Evaluating Consumption CAPM under Heterogeneous Preferences

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Abstract

We construct Consumption CAPM pricing kernel under heterogeneous Epstein-Zin-Weil (EZW) preferences setup. We show that except under some strict assumptions, aggregation does not hold and individual level information is necessary to price assets. Assuming complete market, we estimate preferences parameters for heterogeneous agents using households level consumption and wealth data from Panel Study of Income Dynamics (PSID) database. Contrast to the literature, we get relatively larger Elasticity of Inter-temporal Substitution (EIS) and much smaller Relative Risk Aversion (RRA) parameters. We then calculate excess return for risky assets using Consumption-CAPM pricing kernel with our estimated preferences and individual consumption and wealth data. We show that our heterogeneous preferences pricing kernel improves the representative agent pricing kernel at three fronts, namely idiosyncratic risk factors, heterogeneous factor premia and idiosyncratic characteristics dependent aggregation weights. Each front improves the explanatory power to real world risk premium and cross sectional differences in stock excess returns. Lastly, we demonstrate another advantage of heterogeneous preferences model in terms of accounting for market participation heterogeneity. Including market participants only further improves model’s explanatory power on market risk premium dynamics.


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1 Introduction

In this study, we use Epstein-Zin-Weil (EZW) utility to setup a time-varying heterogeneous preferences model for Consumption CAPM (CCAPM). We prove except under some special cases, we cannot get aggregation result. Hence every household matters in asset pricing equation. We then estimate EZW utility parameters for heterogeneous agents using Panel Study of Income Dynamics (PSID) data under complete market assumption. We show that with heterogeneous agents, we can get relative large Elasticity of Inter-temporal Substitution ($EIS$) and small Relative Risk Aversion ($RRA$) coefficients that are consistent with empirical microeconomic findings. With estimated preferences parameters, we generate predictions for market excess returns. We show that our heterogeneous preferences model improves representative agent model at three fronts, namely idiosyncratic risk factors, heterogeneous factor premia and individual characteristics dependent aggregation weights, which results in a total 62% increase in the explanatory power on risk premium dynamics. We also show heterogeneous preferences models outperform representative agent model in explaining cross sectional differences of stock excess returns for Fama-French 25 size-value sorted portfolios. Our model also has the advantage over representative agent model in the ability to accommodate market participation heterogeneity, limiting households to market participants only can further improve the explanatory power by 21%.

Consumption-based CAPM provides a micro-founded structure to asset prices. It links households’ insurance motivation for consumption smoothing and prices of insurance instruments - risky assets - together. Lucas (1978) and Breeden (1979) first propose this model in a discrete time setup, later Duffie & Zame (1989) extend it into continuous time setup. Numerous studies assuming representative agent, using different aggregate consumption data, reject the Consumption CAPM at the aggregate level, for example Breeden et al. (1989). That means, fluctuations in aggregate consumption level alone, cannot explain the risk pre-
mium. Mehra & Prescott (1985) provides a nice summary of this puzzle. To solve this problem, researchers follow two major approaches.

Most innovations in improving Consumption CAPM are about using better proxies for consumptions, specially proxies with larger volatility. Campbell & Cochrane (1999) look at the consumption level deviating from "habit", Lettau & Ludvigson (2001) construct a new measure of consumption, Aït-Sahalia et al. (2004) use consumption of luxury goods as the proxy, Malloy et al. (2009) focus on stockholders’ consumptions and Savov (2011) uses garbage as the proxy. All those studies assume the existence of a representative agent.

Mankiw & Zelds (1991) started to think about a heterogeneous agent setup, they argue that agents differ from each other due to limited financial market participation for some agents. Constantinides & Duffie (1996) look the same problem from a different angle. They show that in an incomplete market, the second moment of cross sectional consumption distribution matters when agents receive idiosyncratic income shocks. Recently, Gärleanu & Panageas (2015) model heterogeneity with heterogeneous preferences using an overlapping-generations model.

No empirical study is done to test CCAPM directly under heterogeneous agent framework, but there are some studies testing cross-sectional implications of heterogeneous agents CCAPM, they are Brav et al. (2002), Cogley (2002) and Vissing-Jørgensen (2002). However all those three studies use power utility. Under power utility, individual consumption can be aggregated in the CCAPM relation. In other words, only the aggregate consumption growth rate is a risk factor. Therefore effects of heterogeneous agents are limited under power utility setup.

Another branch of utility is recursive utility. Epstein & Zin (1989) and Weil (1989) show a utility function in recursive form and rewrite CCAPM relation using this utility. In this paper, we show that except for some special cases, the EZW utility function preserves heterogeneous agents structure. The traditional aggregation result no longer hold, therefore individual level data is necessary for pricing kernel. Before our study, no empirical study for heterogeneous preferences consumption CAPM with EZW utility has been done.

We contribute to the literature at following aspects. Firstly we show that individual households level data is necessary for asset pricing if we do not assume some strict as-
sumptions about households preferences or growth rates of their consumption and wealth. Secondly, we show that we can get volatile consumption that is needed to answer equity premium puzzle by looking at cross sectional households heterogeneity. Thirdly, we provide a direct empirical comparison between heterogeneous preferences and representative agent consumption CAPM. Lastly, we are the first study to incorporate both preferences heterogeneity and market participation heterogeneity.

Our paper does not intend to directly answer equity premium puzzle, nor to evaluate the absolute performance of consumption CAPM. Instead, we focus on the relative performances of heterogeneous preferences model and representative agent model, and provide some light into the direction of solving the equity premium puzzle eventually.

2 Model

2.1 Representative Agent Benchmark

We adopt an EZW Utility for an endowment economy with complete financial market,

\[ V_t = [(1 - \beta) c_t^{1 - \rho} + \beta \mathbb{E}_t [V_{t+1}^{1-\alpha}]^{1-\alpha}]^{1-\rho}, \]

where \( \beta \) is the subjective discount rate, \( 1/\rho \) is the Elasticity of Inter-temporal Substitution (\(EIS\)) and \( \alpha \) is the Relative Risk Aversion (\(RRA\)). Define wealth \( W_t = \frac{V_t}{MC_t} \), where \( MC_t \) is the marginal utility of consumption; and return on wealth \( R_{t+1}^W = \frac{W_{t+1}}{W_t} \). The stochastic discount factor is

\[ S_{t,t+1} = \beta^\theta (R_{t+1}^W)^{\theta-1} \left( \frac{C_{t+1}}{C_t} \right)^{-\theta/\psi}, \quad (1) \]

where \( \theta = \frac{1-\alpha}{1-\rho} \) and \( \psi = 1/\rho \). Then for any risky or riskless asset \( i, i = 1 \cdots I \), its gross return \( R_{t,t+1}^i \) must satisfy

\[ \mathbb{E}_t S_{t,t+1} R_{t,t+1}^i = 1. \quad (2) \]
Let \( g_{t+1} = \ln \left( \frac{C_{t+1}}{C_t} \right) \) and \( h_{t+1} = \ln R_W^t \) be log aggregate consumption growth rate and log aggregate return on wealth, with variance \( \sigma_C^2 \) and \( \sigma_W^2 \) respectively. The risk-free rate is

\[
 r_{t+1}^f = -\ln \beta + \frac{1}{\psi} E_t[g_{t+1}] - \frac{\theta}{2\psi^2} \sigma_C^2 - \frac{(1 - \theta)}{2} \sigma_W^2,
\]

and expected excess return of any risky asset \( i \) is

\[
 \ln E_t[R_i^{t+1}] - r_{t+1}^f = \frac{\theta}{\psi} \text{Cov}_t[g_{t+1}, r_i^{t+1}] + (1 - \theta) \text{Cov}_t[h_{t+1}, r_i^{t+1}], \quad (3)
\]

where \( r_{t+1}^i = \ln R_i^{t+1} \). Therefore under representative agent case, aggregate consumption growth rate and aggregate return on wealth are risk factors in pricing assets.

2.2 Heterogeneous Preferences Setup

Under heterogeneous preferences setup, for each household \( j, j = 1, \ldots, J \), utility is specified by three idiosyncratic parameters, \( \beta^j, \alpha^j \) and \( \rho^j \) (or equivalently \( \beta^j, \theta^j, \psi^j \)).

\[
 V_t^j = \left[ (1 - \beta^j) c_t^{1-\rho^j} + \beta^j E_t \left[ (V_t^{j+1})^{1-\alpha^j} \right] \right]^{\frac{1}{1-\rho^j}}.
\]

Under this specification, equation (1) now becomes

\[
 S_{t+1} = (\beta^j)^{\theta^j} (R_W^j)^{\theta^j-1} \left( \frac{c_t^{j+1}}{c_t^j} \right)^{-\theta^j/\psi^j}. \quad (4)
\]

Notice that although parameters on right hand side are heterogeneous, the stochastic discount factors are the same. This is because different agents will agree on the price of any asset in the market in equilibrium. Suppose two agents disagree on any asset, it will simultaneously create supply and demand for this asset at a price between their perceived prices and market clearing condition will eliminate the price difference. Arbitrage works. After the trade, both agents’ consumption and wealth change and their opinion on the price for this asset agree now.

Since equation (2) still holds under heterogeneous preferences setup, now for every risky
asset \( i \) and every household \( j \), the expected excess return is

\[
\ln \mathbb{E}_t[R^i_{t+1}] - r^f_{t+1} = \frac{\theta^j}{\psi^j} \text{Cov}_t[g^j_{t+1}, r^i_{t+1}] + (1 - \theta^j) \text{Cov}_t[h^j_{t+1}, r^i_{t+1}] \tag{5}
\]

Equation (5) holds for every household if the household is a market participant. We start by assuming every household is a market participant. This assumption is also implicitly assumed in past literature using aggregate consumption. When using aggregate consumption, every cent in aggregate consumption is included, which means every household is included. However, assuming every household is a market participant does not necessarily mean every household is engaging in the trading for every asset. A household will not trade an asset if their valuation of the asset equals the asset’s equilibrium price under frictionless trading or lies in the price spread under frictional trading.

We later relax the assumption of full market participation and focus on market participants only. This practice shows one of the advantages of our model over the representative agent model which cannot separate market participant from non-participant in aggregate data.

### 2.2.1 Special Cases with Aggregation Result Holds

In this section, two sets of special cases which deliver aggregation result are discussed, such that only aggregate consumption growth rate and aggregate return on wealth serve as the risk factors for asset pricing. Then we argue that those special cases are unlikely to happen in reality.

**Case 1** \( \rho^j = \alpha^j, \forall j \). For every household, the reciprocal of Elasticity of Inter-temporal Substitution equals the Relative Risk Aversion coefficient.

**Proposition 1** With \( \rho^j = \alpha^j (\theta^j = 1), \forall j \),

\[
\ln \mathbb{E}_t[R^i_{t+1}] - r^f_{t+1} = \frac{\mathbb{E}_t[C_{t+1}]}{\sum_{j=1}^J \left( \psi^j / \mathbb{E}_t \left[ \frac{1}{r^i_{t+1}} \right] \right)} \text{Cov}_t \left[ g_{t+1}, r^i_{t+1} \right]
\]

where \( g_{t+1} \) is the aggregate consumption growth rate.
Case 2 \( \alpha_j = 1, \forall j \). Every household has unit Relative Risk Aversion coefficient.

**Proposition 2** With \( \theta^j = 0, \forall j \),

\[
\ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f = \frac{\mathbb{E}_t[W_{t+1}]}{\sum_{j=1}^{J} 1/\mathbb{E}_t[1/w_{t+1}^j]} \text{Cov}_t[h_{t+1}, r_{t+1}^i]
\]

where \( h_{t+1} \) is the aggregate return on wealth.

**Proof:** See appendix.

When \( \rho^j = \alpha^j, \forall j, \theta^j = 1, \forall j \), the EZW utility is reduced to a recursive form of power utility, therefore it is not surprising the aggregation result holds.

But this case is unlikely to happen under our setup. \( \rho^j = \alpha^j, \forall j \) means all \((\rho^j, \alpha^j)_{j=1}^{J}\) locate on the 45 degree line on the 2-dimensional parameter plane. It may hold for some households, but it is an unrealistic restriction for the population. In reality, we expect parameter pairs for households evenly distributed on the plane for all possible combinations. And if there exists one household does not locate on the 45 degree line, this condition no longer holds.

Similar argument holds for the case 2. It is unrealistic for every households to have same relative risk aversion coefficient, especially all equal to 1. If there is one household’s risk aversion coefficient does not equal to 1, this aggregation result no longer holds.

Case 3 \( g_{t+1}^j = g_{t+1}, \forall j \). For each period, consumption growth rates are identical for every households.

**Proposition 3** With \( g_{t+1}^j = g_{t+1}, \forall j \),

\[
\ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f = \frac{\sum_{j=1}^{J} \frac{\theta^j}{(1-\theta^j)\mathbb{E}_t[1/w_{t+1}^j]}}{\sum_{j=1}^{J} \frac{1}{(1-\theta^j)\mathbb{E}_t[1/w_{t+1}^j]}} \text{Cov}_t[g_{t+1}, r_{t+1}^i] + \frac{\mathbb{E}_t[W_{t+1}]}{\sum_{j=1}^{J} \frac{1}{(1-\theta^j)\mathbb{E}_t[1/w_{t+1}^j]}} \text{Cov}_t[h_{t+1}, r_{t+1}^i]
\]

**Proof:** See appendix.
Case 4 $h_{t+1}^j = h_{t+1}, \forall j$. For each period, returns on wealth are identical for every households.

Proposition 4 With $h_{t+1}^j = h_{t+1}, \forall j$,

$$\ln E_t[R_{t+1}^i] - r_{t+1}^i = \frac{\mathbb{E}_t[C_{t+1}]}{\sum_{j=1}^J \frac{\psi^j}{\theta^j \mathbb{E}_t[1/c_{t+1}^j]}} \text{Cov}_t[g_{t+1}, r_{t+1}^i] + \frac{\sum_{j=1}^J \frac{\psi^j (1-\theta^j)}{\theta^j \mathbb{E}_t[1/c_{t+1}^j]}}{\sum_{j=1}^J \frac{\psi^j}{\theta^j \mathbb{E}_t[1/c_{t+1}^j]}} \text{Cov}_t[h_{t+1}, r_{t+1}^i]$$

Proof: See appendix.

Under these two cases, all households have either the same consumption growth rate $g_{t+1}$ or the same rate of return on wealth $h_{t+1}$, then the aggregation result still holds.

But these two cases are unlikely to happen either. We assume complete market in this study, as a result, idiosyncratic risks are fully insured. Households’ consumption growth rates and returns on wealth are fully governed by preference parameters. Same rates require same preference parameters, which is unrealistic and contradicts to our heterogeneous preferences assumption.

2.2.2 General Case without Aggregation

Under general case when $\rho^j \neq \alpha^j$ and $h_{t+1}^j \neq h_{t+1}$ for some $j$, the aggregation result no longer holds. We cannot aggregate the heterogeneous CCAPM equations to a equation with only two aggregate risk factors, we need households’ level data to price assets. Equation (5) holds for every asset $i$ and household $j$. Sum over households $j$, we have

$$\ln E_t[R_{t+1}^i] - r_{t+1}^i = \sum_{j=1}^J \left[ \phi_j^i \frac{\theta_j^i}{\psi_j^i} \text{Cov}_t[g_{t+1}^j, r_{t+1}^i] \right] + \sum_{j=1}^J \left[ \phi_j^i (1-\theta_j^i) \text{Cov}_t[h_{t+1}^j, r_{t+1}^i] \right], \quad (6)$$

where $\phi_j^i$ are aggregation weight for household $j$. Comparing to Equation (3) of CCAPM under representative agent, our model improves on three fronts. Firstly we expand two aggregate risk factors $g_{t+1}$ and $h_{t+1}$ to $2 \times J$ idiosyncratic risk factors, $g_{t+1}^j$ and $h_{t+1}^j$ for $j = 1, \cdots, J$ (RF). Secondly we improve homogeneous factor premia $\theta/\psi$ and $1-\theta$ on consumption and wealth risk factors to heterogeneous factor premia $\theta_j^i/\psi_j^i$ and $1-\theta_j^i$ (FP).
Lastly, we allow individual household characteristics - intensity of market engagement in particular - dependent aggregation weights $\phi^j$ (AW).

Equation (6) highlights some deep economic intuitions. Firstly, instead of aggregate variables, now every household represents two risk factors - their own consumption growth rate and return on wealth. It provides a more comprehensive General Equilibrium-like structure by including every individual households in pricing any risky asset. It provides a theoretical support for multi-factor asset pricing models.

Secondly, individual household’s preference matters. As we can see, not only factor loadings are individual household specific, but also factor premia are household specific. Therefore household’s preference $(\theta^j, \psi^j)$ not only determine risky asset $i$’s factor loadings $\text{Cov}_t[g_{t+1}^j, r_{t+1}^i]$ and $\text{Cov}_t[h_{t+1}^j, r_{t+1}^i]$ on risk factors, but also determine the corresponding factor premia, $\theta^j / \psi^j$ and $1 - \theta^j$. Two households with the same consumption factor loading can still influence the market differently through this heterogeneous factor premia.

Lastly, different households enter the aggregated pricing kernel with different weights. Weights are individual household’s characteristics dependent. More importantly, they have the potential to accommodate market participation heterogeneity. The weight of a market non-participant is simply zero.

\section{Choosing Aggregation Weights}

A choice that needs special consideration is which aggregation weight to use. As we can see from equation (6), different aggregation weights will generate stark different results. An ideal aggregation weight should be the exact measure of the intensity of households’ engagement in the market. However, such measure is difficult to define and acquire. Instead, we need a proxy for it.

By disaggregating the Consumption CAPM equation under representative agent, we have

$$
\ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f = \frac{\theta}{\psi} \text{Cov}_t[g_{t+1}, r_{t+1}^i] + (1 - \theta) \text{Cov}_t[h_{t+1}, r_{t+1}^i]
$$

$$
= \sum_{j=1}^{J} \left[ \mathbb{E}_t \left[ \frac{g_{j+1}}{C_{t+1}} \right] \frac{\theta}{\psi} \text{Cov}_t[g_{j+1}, r_{t+1}^i] \right] + \sum_{j=1}^{J} \left[ \mathbb{E}_t \left[ \frac{w_{j+1}}{W_{t+1}} \right] (1 - \theta) \text{Cov}_t[h_{j+1}, r_{t+1}^i] \right].
$$
As we can see, under representative agent assumption, each household enter the pricing kernel using consumption weight $E_t[c_{t+1}/C_{t+1}]$ for their consumption growth rate and wealth weight $E_t[w_{t+1}/W_{t+1}]$ for their return on wealth. And by representative agent assumption, all rates are the same, so $E_t[c_{t+1}/C_{t+1}] = E_t[w_{t+1}/W_{t+1}] = 1/J$. Therefore, in this paper, we choose three different weights for aggregation weights $\phi_j$. The first is equal weight $1/J$, the second is consumption weight $E_t[c_{t+1}/C_{t+1}]$ and the last is wealth weight $E_t[w_{t+1}/W_{t+1}]$. We further assume that households’ consumption and wealth weights $c_{t}/C_{t}$ and $w_{t}/W_{t}$ follow martingale, $E_t[c_{t+1}/C_{t+1}] = c_{t}/C_{t}$ and $E_t[w_{t+1}/W_{t+1}] = w_{t}/W_{t}$. This assumption allows predictable growth in both consumption and wealth, as long as there is no predictable growth heterogeneity.

### 2.4 Time-varying heterogeneous Preference

With constant heterogeneous preferences parameters $(\theta^i, \psi^j)$, factor premia from each risk factors are constant over time. However it is inconsistent with conditional factor pricing consensus that both risk factor loading and factor premia are time-varying (Jagannathan & Wang (1996)). To incorporate this, time-varying heterogeneous preference parameters $(\theta^j_t, \psi^j_t)$ are introduced. The Consumption CAPM relation now becomes

$$
\ln E_t[R^i_{t+1}] - r^f_{t+1} = \sum_{j=1}^{J} \left[ \phi^j_t \frac{\theta^j_t}{\psi^j_t} \text{Cov}_t[g^j_{t+1}, r^i_{t+1}] \right] + \sum_{j=1}^{J} \left[ \phi^j_t (1 - \theta^j_t) \text{Cov}_t[h^j_{t+1}, r^i_{t+1}] \right].
$$

(7)

The intuition behind this model is clear. The population of heterogeneous agents is characterized by a time-varying two-dimensional distribution of $(\theta, \psi)$. On one side, it governs the distribution of households’ optimal behaviors, like intertemporal consumption decisions and wealth accumulations; on the other side, together with household optimal behavior, it determines the excess return for any risky asset, and consequently the cross sectional distribution of stock returns.
3 Data and Econometric Procedure

3.1 Data

The most commonly used datasets for empirical consumption CAPM studies are Bureau of Economic Analysis’ National Income and Product Accounts Tables (NIPA) dataset and Bureau of Labor Statistics’ Consumer Expenditure Survey (CEX) dataset. The latter is the dataset used for studies of consumption heterogeneity. Although CEX provides relatively high frequency consumption data at quarterly level, it only tracks a household for five consecutive quarters. The lack of panel structure and the lack of households’ wealth information make CEX not suitable for this study. Panel Study of Income Dynamics (PSID) provides panel data for households’ consumption, but prior 1998 data is not as accurate as CEX data. However, after the survey structure changes in 1998, consumption data quality improves a lot. Guo (2010) shows that total consumption from CEX can be predicted quite well from PSID data. In particular, Li et al. (2010) show that PSID consumption data align closely with corresponding measure from the CEX, the ratios of means (PSID/CE) are 1.02 and 1.01, respectively for 2001 and 2003.

This paper uses PSID as data source, and constructs consumption measure following Li et al. (2010)’s method. The household consumption includes 4 major categories, Food, Housing, Transportation and Health Care and 2 minor categories, Education and Childcare. Table 1 provides some descriptive statistics.

<table>
<thead>
<tr>
<th></th>
<th>1999</th>
<th>2001</th>
<th>2003</th>
<th>2005</th>
<th>2007</th>
<th>2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>20,933</td>
<td>23,512</td>
<td>24,920</td>
<td>28,064</td>
<td>30,272</td>
<td>30,198</td>
</tr>
<tr>
<td>Stdev</td>
<td>17,259</td>
<td>19,135</td>
<td>25,770</td>
<td>24,614</td>
<td>23,382</td>
<td>23,968</td>
</tr>
</tbody>
</table>

*Numbers reported in this table are average households' nominal consumption and standard deviation. The unit is US dollars.

To estimate EZW utility, we also need returns on wealth. However, this measure is unobservable itself. Various methods are developed to handle this issue. Epstein & Zin (1991), Bakshi & Naka (1997) and Normandin & St-Amour (1998) use financial wealth as the proxy for the aggregate wealth, but this proxy lacks the part of return on human capital.
Jeong et al. (2015) include human capital for a better approximation of aggregate wealth. Chen et al. (2013) develop a semiparametric method to avoid using any proxy for return on wealth. Bansal et al. (2007) use simulation based method to avoid using proxy as well.

In this study, we use wealth data from PSID directly. Benefit of doing so is that we have a complete panel for households that includes both consumption and wealth - two measures we need to estimate EZW utility parameters. Based on Juster et al. (1999), the wealth data in the PSID lines up well with data from the Fed’s Survey of Consumer Finances (SCF). However, like Epstein & Zin (1991) and other studies, our treatment lacks return on human capital as well, which may bias our estimates. Table 2 provides some descriptive statistics. It is worth notice that our wealth data captures two shrinks of wealth in 2001-2003 and 2009, which precisely matches two economic recessions, the technology bubble and the Great Recession.

Table 2: Descriptive Statistics - Household Wealth

<table>
<thead>
<tr>
<th></th>
<th>1999</th>
<th>2001</th>
<th>2003</th>
<th>2005</th>
<th>2007</th>
<th>2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>123,343</td>
<td>110,918</td>
<td>107,892</td>
<td>124,401</td>
<td>152,827</td>
<td>141,990</td>
</tr>
<tr>
<td>Stdev</td>
<td>1,422,366</td>
<td>771,520</td>
<td>711,844</td>
<td>833,320</td>
<td>1,053,670</td>
<td>1,355,234</td>
</tr>
</tbody>
</table>

Numbers reported in this table are average households’ nominal wealth and standard deviation. The unit is US dollars.

Our sample starts from 1999, when consumption data is much more comprehensive than prior 1998 period, and ends at 2009, the last year before the structure of recording household wealth is changed. PSID surveys are conducted biennially, so from 1999 to 2009, we have 6 periods (5 transition periods) in total. For each period in this sample period, we track exactly same entries for both consumptions and wealth. PSID tracks households from 1968, with unique 1968 IDs for every households. However, most households split into multiple smaller households since 1968, the panel structure only exists for households with same 1968 ID. Therefore, I aggregate small households with the same 1968 ID to construct panel for original households. After deleting households with 0 consumption, our sample contains 2,468 households. Because wealth measures are mostly self-estimated by households, they are very volatile. About half of the sample have negative net wealth, which imposes problems when calculating returns on wealth. Therefore we also delete households with negative net
wealth. We need keep in mind that by doing this, we may create an upward bias on RRA estimates. Our final sample contains 1,384 households. Table 3 shows descriptive statistics for consumption growth rates and returns on wealth for these 1,384 households in our sample.

Table 3: Descriptive Statistics - Growth Rates of Consumption and Wealth

<table>
<thead>
<tr>
<th>Consumption</th>
<th>T=1</th>
<th>T=2</th>
<th>T=3</th>
<th>T=4</th>
<th>T=5</th>
<th>avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.184</td>
<td>0.135</td>
<td>0.140</td>
<td>0.113</td>
<td>0.058</td>
<td>0.126</td>
</tr>
<tr>
<td>Stdev</td>
<td>0.524</td>
<td>0.495</td>
<td>0.484</td>
<td>0.497</td>
<td>0.519</td>
<td>0.504</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wealth</th>
<th>T=1</th>
<th>T=2</th>
<th>T=3</th>
<th>T=4</th>
<th>T=5</th>
<th>avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.181</td>
<td>0.117</td>
<td>0.197</td>
<td>0.164</td>
<td>-0.125</td>
<td>0.107</td>
</tr>
<tr>
<td>Stdev</td>
<td>1.309</td>
<td>1.247</td>
<td>1.208</td>
<td>1.271</td>
<td>1.312</td>
<td>1.269</td>
</tr>
</tbody>
</table>

Numbers reported in this table are average households’ nominal consumption growth rates and return on wealth and corresponding standard deviations. \( T = 1, \ldots, 5 \) are five bennial transition periods from 1999 to 2009.

It is clear that the cross sectional heterogeneity in consumption growth rates is large, while the cross sectional heterogeneity in return on wealth is even larger. It indirectly shows the importance of incorporating heterogeneous agents in asset pricing. Given such high level of cross sectional differences in returns on wealth, using aggregate data generates misleading results.

3.2 Econometric Procedure

To estimate utility parameters, we used Generalized Method of Moments (GMM), developed by Hansen (1982). Since \( \beta \) does not enter the Consumption CAPM equation in determining excess returns, we do not estimate \( \beta \) and simply choose \( \beta = 0.99 \) annually. Unlike past literature running GMM along time for one representative agent using aggregate data, we run GMM cross sectionally for each period using household level data. We have 6 periods data and need 2 period for each estimation. So we run GMM for 5 transition periods.

For every period \( t = 1, \ldots, 5 \), we sort households by wealth and divide them into quartiles \( q = 1, \ldots, 4 \). We assume households in each quartile have the same preference. For each quartile \( q \) at each period \( t \), we run GMM to estimate \((\psi_q^t, \theta_q^t)\). For all households \( j = 1, \ldots, 346 \) in quartile \( q \), their preferences \((\psi_j^t, \theta_j^t) = (\psi_q^t, \theta_q^t)\). We run estimation for \( 4 \times 5 = 20 \) times in total to estimate heterogeneous preferences. For each household, it
may belong to a different quartile at a different period. Therefore, theoretically we may have $4^5 = 1,024$ different sets for $(\psi^j_t, \theta^j_t)^5_{t=1}$, or equivalently 1,024 types of agents, which is relative close to our sample size of 1,384. Therefore, we maximize the heterogeneity in the model in accordance with the availability of data.

For each period $t$ and each wealth quartile $q$, the moment conditions are

$$E_t \left[ \beta^{\theta^j_t} \left( \frac{c^j_{t+1}}{c^j_t} \right)^{-\theta^j_t/\psi^j_t} (R^W_{t+1})^{\theta^j_t-1} (R^m_{t+1} - R^f_{t+1}) \right] = 0, \ \forall j = 1, \cdots, 346,$$  \hspace{1cm} (8)

where $\theta^j_t$ and $\psi^j_t$ are preference parameters to estimate. For households $j = 1, \cdots, 346$ in wealth quartile $q$, $c^j_{t+1}$ and $c^j_t$ are households’ consumptions, $R^W_{t+1}$ is households’ gross return on wealth. $R^m_{t+1}$ is the market return and $R^f_{t+1}$ is the risk free rate.

To make comparison with representative agent model, we also run GMM using all data points for each period $t$, to generate a set of time-varying utility parameter $(\psi_t, \theta_t)^5_{t=1}$ for the homogeneous preference with following moment conditions,

$$E_t \left[ \beta^{\theta_t} \left( \frac{c^j_{t+1}}{c^j_t} \right)^{-\theta_j/\psi_t} (R^W_{t+1})^{\theta_j-1} (R^m_{t+1} - R^f_{t+1}) \right] = 0, \ \forall j = 1, \cdots, 1384,$$  \hspace{1cm} (9)

where $\theta_t$ and $\psi_t$ are representative agent preference parameters to estimate. For households $j = 1, \cdots, 1384$, $c^j_{t+1}$ and $c^j_t$ are households’ consumptions, $R^W_{t+1}$ are households’ gross returns on wealth. $R^m_{t+1}$ is the market return and $R^f_{t+1}$ is the risk free rate.

## 4 Estimation Result

Table 4 shows estimation results. $\{T1, \cdots, T5\}$ represent five biennial transition periods from 1999 to 2009, $\{Q1, \cdots, Q4\}$ represent four wealth quartiles, $Q1$ is the poorest while $Q4$ is the richest. All EIS coefficients are within $[2.0, 6.4]$ range except for the fourth quartile in the last period, which is 22.5. All RRA coefficients are within $[0.03, 0.45]$, which lies between 0 - a risk neutral preference, and 10 - the well respected upper bound for relative risk aversion, except for the third and the fourth quartile in the last period, which are -0.04 and -0.20, respectively. The unusually large EIS at 22.5 and the two risk loving RRA coefficients in
the last transition period is caused by the huge drop in wealth for the rich people from 2007 and 2009. It is an indication of the impact magnitude of the Great Recession to households’ wealth.

Table 4: Estimation Result

<table>
<thead>
<tr>
<th></th>
<th>$EIS$</th>
<th>$\bar{RRA}$</th>
<th>avg.</th>
<th>$Rep$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Q1</td>
<td>Q2</td>
<td>Q3</td>
<td>Q4</td>
</tr>
<tr>
<td>T1</td>
<td>2.79</td>
<td>3.14</td>
<td>3.57</td>
<td>2.03</td>
</tr>
<tr>
<td>T2</td>
<td>2.92</td>
<td>4.27</td>
<td>3.31</td>
<td>4.43</td>
</tr>
<tr>
<td>T3</td>
<td>4.85</td>
<td>3.68</td>
<td>3.91</td>
<td>3.86</td>
</tr>
<tr>
<td>T4</td>
<td>3.88</td>
<td>5.52</td>
<td>3.03</td>
<td>3.85</td>
</tr>
<tr>
<td>T5</td>
<td>6.04</td>
<td>6.38</td>
<td>5.22</td>
<td>22.5</td>
</tr>
<tr>
<td>avg.</td>
<td>4.10</td>
<td>4.60</td>
<td>3.81</td>
<td>7.34</td>
</tr>
</tbody>
</table>

a Numbers reported in this table are estimated Elasticity of Inter-temporal Substitution ($EIS$) and Relative Risk Aversion ($RRA$) parameters for both heterogeneous preferences model and representative agent model. $T_1, \cdots, T_5$ are five bennial transition periods from 1999 to 2009. $Q_1, \cdots, Q_4$ are four wealth quartiles, $Q_1$ being the poorest and $Q_4$ being the richest.

b Estimation is done using Generalized Method of Moments (GMM) with following moment condition,

$$\mathbb{E}_t \left[ \beta \theta^q \left( \frac{c_{jt+1}^q}{c_{jt}^q} \right)^{-\theta_j \psi_j^q} (R_{t+1}^{W,j} \theta_j^{q-1}(R_{m,t+1}^{t} - R_{f,t+1}^{t})) \right] = 0. $$

t = 1, \cdots, 5 are 5 periods and $q = 1, \cdots, 4$ are 4 wealth quartiles. $j$ represents households in wealth quartile $q$. $R_{m,t+1}^{t}$ is market return and $R_{f,t+1}^{t}$ is risk free rate. We use excess market return as test asset.

c $EIS$ parameters for heterogeneous preferences model are recorded in column 2-5, with average for each period being recorded in column 10. $RRA$ parameters for heterogeneous preferences model are recorded in column 6-9, with average for each period being recorded in column 11. Averages for $EIS$ and $RRA$ for each wealth quartile are recorded corresponding columns and in the last row.

d $EIS$ and $RRA$ parameters for representative agent model are recorded in column 12 and 13, with average across 5 periods being recorded in corresponding columns and in the last row.

Across different wealth quartiles, there is no clear pattern for $EIS$. $RRA$ shows strong patterns across different wealth quartiles. With more wealth, $RRA$ tends to be lower, which implies wealthy households are less risk averse. Across periods, we see a weak trend for $RRA$. As time progresses, agents tend to be less risk averse, however, this trend is largely
driven by those two negative $RRA$ coefficients in the last period. We do observe that $EIS$ has an tendency to increase over time for both average of heterogeneous preferences and the representative agent. It implies households are more inter-temporally elastic for consumption. Or in other words, the impact of shocks to households welfare becomes smaller over time.

Comparing $EIS$ and $RRA$ for representative agent with those for heterogeneous preferences, we find in each period, both $EIS$ and $RRA$ for representative agent lie in the middle of the ranges of those two parameters for heterogeneous preferences. The result is not surprising, as heterogeneous preferences successfully captured the variations of those two parameters across different wealth groups, and representative agent preference is a weighted average of heterogeneous preferences.

The most important result is that our estimation generates a much smaller $RRA$ for representative agent than literature. Our $RRA$ lies within the range of $[0.11, 0.36]$, which is smaller than $[1, 8]$ in Jeong et al. (2015) and much smaller than $[17, 60]$ in Chen et al. (2013). The reason for such small $RRA$ is that unlike the literature running GMM along time, we run GMM estimation cross sectionally. Aggregate consumption is really smooth along time, therefore they generate a large $RRA$ as if the agent extremely cares about consumption smoothing, or equivalently extremly care about risk aversion. However, the cross sectional difference in consumption growth rates is huge. Therefore, when we make the assumption that agents have the same preference, GMM sees the different consumption growth rates as the agent having very volatile consumption decisions. In other words, the agent does not care about risk that much. That is the reason why we are able to generate such a small $RRA$ coefficient.

Mehra & Prescott (1985) formally assert equity premium puzzle, which states that size of equity premium is too big to be justified by consumption-based asset pricing theory. More specifically, traditional empirical investigations into consumption CAPM face two issues. Firstly, GMM estimation using aggregate consumption data generates really large $RRA$ value, which is far above the well respected upper bound of 10. Savov (2011) finds an RRA about 17, Parker & Julliard (2005) find it around 66, and Jagannathan & Wang (2007) find it around 88. Secondly, even with such a high $RRA$, consumption CAPM still cannot generate
a risk premium with acceptable size to match the real data. The fundamental reason for both issues is of course that aggregate consumption is too smooth over time.

Our result provides a new angle to solve the equity premium puzzle. Individual level wealth and consumption are much more volatile than aggregate level, and the cross sectional differences are even larger. Many individual level fluctuations are “averaged out” in aggregation. As a result, we are not able to generate reasonably small $RRA$ coefficient and large enough risk premium with this smoothed consumption. To find the volatile consumption and wealth growth, we need heterogeneous agents and individual level data as shown in this study.

However, we need to interpret results of this study with caution. Our low level of $RRA$ and relatively high level of $EIS$ come from huge cross sectional differences in households consumption growth rates and returns of wealth. As aggregate consumption underestimates the fluctuation of true individual household’s consumption growth rates, cross sectional data overstates the fluctuation. Nevertheless, those coefficients still serve for our purpose - comparing consumption CAPM model under heterogeneous preferences with representative agent setup. As long as we treat both models with the same method, we can conduct the comparison.

Another question we need to answer before we move to asset pricing practice is whether agents actually differ with each other in terms of preferences. To achieve this, we run following regression for both $EIS$ and $RRA$,

$$Parameters q_t = \beta_0 + \beta_T \times t + \beta_Q \times q + \varepsilon_t,$$

where $t = 1, \cdots, 5$ and $q = 1, \cdots, 4$ are indices for periods and wealth groups. The regression serves two purposes. Firstly, it captures and confirms trends observed previously. Secondly, the test against $\beta_Q = 0$ is also the test against the null hypothesis that all agents from different wealth groups have the same preferences parameters. As we can see in Table 5, although different wealth groups show no significant difference in elasticities of inter-temporal substitution, they indeed differ in relative risk aversion.
Table 5: Test on Heterogeneity

<table>
<thead>
<tr>
<th></th>
<th>EIS</th>
<th>RRA</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_T$</td>
<td>1.466**</td>
<td>-0.070***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(2.456)</td>
<td>(-4.370)</td>
</tr>
<tr>
<td>$\beta_Q$</td>
<td>0.894</td>
<td>-0.076***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(1.184)</td>
<td>(-3.768)</td>
</tr>
</tbody>
</table>

\(^a\) Numbers reported in this table are coefficients and corresponding t-statistics for the following regression,

$$Parameters_t^q = \beta_0 + \beta_T \times t + \beta_Q \times q + \varepsilon_t^q,$$

where $t = 1, \ldots, 5$ and $q = 1, \ldots, 4$ are indices for periods and wealth groups.

\(^b\) (*) is $p$-value $< 0.1$, (**) is $p$-value $< 0.05$, (***$p$-value $< 0.01$

5 Implied Market Risk Premium

Our estimated preferences parameters vary every two years. We interpolate using the average of two neighbour years to get a set of parameters for 9 consecutive years. Interpolating does not increase the theoretical maximum number of different households, which is still 1,024.

We then use those parameters, together with consumption and wealth growth rates, to calculate implied market risk premium. However, we are not able to calculate time-varying (conditional) factor loadings for idiosyncratic risk factors, $\text{Cov}_t[g_{t+1,j}, r_{t+1}]$ and $\text{Cov}_t[h_{t+1,j}, r_{t+1}]$, due to our sample contains only 5 transition periods for consumption and wealth growth. Therefore, we are not able to calculate implied market risk premium with the form of Equation (7). Instead, in this study, we calculate constant (unconditional) factor loadings for each household, $\text{Cov}_t[g_{t+1,j}, r_{t+1}]$ and $\text{Cov}_t[h_{t+1,j}, r_{t+1}]$, and compute implied market risk premium (IRP) using the following equation,

$$IRP_{t+1} = \sum_{j=1}^{J} \left[ \frac{\phi_i^j \theta_i^j}{\psi_i^j} \text{Cov}[g_{t+1,j}, r_{t+1}^m] \right] + \sum_{j=1}^{J} \left[ \phi_i^j (1 - \theta_i^j) \text{Cov}[h_{t+1,j}, r_{t+1}^m] \right], \hspace{1cm} (11)$$

where $g_{t+1}^j$ is the consumption growth rate and $h_{t+1}^j$ is the return on wealth for household
\( j \), \((\psi_j^t, \theta_j^t)\) are time-varying preference parameters, and \(\phi_j^t\) is the time-varying aggregation weight for household \( j \). \( r_{t+1}^m \) is the market return.

Theoretically, time-varying risk premium comes from two parts, one is the time-varying factor loading or risk exposure, the other is the time varying factor premia or risk price. In this study, we are restricted to focus on the time-varying factor premia, and time-varying aggregation weights, which highlights the benefits of introducing heterogeneous preferences into consumption CAPM. We believe the performance of the model will be better after including time-varying factor loadings.

We exam five models in this study. Two of which are homogeneous preference models and the other three are heterogeneous preferences models. Model 1 is the representative agent model (Rep). We use homogeneous preference parameters \((\psi_t, \theta_t)\) and aggregate consumption and wealth \((C_t, W_t)\), as in equation (12).

\[
\ln \mathbb{E}_t[R_{t+1}^m] - r_{t+1}^f = \frac{\theta_t}{\psi_t} \text{Cov}[g_{t+1}, r_{t+1}^m] + (1 - \theta_t) \text{Cov}[h_{t+1}, r_{t+1}^m],
\]

(12)

where \(g_{t+1}\) is the aggregate consumption growth rate, \(h_{t+1}\) is the aggregate return on wealth, \((\psi_t, \theta_t)\) are time-varying homogeneous preference parameters.

Model 2 is homogeneous preference model with idiosyncratic consumption and wealth (Hom). We use homogeneous preference parameters \((\psi_t, \theta_t)\) and households level consumption and wealth \((c_{jt}, w_{jt})\), and use equal weights \(1/J\) to aggregate, as in equation (13).

\[
\ln \mathbb{E}_t[R_{t+1}^m] - r_{t+1}^f = \sum_{j=1}^{J} \left[ \frac{1}{J} \frac{\theta_t}{\psi_t} \text{Cov}[g_{jt+1}, r_{t+1}^m] \right] + \sum_{j=1}^{J} \left[ \frac{1 - \theta_t}{J} \text{Cov}[h_{jt+1}, r_{t+1}^m] \right].
\]

(13)

Model 3 is heterogeneous preferences model with equal weights (HetE). We use heterogeneous preferences \((\psi_j^t, \theta_j^t)\) and households level consumption and wealth \((c_{jt}, w_{jt})\), and use equal weight \(1/J\) for \(\phi_j^t\) to aggregate.

\[
\ln \mathbb{E}_t[R_{t+1}^m] - r_{t+1}^f = \sum_{j=1}^{J} \left[ \frac{1}{J} \frac{\theta_t}{\psi_j^t} \text{Cov}[g_{jt+1}, r_{t+1}^m] \right] + \sum_{j=1}^{J} \left[ \frac{1 - \theta_t}{J} \text{Cov}[h_{jt+1}, r_{t+1}^m] \right].
\]

(14)
Model 4 is heterogeneous preferences model with consumption weights \((\text{HetC})\), just change the equal weights \(1/J\) in model 3 to consumption weights \(c_j^t/C_t\), where \(c_j^t\) is household \(j\)'s consumption and \(C_t\) is aggregate consumption.

\[
\ln \mathbb{E}_t[R_{t+1}^m] - r_{t+1}^f = \sum_{j=1}^J \left[ \frac{c_j^t}{C_t} \frac{\theta_j^t}{\psi_j^t} \operatorname{Cov}[g_{t+1}^j, r_{t+1}^m] \right] + \sum_{j=1}^J \left[ \frac{c_j^t}{C_t} (1 - \theta_j^t) \operatorname{Cov}[h_{t+1}^j, r_{t+1}^m] \right] . \tag{15}
\]

Model 5 is heterogeneous preferences model with wealth weights \((\text{HetW})\), just change the equal weights \(1/J\) in model 3 to wealth weights \(w_j^t/W_t\), where \(w_j^t\) is household \(j\)'s wealth and \(W_t\) is aggregate wealth.

\[
\ln \mathbb{E}_t[R_{t+1}^m] - r_{t+1}^f = \sum_{j=1}^J \left[ \frac{w_j^t}{W_t} \frac{\theta_j^t}{\psi_j^t} \operatorname{Cov}[g_{t+1}^j, r_{t+1}^m] \right] + \sum_{j=1}^J \left[ \frac{w_j^t}{W_t} (1 - \theta_j^t) \operatorname{Cov}[h_{t+1}^j, r_{t+1}^m] \right] . \tag{16}
\]

Table 6 reports results of implied market risk premium from those five models, as well as actual market risk premium.\(^1\)

<table>
<thead>
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</tr>
</thead>
<tbody>
<tr>
<td>Actual</td>
<td>-17.6%</td>
<td>-15.2%</td>
<td>-22.8%</td>
<td>30.8%</td>
<td>10.7%</td>
<td>3.09%</td>
<td>10.6%</td>
<td>1.04%</td>
<td>-38.3%</td>
<td>-4.19%</td>
</tr>
<tr>
<td>Rep</td>
<td>0.10%</td>
<td>0.09%</td>
<td>0.07%</td>
<td>0.10%</td>
<td>0.14%</td>
<td>0.12%</td>
<td>0.06%</td>
<td>-0.04%</td>
<td>-0.15%</td>
<td>0.06%</td>
</tr>
<tr>
<td>Hom</td>
<td>0.03%</td>
<td>0.03%</td>
<td>0.02%</td>
<td>0.04%</td>
<td>0.06%</td>
<td>0.05%</td>
<td>0.02%</td>
<td>-0.03%</td>
<td>-0.09%</td>
<td>0.02%</td>
</tr>
<tr>
<td>HetE</td>
<td>-0.24%</td>
<td>0.07%</td>
<td>0.25%</td>
<td>0.28%</td>
<td>0.26%</td>
<td>0.15%</td>
<td>0.03%</td>
<td>-0.26%</td>
<td>-0.71%</td>
<td>-0.02%</td>
</tr>
<tr>
<td>HetC</td>
<td>-0.14%</td>
<td>-0.04%</td>
<td>-0.06%</td>
<td>0.01%</td>
<td>0.04%</td>
<td>0.01%</td>
<td>-0.04%</td>
<td>-0.18%</td>
<td>-0.61%</td>
<td>-0.11%</td>
</tr>
<tr>
<td>HetW</td>
<td>-1.07%</td>
<td>-0.11%</td>
<td>-0.43%</td>
<td>0.22%</td>
<td>0.08%</td>
<td>0.04%</td>
<td>-0.05%</td>
<td>-0.28%</td>
<td>-1.38%</td>
<td>-0.23%</td>
</tr>
</tbody>
</table>

\(^a\) Numbers reported in this table are implied risk premium from 2000 to 2008 generated by five models.

\(^b\) Actual is actual risk premium we observe from data. Rep is the representative agent model, using homogeneous preference and aggregate data. Hom is the homogeneous preference model with households level data. HetE is the heterogeneous preferences model with equal weights. HetC is the heterogeneous preferences model with consumption weights. HetW is the heterogeneous preferences model with wealth weights.

A first impression is that implied market risk premium generated by all five models are relatively small comparing to the actual market risk premium. This is the direct result\(^1\) of the way these models are constructed.
of small $RRA$ coefficients we estimated. Therefore this study cannot give straight answers towards equity premium puzzle. Instead, we focus on comparing performances between homogeneous preference models and heterogeneous preferences models, and hopefully this paper can shed some light into the direction to solve the puzzle. However, we need also keep in mind that in the sample period, the first decade of this century, we saw two recessions, the bust of the dotcom bubble and the Global Financial Crisis. As a result, the market risk premium varies dramatically between -38% and 30.8%. Any attempt to capture the fluctuation of that magnitude from consumption perspective would be much more difficult than capturing some more moderate fluctuations of risk premium in a longer time horizon.

Interestingly, neither of the two homogeneous preference models can produce a negative average risk premium like what we see in the data, while all three heterogeneous preferences models successfully generate a negative average risk premium with wealth weights performing the best. As we can see in Figure 1, HetW produces the most volatile risk premium.

Figure 1: Annualized Implied Risk Premium

![Figure 1: Annualized Implied Risk Premium](image)

The average implied risk premium only provides a rough examination on the magnitude aspect of model performances. We further look into each year to see whether each model
successfully predict the sign of actual risk premium. Table 7 records the performance of sign prediction for each model.

Table 7: Sign Accuracy

<table>
<thead>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual</td>
<td>-18%</td>
<td>-15%</td>
<td>-23%</td>
<td>31%</td>
<td>11%</td>
<td>3%</td>
<td>11%</td>
<td>1%</td>
<td>-38%</td>
</tr>
<tr>
<td>Rep</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>v</td>
<td>v</td>
<td>v</td>
<td>x</td>
<td>v</td>
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<td>v</td>
<td>v</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>HetE</td>
<td>v</td>
<td>x</td>
<td>x</td>
<td>v</td>
<td>v</td>
<td>v</td>
<td>v</td>
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<td>x</td>
<td>v</td>
<td>v</td>
<td>v</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

*This table reports sign prediction accuracy for five models in study from 2000 to 2008. ✓ means the model successfully predicts the sign of market risk premium of that year, x means the model unsuccessfully predicts the sign of market risk premium of that year.

b Actual is actual risk premium we observe from data. Rep is the representative agent model, using homogeneous preference and aggregate data. Hom is the homogeneous preference model with household level data. HetE is the heterogeneous preferences model with equal weights. HetC is the heterogeneous preferences model with consumption weights. HetW is the heterogeneous preferences model with wealth weights.

A first look shows that all five models successfully predict the boom from 2003 to 2005 and the Global Financial Crisis in 2008. All five models failed to predict the positive risk premium of 1% in 2007, they all suggest a negative risk premium. If we consider the actual start date of the Global Financial Crisis is mid 2007 and 1% is barely above 0, then this wrong prediction for 2007 isn’t the worst case.

The best performer in predicting signs is HetC, heterogeneous preferences model with consumption weights. It successfully captures the tech bubble, the 2003-2005 boom and the global financial crisis. HetE and HetW are runner-ups, with HetE performing relatively worse in the tech bubble and HetW performing worse in the global financial crisis. Interestingly, neither homogeneous preference model succeed to capture the tech bubble while both succeed to capture the GFC. It raises a question, why?

Recall the natures of those two crisis. The tech bubble starts from a fall of tech companies’ stock prices, which tightens the financial market and it in turn spreads to the real economy. The GFC starts from the drop of housing prices in mid 2006, which causes the
drop in MBS prices and later the fail of Lehman Brothers. The key difference between those two regarding households is that the former starts with the wealth drop of a small group of households, those who are rich and have investments in tech stocks or who work for tech firms, while the latter starts with the wealth drop of the majority of households, anyone who owns real estate.

Mechanically, when we estimate homogeneous preference parameters, the wealth drop of the small group of wealthy households is ignored. All households are included in estimation while the majority do not experience the same wealth drop. As a result, the homogeneous preference parameters provide information representing more on the unaffected majority. Therefore, we cannot see the crisis neither using aggregate data nor using households level data with homogeneous preference. In contrast, if we estimate heterogeneous preferences parameters, we look into different groups of households. Although households with wealth drop in tech bubble is a small group considering all households, they are a relatively larger subgroup in rich households group. As a result, the wealth drop information is captured by rich group’s preferences parameters and later shows up in risk premium when we aggregate across households, and the information is amplified by wealthy group’s relatively large aggregation weights. That also explains why wealth weights outperforms equal weights in predicting the tech bubble.

The story is different for the GFC. We first see the wealth drop for the majority of households. This information is captured when we estimate heterogeneous preferences. Similarly, this information is also captured when we estimate homogeneous preference since the size of the group is large. Therefore, we are able to predict the crisis using both homogeneous preference models and heterogeneous preferences models. The ability to capture the crisis starting from a small group of households is one advantage of heterogeneous preferences models over homogeneous preference models. We may have financial crisis caused by different groups of people, the size of the group can be small or large. Our analysis suggests that homogeneous preference model can only identify potential crisis if the crisis is caused by a large group of households, like the GFC. However, with heterogeneous preferences models, a crisis starts with a smaller group of households may still be identified. It also sheds some light into the study of systemic risk. Different risks may affect different amount of households,
we may use heterogeneous preferences model to study what is the threshold above which a risk will become a systemic risk.

6 Time Series of Risk Premium

We further exam pricing performances of those five models using following Times Series regression equation

\[ Actual_t = \alpha + \beta \times Implied_t + \varepsilon_t, \] (17)

where \( Implied_t \) are model generated market excess returns, \( Actual_t \) are real world market excess returns from data, \( t = 2000, \cdots, 2008 \). Results are recorded in the following Table 8. With a perfect asset pricing model, we expect to have \( \alpha \) equals to 0 as Gibbons et al. (1989) argues, \( \beta \) be positive and equals to 1 and \( R^2 \) goes to 1. As we can see, \( \alpha \) for all five models are small and statistically insignificant from zero, suggesting no unexplained risk premium left. Slopes for neither homogeneous models is statistically significant while all three heterogeneous preferences models' slopes are statistically significantly positive, which suggests those three models are pricing the market risk premium on the right direction. Looking at magnitudes of those coefficients, heterogeneous preferences models have smaller coefficients than homogeneous models, meaning heterogeneous preferences models generate closer to actual risk premium, although the difference with actual data is still large. Again, this paper does not intend to solve the equity premium puzzle with risk premium that is quantitatively close enough to real data, instead this paper focuses on the relative performance between heterogeneous preferences models and representative agent model. Heterogeneous preferences models indeed outperform homogeneous models in terms of generating quantitatively closer to actual risk premium.

We further exam whether our implied risk premium captures the dynamics of actual risk premium by looking at \( R^2 \). As the benchmark, Representative Agent model explains about 30.4% of total variations in risk premium dynamics. Representative agent model has the most restrictive assumption that all households have the same preference and it uses the least informative data - aggregate consumption and wealth, therefore it is not surprising it performs the worst out of all five models. Heterogeneous Preferences with Consumption
Table 8: Market Risk Premium

<table>
<thead>
<tr>
<th></th>
<th>Rep</th>
<th>Hom</th>
<th>HetE</th>
<th>HetC</th>
<th>HetW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>-0.111</td>
<td>-0.083</td>
<td>-0.033</td>
<td>0.042</td>
<td>0.010</td>
</tr>
<tr>
<td>$t$-stat</td>
<td>(-1.500)</td>
<td>(-1.276)</td>
<td>(-0.582)</td>
<td>(0.677)</td>
<td>(0.131)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>126.682</td>
<td>261.170</td>
<td>41.657**</td>
<td>73.719**</td>
<td>21.407*</td>
</tr>
<tr>
<td>$t$-stat</td>
<td>(1.747)</td>
<td>(1.867)</td>
<td>(2.253)</td>
<td>(2.600)</td>
<td>(2.036)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.3037</td>
<td>0.3323</td>
<td>0.4202</td>
<td>0.4912</td>
<td>0.3720</td>
</tr>
</tbody>
</table>

*a* Numbers reported in this table are regression coefficients and their corresponding $t$-statistics, as well as $R^2$ for five models in study using following regression equation,

$$Actual_t = \alpha + \beta \times Implied_t + \varepsilon_t,$$

where $Implied_t$ are implied risk premium at time $t$ generated by models, and $Actual_t$ are actual risk premium at time $t$.

*b* *Rep* is the representative agent model, using homogeneous preference and aggregate data. *Hom* is the homogeneous preference model with households level data. *HetE* is the heterogeneous preferences model with equal weights. *HetC* is the heterogeneous preferences model with consumption weights. *HetW* is the heterogeneous preferences model with wealth weights.

*c* (*) is $p$-value < 0.1, (**) is $p$-value < 0.05

Weights is the best performer, explaining almost half of the dynamics at 49.1%. It is a 61.77% improvement from the benchmark. With our models settings, we are able to decompose this improvement into three parts, each corresponds to one advantage heterogeneous preferences model has over representative agent model, namely Idiosyncratic Risk Factors (RF), Idiosyncratic Factor Premia (FP) and Idiosyncratic Characteristics dependent Aggregation Weights (AW).

Comparing Representative Agent (Rep) model (Equation 12) to Homogeneous Preference with Idiosyncratic Growth Rates (Hom) model (Equation 13), the difference between the two is we introduce Idiosyncratic Risk Factors (RF) in Hom, while Rep uses Aggregate Risk Factors. By doing this, $R^2$ increases from 30.4% to 33.2%, which is a 9.4% increase from Rep at 30.4%.

Comparing Homogeneous Preference with Idiosyncratic Growth Rates (Hom) model (Equation 13) to Heterogeneous Preferences with Equal Weights (HetE) model (Equation...
the difference between the two is we introduce Heterogeneous Factor Premia (FP) in HetE. By doing this, $R^2$ increases from 33.2% to 42.0%, which is a 28.9% increase from Rep at 30.4%.

Comparing Heterogeneous Preferences with Equal Weights (HetE) model (Equation 14) to Heterogeneous Preferences with Consumption Weights (HetC) model (Equation 15), the difference between the two is we introduce Idiosyncratic Characteristics dependent Weights (AW) in HetC. By doing this, $R^2$ increases from 42.0% to 49.1%, which is a 23.4% increase from Rep at 30.4%.

Total improvement in $R^2$ from Representative Agent model (Rep) to Heterogeneous Preferences with Consumption Weights (HetC) model is the sum of all three improvements from those three fronts,

$$
\text{Total Improvement} = 61.77\% = 9.42\% + 28.94\% + 23.41\% .
$$

Another interesting point is that Heterogeneous Preferences with Wealth Weights (HetW) model (Equation 16) performs the best in terms of magnitude, while it performs much worse than using Consumption Weights (HetC) in terms of capturing dynamics. The good performance on magnitude comes from the volatility of return on wealth. Its average standard deviation is 1.27, which is more than double of consumption growth rates’ 0.50. The bad performance in capturing dynamics suggests that Wealth Weight is not a good proxy for the intensity of market engagement, which is counter intuitive as we expect wealthy people to invest more in the market. Brunnermeier & Nagel (2008) find that fluctuations in nominal wealth do not affect households’ portfolio choice much, and our result supports that finding. A big drop in a rich household’s nominal wealth does not mean an equally big drop in his market engagement intensity, instead they may still hold the same portfolio and have the same influence on the market. Consumption, on the other hand, can also measure the intensity if we believe wealthy people also consume more, but it is much less volatile than wealth and is a better proxy for the relatively stable market engagement intensity.

Brav et al. (2002), Cogley (2002) and Vissing-Jørgensen (2002) use the average of individual households’ pricing kernel as the heterogeneous pricing kernel, and use up to third
moments of the cross-sectional distribution of consumption growth rate as risk factors. Our study is a direct extension on those three studies at three aspects. Firstly we use the more generalized Epstein-Zin-Weil utility, rather than using the special case of power utility in those three studies. Secondly, because of the panel structure in our data, we are able to look at the whole sample distribution in consumption growth rates, instead of the first three moments of the cross-sectional distribution of consumption. Thirdly, we adopt the market engagement intensity based aggregation weights rather than using the equal weight in those three studies. Lastly, we allow heterogeneous preference parameters.

7 Account for Market Participation

One advantage of our configuration of Heterogeneous Preferences models over Representative Agent model is that we are able to accommodate for heterogeneous market participations through heterogeneous aggregation weights. To demonstrate this, we look at three types of market participation. One is General Market Participants (GP), defined as households with either an IRA stock account or a non-IRA stock account. This definition includes three different types of households, (i) households with only an IRA account, (ii) households with only a non-IRA account, and (iii) households with both an IRA and a non-IRA account. Another type of market participation is Investors (I), defined as households with a non-IRA stock account, which is a subset of General Market Participants (GP), including type (ii) and (iii) households. Households with a non-IRA account intentionally bet in the market. The last type of market participation is Smart Investors (SI), defined as households with both an IRA and a non-IRA stock account, which is a subset of Investors (I), including type (iii) households only. Those are households not only intentionally bet in the market, they are also sophisticated enough to explore the tax advantage of IRA account. These households are most likely being marginal traders in the market. Following Table 9 records results.

When we limit households to those who participate in stock market, our Heterogeneous Preferences models get further improvements in explaining risk premium dynamics. $R^2$ improve from 49.1% using all households (HetC) to 50.9% using only General Market Participants (HetC/GP), and further to 55.5% by reducing market participants to Investors.
Table 9: Market Participation

<table>
<thead>
<tr>
<th></th>
<th>HetC</th>
<th>HetW</th>
<th>HetC/GP</th>
<th>HetW/GP</th>
<th>HetC/I</th>
<th>HetW/I</th>
<th>HetC/SI</th>
<th>HetW/SI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coef.</td>
<td>73.719**</td>
<td>21.407*</td>
<td>76.717**</td>
<td>21.780*</td>
<td>84.635**</td>
<td>21.244*</td>
<td>87.587**</td>
<td>20.831*</td>
</tr>
<tr>
<td>t-stat</td>
<td>(2.600)</td>
<td>(2.036)</td>
<td>(2.695)</td>
<td>(2.230)</td>
<td>(2.956)</td>
<td>(2.021)</td>
<td>(2.996)</td>
<td>(1.954)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.4913</td>
<td>0.3720</td>
<td>0.5091</td>
<td>0.4153</td>
<td>0.5552</td>
<td>0.3684</td>
<td>0.5619</td>
<td>0.3529</td>
</tr>
</tbody>
</table>

a Numbers reported in this table are regression slopes and their corresponding $t$-statistics, as well as $R^2$ for 2 heterogeneous preferences models in study with 4 different market participation using following regression equation,

$$Actual_t = \alpha + \beta \times Implied_t + \varepsilon_t,$$

where $Implied_t$ are implied risk premium at time $t$ generated by models, and $Actual_t$ are actual risk premium at time $t$.

b Four different market participations are full market participation, general market participants (GP), investors (I) and smart investors (SI). General market participant is defined as households with either an IRA stock account or a non-IRA stock account. Investor is defined as households with a non-IRA stock account. Smart Investor is defined as households with both an IRA stock account and a non-IRA stock account.

c $HetC$ is the heterogeneous preferences model with consumption weights under full market participation. $HetW$ is the heterogeneous preferences model with wealth weights under full market participation. $HetC/GP$ is the heterogeneous preferences model with consumption weights with general market participants only. $HetW/GP$ is the heterogeneous preferences model with wealth weights with general market participants only. $HetC/I$ is the heterogeneous preferences model with consumption weights with investors only. $HetW/I$ is the heterogeneous preferences model with wealth weights with investors only. $HetC/SI$ is the heterogeneous preferences model with consumption weights with smart investors only. $HetW/SI$ is the heterogeneous preferences model with wealth weights with smart investors only.

d $(*)$ is $p$-value < 0.1, $(**)$ is $p$-value < 0.05

The improvement in $R^2$ comes at the cost of a slightly worse performance in magnitude, with relative performance between using Consumption Weights and Wealth Weights holds. A clear trade-off between capturing dynamics and capturing magnitude arise. The optimal weights is therefore an open question for future study.

8 Cross Section of Stock Returns

The next question we try to answer is whether our Heterogeneous Preferences models outperform Representative Agent model in terms of explanatory power on the cross section of stock returns.
stock returns.

Our test assets are Fama-French 25 portfolios sorted by size and book-to-market ratio, developed by Fama & French (1992). Because the number of our risk factors for individual consumption and risk factors are too big (1,384 × 2 = 2,768), we cannot use standard Fama & MacBeth (1973) method. Instead, we calculate individual consumption and wealth risk factor loadings for each test asset, and use time-varying heterogeneous utility parameters to calculate implied excess return for that test asset. Then we run OLS regression on realized excess returns of test asset against the implied excess returns. Given the panel structure of 25 assets and 9 periods, we run three types of regressions. The first one is Pooled-OLS, which utilizes 25 × 9 = 225 data points. The second type of regression is time series regression for all 25 assets. The last type of regression is 9 cross sectional regressions. For the second and the third type, we record average coefficient, average p-value and average $R^2$. We also calculate the $t$-statistics against the null hypothesis that all coefficients are zero. Results are recorded in Table 10

Results we see in time series of risk premium generally hold in cross sectional analysis of stock returns. Firstly, Heterogeneous Preferences models generally have smaller coefficients than Representative Agent model, except for the average coefficient of cross sectional regressions for HetC. Heterogeneous Preferences models do generate closer to actual excess returns than homogeneous preference counterparts. Secondly, Heterogeneous Preferences models capture both more excess return dynamics and cross sectional variations than Representative Agent model, with HetC being the best performer. $R^2$ increases from [12.7%, 25.0%] for Rep to [39.5%, 52.9%] for HetC. The results for HetC and HetW reported in Table 10 does not account for market participation heterogeneity, as the purpose of this analysis is to compare the relative performances of heterogeneous preferences models and representative agent model, with the same market participation assumption and the same data. If we focus on market participants only, results are better for HetC and HetW.
Table 10: Cross Section of Stock Returns

<table>
<thead>
<tr>
<th>Pooled OLS</th>
<th>Rep</th>
<th>Hom</th>
<th>HetE</th>
<th>HetC</th>
<th>HetW</th>
</tr>
</thead>
<tbody>
<tr>
<td>coef</td>
<td>74.343***</td>
<td>74.978***</td>
<td>47.031***</td>
<td>63.369***</td>
<td>23.883***</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.127</td>
<td>0.2261</td>
<td>0.407</td>
<td>0.456</td>
<td>0.319</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time Series</th>
<th>Rep</th>
<th>Hom</th>
<th>HetE</th>
<th>HetC</th>
<th>HetW</th>
</tr>
</thead>
<tbody>
<tr>
<td>avg.coef</td>
<td>178.980***</td>
<td>242.277***</td>
<td>52.803***</td>
<td>84.328***</td>
<td>29.006***</td>
</tr>
<tr>
<td>avg.p-value</td>
<td>0.220</td>
<td>0.073</td>
<td>0.076</td>
<td>0.037</td>
<td>0.121</td>
</tr>
<tr>
<td>avg.$R^2$</td>
<td>0.250</td>
<td>0.417</td>
<td>0.458</td>
<td>0.529</td>
<td>0.414</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cross Section</th>
<th>Rep</th>
<th>Hom</th>
<th>HetE</th>
<th>HetC</th>
<th>HetW</th>
</tr>
</thead>
<tbody>
<tr>
<td>avg.coef</td>
<td>24.919*</td>
<td>28.801*</td>
<td>16.107*</td>
<td>33.897***</td>
<td>8.641</td>
</tr>
<tr>
<td>t-stat</td>
<td>(1.529)</td>
<td>(1.553)</td>
<td>(1.652)</td>
<td>(2.530)</td>
<td>(1.244)</td>
</tr>
<tr>
<td>avg.p-value</td>
<td>0.113</td>
<td>0.073</td>
<td>0.066</td>
<td>0.008</td>
<td>0.151</td>
</tr>
<tr>
<td>avg.$R^2$</td>
<td>0.224</td>
<td>0.304</td>
<td>0.284</td>
<td>0.395</td>
<td>0.247</td>
</tr>
</tbody>
</table>

*a* For Pooled OLS, we run one regression for each model and record corresponding results, using following regression equation,

$$Actual_i^t = \alpha + \beta \times Implied_i^t + \varepsilon_i^t,$$

where $Implied_i^t$ are implied excess return for asset $i$ at time $t$ generated by models, and $Actual_i^t$ are actual excess return for asset $i$ at time $t$. We use 25 Fama-French size-value sorted assets.

*b* For Times Series, we run 25 regressions for each model, one regression for each asset, using following regression equation,

$$Actual_i^t = \alpha + \beta^t \times Implied_i^t + \varepsilon_i^t,$$

We record average of $\beta^t$, average $p$-value and average $R^2$ for those 25 regression. $t$-stat is the $t$-statistics of the t-test against the null that average of $\beta^t$ equals to zero.

*c* For Cross Section, we run 9 regressions for each model, one regression for each period, using following regression equation,

$$Actual_i^t = \alpha + \beta_t \times Implied_i^t + \varepsilon_i^t,$$

We record average of $\beta_t$, average $p$-value and average $R^2$ for those 25 regression. $t$-stat is the $t$-statistics of the t-test against the null that average of $\beta_t$ equals to zero.

*d* *Rep* is the representative agent model, using homogeneous preference and aggregate data. *Hom* is the homogeneous preference model with households level data. *HetE* is the heterogeneous preferences model with equal weights. *HetC* is the heterogeneous preferences model with consumption weights. *HetW* is the heterogeneous preferences model with wealth weights.

*e* (**) is $p$-value $< 0.1$, (**) is $p$-value $< 0.05$, (***) is $p$-value $< 0.01
9 Conclusion

We adopt Epstein-Zin-Weil utility under a heterogeneous preferences environment. We show individual level data is needed to price assets if some strict assumptions for aggregation do not hold. Our heterogeneous preferences model improves representative agent model at three fronts, 1) idiosyncratic risk factors, 2) heterogeneous factor premia and 3) idiosyncratic characteristics dependent aggregation weights.

We group households into four groups by wealth and assume households in the same group have the same preference parameters. Then we use PSID consumption and wealth data to estimate time-varying preference parameters for those four types of households, as well as a representative agent. Because of the large cross sectional heterogeneity in consumption growth rates and returns on wealth, estimation generates relatively large Elasticity of Intertemporal Substitution and small Relative Risk Aversion coefficients. One aspect of equity premium puzzle is that GMM estimation generates unrealistically large relative risk aversion parameter because the aggregate consumption is too smooth. Our result suggests that the large cross sectional difference within households may help bring the size of risk aversion parameter down.

With estimated parameters, we calculate models’ implied risk premium. We look at five models, representative agent, homogeneous preference with individual level data, heterogeneous preferences with equal weights, consumption weights and wealth weights. We find heterogeneous preferences models generally outperform representative agent model by both generating quantitatively closer to actual risk premium and capturing more risk premium dynamics. Using consumption weights, heterogeneous preferences model improves $R^2$ from representative agent model by 61.77%. Out of this 61.77% improvement, idiosyncratic risk factors contribute 9.42%, heterogeneous factor premia contribute 28.94% and idiosyncratic aggregation weights contribute 23.41%.

Heterogeneous preferences models also outperform representative agent model in explaining cross section of stock returns. Using Fama-French 25 size-value sorted portfolios, we show that heterogeneous preferences models generally generates quantitatively closer to actual excess returns and captures more excess returns time series dynamics and cross sectional
Lastly, our heterogeneous preferences models have the advantage over representative agent model in the ability to accommodate heterogeneous market participation. Including market participants only, we further achieve a 23.24% improvement in capturing risk premium dynamics, which brings total improvement from representative agent model to 85.01%.

References


Appendix

A.1 Proof of Propositions

A.1.1 Proof of Proposition 1

Proof. Under $\rho^j = \alpha^j, \forall j$,

$$\ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f = \frac{1}{\psi^j} \text{Cov}_t[g_{t+1}^j, r_{t+1}^i]$$

Rewrite

$$\ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f = \frac{1}{\psi^j} \text{Cov}_t \left[ \ln \left( \frac{c_{t+1}^j}{c_t^j} \right), r_{t+1}^i \right]$$

By Stein’s Lemma,

$$\ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f = \frac{1}{\psi^j} \text{Cov}_t \left[ \left( \frac{c_{t+1}^j}{c_t^j} \right), r_{t+1}^i \right] \mathbb{E}_t \left[ \frac{c_t^j}{c_{t+1}^j} \right] = \frac{1}{\psi^j} \mathbb{E}_t \left[ \frac{1}{c_{t+1}^j} \right] \text{Cov}_t[c_{t+1}^j, r_{t+1}^i]$$
Multiply \( \psi_j / \mathbb{E}_t \left[ \frac{1}{c_{t+1}^j} \right] \) on both sides, we have

\[
\left( \ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f \right) \psi_j / \mathbb{E}_t \left[ \frac{1}{c_{t+1}^j} \right] = \text{Cov}_t[c_{t+1}^j, r_{t+1}^i]
\]

Sum over \( j \),

\[
\left( \ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f \right) \sum_{j=1}^J \left( \psi_j / \mathbb{E}_t \left[ \frac{1}{c_{t+1}^j} \right] \right) = \sum_{j=1}^J \text{Cov}_t[c_{t+1}^j, r_{t+1}^i] = \text{Cov}_t \left[ \sum_{j=1}^J c_{t+1}^j, r_{t+1}^i \right] = \text{Cov}_t[C_{t+1}, r_{t+1}^i]
\]

Rearrange

\[
\ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f = \frac{1}{\sum_{j=1}^J \left( \psi_j / \mathbb{E}_t \left[ \frac{1}{c_{t+1}^j} \right] \right)} \cdot \text{Cov}_t[C_{t+1}, r_{t+1}^i] = \frac{C_t}{\sum_{j=1}^J \left( \psi_j / \mathbb{E}_t \left[ \frac{1}{c_{t+1}^j} \right] \right)} \cdot \text{Cov}_t \left[ \frac{C_{t+1}}{C_t}, r_{t+1}^i \right]
\]

With Stein’s Lemma again,

\[
\ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f = \frac{C_t}{\sum_{j=1}^J \left( \psi_j / \mathbb{E}_t \left[ \frac{1}{c_{t+1}^j} \right] \right)} \cdot \text{Cov}_t \left[ \frac{C_{t+1}}{C_t}, r_{t+1}^i \right]
\]

A.1.2 Proof of Proposition 2

\textbf{Proof.} Under } \alpha^j = 1, \forall j, \ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f = \text{Cov}_t[h_{t+1}^i, r_{t+1}^i]
Rewrite
\[
\ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f = \text{Cov}_t \left[ \ln \left( \frac{w_{t+1}^j}{w_t^j - c_t^j} \right), r_{t+1}^i \right]
\]

By Stein’s Lemma,
\[
\ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f = \frac{w_t^j - c_t^j}{\mathbb{E}_t[w_{t+1}^j]} \text{Cov}_t \left[ \left( \frac{w_{t+1}^j}{w_t^j - c_t^j} \right), r_{t+1}^i \right] = \mathbb{E}_t \left[ \frac{1}{w_{t+1}^j} \right] \text{Cov}_t[w_{t+1}^j, r_{t+1}^i]
\]

Multiply \(1/\mathbb{E}_t[1/w_{t+1}^j]\) on both sides, we have
\[
\left( \ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f \right) \frac{1}{\mathbb{E}_t[1/w_{t+1}^j]} = \text{Cov}_t[w_{t+1}^j, r_{t+1}^i]
\]

Sum over \(j\),
\[
\left( \ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f \right) \sum_{j=1}^J \frac{1}{\mathbb{E}_t[1/w_{t+1}^j]} = \sum_{j=1}^J \text{Cov}_t[w_{t+1}^j, r_{t+1}^i] = \text{Cov}_t \left[ \sum_{j=1}^J w_{t+1}^j, r_{t+1}^i \right] = \text{Cov}_t[W_{t+1}, r_{t+1}^i]
\]

Rearrange
\[
\ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f = \frac{1}{\sum_{j=1}^J 1/\mathbb{E}_t[1/w_{t+1}^j]} \text{Cov}_t[W_{t+1}, r_{t+1}^i]
\]
\[
= \frac{\mathbb{E}_t[W_{t+1}] W_t - C_t}{\sum_{j=1}^J 1/\mathbb{E}_t[1/w_{t+1}^j] \mathbb{E}_t[W_{t+1}]} \text{Cov}_t \left[ \frac{W_{t+1}}{W_t - C_t}, r_{t+1}^i \right]
\]

With Stein’s Lemma again,
\[
\ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f = \frac{\mathbb{E}_t[W_{t+1}]}{\sum_{j=1}^J 1/\mathbb{E}_t[1/w_{t+1}^j]} \text{Cov}_t \left[ \ln \left( \frac{W_{t+1}}{W_t - C_t} \right), r_{t+1}^i \right]
\]

\[\square\]
A.1.3 Proof of Proposition 3

Proof. Under \( g_{t+1}^j = g_t, \forall j \),

\[
\ln \mathbb{E}_t[\mathbb{R}_{T+1}^i] - r_{T+1} = \frac{\theta^j}{\psi^j} \text{Cov}_t[g_{t+1}, r_{t+1}^i] + (1 - \theta^j) \text{Cov}_t[h_{t+1}, r_{t+1}^i]
\]

By Stein’s Lemma,

\[
\ln \mathbb{E}_t[\mathbb{R}_{T+1}^i] - r_{T+1} = \frac{\theta^j}{\psi^j} \text{Cov}_t[g_{t+1}, r_{t+1}^i] + (1 - \theta^j) \text{Cov}_t \left[ \ln \left( \frac{w_{t+1}^j}{w_t^j - c_t^j} \right), r_{t+1}^i \right]
\]

Rearrange we have

\[
\left( \ln \mathbb{E}_t[\mathbb{R}_{T+1}^i] - r_{T+1} \right) \frac{1}{(1 - \theta^j)\mathbb{E}_t[1/w_{t+1}^j]} = \text{Cov}_t[g_{t+1}, r_{t+1}^i] \frac{\theta^j}{\psi^j (1 - \theta^j)\mathbb{E}_t[1/w_{t+1}^j]} = \text{Cov}_t[w_{t+1}^j, r_{t+1}^i]
\]

Sum over \( j \),

\[
\left( \ln \mathbb{E}_t[\mathbb{R}_{T+1}^i] - r_{T+1} \right) \sum_{j=1}^J \frac{1}{(1 - \theta^j)\mathbb{E}_t[1/w_{t+1}^j]} = \text{Cov}_t[g_{t+1}, r_{t+1}^i] \sum_{j=1}^J \frac{\theta^j}{\psi^j (1 - \theta^j)\mathbb{E}_t[1/w_{t+1}^j]}
\]

\[
= \sum_{j=1}^J \text{Cov}_t[w_{t+1}^j, r_{t+1}^i] = \text{Cov}_t[W_{T+1}, r_{T+1}^i]
\]

Rearrange

\[
\ln \mathbb{E}_t[\mathbb{R}_{T+1}^i] - r_{T+1} = \sum_{j=1}^J \frac{\theta^j}{\psi^j (1 - \theta^j)\mathbb{E}_t[1/w_{t+1}^j]} \text{Cov}_t[g_{t+1}, r_{t+1}^i] + \sum_{j=1}^J \frac{1}{(1 - \theta^j)\mathbb{E}_t[1/w_{t+1}^j]} \text{Cov}_t[W_{T+1}, r_{t+1}^i]
\]

\[
= \sum_{j=1}^J \frac{\theta^j}{\psi^j (1 - \theta^j)\mathbb{E}_t[1/w_{t+1}^j]} \text{Cov}_t[g_{t+1}, r_{t+1}^i]
\]

\[
+ \sum_{j=1}^J \frac{\mathbb{E}_t[W_{T+1}]}{(1 - \theta^j)\mathbb{E}_t[1/w_{t+1}^j]} \frac{W_t - C_t}{\mathbb{E}_t[W_{T+1}]} \text{Cov}_t \left[ W_{T+1}, r_{t+1}^i \right]
\]

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By Stein’s Lemma again,

\[
\ln \mathbb{E}_t[R^i_{t+1}] - r^f_{t+1} = \sum_{j=1}^{J} \frac{\theta^j}{\psi^j(1-\theta^j)\mathbb{E}_t[1/w^j_{t+1}]} \text{Cov}_t[g^j_{t+1}, r^i_{t+1}] + \sum_{j=1}^{J} \frac{1}{(1-\theta^j)\mathbb{E}_t[1/w^j_{t+1}]} \text{Cov}_t \left[ \ln \left( \frac{W_{t+1}}{W_t - C_t} \right), r^i_{t+1} \right] + \sum_{j=1}^{J} \frac{\mathbb{E}_t[W_{t+1}]}{(1-\theta^j)\mathbb{E}_t[1/w^j_{t+1}]} \text{Cov}_t \left[ g^j_{t+1}, r^i_{t+1} \right] + \sum_{j=1}^{J} \frac{1}{(1-\theta^j)\mathbb{E}_t[1/w^j_{t+1}]} \text{Cov}_t \left[ h^j_{t+1}, r^i_{t+1} \right]
\]

A.1.4 Proof of Proposition 4

Proof. Under \( h^j_{t+1} = h_{t+1}, \forall j \),

\[
\ln \mathbb{E}_t[R^i_{t+1}] - r^f_{t+1} = \frac{\theta^j}{\psi^j} \text{Cov}_t[g^j_{t+1}, r^i_{t+1}] + (1 - \theta^j) \text{Cov}_t[h_{t+1}, r^i_{t+1}]
\]

By Stein’s Lemma,

\[
\ln \mathbb{E}_t[R^i_{t+1}] - r^f_{t+1} = \frac{\theta^j}{\psi^j} \text{Cov}_t \left[ \ln \left( \frac{c^j_{t+1}}{c^j_{t}} \right), r^i_{t+1} \right] + (1 - \theta^j) \text{Cov}_t[h_{t+1}, r^i_{t+1}]
\]

Rearrange we have

\[
\left( \ln \mathbb{E}_t[R^i_{t+1}] - r^f_{t+1} \right) \frac{\psi^j}{\theta^j \mathbb{E}_t[1/c^j_{t+1}]} - \text{Cov}_t[h_{t+1}, r^i_{t+1}] \frac{\psi^j(1 - \theta^j)}{\theta^j \mathbb{E}_t[1/c^j_{t+1}]} = \text{Cov}_t[c^j_{t+1}, r^i_{t+1}]
\]

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Sum over \( j \),

\[
\left( \ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f \right) \sum_{j=1}^J \frac{\psi^j}{\theta^j \mathbb{E}_t[1/c_{t+1}^j]} - \text{Cov}_t[h_{t+1}, r_{t+1}^i] \sum_{j=1}^J \frac{\psi^j(1 - \theta^j)}{\theta^j \mathbb{E}_t[1/c_{t+1}^j]} = \sum_{j=1}^J \text{Cov}_t[c_{t+1}^j, r_{t+1}^i] = \text{Cov}_t[C_{t+1}, r_{t+1}^i]
\]

Rearrange

\[
\ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f = \frac{1}{\sum_{j=1}^J \frac{\psi^j}{\theta^j \mathbb{E}_t[1/c_{t+1}^j]}} \text{Cov}_t[C_{t+1}, r_{t+1}^i] + \frac{\sum_{j=1}^J \frac{\psi^j(1 - \theta^j)}{\theta^j \mathbb{E}_t[1/c_{t+1}^j]}}{\sum_{j=1}^J \frac{\psi^j}{\theta^j \mathbb{E}_t[1/c_{t+1}^j]}} \text{Cov}_t[h_{t+1}, r_{t+1}^i]
\]

\[
= \frac{\mathbb{E}_t[C_{t+1}]}{\sum_{j=1}^J \frac{\psi^j}{\theta^j \mathbb{E}_t[1/c_{t+1}^j]}} \text{Cov}_t \left[ \frac{C_{t+1}}{C_t}, r_{t+1}^i \right] + \frac{\sum_{j=1}^J \frac{\psi^j(1 - \theta^j)}{\theta^j \mathbb{E}_t[1/c_{t+1}^j]}}{\sum_{j=1}^J \frac{\psi^j}{\theta^j \mathbb{E}_t[1/c_{t+1}^j]}} \text{Cov}_t[h_{t+1}, r_{t+1}^i]
\]

By Stein’s Lemma again,

\[
\ln \mathbb{E}_t[R_{t+1}^i] - r_{t+1}^f = \frac{\mathbb{E}_t[C_{t+1}]}{\sum_{j=1}^J \frac{\psi^j}{\theta^j \mathbb{E}_t[1/c_{t+1}^j]}} \text{Cov}_t \left[ \frac{C_{t+1}}{C_t}, r_{t+1}^i \right] + \frac{\sum_{j=1}^J \frac{\psi^j(1 - \theta^j)}{\theta^j \mathbb{E}_t[1/c_{t+1}^j]}}{\sum_{j=1}^J \frac{\psi^j}{\theta^j \mathbb{E}_t[1/c_{t+1}^j]}} \text{Cov}_t[h_{t+1}, r_{t+1}^i]
\]

\[
= \frac{\mathbb{E}_t[C_{t+1}]}{\sum_{j=1}^J \frac{\psi^j}{\theta^j \mathbb{E}_t[1/c_{t+1}^j]}} \text{Cov}_t[g_{t+1}, r_{t+1}^i] + \frac{\sum_{j=1}^J \frac{\psi^j(1 - \theta^j)}{\theta^j \mathbb{E}_t[1/c_{t+1}^j]}}{\sum_{j=1}^J \frac{\psi^j}{\theta^j \mathbb{E}_t[1/c_{t+1}^j]}} \text{Cov}_t[h_{t+1}, r_{t+1}^i]
\]

\[
\square
\]

**A.2 Ambiguity Aversion**

Bansal & Yaron (2004) utilize Epstein-Zin-Weil utility and propose a long run risk model that better explains some quantitative characteristics of equity market. This is a significant advancement in solving equity premium puzzle. However, Epstein et al. (2014) find a new quantitative issue arise with the model. Epstein-Zin-Weil utility separates Elasticity of Inter-temporal Substitution (EIS) and Relative Risk Aversion (RRA) parameters, the relative size of those parameters implies agent’s attitude towards ambiguity aversion, or in other words, agent’s preference regarding early resolution of risk. More specifically, Epstein et al. (2014) state that if an agent’s \( EIS \times RRA > 1 \), he will prefer early resolution of risk; if
$EIS \times RRA < 1$, he will prefer late resolution of risk; if $EIS \times RRA = 1$, under which the Epstein-Zin-Weil utility is reduced to power utility, he has no preference towards the timing of risk resolution.

We also check our heterogeneous agents’ preferences on early resolution of risk. Following Table 11 records size of $EIS \times RRA$ for different agents.

Table 11: Early Resolution of Risk

<table>
<thead>
<tr>
<th>$EIS \times RRA$</th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>avg.</th>
<th>Rep</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>1.25</td>
<td>1.07</td>
<td>0.96</td>
<td>0.79</td>
<td>1.02</td>
<td>1.04</td>
</tr>
<tr>
<td>T2</td>
<td>1.23</td>
<td>0.94</td>
<td>0.86</td>
<td>0.13</td>
<td>0.79</td>
<td>1.03</td>
</tr>
<tr>
<td>T3</td>
<td>1.89</td>
<td>0.99</td>
<td>0.74</td>
<td>0.81</td>
<td>1.11</td>
<td>1.10</td>
</tr>
<tr>
<td>T4</td>
<td>1.36</td>
<td>1.10</td>
<td>0.94</td>
<td>0.58</td>
<td>1.00</td>
<td>1.06</td>
</tr>
<tr>
<td>T5</td>
<td>1.21</td>
<td>0.38</td>
<td>-0.21</td>
<td>-4.50</td>
<td>-0.78</td>
<td>0.68</td>
</tr>
<tr>
<td>avg.</td>
<td>1.39</td>
<td>0.90</td>
<td>0.66</td>
<td>-0.44</td>
<td>0.63</td>
<td>0.79</td>
</tr>
</tbody>
</table>

\( a \) Numbers reported in this table are estimated Elasticity of Inter-temporal Substitution (EIS) times Relative Risk Aversion (RRA) parameters ($EIS \times RRA$) for both heterogeneous preferences model and representative agent model.

\( b \) $T_1, \ldots, T_5$ are five bennial transition periods from 1999 to 2009. $Q_1, \ldots, Q_4$ are four wealth quartiles, $Q_1$ being the poorest and $Q_4$ being the richest. Rep is representative agent.

\( c \) Averages across different wealth groups for each period are recorded in column 6 and averages across different periods for each wealth groups are recorded in row 7.

Like preferences parameters, we want to check whether agents do differ with each other. To achieve this, we run following regression and record results in Table 12.

\[
EIS_i^q \times RRA_i^q = \beta_0 + \beta_T \times t + \beta_Q \times q + \varepsilon_i^q.
\]

As we can see, our estimated households do differ in terms of attitude towards early or late resolution of risk. Poor people prefer early resolution while rich people prefer late resolution. It is another proof of the existence of heterogeneous preferences and different households with different wealth do have different Preferences.
Table 12: Early Resolution of Risk

<table>
<thead>
<tr>
<th></th>
<th>$EIS \times RRA$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_T$</td>
<td>$-0.339^*$</td>
</tr>
<tr>
<td>$t$-stat</td>
<td>(-2.042)</td>
</tr>
<tr>
<td>$\beta_Q$</td>
<td>$-0.572^{**}$</td>
</tr>
<tr>
<td>$t$-stat</td>
<td>(-2.722)</td>
</tr>
</tbody>
</table>

Numbers reported in this table are coefficients and corresponding $t$-statistics for the following regression,

$$\text{Parameters}_q^t = \beta_0 + \beta_T \times t + \beta_Q \times q + \epsilon_q^t,$$

where $t = 1, \cdots, 5$ and $q = 1, \cdots, 4$ are indices for periods and wealth groups.

$(^*)$ is $p$-value $< 0.1$, $(^{**})$ is $p$-value $< 0.05$, $(^{***})$ is $p$-value $< 0.01$

A.3 Pricing Kernel Performance without Tech Bubble

A question we are addressing here is whether the relatively worse performances of representative agent model is solely because of its failure to capture the tech bubble. We then exclude 2000-2002 tech bubble and look at the rest of the sample. Results recorded in following Table 13.

Overall, heterogeneous preferences models still outperform representative agent model in terms of both capturing market excess return magnitude and dynamics. It signals the importance of heterogeneous preferences models in asset pricing. Heterogeneous preferences models not only be able to capture some crisis which representative agent model is unable to capture, they can also provide more information about asset prices in periods when representative agent model can actually work well.

We further look into details, by excluding tech bubble, we increase $R^2$ for all five models. It is not surprising for Rep and Hom to get an increase as they both fail to capture the tech bubble, as a result, $R^2$’s for those two models increase to almost 70%. It is surprising though that even three heterogeneous preferences models achieve an increase in $R^2$, to more than 80% for HetE and HetC and more than 90% for HetW. It suggests that including tech bubble actually worsens the performance of heterogeneous preferences models. Considering
Table 13: Market Risk Premium 2003-2008

<table>
<thead>
<tr>
<th></th>
<th>Rep/6Y</th>
<th>Hom/6Y</th>
<th>HetE/6Y</th>
<th>HetC/6Y</th>
<th>HetW/6Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>β</td>
<td>168.953**</td>
<td>327.612**</td>
<td>54.575**</td>
<td>83.174**</td>
<td>37.010***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(2.928)</td>
<td>(2.866)</td>
<td>(4.517)</td>
<td>(4.366)</td>
<td>(6.325)</td>
</tr>
<tr>
<td>R²</td>
<td>0.6818</td>
<td>0.6755</td>
<td>0.8361</td>
<td>0.8265</td>
<td>0.9091</td>
</tr>
</tbody>
</table>

a Numbers reported in this table are regression coefficients and their corresponding t-statistics, as well as R² for five models in study using following regression equation,

$$\text{Actual}_t = \alpha + \beta \times \text{Implied}_t + \varepsilon_t,$$

where Implied$_t$ are implied risk premium at time $t$ generated by models, and Actual$_t$ are actual risk premium at time $t$. $t$ is from 2003 to 2008.

b Rep/6Y is the representative agent model, using homogeneous preference and aggregate data. Hom/6Y is the homogeneous preference model with households level data. HetE/6Y is the heterogeneous preferences model with equal weights. HetC/6Y is the heterogeneous preferences model with consumption weights. HetW/6Y is the heterogeneous preferences model with wealth weights.

c (*) is $p$-value $< 0.1$, (**) is $p$-value $< 0.05$, (***) is $p$-value $< 0.01$.

the relative performance between representative agent model and heterogeneous preferences models, it is reasonable to believe our level of heterogeneity of 4 wealth groups does not fully capture information about the tech bubble.