To Integrate or Not To Integrate?
The Role of Country Size

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Abstract
Public goods are an important aspect to consider when countries integrated economically. Using a Hotelling style model with elements from Alesina and Spolaore (2003), we analyse the effects of economic integration between two countries, the only difference between them being size. We consider two sorts of integration, one where the countries fully merge to form a new country (Country Formation) and one where country integrate but retain their own sovereignty (Unions). For Country Formation, we allow one capital in the new country. Here we find that building a more efficiently located capital will yield the most benefits but retaining the larger country’s capital will be the voter’s choice. For unions, we find that if the two countries’ capitals are retained, the smaller country benefits. The smaller country will vote for integration and the larger country will not. We extend this by looking at the benefits from choosing a level of infrastructure to reduce transport costs. The country can either choose the infrastructure independently or as an integrated country. We find that when size differentials are not too big, both countries’ voters will want to integrated and both countries stand to benefit as a result. We also find that infrastructure traps will exist in some cases which will result in no infrastructure investments.

1 Introduction
Countries integrate and form unions for both economic and political reasons. For example, some provinces and territories have integrated into federal states such as Australia, Canada and the United States of America. Sometimes, countries get absorbed by bigger ones, this we saw after the First World War with the Treaty of Versailles. Countries which were separated may reunite, like

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Germany and Yemen. Groups of countries may band together for common interests, such as the formation of the European Union (EU). This is often not an easy process as many parties need to be satisfied with the terms of merger.

Integrated countries or federal states in the usual sense will see the central government making most decisions, the EU on the other hand is a loose confederation of independent states. We can classify the integration of countries into two broad categories, integration which results in the formation of a new country (which we will call "Country formation") and unions in the looser sense such as the EU (which we will call "Unions").

The question this paper tries to address is whether country size and the size differentials between the countries affect the decision to integrate. We do this from a public good perspective. The recent ratification process of the European Constitution may give us some insight on this. The constitution can be seen as a huge step towards European integration. Larger countries such as France, England and the Netherlands have shown dislike for the constitution. Luxemburg on the other hand has welcomed it. This hints to us that size could matter to some degree when it comes to preference for integration.

To model this sort of behaviour, an obvious choice would be Hotelling type location models. In Alesina and Spolaore (2003)'s size of nations model, formation of countries are examined with a Hotelling type setup. The world lies on a straight line and the number and size of the nations formed is based on different factors such as fixed costs and transport costs.

In their model, Alesina and Spolaore find the equilibrium and efficient number of nations that should be formed in the world. It was established that the public good (i.e. the capital) will be optimally located in the centre of the country. Our paper adopts this concept as a major assumption. Most of the initial framework in our paper is based on the size of nations model, see appendix for more details.

Following Alesina and Spolaore (2003), regionalism is explored in Goyal and Staal (2004). Regional preferences were defined in terms of size, location and diversity. This will in turn define the number of resultant countries. Alesina and Spolaore extend their paper in Alesina, Spolaore and Wacziarg (2000), the relationship between openness in trade and the equilibrium size and number of nations were explored.

The literature mentioned analysed country size by looking at what is the most efficient and what will be the equilibrium. Tharakan and Thisse (2002) takes on a different approach and looks at how benefits, from openness to trade, differ between countries when they have different sizes. Country size was found to play an important part in benefits from trade. They look at countries already formed and not how they should be formed.

Since the world is formed into nations only once and is left to evolve over time, it makes more sense to look at integration of countries already formed than to look at formation of these countries. Here, we look at integration which would result in the sharing of the public good. This would include both the costs of providing the public good and the benefits which the citizens can derive from it. In this paper, we analyse what happens when two countries of different
size integrates, from a public good perspective.

This paper has two main sections. In the first, we will look at country formation and the location of the new country’s capital. In the second section, we look at unions and conditions which will benefit all parties in the union. In the second section, we introduce an infrastructure choice where infrastructure can reduce transport costs of individuals. Here, we borrow in part from Ghosh and Meagher (2005), where infrastructure investments are treated as a device to reduce the magnitude of transport cost in a Hotelling style model.

1.1 Basic setup

We use a framework with 2 countries of unequal size with their populations distributed uniformly; a geographically smaller country would also have a smaller population. A country’s gain would depend largely on the size of their direct neighbours. Therefore, we not only need to look at different sizes but specifically adjacent countries with different sizes. This framework will help us examine how country size affects the decision to integrate. We can investigate who are the winners and losers, if integration will occur and under what circumstances it will occur. We do this through two points of view, one from the social planner’s perspective and one from the voter’s perspective.

All analysis from this point forward will start from the following: the world will consist of two countries lying on a straight line. The smaller country will be on the left and of size 1. The larger country will be on the right and of size S. The smaller country will be country 1 and the larger country, country 2. All variables pertaining to the country will be subscripted with its respective number. Following Alesina and Spolaore (2003), each country’s capital is located in the middle of the country, this will be the most efficient location as the population is uniformly distributed. See Figure 1. It is assumed that each country incur a fixed cost of \( k \) (of maintaining a capital) and individuals incur a transport cost of \( \alpha \) (of being away from the capital). We analyse the changes to costs and benefits when a country goes from non-integrated to integrated.

Figure 1: Basic Setup (Showing transport costs and size differentials)
2 Country formation

In this section, we look at what happens when a new country is formed. When the countries form a new country like the reunification of Germany, we can assume that eventually there will be only one capital. The question to pose would then be where to locate the capital. We look at three possible alternatives, when either one of the capitals is retained or when a new capital is created. These alternatives will be explored in the following sub-sections.

When not integrated, each country has to bear the cost of maintaining its own capital. When integrated, the two countries share the costs of maintaining one capital. The trade-off from savings in fixed costs are the transport costs of the individuals, this may change when integration occurs as the location of the capital they use may change.

Costs before Integration

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<td>Total costs of $C_1$</td>
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**Proposition 1**

i) For overall welfare to improve, the parameter $k$ has to be sufficiently large. Therefore, there exists a minimum level of $k$, $k^*$, where integration will improve overall welfare if $k > k^*$.

ii) $k$ also has to be sufficiently large in order for the individual to experience a reduction in transport costs. Therefore, there exists a minimum level of $k$, $\hat{k}$, where integration will be voted for if $k > \hat{k}$.

iii) When $\hat{k} > k > k^*$, voting is inefficient as all $k$s in this range will see an increase in welfare if the countries integrate but will not be high enough for the median voter to benefit.

2.1 Retain smaller country’s capital

If $C_1$’s capital is retained, the individuals in $C_1$ do not face a change in transport costs, they face lower fixed costs as it is now shared. Therefore, all individuals in $C_1$ benefit. In $C_2$, most of the country’s individuals now face higher transport costs, only some individuals at the border will save in transport costs. See Figure 2.

**Lemma 1**

i) Integration will improve overall welfare if $k > k^*_{1r}$

ii) Integration will be voted for if $k > \hat{k}_{r1}$

iii) When $\hat{k}_{r1} > k > k^*_{r1}$, voting is inefficient.

Social planner’s perspective (Changes in total costs):

For the social planner to prefer to integrate, overall welfare must improve. Therefore, overall costs have to reduce. This will be true when $k > \frac{\alpha (2S + S^2)}{4}$. The minimum $k$ needed for overall welfare to be positive (when capital $s$ is retained) is
Figure 2: Transport costs when Capital 1 is retained.

\[ k_{r1}^* = \frac{\alpha(2S + S^2)}{4}. \]  

(1)

\( k_{r1}^* \) is increasing with \( S \).

**Voter’s perspective (Median voter’s choice in each country):**

Since all individuals in \( C_1 \) benefit, all voters in \( C_1 \) will vote for integration. In \( C_2 \), the median voter will vote for integration if \( k > \frac{\alpha S(S+1)^2}{2} \). Since, we need both countries to vote for integration, the minimum \( k \) needed for the vote to pass (when capital 1 is retained) is

\[ \tilde{k}_{r1} = \frac{\alpha S(S + 1)^2}{2}. \]  

(2)

\( \tilde{k}_{r1} \) is increasing with \( S \).

\( \tilde{k}_{r1} > k_{r1}^* \). When \( \tilde{k}_{r1} > k > k_{r1}^* \), overall welfare is positive and the vote will not pass. Voting at this point will be inefficient as integration will improve overall welfare but voters will not vote for it. □

### 2.2 Retain larger country’s capital

If \( C_2 \)’s capital is retained, the individuals in \( C_2 \) do not face a change in transport costs, they also face lower fixed costs as it is now shared. Therefore,
all individuals in $C_2$ benefit. In $C_1$, all individuals now face higher transport costs. See Figure 3.

![Figure 3: Transport costs when Capital 2 is retained.](image)

**Lemma 2**

i) Integration will improve overall welfare if $k > k_{r_2}^*$

ii) Integration will be voted for if $k > k_{r_2}$

iii) When $k_{r_2} > k > k_{r_2}^*$, voting is inefficient.

**Social planner’s perspective (Changes in total costs):**

For the social planner to prefer to integrate, overall welfare must improve. Therefore, overall costs have to reduce. This will be true when if $k > \alpha \left( \frac{1+2S}{1} \right)$. The minimum $k$ needed for overall welfare to be positive (when capital 2 is retained) is

$$k_{r_2}^* = \alpha \left( \frac{1+2S}{4} \right).$$

$k_{r_2}^*$ is increasing with $S$.

**Voter’s perspective (Median voter’s choice in each country):**

Since all individuals in $C_2$ benefit, all voters in $C_2$ will vote for integration. In $C_1$, the median voter will vote for integration if $k > \frac{\alpha(S+1)^2}{2S}$. Since, we need both countries to vote for integration, the minimum $k$ needed for the vote to pass (when capital 2 is retained) is

$$\tilde{k}_{r_2} = \frac{\alpha(S+1)^2}{2S}.$$
\( \tilde{k}_{r2} \) is increasing with S.  
\( \tilde{k}_{r2} > k_{r2}^* \), when \( \tilde{k}_{r2} > k > k_{r2}^* \), overall welfare is positive and the vote will not pass. Voting at this point will be inefficient as integration will improve overall welfare but voters will not vote for it. \( \Box \)

### 2.3 Relocate and build new capital

If the capital was to be relocated to a more efficient point (i.e., minimizes total transports costs), it will be located in the middle of the world, at \( \frac{1+S}{2} \). Transport costs now changes for individuals in both countries. See Figure 4.

![Figure 4: Transport costs if new capital is built.](image)

**Lemma 3**  

i) Integration will improve overall welfare if \( k > k_{new}^* \)  
ii) Integration will be voted for if \( k > k_{new} \)  
iii) When \( k_{new} > k > k_{new}^* \), voting is inefficient.

**Social planner’s perspective (Changes in total costs):**

For the social planner to prefer to integrate, overall welfare must improve. Therefore, overall costs have to reduce. This will be true when if \( k > \frac{\alpha S}{2} \). The minimum k needed for overall welfare to be positive (when new capital is chosen) is

\[
 k_{new}^* = \frac{\alpha S}{2}. \tag{5}
\]

\( k_{new}^* \) is increasing with S.

**Voter’s perspective (Median voter’s choice in each country):**

In \( C_1 \), the median voter will vote for integration if \( k > \alpha \left( \frac{1+S}{2} \right) \). In \( C_2 \), the median voter will only vote for integration if \( k > \frac{\alpha(3+S)}{2} \). Since, we need both
countries to vote for integration, the minimum $k$ needed for the vote to pass (new capital is chosen) is

$$
\hat{k}_{\text{new}} = \frac{\alpha(S + S^2)}{2}.
$$

(6)

as $\frac{\alpha(S + S^2)}{2} > \alpha \left( \frac{1+S}{2} \right)$. $\hat{k}_{\text{new}}$ is increasing with $S$.

$\hat{k}_{\text{new}} > k^*_{\text{new}}$, when $\hat{k}_{\text{new}} > k > k^*_{\text{new}}$, overall welfare is positive and the vote will not pass. Voting at this point will be inefficient as integration will improve overall welfare but voters will not vote for it.$\square$

2.4 Comparing the $k$s

Proposition 2 Building a new capital is most likely to ensure welfare benefits and retaining the larger country’s capital is most likely to ensure voters’ approval.

When we rank the different levels of $k$, we find the following (smallest on the left, largest on the right):

\[
\begin{align*}
    k^*_{\text{new}} &< k^*_{rl} < k^*_{rs} < \tilde{k}_{rl} < \tilde{k}_{\text{new}} < \tilde{k}_{rs}
\end{align*}
\]

Figure 5: Comparing $k$, For $1.618 > S > 2.2143$

Since all the $k$ values are increasing with $S$, the lower the $k$, the less restrictions there are on the parameters of $S$. Building a new capital requires the least parameter restrictions on $k$ for there to be welfare benefits. Retaining the larger country’s capital comes second and retaining the smaller country’s capital requires the most restrictions on $k$. Building a new capital will be the most likely alternative to ensure welfare benefits. Therefore, the social planner should choose this alternative if voters’ approval is not required.

\[
\begin{align*}
    k^*_{\text{new}} &< k^*_{rl} < \tilde{k}_{rl} < k^*_{rs} < \tilde{k}_{\text{new}} < \tilde{k}_{rs}
\end{align*}
\]

Figure 6: Comparing $k$, For $S < 1.618$ or $S > 2.2143$
For voters to vote for integration, retaining the larger country’s capital requires the least parameter restrictions on $k$. Building a new capital comes second and retaining the smaller country’s capital requires the most restrictions on $k$. This means that retaining the larger country’s capital is most likely to be approved by the countries’ voters as it requires the lowest level of parameterization. The social planner should choose this alternative if voters’ approval is required. □

3 Unions

In this section, we look at union of countries such as the EU. It is logical to assume that the capitals of these countries would not be changed as each country would retain its own sovereignty. We first start with the basic model and then extend it to include an infrastructure choice.

3.1 Base case

Proposition 3
i) When the 2 capitals are retained, integration will improve overall welfare.
ii) Majority of voters in the larger country will not vote for integration.

If the 2 original capitals are retained, individuals, in $C_2$, who were too far from their capital may now find $C_1$’s capital nearer and use it instead. The population between the 2 capitals will split into half and use their nearest capital. There is a reduction in transport costs faced by $C_2$ as observed in Figure 7, (changes in transport costs shown by the dotted lines). The combined country will now share the fixed costs of $2k$.

We notice that $C_1$ make a gain on the reduction in fixed costs. $C_2$ on the other hand makes a loss on the increase in fixed costs and a gain from transport.
cost reductions. The gains from the smaller country covers the losses from the larger country, which leaves a gain in transport cost reductions in the larger country. This means that overall welfare is better. Less than half the population in the larger country will make a net gain, this means that the majority will not vote for integration.

3.2 Infrastructure investment

In the previous sections we found that politically, integration is possible with one capital. In the case where both capitals are retained, integration will not be voted through. This does not explain real life cases where economic unions do exist. In this section, we add an infrastructure investment choice, to the case where both capitals are retained. The basic setup is again modified from the basic utility functions found in Alesina and Spolaore (2003). The idea of infrastructure is borrowed from Ghosh and Meagher (2005).

Infrastructure is a public good which can improve access to other public goods; better roads would reduce the time it takes to get to the capital. Governments often have to make choices on how much infrastructure to invest in. This section analyses the choice of a level of infrastructure which will reduce a country’s transport cost \( (\alpha) \). When the countries are not integrated, they make separate choices of infrastructure. When they are integrated, they choose a level of infrastructure together. The same basic structure is used from the previous model, instead of looking at the sharing of fixed costs; we now look at the benefits from making an infrastructure investment. We now find cases where it is possible to retain both capitals and have both countries vote for integration.

The effect a level \( I_j \) of infrastructure will have on the countries if they are not integrated is shown in Figure 8. \( I_j \) effectively lowers transport costs for all individuals.

![Figure 8: Effects of infrastructure investment on transport costs](image)

When the countries are integrated, they share a common level of \( I \) which is
different from their original $I_j$, some individuals in $C_L$ also benefit by going to
$C_\gamma$’s capital. This is shown by the dotted lines in Figure 9.

Figure 9: Difference between choosing infrastructure separately and as an inte-
grated country.

The level of infrastructure chosen will depend on how it is chosen. We
consider 2 alternatives, a voting outcome (where voters vote on a level of in-
frastructure) and a social optimum outcome (where a social planner chooses the
level of infrastructure). There will also be a cost associated with the infrastruc-
ture, $\beta I \ast \text{(size of the country)} + \gamma I^2$. $\beta$ is the component variable with the size
of the country and $\gamma$ varies only with the level of $I$. We will first examine the
case where $\beta$ is normalized to zero.

3.2.1 Choosing the level of investment

When not integrated,

When not integrated, we find that the social planner will choose the same
levels of $I$ as the median voter. We find this by maximising utility (total utility
for the social planner and individual utility for the median voter). When not
integrated, the median voter is located in between the capital and the edge of
the country, $l_m = s_j/4$, where $l$ denotes the distance away from the capital. In
$C_1$, $l_{m1} = 1/4$. In $C_2$, $l_{m2} = S/4$. See Appendix for details on finding the location
of the median voter. $C_1$ will choose $I_1 = \frac{1}{8}\gamma$. $C_2$ will choose $I_2 = \frac{1}{8}\frac{s^2}{\gamma}$.

When integrated,

Social planner: The social planner will choose the level of $I$ by maximizing
total benefits. When integrated, the social planner maximises total world utility
and will choose $I = \frac{1}{32\gamma} (2S + 3S^2 + 3)$

Median Voter: When considering the voting outcome for a choice of $I$, we
take the median voter’s choice. To find his choice, we need to find $l_m$ (the
distance the median voter is away from the capital. The median voter’s choice
becomes a maximization of $I_j l_m - \gamma \frac{I_j^2}{\text{sizeofcountry}}$. See Appendix for details on finding the location of the median voter.

When integrated, there are 2 possible cases for $l_m$ (The distance the median voter is away from his nearest capital.) When $S \leq 3$, $l_m = (1 + S)/8$. When $S > 3$, $l_m = S/6$.

Since the median voter can have 2 locations given the size of the country, we need to consider the $I$ chosen for these 2 cases.

In Voting Case 1, the infrastructure level chosen when $S \leq 3 = \frac{1}{16} \left(2S + S^2 + 1\right)$

In Voting Case 2, the infrastructure level chosen when $S > 3 = \frac{1}{12\gamma} (S + S^2)$

### 3.2.2 Social welfare

**Proposition 4** Overall welfare is better with integration when $S$ is upper bounded by $S^*$.

When the social planner chooses the level of $I$, overall welfare will be better as long as $S < 2.9416$. Where $S_{SO}^* = 2.9416$.

When the voters choose the level of $I$ and $S \leq 3$, overall welfare will be better as long as $S < 2.8901$. Where $S_{V1}^* = 2.8901$.

When the voters choose the level of $I$ and $S > 3$, overall welfare will be better as long as $S < 2.8814$. This is contradictory, therefore, when $S > 3$, overall welfare will not be better than if they were none integrated.

### 3.2.3 The voting decision

**Proposition 5** Majorities in both countries will prefer integration when $S$ is upper bounded by $\tilde{S}$.

A country will vote for integration if the median voter finds it beneficial to do so. Therefore, we need to look at the change in benefits of the median voter if integration is chosen. The changes in benefits for the median voter are:

\[ (I_i l_m - \gamma \frac{I_i^2}{1+S}) - (I_n l_m - \gamma \frac{I_n^2}{\text{sizeofcountry}}) \]

Where $I_i$ is the $I$ chosen if integrated and $I_n$ is the $I$ chosen if not integrated. The median voter in $C_1$ is located at $(1/4)$ and in $C_2$ is located in $(S/4)$.

When the social planner chooses the level of $I$, voters in $C_1$ will prefer integration when $S < 2.3935$, and in $C_2$ $S$ needs to be $< 13.406$. Since we need both countries’ voters to prefer integration, $S$ needs to be $< 2.3935$. Where $\tilde{S}_{SO} = 2.3935$.

When the voters choose the level of $I$ and $S \leq 3$, voters in $C_1$ will prefer integration when $S < 2.7093$, and in $C_2$ $S$ needs to be $< 5.1563$. Since we need both countries’ voters to prefer integration, $S$ needs to be $< 2.7093$. Where $\tilde{S}_{V1} = 2.7093$.

When the voters choose the level of $I$ and $S > 3$, voters in $C_1$ will prefer integration when $S < 2.7868$, and in $C_2$ $S$ needs to be $< 8$. Since we need both countries’ voters to prefer integration, $S$ needs to be $< 2.7093$. This is contradictory to the condition where $S > 3$, therefore, $C_1$ will never vote for integration here and thus integration will never be voted through. \(\square\)
3.3 Infrastructure traps

In this section, we allow $\beta$ (the cost component variable with the size of the country) to vary from zero. We find that as $\beta$ gets higher, there will exist infrastructure traps (upper limits on $\beta$, which will cause zero levels of investment. See Appendix for more details. We find that at certain levels of $\beta$, the large country will not choose to invest in infrastructure on its own but will do so as an integrated country. Conversely, we find that the small country will choose a level of infrastructure investment on its own but not as an integrated country. This is interesting as integration provides the large country an incentive to invest in infrastructure and a disincentive for the small country. Infrastructure traps get larger in the small country and smaller in the large country.

4 Conclusion

If we think of the countries integrating to form a new country, it is reasonable for the new country to have one capital. If this is the case, retaining the capital of the larger country is the most popular choice and would be most likely to be voted through. We have seen this historically with Germany where the capital in Berlin was retained. Voter’s choice aside, relocating the capital to a more central location would produce the largest welfare benefits.

If countries are to share the fixed costs of maintaining their capitals, the larger country would not vote for integration. This is despite of the fact that overall welfare will increase as a result. This seems to fit with the EU scenario, larger countries had rejected the constitution and smaller ones such as Luxembourg have voted it through. This suggests that individual voting may not be the best method when making integration decisions. Therefore, it may be better that such decision be left to a central governing body which can unbiasedly analyse the overall welfare effects of integration.

Policy makers can make voters more willing to vote for integration by adding an infrastructure choice into the mix. This is especially true when the size differential between the two countries is not too big. When the size difference between the countries is too big, and the level of infrastructure is decided upon via voting, then no country will prefer to integrate. This gives insight to why countries may prefer to unionize (e.g. the EU), as such union are seen as not only as a means to share the fixed costs of the public good but also as a means to cooperate in future changes to public goods (via improvements in infrastructure).

When the variable costs of infrastructure gets high, there may be infrastructure traps where no investment is made on infrastructure. An interesting observation from this is that large countries may not want to invest in infrastructure as an individual country but will do so as an integrated one. This may be because there are now more people to spread the costs over. On the other hand, the smaller country may invest in infrastructure as an individual country but not as an integrated one. This may be due to the fact that the optimum level
of infrastructure is now perceived as too high for the smaller country.

Integration can lead to overall efficiency with transport costs, but we often find that voters may not like to integrate. It is possible to include terms of integration which will give an overall benefit and have majorities prefer it. When policy makers make the decision to integrate, they should consider this when putting it to voters.
5 Appendix

A Alesina and Spolaore’s Size of Nations Model

The basic setup of this paper’s model is adapted from Alesina and Spolaore’s size of nations model. Presented here, is a simplified version of parts of the model used in this paper. The world population has a mass of one are distributed uniformly on a straight line. An individual’s utility on the line is given as:

\[ u_i = y - t_i + g - al_i \]

\( y \) is income, \( t_i \) are taxes paid by the individual, \( g \) is the gains from the public good, \( a \) is the transport cost and \( l_i \) is the distance the individual is away from the public good.

Total taxes must cover the cost of the public good, \( \int t_i\,di = k + gs \), where \( s \) is the size of the country. The sum of everybody’s utility is given as \( \int u_i\,di = y - (kN + g + a\int l_i\,di) \) where \( N \) is the number of countries in the world. The social planner chooses the optimum number of nations by maximizing total utility. The optimal number of nations is given as \( N^* = \sqrt{4a/k} \). Since The size of the world is 1, the size of each nation is \( 1/N^* \)

B Infrastructure investment: Finding the median voter

If the two countries choose not to integrate, individuals at the borders will face the highest transport costs, see Figure 10.

![Figure 10: Transport costs when not integrated](image-url)
When the 2 countries merge, transport costs will change for individuals in country 2 who are near the border of country 1. Figure 11 illustrates this.

![Figure 11: Transport costs when integrated](image)

To determine the voting equilibrium when voters vote on a level of I to reduce their transport costs, we need to find the person with the median transport costs (the median voter). \( l_m = \) the distance the median voter is away from the capital.

### B.1 Location of the median voter when not integrated

This is quite straightforward when the two countries do not integrate. The transport costs a country will face, when not integrated, is illustrated by Figure 12. The distribution of costs can be represented by Figure 13.

We need only look at the distribution of transport costs; the median voter is located where the area of the distribution is half of the size of the country. The median voter is located at \( s_j/4 \) from the capital, \( l_m = s_j/4 \).
Figure 12: Transport cost of country $j$ when not integrated

Figure 13: Distribution of transport costs when not integrated
B.2 Location of the median voter when integrated

The median voter is harder to find if the 2 countries choose to integrate. We find that they now face asymmetric transport costs. With the assumption of linear transport costs, we can identify three symmetric groups of areas, a, b and c, see Figure 14.

Figure 14: Transport costs when integrated (Classified into three groups)

The distribution of transport costs can be represented by Figure 15.

The median voter is located where the area of the distribution is half of the size of the two countries. Since the total length of the countries is 1+S, this area needs to be (1+S)/2.

\[ l_m \] is the distance the median voter is from his nearest capital. The median voter will be between 0 and 1/2, when \[ 4l_m = (s_1 + s_2)/2 \], and be between 1/2 and (1+S)/4, when \[ 4s_1/2 + 3l_m = (s_1 + s_2)/2 \].

This is falls in area A with \( l_m = (1+S)/8 \) (when \( S \leq 3 \)) or in area B where \( l_m = (S/6) \) (when \( S > 3 \)). The median voter will not be located in area C as (area A + area B) > (1+S)/2.
C \textbf{Allowing }\beta \textbf{ to vary}

C.1 Voting Outcome

When considering the voting outcome for a choice of $I$, we take the median voter’s choice. To find his choice, we need to find $l_m$ (the distance the median voter is away from the capital). The median voter’s choice becomes a maximization of $I_j l_m - \beta I_j^2$.

When not integrated, the median voter is located in the middle of the country, $l_m = s_j/4$. In $C_1$, $l_{m1} = 1/4$. In $C_2$, $l_{m2} = S/4$. When integrated, there are 2 possible cases for $l_m$ (The distance the median voter is away from his nearest capital.) When $S \leq 3$, $l_m = (1 + S)/8$. When $S > 3$, $l_m = S/6$.

C.1.1 Voting outcome when the countries remain separate

The infrastructure level chosen by $C_1$ via voting is

$$I_1 = \begin{cases} \frac{1}{2}, (1 - \beta) & \text{if } \beta < \frac{1}{4} \\ 0 & \text{if } \beta \geq \frac{1}{4} \end{cases} \quad (7)$$

There is an upper limit on $\beta = \frac{1}{4}$

$$\beta_1 = \frac{1}{4} \quad (8)$$

, as the I chosen cannot be negative.
The infrastructure level chosen by $C_2$ via voting is:

\[ I_2 = \begin{cases} \frac{1}{2\gamma} \left( \frac{S^2}{4} - S \beta \right) & \text{if } \beta < \frac{S}{4} \\ 0 & \text{if } \beta \geq \frac{S}{4} \end{cases} \]  

(9)

There is an upper limit on $\beta$:

\[ \beta_2 = \frac{S}{4} \]  

(10)

as the $I$ chosen cannot be negative.

C.1.2 Voting outcome when the countries integrate

Since the median voter can have 2 locations given the size of the country, we need to consider the $I$ chosen for these 2 cases. In Voting Case 1, the infrastructure level chosen when $S \leq 3 = I_{i(v1)}$

\[ I_{i(v1)} = \begin{cases} \frac{S+1}{16\gamma} \left( S + 1 - 8\beta \right) & \text{if } \beta < \frac{S+1}{8} \\ 0 & \text{if } \beta \geq \frac{S+1}{8} \end{cases} \]  

(11)

There is an upper limit on $\beta$:

\[ \beta_{i(v1)} = \frac{S+1}{8} \]  

(12)

as the $I$ chosen cannot be negative.

In Voting Case 2, the infrastructure level chosen when $S \leq 3 = I_{i(v2)}$

\[ I_{i(v2)} = \begin{cases} \frac{1}{12\gamma} \left( S^2 + S + \beta(6S - 6) \right) & \text{if } \beta < \frac{S^2+8}{6-6S} \\ 0 & \text{if } \beta \geq \frac{S^2+8}{6-6S} \end{cases} \]  

(13)

There is an upper limit on $\beta$:

\[ \beta_{i(v2)} = \frac{S^2 + S}{6 - 6S} \]  

(14)

as the $I$ chosen cannot be negative. $\beta_{i(v2)} < 0$ when $S > 1$, since $\beta > 0$, $I_{i(v2)}$ will always be zero.

C.2 Social Optimum

The social planner will choose the level of $I$ by maximizing total benefits.
C.2.1 When not integrated,

When not integrated, we find that the social planner will choose the same levels of I as the median voter. Therefore, the Is and the upper limits on $\beta$s are the same as the voting outcome when not integrated.

C.2.2 When integrated,

The infrastructure level chosen by the social planner is

$$I_{i_{(so)}} = \begin{cases} 
\frac{1}{327} (3S^2 + 2S + 3 - \beta(6S + 16)) & \text{if } \beta < \frac{3S^2 + 2S + 3}{16 + 16S} \\
0 & \text{if } \beta \geq \frac{3S^2 + 2S + 3}{16 + 16S} 
\end{cases} \quad (15)$$

There is an upper limit on $\beta$

$$\beta_{i_{(so)}} = \frac{3S^2 + 2S + 3}{16 + 16S} \quad (16)$$

, as the I chosen cannot be negative.

Since the upper limits on $\beta$ determine the level of Is chosen, the relationship between the betas will determine what levels of Is will be chosen by each country when not integrated and by the new country when integrated.

For voting outcome 1
$$\beta_L > \beta_{i_{(v1)}} > \beta_s$$

For voting outcome 2
$$\beta_L > \beta_s > 0 > \beta_{i_{(v2)}}$$

For social optimum
$$\beta_L > \beta_{i_{(so)}} > \beta_s$$

There are ranges of $\beta$ where a country may choose a level of I when not integrated and none when integrated, and vice versa.

References


Tharakan, J. and Thisse, J.-F. (2002), The importance of being small. Or when countries are areas and not points. Regional Science and Urban Economics 32, 381-408