The Licensing of Complementary Innovations and the Threat of Litigation*

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Abstract

It has been commonly argued that in Standard Setting Organizations the large number of inventors that hold patents for technologies that are essential for development of compatible products leads to excessively large royalties. This well-known Cournot-complements or Royalty-Stacking effect would hurt efficiency and downstream competition. In this paper we show that when we consider patent litigation and introduce heterogeneity in the portfolio of different firms these results change substantially due to what we denote the Inverse-Cournot effect. We show that the higher the total royalty that a downstream producer pays, the higher the royalty that small patent holders restricted by the threat of litigation of that downstream producer will charge. This effect generates a moderation force in the royalty that unconstrained large patent holders will charge that may overturn some of the standard predictions in the literature. Furthermore, we show that our model has implications for patent aggregation, patent pools, and vertical mergers consistent with what has been observed in the mobile telecommunications industry, typically considered an example of royalty stacking.

JEL codes: L15, L24, O31, O34.


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1 Introduction

The fundamental nature of the patent system is under debate among claims on whether it fosters or hurts innovation. The main concerns focus on the impact of patent enforcement in the ICT industry. ICT products, such as laptops, tablets or smartphones, use multiple technologies covered by complementary patents. The royalties that must be paid for multiple patented technologies in a single product summed together are said to form a harmful “royalty stack” (Lemley and Shapiro, 2007). The use of injunction threats in an industry characterized by royalty stacking leads, it is argued, to the systematic overcompensation of patent owners. This in turn is claimed to result in excessively high end-product prices and a reduction in the incentives to invest and innovate in product markets.

The arguments supporting royalty stacking and the need for a profound reform of the patent system rely on theoretical models which reformulate the well-known Cournot complements problem in a licensing framework. Cournot (1838) showed that consumers are better off when all products that are complementary from a demand viewpoint are produced and marketed by a single firm. In industries where each single product is covered by multiple patents, when each patent holder considers the royalty to charge, it may not fully take into account that an increase in its royalty is likely to result in a cumulative royalty rate that may be too high according to both the licensee and its consumers. Since this negative externality (or Cournot effect) will be ignored by all patent holders, the royalty stack may prove inefficiently high. For this reason, papers such as Lerner and Tirole (2004) have argued that “patent pools” which consolidate complementary patent rights into a single bundle are generally considered welfare-enhancing.

The Cournot effect also explains current concerns with the emergence of “patent privateers,” firms that spin off patents for others to assert them. Lemley and Melamed
argue that “patent reformers and antitrust authorities should worry less about aggregation of patent rights and more about disaggregation of those rights, sometimes accomplished by spinning them out to others.” Similarly, “patent trolls” or “patent assertion entities” (PAEs) - i.e. patent owners whose primary business is to enforce patents to collect royalties - are accused of imposing disproportionate litigation costs and extracting excessive patent royalties and damage awards because the existing patent system allows them to leverage even relatively small portfolios of “weak patents.” The America Invents Act (AIA) enacted by the US Congress in 2011 was designed in part to deal with the problems created by trolls. Patent reform is now back in Congress with new legislation (the Innovation Act) intended to apply specifically to patent trolls.

The call for reform is not unanimous: some consider that royalty stacking is a theoretical possibility without empirical support. The controversy about the empirical relevance of royalty stacking, or about the economic implications of the activity of patent trolls, is raging. In March 2015, a group of distinguished lawyers and economists wrote a letter to Congress arguing that “a large and increasing body of evidence indicates that the net effect of patent litigation is to raise the cost of litigation and inhibit technological progress, subverting the very purpose of the patent system.” In response, another group of lawyers and economists wrote to Congress to express their “deep concerns with the many flawed, unreliable, or incomplete studies about the American patent system that have been provided to members of Congress.”

1 A weak patent is defined as a patent that may well be invalid, but nobody knows for sure without conclusive litigation (see Farrell and Shapiro (2008)).
to hinder the implementation of mobile communication standards and the development of new standards and innovations. The US Court of Appeals for the Federal Circuit in Ericsson v D. Link stated: “The best word to describe [the] royalty stacking argument is theoretical.”

The absence of (clear-cut) evidence in support of royalty stacking is puzzling given that the theoretical foundations of this hypothesis has remained unchallenged. In this paper we develop a model of licensing complementary innovations under the threat of litigation that explains the circumstances under which royalty stacking is likely to be a problem in practice. This model departs from the extant literature in only one natural dimension; we assume that manufacturers of products covered by multiple patented technologies may challenge in court the patents that cover their products and, crucially, the likelihood that a judge rules in favor of the patent holder is increasing in the number and quality of its patents. This assumption is reasonable. Downstream manufacturers commonly challenge the validity of the patents that cover their products when they litigate in court the licensing terms offered by patent holders. This strategy is less likely to succeed if the patent holder holds a large and valuable patent portfolio.

Patent holders with large and high quality patent portfolios will not be constrained by the threat of litigation when setting royalty rates. On the contrary, owners of weak portfolios will have to moderate their royalty claims in order to avoid litigation over patent validity. Since setting up and running an active licensing program involves non-trivial fixed costs, the number of active licensors might be smaller than the number of patent holders. Developers with weak patent portfolios will not find it profitable to become active licensors. The number of active licensors will be smaller when only a few patent holders have strong patent portfolios.

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More interestingly, our analysis shows that the ability of a patent owner to charge a high royalty without triggering litigation depends on the aggregate royalty charged by all other patent holders: the higher that aggregate rate, the higher the royalty that any patent holder can charge. The intuition is that the higher the aggregate rate charge by other owners, the less likely it is that the gains from litigation cover the legal costs of challenging the validity of the patents owned by a given licensor. This positive relationship is a novel effect that we denote as the **Inverse Cournot Effect**. This effect provides incentives for unconstrained patent holders (i.e. with strong portfolios) to cut down their royalty rates to force patent holders with weak portfolios to charge lower royalties or else to face litigation. In so doing, the Inverse Cournot Effect offsets the royalty stacking effect (or Cournot effect). We find that the Inverse Cournot Effect becomes more important when patent holdings are more skewed. As a result, the total royalty rate decreases when patent holdings are more skewed.

Standard royalty-stacking models suggest that consolidating (complementary) patent holdings in a few hands are unambiguously welfare increasing because they reduce the overall royalty burden and end-consumer prices. This consolidation increases firm profits and social welfare. Our model suggests another motivation for this consolidations. Aggregating weak portfolios will allow patent holders to charge higher royalty rates by mitigating the threat of litigation. The motivation for this consolidation goes against social welfare. That is, we show that this consolidation is profitable if and only if it raises the total royalty rate.

Finally, we also study the incentives for downstream producers to integrate upstream developers. We show that this integration will be profitable when targeting large developers because it not only reduces the double-marginalization problem but also because, due to the Inverse Cournot Effect, it forces other smaller developers to lower their royalty, reducing the royalty stack.
Our model has a number of testable implications, which provide the means to falsify it. Consider different technological standards, each of the covered by multiple patents, but exhibiting different distributions of patent holdings. We could rank these standards according to the skewness of such distributions. Our theory predicts that the cumulative royalty rate should be lower when the distribution of patent holdings is more skewed, i.e. when a few patent holders accumulate most valuable patents. The effect of patent holding heterogeneity on the total aggregated royalty rate will be smaller when downstream competition is fierce. Our theory also predicts that patent pools combining strong patent portfolios will be more common in industries with a less skewed distribution of patent holdings. When downstream competition is stronger a patent pool involving the whole industry is more likely to form, since licensor profits are more similar, irrespective of the strength of their portfolios, and the total royalty is likely to be higher. Regarding patent acquisitions, our models implies that downstream manufacturers will tend to acquire large and valuable patent portfolios and that such acquisitions will be more likely in standards exhibiting a less skewed distribution of patent holdings. Instead, we predict that PAEs will be more active in highly skewed standards and will focus their acquisition activity in the aggregation of weak patent portfolios.

We have tested some of these propositions using data from the 2G, 3G and 4G mobile communication standards. Each of these standards comprises many complementary “standard-essential patents” (SEPs). The distribution of patent holdings is very different in each case, however: the distribution of 4G SEPs is significantly less skewed than the distribution of 2G and 3G SEPs. (The top 2 owners of 2G SEPs have 42% of all 2G SEPs. The corresponding figures for 3G and 4G are 30% and 23%, respectively.) We do not have data on the total aggregate royalty rate for each of these standards, but we note that the only examples of excessive royalty rates charged by owners of weak patent
portfolios concern downstream industries with fierce downstream competition.

We do have data on patent pools and patent acquisitions. Consistently with our theory, we observe that the only patent pool arrangement involving owners of large patent portfolios involved 4G SEPs. All 3G patent pools were integrated by companies with small SEP portfolios. The proportion of all 4G SEPs covered by pools was 56.1%, which is much higher than the corresponding figure for the 3G standard: 8.1%. Also in line with our theory we find that (a) the percentage of acquisition deals involving PAEs as buyers was highest in the more skewed standard, 2G, and lowest in the less skewed standard, 4G; and (b) the number and percentage of deals involving traditional industry participants (i.e. implementers with or without patent portfolios) was highest in the less skewed standard, 4G, and lowest in the most skewed standard, 2G.

1.1 Literature Review

The literature on standard setting organizations (SSOs) has emphasized the fact that the licensing of complementary and essential patents by many developers could give rise to a royalty stacking problem. This concern, related to the standard Cournot complements problem was introduced in this context by Shapiro (2001). Spulber (2014), however, shows that this is not a general result and quantity competition would generate the cooperative outcome and, once R&D is endogeneized, more innovation.

Our paper is also related to a long literature on patent litigation, including papers like Llobet (2003) and Farrell and Shapiro (2008). More recent works have focused on SSOs and captured the conflict between producers and Non-Practicing Entities (NPEs). This is the case, for example, of Choi and Gerlach (2015a) that studies the information externalities that arise when a NPE sequentially litigates against several producers.

The paper closest to ours are Choi and Gerlach (2014) and, particularly, Bourreau (2014) notes that most patent trolls target firms selling less than $100 million a year.
et al. (2015). The former studies the incentives for firms to acquire patent portfolios as a function of existing patent holdings of the different firms. In the latter, the authors study licensing and litigation in SSOs, as well as the decisions of firms to sell their IP to other innovators. The main important difference with our paper, however, is that in their setup litigation occurs ex-post. As a result, the total quantity produced does not depend on the outcome of this litigation and the damages paid are constant. This assumption severs the link between the licensing decision of different patent holders eliminating the Inverse Cournot Effect that plays an important role in our model. Instead, in that paper the interaction arises from ex-ante competition.

Finally, our paper is related to the literature on patent pools. Lerner and Tirole (2004) devise a mechanism to weed out welfare-decreasing patent pools that include substitute patents and might induce collusion from welfare-increasing ones that include only complements. This mechanism consists of allowing firms to offer a price to license their patents together and separately. This rule leads to a unique equilibrium only when there are two patent holders. Boutin (2014) provides additional conditions on independent-licensing to guarantee that there exists a unique equilibrium in which welfare-decreasing pools will not emerge. Our work contributes to this literature by showing that even if we restrict ourselves to complementary patents, patent pools may not increase welfare if they include small patent holders which increase their chances in court when they bundle their patents, making royalty stacking more significant. Choi and Gerlach (2015b) develop a model of patent litigation of weak patent holders and endogenous formation of pools that delivers a similar insight. In their model, however, the interaction between large and small patent holders outside of the pool is absent.

\footnote{Lerner and Tirole (2015) generalizes the previous argument to SSOs.}
2 The Model

Consider a market in which a downstream monopolist, firm $D$, produces a good the demand for which can be represented by the function $D(p)$, where $p$ is the price that final consumers pay. This demand is downward slopping and twice continuously differentiable. The production of the good requires firm $D$ to obtain a license for the technologies of $N$ different pure upstream developers. Each developer $i \in \{1, \ldots, N\}$ holds a patent portfolio of $x_i$ patents relevant for its own technology. Assume that $x_1 \geq x_2 \geq \ldots \geq x_N$.

Each upstream developer charges a per-unit royalty $r_i$ to license the patents necessary to make use of its technology. We denote the total royalty that the downstream developer pays as $R = \sum_{i=1}^{N} r_i$. We assume that there is no further cost of production so that the marginal cost is also equal to $R$.

The royalty rate for technology $i$ is set by upstream developer $i$ as a take-it-or-leave-it offer. The downstream producer, however, might challenge in court the patents that cover the technology. Litigation between the downstream monopolist and any upstream developer involves legal costs $L_D$ and $L_U$, respectively. The success in court of the upstream patent holder is based on its number of patents. In particular, the probability that a judge rules in favor of patent holder $i$, denoted as $g(x_i)$, is assumed to be increasing in $x_i$. This assumption can be justified on several grounds. First, one of the most common ways for a downstream developer to dispute in court the licensing terms offered is to challenge the validity of the patents that cover the technology. This strategy is less likely to succeed if the developer holds more patents (or if they are more valuable), increasing the chances in court of the developer. Second, developers do not typically defend their innovations with all their patent portfolio but, rather, they choose the patents that are

\footnote{For simplicity we abstract from situations in which upstream developers own patented technologies that might be infringed by other upstream developers.}

\footnote{As pointed out in Llobet and Padilla (2014) royalty-stacking problems are aggravated under per-unit royalties compared to the more frequent ad-valorem royalties, based on firm revenue. As it will become clear later in the paper, assuming ad-valorem royalties should strengthen the results.}
most likely to be upheld in court or that are more relevant for the disputed application. Thus, a developer is more likely to find a suitable patent for litigation if choosing from a larger patent portfolio. Finally, the model is isomorphic to one in which each upstream developer $i$ holds a unique patent of quality (or a number of patents of weighted quality) $x_i$. To the extent that more substantial innovations translate into stronger patents, we can interpret the increasing function $g(x)$ as a reflection of this relationship.

The timing of the model is as follows. First, all upstream developers choose their royalty rate simultaneously. In the second stage the downstream producer chooses which patents to litigate (if any). In the final stage, the downstream firm chooses the price for the final good.

In order to set the stage for the model we start with the case in which legal costs are sufficiently high so that litigation never plays a role in the model. This assumption will give raise to the standard Royalty Stacking result in the literature that we reproduce in the following section.

### 2.1 High Downstream Litigation Costs

Consider the case in which $L_D$ is sufficiently high so that it would never be profitable for the downstream producer to go to court. Upstream developers would not be constrained in any way by litigation. This implies that the stages of the game can be collapsed into two, as the second stage becomes irrelevant. First, developers simultaneously choose their royalty rate. The downstream producer observes the royalty and chooses the final price to maximize profits.

The expression for profits of the downstream producer, given a total royalty $R$, arises from

$$\Pi_D(R) = \max_p (p - R)D(p).$$

Standard calculations show that the optimal price $p^M(R)$ is increasing in $R$ and, therefore,
the profit function is decreasing and convex in \( R \): \( \Pi'_D(R) = -D(p) < 0 \) and \( \Pi''_D(R) = -D'(p) \frac{dp^M}{dR}(R) > 0 \).

In the first stage, each upstream developer \( i \) chooses its royalty \( r_i \) to maximize

\[
\max_{r_i} r_i D \left( p^M(R_{-i} + r_i) \right),
\]

where we denote as \( R_{-i} = \sum_{j \neq i} r_j \) the total royalty payments from licensees other than licensee \( i \). Throughout the paper we will define upstream profits as \( \Pi_U(r_i, R_{-i}) = r_i D \left( p^M(R_{-i} + r_i) \right) \).

In order to guarantee that the problem is well behaved and the first-order conditions are necessary and sufficient we maintain throughout the paper the following standard regularity assumption.

**Assumption 1.** The marginal return from the aggregate royalty rate,

\[
D(p^M(R)) + RD'(p^M(R)) \frac{dp^M}{dR}
\]

is decreasing in \( R \).

Patent holdings are only relevant to the extent that they can affect the probability that the patent holder wins in court. If the downstream producer cannot challenge the patents that protect the different technologies, all upstream producers become identical. All technologies are perfect complements, reinforcing the market power of upstream developers, independently of the strength of each firm’s patent portfolio. Under the previous assumption we have that the model has a unique symmetric equilibrium in which all firms choose a royalty \( r_i = r^* \), arising from the first-order condition of the problem in (1),

\[
D(p^M(R^*)) + r^* D'(p^M(R^*)) \frac{dp^M}{dR} = 0.
\]

**Proposition 1** (Royalty Stacking). If litigation is sufficiently costly for the downstream producer, in the unique equilibrium of the game all firms choose \( r^*_i = r^* \), defined by (2),
independent of each firm’s patent portfolio. In this equilibrium \( r^*(N) \) is decreasing in \( N \) but \( R^*(N) \) is increasing in \( N \).

This result is a version of the Cournot complements problem, in which firms choosing quantities of complementary products end up producing too little compared even to a monopolist’s quantity. The intuition is standard and it has been emphasized in papers like [Lemley and Shapiro (2007)](http://example.com) under the name of royalty-stacking. In particular, it is easy to show that when technologies are complementary, as in the case we illustrate here, royalty rates become strategic substitutes. When developer \( i \) chooses its royalty it does not internalize the fact that the decrease in sales of the final product not only affects its own royalty revenues but also the revenue of all other developers. This negative externality implies that the aggregate royalty is higher than the one these firms would choose if they could coordinate and choose the total royalty together. Furthermore, whereas in that case the total royalty would be independent of the number of firms (or the number of patents), in the equilibrium we observe that the royalty-stacking problem becomes more severe the more fragmented are patent portfolios in the hands of more firms. Throughout the paper we denote the strategic-substitutes relationship that leads to this distortion the Cournot Effect.

The previous result also shows that each developer obtains in equilibrium the same royalty, which is independent of the size of its patent portfolio or the quality of its patents. However, this result is at odds with existing evidence suggesting that firms that hold better patents receive higher royalty payments. This pattern will emerge next, once enforcement becomes an issue.

### 2.2 The Threat of Patent Litigation

We now study the situation in which the downstream producer faces legal costs sufficiently low so that the equilibrium candidate derived in Proposition 1 does not emerge. The
reason is that under those royalty rates the downstream producer would be interested in litigating the developers with a weaker patent portfolio. As we will see next, upstream developers will respond to this threat, when credible, by lowering their royalty rate.

We solve the game by backwards induction. In the last stage the downstream producer chooses the price as in the previous case, leading to an equilibrium value $p^M(R)$. We now analyze the effect of the threat of litigation on the licensing decision.

2.2.1 The Patent Litigation Decision

Consider the decision of the downstream producer to litigate the royalty terms set by upstream developer $i$. If the downstream producer decides to accept the terms set by developer $i$, profits for the downstream firm and the developer correspond to $\Pi_D(R_{-i}+r_i)$ and $\Pi_U(r_i,R_{-i})$. If the patents are litigated and the court rules in favor of the patent holder – an outcome that occurs with probability $g(x_i)$ –, profits become $\Pi_D(R_{-i}+r_i)-L_D$ and $\Pi_U(r_i,R_{-i})-L_U$ for the downstream producer and upstream developer $i$ since the original situation is preserved but legal costs are incurred. Instead, if the court considers that there has been no infringement, ruling in favor of the downstream producer – an outcome that arises with probability $1-g(x_i)$ –, profits for the producer and upstream developer $i$ correspond to $\Pi_D(R_{-i})-L_D$ and $-L_U$, respectively. That is, the upstream firm loses the possibility to charge any royalty, leading to 0 profits, gross of legal costs.

As a result, the downstream developer decides to litigate upstream developer $i$ if and only if

$$(1-g(x_i)) [\Pi_D(R_{-i}) - \Pi_D(R_{-i} + r_i)] \geq L_D.$$ 

In other words, litigation will occur as long as the expected gains from avoiding to pay a license for the patents of developer $i$ are higher than the legal costs involved.\footnote{Notice that litigation in this model is related to whether the patents owned by developer $i$ are valid and have been infringed or not. We are abstracting from litigation arising due to the non-payment of the obligations related to the license. That case would entail lower litigation costs and, for simplicity, we assume that as a result the upstream developer would always sue if the payment is not made.} We
define the highest royalty that makes the downstream producer indifferent between going to court or not as \( \bar{r}_i \),

\[
\Pi_D(R_{-i}) - \Pi_D(R_{-i} + \bar{r}_i) = \Lambda_i,
\]

where we denote as \( \Lambda_i \equiv \frac{L_D}{1 - g(x_i)} \) as the litigation hurdle that the downstream producer must overcome to invalidate the patent. Of course, this hurdle is increasing in its legal costs and decreasing in the probability of winning in court.

The next lemma characterizes how this ceiling on the royalty rate depends on the parameters of the model.

**Lemma 1.** The downstream producer will litigate upstream patent holder \( i \) if \( r_i \geq \bar{r}_i(x_i, R_{-i}) \), as defined by (3). This threshold royalty \( \bar{r}_i \) is strictly increasing in \( R_{-i} \) and \( \Lambda_i \).

Obviously, the highest royalty that the upstream developer can charge without triggering litigation is increasing in the litigation hurdle, since a stronger patent portfolio and higher downstream legal costs decrease the returns from going to court.

More interestingly, the previous lemma also illustrates the fact that the ability of an upstream developer to charge a high royalty without triggering litigation depends on the royalty charged by other upstream developers. In particular, the higher the royalty that other developers charge, the higher the royalty that firm \( i \) can charge. The intuition is that if \( R_{-i} \) is high, profits for the downstream producer are low, independently of whether the patents of firm \( i \) are upheld in court or not. Thus, the higher the aggregate royalty of other firms the less likely it is that the gains from litigation cover the legal costs involved. This positive relationship between the royalty rate that firms charge is a novel effect that we denote the **Inverse Cournot Effect**. This effect will play an important role in the results below as a counterbalancing force to the conventional royalty-stacking (or standard Cournot) effect.
2.2.2 The Equilibrium Royalty

In the choice of the royalty an upstream developer faces two possibilities. First, it might choose a royalty $r_i$ lower than $\bar{r}_i$ and discourage litigation from the downstream producer, obtaining profits $\Pi_U(R_i + r_i)$. Alternatively, if a royalty $r_i > \bar{r}_i$ is chosen the downstream producer will fight the patents in court and the developer will obtain expected profits $g(x_i)\Pi_U(R_i + r_i) - L_U$. Notice that since the probability of court victory does not depend on $r_i$ upstream developer $i$ will only consider two options. It will either choose $\bar{r}_i$ and avert litigation or choose the royalty that maximizes profits abstracting from litigation, $\hat{r}_i$, which arises from

$$\hat{r}_i \in \arg \max_{r_i} r_i D \left( p^M (R_i + r_i) \right).$$

From Lemma 1, if firm $x_i$ is sufficiently high we have that $\hat{r}_i < \bar{r}_i$, and the developer will just choose $\hat{r}_i$ since the profit maximizing royalty does not trigger litigation. If $x_i$ is small and $\hat{r}_i > \bar{r}_i$ the developer will choose $\hat{r}_i$ if and only if

$$g(x_i)\Pi_U(\hat{r}_i, R_{-i}) - L_U > \Pi_U(\bar{r}_i, R_{-i}).$$

In other words it will litigate if the profits of doing so are higher than the profits from offering the highest royalty that the downstream producer would accept. It is only in this latter case that litigation would emerge in equilibrium. The specific region of values of $x_i$ for which litigation would arise, however, depends on the shape of the demand function and the probability of success in court. It is clear that if $x_i$ is such that $\bar{r}_i$ is lower than $\hat{r}_i$ but very similar, litigation would not emerge, since the difference in profits would be unlikely to compensate the legal costs involved. Similarly, if $x_i$ were very low and the developer anticipated a very small probability of success it would not find worthwhile to go to court.

In order to simplify the analysis we will focus on the case in which litigation destroys value, which can be understood as a case of high legal costs.
Assumption 2. The total surplus from litigation is negative.

This assumption is naturally satisfied in models of settlement and litigation, in which agents decide how to divide a fixed amount.\footnote{In those models, settlement arises as a way to avoid value-destroying litigation, due to the costs it entails. For this reason, it is typically assumed that litigation occurs either because agents have different beliefs about the probability they might succeed in court or because firms hold private information on the strength of their case.} This is not the case here and, absent the previous assumption, litigation could occur in equilibrium. As an illustration, consider the extreme case without legal costs. It is immediate that in such a setup litigation would always add social value. The reason is that courts would leave the market outcome unchanged with probability \( g(x_i) \) but with probability \( 1 - g(x_i) \) they would rule in favor of the downstream producer, reducing the royalty to 0 and eliminating the double-marginalization distortion. The former outcome would leave surplus unchanged but the latter would increase total surplus. As a result, litigation could occur in equilibrium because the upstream developer would gamble hoping that courts would enforce the monopoly royalty \( \hat{r}_i \) while, at the same time, the downstream producer would hope that the patents were considered invalid in court, reducing the aggregate royalty.

Assumption 2 implies that legal costs are significant and, as a result, they cannot be compensated with the benefits from eliminating double marginalization. As a result, litigation will never emerge in the equilibrium of the model.

Therefore, we consider the case in which legal costs for the downstream producer take an intermediate value, sufficiently high so that litigation does not emerge in equilibrium but low enough so that litigation is a threat for upstream developers with weak patent portfolios. In that case, upstream developers will be divided in two classes. Those with small patent holdings, \( i > \hat{n} \), will be constrained by the litigation threat. Those with large patent holdings, \( i \leq \hat{n} \) will choose the royalty, \( r^* \), arising from (4) so that

\[
D(p^M(R^*)) + r^* D'(p^M(R^*)) \frac{dp^M}{dR} = 0, \tag{5}
\]
where
\[
R^* = \tilde{n}r^* + \sum_{i > \tilde{n}} \tilde{r}_i(x_i, R^* - r_i^*).
\]

The next proposition summarizes the previous discussion.

**Proposition 2.** In the equilibrium of the game, there is a developer \( \tilde{n} \in [1, N] \) such that those with larger patent holdings, \( i \leq \tilde{n} \), choose the monopoly royalty rate \( r_i^* = r^* \), defined in (5). Developers with smaller patent holdings, \( i > \tilde{n} \), choose \( r_i^* = \tilde{r}_i(x_i, R^* - r_i^*) \).

The strategic behavior of upstream developers depends on their patent portfolio. Those with large patent portfolios, \( i \leq \tilde{n} \), respond to the royalty of other developers according to the standard Cournot Effect contributing to the royalty-stacking problem. Those with \( i > \tilde{n} \) are actually limited in the royalty they can charge. Interestingly, the Inverse Cournot Effect indicates that the fewer the unconstrained patent holders and, as a consequence, the lower the aggregate royalty, the lower will be the maximum royalty that small patent holders can charge. This effect mitigates the royalty-stacking problem.

The previous expressions are unlikely to lead to explicit solutions for the royalty rates that each firm charges, since they depend in turn on the aggregate royalty. For this reason, we now illustrate these two effects in a very simple case with two types of patent holders, depending on whether they have a large or small patent portfolio. We later show the results of numerical examples for a more general case.

In Appendix A we study the effect of introducing downstream competition. As we show there, more competition makes litigation less relevant for two reasons. First, it creates a free-riding effect, as invalidated patents reduce the total royalty that all downstream producers pay but the cost is only incurred by one firm. Second, because firms incur in lower mark-ups and lower sales when competition increases, the difference in profits between invalidating the patents of a downstream competitor or not decreases and so do the incentives to litigate. Thus, more downstream competition makes the litig...
gation margin less relevant, increasing the royalty-stacking problem (even if we abstract from the standard double-marginalization argument). Because adding competition does not have qualitative implications for the main results of the paper we focus for simplicity on the downstream monopoly case.

2.3 The Two-Type Case

Assume that of the total $N$ patent holders a subset $N_H$ are large firms, with $x$ sufficiently high so that they will never be challenged in court. From Proposition 2, heterogeneity in patent holdings is irrelevant for those firms and, for simplicity, we will assume that all have $x = x_H$, with $g(x_H) = 1$. That is, they will always succeed in court. The remaining patent holders, a subset $N_L = N - N_H$ have patent holdings $x_L$, sufficiently low so that in equilibrium they might be constrained. In order to obtain analytic results we will focus on the linear demand case, $D(p) = 1 - p$. In that case, the final price and profits of the downstream producer as a function of the aggregate royalty, $R$, can be written as

$$p^M(R) = \frac{1 + R}{2} \text{ and } \Pi_D(R) = \left(\frac{1 - R}{2}\right)^2.$$  

Standard calculations along the lines of those in Proposition 2 allow us to show that small patent holders will be constrained as long as

$$\Lambda_L \leq \frac{3}{4(N + 1)^2},$$

where $\Lambda_L = \frac{L_D}{1 - g(x_L)}$. Under condition (6), the equilibrium royalty rate for developers with a large portfolio can be obtained from equation (5) as

$$r_H^* = 1 - \frac{N_Lr_L^*}{N_H + 1}.$$  

Similarly, in equilibrium $r_L^*$ corresponds to the maximum royalty $\bar{r}$ defined in equation (3), which can be written as the smallest solution to

$$(2N_L - 1)r_L^2 + 2(N_Hr_H - 1)r_L + 4\Lambda_L = 0.$$  

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Figure 1: Equilibrium Royalty rates for large and small patent holders. The Cournot Effect, in equation (7), corresponds to the solid line. The Inverse Cournot Effect, in equation (8), corresponds to the solid line. Parameter values are \( N_H = N_L = 1 \) and \( \Lambda = 0.07 \).

Obviously, the equilibrium royalty can be computed as \( R^* = N_H r^*_H + N_L r^*_L \).

Equations (7) and (8) are depicted in Figure 1 and they illustrate the workings of the model. As expected, the royalty rate of the large patent holders decreases with the royalty of the small ones due to the standard Cournot Effect. However, a higher \( r_H \) leads to a higher \( r_L \) due to the Inverse Cournot Effect. As we saw in Lemma 1, the higher is the total royalty the less constrained developers with small patent portfolios become, which allows them to raise their royalty rate.

Compared to the case in which litigation is not binding, small patent holders choose a lower royalty which results, due to the Cournot Effect, on a higher royalty from large patent holders. This change is not one-to-one and the equilibrium total royalty is lower than the one it would emerge when litigation is not binding.

The previous figure is also useful in order to understand the effect of changes in the parameters. For instance, an increase in \( \Lambda_L \) leads to an upward shift in the curve that determines \( r^*_L \). This shift increases the equilibrium royalty rate for the small patent
holders while decreasing the rate that large ones charge. For the reason stated before, this increase in $\Lambda$ leads to a increase in the aggregate royalty.

2.4 The Distribution of Patents among Developers

In this section we study how changes in the distribution of patents among firms affects the equilibrium royalty. We know from Proposition 1 that when legal costs are high this distribution should be inconsequential for the equilibrium royalty rate. However, once legal costs do not rule out litigation the Inverse Cournot Effect might operate in the opposite direction, reducing the room for developers with a small patent portfolio to charge a high royalty rate. The more asymmetric patent ownership becomes the stronger will be this second effect and the lower will become the aggregate royalty.

We consider the case in which the number of developers $N$ is large but their decisions still depend on strategic considerations. Our simulations, however, will approximate this case with a situation in which there is a continuum of heterogeneous developers. In particular, these firms are indexed by $s \in [0, 1]$. We denote as $x(s)$ the number of patents of firm $s$. As in the discrete case, $x$ is decreasing in $s$. That is, the lower the $s$ the stronger the patent portfolio of a developer. We normalize the total mass of patents in

\[ x(s) \]

Figure 2: Distribution of patent holders for different values of $b$. 

\[ s \times (s) \]

\[ b = 1.5 \]

\[ b = 2.5 \]

\[ b = 3.5 \]
the market to 1. Thus,
\[ \int_0^1 x(s) ds = 1. \]

In order to easily account for the asymmetry in patent holdings we assume that \( x(s) \) is generated according to a Kumaraswamy distribution with density
\[ x(s) = b(1 - s)^{b-1}. \]

This is a useful distribution for our purposes because with a unique parameter it allows us to spawn a wide variety of heterogeneous patent holdings. In particular, the value \( b = 1 \) corresponds to the case of a uniform distribution, implying equal patent holdings. As illustrated in Figure 2, higher values of \( b \) represent more skewed distributions so that lower values of \( s \) get a larger proportion of total patents.

The total royalty rate that the downstream firm pays can be written as
\[ R = \int_0^1 r(s) ds, \]
where \( r(s) \) indicates the patent chosen by the patent holder indexed by \( s \).

The simulation in Figure 3 illustrates the equilibrium of the model under different values of \( b \). The panel below describes the royalties that emerge as patent holdings become more skewed. When \( b \) is low, we observe that most firms charge the same rate while a few with a small patent portfolio (those with \( s > \tilde{s} \)) are somehow constrained and reduce their royalty rate. When patent holdings become more skewed the proportion of unrestricted developers is reduced, as illustrated in the second panel of the figure, through the decrease in \( \tilde{s} \). This decrease in \( \tilde{s} \) affects the equilibrium royalty rate profile in a very different way for those developers that are restricted and unrestricted. For the latter, the more skewed distribution of patents among firms implies that those with a high value of \( s \) now have a small patent portfolio. Their chances in court are lower and, as stated in Lemma 1, the highest rate they can charge decreases. Large patent holders,
Figure 3: Equilibrium royalties and proportion of firms that are not limited by litigation (that is, $s < \tilde{s}$ for different values of $b$). Demand is assumed to be $D(p) = 1 - p$, $L_D = 0.01$, and a patent holder’s probability of success is $g(x) = \min\{x, 1\}$.

However, increase their royalty due to the standard reasons. Royalty rates are strategic substitutes, and thus, they respond to the larger proportion of restricted developers and their lower total royalty rate by increasing their individual royalty. Interestingly, the top panel shows that the combination of both opposing effects leads to a total royalty rate $R$ that decreases as $b$ increases. The reason is that the Inverse Cournot Effect becomes more important when patent holdings are more skewed. Developers with a small value of $s$ respond to a decrease in the royalty of other developers with a less than equal increase in their own royalty, reducing the aggregate royalty as a result.

Finally, it is worth to notice that our model overestimates the effect of small patent holders by assuming that there is no cost of licensing patents. In Figure 4 we introduce such a cost. We assume that developers have a cost $k \geq 0$ of licensing their patents.
As expected, if this cost is introduced developers with a small patent portfolio will not enforce their patents and receive no royalty revenues. Although unconstrained developers increase their royalty as a result of the lower number of developers willing to license, this increase is not enough to compensate the previous effect. As a result, the aggregate royalty is decreasing in the cost $k$. Furthermore, notice that because this aggregate royalty is lower, and due to Lemma 1, developers with intermediate values of $x$ will also decrease their royalty rate. This decrease is small and, thus, difficult to appreciate in the figure.

3 Portfolio Consolidation

A standard conclusion of the royalty-stacking argument outlined in Proposition 1 is that public interventions aimed at consolidating patent holdings in the hands of a few firms would typically be welfare increasing as they would lead to a lower total royalty and lower final prices. The typical example of such an intervention is the promotion of patent pools. These arrangements comprise firms – upstream developers in the context of our model – that license their technology together. Although developers retain ownership of their patents, the fact that they determine the pricing as a bundle reduces the royalty
stacking problem since fewer firms are involved in the negotiation.

Consolidation can also arise from a transfer of ownership. Two typical examples, although different in many dimensions, are patent acquisitions and the emergence of patent assertion entities (PAEs) that aggregate patents for enforcement purposes. For most of the discussion in this section these different arrangements will have similar implications and we will refer to them generically as consolidation.

In particular, we show that once heterogeneity in patent ownership translates into different chances in court, the standard royalty-stacking result does not necessarily hold anymore. As the very simple case that we discuss next illustrates, portfolio consolidation among developers with a small patent portfolio might, in fact, have the opposite effect. It might allow these firms to charge higher royalties that would translate into a higher aggregate royalty and a more acute royalty-stacking problem.

To articulate the discussion we restrict ourselves to the two-type and linear demand case discussed in the previous section. Because the analytic characterization of the effect of each of these types of consolidation is quite cumbersome, we will focus here on the case of three upstream heterogeneous developers – \( N = 3, N_H \geq 1, \) and \( N_L \geq 1 \) – which delivers most of the relevant insights.

Patent consolidation can operate in three different ways depending on whether it involves large patent holders only, small ones only, or a combination of both. Furthermore, notice that, from equation (6), if small patent holders are restricted when \( N = 3 \), which occurs when \( \Lambda_L \leq 3/64 \), they will also be constrained after consolidation takes place.

Let’s start with the case in which two large patent holders decide to consolidate their portfolios which, of course, can only occur when \( N_H = 2 \).

**Lemma 2.** The consolidation between the two large patent holders always leads to an decrease in the aggregate royalty and an increase in the total profits of these firms.
To understand this result, first notice that the sum of their portfolios will not increase their chances in court, given that \( g(x_H) = 1 \). Thus, the effect of the merger arises only from changes in the royalty rate.

In terms of profits, when the two large patent holders consolidate we have that the Cournot and the Inverse Cournot effect operate in the same direction. Consolidation reduces the royalty stack because it eliminates one of the patent holders. Furthermore, since downstream profits are now higher the incentives to litigate of the producer increase, reducing \( r_L \) and, thus, making \( R \) even lower, which benefits the consolidated firm.

The consolidation between a large and a small developer has similar features. As in the previous case, it does not increase the probability to prevail in court for the large firm and all the results will depend on the effect over the equilibrium royalty. As we will see next, however, the profitability of this consolidation will depend on the initial distribution of patent holdings.

**Lemma 3.** The consolidation between a large patent holder and a small one always decreases the aggregate royalty. It will be profitable if and only if before consolidation \( N_H = 1 \) and \( N_L = 2 \).

Regardless of whether in the original situation there were two large patent holders or two small ones the result of the consolidation is always a decrease in the royalty rate. This is, of course a natural consequence of the Cournot effect as a result of the decrease in the number of upstream developers.

The reason for the different profitability in each case has to do with the Inverse Cournot Effect. Start with the case in which before consolidation there was only one large developer. Suppose that after the consolidation the large patent holder chooses a royalty that replicates the initial sum of the royalty charged by both firms, \( r'_L = r^*_H + r^*_L \).

It is clear that this strategy leads to the same aggregate royalty, since the remaining
small developer is as constrained as in the original situation. Profits remain unchanged. However, due to royalty stacking the price is above the monopoly one. Thus, the large patent holder will benefit from lowering its own royalty and, due to the Inverse Royalty effect, this will also lead to a lower royalty chosen by the small developer, making this move profitable.

When there are originally two large developers consolidation is not profitable. The reason is that by taking one small developer off the market the aggregate royalty falls but this decrease is partially compensated by an increase in the royalty of the other large (and therefore unconstrained) patent holder. Thus, the gains from lowering the aggregate royalty are not appropriated by the consolidating parties.

This result is the counterpart of the classical result highlighted in the context of firm mergers in papers such as Farrell and Shapiro (1990). They show that, under Cournot competition, a merger between two of the three identical firms in the market increases total profits since it raises the price in the final market but, as in our context, it leads to lower profits for the merging parties unless substantial cost synergies come about. As a result, firms typically have insufficient incentives to merge since some of the profits from doing so accrue to their competitors. This implication translates quite naturally to our setup.

Finally, when two small patent holders consolidate – that is, initially $N_H = 1$ and $N_L = 2$ – the effect on the strength of the patent portfolios becomes relevant. The two portfolios turn into a larger one $x_C = 2x_L$ operated by entity $C$. The characterization of the equilibrium can now take two different forms depending on whether the new litigation hurdle

$$\Lambda_C \equiv \frac{L_D}{1 - g(x_C)}$$

is greater than 1/12, which determines whether the consolidated firm can discourage the downstream producer to litigate when it charges the unrestricted royalty rate or not.
Comparing the aggregate royalty when three firms are present in the market and when
the two small (and symmetric) ones are consolidated we can derive the following result.

**Lemma 4.** Patent consolidation by the small firms results in higher profits if and only if
the total royalty increases. This increase occurs if the success probability in court of the
smaller players was low before the merger, $\Lambda_L < \bar{\Lambda}$, where

$$\bar{\Lambda} = \begin{cases} \frac{A}{2} - \Lambda_C^2 & \text{if } \Lambda_C < \frac{1}{12}, \\ \frac{1}{12} - \frac{5}{144} & \text{if } \Lambda_C \geq \frac{1}{12}, \end{cases}$$

which is a weakly increasing function of $x_C$ and $L_D$.

The intuition for why the merger is profitable if and only if the aggregate royalty
increases can be explained as follows. Denote as $(r^*_H, r^*_L, R^*)$ and $(r'_H, r'_C, R')$ the royalty
of the large, the small patent holder, and the total royalty before and after the merger,
respectively. For simplicity, let’s focus on the case in which firm $C$ is still constrained
and suppose that $R^* = R'$ so that quantity stays unchanged after the consolidation. In
that case, profits of the small firms will not change after the consolidation takes place
if and only if $r'_C = 2r^*_L$ and this will be possible if at that value $\Lambda_C$ is such that the
downstream firm prefers not to litigate. In that situation it is immediate from equation
(7) that $r'_H = r'_H$, constituting an equilibrium. Suppose now that $\Lambda_C$ increases beyond
this point. As a result, profits for $C$ increase after consolidation, since $r'_C$ will raise and, in
response, since $r'_H$ and $r'_C$ are strategic substitutes, $r'_H$ will be reduced although, as usual,
this reduction will be less than one-to-one, raising the equilibrium aggregate royalty.

The previous argument also illustrates the fact that the consolidation becomes prof-
itable because it reduces the profits of the large patent holder which faces a lower volume
in the final market and must also choose a lower royalty.

The region of parameters for which the total royalty increases after consolidation can
be understood as the combination of two effects. On the one hand, we know from the
numerical results in the previous section that when patent holdings are very asymmetric
the aggregate royalty tends to be lower due to the limit that litigation places on firms with small patent portfolios. On the other hand, the royalty-stacking argument shows that the more concentrated is patent ownership the more firms will internalize the externality that a high royalty generates on total output.

The first effect is more likely to arise the lower is \( \Lambda_L \) (or more precisely \( g(x_L) \)), which translates the asymmetry in patent holdings into an asymmetry in equilibrium royalties and, thus, a lower aggregate royalty. The second effect becomes less important the higher is \( \Lambda_C \) (or more precisely \( g(x_C) \)) and, thus, the less firms need to lower their royalty as a result of the litigation threat after their patents are aggregated. When both conditions concur, consolidation delivers a higher total royalty when patents are consolidated, benefiting the involved parties.

The next result summarizes the previous lemmas and characterizes the kinds of consolidation we might observe in equilibrium.

Proposition 3. Consider a linear demand \( D(p) = 1 - p \) and an initial situation with three heterogeneous upstream developers. Then,

1. any consolidation involving a large patent holder will reduce the royalty stack but it will only occur if it involves all large patent holders.

2. a consolidation between small developers will be profitable if and only if it increases the royalty stack.

An important conclusion is that although welfare-increasing consolidation involving large patent holders might not always occur, welfare-decreasing one involving small patent holders will always take place. Although we do not claim that these results are necessarily general they illustrate how the Cournot and Inverse Cournot Effect interact and help us understand under which conditions consolidations will take place and their welfare implications. Compare, for example, two distributions of patent holdings with a constant
number of firms. We should expect consolidation involving large patent holders to be more likely in the more skewed one. The reason is that in that distribution the Cournot Effect is less likely to operate, since fewer unconstrained developers benefit from the decrease of the number of upstream developers. Furthermore, the Inverse Cournot Effect is also more powerful when the distribution is more skewed. The reason is that after consolidation small developers become even more constrained by the lower aggregate royalty which forces them to reduce their own royalty rate further. This change benefits large developers and among them the consolidating ones.

The previous comparison also suggests that we should see a different pattern of consolidation involving large patent holders depending on the distribution of these patent holdings. When these holdings are very skewed, deals between a few firms would become profitable since the Cournot Effect would be small and these firms would benefit from the large Inverse Cournot Effect. On the contrary, when patent holdings are more similar deals among few firms would typically become unprofitable and we should see only larger deals, comprising a large number of large developers. These deals are likely to be implemented in different ways in either case. For small consolidations patent acquisitions are more likely, whereas for large ones patent pools seem a more operational mechanism.

Similarly, mergers among small patent holders or the presence of patent aggregators should also be more likely when patent holdings are more skewed. The reason is that in those cases the aggregate royalty is lower and small patent holders are likely to be more constrained in their royalty rate before the merger.

Finally, from a policy point of view, the previous logic would suggest that competition authorities should not expect welfare-enhancing consolidation to arise in equilibrium as often as it ought to. This lack of incentives would justify interventions like the block exemption granted to patent pools in the 2014 EU guidelines for technology transfer.
agreements. The previous proposition indicates, however, that patent pools or patent acquisitions might only be welfare-enhancing if they involve large patent holders either alone or in combination with small ones since in that case they are motivated by the desire to alleviate the royalty-stacking problem. In contrast, patent pools or, more likely, patent acquisitions by small patentholders are motivated by the interest of increasing the royalty that those firms may charge and since they are only profitable if they increase the royalty stack, they reduce social welfare along the way.

4 Vertical Mergers

Not all agreements involve developers only. Very often firms engage in vertical mergers. That is, downstream producers are integrated with upstream developers. In this section we study when this integration will be profitable, its characteristics, and how this would change depending on the level of heterogeneity in patent holdings.

In particular, consider the acquisition (or the merger) of one of the upstream developers by the downstream producer. This merger will generate a positive effect for the vertically-integrated firm as the double-marginalization problem will be mitigated. That is, the patent portfolio will be used by the downstream producer at a royalty rate of 0, which will lower the downstream price towards the monopoly price, increasing total profits for the integrated firm. A second effect will emerge in as much as the merger changes the royalty charged by the remaining upstream developers. The direction of this effect will depend on the strength of the Cournot and the Inverse Cournot Effect. The decrease of the aggregate royalty will have an opposite effect on the remaining large and small patent holders. For the former, the lower royalty stack leads to an increase in the royalty, reducing the profitability of the merger. For the latter, the litigation constraint will become tighter and they will reduce their royalty in order to avoid litigation. The
different reaction of other upstream developers will imply that the downstream producer will tend to prefer to merge with a large patent holder.

We illustrate this simple intuition using the same example we discussed in the previous section. We consider the case of three upstream developers that could have either a large portfolio of size $x_H$ that guarantees that they will never be litigated or a size $x_L$ and, as defined earlier, constitute a litigation hurdle $\Lambda_L$ for the downstream producer.

We analyze the same two market configurations we discussed in the previous section in the next proposition.

**Proposition 4.** Consider a linear demand $D(p) = 1 - p$ and an initial situation with three heterogeneous upstream developers. A vertical merger will be profitable if and only if $N_H = 1$ and $N_L = 2$ and it involves the downstream producer and the large upstream patent holder. That merger will decrease the equilibrium royalty.

The results are, thus, consistent with the previous intuition, indicating that vertical integration in case it occurs will involve only large developers. Nevertheless, this integration might not always take place. In particular, when there are several upstream large unconstrained developers, integration might not be profitable. The reason is that by vertically integrating both firms upstream profits are eliminated. However, this increase is not compensated with the reduction in double marginalization, since other upstream patent holders respond to the smaller number of developers by increasing their royalty and capturing some of the gains from the lower royalty stack. Notice, however, that although the merger would not occur, it would always have been socially beneficial since it would have led to a decrease in the equilibrium royalty.

As in the previous section, these results can be extrapolated to more general patent holding structures. In particular, when patent holdings are very skewed we should expect vertical mergers to be more likely as the Inverse Cournot Effect is likely to prevail. When
patent holdings are more equal these mergers are less likely to take place or, alternatively, they will need to involve more firms and more patents in order to weaken the Cournot Effect.

5 Concluding Remarks

The model we have presented in this paper delivers results that have significant public policy implications that we discuss next.

Royalty stacking. The royalty stack is not proportional to the number of complementary patents reading a given technology. It is also not proportional to the number of patent holders. This is because (a) some patent holders – those with very weak patent portfolios – will prefer not to enforce their patent rights; (b) active licensors with weak patent portfolios will be unable to charge high royalties due to the threat of litigation; and, due to the Inverse Cournot Effect; (c) active licensors with strong patent portfolios will have an incentive to limit their royalty demands because that weakens the position of the active licensors with weak patent portfolios, lowers their royalties and end-consumer prices and hence increases the volume over which the owners of strong patent portfolios can apply their high rates. Royalty stacking is more likely to be a problem when downstream competition is strong since, in those industries, manufacturers have a limited incentive to litigate and thus patent holders are not constrained by the threat of litigation.

Patent pools. Current antitrust policy, following Lerner and Tirole (2004, 2015), distinguishes between patent pools involving complementary patents, which are regarded as procompetitive, and patent pools involving substitute patents, which are considered anticompetitive unless licensors make their patents available unbundled. We find that while pools of complementary strong patent portfolios are welfare increasing, patent pools comprising complementary weak patent portfolios will increase, rather than mitigate, the
royalty stack.

**Horizontal patent acquisitions.** Our model shows that horizontal mergers between owners of strong patent portfolios will lead to reductions in the total royalty rate and, therefore, will prove welfare increasing. The acquisition of a weak patent portfolio by one or more owners of strong patent portfolios will also reduce the total royalty rate. On the contrary, the acquisition of a weak patent portfolio by a patent owner or an aggregator with a weak patent portfolio will have the opposite effect, raising the total royalty and causing a reduction in welfare. Competition authorities should expect to see the harmful aggregation of weak patent portfolios in equilibrium. The reason is that aggregation of patent portfolios and the higher (combined) royalty that the merged entity can charge which will trigger a reduction of the royalty rates set by the owners of stronger patent portfolios, which will see their position reinforced, increasing the profits of the merged entity.

**Vertical patent acquisitions.** Large downstream producers will have an incentive to acquire strong patent portfolios, but no interest in weak patent portfolios. By acquiring a strong patent portfolio, a downstream manufacturer not only internalizes the royalty set by the owner of that portfolio, it also reduces, by virtue of the Inverse Cournot Effect, the royalty rates charged by all litigation-constrained patent owners.
References


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**Spulber, Daniel F.**, “Incentives to Innovate with Complementary Inventions,” 2014, working paper.
A Multiple Downstream Producers

Consider now the case in which $M$ firms compete downstream. For simplicity assume that these firms are identical and compete in quantities. In the first stage patent holders simultaneously choose their royalty rate.

As it is well known, the equilibrium price in the final market can be characterized as

$$\frac{p^* - R}{p^*} = \frac{1}{M\eta},$$

(9)

where $\eta$ is the demand elasticity. The equilibrium price, $p^*(R, M)$, is a decreasing function of the number of firms, as well as an increasing function of the total royalty rate. Furthermore, we make the usual assumption that the marginal revenue of each firm is decreasing in total quantity so that individual quantities decreases as the number of competitors increases.

The profits of downstream producer $j = 1,..M$ will depend on $R$ and $M$ since we have that

$$\Pi_{D_j}(R, M) = (p^*(R, M) - R)D(p^*(R, M)).$$

Let’s consider now the decision of producer $j$ to litigate. As opposed to what we found in the baseline model, the decision here will depend not only on the royalty rate of developer $i$ but also on the decision of all other downstream producers. A free-riding problem arises in the sense that when a patent is invalidated in court the aggregate royalty that all downstream producers pay is reduced. Legal costs, however, are incurred by only one party.

In our analysis we abstract from this issue and instead we discuss the incentives that a downstream producer has to litigate an upstream developer under the expectation that it will not be sued by any other downstream developer. This is the most favorable case for litigation to emerge and it allows us to focus on the individual incentives that firms have to litigate. As in the previous case, downstream developer $j$ would choose to litigate if

$$\Pi_{D_j}(R_{-i}, M) - \Pi_{D_j}(R_{-i} + r_i, M) \leq \Lambda_i,$$

(10)

which, of course, now depends on the number of downstream competitors. Under standard conditions, the highest royalty that upstream developer $i$ will charge and avert litigation, $\tilde{r}_i$, will be increasing in $M$. 

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Lemma 5. The highest royalty that developer $i$ can charge without inducing litigation, $\bar{r}_i$, is increasing in $R_i$, $M$, and $\Lambda_i$.

Assumption 2 guarantees that upstream developers would like to avoid litigation, meaning that they would always choose a royalty that satisfied the previous constraint. That constraint would only be binding for developers with low patent holdings. For the rest, the equilibrium royalty would emerge from

$$\max_{r_i} r_i D(p^*(R, M)).$$

Since obtaining general results in this case is difficult we rely on the intuitions drawn from two typical examples: a linear and a isoelastic demand function.

Example 1 (Linear demand). Assume $D(p) = 1 - p$. Using (9) we obtain the equilibrium price as

$$p^*(R, M) = \frac{1 + MR}{M + 1}.$$  

Upstream developer $i = 1, \ldots, N$ chooses the royalty to maximize

$$\max_{r_i \leq \bar{r}_i} r_i D(p^*(R, M)) = \max_{r_i} r_i M \frac{1 - R}{M + 1},$$

where $\bar{r}_i$ arises from (10). As a result, developers with large patent portfolios choose $r_i^* = \frac{1 - R}{2}$, independent of $M$.

Example 2 (Isoelastic demand). Assume that demand is $D(p) = p^{-\eta}$, for $\eta > 1$. In that case, the equilibrium price is $p^*(R, M) = \frac{M \eta}{M \eta - 1} R$. Upstream developer $i = 1, \ldots, N$ chooses the royalty to maximize

$$\max_{r_i \leq \bar{r}_i} r_i D(p^*(R, M)) = \max_{r_i} r_i M \left( \frac{M \eta}{M \eta - 1} \right)^{-\eta} R^{-\eta}.$$  

As before, all developers unconstrained by litigation will choose a royalty independent of $M$, $r_i^* = \frac{R - i}{\eta - 1}$.  

In both examples the royalty of the unconstrained developers is independent of $M$, simplifying the comparative statics exercise that studies the effects of an increase in downstream competition in the equilibrium royalties.\footnote{More generally this result holds as long as the semi-elasticity of the demand with respect to $R$, $\frac{D'(p^*)}{D(p^*)} \frac{\partial p^*}{\partial R}$, is independent of $M$.} In particular, increases in $M$ rise...
\( \bar{r} \), implying that developers with small patent holdings can increase their royalty rate. As a result, fewer developers are constrained by litigation.

With the previous result in mind one can anticipate how the effects of the heterogeneity on patent holdings change when downstream competition increases. We should observe that more downstream competition is associated with a decrease in the effect of patent holding heterogeneity on the total aggregated royalty. In order to see this effect, it is useful to compare the two extreme cases. As it transpires from the previous results, when downstream competition is very strong, developers are not constrained in any way by litigation and, therefore, patent holdings do not change the equilibrium outcome. At the other extreme, we saw in the previous section that when there is no downstream competition, more heterogeneous holdings imply a lower aggregate royalty.

The previous result also indicates that the benefits from forming a patent pool are very different depending on the downstream market structure. Our model suggests that, for a given level of patent holding heterogeneity, when downstream competition is stronger a patent pool would be more likely to be formed, since developer profits are more similar and the total royalty is likely otherwise to be higher.

B Proofs

**Proof of Proposition 1** When \( L_D \to \infty \), the equilibrium royalty is determined by

\[
D(p^M(N \ast r^*)) + r^* D'(p^M(N \ast r^*)) \frac{dp^M}{dR} = 0.
\]

Using the Implicit Function Theorem to show that \( r^* \) is decreasing in \( N \) it is enough to verify that the sign of the derivative with respect to \( N \) is negative. In this case, the derivative becomes

\[
r^* \left[ D'(p^M(R)) \frac{dp^M}{dR} + r \frac{d}{dR} \left( D'(p^M(R)) \frac{dp^M}{dR} \right) \right] \\
\leq r^* D'(p^M(R)) \frac{dp^M}{dR} \left( 1 - \frac{2}{N} \right)
\]

where the last inequality arises from Assumption 1. As a result this derivative is negative for \( N \geq 2 \).

Regarding the effect of \( N \) on the total royalty, \( R \), notice that the first-order condition can be written as

\[
D(p^M(R^*)) + \frac{R^*}{N} D'(p^M(R^*)) \frac{dp^M}{dR} = 0,
\]
where $R^* = N \ast r^*$. It is easy to check that the derivative with respect to $R$ is negative. The derivative with respect to $N$ is

$$-\frac{1}{N^2} D'(p^M(R^*)) \frac{dp^M}{dR} > 0,$$

and using again the Implicit Function Theorem we obtain the desired result.

Proof of Lemma 1: The arguments in the text imply that equation (3) determines the maximum royalty that will not trigger downstream litigation. From this equation we can see that

$$\frac{d\bar{r}_i}{d\Lambda_i} = \frac{1}{\Pi'_D(R_i + \bar{r}_i)} > 0,$$

$$\frac{d\bar{r}_i}{dR_{-i}} = -\frac{\left[\Pi'_D(R_i) - \Pi'_D(R_i + \bar{r}_i)\right]}{\Pi'_D(R_i + \bar{r}_i)} > 0.$$

Proof of Proposition 2: We first need to show that the problem of an unconstrained developer is concave. In particular, we want to show that

$$D(p^M(R)) + r_i D'(p^M(R)) \frac{dp^M}{dR}$$

is decreasing in $r_i$. The second derivative can be written as

$$2D'(p^M(R)) \frac{dp^M}{dR} + r_i \frac{d}{dR}\left(D'(p^M(R)) \frac{dp^M}{dR}\right).$$

Towards a contradiction, notice that since the first term is negative, a necessary condition for this derivative to be positive is that the second term is positive. In that case, though

$$2D'(p^M(R)) \frac{dp^M}{dR} + r_i \frac{d}{dR}\left(D'(p^M(R)) \frac{dp^M}{dR}\right) < 2D'(p^M(R)) \frac{dp^M}{dR} + R \frac{d}{dR}\left(D'(p^M(R)) \frac{dp^M}{dR}\right) < 0,$$

where the last term comes from Assumption 1. Thus, the profit function is concave.

Since the unconstrained optimal licensing rate is independent of $x_i$, all firms would choose the same royalty if possible. Otherwise, they will choose the highest one satisfying Lemma 1. The highest $i$ for which the constraint is not binding determines $\bar{n}$. Proof of Lemma 2: Before the consolidation, we have, using (7) and (8), that

$$r'_{iL} = \sqrt{12\Lambda_L + 1} - 1, \quad r'_{iH} = \frac{2 - \sqrt{12\Lambda_L + 1}}{3},$$

and $R^* = \frac{1 + \sqrt{12\Lambda_L + 1}}{3}$. After the merger royalties become $r'_{iH} = \frac{1}{2} - 2\Lambda_L, r'_{iL} = 4\Lambda_L,$ and $R' = \frac{1}{2} + 2\Lambda_L$. It is immediate that $R^* > R'$ if and only if $\Lambda_L < \frac{1}{4}$. 

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Regarding profits, notice that \( \Pi_H = r_H \frac{1-R}{2} \). Replacing the previous expressions we have that profits change from \( \Pi^*_H = \frac{(2-\sqrt{2}L+1)^2}{18} \) before the merger to \( \Pi'_H = \frac{(1-4L)^2}{8} \) afterwards. We have that \( \Pi'_H > 2\Pi_H \) for all values of \( \Lambda_L \) for which the small firm was constrained before the merger.

**Proof of Lemma 3:** Consider the case in which before consolidation \( N_H = 2 \) and \( N_L = 1 \). We have, using (7) and (8), that \( r_L^* = \sqrt{2}L + 1 - r = \frac{2-\sqrt{2}L+1}{3} \), and \( R^* = \frac{1+\sqrt{2}L+1}{3} \). After the merger we have \( r_H^* = \frac{1}{3} \), and \( R' = \frac{2}{3} \). It is immediate that \( R^* > R' \) for all \( \Lambda_L > 0 \).

Regarding the profits notice that \( \Pi_H = r_H \frac{1-R}{2} \) and \( \Pi_L = r_L \frac{1-R}{2} \). Replacing the previous expressions we have that profits change from \( \Pi^*_H = \frac{(2-\sqrt{2}L+1)^2}{18} \) and \( \Pi^*_L = \frac{(2-\sqrt{2}L+1)(\sqrt{2}L+1-1)}{6} \) to \( \Pi'_H = \frac{1}{18} \). It can be easily shown that \( \Pi'_H < \Pi_H + \Pi^*_L \) for all \( L \) and, thus, the merger will not be profitable.

Consider now the case in which before consolidation \( N_H = 1 \) and \( N_L = 1 \). We have, using (7) and (8), that \( r_H^* = \frac{\sqrt{1-16L}}{2} \), \( r_L^* = \frac{1-\sqrt{1-16L}}{2} \), and \( R^* = \frac{2-\sqrt{1-16L}}{2} \). After the merger we have \( r_H^* = \frac{1}{2} - 2\Lambda_L \), \( r_L^* = 4\Lambda_L \), and \( R' = \frac{1}{2} + 2\Lambda_L \). It is immediate that \( R^* > R' \) for all \( \Lambda_L > 0 \).

Regarding the profits notice that \( \Pi_H = r_H \frac{1-R}{2} \) and \( \Pi_L = r_L \frac{1-R}{2} \). Replacing the previous expressions we have that profits change from \( \Pi^*_H = \frac{1-16L}{8} \) and \( \Pi^*_L = \frac{(1-\sqrt{1-16L})\sqrt{1-16L}}{8} \) to \( \Pi'_H = \frac{(1-4L)^2}{8} \). It can be shown that \( \Pi'_H > \Pi^*_H + \Pi^*_L \) if and only if \( 1-16L < (1-4L)^4 \).

After some algebra we obtain that

\[
(1-4L)^4 = 1-16L + 32\Lambda_L^2 [3 + 8\Lambda_L^2 - 8L]
\]

and the second term is always positive for \( L \in \left[0, \frac{3}{64}\right] \). Thus, consolidation will be profitable.

**Proof of Lemma 4:** Before consolidation, we have, using (7) and (8), that \( r_H^* = \frac{\sqrt{1-16L}}{2} \), \( r_L^* = \frac{1-\sqrt{1-16L}}{2} \), and \( R^* = \frac{2-\sqrt{1-16L}}{2} \). After consolidation we have, when \( \Lambda_L < \frac{1}{12} \), that \( r_H^* = \frac{1}{2} - 2\Lambda_L \), \( r'_L = 4\Lambda_L \), and \( R^* = \frac{1}{2} + 2\Lambda_L \). If \( \Lambda_L > \frac{1}{12} \), then \( R'_H = r'_L = \frac{1}{3} \) and \( R' = \frac{2}{3} \). The comparison of the total royalty in both cases delivers the result.

Regarding profits, we can write \( \Pi'_L = \frac{\sqrt{1-16L}(1-\sqrt{1-16L})}{8} \). After consolidation, if \( \Lambda_C < \frac{1}{12} \), \( \Pi'_L = \Lambda_C(1-4\Lambda_C) \). Notice that \( \Pi'_L \geq 2\Pi^*_L \) if and only if either \( \frac{1}{2} > 4\Lambda_C \geq \sqrt{1-16L} \) or \( \frac{1}{2} \leq 4\Lambda_C \leq \sqrt{1-16L} \). The second condition cannot occur since \( \Lambda_C < \frac{1}{12} \) and thus we
have that it requires \( 4\Lambda_C \geq \sqrt{1 - 16\Lambda_L} \) which holds if and only if \( \Lambda_L \leq \bar{\Lambda}_L \) as stated. If \( \Lambda_C \geq \frac{1}{12} \) then \( \Pi_P = \frac{1}{18} \). These profits are higher than \( 2\Pi_L^* \) if and only if \( \frac{2}{3} \leq \sqrt{1 - 16\Lambda_L} \) or \( \Lambda_L \leq \frac{5}{144} = \bar{\Lambda}_L \).

**Proof of Proposition 3:** Immediate from Lemma 2, 3, and 4.

**Proof of Proposition 4:** In all cases, a vertical merger implies that the upstream division of the integrated firm will charge a royalty of 0 and, thus, all profits will come from the final market operation.

Consider first the case in which \( N_H = 2 \) and \( N_L = 1 \). Profits before the merger can be computed as \( \Pi_D^* = \frac{(2 - \sqrt{1 + 12\Lambda_L})^2}{36} \), \( \Pi_H^* = \frac{(2 - \sqrt{1 + 12\Lambda_L})^2}{18} \), and \( \Pi_L^* = \frac{(2 - \sqrt{1 + 12\Lambda_L})(\sqrt{1 + 12\Lambda_L} - 1)}{6} \).

- If the merger involves the downstream producer and one of the upstream developers with a large portfolio, profits become \( \Pi_D' = \frac{(1 - 4\Lambda_L)^2}{16} \). Some calculations show that these profits cannot be larger than the sum of the individual ones before the merger since that would occur if \( 5 + 12\Lambda_L < 4\sqrt{1 + 12\Lambda_L} \) which cannot occur for a positive \( \Lambda_L \).

- If the merger involves an upstream producer with a small portfolio, profits become \( \Pi_D' = \frac{1}{36} \). This is independent of \( \Lambda_L \), as no patent holder with \( x_L \) chooses \( r_L \).

Comparing those profits with those of the individual firms we have that they are lower for all \( \Lambda_L \in [0, 14/75] \), higher than the maximum value \( \frac{3}{64} \).

Consider now the case in which \( N_H = 1 \) and \( N_L = 2 \). Profits before the merger can be computed as \( \Pi_D = \frac{1}{16} - \Lambda_L \), \( \Pi_H = \frac{1}{8} - 2\Lambda_L \), and \( \Pi_L^* = \frac{\sqrt{1 - 16\Lambda_L} - 1 + 16\Lambda_L}{8} \).

- If the merger involves the large upstream developer and the downstream producer, profits become \( \Pi_D' = \frac{(1 - 16\Lambda_L)^2}{36} \). It can be shown that these profits are higher than \( \Pi_D^* + \Pi_H^* = \frac{3}{16}(1 - 16\Lambda_L) \) for all \( \Lambda_L \in [0, 3/64] \). Hence, this vertical aggregation is profitable.

- If the merger involves one of the small upstream developers, profits become, \( \Pi_D' = \frac{(1 - 4\Lambda_L)^2}{16} \). These profits are lower than \( \Pi_D^* + \Pi_L^* = \frac{2\sqrt{1 - 16\Lambda_L} - 1 + 16\Lambda_L}{16} \) for all \( \Lambda_L \in [0, 3/64] \). Hence, this vertical aggregation is not profitable.

**Proof of Lemma 5:** The comparative statics with respect to \( R_i \), \( L_D \), and \( x_i \) are identical as the ones in Lemma 1 and for this reason omitted. Regarding the effect of \( M \)
notice that
\[
\frac{\partial \Pi_{D_j}}{\partial R} = -D(p^*(R, M))
\]
and
\[
\frac{\partial \Pi_{D_j}}{\partial R \partial M} = D'(p^*(R, M)) \frac{\partial p^*}{\partial M} < 0
\]
so that the higher the number of $M$ the lower the effect on profits of changes in $R$, which implies that incentives to litigate decrease as $M$ increases. As a result, $\bar{r}_i$ increases with $M$. \qed