a representative agent with recursive utility who seeks to make robust consumption and portfolio decisions and shows that the demand for equities is significantly reduced. Maenhout (2006) extends the robust portfolio allocation analysis by allowing for a time-varying mean-reverting risk premium and shows that while the desire for robustness lowers the total equity share, the proportion of the inter-temporal hedging demand is increased. Consumption and investment problems in a stochastic opportunity setting where the agent

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\(^3\)See Anderson et al. (2003), Hansen et al. (2006), Cogley et al. (2008), Hansen and Sargent (2008), Hansen and Sargent (2010) and Hansen and Sargent (2011). For a discussion and comparison of the max-min expected utility of Gilboa and Schmeidler (1989) and robust control theory, see Hansen and Sargent (2001) and Hansen et al. (2006).
exhibits recursive utility and EIS is unequal to one, can only be solved using numerical approximation techniques (see for instance Campbell et al. (2004)). With jumps, Liu et al. (2005) employ a pure-exchange economy framework with a representative agent who faces model uncertainty with respect to rare events in the underlying aggregate endowment in order to study the equilibrium equity price. However the focus of that paper is not on portfolio policies: the representative agent sets his share of wealth in the risky asset to always be equal to one. Drechsler (2013) uses a robust framework in an equilibrium model where the risky assets follow a jump-diffusion process. Since it focuses on an aggregate endowment economy populated by a representative agent, the optimal portfolio allocation for the investor is to put all his wealth in the consumption claim.

In this paper, we derive robust optimal consumption and portfolio policies of an investor with recursive preferences, when the underlying risky asset follows a Lévy jump-diffusive process. Using robust control, we allow for model misspecification with respect to both the drift and jump intensity parameters. These are the two parameters which are the hardest to estimate on the basis of historical data on asset returns, hence our focus on them. We assume a constant opportunity set, i.e., we abstract from time-varying coefficients on the risky asset or on the entropy growth bound. In this setting, we derive both robust consumption and portfolio policies in fully closed form. Additionally, we derive a semi-closed form formula for the detection-error probabilities, i.e., for the likelihood that the investor selects the wrong model based on a time series of past asset returns. We can therefore determine an upper bound for the set of alternative models which are reasonably close to the investor’s reference model. These new results allow us to analyze how the investor’s risk aversion, preference for the timing of resolution of uncertainty and the degree of misspecification of the drift and jump intensity parameters all have different implications for his robust optimal policies.

The remainder of the paper is organized as follows. Section 2 introduces the general robust consumption and portfolio allocation problem. Section 3 derives optimal consumption policies and robust portfolio weights under both drift and jump intensity perturbation in closed-form. In section 4 we make use of these explicit expressions in order to conduct a straightforward comparative static analysis with respect to the asset price parameters and the desired degree of robustness. Section 5 derives a semi-explicit formula for the detection-error probability when the underlying measure change follows a Lévy jump-diffusive process. In section 6, we analyze the impact of uncertainty by computing certainty equivalents of wealth. Section 7 concludes. The Appendix contains additional models and investor preferences for which explicit robust solutions can also be obtained, as well as all the proofs.
2 The Robust Consumption and Portfolio Allocation Problem with Jumps

We consider an infinite horizon recursive utility maximization problem where the investor chooses his consumption level and allocates his funds between a riskless and a risky asset subject to both diffusive and jump risks. The investor has a particular dynamic model in mind which presumably represents his best estimate of the risky asset dynamics under a benchmark or reference probability measure. However, the investor fears that this model is potentially misspecified: he believes that the true model could lie in a larger set of alternative models that are statistically difficult to distinguish from his reference model. In order to mitigate the effect of potential model misspecification on his utility, the investor wants to choose optimal consumption and portfolio holdings that are robust with respect to perturbations of his reference model. Ultimately, he seeks to make optimal decisions given that he knows enough to not fully trust his base model.

The first issue is to specify the set of alternative models for asset returns which the investor considers as plausible alternatives. Following the Hansen-Sargent robust control approach, the investor considers a set of alternative models consisting of those models which are at a finite distance from the reference model, in the relative entropy (or Kullback-Leibler distance) sense. Models within this set should be difficult to distinguish statistically by the investor. Using the Lagrange approach, we then convert the entropy constraint into a penalty on perturbations from the reference model and finally use stochastic optimal control to derive the optimal robust portfolio weights and consumption policy of the investor in closed-form.

2.1 Asset Price Dynamics under the Reference and Robust Measures

We assume a complete, filtered probability space \((\Omega, \mathcal{F}, \mathbb{P} = (\mathcal{F}_t)_{t \geq 0}, \mathbb{P})\) satisfying the usual assumption. \(\mathbb{P}\) denotes the reference (or physical) probability measure. The investment set available to the investor at time \(t \geq 0\) consists of a riskless (locally deterministic) asset with price \(S_{0,t}\) and a risky asset with price \(S_{1,t}\) following an exponential Lévy process. More specifically, the dynamics of the two assets are given by

\[
\begin{align*}
\frac{dS_{0,t}}{S_{0,t}} &= r dt, \quad S_{0,0} > 0 \quad (1) \\
\frac{dS_{1,t}}{S_{1,t-}} &= (r + R) dt + \sigma dB_t + Jd\tilde{Y}_t, \quad S_{1,0} > 0, \quad \mathbb{P} - a.s. \quad (2)
\end{align*}
\]

where \(r \geq 0\) is the riskless return, \(R \in \mathbb{R}\) denotes the excess return of the risky asset over the riskfree asset, \(\sigma > 0\) is the volatility parameter, \(B_t = (B_t)_{t \geq 0}\) is a Brownian motion under \(\mathbb{P}\) and \(J \in (-1, 1)\) is a jump scaling factor. \(\tilde{Y}_t = Y_t - \Lambda_t\) is a compensated pure jump process with Lévy measure \(\lambda \nu(dz)\), where \(\lambda \geq 0\) is a fixed jump intensity parameter. The Lévy measure \(\nu\)
satisfies \( \int_{\mathbb{R}} \min(1, |z|) \nu(dz) < \infty \), so that jumps have finite variation. We write \( \Lambda_t = \lambda \kappa t \) where \( \kappa = \mathbb{E}[Z_t] < \infty \) denotes the predictable compensator of the jump process \( Y_t \).

In the sequel, we make \( Y_t \) a compensated compound Poisson process, i.e. \( \tilde{Y}_t = \sum_{n=1}^{N_t} Z_n - \Lambda_t \), where \( N_t \) is a scalar Poisson process with jump intensity, or arrival rate, \( \lambda \). The jump sizes \( Z_n \) are independent of \( N_t \) and are assumed to be i.i.d. with Lévy measure \( \nu(dz) \). Given these assumptions, the dynamics of the risky asset under the reference measure can be expressed as

\[
\frac{dS_{1,t}}{S_{1,t-}} = (r + R)dt + \sigma d\tilde{B}_t + \int \nu(dz) d\Lambda_t,
\]

\( S_{1,0} > 0, \quad \mathbb{P} \text{-a.s.} \) (3)

In order to introduce model misspecification, we need to specify a set of alternative or worst-case robust dynamics which are statistically close to the reference dynamics in (3). For this purpose, consider an equivalent probability measure \( \mathbb{P}^\theta \) which is absolutely continuous with respect to the reference measure \( \mathbb{P} \). We call \( \mathbb{P}^\theta \) the robust (or perturbed) measure. The investor considers alternative models under the robust measure \( \mathbb{P}^\theta \) which take the general form

\[
\frac{dS_{1,t}}{S_{1,t-}} = (r + R + \sigma h_t - \lambda^\theta J \int_{\mathbb{R}} z \nu^\theta(dz)) dt + \sigma d\tilde{B}_t^\theta + \int \nu^\theta(dz) d\Lambda_t^\theta, \quad S_{1,0} > 0, \quad \mathbb{P}^\theta \text{-a.s.} \] (4)

which has the effect of making the investor uncertain about his expected return and the arrival rate of the jumps. First, the drift has changed from \( \mathbb{P} \) to \( \mathbb{P}^\theta \), with \( (h_t)_{t \geq 0} \) a continuous \( \mathcal{F}_t \)-measurable function of the Markovian state \( S_{1,t} \) with the same dimensionality as the one-dimensional Brownian motion. In what follows, we refer to \( h_t \) as a drift perturbation function, since it perturbs the drift dynamics of the risky asset under \( \mathbb{P} \) without affecting the jump component \( \tilde{Y}_t \). Second, the stochastic process \( B_t^\theta = (B_t^\theta)_{0 \leq t} \) remains a Brownian motion but now under the perturbed or robust measure \( \mathbb{P}^\theta \). Third, another set of perturbations affect the jump component \( \tilde{Y}_t^\theta \), namely its jump intensity and jump size distribution under \( \mathbb{P}^\theta \). The jump intensity \( \lambda \) is transformed into \( \lambda^\theta \) under the robust probability measure as follows

\[
\lambda^\theta = e^a \lambda, \quad a \in \mathbb{R}
\]

where \( a \) is a scalar jump intensity perturbation parameter that amplifies or diminishes the jump intensity.

In this setup, perturbing the jump intensity has two effects on the risky assets dynamics. It both alters the drift and changes the frequency of jumps occurring in the Poisson process \( N_t \). Consider the case when there are only negative jumps in the stock price dynamics of (4), for example by assuming that the jump size \( z \) has positive support and \( J \in (-1, 0) \). Under this assumption,
increasing the jump intensity leads to more frequent negative jumps but also, due to compensation, raises the expected return on the risky asset, consistently with the empirical risk-return trade-off observed in empirical data. In other words, compensating the jump process leads to the stock price \(S_t\) carrying a risk premium for intensity misspecification. As we later show, this has implications as to how jump risk will affect optimal portfolio holdings. For instance, in a diffusive setting, i.e. if we set \(\lambda = 0\) in (4), and the investor is only concerned about potential drift misspecification, the optimal portfolio weight in the risky asset is simply reduced, as model misspecification concerns lead to a decrease in the expected return while leaving volatility unchanged. However, in the robust setting here, there exists a trade-off between higher frequency of jumps occurring and simultaneously higher jump risk compensation.

The jump size distribution under \(P^\vartheta\) has Lévy measure

\[
\nu^\vartheta(dz) = \nu(dz; b), \ b \in \mathbb{R}^L, \ L \geq 1
\]

where \(b\) is a set of possibly vector valued perturbation parameters. For instance, if the jump size distribution is normal, \(Z_n \sim N(\mu, \sigma^2)\), a jump size perturbed model may read \(Z_n \sim N(\mu + \delta \mu, \sigma^2 v \sigma)\), where \(\delta \mu \in \mathbb{R}\) shifts the mean from \(\mu\) under \(P\) to \(\mu + \delta \mu\) under \(P^\vartheta\) and likewise the variance is scaled by \(v \sigma^2 > 0\). So for \(\alpha = \delta \mu = 0\) and \(v \sigma = 1\) we get back the jump distributions of the reference model under the measure \(P\).

### 2.2 Change of Measure

The dynamics of the compensated exponential Lévy process under the reference and robust measures are linked through a specific likelihood ratio or Radon-Nikodym density process \(\vartheta_t\). This density process not only changes the dynamics of the risky asset but also, as will be shown in the next section, restricts the set of alternative models that are statistically difficult to distinguish from the reference model. Fix \(T > 0\) and define

\[
\vartheta_t = (\vartheta_t)_{t \in [0,T]} = \frac{dP^\vartheta}{dP} \bigg|_{\mathcal{F}_t} = \vartheta_t^D \vartheta_t^J
\]

where \(\vartheta_t^D\) is a \((\mathcal{F}_t, P)\)-martingale that defines the measure change of the continuous part of the stochastic process and \(\vartheta_t^J\), also a \((\mathcal{F}_t, P)\)-martingale, that defines the measure change of the discontinuous or jump part. The change of measures of the diffusive and jump part factor only when the continuous and the jump part of the stochastic process are independent, which is the case in the setting we employ above, since \([N, W]_t = 0\).

In a jump diffusive setting, where jumps follow a compound Poisson process, there are three ways to change the measure, i.e. from \(P\), the reference measure to \(P^\vartheta\), the robust measure:

1. Measure change of the diffusive part through \(\vartheta_t^D\) affecting the drift and the Brownian motion.
2. Measure change of the jump part through $\vartheta_t^J$ by changing the jump intensity of the process under $\mathbb{P}^\vartheta$.

3. Measure change of the jump part through $\vartheta_t^J$ by changing the jump size of the process under $\mathbb{P}^\vartheta$.

From Girsanov’s theorem for Itô semimartingales, for the diffusive drift measure change $\vartheta_t^D$, with $(h_t)_{t \geq 0}$ being a progressively measurable process, we have that

$$B_t^\vartheta = B_t - \int_0^t h_s ds$$

is a Brownian motion with respect to the measure $\mathbb{P}^\vartheta$. Then it follows that an absolutely continuous change of measure can be represented by an exponential $(\mathbb{P}, \mathcal{F}_t)$-martingale $\vartheta_t^D$ satisfying

$$\vartheta_t^D = \exp \left\{ \int_0^t h_s dB_s - \frac{1}{2} \int_0^t h_s^2 ds \right\}, \quad \mathbb{E}_0 [\vartheta_t] = \vartheta_0 = 1,$$

where $\mathbb{E}_t[\cdot] = \mathbb{E}[\cdot | \mathcal{F}_t]$ denotes the conditional expectation up to time $t$ with respect to the measure $\mathbb{P}$ and likewise we define by $\mathbb{E}_t^\vartheta [\cdot]$ the conditional expectation up to time $t$ with respect to the robust measure $\mathbb{P}^\vartheta$.

Concerning the jump part of the risky asset, we let $N_t = (N_t)_{t \in [0,T]}$ be a Poisson process with jump intensity $\lambda > 0$ on the probability space $(\Omega, \mathcal{F}_t, \mathbb{P})$ and $T > 0$ again fixed. We want to change the intensity $\lambda$ of the Poisson process $(N_t)_{t \in [0,T]}$ on $\mathbb{P}$ to a jump intensity $\lambda^\vartheta$ under the robust measure $\mathbb{P}^\vartheta$. Likewise, we want to perturb the Lévy measure such that the jumps have distribution $\nu^\vartheta(dz) := \nu(dz; b)$ under the robust measure. The appropriate measure change $\vartheta^J = (\vartheta^J_t)_{t \in [0,T]}$ is

$$d\frac{d\mathbb{P}^\vartheta}{d\mathbb{P}} \bigg|_{\mathcal{F}_t} = \vartheta^J_t = e^{(\lambda - \lambda^\vartheta)t} \prod_{n=1}^{N_t} \frac{\lambda^\vartheta}{\lambda} \frac{\nu^\vartheta(Z_n)}{\nu(Z_n)}, \quad \mathbb{E} [\vartheta^J_t] = \vartheta^J_0 = 1, \quad \mathbb{P} - a.s.$$

which is a $(\mathcal{F}_t, \mathbb{P}^\vartheta)$-martingale and satisfies

$$d\vartheta^J_t = \vartheta^J_t \left( H_t - \lambda^\vartheta \right) - \lambda^\vartheta d(N_t - \lambda t), \quad H_t = \sum_{n=1}^{N_t} \frac{\lambda^\vartheta}{\lambda} \frac{\nu^\vartheta(Z_n)}{\nu(Z_n)}.$$

where $H_t$ is a compound Poisson process and $H_t - \lambda^\vartheta t$ is a $(\mathcal{F}_t, \mathbb{P}^\vartheta)$-martingale. Therefore, the density process in (10) is a right-continuous, adapted process with left limits (càdlàg).

### 2.3 Relative Entropy Growth Bounds and Detection-Error Probabilities

By changing the perturbation parameters $a$, $b$ and $h_t$ in (5), (6) and (8) we control the discrepancy between the dynamics of the risky asset under the reference measure with respect to its dynamics under the robust measure. Therefore, the more $h_t$, $a$ and $b$ deviate from their no-perturbation values,
the more different the dynamics of the risky asset become under the reference with respect to the robust measure: In other words, the set of alternative models expands. However, the possible set of models under consideration has to be restricted to a subset of models which are statistically difficult to distinguish from the reference model. A natural statistical tool to measure distances between two probability distributions is relative entropy, and it is well suited for the purpose of defining alternative models in robust control (see Anderson et al. (2003)).

The alternative set of possible models that are similar in a statistical sense are linked to the measure change $\vartheta_t$. Given two probability measures $P$ and $P^\vartheta$, growth in entropy of $P$ relative to $P$ over the time interval $[t, t + \Delta t]$ is defined as

$$
G(t, t + \Delta t) \equiv \mathbb{E}_t^\vartheta \left[ \log \left( \frac{\vartheta_{t+\Delta t}}{\vartheta_t} \right) \right], \quad \mathcal{R}(\vartheta_t) \equiv \lim_{\Delta t \to 0} \frac{G(t, t + \Delta t)}{\Delta t} \quad \forall t \geq 0
$$

Thus the set of admissible model misspecification can be characterized as

$$
\{ \vartheta_t : \mathcal{R}(\vartheta_t) \leq \eta, \forall t \geq 0, \eta \geq 0 \}
$$

where $\eta$ is a constant that defines an upper bound on the set of alternative models. As $\eta \to 0$, the investor becomes fully confident about his reference model, while increasing $\eta$ expands the set of alternative models that are statistically further away from the reference model, so overall model uncertainty increases. Due to the independence of the diffusive and the jump component, the measure change is given by $\vartheta_t = \vartheta_t^D \vartheta_t^J$, which implies that relative entropy growth is simply the sum of the two components (drift and jump), namely $\mathcal{R}(\vartheta_t) = \mathcal{R}(\vartheta_t^D) + \mathcal{R}(\vartheta_t^J)$. Therefore, by varying the perturbation function $h_t$ and perturbation parameters $a$ and $b$ we regulate the space of admissible models within the set $[0, \eta]$, $\forall t \geq 0$.

An immediate question that arises in this context is: What is a reasonable value for $\eta$? Anderson et al. (2003) provide a statistical tool for model detection based on the log of the measure change $\vartheta_t$ in the form of detection-error probabilities in order to quantify the amount of model uncertainty that seems plausible to the investor. The basic intuition behind this test statistic is, given the right model is $P$ and a finite time series sample of the state variable (in our case the risky asset) of length $T - t$, how likely will the investor mistakenly assume the data have been generated by model $P^\vartheta$ instead of the true model $P$. The detection-error probability is quantifying the likelihood that the investor is going to select the wrong model. Thus if the true model is $P$, the investor will falsely reject it for model $P^\vartheta$ based on a time series sample of length $T - t$ whenever $\log(\vartheta_T) > 0$. Conversely, if the true model is $P^\vartheta$ he will erroneously reject it for model $P$ whenever $\log(\vartheta_T) < 0$.

### 2.4 Wealth Dynamics and Robust Utility Maximization

We denote by $X_t = (X)_{t \geq 0}$ the investor’s wealth at time $t$. Let $\omega_{0,t} = w_{0,t}/X_t$ be the percentage of wealth (or portfolio weight) invested in the risk free asset and $\omega_{1,t} = w_{1,t}/X_t$ be the percentage of
wealth invested in the risky asset; in other words, \( w_{i,t}, i \in \{0, 1\} \) is the absolute amount of money invested into asset \( i \). The portfolio weights are adapted predictable càdlàg processes and satisfy \( \omega_{0,t} + \omega_{1,t} = 1 \). The investor consumes at an instantaneous rate \( C_t \) at time \( t \). Under the robust dynamics given in (4) his wealth evolves according to

\[
dX_t = \omega_{0,t} X_t \frac{dS_{0,t}}{S_{0,t}} + \omega_{1,t} X_t \frac{dS_{1,t}}{S_{1,t}} - C_t dt
\]

\[
dX_t = \left[ X_t \left( r + \omega_t R + \sigma h_t \omega_t - \omega_t \lambda^\theta \int_{\mathbb{R}} z \nu^\theta(dz) \right) - C_t \right] dt + \omega_t X_t \sigma dB_t^\theta + \omega_t X_t JdY_t^\theta, \quad (14)
\]

with \( X_0 > 0, \mathbb{P}^\theta - \text{a.s.} \) and we have set \( \omega_t = \omega_{1,t} \).

As for the preference specification, we equip our representative agent with recursive utility\(^4\) which separates preferences over the timing of the resolution of uncertainty, i.e., the elasticity of intertemporal substitution, from risk aversion. Those two preference parameters measure distinct features of the agent’s utility. Whereas risk aversion measures the degree to which the agent dislikes consumption fluctuations due the randomness of future states, the elasticity of intertemporal substitution measures the agent’s aversion to consumption fluctuations across time in a deterministic world. The lower the value of the parameter \( \psi \) is, the higher the distaste of the representative agent for substituting current for future consumption in response to changes in investment opportunities. More formally speaking, let \( V_t \) denote the continuation value, which represents utility over future consumption. In our robust control framework, recursive utility is defined as the solution to

\[
V_t := \mathbb{E}_t^\theta \left[ \int_t^\infty f(C_s, V_s) ds \right]
\]

(15)

where \( \mathbb{E}_t^\theta [\cdot] \) denotes the \( \mathcal{F}_t \)-conditional expectation with respect to the robust measure \( \mathbb{P}^\theta \) and the normalized aggregator of consumption and continuation value is given by

\[
f(C, V) = \frac{\beta(1 - \gamma)V}{1 - 1/\psi} \left( \frac{C}{((1 - \gamma)V)^{1/\gamma}} \right)^{1-1/\psi} - 1
\]

(16)

where \( \beta \in (0, \infty) \) denoting the investor’s discount rate, \( \gamma > 0 \) is the coefficient of relative risk aversion and \( \psi > 0 \) the elasticity of intertemporal substitution (EIS).

As we will discuss later, the separation of risk aversion and elasticity of intertemporal substitution will induce a preference for the timing of resolution of uncertainty which has important implications for the optimal consumption of the representative agent in the robust setting. The agent has a preference for early resolution of uncertainty when the risk aversion effect dominates the intertemporal substitution effect, i.e. \( \gamma > 1/\psi \). Such preferences imply that, when the investor is facing high future income uncertainty, it may be optimal to reduce current consumption and increase savings to

\(^4\)This utility specification is also known as stochastic differential utility see Duffie and Epstein (1992) which is the continuous time equivalent of the discrete time recursive utility formulated in Epstein and Zin (1989) and Weil (1990).
buffer future income shocks. Contrarily, when the intertemporal substitution effect dominates the risk aversion effect, i.e. \( \gamma < 1/\psi \), then the agent cares more about current as opposed to future consumption.

In the special case where the agent is indifferent with respect to the timing of resolution of uncertainty, i.e. \( \gamma = 1/\psi \), we recover the standard time and state separable CRRA preferences. To see this, let \( \psi \to 1/\gamma \) in (16) such that the aggregator reduces to

\[
f(C, V; \psi = 1/\gamma) = \beta \left( \frac{C^{1-\gamma}}{1-\gamma} - V \right)
\]

which can be shown to be ordinary equivalent to the standard additive power utility given by (see Duffie and Epstein (1992) for details)

\[
\mathbb{E}_t^\vartheta \left[ \int_t^\infty e^{-\beta s} \frac{C_s^{1-\gamma}}{1-\gamma} ds \right]
\]

(18)

Other frequently employed limiting cases include \( \gamma \to 1 \) which results into log utility

\[
\mathbb{E}_t^\vartheta \left[ \int_t^\infty e^{-\beta s} \log(C_s) ds \right]
\]

(19)

or when EIS approaches 1, in which case the normalized aggregator reduces to

\[
f(C, V; \psi = 1) = \beta V [(1 - \gamma) \log(C) + \log(V(1 - \gamma))].
\]

(20)

This simplified aggregator allows for simplified solutions in several consumption and investment problems: see, e.g., Campbell et al. (2004), Wachter (2013) or Drechsler (2013).

Next, given the wealth dynamics under the robust measure in (14) and the recursive preference specification given in (15) and (16), we formulate the robust decision making problem of the investor. As a first step, the investor’s problem is to choose a set of worst-case drift perturbation functions \( \{h_s\}_{t \leq s < \infty} \) and worst-case jump intensity \( a \) and jump size parameters \( b \in \mathbb{R}^L \). As a second step, the investor has to select admissible consumption and portfolio holdings \( \{C_s, \omega_s\}_{t \leq s < \infty} \) that maximize his expected utility of consumption under the worst-case scenario. The robust consumption and portfolio policies are the solution of the following max-min problem

\[
\max_{\{C_s, \omega_s\}_{t \leq s < \infty}} \min_{\{h_s\}_{t \leq s < \infty}, a, b} \mathbb{E}_t^\vartheta \left[ \int_t^\infty f(C_s, V_s) ds \right]
\]

(21)

subject to the wealth dynamics in (14) and the entropy growth constraint

\[
\mathcal{R}(\vartheta_t) \leq \eta, \quad \forall t \geq 0.
\]

(22)

We define the time-homogeneous value function

\[
V(X_t) = \max_{\{C_s, \omega_s\}_{t \leq s < \infty}} \min_{\{h_s\}_{t \leq s < \infty}, a, b} \mathbb{E}_t^\vartheta \left[ \int_t^\infty f(C_s, V_s) ds \right]
\]

(23)
associated with the optimal stochastic robust control problem in (21). Using Itô’s formula for semi-
martingales, the perturbed Hamilton-Jacobi-Bellman (HJB) equation characterizing the optimal
robust consumption and portfolio allocation problem is given by

\[ 0 = \max_{\{C_t, \omega_t\}} \min_{\{h_t, a, b\}} f(C_t, V_t) + \]
\[ + \frac{\partial V(X_t, t)}{\partial X} \left[ X_t \left( r + \omega_t \left( R + \sigma h_t - \lambda^\theta J \int R \nu(dz, b) \right) \right) - C_t \right] \]
\[ + \frac{1}{2} \frac{\partial^2 V}{\partial X^2} X_t^2 \sigma^2 + \lambda e^a \int R \left[ V(X_{t_-} + X_{t_-} \omega_t Jz, t) - V(X_{t_-}, t) \right] \nu(dz, b) \]

subject to

\[ R(\vartheta_t) \leq \eta \] (25)

and the transversality condition

\[ \lim_{t \to \infty} \mathbb{E}^\theta [V(X_t, t)] = 0. \] (26)

In order to solve the investor’s robust consumption and investment problem, we employ a standard
Lagrange technique for constraint optimization. The only modification compared to the standard
Lagrangian method is that we scale the entropy penalty term as follows

\[ \theta_t = \theta(X_t) = \tilde{\theta}(1 - \gamma) V(t, X_t) > 0, \quad \tilde{\theta} \in \mathbb{R}_+. \] (27)

This specification renders the Lagrange multiplier state-dependent in wealth which will allow us to
find a closed form solution to the consumers’ consumption and investment problem. Moreover, in
order for the problem in (24) to admit a solution, we have to restrict the domain of the elasticity
of inter temporal substitution \( \psi \) and the coefficient of risk aversion \( \gamma \). As shown in Schroder and
Skiadas (1989) or Maenhout (2004), the necessary conditions are

\[ \frac{\gamma - \frac{1}{\psi}}{1 - \frac{1}{\psi}} < 1, \quad \text{and} \quad \frac{1}{\psi} > \max \left( 0, \frac{\gamma - \frac{1}{\psi}}{1 - \gamma} \right), \quad \text{if} \quad \psi \neq 1 \]

(28)

To find a solution of the problem in (24) subject to the entropy growth constraint in (25), we
denote by \( \mathcal{L} := \mathcal{L}(C_t, \omega_t, h_t, a, b, \theta_t) \) the Lagrangian associated to the problem given in (24) and by \( \theta_t \)
the corresponding state-dependent Lagrange-multiplier of the entropy growth constraint in (25). We
first determine the optimal robust control policies \( h^*_t, a^* \) and \( b^* \), by solving a constraint optimization
problem. Then the first order optimality conditions for the minimization part of (24) are given by

\[ \frac{\partial \mathcal{L}}{\partial h_t} = \sigma \omega_t - \frac{\partial}{\partial h_t} \theta_t (\mathcal{R}(\vartheta_t) - \eta) = 0, \] (29)
\[ \frac{\partial \mathcal{L}}{\partial a} = \frac{\partial}{\partial a} \lambda e^a \int R \left[ V(X_{t_-} + X_{t_-} \omega_t Jz) - V(X_{t_-}) \right] \nu(dz, b) - \frac{\partial}{\partial a} \theta_t (\mathcal{R}(\vartheta_t) - \eta) = 0, \] (30)
\[ \frac{\partial \mathcal{L}}{\partial b_t} = \frac{\partial}{\partial b_t} \lambda e^a \int R \left[ V(X_{t_-} + X_{t_-} \omega_t Jz) - V(X_{t_-}) \right] \nu(dz, b_t) - \frac{\partial}{\partial b_t} \theta_t (\mathcal{R}(\vartheta_t) - \eta) = 0, \] (31)
\[ l = 1, \ldots, L, \]
\[ \frac{\partial L}{\partial \theta} = R(\theta_t) - \eta = 0, \quad \theta_t \geq 0, \quad \theta_t (R(\theta_t) - \eta) = 0. \]  

Each equation in (29) to (32) summarizes two opposite effects. For instance, from (29), the left term \( \partial \mathbb{E}^\theta [V(X_t)] / \partial h_t = \sigma \omega_t \) characterizes the marginal impact on the investor’s utility that results from increasing perturbation. The right term in (29) \( (\partial / \partial h_t) \theta (R(\theta_t) - \eta) \) captures the associated increase in detectability of the robust model. Thus under the robust measure, the perturbation of the reference model is such that its effect is most harmful to the investor’s utility while simultaneously remaining difficult to detect statistically.

### 3 Explicit Robust Consumption and Portfolio Weights Under Drift and Jump Intensity Perturbations

To solve the problem, from the first order conditions in (29) to (32) we obtain the optimal amount of perturbation of the drift component \( h^*_t \), jump intensity \( a^* \) and size perturbation \( b^* \in \mathbb{R}^L \), that satisfy the entropy growth constraint, the complementary slackness condition and non-negativity constraint of \( \theta^* \) in (32). Having obtained a set of optimal robust control parameters \( \{h^*_t, a^*, b^*\} \), we then plug them back into the Lagrangian and solve the corresponding perturbed HJB equation for the optimal consumption policy \( C^*_t \) and portfolio weights \( \omega^* \).

Given \( \{h^*_t, a^*, b^*\} \), the first order condition for the investor’s optimal consumption and portfolio policies are given by,

\[
\frac{\partial L(h^*_t, a^*, b^*, \theta^*)}{\partial C_t} = \frac{\partial f(C_t, V_t)}{\partial C_t} = \frac{\partial V}{\partial X}, \\
\frac{\partial L(h^*_t, a^*, b^*, \theta^*)}{\partial \omega_t} = \frac{\partial}{\partial \omega_t} \left[ \frac{\partial V(X_t)}{\partial X} \right] X_t \left( r + \omega_t R + \sigma h_t \omega_t - \omega_t \lambda \theta^* J \int \nu(dz) \right) - C_t \\
+ \frac{1}{2} \frac{\partial^2 V(X_t)}{\partial X^2} \omega_t^2 \sigma^2 + \lambda e^{a^*} \int \left[ V(X_{t-} + X_{t-} \omega_t Jz) - V(X_{t-}) \right] \nu(dz; b^*) 
\]

Equation (33) represents the envelope condition for optimal consumption. From (34) we obtain the optimal portfolio allocations as a function of the perturbation parameters \( \{h^*_t, a^*, b^*\} \). We are now going to discuss the case when both drift and jump intensity are being distorted.

In order to derive more explicit results, we need to make some further assumptions about the Lévy measure characterizing the jump sizes and the amount of perturbation of the reference model we allow for. We do not perturb the Lévy measure, in other words, the jump size distribution is the same under both measures, i.e. \( \nu(dz; b) = \nu(dz) \). We only perturb the arrival rate of the jumps.

Lastly, we derive the measure change, in order to fully characterize the set of alternative models. When there is no jump size perturbation, the Radon-Nikodym derivative in (11) reduces to

\[
d\tilde{\theta}_t^l = (e^\alpha - 1)\partial t_+ d\tilde{N}_t, \quad \partial_0^l = 1 \text{ with } \tilde{N}_t = N_t - \lambda t
\]
whose explicit solution is given by

\[ \vartheta^J_t = \exp\{aN_t - \lambda(e^a - 1)t\}, \quad \vartheta^J_0 = 1. \] (36)

Therefore, together with (9) characterizing the measure change of the diffusive part we arrive at

\[ \mathcal{R}(\vartheta_t) = \mathcal{R}(\vartheta^D_t) + \mathcal{R}(\vartheta^J_t) = \frac{1}{2}h^2 + e^a\lambda(a - 1) + \lambda. \] (37)

The investor's consumption and portfolio choice problem is summarized by (24) and (37), which limits the set of alternative models. The solution to this problem is given by a two step-procedure. In a first step, which corresponds to the min-part in (24), the investor has to decide how rich the alternative set of models is that he considers reasonably close to his reference model. In doing so, he specifies his preference for robustness with respect to small perturbations of his reference model by optimally choosing \( \{h^*, a^*\} \). In a second step, the investor has to decide on his optimal consumption and portfolio policies. In Theorem 1 below, we state the optimal consumption policy for our representative investor.

**Theorem 1. (Optimal Consumption)** With recursive preferences, from the first order condition of consumption in (33) we obtain

\[ C^*_t = C^*_t(h^*, a^*, \omega^*) = \beta^\psi K(h^*, a^*, \omega^*)X_t \] (38)

where we require that \( X_t > 0 \) such that consumption remains non-negative. Then given \( \{h^*, a^*\} \), evaluating (A.2) at the optimal consumption \( C^*_t \) and portfolio holdings \( \omega^*_t \) the constant \( K \) is given by

\[ K^* := K(h^*, a^*, \omega^*) = \beta^{1-\psi} + (1 - \psi)\beta^{-\psi}g(h^*, a^*, \omega^*) \] (39)

\[ g(h^*, a^*, \omega^*) := r + \omega^* \left( R + \sigma h^* - J\lambda e^{a^*} \int \nu(dz) \right) - \frac{\gamma}{2} \sigma^2 \omega^{a^*} \]

\[ - \frac{\lambda e^{a^*}}{1 - \gamma} \int R \left[ (1 + \omega^*Jz)^{1-\gamma} - 1 \right] \nu(dz) \] (40)

which is positive as long as the conditions in (28) are satisfied.

**Proof.** See Appendix A.1.

Theorem 1 shows that the optimal consumption to wealth ratio is constant, and, from (38) and (40), it follows that this ratio is affected by robustness concerns. The constant will be fully determined once we have first solved for the optimal perturbation parameters \( \{h^*, a^*\} \) and secondly obtained the optimal portfolio holdings \( \omega^* \).

Finally, given optimal consumption in (38) and the constant in (40) we can now check that the transversality condition is satisfied. Taking unconditional expectations of the value function guess (A.1) with respect to the robust measure we have that

\[ \mathbb{E}^\vartheta [V(X^*_t)] = \frac{K^*(1 - \psi)}{(1 - \gamma)} \mathbb{E}^\vartheta \left[ X^*_t(1 - \gamma) \right] \] (41)
This conditional expectation can be explicitly computed by applying Itô’s formula for semimartingales to the function \( f(x) = x^{1-\gamma} \), and using the wealth dynamics and optimal consumption as given in (14) and (39), respectively. More precisely, the unconditional expectation with respect to the robust measure \( \mathbb{P}^{\vartheta} \), i.e. \( \mathbb{E}^{\vartheta}[X_t^{(1-\gamma)}] =: \varphi(t) \), solves the following ordinary differential equation

\[
\varphi'(t) = \frac{d\mathbb{E}^{\vartheta}[X_t^{(1-\gamma)}]}{dt} = \psi(1-\gamma)(g-\beta)\varphi(t), \quad \varphi(0) = \frac{X_0^{(1-\gamma)}}{1-\gamma}
\]

from which it follows that \( \mathbb{E}^{\vartheta}[X_u^{(1-\gamma)}] = e^{-(g-\beta)(\gamma-1)\psi t}X_0^{(1-\gamma)}(1-\gamma) \psi t \) converges to zero exponentially fast as \( t \to \infty \) provided that \( (g-\beta)(\gamma-1) > 0 \), which establishes that the transversality condition is satisfied.

### 3.1 Closed Form Robust Portfolio Weights

In order to obtain fully explicit portfolio weights, we further specify the level of risk aversion, the Lévy measure and the entropy growth constraints. In the sequel, we focus on a Lévy measure \( \nu(dz) \) defined on \((0,1]\) and restrict the deterministic jump scaling factor \( J \) to the open interval \((-1,0)\). This implies that we only consider negative jumps in the asset price dynamics. In practice, those are the harmful jumps a risk-averse investor is most concerned about. We set \( \nu(dz) \) to follow a power law under both measures, i.e.

\[
\nu^{\vartheta}(dz) = \nu(dz) = dz/z, \quad \text{if } z \in (0,1].
\]

Concerning the treatment of the entropy growth constraints, since total relative entropy separates into a diffusive and a jump part, i.e. \( R(\vartheta_t) = R(\vartheta^D_t) + R(\vartheta^J_t) \), we can treat entropy growth with respect to the drift and jump parts independently. However, this implies that the total maximal amount of robustness \( \eta \) is the sum of the maximal amount of robustness with respect to drift misspecification, denoted by \( \eta^D \) and the maximal amount of robustness with respect to jump intensity misspecification, denoted by \( \eta^J \). Therefore \( \eta = \eta^D + \eta^J \) and we write \( \eta^D = \zeta^D \eta \) and thus \( \eta^J = (1 - \zeta^D) \eta \), where \( \zeta^D \in [0,1] \) denotes the share of total amount of robustness that is allocated to drift perturbation. When \( \zeta^D = 0 \) the investor has full confidence in his drift specification and all the robustness concerns relate to the jump intensity. Likewise, when \( \zeta^D = 1 \), the investor believes that there is no jump intensity misspecification and thus is only concerned that the drift might be:

\[
\text{Below and in the appendix, we describe other models (including other preferences and jump distributions) where robust policies can be obtained in closed-form.}
\]

\[
\text{A similar idea has also been used by Chen and Epstein (2002), Sbuelz and Trojani (2008), Ulrich (2010), Ulrich (2012) to solve a model ambiguity problem when the risks are only diffusive. In our case, a joint entropy growth constraint gives rise to first order conditions of the drift and the jump intensity perturbation parameter which cannot be solved analytically and one needs to rely on numerical techniques to solve the system.}
\]
missspecified. With this decomposition, the entropy growth constraints are given by
\[ R(\vartheta^D_t) \leq \eta^D, \quad R(\vartheta^J_t) \leq \eta^J, \quad \forall t \geq 0. \] (44)

Finally, having specified the entropy growth sets for drift and jump perturbation, we can solve for the optimal portfolio weights in fully closed-form for some explicit choices of risk aversion \( \gamma \).

**Theorem 2. (Optimal Portfolio Weights)** For \( \gamma = 2 \) and the jump sizes governed by the power law in (43), we obtain the following fully closed-form robust portfolio weights as given below
\[
\omega^*(h^*, a^*) = \frac{h_1(\eta) + H(\eta)}{4J\sigma^2}, \quad H(\eta) := \sqrt{h_1(\eta) + h_2^2(\eta)}
\] (45)
\[
h_1(\eta) := J (R + h^*\sigma) - J^2\lambda e^{a^*} - 2\sigma^2, \quad h_2(\eta) := 8\sigma^2\lambda J^2 e^{a^*}
\]
\[
h^* = -\sqrt{2\eta^D}, \quad \forall \eta^D \geq 0, \quad a^* = 1 + W \left( \frac{\eta^J - \lambda}{e\lambda} \right) \geq 0, \quad \forall \eta^J \geq 0, \lambda \in (0, \infty) \] (46)

where \( W ((\eta^J - \lambda) / (e\lambda)) = W (\cdot; \lambda) \) denotes Lambert’s \( W \) function, \( e \) is Euler’s constant and we require that the solvency constraint \( |\omega^* J| < 1 \) holds at the optimal portfolio allocation \( \omega^* \). Note that \( \lim_{\eta \to 0} W (\eta; \lambda) = W (-1/e; \lambda) = -1 \) so that \( \lambda^0 = \lambda \) we are indeed back to the case where there are no robustness concerns of the investor with respect to the jump intensity parameter.

**Proof.** See Appendix A.2. \( \square \)

From (45) we can infer that the optimal portfolio allocation is independent of the investor’s EIS. This result follows because we consider a constant opportunity set, i.e. no time variation in the expected return, volatility, jump intensity of the risky asset or model uncertainty \( \eta \). Moreover, Theorem 2 shows that the optimal portfolio weights are highly non-linear functions of the optimal perturbation parameters \( h^* \) and \( a^* \) rendering precise comparative static analysis difficult. For this reason, in Figure 2 we plot the optimal robust portfolio weights as a function of the share of total robustness assigned to drift perturbation \( \zeta^D \) to assess the quantitative impact of drift versus jump uncertainty. As expected, the robust portfolio allocation is such that the amount invested into the risky asset is lower compared to the case when the investor has full confidence in his reference model (\( \eta = 0 \)). Further, regardless of the share of \( \eta \) allocated to drift or jump intensity robustness, the optimal portfolio weights are more sensitive when \( \eta \) is low compared to the case when \( \eta \) is large. This implies that the marginal effect of increasing the total amount of robustness on \( \omega^*(h^*, a^*) \) is declining. Next, comparing the cases of only drift \( \zeta^D = 1 \) vs. only jump intensity \( \zeta^D = 0 \) perturbations shows that \( \omega^* \) is reduced more in the case when there are solely concerns about potential drift as opposed to only jump intensity misspecification. However, this is a direct consequence of compensating the Lévy process as the expected return on the risky asset is higher due to adding the compensator.

Moreover, the asymptotic behavior of the portfolio weights when the total amount of robustness tends to infinity, shows that
\[
\lim_{\eta \to \infty} \omega^*(h^*, a^*) = -\infty
\]
which implies that regardless of whether the drift or the jump intensity is misspecified or both, the investor optimally chooses to short the risky asset (up until the solvency constraint $|\omega^* J| < 1$ holds) when the amount of model uncertainty becomes extreme. Next, we illustrate how vastly different the robust and non-robust portfolio allocations can become when uncertainty about the expected return on the risky asset increases. Suppose that the true excess return $R^*$ and jump intensity $\lambda^*$ are both unknown to the investor. As uncertainty $\eta$ increases, so does the potential expected return range of the risky asset.\(^7\) A non-robust investor can potentially pick any portfolio weight within the (wide) range between the upper and lower bounds represented by the dashed lines in Figure 3: effectively, a non-robust investor could choose any value from shorting the asset to leveraging heavily into the risky asset as implied by the optimal portfolio allocations based on the bounds $(R_*, \lambda)$ and $(\bar{R}, \bar{\lambda})$. In other words, the non-robust portfolio allocations under linearly increasing uncertainty can deviate substantially from the optimal portfolio weight had the true parameters $(R^*, \lambda^*)$ been known. By contrast, using the closed form portfolio weights in (45), we see in the same figure that the resulting optimal robust investment in the risky asset remains quite stable as $\eta$ increases.

We also see in Figure 3 that the robust investor is systematically more conservative than the non-robust robust investor in the hypothetical full information scenario where $(R^*, \lambda^*)$ are known: this is consistent with the results obtained in the purely diffusive case (see Ceria and Stubbs (2006) or Scherer (2007)). Moreover, comparing the case when the investor has full confidence in the jump intensity specification (i.e. $\zeta_D = 1$) to the other extreme case where the investor has full confidence in the drift specification (i.e. $\zeta_D = 0$), the figure shows that the optimal amount invested into the risky asset is reduced more in the former compared to the latter case (comparing the solid red to dashed blue line). More specifically, whereas in the former case, the share of wealth invested into the risky asset drops only slightly from 66% to 65% as $\eta$ increases, it falls from 66% to 35% in the latter case. The presence of a jump risk premium partially compensates the impact of robust decision making with respect to diffusive uncertainty.\(^8\) Indeed the optimal portfolio weight, in the case the investor is concerned about both drift and jump intensity misspecification ($\zeta_D = 0.5$), declines less in uncertainty compared to the case where the investor is only concerned about potential drift misspecification of his model (i.e. $\zeta_D = 0$).

### 3.2 Further Models with Closed-Form Solutions

Before analyzing further the properties of the optimal robust policies, we note that there exist other situations which lead to fully explicit portfolio weights. For CRRA preferences with $\gamma = 3$ or 4, the

\[^7\]We capture the relationship between the expected return and uncertainty in $\eta$ using a first order Taylor expansion of the drift in (4), so that the expected return increases (decreases) linearly in $\eta$.

\[^8\]Our analysis shows, that the optimal allocation in the risky asset is higher when the stock price follows a compensated exponential jump diffusive process, compared to the case when the risky asset follows its non compensated counterpart.
first order conditions for optimal portfolio holdings lead to a cubic (resp. quartic) equation, which given that $|wJ| < 1$, is solvable in closed form using standard methods. Using the same dynamics of the stock price process as in (4), we can derive fully explicit portfolio weights if we assume that the jump sizes are uniformly distributed on $(0, 1)$. An interesting example is when the Poisson process is not compensated, meaning $Y_t = \sum_{n=1}^{N_t} Z_n$ and the i.i.d. jumps $Z_n$ have symmetric Lévy measure about zero, i.e.

$$\lambda_\nu(dz) = \begin{cases} 
\lambda dz/z & \text{if } z \in (0, 1], \\
-\lambda dz/z & \text{if } z \in [-1, 0)
\end{cases}$$

so that the underlying asset exhibits both positive and negative jumps (see Appendix B.1 for a discussion). The quantitative behavior of the optimal portfolio weights is very similar to the compensated Lévy case.

Another interesting example is given when the jumps are negative (no positive jumps) and we do not compensate the Lévy - process $S_t$. In this case, the portfolio weights are more sensitive to jump intensity perturbation, meaning the fraction of wealth invested into the risky asset is not only lower, but also decreases in $\eta_J$ substantially faster compared to the compensated Lévy case.

Lastly, in Appendix B.2 we solve the robust consumption and investment problem when the investor has exponential utility.

### 3.3 Optimal Consumption Policy under Uncertainty

Having obtained explicit portfolio weights, we now investigate the impact of uncertainty on the investor’s optimal consumption policy. Recursive utility allow us to study the investor’s preferences for the timing of resolution of uncertainty, as risk aversion and intertemporal substitution of consumption are disentangled. For this reason, we focus our discussion of the investor’s optimal consumption policy as given in (39) first, around the level of his elasticity of intertemporal substitution, and secondly, we compare how drift vs jump intensity misspecification affect the investor’s consumption behavior. More precisely, since the investors’ wealth is also affected by the optimal portfolio policy $\omega^*(h^*, a^*)$, we plot in Figure 4 the consumption to wealth ratio which is constant and given by $K^*$ (see (39)).

A first conclusion that can be drawn from Figure 4 is that regardless of the level of EIS and the type of model misspecification we consider, the investor consumes less if there is uncertainty. This follows because under the robust measure, the investor becomes more pessimistic about his future wealth prospects and therefore optimally reduces consumption. Furthermore, from Panel A we see that relative to an investor with CRRA utility, an investor with preferences for early resolution of uncertainty consumes less no matter how large uncertainty $\eta$ is. This result is intuitive, as a consumer with preference for early resolution of uncertainty cares more about future fluctuations of his wealth (risk aversion) than substitution of consumption across time and therefore optimally chooses to
decrease current consumption. On the other hand, if the EIS effect dominates the risk aversion effect, the consumer cares more about current than future consumption and therefore optimally chooses to increase current consumption relative to an investor with time and state separable CRRA utility. Moreover, Panel B shows that the optimal consumption-wealth ratio is relatively insensitive with respect to uncertainty if there are only concerns about jump intensity misspecification.

Contrarily, in the other extreme case, where only the drift $R$ is unknown, the agents’ consumption-wealth ratio is significantly reduced when uncertainty increases. In the more realistic case where the investor does neither know the true expected return $R$ nor the true jump intensity $\lambda$, the Figure shows that the optimal consumption-wealth ratio exhibits intermediate sensitivity to uncertainty. However, the decline in the consumption-wealth ratio appears to be closer to the reduction in the consumption-wealth ratio where only the drift is unknown, i.e. $\zeta^D = 1$, compared to decline in the consumption-wealth ratio when only the jump intensity is unknown, i.e. $\zeta^D = 0$. This differential sensitivity of optimal the consumption-wealth ratio with respect to the type of uncertainty can be attributed to the findings in Figure 2 where we have carried out a similar study varying $\zeta^D$. As discussed there, since we compensate the jump process and therefore introduce a trade off between higher frequency of jumps occurring and higher expected return (through the compensator), the portfolio weights become less sensitive to jump intensity misspecification. As Panel B in Figure 4 reveals, a similar conclusion can be drawn for the optimal consumption-wealth ratio.

In light of these findings, it appears to be the case that our framework not only allows us to address the criticism of robust portfolio weights being too conservative (at least in the diffusive case), but also that robust consumption (relative to wealth) is less conservative compared to the diffusive case where there is only drift uncertainty. To put these results into perspective, we can consider a hypothetical scenario where the agent knows the true expected return $R^*$ and jump intensity $\lambda^*$ in order to quantify the impact of uncertainty on the agent’s optimal consumption-wealth ratio. The full information case corresponds to a consumption-wealth level of 1.6915 as shown by the dashed dotted purple line in Figure 4. If there is uncertainty (i.e. $\eta = 0.015$), the optimal consumption-wealth ratio falls to 1.6777 in the case where only the jump intensity is unknown, it drops to 1.1778 if both the expected return and jump intensity are unknown and it falls to 1.0654 in the special case when only the expected return of the risky asset is unknown. These numbers show that the agent incurs a loss in terms of consumption as a fraction of wealth due to uncertainty.
4  Sensitivity Analysis of Optimal Portfolio Weights and Implications

4.1 Comparison with Optimal Non-Robust Policies

Given the explicit portfolio weights derived in the previous section, we can now analyze their sensitivity with respect to both the asset price parameters \( R, \sigma, J, \lambda \) of the model and the entropy growth parameters \( \eta^D \) and \( \eta^J \). We start our analysis by comparing \( \omega^* \) in (45) to the classical Merton solution with and without robustness concerns, which can be obtained by letting \( \lambda \to 0 \) above. In this case the risks are only diffusive, and since we have set \( \gamma = 2 \) the optimal portfolio holding \( \omega^* \) reduces to

\[
\omega^*_{RM} = \frac{R - \sqrt{2\eta^D \sigma}}{2\sigma^2}
\]

where the subscript RM refers to the robust Merton portfolio allocation. The standard Merton policy is given by

\[
\omega^*_M = \frac{R}{2\sigma}
\]

obtained by setting \( \eta^D = 0 \) in (48).

From (48), one can see that an investor who is concerned about potential model misspecification will always invest less into the risky asset than an investor who is not. Next, going back to the robust jump diffusive weights in (45), for any \( \lambda > 0 \), we have that \( \omega^* = 0 \) whenever \( R = \sqrt{2\eta^D \sigma} \). This is intuitive as when the expected return under the robust measure is zero and since \( \mathbb{E}_t^\vartheta[\tilde{Y}_t] = 0 \), the investor has no benefit in allocating funds into the risky asset. Note that

\[
\frac{\partial \omega^*}{\partial R} = \frac{1}{4\sigma^2} (1 + F(\eta)) > 0, \quad F(\cdot) := \frac{h_1(\cdot)}{H(\cdot)}: \eta \to (-1, 1).
\]

from which we see that the inequality in (50) will always be satisfied under both measures. Thus increasing the expected return will lead to an increase of investment into the risky asset. Next, the limit of the optimal robust jump diffusive weights when volatility tends to infinity is 0, i.e.

\[
\lim_{\sigma \to \infty} \omega^*(h^*, a^*) = 0
\]

The rate at which the portfolio weights approach zero is affected by the amount of drift perturbation \( \eta^D \) in the following way,

\[
\omega^* |_{\sigma \to \infty} \sim -\sqrt{\frac{\eta^D}{2\sigma^2}} + O\left(\frac{1}{\sigma^2}\right)
\]

suggesting that the rate at which the portfolio weights approach zero is lower whenever there is doubt about the drift specification of the model. Furthermore, the partial derivative of \( \omega^* \) with respect to the jump intensity \( \lambda \) is

\[
\frac{\partial \omega^*}{\partial \lambda} = \frac{J \left( e^{1+W\left(\frac{\eta^J - \lambda}{e\lambda}\right)} - 1 \right) (h_2(\eta) + H(\eta) - 4\sigma^2)}{4\sigma^2H \left( 1 + W\left(\frac{\eta^J - \lambda}{e\lambda}\right) \right)} < 0
\]
which implies that the investor will optimally reduce his share invested into the risky asset whenever the likelihood of jumps occurring increases. This observation might at first sight seem to be intuitive, but since we are compensating the jump component, an increase in jump intensity not only leads to more jumps, but also increases the jump risk premium which raises the expected return.

Furthermore, for $\lambda \to \infty$ the optimal portfolio weight approach zero at the following rate

$$\omega^*|_{\lambda \to \infty} \sim \frac{1}{\lambda} \frac{R - \sqrt{2D\sigma}}{f^2} e^{-\lambda W} + O \left( \frac{1}{\lambda^2} \right)$$

(53)

Thus, whenever $R - \sqrt{2D\sigma} > 0$, $\omega^* > 0$ the optimal portfolio weight approaches zero from positive territory and vice versa when $R - \sqrt{2D\sigma} < 0$. Further, under the robust measure the rate is lower due to both drift (downward level shift) and jump intensity perturbation (downward scaling).

4.2 Sensitivity to the Desired Amount of Robustness

Next, we analyze the sensitivity of the optimal robust portfolio weights with respect to $\eta$ in detail.

**Proposition 1.** (Monotonicity of Optimal Portfolio Weights with respect to Uncertainty) The partial derivative of optimal portfolio weights $\omega^*$ with respect to uncertainty $\eta$ is negative for any $\eta > 0$. Specifically,

$$\frac{\partial \omega^*}{\partial \eta} = -\left( \frac{\sqrt{\zeta^D}}{4\sqrt{2\eta\sigma}} + \frac{J (1 - \zeta^D)}{4\sigma^2 \left( 1 + W \left( \frac{\eta^D - \lambda}{e\lambda} \right) \right)} \right) (1 + F(\eta)) + \frac{J (1 - \zeta^D)}{\left( 1 + W \left( \frac{\eta^D - \lambda}{e\lambda} \right) \right) H(\eta)} < 0$$

(54)

where $F(\cdot) = \frac{h_2(\cdot)}{H(\cdot)} : \eta \to (-1, 1)$ and $\zeta^D \in [0, 1]$. 

**Proof.** See Appendix C.

Proposition 1 implies that the investor will optimally hold less of the risky asset when uncertainty is introduced despite the higher jump risk premium. Additionally, evaluating (54) above for $\eta = 0$, we can infer using L’Hospital’s rule that

$$\frac{\partial \omega^*}{\partial \eta} \bigg|_{\eta=0} = -\infty$$

(55)

which implies that optimal portfolio weights are highly sensitive when robustness concerns are first introduced into the reference model. Furthermore, one can show that

$$\frac{\partial \omega^*}{\partial \eta} \bigg|_{\eta=\infty} = 0$$

(56)

which implies that the marginal impact of increasing model uncertainty diminishes as $\eta$ increases.

In the next figure, we plot the robust jump diffusive and the jump diffusive portfolio weights for a wide range of the parameters $R, \sigma, J$ and $\lambda$. Figure 5 show that, when robustness is sought after,
the amount invested into the risky asset is always lower than when the investor has full confidence in his reference model. From Panel A we see that increasing volatility of the risky asset leads the investor to optimally decrease $\omega^*(h^*, a^*)$ and even short the asset whenever $R$ is sufficiently low. Panel B shows that when both $J$ and $\lambda$ are high, then the investor simply allocates all his funds into the risk free asset. Thus, the model can capture the well-documented empirical flight-to-quality behavior when jump risk $\lambda$ is high (or when jump sizes are large, i.e., $J$ approaches $-1$). However, contrary to the case when jumps are only negative and the process is not compensated which leads the investor to optimally short the risky asset, i.e. $\omega^* \rightarrow -\infty$ as $\lambda \rightarrow \infty$ (as in the non-robust setting of Aït-Sahalia et al. (2009)), now the robust investor does not short the risky asset whenever $J$ and $\lambda$ are sufficiently high, but instead optimally chooses $\omega^* = 0$.

Finally, if either the jump scaling parameter or the intensity approach zero, i.e., the risky assets dynamics converges to a purely diffusive process, $\omega^*(h^*, a^*)$ increases non-linearly and the gap between the robust and non-robust portfolio weights widens. This suggests that for low jump risks $\lambda$ decreasing and simultaneously low jump size scaling $J \rightarrow 0$, perturbations of the reference model have a higher impact on $\omega^*(h^*, a^*)$, meaning that there is a significant drop in the amount invested into the risky asset, compared to the case when both $J$ and $\lambda$ are high.

5 Detection-Error Probability for Lévy Jump-Diffusive Processes

The robust portfolio weights derived in the previous sections crucially depend on the amount of model uncertainty $\eta$ we allow for. How should $\eta$ be chosen? In order to quantify how much uncertainty seems reasonable to the investor, we make use of detection-error probabilities as suggested by Anderson et al. (2003). More formally, let $\xi_t := \log (\vartheta_t)$ and $\mathcal{F}_t$ be the filtration with respect to which the probabilities and expectations are conditioned, the detection-error probability $\pi(t, T; \eta)$ is defined as the conditional probability at time $t$ of making a detection error given a sample of length $T > 0$,

$$\pi(t, T; \eta) = \frac{1}{2} \left( \mathbb{P} [\xi_T > 0 | \mathcal{F}_t] + \mathbb{P}^\vartheta [\xi_T < 0 | \mathcal{F}_t] \right), \quad 0 \leq t \leq T. \quad (57)$$

Therefore, as $\eta$ increases, so does the set of admissible models for the risky asset under $\mathbb{P}$ and $\mathbb{P}^\vartheta$, thereby causing the detection-error probability to decrease towards zero. Thus the larger $\eta$, the easier statistical discrimination between the model dynamics under $\mathbb{P}$ and $\mathbb{P}^\vartheta$ becomes. Anderson et al. (2003) suggest to choose $\eta$ such that the detection-error probability is at least 10%. Note that when $\pi(t, T; \eta) = 0.5$ the models are statistically indistinguishable.

We now derive an expression for the conditional probabilities in (57) by means of Fourier transformation of the conditional probability measures. The conditional characteristic functions of $\xi_T$, under the reference measure $\mathbb{P}$, denoted by $\phi_{\mathbb{P}}(u, t, T)$ and $\phi_{\mathbb{P}^\vartheta}(u, t, T)$ under the robust measure $\mathbb{P}^\vartheta$ are given by

$$\phi_{\mathbb{P}}(u, t, T) = \mathbb{E}^\mathbb{P} \left[ e^{iu\xi_T | \mathcal{F}_t} \right] = \mathbb{E}^\mathbb{P} [\vartheta_T^{iu} | \mathcal{F}_t]$$

(58)
\[
\phi_P(u, t, T) = \mathbb{E}^P \left[ e^{iu\xi_T} | \mathcal{F}_t \right] = \mathbb{E}^P \left[ \vartheta^{iu}_T | \mathcal{F}_t \right] \tag{59}
\]

where \( i = \sqrt{-1} \) and \( u \in \mathbb{R} \) is the transform variable. Using a simple measure change of the form

\[
\phi_P(u, t, T) = \int_{\omega \in \Omega} \vartheta^{iu}_T d\mathbb{P}^\omega(\omega) = \mathbb{E}^P \left[ \vartheta^{1+iu}_T | \mathcal{F}_t \right] \tag{60}
\]

the Fourier transform under the robust measure can be obtained by integrating with respect to the reference measure. By an application of Feynman-Kac’s theorem, we can compute the conditional expectations (58) and (60) by solving a partial differential difference equation (PDDE) with appropriate boundary conditions. If we were to also perturb the jump sizes, the measure change would satisfy a partial integral-differential equation.

In order to derive this PDDE for both conditional Fourier transforms we need the dynamics of the measure change under \( P \) given the optimal perturbation policies \( h_t^* \) and \( a^* \). An application of Itô’s product formula for semimartingales to the optimally perturbed \( \vartheta_t^* \) shows that

\[
\vartheta_t^* = \vartheta_0^* + \int_0^t h_s^* dB_s + \int_0^t \left( \frac{1}{2} h_s^* + \lambda \left( e^{a^*} - 1 \right) \right) d\tilde{N}_s - \int_0^t a^* \tilde{N}_s dt
\]

Then applying Itô’s formula to (9) and using (35) we obtain

\[
0 = \mathcal{A} \phi_P(u, t, T) = \phi_P(u, t, T) - \frac{\partial^2 \phi_P}{\partial \xi^2} h_t^* + \lambda (\vartheta_P(u, t, T) - \vartheta_P(u, t-, T)) \tag{63}
\]

subject to the following boundary condition

\[
\vartheta_P(u, T, T) = \vartheta_T^{iu} \tag{64}
\]

Likewise the PDDE for \( \phi_P(u, t, T) = \phi_P(u, t, T) \) is equivalent and given by

\[
0 = \mathcal{A} \phi_P(u, t, T)
\]

23
\[
\frac{\partial \phi_{\mathbb{P}^{\vartheta}}}{\partial t} - \frac{\partial \phi_{\mathbb{P}^{\vartheta}}}{\partial \xi} \left( \frac{1}{2} h_t^2 + \lambda \left( e^{\alpha^*} - 1 \right) \right) + \frac{1}{2} \frac{\partial^2 \phi_{\mathbb{P}^{\vartheta}}}{\partial \xi^2} h_t^2 + \lambda \left( \phi_{\mathbb{P}^{\vartheta}}(u, t, T) - \phi_{\mathbb{P}^{\vartheta}}(u, t, -T) \right) = 0
\]

(65)

subject to a different boundary condition which is given by

\[
\phi_{\mathbb{P}^{\vartheta}}(u, T, T) = \vartheta + iu.
\]

The PDDE in (65) admits an unique exponential affine solution of the form

\[
\phi_{\mathbb{P}^{\vartheta}}(u, t, T) = e^{\alpha(T-t) + \beta(T-t)\xi_t}
\]

(66)

where \( \alpha(\cdot) \) and \( \beta(\cdot) \) are both deterministic functions of time to maturity \( \tau = T - t \). Inserting the conjecture of (66) into (63) gives

\[
0 = -\alpha' - \beta' \xi_t - \beta \left( \frac{1}{2} h_t^2 + \lambda \left( e^{\alpha^*} - 1 \right) \right) + \frac{1}{2} h_t^2 \beta^2 + \lambda \left( e^{\beta a^*} - 1 \right).
\]

Using the fact that this equation has to hold for all \( \xi_t \) and the constant terms we get two equations, namely

\[
\beta(T-t) = \beta = \int_t^T \beta' = K \rightarrow K = iu = \beta
\]

(67)

\[
\alpha(T-t) = \int_t^T \alpha' = \left[ -\beta \left( \frac{1}{2} h_t^2 + \lambda \left( e^{\alpha^*} - 1 \right) \right) + \frac{1}{2} h_t^2 \beta^2 + \lambda \left( e^{\beta a^*} - 1 \right) \right] (T-t)
\]

(68)

where the expression for \( \beta \) in (67) follows from the boundary condition in (64) and \( \alpha(0) = 0 \). The solution for (65) is identical except that \( \beta = 1 + iu \). The following Proposition summarizes the semi closed-form solution to compute the detection-error probability.

**Proposition 2.** (Detection-Error Probability) Given conditional characteristic functions \( \phi_{\mathbb{P}^{\vartheta}}(u, t, T) \) and \( \phi_{\mathbb{P}}(u, t, T) \), the detection-error probability in (57) based on a sample of length \( T - t \) is given by

\[
\pi(t, T; \eta) = \frac{1}{2} - \frac{1}{2\pi} \int_0^\infty \left( \text{Re} \left[ \phi_{\mathbb{P}^{\vartheta}}(u, 0, T) \right] - \text{Re} \left[ \phi_{\mathbb{P}}(u, 0, T) \right] \right) du
\]

(69)

where \( \text{Re}(\cdot) \) denotes the real part of a complex number.

**Proof.** See Appendix D. \( \square \)

In Figure 6, we plot the detection-error probability as a function of the robustness parameter \( \eta \) using monthly and weekly sampled data points over a time period of one year \( T = 1 \).

As expected, \( \pi(t, T; \eta) \) is monotonously decreasing in \( \eta \), which is intuitive as when the set of alternative models increases, the dynamics of the process under \( \mathbb{P} \) becomes more distinct from the dynamics of the process under \( \mathbb{P}^{\vartheta} \). Furthermore, the two graphs in Figure 6 imply that making an error in classifying whether a given time series was generated under the reference or the robust measure is smaller in the case when we have weekly data available as opposed to only monthly data. This is due to the fact that it becomes increasingly easier to distinguish between two stochastic processes (one under the reference and the other under the robust measure), the longer the available time series is. The same conclusion can be drawn when increasing the jump intensity parameter \( \lambda \).
6 Analyzing the Impact of Uncertainty

In the following sections we investigate the impact of drift and jump uncertainty on the investor’s utility by computing certainty equivalents of wealth. In order to do so, we select a set of benchmark parameters we will later on use to quantify the maximum amount of model uncertainty which will crucially impact the optimal consumption and portfolio policies as well as the certainty equivalent comparison.

6.1 Model Parameters and Calibration of Detection Error Probability

The benchmark parameters for our study, as well as the alternative parameters, which we use for comparative statics, are given in Table 1. In order to fully specify the dynamics under the robust measure $\mathbb{P}^\theta$ we have to quantify the maximal amount of robustness $\eta = \eta^D + \eta^J$. As discussed above, we choose $\eta$ such that the target detection-error probability is $\bar{\pi} = 10\%$ which is the value suggested by Anderson et al. (2003), i.e.

$$\arg \min_{\eta \in (0, \infty)} ||\pi(t, T_d; \eta) - \bar{\pi}||, \quad 0 \leq t \leq T_d.$$ 

where $\pi(t, T_d; \eta) = \frac{1}{2} (\mathbb{P}[\xi_T > 0|\mathcal{F}_t] + \mathbb{P}^\theta[\xi_T < 0|\mathcal{F}_t])$ and $T_d$ is the time series length. We also vary $\eta^D$ as well as $\eta^J$ in order to quantify the effect of changing the absolute level of drift against jump intensity perturbation. In the extreme case when $\eta^D = 0$ ($\eta^J = 0$), the investor has full confidence in the drift (jump intensity) estimate of the stock price process. Furthermore, we vary the time series length where we assume 252 days of trading days per year, the investor has at his disposal to quantify the maximum amount of model uncertainty $\eta$. Lastly, we specify two jump size distributions, namely the power law in (47) and uniform jumps on $(0,1)$ for which fully closed form solutions of the portfolio weights are obtainable as we showed above. In the case when jumps are uniformly distributed on $(0,1)$, fully closed form portfolio weights can be obtained for risk aversion $\gamma = 3, 4$.

6.2 Wealth Loss Due to Uncertainty: A Certainty Equivalent Comparison

In this section, we quantify the wealth loss the agent incurs due to not knowing the exact values of the excess return $R$ and jump intensity $\lambda$. As the notion of utility is only ordinal, we compute the certainty equivalent of wealth in order to give a sensible quantitative answer as to how much the investor would be willing to pay in order to use the correct robust jump diffusive policy $\omega^*(h^*, a^*)$ as given in (45). We compare four different portfolio strategies by computing their certainty equivalents. Those portfolio policies include, the robust jump diffusive portfolio policy $\omega_{JD}^*(h^*, a^*)$, its non robust counterpart $\omega_{JD}^*(h^* = 0, a^* = 0)$, the robust Merton solution (48) where $\eta^D = \eta$ and the standard Merton solution with $\eta = 0$ as given in (49).
In order to derive an expression for the certainty equivalent, recall that at the optimum, the continuation value of utility is equal to the value function \( V(X_t) \) evaluated at the optimal robust policies \( h^*, a^* \), asset allocation \( \omega^* \) and consumption policy \( C^* \). Then, the certainty equivalent \( m \) is the solution to

\[
V_{r,JD}((1 - m)X_t) = V_i(X_t),
\]

(70)

where \( V_{r,JD}(X_t) \) and \( V_i(X_t) \) denote the value function when using the optimal robust jump diffusive solution \( \omega^*(h^*, a^*) \) and \( V_i(X_t) \) is the value function when using one of the suboptimal portfolio policies \( \omega^*_i, i \in \{M, rM, JD\} \). Using the value function guess in (A.1) and solving equation (70) for the certainty equivalent gives the following expression

\[
m = 1 - \left( \frac{K_{r,JD}}{K_i} \right)^{\frac{1}{1 - \psi}}, \quad i \in \{M, rM, JD\}
\]

(71)

where \( K_{r,JD} \) denotes the constant under the robust measure \( \mathbb{P}^\theta \) as given in (40) and \( K_i \) is the corresponding constant using the suboptimal strategies \( i \in \{M, rM, JD\} \). Notice that we have fixed \( \gamma = 2 \), from which it follows by the conditions given in (28) that the elasticity of inter temporal substitution \( \psi \) has to lie in the open interval \((0, 1)\). Furthermore, the constant \( K \) is globally concave in the portfolio weights \( \omega \) which implies that the optimal robust jump diffusive weight \( \omega^*(h^*, a^*) \) is the unique maximizer.

In other words, we must have that \( K_i < K_{r,JD} \) from which it follows by (71) that

\[
0 < m < 1, \quad \text{if} \ i \neq rJD
\]

(72)

Therefore, the certainty equivalent \( m \) gives the certain percentage of wealth the investor is willing to give up in order to use the optimal robust jump diffusive policy \( \omega^*(h^*, a^*) \). In Figure 7, we plot for each suboptimal portfolio policy \( i \in \{M, rM, JD\} \) its corresponding certainty equivalent of wealth.

A first conclusion that can be drawn from Figure 7 is that, in general, parameter uncertainty causes large wealth losses. For instance, in Panel A, the percentage of wealth the investor is willing to give up in order to use the correct robust jump diffusive portfolio policy ranges from 35% to 40% for \( \eta = 0.0132 \). However, the wealth loss varies considerably across the portfolio strategies and parameter values (see for instance Panel C or F when \( \zeta^D = 0 \)). Comparing Panels A, C and E where we vary the EIS, we can conclude that the investor is willing to give up more wealth (i.e. higher certainty equivalent) in the case where he has a preference for early resolution of uncertainty \( (\psi = 0.75) \) compared to the case when he has a preference for late resolution of uncertainty \( (\psi = 0.25) \) regardless which portfolio policy is being employed. Moreover the magnitudes of how much wealth the investor is willing to give up differ considerably depending on the portfolio policy applied. For instance, using the robust Merton solution shows that the certainty equivalent ranges somewhere between zero and 2.25%, which compared to the other portfolio policies (Merton and jump diffusive) is considerably
smaller. Moreover, increasing uncertainty does not necessarily imply that the certainty equivalent increases (see Panel C). Whereas the ranking of the certainty equivalents in the case we vary the investor’s EIS does not change (dotted blue line is always above the dashed black and solid red line), in the case where we investigate different types of model uncertainty, we find very mixed results. For instance, considering only drift perturbation $\zeta^D = 1$, we find that under the Merton solution, the certainty equivalent is substantially larger than compared to the robust Merton solution.

6.3 Comparative Static Analysis

In this section, we vary all the remaining parameters of the model and report the optimal portfolio weights (for all the four portfolio strategies mentioned above), the optimal consumption-wealth ratio and the certainty equivalent. We start by varying the amount of data the investor has available for determining the optimal amount of uncertainty $\eta$. The reason why we consider differential time series lengths for quantifying the amount of maximal robustness and the investment horizon is due to the fact mentioned in Section 5 (see Figure 6), namely that statistical discrimination between two processes becomes easier the longer the available time series is.

Table 3 summarizes the results. Using daily data, the first column labeled 'Weights' shows that the standard optimal Merton weight $\omega^*_M$ is the highest holding of the risky asset, whereas its robust counterpart $\omega^*_rM$ has the lowest holding in the risky asset. This is a direct consequence of setting $\eta^D = \eta$ in which case the maximum amount of model uncertainty concerns only the drift part and therefore lowers to excess return considerably (see column 'Drift'). The optimal jump diffusive and robust jump diffusive weights lie in between the two Merton weights. Furthermore, the column $C_t/X_t$ shows that the consumption - wealth ratio is lowest for the standard Merton portfolio strategy, which translates into a relatively high share of wealth the investor is willing to give up in order to have access to the correct robust jump diffusive policy (about 20%, see column labeled 'CE'). Comparing the drift of the risky asset for the two diffusive solutions Merton and robust Merton (no jumps), against the drift of the risky asset in the jump diffusive framework we observe that, under uncertainty seems lower in the latter compared to the former case. This can be explained as follows: since we are compensating the negative jumps, a higher jump intensity not only increases the likelihood of jumps occurring, but also raises the expected return as the compensator adds a jump risk premium to the drift of the stock price. Following this line of argumentation, this implies that under the robust measure, the increase in jump intensity (7.6%) partially offsets the negative impact of the diffusive adjustment $h^* = -\sqrt{2\eta^D}$, due to the larger jump risk premium. This leads to a lower decline of the drift of the risky assets and therefore explains the relatively higher optimal robust jump-diffusive portfolio weight compared to the robust Merton weight.

The bottom part of the table shows the results in the case the investor has fewer data points available to assess the maximal amount of model uncertainty. Considering weekly and daily data
and evaluate (70) such that the detection-error probability is 10%, we find that $\eta = 0.0132$ and $\eta = 0.0645$, which corresponds to a 4.9-fold increase in the amount of model uncertainty. Furthermore, comparing the optimal drift $h^*$ and jump intensity $a^*$ perturbation parameters for the two time series lengths we find that $h^* = 0.075$ and $a^* = 0.073$, if we use daily data, as opposed to $h^* = 0.254$ and $a^* = 0.235$ if we use weekly data. Hence, if the investor has fewer data points available, the estimated maximal amount of model uncertainty is considerably higher, as it is more difficult two distinguish between two realized sample paths obtained over a shorter time period. For precisely this reason, the steep reduction in portfolio holdings of the robust Merton and robust jump-diffusive solution in the case the investor has only weekly data available is not surprising.

In conclusion, model uncertainty is higher when the investor has fewer data points available which translates into more conservative portfolio holdings (compare Merton and robust Merton, or jump-diffusive and robust jump diffusive portfolio holdings), reduced consumption-wealth ratios and higher wealth loss as measured by the certainty equivalent.

The last part of this section is devoted to further comparative static analysis with respect to the remaining parameters of the model. In Table 3 we summarize the results. Comparing the optimal portfolio weights, we see that their ordering is preserved regardless of which market scenario we consider. More precisely, the share of wealth invested into the risky asset is always highest for the Merton case, second and third highest for the jump-diffusive and robust jump-diffusive solution, respectively and lowest for the robust Merton solution. The same conclusion can be drawn for the optimal consumption-wealth ratio and certainty equivalents. In light of the optimal consumption policy given in (38), this result is not that surprising as the consumption-wealth ratio is directly affected by the optimal portfolio choice.

Furthermore, there seems to be considerable variation as to how much the investor is willing to give up of his wealth in order to use the correct robust jump-diffusive portfolio policy. For instance, in the case when jump risk is very high (see $\lambda = 5$), we see that the investor is willing to forgo almost 80% of his wealth (78.4%) if he is currently employing a Merton portfolio allocation strategy and only 0.2% if he is using the robust Merton portfolio policy. In general, comparing the certainty equivalents across market scenarios we can conclude that they are low for the case the investor is allocating his wealth according to a robust Merton portfolio policy, and they are high in the case the investor is using the standard Merton solution. This result is not surprising as the robust Merton solution is comparatively close to the optimal robust jump-diffusive solution regardless of which market scenario we consider. However, this result changes if we vary the composition of model uncertainty, as measured by the parameter $\zeta^D$. Lastly, comparing the optimal jump-diffusive solution to its robust counterpart, we can conclude that the amount invested into the risky asset is about half in the latter compared the former case. The certainty equivalent for the optimal jump-diffusive

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9This is about equivalent to $252/52 \approx 5$, i.e. the number of daily compared to weekly data points.
solution indicates that the investor is willing to give up between 9.3% to 28.3% of his wealth to switch to the optimal robust jump-diffusive portfolio policy.

7 Conclusion

In this paper, we study a robust optimal consumption and portfolio choice problem where the underlying risky asset follows a Lévy process. We introduce model misspecification concerning the drift and jump intensity parameters and derive explicit expressions for optimal consumption and portfolio rules. Additionally, we introduce an intensity misspecification premium which has important implications for robust portfolio allocations. This is achieved by simply compensating the jump component of the exponential Lévy model we employ in this paper. Such a model assumption implies, that under the robust measure, the drift of the risky asset is reduced under the robust measure, which is a standard result when doing asset pricing with the robust control. However, the reduction is smaller as through compensating the jump component, we add a jump intensity misspecification premium that increases the drift of the risky asset. A direct consequence of this assumption is that the optimal allocation in the risky asset is higher under the assumption that the stock price follows a compensated exponential jump diffusive process, compared to the case when the risky asset follows a non compensated jump diffusive process which is the standard assumption on the robust dynamics of the process. This modeling specification helps to address the commonly stated critic that robust portfolio allocations are too conservative, meaning that the weight in the risky asset is too low, and therefore the investor forgoes much of the upside potential (for a discussion see Ceria and Stubbs (2006) or Scherer (2007)). Furthermore, the addition of a jump compensator to the drift adds more realism to the model as in times of higher uncertainty, stocks become more risky but offer a higher risk compensation in order to induce the investor to hold them.

Additionally, we derive a semi-closed form solution for detection-error probabilities. Our analysis shows that the detection-error probability is very sensitive to the length of data we have available, which is consistent with the fact that for two stochastic processes, one generated under the reference and another one generated under the robust measure, it becomes easier to statistically discriminate one from another the longer the available time series of observations is.

The representative investor has recursive utility, which allows to study the differential impact of substitution of consumption across time and aversion to risk on the optimal consumption(-wealth) policy. We find that, if the investor exhibits preference for early resolution of uncertainty, the optimal consumption-wealth ratio is lower regardless of the level of model uncertainty, compared to an investor with standard CRRA preferences. Moreover, we show that model uncertainty leads to significant wealth loss in certainty equivalent terms. In particular, if the agent has preference for early resolution of uncertainty, this negative wealth effect is amplified as the fraction of wealth he
is willing to give up in order to have access to the correct, robust jump-diffusive portfolio strategy, increases.
References


Appendix A  Proof of Theorems

Appendix A.1  Proof of Theorem 1

We conjecture a solution of the form

\[ V(x) = \frac{K - \gamma}{1 - \gamma}, \quad \text{with} \quad \frac{\partial V(x)}{\partial x} = (1 - \gamma)V(x)/x, \quad \frac{\partial^2 V(x)}{\partial x^2} = -\gamma(1 - \gamma)V(x)/x^2 \]  

(A.1)

for some constant \( K \). Then, after division by \((1 - \gamma)V(X_t)\) the HJB equation in (24) becomes

\[
0 = \max_{C_t,\omega_t} \min_{h_t,a} \left[ \frac{f(C_t,V_t)}{(1 - \gamma)V(X_t)} - \frac{\beta}{(1 - \gamma)} + \left[ r + \omega_t R + \sigma h_t \omega_t - \omega_t \lambda^\theta J \int_R z \nu(dz) - \frac{C_t}{X_t} \right] \right]
\]

\[
- \frac{1}{2} \gamma \omega_t^2 \sigma^2 + \frac{\lambda^\theta}{(1 - \gamma)} \int_R [(1 + \omega_t Jz)^{1 - \gamma} - 1] \nu(dz), \quad \text{subject to } \mathcal{R}(\theta_t) \leq \eta. \tag{A.2}
\]

Using the explicit expression of the entropy growth constraint in (37), we obtain the optimal perturbation parameters from Equation (29) through (31), which we explicitly solve for in Section 3.1. Given \( \{h_t^*,a^*\} \) and \( \theta_t^* \), \( R + \sigma h_t \omega_t - (1/2) \gamma \omega_t^2 \sigma^2 \) and \( (\lambda^\theta/(1 - \gamma)) \int_R [(1 + \omega_t Jz)^{1 - \gamma} - 1] \nu(dz) \) are both concave in \( \omega_t \), thus any solution to (A.2) will always have a unique maximizer. Note that if we were to allow for a jump size perturbation, the investor would additionally need to decide upon the optimal jump size perturbation \( b^* \). This type of perturbation leads to highly-nonlinear first order conditions of both the jump size perturbation parameter \( b \) as well as for the optimal portfolio holdings which can only be resolved numerically.

The objective function is time independent. This follows because \( R, \sigma, J, \lambda \) and \( \eta \) are constant which implies that the optimal drift perturbation parameter \( h_t \) will also be independent of time, \( h_t = h_t, \forall t \geq 0 \). Furthermore, since

\[
\left[ R + \sigma h_t - \lambda^\theta J \int_R z \nu(dz) \right] \omega_t - \frac{\gamma \omega_t^2 \sigma^2}{2} + \frac{\lambda^\theta}{(1 - \gamma)} \int_R [(1 + \omega_t Jz)^{1 - \gamma} - 1] \nu(dz)
\]

does not depend on the investor’s time \( t \) wealth \( X_t \), the optimal portfolio share will not only be time- but also be state-independent, i. e. \( \omega^*(X_t,t) = \omega^*, \forall t \geq 0 \). On the other hand, the optimal portfolio allocation will of course be a function of the perturbation parameters \( \{h^*,a^*\} \), in other words, \( \omega^* = \omega^*(h^*,a^*) \).

Then, taking a partial derivative of the Lagrangian \( \mathcal{L} \) with respect to consumption gives the optimality condition in Equation (33). Lastly, to determine the constant \( K \) in (39), we evaluate (A.2) at the optimal policies \((C_t^*,\omega^*)\).

Appendix A.2  Proof of Theorem 2

Define as \( \mathcal{L}(\omega_t, h_t, \theta^D, \theta^J) \) the Lagrangian associated to the constraint HJB problem in (A.2) with constant Lagrange multipliers \( \theta^D \) and \( \theta^J \). Then taking \( \gamma = 2 \) we obtain

\[
\mathcal{L}(\omega, h, a, \theta^D, \theta^J) = \omega R + \sigma h \omega - \omega \lambda J e^a - \omega^2 \sigma^2 + \lambda e^a \log(1 + \omega J)
\]
The necessary first order optimality conditions for the robustness parameters \( h \) and \( a \) are

\[
\frac{\partial L(\omega, h, a, \tilde{\theta}^D, \tilde{\theta}^J)}{\partial h} = \sigma_\omega + \tilde{\theta}^D h = 0, \quad \rightarrow h^* = -\frac{\sigma_\omega}{\tilde{\theta}^D}, \quad \tilde{\theta}^D \geq 0
\] (A.4)

\[
\frac{\partial L(\omega, h, a, \tilde{\theta}^D, \tilde{\theta}^J)}{\partial a} = \lambda e^a \left( \lambda - \eta \lambda + \log (1 + J \omega) - \omega J \right) = 0, \quad \rightarrow a = \frac{\omega J - \log (1 + J \omega)}{\tilde{\theta}^J}, \quad \tilde{\theta}^J \geq 0
\] (A.5)

\[
\frac{\partial L(\omega, h, a, \tilde{\theta}^D, \tilde{\theta}^J)}{\partial \tilde{\theta}^D} = \eta^D - \frac{1}{2} h^2 = 0, \quad \rightarrow \tilde{\theta}^{D*} = \pm \sqrt{\frac{\sigma^2 \omega^2}{2 \eta^D}}
\] (A.7)

\[
\frac{\partial L(\omega, h, a, \tilde{\theta}^D, \tilde{\theta}^J)}{\partial \tilde{\theta}^J} = \eta^J - \lambda e^a (a - 1) - \lambda = 0, \quad \rightarrow \tilde{\theta}^{J*} = \frac{\omega J - \log (1 + J \omega)}{1 + W \left( \frac{\eta^J - \lambda}{e \lambda} \right)}, \quad \eta^J \geq \lambda, \lambda \in (0, \infty)
\] (A.8)

From the system of equations (A.4) to (A.8) we find that the optimal drift \( h^* \) and jump intensity \((a^*)\) perturbation parameters are given by

\[
h^* = -\sqrt{2 \eta^D}, \quad \forall \eta^D \geq 0, \quad a^* = 1 + W \left( \frac{\eta^J - \lambda}{e \lambda} \right) \geq 0, \quad \forall \eta^J \geq \lambda, \lambda \in (0, \infty)
\] (A.9)

The function \( W(\cdot, \lambda) \) is plotted in Figure 1. Furthermore, \( \lambda^\theta > \lambda, \forall \eta, \lambda > 0 \). Thus, the robust jump intensity under the robust measure \( \mathbb{P}^\theta \) is always higher than the jump intensity under the reference measure \( \mathbb{P} \). Since \( \lambda > 0 \), \( \lambda^\theta \) is increasing in \( \eta^J \)

\[
\frac{\partial W}{\partial \eta^J} = \frac{W \left( \frac{\eta^J - \lambda}{e \lambda} \right)}{(\eta^J - \lambda)(1 + W \left( \frac{\eta^J - \lambda}{e \lambda} \right))} > 0, \quad \forall \eta^J, \lambda > 0.
\]

This implies that the larger the set of potential models \( \eta^J \uparrow \) is we allow for, the higher the jump frequency under the robust measure becomes.

In order to make sure that the optimal robust control variables \( h^* \) and \( a^* \) are indeed (global) minimizers we need to check the second order optimality conditions. For the optimal drift perturbation parameter \( h^* \) we have

\[
\frac{\partial^2 L(\omega, h, a, \tilde{\theta}^D, \tilde{\theta}^J)}{\partial h^2} = \tilde{\theta}^D
\]

and thus we need \( \tilde{\theta}^D \geq 0 \) such that \( L(\omega, h, a, \tilde{\theta}^D, \tilde{\theta}^J) \) is convex in \( h \) and therefore the solution in (46) is indeed a (global) minimum. This implies \( \tilde{\theta}^D = \sqrt{\frac{\sigma^2 \omega^2}{2 \eta^D}} \geq 0 \) and therefore the second order Lagrange-conditions are satisfied for \( h^* \) and \( \tilde{\theta}^{D*} \).

Along the same line of argument, we have

\[
\tilde{\theta}^{J*} = \frac{\omega J - \log (1 + J^2 \omega)}{1 + W \left( \frac{\eta^J - \lambda}{e \lambda} \right)} \geq 0
\]
for any \( \omega \) satisfying the solvency constraint \( |\omega J| < 1 \) and \( 1 + W \left( \frac{(\eta^J - \lambda)}{e\lambda} \right) \in (0, \infty), \forall \lambda, \eta^J > 0 \), and

\[
\frac{\partial L^2(\omega, h^*, a^*, \tilde{\theta}^{D^*}, \tilde{\theta}^{J^*})}{\partial a^2} = e^a \tilde{\theta}^{J^*} \geq 0
\]

which shows that \( a^* \) is the global minimizer.

Next, using the optimal perturbation parameters \( h^* \) and \( a^* \) we can now solve for the optimal robust portfolio weights in closed form. Given \( L = L(\omega, h^*, a^*, \tilde{\theta}^{D^*}, \tilde{\theta}^{J^*}) \) in (A.3) the first order condition for \( \omega \) is given by

\[
\frac{\partial L}{\partial \omega} = \frac{1}{2} h^2 + \sigma^2 \omega^2 - \lambda J e \left( a - 1 + \frac{\eta^J}{e\lambda} \right) \log (1 + \omega J) = 0
\]

which is a quadratic polynomial in the portfolio weights \( \omega \) whose solution is given in Theorem 2 above in the text.

**Appendix B Extensions: Optimal Robust Portfolio Weights for Other Models**

**Appendix B.1 Joint Drift and Jump Intensity Perturbation with CRRA Utility and Symmetric Jumps**

In this section, we derive explicit portfolio weights when the risky asset follows an exponential \( \text{Lévy} \) process (no jump compensation) under the robust measure \( \mathbb{P}^\theta \) of the form

\[
\frac{dS_{1,t}}{S_{1,t-}} = (r + R + \sigma h_t)dt + \sigma dB^\theta_t + JdY^\theta_t, \quad S_{1,0} > 0,
\]

and jumps follow a symmetric power law distribution as in (47). Then the corresponding Lagrangian reads

\[
L(\omega, h, a, \tilde{\theta}^D, \tilde{\theta}^J) = \omega R + \sigma h \omega - \omega \lambda J e^a - \omega^2 \sigma^2 + \lambda e^a \log (1 + \omega J)
\]

\[
+ \tilde{\theta}^D \left( \frac{1}{2} h^2 - \eta^D \right) + \tilde{\theta}^J \left( \lambda e^a (a - 1) + \lambda - \eta^J \right).
\]

(B.10)

from which we obtain the following first order conditions

\[
\tilde{\theta}^{D^*} = \pm \sqrt{\frac{\sigma^2 \omega^2}{2\eta^D}} \rightarrow h^* = -\sqrt{2\eta^D}
\]

(B.11)

\[
\tilde{\theta}^{J^*} = -\log \left( 1 - J^2 \omega^2 \right) \rightarrow a^* = 1 + W \left( \frac{\eta^J - \lambda}{e\lambda} \right)
\]

(B.12)
Note that $\theta^J(\omega) \geq 0$ for any $\omega$ satisfying the solvency constraint $|\omega^*J| < 1$. Given the optimal perturbation parameters in (B.11) and (B.12), the objective function is

$$L(\omega, h^*, a^*, \tilde{\theta}^D, \tilde{\theta}^J) = \omega R - \sigma \omega \sqrt{2\eta^D} - \omega^2 \sigma^2 + \lambda e^{1+W(\frac{\omega^*J}{\epsilon\lambda})} \log(1 + \omega J)$$

$$+ \lambda e^{1+W(\frac{\omega^*J}{\epsilon\lambda})} \log(1 - \omega J)$$

(B.13)

and the first order condition for the optimal portfolio weight is

$$\frac{\partial L}{\partial \omega} = R - \sigma \sqrt{2\eta^D} - 2\omega \sigma^2 + \lambda e^{1+W(\frac{\omega^*J}{\epsilon\lambda})} J/(1 + \omega J)$$

$$- \lambda e^{1+W(\frac{\omega^*J}{\epsilon\lambda})} J/(1 - \omega J) = 0.$$  

(B.14)

So (B.14) is a cubic polynomial in the portfolio weights $\omega$. Let

$$A = 2J^2 \sigma^2, \quad B = J^2(\sqrt{2\eta^D} - R)$$

$$C^D = -2\left(\sigma^2 + J^2 \lambda e^{1+W(\frac{\omega^*J}{\epsilon\lambda})}\right), \quad D^D = R - \sqrt{2\eta^D}$$

Then (B.14) can be written as $Ax^3 + Bx^2 + Cx + D = 0$. Defining $a^D = B^D/A, b^D = C^D/A, c^D = D^D/A$ and $G^D = (a^D)^2 - 3b^D)/9, H^D = (2a^D)^2 - 9a^D b^D + 27c^D)/54$ the solution to (B.14 subject to the solvency condition $|J\omega| < 1$ is given by

$$\omega^* = -2\sqrt{G^D} \cos\left(\frac{1}{3}H^D(\sqrt{G^D} - \frac{2\pi}{3})\right) - a^D/3$$

(B.15)

**Appendix B.2 Exponential Utility: Closed-form portfolio weights with compensated exponential Lévy dynamics**

In this section we consider an investor with exponential utility of the form

$$U(C) = -\frac{1}{q} e^{-qC}, \quad \text{with CARA coefficient } q > 0$$

where his wealth dynamics under the robust measure evolves as in (14), that is no jump size perturbation. We analyze robust optimal portfolio holdings where the jumps sizes follow an exponential distribution, i.e. $Z \sim \text{Exp}(\xi)$, with $\nu(dz) = f_Z(z; \xi) = \xi e^{-z\xi}, z \geq 0$. Then conjecturing a solution of the form $L(x) = -K/q e^{-qx}$ where $r > 0$ is the risk free rate in (1) and $K$ some constant to be determined, we have

$$\frac{\partial L(x)}{\partial x} = -r q L(x), \quad \frac{\partial^2 L(x)}{\partial x^2} = r^2 q^2 L(x)$$

(B.16)

The corresponding robust control problem is given by

$$0 = \max_{\{C_t, h_t\}} \min_{\{\tilde{h}_t\}, a} U(C_t) - \beta L(X_t) - r q L(X_t) \left[ X_t \left( r + \omega_t \left( R + \sigma h_t - \lambda e^a J \int z(\nu(dz)) \right) \right) - C_t \right]$$

38
subject to
\[ \frac{1}{2} \sigma_t^2 \leq \eta^D, \quad \lambda e^a(a - 1) + \lambda \leq \eta^J, \quad \eta^D \geq 0, \quad \eta^J \geq 0 \] (B.18)

We fix \( J = (-1, 0), \) so that jumps are negative. Dividing (B.17) above by \(-rqL(X_t) > 0\) we obtain
\[ 0 = \max_{\{C_t, \omega_t\}} \min_{\{h_t\}, a} - \frac{U(C_t)}{rqL(X_t)} + \frac{\beta}{qr} + X_t \left( r + \omega_t \left( R + \sigma h_t + \lambda e^a \int z\nu(dz) \right) \right) - C_t \]
\[ - \frac{1}{2} r q \omega_t^2 X_t^2 \sigma^2 - \frac{\lambda e^a}{r q} \int \left[ e^{-r q w z} - 1 \right] \nu(dz) \] (B.19)

subject to
\[ \frac{1}{2} \sigma_t^2 \leq \eta^D, \quad \lambda e^a(a - 1) + \lambda \leq \eta^J, \quad \eta^D \geq 0, \quad \eta^J \geq 0 \] (B.20)

We define as \( \mathcal{L} = \mathcal{L}(C_t, h_t, a, \tilde{\theta}^D, \tilde{\theta}^J) \) the Lagrangian corresponding to the perturbed HJB problem in (B.17) with Lagrange multiplier \( \tilde{\theta}^D \) and \( \tilde{\theta}^J \) for the diffusive and jump intensity part of the entropy constraint respectively. Then we have\(^{10}\)
\[ \mathcal{L}(C_t, w, h_t, a, \tilde{\theta}^D, \tilde{\theta}^J) = - \frac{U(C_t)}{rqL(X_t)} + w \left( R + \sigma h_t + \lambda e^a \int z\nu(dz) \right) - C_t - \frac{1}{2} r q w^2 \sigma^2 \]
\[ - \frac{\lambda e^a}{r q} \int \left[ e^{-r q w z} - 1 \right] \nu(dz) + \tilde{\theta}^D \left( \frac{1}{2} \sigma_t^2 \omega_t^2 - \eta^D \right) + \tilde{\theta}^J \left( \lambda e^a(a - 1) + \lambda - \eta^J \right) \] (B.21)

where \( w = \omega_t X_t \) is the (absolute) amount of money invested into the risky asset. Given the optimal perturbation parameters, the objective function is
\[ \mathcal{L}(C_t, w, h^*, a^*, \tilde{\theta}^D^*, \tilde{\theta}^J^*) = - \frac{U(C_t)}{rqL(X_t)} + \frac{\beta}{qr} + X_t \left( r + \omega_t \left( R + h^* \sigma + \lambda e^{a^*} \int z\nu(dz) \right) \right) - C_t \]
\[ - \frac{1}{2} r q \omega^2 \sigma^2 - \frac{\lambda e^{a^*}}{r q} \int \left[ e^{-r q w z} - 1 \right] \nu(dz) \] (B.22)

Then optimal consumption given (B.22) is
\[ C_t^* = r X_t - \frac{1}{q} \ln(rK) \] (B.23)

Furthermore, in order to determine the constant we evaluate (B.22) and use optimal consumption \( C_t^* \) from above and the optimal robust portfolio holdings \( w(h^*, a^*) \). Next, the constant satisfies,
\[ K = \frac{1}{r} \exp \left\{ 1 - \frac{\beta}{r} - w \left( R + h^* \sigma + J \lambda e^{a^*} \int z\nu(dz) \right) + \frac{1}{2} r q^2 w^2 \sigma^2 + \frac{\lambda e^{a^*}}{r q} \int \left[ e^{-r q w z} - 1 \right] \nu(dz) \} \]

\(^{10}\)The integral with respect to the jump measure in B.19 is only convergent if \( \xi - qr w > 0 \) which is very likely to be satisfied given standard parameter values for risk aversion \( q \in \{1, \ldots, 10\} \), \( r \in [0, 0.05] \) and \( \xi > 10 \) which would correspond to a jump size of 10% or less.
Then, assuming that the jump sizes are exponentially distributed with parameter $\xi$, we obtain that the Lagrangian satisfies

$$
\mathcal{L}(C_t, w, h^*, a^*, \theta^{D*}, \tilde{\theta}^J) = -\frac{U(C_t)}{rqL(X_t)} + \frac{\beta}{qr} + X_t \left( r + \omega_t \left( R + h^* \sigma + \frac{\lambda e^a}{\xi} \right) \right) - C_t
$$

$$
-\frac{1}{2} q r \sigma^2 w^2 - \frac{\lambda e^a w}{\xi - qr w}
$$

(B.24)

As before, it follows that $h^*_t = h^* = -\sqrt{2qD^\xi}, \tilde{\theta}^D = \sqrt{\frac{q^2\sigma^2}{2qD^\xi}}$ and $a^* = 1 + W\left( \frac{\eta^2}{c\lambda} \right), \tilde{\theta}^J = \frac{qrw^2}{\xi(1+W(\frac{\eta^2}{c\lambda}))(\xi-qrw)} \geq 0, \forall w \in \mathbb{R}$. Then, the first order condition for the optimal portfolio weight $w^*$ is given by

$$
-w^* \left( \lambda e^a \left( \frac{1+W(\frac{\eta^2}{c\lambda})}{\xi \xi - qrw} \right) + \lambda q e^a \left( \frac{1+W(\frac{\eta^2}{c\lambda})}{\xi \xi - qrw} \right) - \frac{\lambda e^a w}{\xi - qrw} - \sqrt{2qD^\xi} - q r \sigma^2 w + R = 0
$$

(B.25)

which is a cubic polynomial in the amount invested in the risky asset. Let

$$
A(\xi) = -\xi q^2 r^2 \sigma^2, \quad B^D(\xi) = q^2 r^2 \left( \lambda e^a \left( \frac{1+W(\frac{\eta^2}{c\lambda})}{\xi \xi - qrw} \right) + \lambda q e^a \left( \frac{1+W(\frac{\eta^2}{c\lambda})}{\xi \xi - qrw} \right) - \frac{\lambda e^a w}{\xi - qrw} - \sqrt{2qD^\xi} - q r \sigma^2 w + R \right)
$$

$$
C^D(\xi) = -\xi q r \left( 2e^a \left( \frac{1+W(\frac{\eta^2}{c\lambda})}{\xi \xi - qrw} \right) + \lambda q e^a \left( \frac{1+W(\frac{\eta^2}{c\lambda})}{\xi \xi - qrw} \right) - \frac{\lambda e^a w}{\xi - qrw} - \sqrt{2qD^\xi} - q r \sigma^2 w + R \right)
$$

$$
D^D(\xi) = \xi \left( \sqrt{2qD^\xi} - R \right)
$$

Then (B.25) can be written as $A(\xi)x^3 + B^D(\xi)x^2 + C^D(\xi)x + D^D(\xi) = 0$. Next, we define $a^{D\xi}(\xi) = B^D(\xi)/A(\xi), b^{D\xi}(\xi) = C^D(\xi)/A(\xi), c^{D\xi}(\xi) = D^D(\xi)/A(\xi)$ and $G^{D\xi}(\xi) = \left( a^{D\xi}(\xi)^2 - 3b^{D\xi}(\xi) \right)/9, H^{D\xi}(\xi) = \left( 2a^{D\xi}(\xi)^2 - 9a^{D\xi}(\xi)b^{D\xi}(\xi) + 27c^{D\xi}(\xi) \right)/54$ such that the solution to (B.25) subject to the integral convergence condition $\xi - qrw > 0$ is given by

$$
w^*_E = -2\sqrt{G^{D\xi}(\xi)} \cos \left( \arccos \left( \frac{1}{3} H^{D\xi}(\xi)/\sqrt{G^{D\xi}(\xi)} - \frac{2\pi}{3} \right) \right) - a^{D\xi}(\xi)/3
$$

where $w^*_E$ refers to both drift and jump intensity compensated portfolio weights when the investor is assumed to have exponential utility. In Figure (8) we plot the portfolio weights for different levels of jump sizes $\xi$. A first important observation from Panel A to D is that increasing the size of the jumps ($\xi$ decreasing) reduces the absolute amount invested into the risky asset $w^*_E$ as $\mathbb{E}[\xi] = \mathbb{E}[\xi] = 1/\xi$. A tenfold increase in the coefficient of risk aversion ($q$) leads to a proportional decrease in $w^*_E$. On the other hand, increasing the frequency of jumps ($\lambda$) by a factor of ten, shows that the optimal investment into the risky asset decreases over-proportionally.

Appendix C  Proof of the monotonicity of optimal portfolio weights with respect to uncertainty

In this section, we prove Proposition 1, i.e., show that the derivative of the optimal portfolio weights with respect to $\eta$ is strictly negative for any $\eta \in (0, \infty)$. Let $f'(\eta) := \frac{\partial a^*}{\partial \eta}$ denote the partial derivative
of the optimal portfolio weights with respect to uncertainty where \( \eta \in (0, \tilde{\eta}) \) and \( \tilde{\eta} \) is some finite positive constant. Applying the mean value theorem, we have

\[
f' (\eta) = \frac{f'(\tilde{\eta}) - f'(0)}{\tilde{\eta}} = \frac{-1}{4J\sigma^2\tilde{\eta}} \left[ J^2\lambda \left( e^\frac{\tilde{\eta} - \lambda}{\epsilon x} - 1 \right) + J\sqrt{2\eta^D\sigma} - H(\tilde{\eta}) + H(0) \right]
\]  

(C.26)

In order to prove that the optimal portfolio weights are monotonically declining in uncertainty, we need to show that \( f'(\eta) < 0 \), \( \forall \eta \in (0, \tilde{\eta}) \). In other words, using (C.26) we have to verify the following inequality

\[
H(\tilde{\eta}) - J\sqrt{2\eta^D\sigma} > J^2\lambda \left( e^\frac{\tilde{\eta} - \lambda}{\epsilon x} - 1 \right) + H(0)
\]  

(C.27)

Since both sides of (C.27) are strictly positive we can square them and obtain

\[
0 < H(0)J^2\lambda \left( e^{1+W\left( \frac{\tilde{\eta} - \lambda}{\epsilon x} \right)} - 1 \right) - J\sqrt{2\eta^D\sigma} H(\tilde{\eta}) - \lambda^2J^2 \left( e^{1+W\left( \frac{\tilde{\eta} - \lambda}{\epsilon x} \right)} - 1 \right)^2
+ 2J^2\tilde{\eta}^D\sigma + 8\lambda J^2\sigma^2 \left( 1 + e^{1+W\left( \frac{\tilde{\eta} - \lambda}{\epsilon x} \right)} \right) + h_2^2(\tilde{\eta})
\]  

(C.28)

The only negative term in (C.28) is

\[
- \left( e^{1+W\left( \frac{\tilde{\eta} - \lambda}{\epsilon x} \right)} - 1 \right)^2 = -e^2\left( 1 + W\left( \frac{\tilde{\eta} - \lambda}{\epsilon x} \right) \right) + 2e^{1+W\left( \frac{\tilde{\eta} - \lambda}{\epsilon x} \right)} - 1
\]  

(C.29)

Because \( e^{1+W\left( \frac{\tilde{\eta} - \lambda}{\epsilon x} \right)} > 1 \), \( \forall \tilde{\eta} > 0 \), only the first term in (C.29) is negative. Recalling that \( h_2^2(\tilde{\eta}) \) contains the same term, i.e. \( e^2\left( 1 + W\left( \frac{\tilde{\eta} - \lambda}{\epsilon x} \right) \right) \) but with positive sign immediately implies that the inequality in (C.28) has to hold and therefore shows that the partial derivative of the optimal portfolio weights with respect to uncertainty is indeed negative, i.e. is monotonically declining in \( \eta \).

**Appendix D Derivation of Detection-Error Probability**

The formula for the detection-error probability in (57) shows that we have to compute two probabilities, namely \( P[\xi_T \geq 0|\mathcal{F}_t] \) and \( P^\theta [\xi_T \leq 0|\mathcal{F}_t] \). Let \( \xi := \xi_T \) be a continuous real valued random variable with support on the entire real line \( \mathbb{R} \). Let its cumulative distribution function be denoted \( F_\xi(\cdot) \). Let \( \phi_\xi(u) := \phi_\xi(u, t, T) \) denote the characteristic function under the measure \( P \) (see Equation (58)), the Gil-Palaez inversion theorem which is based on Lévy’s general inversion formula states that

\[
F_\xi(a) = P[\xi \leq a] = \frac{1}{2} - \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-iua}\phi_\xi(-u)}{iu} du, \quad a \in \mathbb{R}
\]  

(D.30)

Furthermore, the real and imaginary part of \( \phi_\xi(u) \) are given by

\[
\Re(\phi_\xi(u)) = \frac{\phi_\xi(u) + \phi_\xi(-u)}{2}, \quad \Im(\phi_\xi(u)) = \frac{\phi_\xi(k) - \phi_\xi(-u)}{2i}
\]
Using the substitution \( u = -s \), the integral part in (D.30) can be rearranged to

\[
\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-iua \phi_P(u)} - e^{-iua \phi_P(-u)}}{iu} du = \frac{1}{2\pi} \int_{-\infty}^{0} \frac{e^{-iua \phi_P(u)} - e^{-iua \phi_P(-u)}}{iu} du + \frac{1}{2\pi} \int_{0}^{\infty} \frac{e^{-iua \phi_P(u)} - e^{-iua \phi_P(-u)}}{iu} du
\]

\[
= -\frac{1}{2\pi} \int_{0}^{\infty} \frac{e^{iua \phi_P(u)} - e^{-iua \phi_P(-u)}}{iu} du
\]

where we have used that \( \int_{a}^{b} f(x) \, dx = -\int_{b}^{a} f(x) \, dx \). Next, using the expression in (D.31), the Gil-Palaez inversion formula can be expressed as

\[
P[\xi \leq a] = \frac{1}{2} + \frac{1}{2\pi} \int_{0}^{\infty} \frac{e^{iua \phi_P(u)} - e^{-iua \phi_P(-u)}}{iu} du
\]

\[
= \frac{1}{2} - \frac{1}{\pi} \int_{0}^{\infty} \Im \left( \frac{e^{-iua \phi_P(u)}}{u} \right) du
\]

\[
= \frac{1}{2} - \frac{1}{\pi} \int_{0}^{\infty} \Re \left( \frac{e^{-iua \phi_P(u)}}{iu} \right) du
\]

where the last equality follows immediately from Euler’s formula, i.e. \( e^{ikx} = \cos(kx) + i \sin(kx) \) and noticing that

\[
\frac{1}{2\pi} \int_{0}^{\infty} \frac{e^{iua \phi_P(u)} - e^{-iua \phi_P(-u)}}{iu} du = \frac{1}{2\pi} \int_{0}^{\infty} \frac{e^{iu(\xi - a)} - e^{i(-u)(\xi - a)}}{iu} du = \frac{-1}{\pi} \int_{0}^{\infty} \int_{\mathbb{R}} \sin(u(\xi - a)) \, du
\]

is real-valued. The last equality follows because the function \( u \to \frac{\cos(u(\xi - a))}{u} \) is odd and therefore \( \int_{-\infty}^{\infty} \frac{\cos(u(\xi - a))}{u} \, du = 0 \). Along the same line of argumentation, we get for the probability \( P[\xi \geq a] \), the following expression

\[
P[\xi \geq a] = \frac{1}{2} + \frac{1}{\pi} \int_{0}^{\infty} \Re \left( \frac{e^{iua \phi_P(u)}}{iu} \right) du, \quad a \in \mathbb{R}
\]

(D.33)

which follows immediately by noting that \( P[\xi \geq a] = 1 - P[\xi \leq a] \). Finally, setting \( a = 0 \) in (D.32) as well as (D.33), the expression for the detection-error probability as given in (69) follows directly.
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Table 1: Benchmark parameter values for Monte Carlo simulations

Notes: The time series length for daily sampled data is 252, whereas for weekly sampled data, the time series length is 52.
### Table 2: Monte Carlo simulation results

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</tbody>
</table>

Notes: The time series length for daily sampled data is 252 observations, whereas for weekly sampled data, the time series length is 52 observations. The column 'Weights' refers to the optimal portfolio holdings, where w<sup>*</sup><sub>M</sub> denote the Merton, w<sup>*</sup><sub>rM</sub> the robust Merton, w<sup>*</sup><sub>JD</sub> the jump diffusive and w<sup>*</sup><sub>rJD</sub> the robust jump diffusive weights, respectively. Columns 'C<sub>t</sub>/X<sub>t</sub>' (multiplied by 100) and 'CE' show the consumption-wealth ratio as given in (38) and the certainty equivalent as in (71), respectively. The column 'Drift' represents the expected return in percent where the Merton drift is R, the robust Merton drift is R - h<sup>*</sup>σ where h<sup>*</sup> = √2η<sup>D</sup> with η<sup>D</sup> = η, the jump diffusive drift is R + Jλ and the robust jump diffusive drift is R - h<sup>*</sup>σ - Jλe<sup>a<sup>*</sup></sup>. The column 'Jump Int.' compares the jump intensities λ (reference measure) and λ<sup>0</sup> (robust measure). The column 'h<sup>*</sup>' shows the optimal drift perturbation for the robust Merton solution with h<sup>*</sup> = √2η<sup>D</sup> where η<sup>D</sup> = η and for the robust jump diffusive solution with h<sup>*</sup> = √2η<sup>D</sup>. The last column 'a<sup>*</sup>' shows the optimal jump intensity perturbation a<sup>*</sup> = 1 + W ((η<sup>J</sup> - λ) / (eλ)) for the robust jump diffusive solution.
### Table 3: Comparative statics

<table>
<thead>
<tr>
<th>Risk Aversion $\gamma = 3$</th>
<th>Error Detect. Prob. $\bar{\pi} = 20%$</th>
<th>Jump Size Dist. $U(0,1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w^*_M$</td>
<td>$w^*_{rM}$</td>
<td>$w^*_JD$</td>
</tr>
<tr>
<td>0.417</td>
<td>0.146</td>
<td>0.368</td>
</tr>
<tr>
<td>1.966</td>
<td>2.124</td>
<td>2.032</td>
</tr>
<tr>
<td>0.15</td>
<td>0.008</td>
<td>0.093</td>
</tr>
<tr>
<td>0.625</td>
<td>0.359</td>
<td>0.552</td>
</tr>
<tr>
<td>1.201</td>
<td>1.334</td>
<td>1.270</td>
</tr>
<tr>
<td>0.191</td>
<td>0.002</td>
<td>0.096</td>
</tr>
<tr>
<td></td>
<td>0.625</td>
<td>0.573</td>
</tr>
<tr>
<td></td>
<td>0.344</td>
<td>0.356</td>
</tr>
<tr>
<td></td>
<td>0.191</td>
<td>0.136</td>
</tr>
</tbody>
</table>

Notes: Unless stated otherwise, the parameter values are those from Table 1. The column 'Weights' refers to the optimal portfolio holdings, where $w^*_M$ denote the Merton, $w^*_{rM}$ the robust Merton, $w^*_JD$ the jump diffusive and the $w^*_{rJD}$ robust jump diffusive weights, respectively. Columns '$C_t/X_t$' (multiplied by 100) and 'CE' show the consumption-wealth ratio as given in (38) and the certainty equivalent as in (71), respectively.
Figure 1: Lambert’s W-Function for various levels of jump intensities $\lambda$.

Notes: On the domain $[-1/e, \infty]$ the function $W(\cdot, \lambda)$ is real for any $\lambda > 0$ and $\eta \in [0, \infty)$. Furthermore, note that for $\eta' < \lambda$ we have $W\left(\frac{\eta'-\lambda}{\lambda^2}\right) < 0$ and for $\eta' > \lambda$ we obtain $W\left(\frac{\eta'-\lambda}{\lambda^2}\right) > 0$. In general, when $\eta$ is real, the function $W$ is not injective on the interval $-1/e \leq \eta < 0$. Therefore, we set $W(\cdot, \lambda) \geq -1$ such that $W(\cdot, \lambda)$ is single valued and therefore represents a well defined function. Lambert’s $W$ function is coded in most computational software (e.g., lambertw in Matlab).
Figure 2: Comparison of optimal robust portfolio weights.

Notes: The optimal robust portfolio weights are given in (45) for varying shares of robustness allocated to the drift misspecification $\zeta_D \in \{0, 1/3, 2/3, 1\}$. The selected parameters values are: $R = 0.05$, $\sigma = 0.1$, $J = -0.1$, $\lambda = 1$, $\gamma = 2$, jump size distribution is given in (43) and $\eta \in [0, 0.1]$. The case $\eta = 0$ corresponds to full confidence in the model (no robustness concerns).
Figure 3: Comparison of robust and non-robust portfolio allocations when either the drift or jump intensity are perturbed.

Notes: The green line represents the optimal portfolio holdings knowing the true parameters $R^*$ and $\lambda^*$. The dashed blue line shows the case when the investor believes that only the diffusive part in Equation (4) is misspecified, i.e. $\zeta_D = 1$ whereas the solid red line shows the case when the investor believes that solely the jump part is misspecified, i.e. $\zeta_D = 0$. The dotted-dashed pink line represents the case when there is both uncertainty about the drift and jump intensity, i.e. $\zeta_D = 0.5$. The upper (lower) bound in black $\mathbb{Z}(Z)$, $Z = R, \lambda$ in both panels have been obtained from a first order Taylor expansion of $\mu_S^0 := \left(r + R + \sigma h_t - \lambda^0 J \int_{\mathbb{R}} z \nu(df)\right)$ around $\eta_0 = \epsilon > 0$, where $\epsilon = 10^{-5}$ and the jump measure is given in Equation (43). The selected parameters values are: $R = 0.05$, $\sigma = 0.2$, $J = -0.1$, $\lambda = 1$, $\gamma = 2$, jump size distribution is given in (43) and $\eta \in [0, 0.01]$. 


Figure 4: Optimal consumption-wealth ratio

Notes: Optimal consumption-wealth ratio for various levels of EIS $\psi$ and drift vs. jump intensity misspecification $\zeta^D \in [0, 1]$: The left Panel shows optimal consumption-wealth ratio as given in (38) when $\zeta^D = 0.5$. The dashed black line corresponds to the case when the investor has preference for late resolution as, i.e. $\gamma < 1/\psi$. The solid red line corresponds to the case where the representative investor has standard CRRA preferences and therefore has no preference for timing of resolution of uncertainty. Lastly, the dotted blue line shows the case when the investor has preferences for early resolution of uncertainty, i.e. $\gamma > 1/\psi$. The right Panel shows the optimal consumption-wealth ratio for various degrees of drift $R$ vs. jump intensity $\lambda$ misspecification when the investor has CRRA preferences, i.e. $\psi = 1/\gamma = 0.5$. The purple line represents optimal consumption in the case when there is no uncertainty, i.e. $\eta = 0$. The dashed black line corresponds to the case when there is model uncertainty only with respect to the jump intensity $\lambda$. The solid red line corresponds to the case when there is model uncertainty with respect to both drift and jump intensity. Lastly, the dotted blue line shows the case when there is model uncertainty only with respect to the expected return $R$. The parameter values are $\beta = 0.01$, $\gamma = 2$, $\psi = 0.5$, $R = 0.05$, $r = 0.01$, $\sigma = 0.2$, $J = -0.1$, $\zeta^D = 0.5$, jump sizes follow the power law in (43) and we multiply the constants by 100.
Figure 5: Comparison of robust and non-robust portfolio allocations when both drift and jump intensity are perturbed.

Notes: Panel A displays the optimal robust jump - diffusive weights $\omega^*(h^*, a^*)$ (in red) and the optimal jump diffusive weights $\omega^*(h^* = 0, a^* = 0)$ (in blue) when both the expected return $R$ and the volatility $\sigma$ vary. Panel B displays optimal robust jump - diffusive weights $\omega^*(h^*, a^*)$ (in red) and optimal jump diffusive weights $\omega^*(h^* = 0, a^* = 0)$ (in blue) when both the jump scaling parameter $J$ and the intensity $\lambda$ vary. Unless stated otherwise, the selected parameters values are: $R = 0.05$, $\sigma = 0.1$, $J = -0.1$, $\lambda = 1$, $\gamma = 2$, $\zeta^D = 0.1$, jump size distribution is given in (43) and $\eta = 0.05$. 

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Figure 6: Detection-error probability based on monthly and weekly data.

Notes: The detection-error probability is $\pi(t, T_d; \eta)$. Time series lengths are $T_d \times 12$ (monthly) and $T_d \times 52$ (weekly) where $T_d = 1$ is one year. The parameters values are: $R = 0.05$, $\sigma = 0.15$, $J = -0.1$, $\lambda = 1$, $\gamma = 2$, $\zeta^D = 0.5$, the jump size distribution is given in (43) and $\eta = 0.1$. 

Figure 7: Comparison of certainty equivalents
Notes: The Panels A, C and E show the certainty equivalent $m$ as given in (71) for each of the portfolio strategies Merton, robust Merton and jump diffusive, i.e. $i \in \{M, rM, JD\}$ relative to the robust jump diffusive solution $rJD$ for various levels of EIS (for all plots we have set $\zeta^D = 0.5$, i.e. we consider uncertainty with respect to both drift and jump intensity). For those panels, the dotted blue line corresponds to the case when the investor has preferences for early resolution of uncertainty, i.e. $\gamma > 1/\psi$. The solid red line corresponds to the case where the investor is indifferent about the timing of resolution of uncertainty, i.e. $\gamma = 1/\psi$ (CRRA utility). Lastly, the dashed red line corresponds to the case when the investor has preferences for late resolution of uncertainty, i.e. $\gamma < 1/\psi$. Panels B, D and F show the certainty equivalent $m$ as given in (71) for each of the portfolio strategies Merton, robust Merton and jump diffusive, i.e. $i \in \{M, rM, JD\}$ relative to the robust jump diffusive solution $rJD$ for various degrees of drift $R$ vs. jump intensity $\lambda$ misspecification, when the investor has CRRA preferences, i.e. $\psi = 0.5$. The dotted blue line corresponds to the case when there is model uncertainty only with respect to the jump intensity $\lambda$. The solid red line corresponds to the case when there is model uncertainty with respect to both drift and jump intensity. Lastly, the dashed black line shows the case when there is model uncertainty only with respect to the expected return $R$. Unless stated otherwise, the parameter values are given in Table 1.
Figure 8: Optimal portfolio holdings with exponential utility

Notes: The underlying risky asset follows compensated Lévy dynamics and both drift and jump intensity are perturbed. The compensated Lévy dynamics are as given in (3). The benchmark parameters values are: $R = 0.075$, $\sigma = 0.2$, $J = 0.1$, $\lambda = 0.2$, $q = 1, 10$, $\zeta_D = 0.5$ and $\eta \in [0, 0.1]$. 