Model Specification and Time-varying Jump Intensity: Evidence from S&P500 Returns and Options

Andrew Carverhill and Dan Luo∗

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ABSTRACT

This paper explores the specifications of jumps for modeling stock price dynamics and cross-sectional option prices. We exploit a long sample of about 16 years of S&P500 returns and option prices for model estimation. We explicitly impose the time-series consistency when jointly fitting the return and option series. We specify a separate jump intensity process which affords a distinct source of uncertainty and persistence level from the volatility process. The models are estimated by MCMC method. Our overall conclusion is that simultaneous jumps in return and volatility are helpful in fitting the return, volatility and jump intensity time series, while time-varying jump intensities improve the cross-section fit of the option prices. In the formulation with time-varying jump intensity, both the mean jump size and standard deviation of jump size premia are strengthened.

I. Introduction

The role of jumps in modeling stock return dynamics and cross-sectional option prices is widely examined in the literature, while the empirical results are disparate. Studies using time series of return data generally support the presence of jumps in returns, but diverge on jumps in volatility. Model performances regarding the cross section fit of option prices even disagree over the extra benefits of adding jumps in returns when volatility is already allowed to be stochastic. The tendency for jumps to cluster in time differs in models assuming either constant or time-varying jump arrival rates.1 Specifications of the jump factors have important implications for the dif-

∗The School of Finance, Shanghai University of Finance and Economics. Email: luo.dan@shufe.edu.cn

fusive volatility risk and jump risk premia, as model misspecifications can result in distorted premium estimates which should not be trusted.

In this paper, we fit five models which fall in the general affine jump diffusion class of Duffie, Pan, and Singleton (2000), which is familiar to and frequently applied by both academics and practitioners. The first model is the stochastic volatility (SV) model of Heston (1993). The SVJ and SVCJ models extend the SV model by introducing jumps in returns and simultaneous jumps in returns and volatility, respectively, both with constant arrival rates. The SVSJ and SVSCJ models make jump intensities in the SVJ and SVCJ models time-varying by specifying separate intensity processes. Therefore, we allow intensity processes to have their own sources of uncertainty and different persistency levels from the volatility processes. Evidences in support of independently specified intensity processes are presented, for instance, in Huang and Wu (2004), Santa-Clara and Yan (2010) and Christoffersen, Jacobs, and Ornthanalai (2012). The SVSCJ model is the most general one considered in this study.

We use 16 years of S&P500 index prices and up to two series of options with the shortest maturities beyond 2 weeks from January 1996 to April 2011 for model estimation. Since jumps rarely occur, long time series of data are crucial to estimate the parameters governing the jump behavior to reasonable accuracy. Volatility and jump intensity dynamics also depend on whether the sample encompasses typical high and low volatility/intensity periods. Time series consistency, that is, the same process that generates the stock prices should also determine the cross-sectional patterns and evolutions of the option prices, is explicitly imposed by restricting several parameters to be equal across the physical and risk-neutral measures. However, we allow the standard deviation of the return jump size to differ across the two measures since no arbitrage pricing does not require their equivalency. Model parameters are estimated using a Bayesian Markov Chain Monte Carlo (MCMC) method which automatically balances information in the returns and option prices through their joint likelihood. The MCMC method is shown to be well

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suited for estimating models with stochastic volatility. The latent state variables, namely, the volatility, jump time, jump size and jump intensity factors are achieved together with the posterior distributions of the parameters. It also generates more erratic volatility paths which might alleviate the tension between the relatively high volatility of volatility parameter, $\sigma_v$, estimated from option prices and the low $\sigma_v$ estimated from time series of returns.\(^3\)

Our empirical findings are summarized as follows. We estimate a stronger leverage effect between stock return and its volatility for all the five models. The correlation between return and volatility shocks ranges from -0.83 to -0.80, while the literature usually estimates the correlation to be around -0.50. This is likely due to a strengthened leverage effect over our sample period. Previous studies of Bakshi, Cao, and Chen (1997), Bates (2000), Pan (2002) and Eraker (2004) use data no later than 1996 when our sample starts.

The literature provides mixed results on the size, direction and statistic significance of the volatility risk premia ($\eta^v$). Interestingly, $\eta^v$ is estimated to be positive and significant in the SVCJ model, but becomes negative and significant in the SVSJ and SVSCJ models with time-varying jump intensity. Identification of $\eta^v$ depends on the implied volatility term structure which tends to be flat at the short maturities of option series used in our estimation (between 15 and 49 days). The jump size premia are estimated to be 0.02 and -0.02 for the SVJ and SVCJ models, respectively. The same premia increase to 0.11 and 0.07 for the SVSJ and SVSCJ models. This contrasts the large premia found in some studies but generally agrees with Eraker (2004) and Broadie, Chernov, and Johannes (2007). The standard deviation of jump size premia are very small in magnitude for the SVJ and SVCJ models, but largely inflated if the jump intensity is allowed to be time-varying. The SVSJ and SVSCJ models afford standard deviation of jump size premia of about 0.18. The large premia imply high kurtoses of the risk-neutral return distributions and more concave implied volatility smile curves. Santa-Clara and Yan (2010) estimates $\sigma_y^O$ to be between 0.16 and 0.28, close in magnitude to ours of about 0.24.

Time series fittings of the return and volatility processes support jumps in volatility. The

\(^3\)For more discussions on the advantages of the MCMC method, see Eraker, Johannes, and Polson (2003) and Eraker (2004). Li, Wells, and Yu (2008) and Yu, Li, and Wells (2011) explore this method to fit Lévy models.
standardized return, volatility and jump intensity residuals measured by the historic Brownian innovations of the their processes should follow standard normal distributions given a correctly specified model. The SVCJ and SVSCJ models produce the lowest absolute skewnesses and kurtoses among the five models. Especially, the kurtosis of the return residuals of the SVCJ model is 3.28 and very close to 3.00 of the standard normal distribution. The skewnesses of the volatility residuals for the SV, SVJ and SVSJ models are unanimously larger than 0.69, while the the SVCJ and SVSCJ models produce skewnesses of 0.20 and 0.07, respectively. The kurtoses of the volatility residuals for the SV, SVJ and SVSJ models are uniformly larger than 6.01, while the the SVCJ and SVSCJ models produce kurtoses of 4.03 and 5.26, respectively. Similar improvement in the higher moments of the jump intensity residuals is achieved for the SVSCJ model over the SVSJ model.

Comparing the SVJ to the SVSJ and the SVCJ to the SVSCJ models, the time series fittings become slightly better or even worse when allowing for time-varying jump intensity. The picture changes markedly for the option pricing fittings, both in-sample (for the options series used in model estimation) and out-of-sample (for the large cross-section of 264922 option contracts). The SVJ and SVCJ models show little improvement in in-sample and out-of-sample option pricing performances over the SV model. However, the SVSJ model reduces the absolute option pricing errors over the SVJ model by 20 cents (63%) in-sample and 20 cents (15%) out-of-sample. The SVSCJ model reduces the absolute option pricing errors over the SVCJ model by 16 cents (60%) in-sample and 25 cents (19%) out-of-sample. The negligible gains obtained by introducing jumps with constant arrival rates in return and volatility to fit the cross-sectional option prices stand in contrast to the as high as 40-50% reductions in option pricing errors reported by Bakshi, Cao, and Chen (1997) and Broadie, Chernov, and Johannes (2007), but generally agree with Eraker (2004). The differences in performances are largely attributable to the extent to which the time-series consistency is imposed. We and Eraker (2004) jointly determine the parameters of the physical and risk-neutral dynamics using the combined return and option data series, while Bakshi, Cao, and Chen (1997) and Broadie, Chernov, and Johannes (2007) calibrate the model parameters to the option prices for each sample date. The modest performance improvements of the SVSJ and
SVSCJ models also contradict the results in Bates (2000), Pan (2002) and Eraker (2004), who find the benefits to be small in magnitude or even mixed. All the three papers consider time-varying jump intensities, while they assume the intensity processes to be perfectly correlated with the volatility processes. Our specification of separate intensity processes allow for distinct sources of uncertainty, as well as different levels of persistence from the volatility processes.

Our paper differs from Bakshi, Cao, and Chen (1997) and Bates (2000) in that we fit our models to joint return and option data while they rely solely on options. Pan (2002) and Eraker (2004) exploit a shorter return and option price data series than ours, and their jump intensity factors are specified to be deterministic and affine functions of volatility. We specify separate intensity processes thus allow jump intensities to have their own sources of uncertainty. Broadie, Chernov, and Johannes (2007) use over 16 years of S&P futures options from 1987 to 2003 for model estimation, with a sample length slightly longer than ours. However, their analysis is complicated by the wildcard feature of the futures options. The jump intensities are constrained to be constant in their models. And they do not impose the time series consistency fully by jointly fitting the physical and risk-neutral dynamics from returns and options, but borrow the constrained parameter estimates from the time series study of Eraker, Johannes, and Polson (2003). Santa-Clara and Yan (2010) and Christoffersen, Jacobs, and Owrthanalai (2012) both explore dynamic jump intensities with separate processes, whereas their model frameworks are non-affine and discrete-time, respectively.

The remainder of this paper is organized as follows. Section 2 discusses our model specifications, measure changes and the corresponding option pricing issues. Section 3 outlines the MCMC scheme we design to simulate the model parameters and latent state variables. Section 4 summarize the index returns and the option series we construct for model estimation. We present the empirical results in Section 5. Section 6 concludes this paper. Detailed MCMC sampling algorithms are provided in the appendix.
II. Models and Option Pricing

Our models for the S&P500 index price dynamics fit in the affine jump-diffusion framework of Duffie, Pan, and Singleton (2000). This class of models is the mainframe of derivatives pricing due to its analytic tractability. In this section, we will outline the five models we adopt for the stock price dynamics and discuss the corresponding option pricing methods.

A. Affine Jump Diffusion Return Process

On a fixed probability space \((\Omega, \mathcal{F}, \mathbb{P})\) with an information filtration \((\mathcal{F}_t)\) satisfying the usual conditions, we assume the dynamics of the logarithmic index price is characterized by the following data-generating process

\[
\begin{align*}
    dY_t &= \mu dt + \sqrt{V_t} dW^{(1)}_t + dJ^v_t, \quad (1) \\
    dV_t &= \kappa_v(\theta_v - V_t) dt + \sigma_v \sqrt{V_t} (\rho dW^{(1)}_t + \sqrt{1 - \rho^2} dW^{(2)}_t) + dJ^v_t, \quad (2) \\
    dh_t &= \kappa_h(\theta_h - h_t) dt + \sigma_h \sqrt{h_t} dW^{(3)}_t, \quad (3)
\end{align*}
\]

where \(Y_t = \log(S_t)\) is the log index price; \(V_t\) is the volatility process and \(h_t\) is the jump intensity process. \(W^{(1)}_t, W^{(2)}_t \) and \(W^{(3)}_t\) are independent Brownian motions. \(\mu\) measures the expected index return. \(\mu\) is assumed to be constant for simplicity.\(^4\) \(\kappa_v\) and \(\kappa_h\) are the mean-reverting speeds of \(V_t\) and \(h_t\), respectively. \(\theta_v\) and \(\theta_h\) are the long-run means of \(V_t\) and \(h_t\), respectively. \(\sigma_v\) and \(\sigma_h\) are the so-called volatility of volatility and volatility of jump intensity parameters, respectively.

The volatility process \(V_t\) extends the "square-root" process in Heston (1993) by adding a jump terms \(J^v_t\). \(\rho\) models the typically negative correlation between return and changes in volatility, or the "leverage effect" discussed in Black (1976) and Christie (1982).

\(^4\)A more elaborate drift is shown to be unimportant by Eraker, Johannes, and Polson (2003). The simple drift has also been adopted by Eraker (2004), Li, Wells, and Yu (2008) and Yu, Li, and Wells (2011).
The jump terms $J^y_t$ and $J^v_t$ follow compound Poisson processes.

$$J^y_t = \sum_{n=1}^{N_t} \xi^y_n, \quad J^v_t = \sum_{n=1}^{N_t} \xi^v_n,$$

where $\xi^y_n \sim \text{Normal}(\mu_y, \sigma^2_y)$ and $\xi^v_n \sim \text{Exponential}(\mu_v)$. Jumps in return and volatility are assumed to be perfectly correlated and arrive simultaneously. Jump sizes $\xi^y_n$ and $\xi^v_n$ can also be correlated. However, it is generally hard to estimate the correlation precisely under $\mathcal{P}$ measure due to rare occurrence of jumps. Under $\mathcal{Q}$ measure, the correlation is hard to pin down since $\mu^Q_{y}$ plays the same role in determining the conditional return distribution. We follow Broadie, Chernov, and Johannes (2007) to assume independence between $\xi^y_n$ and $\xi^v_n$ for simplicity.

$N_t$ follows a Poisson counting process with time varying jump intensity $H_t$. $H_t$ is assumed to be an affine function of $V_t$ and $h_t$.

$$H_t = \lambda_0 + \lambda_1 V_t + h_t.$$  \hspace{1cm} (5)

where $\lambda_0$ and $\lambda_1$ are positive constants. Both $V_t$ and $h_t$ are modeled by autonomous "square-root" processes. Therefore, $H_t$ always stays positive. Inclusion of $V_t$ in $H_t$ is motivated by the intuition that jump intensity might be correlated with volatility. That is, jumps arrive more frequently when the market is more volatile. Note that $H_t$ also has another independent source of uncertainty introduced by $h_t$. Five models with different specifications of the jump components are discussed in this paper.

A.1. SV Model

The stochastic volatility (SV) model of Heston (1993) is achieved by restricting both jump terms and the intensity process $h_t$ to be 0.

A.2. SVJ Model

The stochastic volatility with jump in return (SVJ) model sets $J^r_t$ to be 0. The jump intensity is assumed to be constant and equal to $\lambda_0$. Thus $\lambda_1 = 0$ and $h_t = 0$. 

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A.3. SVCJ Model

The stochastic volatility with correlated jumps in return and volatility (SVCJ) model includes compound Poisson jumps in both return and volatility. The jump intensity is again assumed to be constant and equal to $\lambda_0$. Thus $\lambda_1 = 0$ and $h_t = 0$.

A.4. SVSJ Model

The SVJ with stochastic jump intensity (SVSJ) model explores the time variation in the jump intensity. It is attained by restrain $J^Y_t$ to be 0. Here, we further assume $\lambda_0 = 0$ since the long-run mean of $h_t$, $\theta_h$, serve a similar function.

A.5. SVSCJ Model

The SVCJ with stochastic jump intensity (SVSCJ) model extends the SVSJ model by allowing simultaneous jumps in volatility. It is the most general model considered in this paper. We let $\lambda_0 = 0$ for the same reason as in the SVSJ model.

B. Change of Measure

The market in Black and Scholes (1973) is complete with respect to the riskfree bank account and the underlying stock. The market generated by (1) to (3) is incomplete due to the random jump sizes in return and volatility, even with additionally a finite number of option contracts. The state-price density is not unique. Following Pan (2002), we define market price of Brownian shocks as

$$ \zeta_t^{(1)} = \eta^y \sqrt{V_t}, \quad \zeta_t^{(2)} = -\frac{1}{\sqrt{1 - \rho^2}} \left( \rho \eta^y + \frac{\eta^v}{\sigma_v} \right) \sqrt{V_t}, $$

where $\eta^y$ and $\eta^v$ are constants. The state-price density of the jump component is:

$$ U_t = \prod_{n=1}^{N_t} \left( \frac{H^Q_n \pi Q(\tau_n, \xi_n)}{H_{\tau_n} \pi(\tau_n, \xi_n)} \right) \exp \left( \int_0^t \left\{ \int_{\xi} \left[ H_s \pi(s, \xi) - H^Q_s \pi Q(s, \xi) \right] d\xi \right\} ds \right). $$

8
where $\xi = (\xi^y, \xi^v)$ are the sizes of jumps. $\pi$ and $\pi^Q$ are the physical and risk-neutral jump size distributions, respectively. Here, we assume $\pi^Q(\xi^y) = \text{Normal}(\mu^Q_y, (\sigma^Q_y)^2)$ and $\pi^Q(\xi^v) = \text{Exponential}(\mu^Q_v)$. The risk-neutral jump intensity $H^Q_t = cH_t$, where $c$ is a constant coefficient. This allows a premium for jump timing uncertainty. Although the changes of measure for the Brownian shocks only adjusts the drift, the change of measure for jumps is quite flexible. In our models, all the parameters ($\mu_y, \sigma_y, \mu_v, c$) are allowed to be different across both measures. However, due to empirical identification problems discussed in Broadie, Chernov, and Johannes (2007), Eraker (2004), Pan (2002) and Yu, Li, and Wells (2011), we restrict $c = 1$ and $\mu^Q_v = \mu_v$. The constraint $\sigma^Q_y = \sigma_y$ is imposed by equilibrium models such as Bates (1988) and Naik and Lee (1990), but it is not required by no-arbitrage pricing.

The Radon-Nikodym derivatives are

$$\frac{dQ}{dP}|_t = \exp\left\{-2 \sum_{i=1}^{N} \int_0^t \xi_s^{(i)} dW^{(i)}_s - \frac{1}{2} \sum_{i=1}^{2N} \left( \int_0^t (\xi_s^{(i)})^2 ds \right)\right\} U_t.$$  

(8)

The risk-neutral return dynamics are

$$dY_t = (r_t - \delta_t - 0.5V_t - H_t \bar{\mu}^Q_j)dt + \sqrt{V_t}dW^{(1)}_t(Q) + dJ^Y_t(Q),$$  

(9)

$$dV_t = [\kappa_v(\theta_v - V_t) + \eta^v V_t]dt + \sigma_v \sqrt{V_t}(\rho dW^{(1)}_t(Q) + \sqrt{1-\rho^2}dW^{(2)}_t(Q)) + dJ^v_t(Q),$$  

(10)

$$dh_t = \kappa_h(\theta_h - h_t)dt + \sigma_h \sqrt{h_t}dW^{(3)}_t,$$  

(11)

where $r_t$ is the riskfree interest rate; $\delta_t$ is the dividend yield. $H_t = \lambda_0 + \lambda_1 V_t + h_t$ is the jump intensity; $\bar{\mu}^Q_j = \exp(\mu^Q_v + 0.5(\sigma^Q_v)^2) - 1$ is the risk-neutral jump compensator in returns. Both $r_t$ and $\delta_t$ are assumed to be constant for simplicity.

C. Option pricing

Under the equivalent probability measure $Q$, the price of a European call option, $C_t$, can be computed as follows.

$$C_t = E^Q(e^{-\rho(T-t)}\max(S_T - K, 0) | \mathcal{F}_t).$$  

(12)

Following Duffie, Pan, and Singleton (2000), the complex-valued ODEs for $\alpha$ and $\beta$ are

\[
\begin{align*}
\dot{\beta}_1(t) &= 0, \\
\dot{\beta}_2(t) &= 0.5\beta_1(1 - \beta_1) + \lambda_1 \mu_1 \tilde{\beta}_1 + (\kappa_v - \eta^v - \rho \sigma_v \beta_1)\beta_2 - 0.5\sigma_v^2 \beta_2^2 - \lambda_1 \vartheta(\beta(t)), \\
\dot{\beta}_3(t) &= \mu_j \beta_1 + \kappa_h \beta_2 - 0.5\sigma_h^2 \beta_2^2 - \vartheta(\beta(t)), \\
\dot{\alpha}(t) &= r_t - \kappa_v \theta_v \beta_2 - \kappa_h \theta_h \beta_3 - \lambda_0 \vartheta(\beta(t)),
\end{align*}
\]

where $\vartheta(\beta(t)) = e^{\alpha(t) + \beta(t)} - 1$ is the transform of the jump size distributions. The above ODEs have boundary conditions $\beta(T) = u$ and $\alpha(T) = 0$. For the SV, SVJ, SV CJ, SVSJ models considered in this paper, the ODEs affords analytic solutions. However, for the SVSCJ model, they need to be solved by numerical methods like the fourth-order Runge-Kutta.

Once we get $\alpha$ and $\beta$, the option price is given as

\[
C_t(d, X_t, K, T, \Theta) = G_{d,-d}(-\ln(K); X_t, t, T, \Theta) - KG_{0,-d}(-\ln(K); X_t, t, T, \Theta),
\]

where $d = (1, 0, 0)^T$; $K$ is the strike price; $T$ is the expiring date; $\Theta$ is the vector which stacks all the risk-neutral parameters. $X_t = (Y_t, V_t, h_t)^T$ is the state variables at time $t$. The $G_{a,b}(\cdot; X_t, t, T, \Theta)$ function is given by

\[
G_{a,b}(y; X_t, t, T, \Theta) = \frac{\psi^\Theta(a, X_t, t, T)}{2} - \frac{1}{\pi} \int_0^\infty \frac{\text{Im}[\psi^\Theta(a + ivb, X_t, t, T)e^{-ivy}]}{v} dv,
\]

\[
\psi^\Theta(u, X_t, t, T) = e^{\alpha(t) + \beta(t)T} X_t,
\]
III. Econometric Methodology


We use joint S&P500 index returns and options to estimate our models. The index returns are denoted by $Y_t$. We form two option series for model identification: One at-the-money and one out-of-the-money with strike-to-forward-price ratio close to 0.95. The option price series are denoted by $C_t$. In our framework, the price of a European call option is a function (denoted by $F$) of the state variable $X_t = (Y_t, V_t, h_t)$, the contract variables $K$ and $T - t$, and the parameters stacked in $\Theta$. Assuming that the option prices are observed with measurement errors, we have

$$C_t = F(X_t, K, T - t, \Theta) + \omega_t,$$

where $\omega_t \sim Normal(0, (s \cdot BAS_t)^2)$. That is, $\omega_t$ standardized by the option bid ask spread $BAS_t$ is i.i.d. normally distributed with mean 0 and standard deviation $s$. Note that we do not allow time

series dependence in the option price errors since we generally use different option contracts on different days.\footnote{Alternatively, Eraker (2004) constructs the option series by randomly choose one option at its first trading day and follow it till no further trading is observed. He introduces the first-order correlation in the option price errors based on the idea that if pricing error is high on one day, it tends to be high on the next day.} Our construction of the options data series more resembles that of Pan (2002). We use \( Y_t \) and the ATM \( C_t \) to estimate the \( SV, SVJ \) and \( SVCJ \) models. But we include addition OTM option series to estimate the \( SVSJ \) and \( SVSCJ \) models to exploit the different sensitivities of the OTM options from the ATM options to the risk factors.

Our MCMC method relies on a first-order Euler discretization of the continuous time models in (1) through (3). The time-step is chosen to be one day (1/252 years) to reduce the discretization errors.\footnote{The simulation in Eraker, Johannes, and Polson (2003) shows that the errors for daily frequency are small.} The dynamics of the index returns and option prices are generally summarized as follows.

\[
Y_{t+1} = Y_t + \mu \Delta + \sqrt{V_t} \Delta \varepsilon^y_{t+1} + \Delta J^y_{t+1},
\]

\[
V_{t+1} = V_t + \kappa_v (\theta_v - V_t) \Delta + \sigma_v \sqrt{V_t} \Delta \varepsilon^v_{t+1} + \Delta J^v_{t+1},
\]

\[
h_{t+1} = h_t + \kappa_h (\theta_h - h_t) \Delta + \sigma_h \sqrt{h_t} \Delta \varepsilon^h_{t+1},
\]

\[
C_{t+1} = F(X_{t+1}, K, T - t - 1, \Theta) + sBAS_{t+1} \varepsilon^c_{t+1},
\]

where \( \Delta = \frac{1}{252} \); \( \varepsilon^y_{t+1}, \varepsilon^v_{t+1}, \varepsilon^h_{t+1}, \varepsilon^c_{t+1} \sim Normal(0, 1) \) and are not serially correlated. \( corr(\varepsilon^y_{t+1}, \varepsilon^v_{t+1}) = \rho \). \( \varepsilon^h_{t+1} \) and \( \varepsilon^c_{t+1} \) are independent of each other and both independent of \( \varepsilon^y_{t+1} \) and \( \varepsilon^v_{t+1} \).

With repeated applications of the Bayes rule, the joint posterior density function of the option prices, state variables, jump times, jump sizes and parameters naturally attains.

\[
p(C, X, \xi^y, \xi^v, J, \Theta) \propto p(C|X, \Theta)p(X, \xi^y, \xi^v, J|\Theta)p(\Theta).
\]  

To efficiently sample from this density, we derive a Gibbs sampling scheme to reduce the high-
dimension of the problem. For $g = 1, 2, ..., G$ where $G$ is the total number of simulations,

$$V_{t}^{(g)} \sim p(C_t | Y_t, V_{t-1}^{(g)}, h_{t-1}^{(g-1)}, \Theta^{(g-1)})$$

$$\times p(Y_t, V_{t}^{(g)} | Y_{t-1}, V_{t-1}^{(g)}, f_{t-1}^{(g-1)}, (\xi_t)^{(g-1)}, (\xi_t^v)^{(g-1)}, \Theta^{(g-1)})$$

$$\times p(Y_{t+1}, V_{t+1}^{(g)} | Y_t, V_{t}^{(g)} f_{t}^{(g-1)}, (\xi_t)^{(g-1)}, (\xi_t^v)^{(g-1)}, \Theta^{(g-1)})$$

$$\times p(J_{t+1}^{(g-1)} | V_{t}^{(g)}, h_{t}^{(g-1)}, \Theta^{(g-1)}),$$

(26)

$$h_{t}^{(g)} \sim p(C_t | Y_t, V_{t}^{(g)}, h_{t-1}^{(g)}, \Theta^{(g-1)})$$

$$\times p(h_{t}^{(g)} | h_{t-1}^{(g)}, \Theta^{(g-1)}) p(h_{t+1}^{(g)} | h_{t}^{(g)}, \Theta^{(g-1)}) p(J_{t+1}^{(g-1)} | V_{t}^{(g)}, h_{t}^{(g)}, \Theta^{(g-1)}),$$

(27)

$$J_{t}^{(g)} \sim p(Y_t, V_{t}^{(g)} | Y_{t-1}, V_{t-1}^{(g)}, f_{t-1}^{(g-1)}, (\xi_t)^{(g-1)}, (\xi_t^v)^{(g-1)}, \Theta^{(g-1)}) p(J_{t}^{(g)} | V_{t}^{(g)}, h_{t}^{(g)}, \Theta^{(g-1)}),$$

(28)

$$(\xi_t)^{(g)} \sim p(Y_t, V_{t}^{(g)} | Y_{t-1}, V_{t-1}^{(g)}, f_{t}^{(g-1)}, (\xi_t)^{(g-1)}, (\xi_t^v)^{(g-1)}, \Theta^{(g-1)}) p((\xi_t)^{(g)} | \Theta^{(g-1)}),$$

(29)

$$(\xi_t^v)^{(g)} \sim p(Y_t, V_{t}^{(g)} | Y_{t-1}, V_{t-1}^{(g)}, f_{t}^{(g-1)}, (\xi_t)^{(g)}, (\xi_t^v)^{(g-1)}, \Theta^{(g-1)}) p((\xi_t^v)^{(g)} | \Theta^{(g-1)}),$$

(30)

$$\Theta^{(g)} \sim p(C | Y, V^{(g)}, h^{(g)}, \Theta^{(g)}) p(Y, V^{(g)}, h^{(g)}, (\xi)^{(g)}, (\xi^v)^{(g)}, J^{(g)} | \Theta^{(g)}) p(\Theta^{(g)}),$$

(31)

where we have simplified the transition densities of the $V_t$ and $h_t$ by dropping the terms not including $V_t$ and $h_t$ respectively based on their Markov properties. $g = 0$ corresponds to the starting states and initial parameters. Due to the numerical inversion to get the option prices, the state variables and the risk-neutral parameters no longer afford standard conditional distributions from which we can directly draw posterior samples. Instead, a Metropolis-Hastings step is performed for each variable and each parameter over each iteration. Detailed sampling scheme for the general model can be found in the appendix.

IV. Data

The joint returns and options data are obtained from OptionMetrics on Center for Research in Security Prices (CRSP). The daily close prices of the S&P500 index and the index options are sampled from January 4, 1996 till April 29, 2011, with totally 3855 trading days. The risk free
term structure for each day which is derived from BBA LIBOR rates and settlement prices of CME Eurodollar futures is also provided by OptionMetrics. This same index and index options with different sample periods are previously used by Aït-Sahalia and Lo (1998), Aït-Sahalia, Wang, and Yared (2001), Pan (2002), Eraker (2004) and Yu, Li, and Wells (2011), among others.

One price/return series and two option series are constructed for model estimation. The daily S&P500 index close prices are used to form the log price/return series. We apply several standard filters provided in the literature to screening the option prices. To ensure certain interest in the option contracts, we discard options not-traded during the day or with open interest less than 100. To alleviate the effects of price discreteness and bid-ask spreads on the option values, we further exclude options with bid quotes less than $0.1. Since short maturity options are generally more liquid than longer maturity ones, we choose options with the shortest maturity beyond two weeks on each trading day to form the option series. Option prices are then calculated as the mid of the best bid and ask quotes. To reduce any lack of timeliness between the option quotes and the underlying index prices, we back out the implied forward price each day using the put-call parity. The same procedure has been previously applied, for instance, by Aït-Sahalia and Lo (1998) and Eraker (2004).

An at-the-money option series is constructed by picking the option contracts with strike-to-forward price ratios closest to 1.00 each day. An additional out-of-the-money option series is constructed by picking the option contracts with strike-to-forward price ratios closest to 0.95 each day. Options in this additional series have different sensitivity to the risk sources from options in the at-the-money series. Thus, they are supposed to help identify the risk premia and the time varying jump intensity when estimating the SVSJ and SVSCJ models. The idea of putting in an addition out-of-the-money option series to facilitate identification is pursued by Pan (2002) and followed by Santa-Clara and Yan (2010).

The S&P500 index price levels, daily log returns, the S&P500 index option at-the-money implied volatility, together with the CBOE Volatility Index (VIX) are plotted in Figure (1). The daily closing VIX can be downloaded on the CBOE website. A detailed documentation can be found in CBOE (2003).
S&P500 index levels plotted in the top panel of Figure (1) are on average ascending during the sample period. Two lasted market downturns happen during the burst of the Internet Bubble (2001-2002) and the financial crisis (2008-2009). Return spikes are observed in the 2nd panel which plots the index log returns. Clustering of large absolute returns is also evident. The at-the-money option implied volatility plotted in the 3rd panel is very close to the VIX shown in the bottom panel. Both volatility series present time variations as well as high persistence. Figure (1) gives a first motivation for including stochastic volatility and stochastic jump intensity in the return dynamics.

Figure (2) plots the contract variables of the two option series. The top panel shows the time-to-maturity of the options for each day. Both option series choose the shortest maturity contracts beyond 2 weeks thus shares the same time to maturity for a day. Both series show varying strike-to-forward price ratios due to the availability of traded option contracts. However, the strike-to-forward price ratios of the at-the-money option series mainly fluctuate between 0.99 and 1.01. The strike-to-forward price ratios of the out-of-the-money option series mainly fluctuate between 0.94 and 0.96.

Summary statistics of the return and option series are provided in Table (I). During our sample period, the S&P500 index has a average return of 5.0% and volatility of 20.6% (both annualized). Although the returns are slightly negatively skewed, they show a high kurtosis of 10.5. The highest and lowest returns observed are 11.0% and -9.5%, respectively. The option implied forward prices are on average slightly higher than the spot index prices. Since we fixed the moneynesses of the option series, the option strike prices vary according to changes in the forward prices. In general, different option contracts are chosen on different days. The time-to-expirations of the option series ranges between 15 and 49 days, with an average of 30 days and a standard deviation of 9 days. The out-of-the-money option series have a mean value of $9.8, which is lower than the mean at-the-money option price of $24.9. Consistent with a smile in the option implied volatility, the out-of-the-money options have a mean implied volatility of 23.9%, which is higher than the mean at-the-money option implied volatility of 20.0%. The strike-to-forward price ratios of the at-the-money and out-of-the money series have means of 1.0001 and 0.9501,
with standard deviations 0.0028 and 0.0035, respectively.

V. Empirical Results

In this section, we will first present the parameter estimates of the five models considered in this paper. A discussion of the volatility and jump risk premia will be included. We then turn to the volatility, jump size, jump time and jump intensity variables implied from the joint S&P500 index returns and options. The time series fit of the return, volatility and jump intensity processes is analyzed based on the standardized residuals of the processes. Finally, we compare the in-sample and out-of-sample option pricing performances of the models.

A. Parameter estimates

Posterior means and standard deviations of the parameter estimates are reported in Table (II). All the parameters are annualized. The studies of Eraker, Johannes, and Polson (2003), Eraker (2004) and Broadie, Chernov, and Johannes (2007), among others, quote their results based on a standardized time interval of one day. We annualize their results to achieve a direct comparison with ours. The parameters which are determined by the option prices or joint index returns and option prices are pinned down with high accuracy, while the physical parameters relying solely on the returns are less precisely estimated. The least confidence of parameter estimates is found with the parameters determining the sizes of jumps in returns.

The SV model produces the highest long-run mean of the volatility process with $\theta_v = 0.0539$, It corresponds to an annualized volatility of 23.3%, which is higher than the unconditional average of the at-the-money option implied volatility, 20.0%. It is also higher than the estimates reported in other studies. Note that our sample includes an extremely volatile period of the financial crisis when VIX reaches over 75%. The higher long-run mean reflects the relative volatile sample episode. The lowest long-run mean of the volatility process, 0.0214, is achieved with the SVCJ model. It equals to an annualized volatility of 14.6%. A significant proportion of volatil-

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9For instance, the mean-reverting speed and jump intensity are multiplied by a factor of 252.
ity is explained by jumps in volatility, as the SVJ model has an annualized volatility of 22.1%. This is also true for the SVSCJ model, which implies an long-run volatility of 16.3%, lower than 19.4% for the SVSJ model. The mean jump sizes of volatility are 0.0282 and 0.0283, very close to each other, for SVCJ and SVSCJ models, respectively. Assuming volatility is at its long-run level, an average-sized jump leads volatility to increase to 22.3% for the SVCJ model, and to 23.5% for the SVSCJ model.

The mean-reverting speed of the volatility process, $\kappa_v$, ranges from 2.22 to the SVJ model and 5.51 for the SVSCJ model. Two things are noted here. First, the SV and SVJ model have mean-reverting speeds of 2.30 and 2.22, respectively, which are lower than the estimates reported in previous studies. For instance, Pan (2002) and Eraker (2004) find them to be around 5-6. This again indicates the special features of the relatively long sample used in our estimation. The 2005-2006 period is characterized as a relatively tranquil period, followed by the turbulent period from late 2008 to early 2009 with the full unfolding of the financial crisis. These extreme volatility episodes are translate into lower mean-reverting speeds in the models. The other interesting phenomenon is that the diffusion part of the volatility process becomes more smooth when we add in jumps in volatility or admit a separate jump intensity process. The SVCJ and SVSJ models have mean-reverting speeds of 3.89 and 3.56 respectively. This is consistent with our intuition since large upper movements of volatility are recognized as jumps in the SVCJ model or alleviated as the jump intensity increases. The SVSCJ model incorporates both effects and displays the highest mean-reverting speed of 5.51.

The diffusive volatility risk premium equals to the difference between the mean-reverting speeds of the volatility process under the physical and risk-neutral measures. That is, $\eta^v = \kappa_v - \kappa_v^Q$. $\eta^v$ is hard to be tied down. The literature takes different approaches for assessment but the results are disparate. Coval and Shumway (2001) and Bakshi and Kapadia (2003) form delta-neutral option portfolios and expect the sign of the option portfolio returns to coincide with the sign of the volatility risk premium under their continuous-time option hedging strategies. They do find large negative returns associated with their option portfolios. But their results do not disentangle the effects of jump risk premia in price or volatility. Branger and Schlag (2004) argues that these tests
lack statistic power due to discretization errors and possible model mis-specifications in practical situations. Driessen and Maenhout (2006) separately quantify the volatility and jump risk premia using a multi-factor APT style model. The volatility risk premium is shown to be statistically insignificant. Carr and Wu (2009) approximates the risk-neutral return variance or the variance swap rate using a portfolio of options. Together with the realized variance constructed with high-frequency return data, they quantify the variance risk premia on both stock indexes and individual stocks, without the need to specify the dynamic of the underlyings. Formal studies with fully specified underlying price dynamics as well as corresponding option pricing find controversial results, due to different sampling periods, estimation procedures or model specifications. For the SV model, Chernov and Ghysels (2000) estimates $\eta^v$ to be 0.24. Bates (2000) estimates $\eta^v$ to be 2.2. Pan (2002) estimates $\eta^v$ to be 7.6 and significant. Her results imply an explosive volatility dynamics under the risk-neutral measure. Jones (2003a) estimates $\eta^v$ to be -8.2 using the whole sample from 1986 to 2000. But he finds $\eta^v$ to be 7.4 if the earlier part of the sample including the 1987 market crash is excluded. The volatility process is also estimated to be explosive under the risk-neutral measure using the post 1987 data. Eraker (2004) estimates $\eta^v$ to be 2.5 and insignificant. Broadie, Chernov, and Johannes (2007) estimates $\eta^v$ to be -1.3 and insignificant. We estimate $\eta^v$ to be -0.1 and insignificant. The wide variation of the estimates for $\eta^v$ can probably be attributed to the mis-specification of the SV model given the shortcomings of the SV model.\(^{10}\)

Jumps are introduced in return and volatility processes to potentially better capture the dynamics. If a relatively large price move is characterized as a jump in price, it does not directly impact volatility in the SVJ model, or leads to a simultaneous jump in volatility in the SVCJ model. This contrast with the SV model in which both large and small price changes impact volatility through the leverage effect. Bates (2000) estimates $\eta^v$ to be -0.2 for the SVJ model. Pan (2002) estimates $\eta^v$ to be 3.1 and insignificant for the SVJ model.\(^{11}\) Eraker (2004) estimates $\eta^v$

\(^{10}\)The SV model is closely examined by Das and Sundaram (1999). Andersen, Benzoni, and Lund (2002) argues that the high persistence of the volatility process leads to more frequent extreme movements in returns than the observed data suggests for the SV model. Jones (2003a) prefers a stochastic variance model in the CEV class or a model with a time-varying leverage effect to the SV model considered here.

\(^{11}\)For the SVJ model, Bates (2000) and Pan (2002) specify a time-varying jump intensity as an affine function
to be 2.3 and insignificant for the SVJ model, and 3.3 and significant at 5% level for the SVCJ model. Broadie, Chernov, and Johannes (2007) estimates $\eta^v$ to be statistically insignificant for both SVJ and SVCJ models, no matter whether the standard deviations of the sizes of jumps in return are constrained to be the same under the physical and risk neutral measures. We estimate $\eta^v$ to be 0.0 for the SVJ model, and 1.6 and statistically significant for the SVCJ model. To alleviate the problem of the sensitivity of model estimation to sample period chosen and better capture the physical dynamics, we exploit a relatively long sample of joint returns and options for 16 years from 1996 to 2011. Broadie, Chernov, and Johannes (2007) imposes the so-called time-series consistency by constraining some of the parameters, in particular, $\kappa_v, \theta_v, \rho$ and $\sigma_v$, to be the same under both physical and risk-neutral measure. However, they borrow the physical dynamics estimated by Eraker, Johannes, and Polson (2003) for ease of computation and avoid an intensive joint fitting which explicitly balances the return and option data.\textsuperscript{12} Our estimation combines and deliberately weighs the returns and option series and should be effective to learn the model parameters. Our fitting results for SV, SVJ and SVCJ models are nevertheless close to those reported in previous studies. The diffusive volatility risk premium is either statistically insignificant, or small in magnitude.\textsuperscript{13} The SVSJ and SVSCJ model estimate $\eta^v$ to be -4.9 and -3.6, respectively. Both estimates are statistically significant.

Correlation between return and volatility processes, or the leverage effect, is measured by $\rho$. The skewness of the conditional return distribution has the same sign as $\rho$. The kurtosis of the conditional return distribution is increasing with $|\rho|$. Studies based solely on return data, for instance, Andersen, Benzoni, and Lund (2002), Chernov, Gallant, Ghysels, and Tauchen (2003), Eraker, Johannes, and Polson (2003), Eraker (2004) and Li, Wells, and Yu (2008), estimate $\rho$ to range from -0.46 to -0.56. Joint fittings of returns and options in Pan (2002), Jones (2003a), Eraker (2004) and Santa-Clara and Yan (2010), among others, estimate $\rho$ to range from -0.53 to of return variance, while other studies assume that the jump intensity is constant. Comparison of model estimation results should bear this difference in mind.\textsuperscript{12} The joint fitting method has to decide on how many option contracts to be included for the estimation. A large number of option contracts will tilt the likelihood function to weigh heavily on the information in the options and attenuate the importance of the returns.\textsuperscript{13} Eraker (2004) argues that the diffusive volatility risk premium has a small effect on the implied volatility term structure.
The literature shows certain dispersion in the estimates of $\rho$ and the estimates from joint data tend to be more negative than the estimates from solely returns. We estimate $\rho$ to range from -0.80 to -0.83, whether or not the models allow for time-varying jump intensity. The results imply a stronger leverage effect between return and volatility, which reflects more negatively skewed and fat-tailed conditional return distribution during our sample period. The kurtosis of the conditional return distribution is also increasing with $\sigma_v$. The literature estimates $\sigma_v$ to range from 0.16 to 0.30 for joint fitting. Our estimates of $\sigma_v$ fall between 0.38 and 0.46, which are higher than those found in previous studies. Note that the estimates of $\sigma_v$ are smaller in SVSJ and SVSCJ models than those in the other three models. The more negative $\rho$ and relatively high $\sigma_v$ obtained in our estimation thus emphasize the importance of the leverage effect in fitting the conditional return distribution. This will be further elaborated below.

$\lambda_0$ is the frequency of jumps in return and volatility. The estimates confirm that jumps are rare events. We expect there is about 1 jump in 2.5 year as indicated by the SVJ model and about 1 jump per year as indicated by the SVCJ model. The total jump intensity of the SVSJ model, $\lambda_1 V_t + h_t$, ranges from 0.01 to 10.50 with an unconditional average of 0.78. The total jump intensity of the SVSCJ model ranges from 0.01 to 10.20 with an unconditional average of 0.78. As the frequency of the arrival of jumps is allowed to be time-varying, we do find large variations in the jump intensity during our sample period. The average arrival rate is about 1 jump in 1.3 years for both the SVSJ and SVSCJ models. Studies based on time series of returns usually find $\lambda_0$ to be in the range from 0.76 to 1.8.\textsuperscript{14} Studies based on joint data, for instance, Eraker (2004), estimate $\lambda_0$ to be about 0.5, or 1 jump in 2 years. Our estimates of the constant jump intensity is consistent with the literature. Bates (2000) and Pan (2002) specify a time-varying jump intensity which is proportional to the stochastic return variance. Their results afford a more direct comparison with our estimation of the SVSJ and SVSCJ models. Bates (2000) finds an average jump arrival rate of 0.59 or about 1 jump in 1.7 years. Pan (2002) finds an average jump arrival rate of 0.19 or about 1 jump in 5.3 years.\textsuperscript{15} The average jump intensities in our SVCJ and

\textsuperscript{14}Andersen, Benzoni, and Lund (2002) estimate $\lambda_0$ to be 5 with the mean jump size fixed at 0.
\textsuperscript{15}The average jump arrival rates for Bates (2000) and Pan (2002) are calculate as $\lambda_1 \theta_v$ where $\theta_v$ is the long-run mean of the variance process.
SVSCJ models are higher than those obtained by Bates (2000) and Pan (2002). This is likely due to the financial crisis period included in our sample which is characterized with extremely high volatility and jump intensity \( h_t \).

Our five models allow the jump size parameters for the returns to change freely between the physical and risk-neutral measures. The physical jump size parameters, namely, \( \mu_y \) and \( \sigma_y \), are hard to identify since they can only be learnt from returns. Jumps in returns occur rarely and the jump sizes are also uncertain. Negative mean jump size contributes to the negative skewness of the conditional return distribution. And standard deviation of the jump size adds to the kurtosis of the conditional return distribution. Table (II) shows that \( \mu_y \) ranges from -0.03 to 0.03 and \( \sigma_y \) ranges from 0.06 to 0.07. These estimates are broadly consistent with previous studies based on returns. However, there is larger dispersion in the risk-neutral estimates. We estimate \( \mu_y^Q \) to be -0.00 and -0.01 for the SVJ and SVCJ models, respectively. \( \mu_y^Q \) is more negative when we allow for time-varying jump intensity. We estimate \( \mu_y^Q \) to be about -0.10 for the SVSJ and SVSCJ models. \( \sigma_y^Q \) is close to \( \sigma_y \) for the SVJ and SVCJ models. But in the models with time-varying jump intensity, \( \sigma_y^Q \) becomes enlarged and estimated to be 0.25 and 0.23 for the SVSJ and SVSCJ models, respectively. These estimates are higher than those in Broadie, Chernov, and Johannes (2007) which assumes constant jump intensity, but close to those in Santa-Clara and Yan (2010) which specifies a separately jump intensity process. The standard deviations of the risk-neutral jump size parameter estimates are smaller than their physical counterparts, especially for the SVSJ and SVSCJ models. The jumps in volatility have an average size of 0.028 for both the SVCJ and SVSCJ models. Assuming return variance is at their long-run mean level, an average-sized jump in volatility lead \( \sqrt{V_t} \) to increase from 0.146 to 0.223 in the SVCJ model, from 0.163 to 0.235 in the SVSCJ model. To sum up, we find modest jump risk premium in returns. The risk-neutral mean jump sizes of returns are close to 0 in the SVJ and SVCJ models. The negative skewness of the risk-neutral conditional distribution of returns mainly comes from the leverage effect, or the low \( \rho \). The jump premia are larger with time-varying jump intensity. In particular, \( \mu_y - \mu_y^Q \) euqals 0.11 for the SVSJ model and 0.07 for the SVSCJ model. These are close to the premia estimated, for instance, by Eraker (2004) and Broadie, Chernov, and Johannes (2007), but
smaller than Pan (2002).

B. The latent volatility and jump variables

The estimated volatility processes of the five models considered are plotted in Figure (3). When the jump intensities are allowed to be time-varying in the SVSJ and SVSCJ models, the volatility processes become smoother than in the SVJ and SVCJ models with constant jump intensity. The extremely high return variance during the financial crisis period estimated in the SV, SVJ and SVCJ models is reduced to about one half in the SVSJ and SVSCJ models. This is also reflected in the estimated volatility of volatility parameter, $\sigma_v$. $\sigma_v$ is estimated to be 0.45 to 0.46 for the SV and SVJ models. When we admit jumps to account for large positive moves in volatility, $\sigma_v$ is reduced to 0.43 for the SVCJ model. The lowest $\sigma_v$ is achieved with the SVSJ and SVSCJ models, with estimates around 0.38.

The time-series consistency of the parameter estimates is discussed by Bates (2000), Pan (2002), Eraker (2004) and Broadie, Chernov, and Johannes (2007), among others. Specifically, the change of measure requires $\rho$ and $\sigma_v$ to be the same under the physical and risk-neutral measures. However, studies based on returns usual find $\sigma_v$ to be much smaller than those reported by studies based on option prices. For instance, the summary of Table I in Broadie, Chernov, and Johannes (2007) shows that $\sigma_v$ is estimated to be no larger than 0.14 in the studies of return time series. Studies of option prices in Bates (2000) and Bakshi, Cao, and Chen (1997) find $\sigma_v$ to be larger than 0.32. Eraker (2004) argues that $\sigma_v$ estimated from the posterior volatility time series is consistent with the estimates based on joint fitting with returns and options. Thus no mismatch across measure is found. Our estimates of $\sigma_v$ are obtained through the effective joint fitting of returns and options. Our estimates are higher than those in previous studies of Pan (2002) and Eraker (2004) which also exploit joint data for model estimation. This indicates the volatile sample periods we have included in the estimation. Pan (2002) uses S&P500 index options from January 1989 to December 1996. Eraker (2004) uses S&P500 index options from January 1987 to December 1990. Both miss the volatile periods after 1996, i.e. the financial crisis.

The filtered jumps are shown in Figure (5). Fewer jump are identified in the SVJ model than
in the other four models. We see about 5 large jumps in returns in the upper-left panel of Figure (5). The $\lambda_0$ estimate of SVJ predicts about 6 jumps in 16 years. When we introduce jumps in volatility or allow jump intensity to vary over time, we see increased number of jumps identified in the SVCJ, SVSJ and SVSCJ models. Clustering of jump arrivals is also evident, especially in the financial crisis period. The SV, SVJ and SVCJ models explain the crisis mainly as a "market turbulence" with highly-inflated volatility. However, the other two models attribute this extreme period partly to clustered and simultaneous jumps in return and volatility. The jump intensity process $h_t$ is shown in Figure (4). It shows large fluctuations during our sample period. $\sigma_h$ is estimated to be 2.8 and 2.7 for the SVSJ and SVSCJ models, respectively. The intensity process shows distinct variations compared with the volatility process. By specifying a separate process, we allow the jump intensity to have its own sources of uncertainty. $h_t$ is estimated to mean-reverting at a higher speed in the SVSJ model. But it is estimated to be more persistent in the SVSCJ model. The long mean of $h_t$ is estimated to be 0.13 and 0.16 in the SVSJ and SVSCJ models, respectively.

C. Performances in fitting the return, volatility and jump intensity time series

We examine the performances of the models to fit the S&P500 index returns, volatility and jump intensity dynamics based on the daily standardized residuals of their processes. According to Equation (21) to (23) and the parameter estimates in Table (II), the historic Brownian increments of the return, volatility and jump intensity processes can be calculated as follows.

$$
\varepsilon^y_{t+1} = \frac{Y_{t+1} - Y_t - \mu \Delta - \Delta J^y_{t+1}}{\sqrt{V_t \Delta}},
$$

$$
\varepsilon^v_{t+1} = \frac{V_{t+1} - V_t - \kappa_v (\theta_v - V_t) \Delta - \Delta J^v_{t+1}}{\sigma_v \sqrt{V_t \Delta}},
$$

$$
\varepsilon^h_{t+1} = \frac{h_{t+1} - h_t - \kappa_h (\theta_h - h_t) \Delta}{\sigma_h \sqrt{h_t \Delta}}.
$$

Given that our models are correctly specified, $\varepsilon^y_{t+1}$, $\varepsilon^v_{t+1}$ and $\varepsilon^h_{t+1}$ should follow standard normal distributions.

The return residuals, $\varepsilon^y_t$, for the five models are plotted in Figure (6). We see that the SVCJ
model fit the returns the best in terms that it does not generate large return residuals. The SV, SVJ and SVSJ models produce large return residuals on October 27, 1997 with a return of -7.1%,\(^{16}\) and on February 27, 2007 with a return of -3.5%. The spot volatility is about 1.5% on October 27, 1997, and 0.6% on February 27, 2007. The large residual on October 27, 1997 is due to large return, while the large residual on February 27, 2007 is due to low volatility. The SVSCJ model show one large residual on October 28, 1997. A upward jump in volatility explains the large negative return on October 27, 1997. However, the volatility process cannot jump down in our model. The subsequent rapid drop in volatility is shown as a large negative residual in volatility and large positive residual in return on October 28, 1997. The residual statistics are presented in Table (III). All the return residuals of the five models have means close to 0. The standard deviation of the residuals are most close to 1 for the SVSJ and SVSCJ models. The SV and SVCJ models have the smallest standard deviation of about 0.90. All the five models have slightly negative skewnesses, ranging from -0.11 for the SVSCJ model to -0.43 for the SVSJ model. The kurtoses of the five models are relatively high, ranging from 3.3 for the SVCJ model to 4.2 for the SVSJ model. The SVSJ and SVSCJ Models with simultaneous jumps in return and volatility tend to fit the return time series better than the SV model without jumps or the SVJ and SVSJ models with only jumps in returns.

The volatility residuals, \(\varepsilon^v_t\), are plotted in Figure (7) and the jump intensity residuals, \(\varepsilon^h_t\), are plotted in the bottom two panels of Figure (4). We see that all models generate large volatility residuals around October 27, 1997. The SVCJ and SVSCJ models produce less spikes in volatility residuals than the SV, SVJ and SVSJ models in other time periods. The high spike in the residuals of the jump intensity process of the SVSJ model around October 27, 1997 is largely reduced for the SVSCJ model. The residual statistics in Table (III) show that the SVCJ and SVSCJ models have skewnesses of volatility residuals of 0.20 and 0.07, respectively, while the other three models have skewness no less than 0.69. The SVSCJ model also has a smaller skewness of jump intensity residuals of 0.72 than the SVSJ model with a skewness of 1.35. The kurtoses of the volatility

\(^{16}\)This mini-crash is also discussed, for instance, in Broadie, Chernov, and Johannes (2007). The option implied volatility changed from 26% to 40% on this day.
residuals of the SVCJ and SVSCJ models are 4.0 and 5.3, respectively. The other three models have kurtoses no less than 6.0. The kurtoses of the jump intensity residuals are more striking, the kurtosis of the SVSJ model is about 11.6 and about twice the kurtosis of the SVSCJ model. The results confirm that jumps in volatility is important to explain the dynamics of the volatility and jump intensity processes.

Our study of the return, volatility and jump intensity time series is broadly consistent with Eraker (2004) and Broadie, Chernov, and Johannes (2007). We find evidence in support of jumps in volatility to explain the physical dynamics. Our models still have difficulties accounting for the large negative return on October 27, 1997 when volatility cannot jump. Even when we admit simultaneous jumps in return and volatility, the sharp decrease in volatility after October 27, 1997 produce a large volatility residual since the volatility process cannot jump down. Comparing the SVJ with the SVSJ model, and the SVCJ to the SVSCJ model, we see that allowing the jump intensity to time-varying does not improve the time series fitting much.

D. Performances in fitting the options

Superior performances in fitting the physical return, volatility and jump intensity dynamics do not necessarily lead to better option pricing. The literature on time series studies mainly advocates the inclusion of jumps in both return and volatility. However, the evidence from options are a bit controversial on the relationship between model complexity and pricing performance. Bates (2000), Pan (2002) and Eraker (2004) all find economically small benefits for including jump in return and volatility. However, Bakshi, Cao, and Chen (1997) reports an improvement of about 40% in pricing errors of the SVJ model over the SV model. Broadie, Chernov, and Johannes (2007) also find strong support for jumps in return and modest evidence for jumps in volatility. In this section, we will discuss the in-sample fitting of the five models to the at-the-money (and 5% out-of-the-money) option series and the out-of-sample fitting of the large cross-section of options.

Table (IV) summarizes the in-sample option pricing errors measured by absolute pricing errors, which are the absolute differences between market and model implied option prices, and
percentage pricing errors, which are the absolute pricing errors scaled by the market prices of the options. The SV and SVJ models provide similar in-sample fitting: The absolute pricing errors are about 31 cents on average (The mean of the option prices is $24.91 and the mean of the bid-ask spreads is $1.63.). The percentage pricing errors of the SV and SVJ models are also similar and equal to 1.1% (The average of the bid-ask spreads divided by the option prices is 6.7%). The average pricing errors are smaller than the bid-ask spread, both in absolute value and percentage. The SVCJ model has a mean absolute pricing error of 27 cents, which is about 12% lower over the SV and SVJ models. The percentage pricing errors of the SVCJ model is 1.0% and about 9% lower than those of the SV and SVJ models. The greatest improvement in the in-sample fitting is achieved by the SVSJ and SVSCJ models. The SVSJ and SVSCJ models produce absolute pricing errors of about 11 cents and percentage pricing errors of about 0.5%. Allowing the jump intensity to be time-varying, we obtain a 63% reduction in absolute pricing errors of the SVSJ model over the SVJ model, and a 60% reduction of the SVSCJ model over the SVCJ model.

Now we turn to the out-of-the-sample option pricing performances for the cross-section of totally 264922 options. The model fitting of the volatility smirks on low, medium and high volatility days is illustrated in Figure (8). We can see that all the five models fit the implied volatility smirk well for the low volatility day, except that the SV, SVJ and SVCJ models do not generate enough steepness for deep-out-of-the-money put options and that the SV model misses the hook for the out-of-the-money call options. For the medium volatility day, the SV, SVJ and SVCJ models all produce too flat smirk curves. The SVSJ and SVSCJ models matches the smirk reasonably well, while over-shoot a bit at both ends. On the high volatility day, the SV, SVJ and SVCJ models cannot match closely the at-the-money options. The smile curves generated by the three models are too flat, either. The SVSJ and SVSCJ models again show much flexibility in matching the smirk curve on this day with extremely high volatility.

The absolute option pricing errors are reported in Table (V), for the whole cross-section and subsamples grouped according to time-to-expiration and moneyness (strike to forward price ra-

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17The low, medium and high volatility days correspond to July 27, 2005, December 1, 1999 and November 20, 2008, with Black-Scholes option implied volatility 8.5%, 18.9% and 76.4%, respectively.
The whole sample consists options with moneynesses ranging from 0.90 to 1.07 and maturities less than 3 months. Let's first look at the overall performances of the five models. The SV, SVJ, and SVCJ models produce average absolute pricing errors of $1.32, $1.31 and $1.32, respectively. The SVSJ model achieves an improvement of about 20 cents (or 15%) and the SVSCJ model reduces the absolute pricing errors by about 25 cents (or 19%). Thus, the greatest benefits are obtained by allowing a separate and time-varying jump intensity process. Introduction of jumps in volatility provides extra improvement. If we look at all the options grouped by moneynesses, we see that the superior performances of the SVSJ and SVSCJ model mainly originate from the out-of-the-money put options and the at-the-money options. These two models actually perform worse than the other three models for options with moneynesses larger than 1.00. When we group all the options according to time-to-maturity, the SVSJ and SVSCJ models perform better than the other three models for options with maturities less than 2 months, but worse for options with maturity between 2 and 3 months. We further group options by maturities and moneynesses, the results confirms that the largest improvement of option pricing performances of the SVSJ and SVSCJ models is found at the upper-left corner with lowest maturity and moneyness levels, and the worst option pricing performances of the SVSJ and SVSCJ models are found at the lower-right corner with highest maturity and moneyness levels.

Our results point to the inclusion of a separate and time-varying jump intensity process, which help to fit the option prices both in sample and out-of-sample. The SVSJ and SVSCJ models also display advantages to generate the implied volatility smirk patterns on high volatility days over the SV model without jumps and SVJ and SVCJ models with constant jump arrival rates. Pan (2002) also emphasizes the importance of the time-varying jump intensity in reconciling the joint physical and risk-neutral dynamics and fitting the cross-section of option prices. The failure of the SV model to match the cross-section of option is broadly consistent with the literature. The square root volatility process cannot rise fast enough to account for occasional surge of volatility in the sample. The difficulties of models with constant jump arrival rates to generate steep implied volatility curves are also found by Eraker (2004). However, the SVSJ and SVSCJ models still tend to over-price the deep-out-of-the-money put and call options.
VI. Conclusion

In this paper, we study the specification issues regarding jumps in return and volatility processes for modeling the stock price dynamics and associated option prices. To afford a proper representation of the jump behavior and volatility patterns, we use a long time series of 16 years of S&P500 index prices and up to two option price series for model estimation. We impose the time series consistency by restricting several parameters to be equal across the physical and risk-neutral measures, and fit our models to the joint return and option data. The arrival rates of jumps are allowed to be time-varying by specifying separate intensity processes which bear distinct source of uncertainty and persistence levels to the volatility processes.

Our results show that simultaneous jumps in return and volatility are important to explain the stock price and volatility time series, while jumps in returns solely do not significantly enhance the time series fitting when volatility is already stochastic. However, the cross-sectional option pricing performances show that time variability of the jump intensity processes play an important role. The SVSJ and SVSCJ models afford improvements of 15-19% in pricing errors over the SVJ and SVCJ models for the whole cross-section of options, while the SCJ and SVCJ models produce only negligible in-sample and out-of-sample reductions in option pricing errors over the SV model.
References


Appendix

In this appendix, I provide the detailed MCMC method to estimate the general model shown in (1) to (3) and (9) to (11). Due to the numerical integration we perform to get the option values, each of the risk-neutral parameters requests a Metropolis step. However, parameters and state variables that do not enter the option pricing formula can be directly sampled based on their standard known posterior distributions. We sample the model parameters and state variables iteratively.

A. Parameters that do not appear in the option pricing formula

- The posterior of $\mu$ is normal.

$$\mu \sim N\left(\frac{S}{W}, \frac{1}{W}\right),$$  \hspace{1cm} (35)

where

$$W = \frac{\Delta_t}{(1-\rho^2)} \sum_{t=1}^{T-1} \frac{1}{V_t} + \frac{1}{M^2},$$

$$S = \frac{1}{(1-\rho^2)} \sum_{t=1}^{T-1} \frac{(A_{t+1} - \frac{B_{t+1}}{\sigma_v})}{v_t} + \frac{m}{M^2},$$

$$A_{t+1} = Y_{t+1} - Y_t - \Delta N_{t+1} \xi_{t+1}^y,$$

$$B_{t+1} = V_{t+1} - V_t - (\kappa_v \theta_v - \kappa_v V_t) \Delta_t - \Delta N_{t+1} \xi_{t+1}^v. \quad \Delta_t \text{ is the time-discretization interval which corresponds to 1 day or } \frac{1}{252} \text{ years.}$$

$\Delta N_{t+1} = 1$ indicates the arrival of a jump during the interval $(t \Delta_t, (t+1) \Delta_t)$. The prior distribution of $\mu$ is $N(m, M^2)$.

- The posterior of $\kappa_v$ follows a truncated normal distribution.

$$\kappa_v \sim N\left(\frac{S}{W}, \frac{1}{W}\right)1_{\kappa_v > 0},$$ \hspace{1cm} (36)

where

$$W = \frac{\Delta_t}{(1-\rho^2)} \sum_{t=1}^{T-1} V_t + \frac{1}{M^2},$$

$$S = \frac{1}{(1-\rho^2)} \sum_{t=1}^{T-1} \frac{(\rho \sigma_v A_{t+1} - B_{t+1})}{M^2},$$

$$A_{t+1} = Y_{t+1} - Y_t - \mu \Delta_t - \Delta N_{t+1} \xi_{t+1}^y,$$

$$B_{t+1} = V_{t+1} - V_t - (\kappa_v \theta_v) \Delta_t - \Delta N_{t+1} \xi_{t+1}^v. \quad \text{The prior distribution of } \kappa_v \text{ is } N(m, M^2)1_{\kappa_v > 0}. \hspace{1cm} (37)$$

- The posterior of $\mu_y$ is normal.

$$\mu_y \sim N\left(\frac{S}{W}, \frac{1}{W}\right).$$

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18Intermediate unobservable data points between $t \Delta_t$ and $(t+1) \Delta_t$ can be simulated by treating them as missing data. See Eraker (2001). Jones (2003b) uses this method to reduce discretization errors when estimating the short rate process with nonlinear drift.

19Let $F(x)$ be the CDF of the normal distribution $N(\bar{\mu}, \bar{\sigma}^2)$. The truncated normal $N(\bar{\mu}, \bar{\sigma}^2)1_{x > 0}$ can be sampled in two steps. (1) Draw $u \sim \text{uniform}(0, 1)$. (2) Get $x = F^{-1}((1 - F(0))u + F(0))$. \hspace{1cm} (35)
where \( W = \frac{T-1}{\sigma_y^2} + \frac{1}{M^2} \) and \( S = \frac{\sum_{t=1}^{T-1} \xi_{t+1}}{\sigma_y^2} + \frac{m}{M^2} \). The prior distribution of \( \mu_y \) is \( N(m, M^2) \).

- The posterior of \( \sigma_y \) follows an inverse gamma distribution.

\[
\sigma_y^2 \sim IG \left( \frac{T-1}{2} + m, \frac{1}{\frac{1}{2} \sum_{t=1}^{T-1} (\xi_{t+1} - \mu_y)^2 + \frac{1}{M^2}} \right),
\]

The prior distribution of \( \sigma_y \) is \( IG(m, M) \).

- The posterior of \( s \) follows an inverse gamma distribution.

\[
s^2 \sim IG \left( T + m, \frac{1}{\frac{1}{2} \left( \sum_{t=1}^{T-1} \left( \frac{C_t - F_t^C}{BAS_t^C} \right)^2 \right) \sum_{t=1}^{T} \left( \frac{P_t - F_t^P}{BAS_t^P} \right)^2 + \frac{1}{M^2}} \right),
\]

where the value of the out-of-the-money put option, \( F_t^P \), is obtained from the put-call parity.

The prior distribution of \( s \) is \( IG(m, M) \).

### B. Parameters that appear in the option pricing formula

- The posterior of \( \kappa_v^O = \kappa - \eta_v \) is proportional to

\[
\prod_{t=1}^{T} \exp \left( - \frac{1}{2s^2} \left( \frac{(C_t - F_t^C)^2}{(BAS_t^C)^2} + \frac{(P_t - F_t^P)^2}{(BAS_t^P)^2} \right) \right) \times N(m, M^2) 1_{\kappa_v^O>0}.
\]

The prior distribution of \( \kappa_v^O \) is \( N(m, M^2) 1_{\kappa_v^O>0} \).

- The posterior of \( (\kappa_v, \theta_v) \) is proportional to

\[
\prod_{t=1}^{T} \exp \left( - \frac{1}{2s^2} \left( \frac{(C_t - F_t^C)^2}{(BAS_t^C)^2} + \frac{(P_t - F_t^P)^2}{(BAS_t^P)^2} \right) \right) \times N \left( \frac{S}{W}, \frac{1}{W} \right) 1_{(\kappa_v, \theta_v)>0},
\]

where \( W = \frac{\Delta_t}{(1-p^2)\sigma_v^2} \sum_{t=1}^{T-1} \frac{1}{V_t} + \frac{1}{M^2}, \ S = \frac{1}{(1-p^2)\sigma_v^2} \sum_{t=1}^{T-1} \frac{B_{t+1} - p\sigma_v A_{t+1}}{V_t} + \frac{m}{M^2}, \ A_{t+1} = Y_{t+1} - Y_t - \mu \Delta - \Delta N_{t+1} \xi_{t+1}^y, \ B_{t+1} = V_{t+1} - V_t + \kappa_v V_t \Delta_t - \Delta N_{t+1} \xi_{t+1}^y \). The prior distribution of \( (\kappa_v, \theta_v) \) is \( N(m, M^2) 1_{(\kappa_v, \theta_v)>0} \).
The posterior of $\sigma_v^2$ is proportional to
\[
\prod_{t=1}^T \exp\left( -\frac{1}{2\sigma_v^2}\left(\frac{(C_t - F_P^C)^2}{(BAS_t^C)^2} + \frac{(P_t - F_P^P)^2}{(BAS_t^P)^2}\right) \right) \times \exp\left( -\frac{\rho}{(1-\rho^2)}\frac{1}{\sigma_v^2} \left( \sum_{t=1}^{T-1} A_{t+1}B_{t+1} \right) \right) \\
\times \left( \frac{1}{\sigma_v^2} \right)^{\frac{T-1}{2}+m} \exp\left( -\frac{\sum_{t=1}^{T-1} B_{t+1}^2}{2(1-\rho^2)} + \frac{1}{M} \frac{1}{\sigma_v^2} \right),
\] (42)
where $A_{t+1} = \frac{Y_{t+1} - Y_t - \mu \Delta - \Delta N_{t+1} \xi_{t+1}}{\sqrt{\sigma_v^2}}$, $B_{t+1} = \frac{V_{t+1} - V_t - (\kappa_i \theta_i, - \kappa_i V_t) \Delta_{t+1} - \Delta N_{t+1} \xi_{t+1}}{\sigma_v \sqrt{\sigma_v}}$. The prior distribution of $\sigma_v^2$ is $IG(m,M)$.

The posterior of $\rho$ is proportional to
\[
\prod_{t=1}^T \exp\left( -\frac{1}{2s^2}\left(\frac{(C_t - F_P^C)^2}{(BAS_t^C)^2} + \frac{(P_t - F_P^P)^2}{(BAS_t^P)^2}\right) \right) \\
\times (1-\rho^2)^{-\frac{T-1}{2}} \exp\left( -\frac{1}{2(1-\rho^2)} \left( \sum_{t=1}^{T-1} A_{t+1}^2 - 2\rho A_{t+1}B_{t+1} + B_{t+1}^2 \right) \right) \exp\left( -\frac{(z-m)^2}{2M^2} \right),
\] (43)
where $z = \frac{1}{2} \log \frac{1+\rho}{1-\rho}$ is the "Fisher’s Z transform". $A_{t+1} = \frac{Y_{t+1} - Y_t - \mu \Delta - \Delta N_{t+1} \xi_{t+1}}{\sqrt{\sigma_v^2}}$, $B_{t+1} = \frac{V_{t+1} - V_t - (\kappa_i \theta_i, - \kappa_i V_t) \Delta_{t+1} - \Delta N_{t+1} \xi_{t+1}}{\sigma_v \sqrt{\sigma_v}}$.
"Fisher’s Z transform" is applied to make the skewed Pearson’s correlation close to normal.

The prior distribution of $z$ is $N(m,M^2)$.

The posterior of $\mu_y^Q$ is proportional to
\[
\prod_{t=1}^T \exp\left( -\frac{1}{2s^2}\left(\frac{(C_t - F_P^C)^2}{(BAS_t^C)^2} + \frac{(P_t - F_P^P)^2}{(BAS_t^P)^2}\right) \right) \times N(m,M^2).
\] (44)

The prior distribution of $\mu_y^Q$ is $N(m,M^2)$.

The posterior of $\mu_v^Q$ is proportional to
\[
\prod_{t=1}^T \exp\left( -\frac{1}{2s^2}\left(\frac{(C_t - F_P^C)^2}{(BAS_t^C)^2} + \frac{(P_t - F_P^P)^2}{(BAS_t^P)^2}\right) \right) \times \left( \frac{1}{\mu_v^Q} \right)^{T-1+m} \exp\left( -\frac{\sum_{t=1}^{T-1} \xi_{t+1} + \frac{1}{M}}{\mu_v^Q} \right),
\] (45)

The prior distribution of $\mu_v^Q$ is $IG(m,M)$. 

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• The posterior of $\sigma_y^2$ is proportional to

$$\prod_{t=1}^{T} \exp\left(-\frac{1}{2\delta^2}\left(\frac{(C_t - F_t^C)^2}{(BAS_t^C)^2} + \frac{(P_t - F_t^P)^2}{(BAS_t^P)^2}\right)\right) \times \left(\frac{1}{\sigma_y^{O2}}\right)^m \exp\left(-\frac{1}{\sigma_y^{O2}}\right)$$

(46)

The prior distribution of $\sigma_y^2$ is $IG(m, M)$.

• The posterior of $\kappa_h$ is proportional to

$$\prod_{t=1}^{T} \exp\left(-\frac{1}{2\delta^2}\left(\frac{(C_t - F_t^C)^2}{(BAS_t^C)^2} + \frac{(P_t - F_t^P)^2}{(BAS_t^P)^2}\right)\right) \times N\left(\frac{S}{W}, \frac{1}{W}\right) 1_{\kappa_h > 0},$$

(47)

where $W = \frac{\Delta_t}{\sigma_h^2} \sum_{t=1}^{T-1} h_t + \frac{1}{M^2}$, $S = -\frac{1}{\sigma_h^2} \sum_{t=1}^{T-1} B_{t+1} + \frac{m}{M^2}$, $B_{t+1} = h_{t+1} - h_t - \kappa_h \Delta_t$. The prior distribution of $\kappa_h$ is $N(m, M^2) 1_{\kappa_h > 0}$.

• The posterior of $(\kappa_h \theta_h)$ is proportional to

$$\prod_{t=1}^{T} \exp\left(-\frac{1}{2\delta^2}\left(\frac{(C_t - F_t^C)^2}{(BAS_t^C)^2} + \frac{(P_t - F_t^P)^2}{(BAS_t^P)^2}\right)\right) \times N\left(\frac{S}{W}, \frac{1}{W}\right) 1_{(\kappa_h \theta_h) > 0},$$

(48)

where $W = \frac{\Delta_t}{\sigma_h^2} \sum_{t=1}^{T-1} h_t + \frac{1}{M^2}$, $S = \frac{1}{\sigma_h^2} \sum_{t=1}^{T-1} B_{t+1} + \frac{m}{M^2}$, $B_{t+1} = h_{t+1} - h_t + \kappa_h h_t \Delta_t$. The prior distribution of $(\kappa_h \theta_h)$ is $N(m, M^2) 1_{(\kappa_h \theta_h) > 0}$.

• The posterior of $\sigma_h^2$ is proportional to

$$\prod_{t=1}^{T} \exp\left(-\frac{1}{2\delta^2}\left(\frac{(C_t - F_t^C)^2}{(BAS_t^C)^2} + \frac{(P_t - F_t^P)^2}{(BAS_t^P)^2}\right)\right) \times \left(\frac{1}{\sigma_h^{2}}\right)^{T+1} \exp\left(-\frac{1}{\sigma_h^{2}}\right)$$

(49)

where $B_{t+1} = \frac{h_{t+1} - h_t - \kappa_h \theta_h \Delta_t}{\sqrt{h_t \Delta_t}}$. The prior distribution of $\sigma_h^2$ is $IG(m, M)$.
• The posterior of $\lambda_0$ is proportional to

$$
\prod_{t=1}^{T} \exp \left( -\frac{1}{2s^2} \left( \frac{(C_t - F_C^t)^2}{(BAS_C^t)^2} + \frac{(P_t - F_P^t)^2}{(BAS_P^t)^2} \right) \right) N(m, M^2) 1_{\lambda_0 > 0} 
\times \prod_{t: \Delta N_{t+1} = 1} (\lambda_0 + \lambda_1 V_t + h_t) \Delta_t \prod_{t: \Delta N_{t+1} = 0} (1 - (\lambda_0 + \lambda_1 V_t + h_t) \Delta_t),
$$

(50)

The prior distribution of $\lambda_0$ is $N(m, M^2) 1_{\lambda_0 > 0}$.

• The posterior of $\lambda_1$ is proportional to

$$
\prod_{t=1}^{T} \exp \left( -\frac{1}{2s^2} \left( \frac{(C_t - F_C^t)^2}{(BAS_C^t)^2} + \frac{(P_t - F_P^t)^2}{(BAS_P^t)^2} \right) \right) N(m, M^2) 1_{\lambda_1 > 0} 
\times \prod_{t: \Delta N_{t+1} = 1} (\lambda_0 + \lambda_1 V_t + h_t) \Delta_t \prod_{t: \Delta N_{t+1} = 0} (1 - (\lambda_0 + \lambda_1 V_t + h_t) \Delta_t),
$$

(51)

The prior distribution of $\lambda_1$ is $N(m, M^2) 1_{\lambda_1 > 0}$.

C. State variables that do not appear in the option pricing formula

• For $1 \leq t \leq T - 1$, the posterior of $\xi_{t+1}^\gamma$ is normal.

$$
\xi_{t+1}^\gamma \sim N \left( \frac{S}{W}, \frac{1}{W} \right),
$$

(52)

where $W = \frac{\Delta N_{t+1} \Delta_t}{(1 - \rho^2) V_t} + \frac{1}{\sigma^2}$, $S = \frac{\Delta N_{t+1} \Delta_t}{(1 - \rho^2) V_t} (A_{t+1} - \frac{\rho}{\sigma^2} B_{t+1}) + \frac{\mu_v}{\sigma^2}$, $A_{t+1} = Y_{t+1} - Y_t - \mu \Delta_t$, $B_{t+1} = V_{t+1} - V_t - (\kappa_v \theta_v - \kappa_v V_t) \Delta_t - \Delta N_{t+1} \xi_{t+1}^\gamma$.

• For $1 \leq t \leq T - 1$, the posterior of $\xi_{t+1}^\nu$ is exponential with mean $\mu_v$ conditioning on $\Delta N_{t+1} = 0$. When $\Delta N_{t+1} = 1$, the posterior of $\xi_{t+1}^\nu$ is normal.

$$
\xi_{t+1}^\nu \sim N \left( \frac{S}{W}, \frac{1}{W} \right) 1_{\xi_{t+1}^\nu > 0}, \text{ with } \Delta N_{t+1} = 1,
$$

(53)

where $W = \frac{\Delta_t}{(1 - \rho^2) \sigma^2 V_t} + \frac{1}{\sigma^2}$, $S = \frac{\Delta_t}{(1 - \rho^2) \sigma^2 V_t} (B_{t+1} - \rho \sigma^2 A_{t+1}) + \frac{\mu_v}{\sigma^2}$, $A_{t+1} = Y_{t+1} - Y_t - \mu \Delta_t - \xi_{t+1}^\nu$, $B_{t+1} = V_{t+1} - V_t - (\kappa_v \theta_v - \kappa_v V_t) \Delta_t$. 39
• For $1 \leq t \leq T - 1$, the posterior of $\Delta N_{t+1}$ is Bernoulli.

$$\Delta N_{t+1} \sim \text{Bernoulli}(\frac{\alpha_1}{\alpha_1 + \alpha_2}),$$  \hspace{1cm} (54)

where $\alpha_1 = (\lambda_0 + \lambda_1 V_t + h_t)\Delta_t e^{-\frac{1}{2(1-\rho^2)}(A_1^2 - 2\rho A_2 + B_2^2)}$, $\alpha_2 = (1 - (\lambda_0 + \lambda_1 V_t + h_t)\Delta_t) e^{-\frac{1}{2(1-\rho^2)}(A_1^2 - 2\rho A_2 + B_2^2)}$, $A_1 = Y_{t+1} - Y_t - \mu \Delta_t - \xi^v_{t+1}$, $A_2 = Y_{t+1} - Y_t - \mu \Delta_t$, $B_1 = V_{t+1} - V_t - (\kappa_v \theta_v - \kappa_v V_t) \Delta_t - \xi^v_{t+1}$, $B_2 = V_{t+1} - V_t - (\kappa_v \theta_v - \kappa_v V_t) \Delta_t$.

D. State variables that appear in the option pricing formula

• For $2 \leq t \leq T - 1$, the posterior of $V_t$ is proportional to

$$\exp\left(-\frac{1}{2s^2}\left(\frac{(C_t - FC_t)^2}{(BAS_t C_t)^2} + \frac{(P_t - FP_t)^2}{(BAS_t P_t)^2}\right)\right)$$

$$\times \exp\left(-\frac{B_t^2 - 2\rho B_t A_t}{2(1-\rho^2)}\right) \times \frac{1}{V_t} \exp\left(-\frac{B_{t+1}^2 - 2\rho B_{t+1} A_{t+1} + A_{t+1}^2}{2(1-\rho^2)}\right)$$

$$\times ((\lambda_0 + \lambda_1 V_t + h_t)\Delta_t 1_{\Delta N_{t+1}=1} + (1 - (\lambda_0 + \lambda_1 V_t + h_t)\Delta_t) 1_{\Delta N_{t+1}=0}),$$  \hspace{1cm} (55)

where $A_t = \frac{Y_{t+1} - Y_t - \mu \Delta_t - \Delta N_{t+1} \xi^v_t}{\sqrt{V_t} h_t}$, $A_{t+1} = \frac{Y_{t+1} - Y_t - \mu \Delta_t - \Delta N_{t+1} \xi^v_t}{\sqrt{V_{t+1}} h_{t+1}}$, $B_t = \frac{V_t - V_{t+1} - (k_v \theta_v - k_v V_t) \Delta_t - \Delta N_{t+1} \xi^v_t}{\sigma_v \sqrt{V_t} h_t}$, $B_{t+1} = \frac{V_{t+1} - V_t - (k_v \theta_v - k_v V_t) \Delta_t - \Delta N_{t+1} \xi^v_t}{\sigma_v \sqrt{V_{t+1}} h_{t+1}}$. The posterior of $V_t$ for $t = 1$ and $T$ can be derived by modifying the transition densities.

• For $2 \leq t \leq T - 1$, the posterior of $h_t$ is proportional to

$$\exp\left(-\frac{1}{2s^2}\left(\frac{(C_t - FC_t)^2}{(BAS_t C_t)^2} + \frac{(P_t - FP_t)^2}{(BAS_t P_t)^2}\right)\right)$$

$$\times \exp\left(-\frac{B_t^2}{2}\right) \times \frac{1}{h_t} \exp\left(-\frac{B_{t+1}^2}{2}\right)$$

$$\times ((\lambda_0 + \lambda_1 V_t + h_t)\Delta_t 1_{\Delta N_{t+1}=1} + (1 - (\lambda_0 + \lambda_1 V_t + h_t)\Delta_t) 1_{\Delta N_{t+1}=0}),$$  \hspace{1cm} (56)

where $B_t = \frac{h_t - h_{t-1} - (k_v \theta_v - k_v h_{t-1}) \Delta_t}{\sigma_h \sqrt{h_{t-1} \Delta_t}}$, $B_{t+1} = \frac{h_{t+1} - h_t - (k_v \theta_v - k_v h_t) \Delta_t}{\sigma_h \sqrt{h_t \Delta_t}}$. The posterior of $h_t$ for $t = 1$ and $T$ can be derived by modifying the transition densities.
Figure 1: Daily S&P500 index, returns and implied volatility. The top two panels show the daily S&P500 index levels and returns, respectively. The third panel plots the implied volatilities of at-the-money S&P500 index options with the shortest maturities beyond two weeks. The last panel graphs the Volatility Index (VIX) levels. The sample period is from January 4, 1996 to April 29, 2011.
Figure 2: Time series of time-to-maturity and moneyness. The top panel shows the time-to-maturities of option series used in the estimation. The middle and bottom panels show the moneyness levels of the two option series, with strike-to-forward price ratios centered around 1.00 and 0.95, respectively. The sample period is from January 4, 1996 to April 29, 2011.
Figure 3: Posterior volatility implied from joint S&P500 returns and options. This figure plots volatilities estimated by the SV, SVJ, SVCJ and SVSJ models, respectively. The sample period is from January 4, 1996 to April 29, 2011.
Figure 4: Posterior jump intensity and its residuals implied from joint S&P500 returns and options.
The top two panels plot the separate jump intensity factor $h_t$ filtered from joint S&P500 returns and options by the SVSJ and SVSCJ models, respectively. The bottom two panels plot the innovations of $h_t$ filtered by the SVSJ and SVSCJ models, respectively. The sample period is from January 4, 1996 to April 29, 2011.
Figure 5: Estimated jumps in S&P500 returns. This figure plots jumps filtered from joint S&P500 returns and options by the SVJ, SVCJ and SVSJ models, respectively. The sample period is from January 4, 1996 to April 29, 2011.
Figure 6: Return residuals. This figure plots standardized log return innovations estimated from joint S&P500 returns and options by the SV, SVJ, SVCJ and SVSJ models, respectively. The sample period is from January 4, 1996 to April 29, 2011.
Figure 7: Volatility residuals. This figure plots standardized volatility innovations estimated from joint S&P500 returns and options by the SV, SVJ, SVCJ and SVSJ models, respectively. The sample period is from January 4, 1996 to April 29, 2011.
Figure 8: Model and market implied volatility smiles. This figure plots the Black-Scholes implied volatility smiles for option prices generated by models studied in this paper and those observed in the market, conditional on the at-the-money implied volatility and time-to-maturity. The lowest, median and highest volatility days during the sample period are listed from top to bottom. The shortest and 2nd shortest maturity options on the specific days are listed at the left and right, respectively. The market data of call options are marked by "×" and put options by "□".
Table I
Summary statistics of the S&P500 return series and two option series

This table reports summary statistics of the S&P500 index returns and option prices within the period from January 4, 1996 to April 29, 2011. Panel A provides summary statistics of the S&P500 continuously compounded daily returns. Panel A provides summary statistics of time-to-maturity, spot and forward index price, option price, implied volatility, strike price and moneyness (strike-to-forward ratio) of the two S&P500 index option series used in the estimation. We used options with the shortest maturities beyond 2 weeks, and with moneyness closest to 1 for the ATM call option series, closest to 0.95 for the OTM put option series, respectively. In general, different option contracts are used on different days during our sample period for both option series.


<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>S.t. Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P500</td>
<td>0.0002</td>
<td>0.013</td>
<td>-0.1924</td>
<td>10.5191</td>
<td>-0.0947</td>
<td>0.1096</td>
</tr>
</tbody>
</table>

Panel B. Summary statistics of the short term S&P500 index options used in the model fitting from January 4, 1996 to April 29, 2011.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>S.t. Dev.</th>
<th>Min.</th>
<th>Max.</th>
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</thead>
<tbody>
<tr>
<td>Time-to-maturity</td>
<td>29.77</td>
<td>29.00</td>
<td>9.15</td>
<td>15.00</td>
<td>49.00</td>
</tr>
<tr>
<td>Spot price</td>
<td>1134.76</td>
<td>1145.98</td>
<td>224.27</td>
<td>598.48</td>
<td>1565.15</td>
</tr>
<tr>
<td>Forward price</td>
<td>1136.53</td>
<td>1146.30</td>
<td>225.55</td>
<td>599.56</td>
<td>1571.51</td>
</tr>
<tr>
<td>The at-the-money call option series</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Option price</td>
<td>24.91</td>
<td>23.50</td>
<td>10.15</td>
<td>5.00</td>
<td>77.65</td>
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<tr>
<td>Implied volatility</td>
<td>0.1997</td>
<td>0.1887</td>
<td>0.0806</td>
<td>0.0831</td>
<td>0.7640</td>
</tr>
<tr>
<td>Strike price</td>
<td>1136.54</td>
<td>1150.00</td>
<td>225.49</td>
<td>600.00</td>
<td>1570.00</td>
</tr>
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<td>Moneyness (Strike/Forward)</td>
<td>1.0000</td>
<td>1.0001</td>
<td>0.0028</td>
<td>0.9716</td>
<td>1.0151</td>
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<td>The out-of-the-money put option series</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>Option price</td>
<td>9.79</td>
<td>7.95</td>
<td>7.44</td>
<td>0.60</td>
<td>58.45</td>
</tr>
<tr>
<td>Implied volatility</td>
<td>0.2387</td>
<td>0.2257</td>
<td>0.0808</td>
<td>0.1248</td>
<td>0.7999</td>
</tr>
<tr>
<td>Strike price</td>
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<td>1090.00</td>
<td>214.30</td>
<td>570.00</td>
<td>1495.00</td>
</tr>
<tr>
<td>Moneyness (Strike/Forward)</td>
<td>0.9501</td>
<td>0.9501</td>
<td>0.0035</td>
<td>0.9357</td>
<td>0.9726</td>
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Table II
Parameter estimates

This table reports posterior means and standard deviations of the parameter estimates based on joint S&P 500 returns and options data from January 4, 1996 to April 29, 2011. The models and parameterizations are provided in Section II. All the parameter estimates are annualized.

<table>
<thead>
<tr>
<th></th>
<th>SV</th>
<th>SVJ</th>
<th>SVCJ</th>
<th>SVSJ</th>
<th>SVSCJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_v$</td>
<td>0.0539</td>
<td>0.049</td>
<td>0.0214</td>
<td>0.0377</td>
<td>0.0267</td>
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<tr>
<td>(0.0098)</td>
<td>(0.0095)</td>
<td>(0.0025)</td>
<td>(0.0045)</td>
<td>(0.0022)</td>
<td></td>
</tr>
<tr>
<td>$\theta_h$</td>
<td>0.128</td>
<td>0.1621</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.0205)</td>
<td>(0.0130)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa_v$</td>
<td>2.2959</td>
<td>2.2165</td>
<td>3.8875</td>
<td>3.5565</td>
<td>5.5078</td>
</tr>
<tr>
<td>(0.3798)</td>
<td>(0.3830)</td>
<td>(0.3946)</td>
<td>(0.4104)</td>
<td>(0.4403)</td>
<td></td>
</tr>
<tr>
<td>$\kappa_v^Q$</td>
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<td>2.2056</td>
<td>2.2555</td>
<td>8.4728</td>
<td>9.1424</td>
</tr>
<tr>
<td>(0.1633)</td>
<td>(0.2859)</td>
<td>(0.1946)</td>
<td>(0.2128)</td>
<td>(0.1657)</td>
<td></td>
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<tr>
<td>$\eta'$</td>
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<td>0.0109</td>
<td>1.6319</td>
<td>-4.9162</td>
<td>-3.6346</td>
</tr>
<tr>
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<td>(0.4286)</td>
<td>(0.4544)</td>
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<td>4.6884</td>
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<td>(0.2254)</td>
<td></td>
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<tr>
<td>$\sigma_v$</td>
<td>0.4587</td>
<td>0.4545</td>
<td>0.4297</td>
<td>0.3814</td>
<td>0.3846</td>
</tr>
<tr>
<td>(0.0067)</td>
<td>(0.0080)</td>
<td>(0.0064)</td>
<td>(0.0055)</td>
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<tr>
<td>$\sigma_h$</td>
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<td>(0.0508)</td>
<td>(0.0440)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>-0.8298</td>
<td>-0.8292</td>
<td>-0.819</td>
<td>-0.8125</td>
<td>-0.7978</td>
</tr>
<tr>
<td>(0.0061)</td>
<td>(0.0067)</td>
<td>(0.0066)</td>
<td>(0.0067)</td>
<td>(0.0065)</td>
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</tr>
<tr>
<td>$\mu_y$</td>
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<td>-0.0288</td>
<td>0.0169</td>
<td>-0.0309</td>
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</tr>
<tr>
<td>(0.0285)</td>
<td>(0.0123)</td>
<td>(0.0207)</td>
<td>(0.0118)</td>
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<tr>
<td>$\mu_y^Q$</td>
<td>-0.0036</td>
<td>-0.0074</td>
<td>-0.0978</td>
<td>-0.1013</td>
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</tr>
<tr>
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<td>(0.0090)</td>
<td>(0.0023)</td>
<td>(0.0015)</td>
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</tr>
<tr>
<td>$\sigma_y$</td>
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<td>0.0729</td>
<td>0.0593</td>
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<tr>
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<tr>
<td>$\sigma_y^Q$</td>
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<td>0.0553</td>
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<tr>
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<td>(0.0045)</td>
<td>(0.0037)</td>
<td>(0.0017)</td>
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<tr>
<td>$\mu_v$</td>
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<td>0.0283</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>(0.0036)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>0.3853</td>
<td>0.8756</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.0743)</td>
<td>(0.1049)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>1.5489</td>
<td>1.6826</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0.2502)</td>
<td>(0.1583)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>0.3867</td>
<td>0.3872</td>
<td>0.3396</td>
<td>0.1854</td>
<td>0.1779</td>
</tr>
<tr>
<td>(0.0103)</td>
<td>(0.0106)</td>
<td>(0.0101)</td>
<td>(0.0040)</td>
<td>(0.0038)</td>
<td></td>
</tr>
</tbody>
</table>
Table III  
**Residual statistics**

This table reports the mean, standard deviation, skewness and kurtosis of the estimated return, volatility and jump intensity residuals. The residuals of the five models are measured as the standardized daily Brownian innovations.

<table>
<thead>
<tr>
<th></th>
<th>SV</th>
<th>SVJ</th>
<th>SVCJ</th>
<th>SVSJ</th>
<th>SVSCJ</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>0.0066</td>
<td>-0.0006</td>
<td>-0.0211</td>
<td>0.0060</td>
<td>0.0082</td>
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<tr>
<td><strong>Standard deviation</strong></td>
<td>0.9020</td>
<td>0.9156</td>
<td>0.8988</td>
<td>1.0080</td>
<td>0.9720</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>-0.3787</td>
<td>-0.4162</td>
<td>-0.2164</td>
<td>-0.4276</td>
<td>-0.1143</td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
<td>4.0964</td>
<td>4.0382</td>
<td>3.2775</td>
<td>4.2396</td>
<td>3.7030</td>
</tr>
<tr>
<td><strong>Volatility residuals</strong></td>
<td>-0.0095</td>
<td>-0.0017</td>
<td>0.0266</td>
<td>-0.0095</td>
<td>-0.0064</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>0.8652</td>
<td>0.8745</td>
<td>0.8597</td>
<td>0.9261</td>
<td>0.8994</td>
</tr>
<tr>
<td><strong>Standard deviation</strong></td>
<td>0.7809</td>
<td>0.7875</td>
<td>0.1953</td>
<td>0.6868</td>
<td>0.0674</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>6.2479</td>
<td>6.3037</td>
<td>4.0291</td>
<td>6.0065</td>
<td>5.2574</td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
<td>0.1010</td>
<td>0.0895</td>
<td>0.8919</td>
<td>0.8851</td>
<td>0.7206</td>
</tr>
<tr>
<td><strong>Jump intensity residuals</strong></td>
<td>1.3520</td>
<td>0.7206</td>
<td>11.5739</td>
<td>6.2030</td>
<td></td>
</tr>
</tbody>
</table>
Table IV

**In-sample option pricing errors**

This table reports the in-sample performances of the five models for the pricing of the S&P500 index options. SV, SVJ and SVCJ models use the at-the-money option series. SVSJ and SVSCJ models use an additional out-of-the-money option series with strike-to-forward price ratio closest to 0.95. The absolute pricing errors are measured as the absolute differences between the market prices and model implied option prices in dollars. The percentage errors are the absolute errors divided by the market observed option prices.

<table>
<thead>
<tr>
<th></th>
<th>SV</th>
<th>SVJ</th>
<th>SVCJ</th>
<th>SVSJ</th>
<th>SVSCJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute (in dollars)</td>
<td>0.3108</td>
<td>0.3133</td>
<td>0.2734</td>
<td>0.1149</td>
<td>0.1099</td>
</tr>
<tr>
<td></td>
<td>(0.6076)</td>
<td>(0.6102)</td>
<td>(0.5105)</td>
<td>(0.2203)</td>
<td>(0.2063)</td>
</tr>
<tr>
<td>Percentage</td>
<td>0.0112</td>
<td>0.0113</td>
<td>0.0099</td>
<td>0.0034</td>
<td>0.0032</td>
</tr>
<tr>
<td></td>
<td>(0.0145)</td>
<td>(0.0146)</td>
<td>(0.0124)</td>
<td>(0.0053)</td>
<td>(0.0048)</td>
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</tbody>
</table>
Table V  
**Out-of-sample option pricing errors**

This table reports the out-of-sample performances of the five models for the pricing of the S&P500 index options. Results are shown conditional on strike-to-forward price ratios and time-to-maturity. The pricing errors are measured as the absolute differences between the market prices and model implied option prices in dollars. A simple arithmetic average is calculated for each moneyness and maturity group. We also report the number of options within each group.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Strike-to-forward price ratio</th>
<th>0.90-0.93</th>
<th>0.93-0.97</th>
<th>0.97-1.00</th>
<th>1.00-1.03</th>
<th>1.03-1.07</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;1 m</td>
<td>#</td>
<td>10903</td>
<td>21164</td>
<td>21770</td>
<td>21175</td>
<td>17543</td>
<td>92555</td>
</tr>
<tr>
<td></td>
<td>SV</td>
<td>1.53</td>
<td>1.35</td>
<td>0.64</td>
<td>0.56</td>
<td>0.82</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>SVJ</td>
<td>1.49</td>
<td>1.31</td>
<td>0.63</td>
<td>0.55</td>
<td>0.80</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>SVCJ</td>
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<td>1.29</td>
<td>0.62</td>
<td>0.53</td>
<td>0.85</td>
<td>0.90</td>
</tr>
<tr>
<td></td>
<td>SVSJ</td>
<td>0.62</td>
<td>0.26</td>
<td>0.30</td>
<td>0.43</td>
<td>1.18</td>
<td>0.52</td>
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<tr>
<td></td>
<td>SVSCJ</td>
<td>0.59</td>
<td>0.25</td>
<td>0.29</td>
<td>0.41</td>
<td>1.05</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>#</td>
<td>13284</td>
<td>24385</td>
<td>26768</td>
<td>26886</td>
<td>22852</td>
<td>114175</td>
</tr>
<tr>
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<td>SV</td>
<td>2.02</td>
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<td>1.04</td>
<td>0.88</td>
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<td>SVJ</td>
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<td>1.57</td>
<td>1.04</td>
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<td>1.20</td>
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<td>0.90</td>
<td>0.96</td>
<td>1.38</td>
<td>0.96</td>
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<tr>
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<td>2.36</td>
<td>2.05</td>
<td>2.26</td>
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<td>2.31</td>
<td>2.01</td>
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<tr>
<td></td>
<td>#</td>
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<td>62526</td>
<td>51464</td>
<td>264922</td>
</tr>
<tr>
<td>1-2 m</td>
<td>SV</td>
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<td>1.65</td>
<td>1.15</td>
<td>0.99</td>
<td>1.16</td>
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</tr>
<tr>
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<td>1.95</td>
<td>1.64</td>
<td>1.15</td>
<td>0.99</td>
<td>1.14</td>
<td>1.31</td>
</tr>
<tr>
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<td>SVCJ</td>
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<td>1.15</td>
<td>0.98</td>
<td>1.20</td>
<td>1.32</td>
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<td>SVCJ</td>
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<td>1.15</td>
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