E-commerce, Productivity Linkages between Sectors, and Fluctuations in Economic Structure

Mei Wen
University of Sydney & Australian National University

December 2005

Abstract: This paper develops a general equilibrium model with general number of goods to explore the productivity linkages between the IT sector and the other sectors through e-commerce. As development of the IT sector can reduce setup and maintenance cost for e-commerce while improving unit transaction efficiency, it is found that increases in productivity in the IT sector can generate fluctuations in economic structure and productivity. This may provide explanations for nationwide productivity increase in US in the 1990s as well as fluctuations observed since 2000. In addition, it is found that the more significant the economies of specialization or the larger the scope for further division of labor, the more likely development of the IT sector will promote the economy into structures with higher levels of division of labor and higher productivity.

Key Words: transaction cost, economies of specialization, information technology, level of division of labor.

JEL Classifications: O12, O33, D58

Acknowledgement: The author is grateful to Prema-chandra Athukorala, Franklin Fisher, John Freebairn, Stephen King, Peter Lloyd, Andrew McLennan, Yew-Kwang Ng and Rohan Pitchford for helpful comments. Editorial help from Carol Kavanagh is highly appreciated. Any remaining errors are my own.
1 Introduction

It has been widely recognized that the productivity growth in the 1990s in many OCED countries, especially US, is largely attributable to the development of ICT sector. One significant economic application with the new technology wave is e-commerce. E-commerce has been changing the global economic outlook radically since the past decade. It is documented that the significant acceleration of productivity growth of the US since 1995 can be largely explained by increasing use of information technology, and in particular, adoption of e-commerce.

However, since 2000, there have been fluctuations in stock prices in both new economy sectors and many conventional sectors, which have not only made investors more cautious, but also shaken people’s confidence in the positive economic impact of the development of IT sector. Changes in expectations and investment attitudes can be partly due to financial bubbles and partly due to lack of theoretical works, which can reveal possible productivity linkages between IT and the other sectors. This paper is to study such productivity linkages and changes in economic structure through e-commerce in a general equilibrium model.

E-commerce, as applications of information technology to commercial areas, is believed to reduce unit transaction costs through the substitution of computer data processing and internet connections for labor services in the production of transactions (Lucking-Reily and Spulber, 2001). Cost reduction comes from more efficient searching for trading partners, more efficient communication, quicker negotiation and confirmation of transactions, faster data processing and product ordering process, reduction of inventories, and even transaction automation. But sellers need to purchase computers and software to setup a local network, and hire IT experts to maintain the services needed. Even after a local network is set up and is used for e-commerce, many work units do upgrade their computers and systems once for a while. Therefore, while e-commerce increases unit transaction efficiency, product sellers incur a fixed cost.

---

1 See Gretton et al., 2002, Nordhaus, 2001, UN, 2004 and 2001a, and United States, 2001b, for example.
3 See Shapiro and Varian (1999), and Freebairn (2002), for example.
Kurz (1974) points out that once transaction technology is considered, the focus of general equilibrium analysis shifts from the primitive concepts of “allocations” to “exchange vectors”. The choice of economic activities is an economic issue of its own. The structure of the transaction technology becomes critical and the study of equilibrium requires a deep analysis of the transaction possibility set. During the past two decades, a large amount of general equilibrium analysis with consideration of transaction cost have been developed, pioneered by Yang (1988), Krugman (1991), Becker and Murphy (1992), and Yang and Ng (1993), etc. For example, elucidated mathematically in Yang and Ng (1993) and Yang (2001), demand and supply are two sides of the same coin (i.e. division of labor according to Young, 1928). They are all different facets of the tradeoffs between the benefits of specialized production and increasing transaction costs due to increasing trade interdependency. A large amount of general equilibrium models with consideration of transaction costs in these two books formally revive Adam Smith’s proposition that “division of labor is limited by the market” and Allyn Young’s proposition that “the size of the market is determined by level of division of labor”. Indeed, in so called “inframarginal analysis”, general equilibrium solution includes not only how much each good individuals consume and purchase from the market, but also what each consumer-producer produces, what to sell to the market and what to buy from the market. New market for newly traded goods emerges when benefits from further specialized production outweigh the increasing transaction costs from higher trade interdependency.

E-commerce does generate new market segment through reductions in transaction cost. Although deliver of physical products is still necessary and the transportation costs involved can hardly be replaced, searching costs for desired products, time spent on finding more appropriate trading partners, communication costs, negotiation costs and contracting costs would all be significantly reduced through e-commerce. Asynchronized transactions for digital products and transaction automation in many service industries like online ticketing and hotel booking can save huge amount of transaction costs. Therefore, people’s decision on adoption of e-commerce should be a general equilibrium outcome with the changing transaction technology of e-commerce and hence the development of applied IT
technologies into e-commerce areas. Subsequently, a natural approach to study adoption of e-commerce and its economy-wide impact is to apply the general equilibrium framework with transaction cost.

Following the efforts of endogenizing transaction cost in general equilibrium studies in Chu and Tsai (1997) and Wen (1997), this paper will explore the economic impact of e-commerce by considering the aggregate effect of reduction in unit transaction cost and possible reduction in fixed set up and maintenance cost for e-commerce with different features of IT development stages. If e-commerce is widely used, development of applied commercial software and firm-specific transaction website design can gradually and significantly reduce unit transaction cost, which makes total transaction costs at higher trade volume lower. And development of CPU can also fasten information processing and speed of computing. These changes encourage specialization and division of labor. However, innovations in computer hardware and large-scale production of them may substantially reduce the fixed cost for e-commerce, which slows the average cost decline with production scale and make further specialization and division of labor less favorable, *Ceteris Paribus*. Henceforth, the tradeoff between the benefit from further specialization (and division of labor) and increased costs from higher trade dependency can be contingent on the feature of development of IT sector. In a period of fast software development with no significant reduction in fixed cost for e-commerce, more specialized production, fast division of labor and a significant increase in productivity maybe observed. In another period, the unit transaction cost can be relatively stable but innovations in hardware may reduce the fixed cost considerably. Then, economic structure can change towards a lower level of division of labor. In this case, short-run zero or negative productivity growth outside the IT sector would be observed. In other words, development of the IT sector can cause fluctuations in economic structure and productivity, although it will increase income.

Therefore, next section will lay out a general equilibrium model to demonstrate how an increase in transaction efficiency through improvement of conventional transaction and transportation infrastructure can trigger division of labor and productivity increase. And section 3 introduces e-commerce into the model. It shows how the adoption of e-commerce
can be a general equilibrium choice and how adoption of e-commerce would affect productivity. While productivity linkage of IT sector and the other sectors through e-commerce as well as concurrent economic structural changes are discussed in section 4, the section will also investigate how potential fluctuations in productivity and economic structure can be eased. Conclusions then follow in section 5.

2 The Model

Consider an economy with M identical consumer-producers. Each individual is endowed with one unit of labor. There are $m$ final goods: $x_j, j = 1, 2, \cdots, m$. Depending on the awareness of trade possibility among neighbors, transportation and other infrastructure conditions, people can choose how many of the $m$ goods to self-provide, how many goods to purchase from the market, what product to sell in exchange for the purchased goods as well as his/her labor allocation into different productions.

**Assumption 1:** The production function of $x_j$ is $f(l_j) = l_j - A, \ j = 1, 2, \cdots, m$, where $0 < A < \frac{1}{m}$ is the fixed learning cost. In other words, production of final goods exhibits economics of specialization.

**Assumption 2:** Individual’s utility function is $U = \prod_{j=1}^{m} X_j, \ X_j$ is his/her consumption of final goods $x_j$.

**Assumption 3:** Transaction technology of a final good $x_j$ is assumed to be the following: when a consumer purchases $x_j^d$ units of good $x_j$, he will receive only $kx_j^d$ units, and $(1-k)x_j^d$ units are costs absorbed into transaction and transportation process. This is typical ice-burger transaction technology. Meanwhile, assume that a seller of good $x_j$ needs to prepare $(1+(1-k))x_j^s = (2-k)x_j^s$ units for delivering $x_j^s$ units of good $x_j$ to his trading partner. The same trading technology applies to all traded goods, e.g. $j = 1, 2, \cdots, m$.

**Assumption 4:** Development of IT sector can improve unit transaction efficiency through e-commerce in such a way that $e = k + S_{it}(1-k)$, where $k$ and $e$ are unit transaction efficiency before and after e-commerce, $(1-k)$ is the scope for transaction efficiency improvement before e-commerce, and $S_{it}$ is the per capita resources inputted into the IT sector for technological advancement.
**Assumption 5:** Due to high learning costs and the requirement of rapid knowledge upgrading, IT experts produce the non-traded final goods only for self-consumption so that he can input the rest of his/her labor endowment into IT production.

**Assumption 6:** To make the problem interesting, it is assumed that \( m \) is sufficiently large so that the scope for division of labor will not be exhausted with conventional transaction technology or with e-commerce. Hence,

\[
\max \left\{ \frac{m}{\ln((2-k)/k)}, \frac{m}{\ln((2-e)/e)} + \frac{D(2-e)}{A} \right\} < \frac{1}{A} - 1
\]

is assumed.

### Possible Economic Structures

1. **Autarky:** When transaction costs are prohibitively high, people can be trapped in autarky in which everyone has to self-provide each and every final good. With assumptions 1 and 2, utility maximization leads to individual labor allocation

\[
\{l_j, j = 1, 2, \ldots, m \} = \{\frac{1}{m}, j = 1, 2, \ldots, m \}.
\]

The maximum utility level is \( \frac{1}{m^m(1-mA)^m} \).

2. **Division of Labor without E-commerce:** Denote the number of traded goods as \( n \) \((1 < n \leq m)\). For simplicity in notation, just take the first \( n \) goods as the traded goods. Due to \textit{ex ante} identical consumer-producers, people partially specialized in the production of one of the traded goods will be symmetric to those partially specialized in the production of another traded good. Similar to the proof in Wen (1998), it is easy to show that the activities of each consumer-producer will have the following features: (1) he/she will not buy and sell the same good due to positive transaction costs, (2) he/she will not self-provide and purchase the same good,\(^4\) and (3) he/she will not sell more than one good because of economies of specialization. Therefore, a consumer-producer partially specializes in production of good \( x_i (i = 1, 2, \ldots, n) \) will solve the following utility maximization problem

\(^4\) Self-provision of the good indicates that the unit cost of self-provision will not be higher than the market price of the good plus the unit transaction cost, and further self-provision will be at even a lower cost due to economies of specialization, hence purchase of the good is less economical than self-provision.
max \( U_{x_t} = x_i \prod_{j, i=1}^{n} (k^{d_j}) \prod_{j=1}^{m} x_j \)

s.t. \( x_i + (2 - k)x_i^s = l_{x_i} - A \)

\( x_j = l_{x_j} - A, \quad j = n + 1, n + 2, \ldots, m \)  

\( l_{x_j} + \sum_{j=n+1}^{m} l_{x_j} = 1 \)

\( \sum_{j,i=1}^{n} P_j x_j^d \leq p_i x_i^s \)

The utility maximization (1) leads to the optimal individual labor allocation, consumption plan of each and every good, trade plans and indirect utility as specified in Table 1 of Appendix A1. They are usually functions of unit transaction efficiency \( k \) and relative prices of the traded goods. As there is free choice of profession (reflected by the choice of production of the good to partially specialize in), utility equalization between different specialists can enable us to solve out the equilibrium relative prices of the traded goods. In addition, market clearing condition of the traded goods will generate the equilibrium relative number of specialists.\(^5\)

The number of traded goods \( n \) is endogenized to maximize the utility, which is given by

\[
n = \frac{m}{\ln((2-k)/k)} + (m+1) - \frac{1}{A}
\]

The optimal \( n \) is smaller than \( m \) under the assumption 6. The corner equilibrium utility at unit transaction efficiency \( k \) through partial division of labor is

\[
U_{PDO} = \left(\frac{k}{2-k}\right)^{\frac{m}{\ln((2-k)/k)} + \frac{1}{A}} \cdot \frac{A^m}{(\ln((2-k)/k))^m}
\]

\(^5\) Let \( p_1 = 1 \), utility equalization leads to the equilibrium price \( p_1 = p_2 = \cdots = p_n = 1 \). Meanwhile, the market clearing conditions lead to division of the population \( M_1 = M_2 = \cdots = M_n = M / n \), where \( M_i (i = 1, 2, \ldots, n) \) is the population of \( x_i \) sellers. The relative prices of traded goods and the relative number of specialists work as a self-adjustment mechanism before the equilibrium is reached. See Wen and King (2004) for a detailed description on this.
In addition, define size of the market $SM$ as total trade value. It can be derived that

$$SM = \frac{n(n-1)}{(2-k)m} \left[ (1-(m-n+1)A) \right. $$

$$\left. = \frac{A}{(2-k)\ln((2-k)/k)} \left( \frac{m}{\ln((2-k)/k)} + (m+1) \right) \left( \frac{m}{\ln((2-k)/k)} + m - \frac{1}{A} \right) \right) \quad (4).$$

(3) **Division of Labor with E-commerce:** it is assumed that when people simultaneously adopt e-commerce with a fixed setting up and maintenance cost $D$, unit transaction efficiency will improve from $k$ to $e$ according to assumption 4. In these kinds of economic structure, groups of people specialize in different final goods productions are symmetric to each other. However, decision problem of IT experts is asymmetric to the final good producers. A typical final good $x_i \ (i = 1, 2, \cdots, n^*)$ seller shall solve the following problem:

$$\max U_{x_i} = x_i \prod_{j=x_i,j=1}^{n^*} (ex_i^d) \prod_{j=n^*+1}^{m} x_j$$

s.t. \quad $x_i + (2-e)x_i^e = l_{x_i} - A$

$$x_j = l_{x_j} - A, \quad j = n^*+1, n^*+2, \cdots, m$$

$$l_{x_i} + \sum_{j=n^*+1}^{m} l_{x_j} = 1$$

$$\sum_{j=x_i,j=1}^{n^*} P_j x_j^d \leq P_i x_i^e - D$$

where $n^*$ is the number of traded good in new transaction conditions, which is endogenized with adoption of e-commerce. Meanwhile, an IT expert only self-provides non-traded final goods and has to input the rest of his/her labor endowments into IT production to maximize utility as follows:

$$\max U_{it} = \prod_{j=1}^{n^*} (ex_i^d) \prod_{j=n^*+1}^{m} x_j$$

s.t. \quad $x_j = l_{x_j} - A, \quad j = n^*+1, n^*+2, \cdots, m$

$$l_{x_i} + \sum_{j=n^*+1}^{m} l_{x_j} = 1$$

$$\sum_{j=x_i,j=1}^{n^*} P_j x_j^d \leq I_e l_{it}$$

where $I_e$ is the wage rate of IT experts. Solutions to problems (5) and (6) are summarized in Appendix A2.
In the current economy, there are \( n^* + 1 \) groups of people: \( n^* \) (1 < \( n^* < m \)) groups of final goods sellers and a group of IT experts. With free choice of specialization, utility equalizations across the different groups determine the relative prices of the traded goods and the wage rate of the IT expert. These conditions lead to \( p_i = p_j = 1, (i, j = 1, 2, \cdots, n^*) \)

and \( I_e = \frac{1}{(e(2-e))(n^*-1)^{n^*/n^*}} \left( \frac{1-(m-n^*+1)A-D(2-e)}{1-(m-n^*)A} \right)^{n^*/n^*} \), where the optimal number of traded goods under the assumption 6 is

\[
n^* = \frac{m}{\ln((2-e)/e)} + (m+1) - \frac{1-D(2-e)}{A}, \text{ when } e \leq 2 \left( 1 + \exp \left( \frac{mA}{1 - A - D(2-e)} \right) \right)^{-1} \tag{7}.
\]

Meanwhile, market clearing conditions and the income equation of IT sector determine the partition of groups as follows:

\[
\sum_{i=1}^{n^*} M_{x_i} + M_\mu = M \quad \text{(Division of the population)}
\]

\[
M_{x_i} = \sum_{j=1, j\neq i}^{n^*} \frac{x_{ij}}{x_i} M_{x_j} + x_{\mu i} M_\mu \quad \text{(Market clearing condition for good } x_i, i = 1, 2, \cdots, (n^* - 1))
\]

\[
\sum_{i=1}^{n^*} M_{x_i} D = I_e \frac{1}{n^*} M_\mu \quad \text{(Income equation of IT sector)} \tag{8},
\]

where \( M_\mu \) and \( M_{x_i} \) are the number of IT experts and the number of \( x_i \) sellers, respectively, \( i = 1, 2, \cdots, n^* \). Therefore, the relative number of each group to the whole population will be

\[
M_{\mu i} = \frac{mD(e(2-e))^{(n^*-1)/n^*} \left( 1-(m-n^*)A \right)^{(n^*-1)/n^*}}{M n^* (mA/\ln((2-e)/e))^{m/n^*} + mD(e(2-e))^{(n^*-1)/n^*} \left( 1-(m-n^*)A \right)^{(n^*-1)/n^*}} \]

\[
M_{x_i} = \frac{(mA/\ln((2-e)/e))^{m/n^*} + mD(e(2-e))^{(n^*-1)/n^*} \left( 1-(m-n^*)A \right)^{(n^*-1)/n^*}}{M n^* (mA/\ln((2-e)/e))^{m/n^*} + mD(e(2-e))^{(n^*-1)/n^*} \left( 1-(m-n^*)A \right)^{(n^*-1)/n^*}}, \quad i = 1, 2, \cdots, n^* \tag{9},
\]

where \( n^* \) is given by (7).

The utility level each consumer-producer enjoys in this economic structure is thus

\[
U_{PDN} = \frac{e^{(n^*-1)}}{m^n (2-e)^{(n^*-1)}} [1 - (m-n^*+1)A - D(2-e)]^n
\]
General Equilibrium Economic Structure

While “division of labor is limited by the market” (Smith, 1776) and “size of the market is determined by the level of division of labor” (Young, 1928), demand and supply are really two sides of the same coin - social level of division of labor (Yang, 1994), which is determined by the tradeoffs between benefits from specialization and increasing trade costs due to increasing trade dependency. Formally, the general equilibrium economic structure is defined to be the one which generates the highest utility level among all possible economic structures discussed above. Due to free choice of profession and subsequent utility equalization among different groups of people in a given economic structure of division of labor, the general equilibrium economic structure is Pareto optimal. Therefore, the following proposition can be established.

**Proposition 1.** The unique general equilibrium economic structure will be Autarky iff the following inequality (11) holds; the general equilibrium economic structure will be division of labor without e-commerce iff the following inequality (12) holds; and the general equilibrium economic structure will be division of labor with e-commerce iff the following inequality (13) holds, where inequalities (11), (12) and (13) are

\[
\left( \frac{1}{mA} - 1 \right)^m > \max \left\{ \frac{1}{(\ln((2 - k)/k))^m} \left( \frac{k}{2 - k} \right)^m \left( \frac{m}{\ln((2 - k)/k)} \right)^{m-1} \right\},
\]

(11),

\[
\frac{1}{(\ln((2 - k)/k))^m} \left( \frac{k}{2 - k} \right)^m \left( \frac{1}{mA} - 1 \right)^m > \max \left\{ \frac{1}{(\ln((2 - e)/e))^m} \left( \frac{e}{2 - e} \right)^m \left( \frac{m}{\ln((2 - e)/e)} \right)^{m-1} \right\}
\]

(12), and
Proof: Direct comparison between the income function of Autarky \( U_A = \left( \frac{1}{m} \right) (1-mA)^n \), the income function \( U_{PDO} \) of the optimal structure among structures of division of labor without e-commerce (i.e. formula (3)), and the indirect utility function \( U_{PDN} \) of the optimal structure among structures of division of labor with e-commerce (i.e. formula (10)) gives \( U_A > \max \{ U_{PDO}, U_{PDN} \} \) iff (11) holds; \( U_{PDO} > \max \{ U_A, U_{PDN} \} \) iff (12) holds; and \( U_{PDN} > \max \{ U_A, U_{PDO} \} \) iff (13) holds. Therefore, proposition 1 follows according to the definition of general equilibrium economic structure.

\[
\frac{1}{(\ln((2-e)/e))^{m}} \left( \frac{e}{2-e} \right)^{m \ln((2-e)/e) + m - \frac{1}{A}} \left( \ln((2-k)/k) \right)^{m}\left( \frac{k}{2-k} \right)^{m \ln((2-k)/k) + m - \frac{1}{A}} \right) ^{\left( \frac{m}{\ln((2-e)/e)} + m - \frac{1}{A} \right)}
\]

(13).

**Corollary 1.** For optimal economic structure with e-commerce to be a unique general equilibrium structure, unit transaction efficiency with e-commerce has to be higher than unit transaction efficiency without e-commerce.

**Proof.** Suppose \( e \leq k < 1 \). Let 
\[
h(x) = \left( \frac{2-x}{x} \right)^{m \ln((2-x)/x) + m - \frac{1}{A}} \cdot \left( \ln((2-x)/x) \right)^{m}.
\]
It can be derived that
\[
h'(x) = h(x) \left( - \frac{D}{A} \ln \left( \frac{2-x}{x} \right) - \left( m \ln((2-x)/x) + m - \frac{1}{A} \right) \frac{2}{x(2-x)} \right) < 0 \text{ when } 0 < x < 1.
\]
Hence, \( \frac{1}{h(k)} \geq \frac{1}{h(e)} \), i.e.
\[
U_{PDO} = \left( \frac{k}{2-k} \right)^{m \ln((2-k)/k) + m - \frac{1}{A}} \cdot \frac{A^m}{(\ln((2-k)/k))^m} \geq \left( \frac{e}{2-e} \right)^{m \ln((2-e)/e) + m - \frac{1}{A}} \cdot \frac{A^m}{(\ln((2-e)/e))^m} \cdot \left( \frac{e}{2-e} \right)^{D(2-e)} = U_{PDN}.
\]
(14)

This contradicts to the fact that the economic structure with e-commerce is the unique general equilibrium structure (i.e. \( U_{PDN} > \max \{ U_A, U_{PDO} \} \)). Therefore, by counterargument, \( e > k \) has to be held for the economic structure with e-commerce to be the general equilibrium. QED.
Intuitively, as e-commerce incurs a fixed set up and maintenance cost \( D \), if there is no improvement in unit transaction efficiency with e-commerce, it is impossible for e-commerce to be endogenously adopted.

**Proposition 2 (The existence of a general equilibrium).** Given the parameters \( m, A, D \) satisfying assumptions 1 and 6, and unit transaction efficiency \( 0 < k, e < 1 \), there will be a general equilibrium economic structure.

Proof: Let

\[
\left( \frac{1}{mA} - 1 \right)^m = \frac{1}{(ln((2-k)/k))^m} \left( \frac{k}{2-k} \right)^{\frac{m}{ln((2-k)/k)+\frac{1}{A}}} \]

be (15),

\[
\left( \frac{1}{mA} - 1 \right)^m = \frac{1}{(ln((2-e)/e))^m} \left( \frac{e}{2-e} \right)^{\frac{m}{ln((2-e)/e)+\frac{1-D/2-e}{A}}} \]

be (16),

\[
\frac{1}{(ln((2-k)/k))^m} \left( \frac{k}{2-k} \right)^{\frac{m}{ln((2-k)/k)+\frac{1}{A}}} = \frac{1}{(ln((2-e)/e))^m} \left( \frac{e}{2-e} \right)^{\frac{m}{ln((2-e)/e)+\frac{1-D/2-e}{A}}} \]

be (17), and

\[
\left( \frac{1}{mA} - 1 \right)^m = \frac{1}{(ln((2-k)/k))^m} \left( \frac{k}{2-k} \right)^{\frac{m}{ln((2-k)/k)+\frac{1}{A}}} = \frac{1}{(ln((2-e)/e))^m} \left( \frac{e}{2-e} \right)^{\frac{m}{ln((2-e)/e)+\frac{1-D/2-e}{A}}} \]

be (18).

Given the parameters \( m, A, D \) satisfying assumptions 1 and 6, and unit transaction efficiency \( 0 < k, e < 1 \), if (18) holds, the all economic structures are general equilibrium structure (multi-equilibria case). Otherwise, one of (11), (12), (13), (15), (16), and (17) has to be hold. In the cases of one of the inequalities (11), (12) and (13) holds, the other two inequalities can not hold and there is a unique general equilibrium economic structure as stated in proposition 1. In the case that none of (11), (12) and (13) holds, one of the equalities (15), (16), and (17) must hold. And the corresponding economic structures which generate the equal utility are both general equilibrium structures. QED.

3 Endogenous Adoption of E-commerce and Increase in Productivity
As stated in the propositions 1 and 2, this paper shows that e-commerce can be endogenously adopted as a general equilibrium outcome. One necessary condition for e-commerce to be adopted is that some economic structure with e-commerce has to generate a higher utility level than the economic structures with conventional transaction technology, i.e. $U_{PDN} > U_{PDO}$, or say inequality (14) does not hold. Let

$$s(D, m, A) = \left( \frac{2-e}{e(2-k)} \right)^{(m-1)A} \cdot \left( \frac{2-e}{e} \right)^{\frac{m}{\ln((2-e)/e)}} \cdot \left( \frac{2-k}{k} \right)^{-m} \cdot \left( \frac{\ln((2-e)/e)}{\ln((2-k)/k)} \right)^{m} \cdot \left( \frac{2-e}{e} \right)^{D(2-e)}.$$  

Then (14) is equivalent to $s(D, m, A) \geq 1$ and $U_{PDN} > U_{PDO}$ equivalent to $s(D, m, A) < 1$. As $D, m,$ and $A$ reflect the fixed cost for e-commerce, scope for division of labor, and fixed learning cost in production (generating economies of specialization), respectively. It is interesting to have the following proposition 3.

**Proposition 3.** When $e > k$, we have $\frac{\partial s}{\partial m} < 0$, $\frac{\partial s}{\partial A} < 0$, $\frac{\partial s}{\partial e} < 0$, but $\frac{\partial s}{\partial D} > 0$.

**Proof:** Note that

$$\ln s(D, m, A) = \left( m - \frac{1}{A} \right) \left[ \ln((2-e)/e) + \ln k - \ln(2-k) - \ln e \right] + m \left[ \ln((2-e)/e) - \ln((2-k)/k) \right]$$

$$+ \frac{D(2-e)}{A} \ln \left( \frac{2-e}{e} \right).$$

$$\frac{\partial \ln s(D, m, A)}{\partial m} = \left[ \ln(2-e) + \ln k - \ln(2-k) - \ln e \right] + \left[ \ln((2-e)/e) - \ln((2-k)/k) \right] < 0 \text{ when } e > k,$$

$$\frac{\partial \ln s(D, m, A)}{\partial A} = \frac{1}{A^2} \left[ \ln((2-e)/e) - \ln((2-k)/k) \right] - \frac{D(2-e)}{A^2} \ln \left( \frac{2-e}{e} \right) < 0 \text{ when } e > k,$$

$$\frac{\partial \ln s(D, m, A)}{\partial e} = -\frac{2}{e(2-e)} \left\{ \left( m - \frac{1}{A} \right) + \frac{m}{\ln((2-e)/e)} + \frac{D(2-e)}{A} \right\} - \frac{D}{A} \ln \left( \frac{2-e}{e} \right) < 0,$$

and

$$\frac{\partial \ln s(D, m, A)}{\partial D} = \frac{D(2-e)}{A} \ln \left( \frac{2-e}{e} \right) > 0.$$ As $s(D, m, A) > 0$, we have proposition 4.

QED.

Proposition 3 indicates that *ceteris paribus*, the larger the $m$, the higher the $A$, the greater the $e$, or the smaller the $D$, the more likely $s(D, m, A) < 1$ ($U_{PDN} > U_{PDO}$). Therefore, compared
with conventional transaction technology, it is more likely for e-commerce to be endogenously adopted the more significant economies of specialization the productions exhibit, the larger the scope for division of labor, the more significant the improvement in unit transaction efficiency through e-commerce, or the lower the setup and maintenance cost, *ceteris paribus*.

**Proposition 4.** If the number of final goods is sufficiently large in term of

\[
\left( \frac{1}{A} \right) m > m = \frac{1}{A} \max \left\{ \frac{1}{2} D(2 - k) k \ln(2 - k) - \ln k \right\}, \arg \left\{ \frac{1}{\exp(mA/(1 - mA))} - 2D \frac{mA}{(1-mA)^2} = 0 \right\},
\]

the general equilibrium number of traded goods increases with improvements in unit transaction efficiency until complete division of labor has reached, *ceteris paribus*.

**Proof:**

1) If the general equilibrium economic structure is division of labor without e-commerce, i.e. before adoption of e-commerce, but unit transaction efficiency is high enough to attract people to go out of autarky and start to trade with each other, the optimal number of traded good is given by (2), i.e.

\[
n = \frac{m}{\ln((2-k)/k)} + (m+1) - \frac{1}{A}.
\]

As \( \frac{dn}{dk} = \frac{2m}{k(2-k)(\ln((2-k)/k))^2} > 0 \), the general equilibrium number of traded goods increases with improvement in unit transaction efficiency.

2) When e-commerce sufficiently improves unit transaction efficiency so that e-commerce is adopted endogenously, equilibrium number of traded goods is

\[
n^* = \frac{m}{\ln((2-e)/e)} + (m+1) - \frac{1 - D(2-e)}{A} > 0.
\]

As \( n^* - n = m \left( \frac{1}{\ln((2-e)/e)} - \frac{1}{\ln((2-k)/k)} \right) + \frac{D(2-e)}{A} > 0 \), number of equilibrium traded goods increases with the improvement in unit transaction efficiency.

3) If the general equilibrium economic structure is division of labor with e-commerce, then e-commerce has been adopted. In this case, the optimal number of traded good is given by (7).

Before complete division of labor has reached (i.e. \( n^* < m \)),

13
\[ n^* = \frac{m}{\ln((2-e)/e)} + (m+1) - \frac{1-D(2-e)}{A}. \]

As \[ \frac{\partial n^*}{\partial e} = \frac{2m}{e(2-e)(\ln((2-e)/e))^2} - \frac{D}{A} > 0 \]
the general equilibrium number of traded goods increases with improvements in unit transaction efficiency, *ceteris paribus*. QED.

As viewed by Smith (1776) and Young (1928), economic development is a process of evolution in division of labor. When there are economies of specialization, higher individual specialization level and higher level of social division of labor brings about higher productivity. Here, the equilibrium number of traded goods reflects both the optimal social level of division of labor and the optimal number of final good sectors. This proposition indicates that equilibrium level of division of labor rises with adoption of e-commerce. In addition, the optimal number of final good sectors increases with improvements in unit transaction efficiency, *ceteris paribus*.

Mentioned in the introduction, e-commerce has contributed significantly to productivity growth since the 1990s. Denote the unit production cost in each traded final good and self provided final good production as \( u_c \) and \( u_c \), respectively. Further use superscript * to denote the variables after adoption of e-commerce. We can formally have

**Proposition 5.** \( u_c^* - u_c < 0 \), \( u_c^* - u_c < 0 \), and \( u_c^* - u_c < 0 \).

Proof: In the economic structure with adoption of e-commerce, the average production cost of traded goods (labor input per unit of output) is

\[ u_c^* = 1 + \frac{mA}{n^*(1-(m-n^*)+1)A + (m-n^*)D(2-e)} \quad (19) \]

while the average cost of self-provided good is \( u_c^* = 1 + \frac{mA}{(1-(m-n^*)+1)A - D(2-e)} \). However, in the economy with conventional transaction facilities, the average production cost of traded

---

\( ^6 \) See Appendix A3 for proof.
goods is \( uc_i = 1 + \frac{mA}{n(1 - (m - n + 1)A)} \) and the average production cost of self-provided good is \( uc_s = 1 + \frac{mA}{(1 - (m - n + 1)A)} \). Therefore, for self-provided goods, the cost reduction is shown by \( uc_s^* - uc_s = \frac{mA}{(1 - (m - n^* + 1)A) - D(2 - e)} - \frac{mA}{(1 - (m - n + 1)A)} \)

\[ = \ln((2 - e) / e) - \ln((2 - k) / k) < 0. \]

Because \( n^* > n \geq 2 \), for formerly traded goods, the average production cost also reduces as \( uc_i^* - uc_i = \frac{mA}{n^*(1 - (m - n^* + 1)A) + (m - n^*)D(2 - e)} - \frac{mA}{n(1 - (m - n + 1)A)} \)

\[ = \frac{mA}{mA / \ln((2 - e) / e) + (m - n^* + 1)D(2 - e)} - \ln((2 - k) / k) \]

\[ < \ln((2 - e) / e) - \ln((2 - k) / k) < 0 \]

Since \( n \geq 2 \), for newly traded goods, this gives \( uc_i^* < uc_i < uc_s \), hence \( uc_i^* - uc_s < uc_i^* - uc_i < 0 \). QED.

As productivity is the reciprocal of average cost, a reduction in production cost implies an increase in productivity. It is worth noting that unlike the improvement in unit transaction efficiency from e-commerce, triggered productivity increases in the final good sectors are not straightforward.

4 Development of the IT sector and Fluctuations in Economic Structure and Productivity

Now let us focus our attention on what would happen if the economy is shifted to internet based e-commerce. After e-commerce has been adopted in the economy, the equilibrium number of traded goods, the unit production cost of traded final goods, and the maximum income level is represented by (7), (19) and (10), respectively. Regarding (7) and (19), the following propositions can be derived.
Proposition 6. When taking $n^*$ as a function of four variables, i.e. $n^* = n^*(e, D, A, m)$, we have \[
\frac{\partial n^*}{\partial m} > 0, \quad \frac{\partial n^*}{\partial A} > 0, \quad \frac{\partial n^*}{\partial e} > 0, \quad \text{and} \quad \frac{\partial n^*}{\partial D} > 0.
\]

Proof: the deduction is straightforward as

\[
\frac{\partial n^* (e, D, A, m)}{\partial m} = \frac{1}{\ln((2-e)/e)} + 1 > 0
\]
\[
\frac{\partial n^* (e, D, A, m)}{\partial A} = \frac{1 - D(2-e)}{A^2} > 0
\]
\[
\frac{\partial n^* (e, D, A, m)}{\partial e} = \frac{2m}{e(2-e)[\ln((2-e)/e)]^2} - \frac{D}{A} > 0
\]
and \[
\frac{\partial n^* (e, D, A, m)}{\partial D} = \frac{(2-e)}{A} > 0
\]

Corollary 2. The economic structure (i.e. equilibrium number of final goods sectors) can fluctuate with the development of IT sector. In other words, with improvement of unit transaction efficiency (higher $e$, or $de > 0$) and reduction in fixed cost of e-commerce (lower $D$, or $dD < 0$), $dn^*$ can be either positive or negative, depending on which forces dominate. When these two forces dominate alternatively, they can bring about fluctuations in economic structure.\(^7\)

Proposition 7. $duc_i^*$ can be alternatively increase and decrease when $D$ decreases and $e$ increases with the development of IT sector. In other words, productivity in the final goods sectors can fluctuate with development of the IT sector.

Proof: the average production cost of traded goods is

\[
uc_i^* = 1 + \frac{mA}{n^*(1 - (m - n^*) + 1)A + (m - n^*)D(2-e)}
\]

The total differentiation of $uc_i^*$ gives

\(^7\) See appendix A4 for a detailed proof.
\[ d\mathcal{U}^*_i = \frac{\partial\mathcal{U}^*_i}{\partial e} dA + \frac{\partial\mathcal{U}^*_i}{\partial \mathring{n}^*} dm + \frac{\partial\mathcal{U}^*_i}{\partial \mathring{n}^*} dA + \frac{\partial\mathcal{U}^*_i}{\partial \mathring{n}^*} dD \]  

(24),

where

\[ \frac{\partial\mathcal{U}^*_i}{\partial D} + \frac{\partial\mathcal{U}^*_i}{\partial \mathring{n}^*} \cdot \frac{\partial\mathcal{U}^*_i}{\partial D} = \frac{m(2-e)(1+n^*-1)A-D(2-e)}{n^*(1-(m-n^*+1)A)+(m-n^*)D(2-e)} < 0 \]  

(25),

\[ \frac{\partial\mathcal{U}^*_i}{\partial A} + \frac{\partial\mathcal{U}^*_i}{\partial \mathring{n}^*} \cdot \frac{\partial\mathcal{U}^*_i}{\partial A} = \frac{-m\left\{1-(m-n^*+1)A+D(2-e)(2-(n^*-1)A)+D^2(2-e)^2\right\}}{n^*(1-(m-n^*+1)A)+(m-n^*)D(2-e)} A < 0 \]  

(26),

\[ \frac{\partial\mathcal{U}^*_i}{\partial m} + \frac{\partial\mathcal{U}^*_i}{\partial \mathring{n}^*} \cdot \frac{\partial\mathcal{U}^*_i}{\partial m} = \frac{-\left[1-(m-n^*+1)A-D(2-e)\right]\left[1+(n^*-1)A-D(2-e)\right]}{n^*(1-(m-n^*+1)A)+(m-n^*)D(2-e)} < 0 \]  

(27),

and

\[ \frac{\partial\mathcal{U}^*_i}{\partial e} + \frac{\partial\mathcal{U}^*_i}{\partial \mathring{n}^*} \cdot \frac{\partial\mathcal{U}^*_i}{\partial e} = \frac{-m^2}{n^*(1-(m-n^*+1)A)+(m-n^*)D(2-e)^2} \left[ \frac{m}{\ln((2-e)/e)} + n^* \right] \times \frac{2m}{e(2-e)[\ln((2-e)/e)]^2} - \frac{D}{A} < 0 \]  

(28).^8

(24) to (28) tell us that when the development of the IT sector reduces the fixed cost for ecommerce, its direct effect and the effect through directly encouraging a lower level of division of labor make the productivity of the final goods sector decline. However, it can increase unit transaction efficiency which increases the productivity of the final goods.

^8 Relationship (7) has been used for establishing several formulae here.
sector through encouraging further specialization and division of labor. Meanwhile, (26) to (28) indicates that both increases in fixed learning cost in the final goods production and the increase in the number of final goods would reduce unit production cost of final good through encouraging further specialization and division of labor. When the productivity increase of the IT sector is mainly reflected in unit transaction efficiency improvement, it triggers a productivity increase of the final goods sectors as the effect of (26) to (28) dominates the effect of (25). When the productivity increase of the IT sector is mostly reflected in reducing the fixed cost of e-commerce with small changes in $A$ and $m$, it can cause productivity decline in the final goods sectors as effect of (25) dominates. However, if $A$ and $m$ increase to large extents, the effect of (25) is less likely to dominate in (24).

QED.

5 Concluding Remarks

In this paper, a simple general equilibrium model is developed to demonstrate endogenous decisions of e-commerce and its effect on cost savings, economic structure, trade dependency, productivity and income. Interestingly, while development of the IT sector can trigger economic structure change and promote productivity increase, it can also bring about fluctuations in economic structure and productivity.

Since the 2nd half of year 2000, investors have been losing confidence in high-tech shares. However, despite the collapse of many dot.coms, there has been steady development of new online intermediaries along with fundamental changes in the economy with e-commerce. Without direct modeling the emergence of new intermediaries, which desires a separate study, this paper shows that productivity increases in IT sector can have a complex effect on productivity of the other sectors. This paper illustrates the fluctuations in productivity and economic structure through a general equilibrium model which can help people to correctly understand how e-commerce generates its impact on the other sectors, thus lifting investors’ confidence. In addition, as shown in former sections, stronger economies of specialization based on knowledge accumulation and a larger scope of the division of labor based on R&D can ease potential fluctuations, leading the economy to
transform more steadily towards economic structures with higher levels of specialization, higher trade dependency, and higher productivity.

Considering the complex reality where global adoption of e-commerce connects different economies and extends the boundary of markets, thereby enlarging the scope for further division of labor, the impact of e-commerce in raising productivity could be expected to be even stronger.
References


## Appendix

### A1 Table 1 Optimal Solutions of Problem (1)

<table>
<thead>
<tr>
<th>Optimal solution</th>
<th>$x_i$ seller</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor allocation</td>
<td>$l_{x_i}$, $l_{x_j}$ ($j = 1, \ldots, n, j \neq i$) and $l_{x_j}$ ($j = n+1, \ldots, m$)</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption portfolio</td>
<td>$x_i$, $kx_j^d$ ($j = 1, \ldots, n, j \neq i$) and $x_j$ ($j = n+1, \ldots, m$)</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Trade plans</td>
<td>$x_i^s$, $x_j^d$ ($j = 1, \ldots, n, j \neq i$)</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Utility</td>
<td></td>
</tr>
</tbody>
</table>

In trade plans, positive means sales and negative means purchase.
### Table 2  Optimal Solutions for Different Configurations in the Economic Structure with E-commerce

<table>
<thead>
<tr>
<th>Optimal solution</th>
<th>$x_i$ seller</th>
<th>IT experts</th>
</tr>
</thead>
</table>
| Labor allocation | $l_{xi}$, $l_{xj}^{j_1,...,n_j
\neq i}$, $l_{xj}^{j_2,...,n_j
\neq i}$ | 0, 0, $\frac{1}{m}(1+n^*A)$, $1-\frac{(m-n^*)}{m}(1+n^*A)$ |
| Consumption portfolio | $x_i$, $e^{x_i}$ | $\frac{e^{l_i}}{mp_i}$, $\frac{e^{l_i}}{mp_j}$, $\frac{1}{m}[1-(m-n^*)A]$ |
| and $x_j$, $(j=n+1,...,n^*)$ |  |  |
| Trade plans | $x_i^d$, $(d_1,...,d_n)$, $x_j^d$, $(d_1,...,d_n)$ | $-\frac{l_e}{mp_i}$, $-\frac{l_e}{mp_j}$, $-\frac{l_e}{mp_j}$ |
| Utility | $e^{(n^*-1)} \frac{e^{(n^*-1)m(2-e)^{n^*}}}{m^m(2-e)^{n^*}p_i^{(m-n^*)+1}\prod_{j=1,j\neq i}^{n^*}p_j} [p_i(1-(m-n^*)A)-D(2-e)]^m$ | $\frac{e^{l_e}}{mp_i^{(m-n^*)+1}\prod_{j=1,j\neq i}^{n^*}p_j} [1-(m-n^*)A]^m$ |

In trade plans, positive value means sale and negative one means purchase.
A3 Proof of \( \frac{\partial n^*}{\partial e} = \frac{2m}{e(2-e)(ln((2-e)/e))^2} - \frac{D}{A} > 0 \)

Proof:

According to Proposition 1 and Corollary 1, when the e-commerce is adopted in the economy, \( e > k \geq \min \left\{ k, 2 \left[ 1 + \exp \left( \frac{mA}{1-mA} \right) \right]^{-1} \right\} \). Let \( x(z) = D(2-z)z[l(2-z) - ln z]^2 \).

\( x'(z) = 2D[l(2-z) - ln z][(1-z)[ln(2-z) - ln z] - 2] < 0 \) when \( z \leq 1 \).

Thus, \( \min \left\{ x(k), x \left( 2 \left[ 1 + \exp \left( \frac{mA}{1-mA} \right) \right]^{-1} \right) \right\} > x(e) = D(2-e)e[ln(2-e) - ln e]^2 \)

Let \( t = \frac{mA}{1-mA} \) and \( g(t) = \frac{1}{\exp(t)} - 2D(t+1) \). Then

\[
\frac{dg(t(mA))}{d(mA)} = g'(t) \cdot \frac{dt}{d(mA)} = \left( - \frac{1}{\exp(t)} - 2D(2t+1) \right) \left( \frac{-1}{(1-mA)^2} \right) > 0.
\]

As \( m > \frac{1}{A} \max \left\{ \frac{1}{2} D(2-k)k[ln(2-k) - ln k]^2, \arg \left( \frac{1}{\exp(mA/(1-mA))} - 2D \frac{mA}{(1-mA)^2} = 0 \right) \right\} \),

\[
2mA > \max \left\{ x(k), x \left( 2 \left[ 1 + \exp \left( \frac{mA}{1-mA} \right) \right]^{-1} \right) \right\} \geq \min \left\{ x(k), x \left( 2 \left[ 1 + \exp \left( \frac{mA}{1-mA} \right) \right]^{-1} \right) \right\} > x(e), \text{ i.e.}
\]

\[
2mA > D(2-e)e[ln(2-e) - ln e]^2.
\]

Therefore, \( \frac{\partial n^*}{\partial k} = \frac{2m}{e(2-e)(ln((2-e)/e))^2} - \frac{D}{A} > 0 \).
Proof of Corollary 2

Total differentiation of $n^*$ can be written as

$$dn^* = \left( \frac{\partial n^*(e, D, A, m)}{\partial D} + \frac{\partial n^*(e, D, A, m)}{\partial e} \cdot \frac{\partial e(D, A, m)}{\partial D} \right) dD$$

$$+ \left( \frac{\partial n^*(e, D, A, m)}{\partial A} + \frac{\partial n^*(e, D, A, m)}{\partial e} \cdot \frac{\partial e(D, A, m)}{\partial A} \right) dA$$

$$+ \left( \frac{\partial n^*(e, D, A, m)}{\partial m} + \frac{\partial n^*(e, D, A, m)}{\partial e} \cdot \frac{\partial e(D, A, m)}{\partial m} \right) dm$$

$$= \frac{\partial n^*(e, D, A, m)}{\partial D} \cdot dD + \frac{\partial n^*(e, D, A, m)}{\partial A} \cdot dA + \frac{\partial n^*(e, D, A, m)}{\partial m} \cdot dm + \frac{\partial n^*(e, D, A, m)}{\partial e} \cdot de$$

[A1],

where

$$\frac{\partial n^*(e, D, A, m)}{\partial D} = \frac{(2 - e)}{A} > 0 \quad [A2],$$

$$\frac{\partial n^*(e, D, A, m)}{\partial A} = \frac{1 - D(2 - e)}{A^2} > 0 \quad [A3],$$

$$\frac{\partial n^*(e, D, A, m)}{\partial m} = \frac{1}{\ln((2 - e)/e)} + 1 > 0 \quad [A4],$$

and

$$\frac{\partial n^*(e, D, A, m)}{\partial e} = \frac{2m}{e(2 - e)[\ln((2 - e)/e)]^2} - \frac{D}{A} > 0 \quad [A5].$$

With economic development, the number of final goods $m$ usually increases with new technology inventions while the fixed learning cost in production also rises due to increasing use of capital goods, i.e. $dm > 0$ and $dA > 0$. This generate direct effects of increasing $n^*$. Meanwhile, IT development in computer hardware and its large scale production reduce the fixed maintenance cost for ecommerce, i.e. $dD < 0$, which generates a direct effect of reducing $n^*$. All these changes generate an indirect effect on the optimal number of traded goods $n^*$ through increasing the unit transaction efficiency $e$. Therefore, $dn^*$ can be either positive or negative depending on the effect reflected in the change in $D$ dominates or the change in $e$ and direct effect of changes in $m$ and A dominate. When the effects in the two opposite directions dominate alternatively with the development of IT
sector, then $n^*$ can increase and decrease alternatively, generating fluctuations in economic structure.