Abstract

This paper analyzes the effects of policy rates on financial intermediaries’ risk taking decisions. We consider an economy in which (i) intermediaries monitor borrowers which lowers their probability of default, (ii) monitoring is not observable which creates a moral hazard problem, and (iii) intermediaries have market power in granting loans. We show that lower policy rates lead to lower intermediation margins and higher risk-taking when intermediaries have low market power, but the result reverses for high market power. We also show that when intermediaries have high market power competition from (nonmonitoring) financial markets results in a U-shaped relationship between policy rates and risk-taking. The paper examines the robustness of these results to introducing heterogeneity in monitoring costs, entry and exit of intermediaries, and funding with deposits and capital.

*JEL Classification:* G21, L13, E52

*Keywords:* Market power, Cournot competition, bank monitoring, credit spreads, risk-taking, monetary policy.

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1 Introduction

Lax monetary conditions leading to low levels of real interest rates have been identified as a key factor originating financial crisis. One common argument on the recent financial crisis is that low monetary policy rates observed before 2007 were a main driver of the subsequent financial collapse. This paper analyzes, from a theoretical perspective, how policy rates can affect the risk-taking decisions of financial institutions. The main objective is to highlight the relevance of the market structure of the financial sector in shaping such relationship.

We show how the effect of policy rates on risk-taking decisions of financial institutions vary depending on the degree of market power of those institutions. In highly competitive financial markets lower rates result in higher risk-taking by financial institutions, while in highly concentrated financial markets lower rates result in lower risk-taking. This result obtains because, although lower policy rates result in lower funding costs for financial institutions, the intensity of the pass-through of lower financing rates to loan rates depends on the market structure. Hence, lower policy rates can lead to lower or higher intermediation margins which in turn determine higher or lower risk-taking incentives for financial intermediaries. We argue that in highly concentrated (competitive) markets lower funding rates result in higher (lower) intermediation margins which reduce (increase) the risk-taking incentives of financial institutions. Therefore, we conclude that, although lower policy rates result in lower loan rates and higher credit supply, the riskiness of such credit can vary depending on the underlying market structure of the financial sector.\footnote{From an empirical perspective, papers like Jimenez et al. (2014) and Iannadou et al (2015) have shown how monetary policy affects both the risk-taking decisions of banks and loan rates, which in turn affect intermediation margins.}

We model a one period risk neutral economy in which a fixed number of financial institutions raise uninsured funding from investors and compete à la Cournot in providing loans to risky penniless entrepreneurs. Financial institutions privately decide the monitoring intensity on their loans, where higher monitoring results in lower probability of loan default. Crucially, we assume that the monitoring decision is unobservable, which creates a standard moral hazard problem between the financial institution and its financiers. The expected
return that deep pocket investors require for their funds is assumed to be equal to an exogenous policy rate (the safe rate), which is our measure of the stance monetary policy. We show how as the policy rate decreases (lax monetary policy) financial institutions increase their loan supply and reduce equilibrium loan rates. However, the intensity of the reduction in the loan rates (pass-through), which in turn determines the equilibrium intermediation margin, depends on their market power. Thus, monitoring decisions are linked to the intermediation margin of financial institutions. In particular, in fairly competitive markets higher safe rates translate into lower bank risk-taking, while in monopolistic markets the relationship reverses sign, that is higher safe rates translate into higher bank risk-taking. In line with the traditional literature linking bank concentration and financial stability, we also show that higher competition results in higher risk-taking for any level of the policy rate.

After stating our main results linking interest rates, market structure, and financial stability, we analyze three relevant aspects of financial markets: (i) the possibility of non-intermediated funding, (ii) asymmetries among financial institutions, and (iii) entry and exit of financial institutions. First, we consider a situation in which entrepreneurs also have the option of accessing direct market finance, assumed to be perfectly competitive and without any monitoring. We show how the equilibrium interest rate that financial institutions can charge is affected by the funding rate that entrepreneurs can obtain in the market. Hence, direct market finance imposes a constraint on the equilibrium loan rates which is more prone to bind in highly concentrated banking markets and when policy rates are low. We show that, when entrepreneurs have the option to access such funding, highly concentrated financial markets exhibit a U-shaped relationship between the policy rate and bank risk-taking. For low (high) levels of the policy rate decreasing such rate increases (decreases) the probability of loan default. In highly competitive banking markets the results of the basic setup hold as direct market funding is not a competitive threat and therefore does not affect the Cournot equilibrium outcome.

We next analyze a situation in which financial institutions differ in their monitoring abilities. We assume that there are two types of institutions: those with high cost of monitoring entrepreneurs and those with low cost of monitoring. We show how, in equilibrium,
financial institutions with higher monitoring costs have lower market shares and their loans have higher probabilities of default. We characterize a situation in which lower policy rates decrease (increase) the market share of those institutions with lower (higher) cost of monitoring and increase (decrease) the probability of loan failure. Hence, we conclude that, in the presence of heterogenous monitoring costs, lower policy rates can have different impact in the probability of loan default of different institutions. By increasing the market share of those institutions with higher cost of monitoring (which grant riskier loans in equilibrium) lower policy rates also affect the equilibrium structure and risk of the financial sector.

We conclude our analysis of financial market structure by taking into account entry and exit decisions in the financial system. We view these entry decisions as a longer run phenomenon than the decisions to grant and monitor loans. Hence, we see this analysis as shedding light in the potential long-term effects of policy rates. We model entry decisions by assuming that financial intermediaries have to pay an ex ante fixed cost to operate. We show that allowing for entry results in higher competition in the financial market, which adds an “entry effect” to our basic results, which increases the probability of loan default. We also show how the results depend on the nature of the cost of entry. In particular, when this cost is increasing in the pre-existing number of intermediaries (in line for example with the literature of entry congestion) then in concentrated markets lower rates decrease the probability of loan default.

Our main setup analyzes a situation in which we ignore (inside) capital of financial intermediaries—outside capital plays essentially the same role as uninsured deposits. As Dell’Ariccia et al. (2014) point out, a relevant determinant of banks’ risk-taking decisions is its capital structure which can be affected by policy rates. Contrary to their results, we find that when the leverage ratio of financial institutions is endogenously determined, market structure is still a relevant variable in shaping how policy rates affect bank risk-taking. Our results differ from those of Dell’Ariccia et al. (2014) because, while they assume an infinitely elastic supply of (inside) equity at a constant mark up above the policy rate, we assume that (inside) equity is increasingly costly to raise. We obtain that for concentrated markets lower policy rates increase leverage (as in their paper) but at the same time decrease (instead of
increase) the probability of loan default.

Overall this paper presents a setup that allows to rationalize how the effect of changes in policy rates on the risk-taking incentives failure of financial intermediaries depend on the market power of those intermediaries. For highly competitive market structures, lower rates result in higher risk-taking, while the result is reversed for concentrated market structures.

**Literature review**  TBC

**Structure of the paper**  Section 2 presents the basic model of Cournot competition in the loan market with uninsured deposits and unobservable monitoring by banks, and analyzes how market power affects the relationship between the safe rate (proxying the stance of monetary policy) and banks’ equilibrium monitoring intensity, which determines the probability of default of their loans. Section 3 examines the robustness of our benchmark results when we incorporate three relevant aspects of competition in the loan market, namely the presence of competitive market lenders that do not monitor borrowers, banks’ heterogeneity in monitoring costs, and banks’ entry (and exit). Section 4 examines the robustness of our benchmark results when we introduce endogenous deposit rates and the possibility of banks raising equity capital. Section 5 contains our concluding remarks. Proofs of the analytical results are in the Appendix.

## 2 The Model

Consider an economy with two dates \( t = 0, 1 \) populated by three types of risk-neutral agents: a continuum of deep pocket investors, a continuum of penniless entrepreneurs, and \( n \) identical financial institutions which we refer to as banks. Investors are characterized by an infinitely elastic supply of funds at an expected return equal to \( R_0 \) (the safe rate). Entrepreneurs have projects that require a unit investment at \( t = 0 \) and yield a stochastic return at \( t = 1 \) given by

\[
\tilde{R} = \begin{cases} 
R, & \text{with probability } 1 - p + m, \\
0, & \text{with probability } p - m,
\end{cases}
\] (1)
where \( p \in (0, 1) \) is the probability of failure in the absence of monitoring, and \( m \in [0, p] \) is the monitoring intensity of the lending bank. While \( p \) is known, \( m \) is not observable, so there is a moral hazard problem. The success return \( R \) is assumed to be a linearly decreasing function of the total amount of loans \( L \), that is

\[
R(L) = a - bL, \tag{2}
\]

where \( a > 0 \) and \( b > 0 \). Competition among entrepreneurs ensures that the success return equals the rate at which they borrow from banks, which means that \( R(L) \) is also the inverse loan demand function. Finally, it is assumed that project returns are perfectly correlated.

Banks compete à la Cournot for loans. Specifically, each bank \( j \) chooses its supply of loans \( l_j \), which determines the total supply of loans \( L = \sum_{j=1}^{n} l_j \) and the loan rate \( R(L) \). Then, banks offer an interest rate \( B(L) \) to the (uninsured) investors,\(^2\) and finally they choose the monitoring intensity \( m(L) \). Monitoring is costly, and the cost function is assumed to take the simple functional form

\[
c(m) = \frac{\gamma}{2} m^2, \tag{3}
\]

where \( \gamma > 0 \).

To characterize the equilibrium of the model we first determine the banks’ borrowing rate \( B(L) \) and monitoring intensity \( m(L) \) as a function of the total supply of loans \( L \). Clearly, the borrowing rate \( B(L) \) has to be smaller than or equal to the loan rate \( R(L) \), for otherwise the banks’ participation constraint would not be satisfied.

The banks’ choice of monitoring is given by

\[
m(L) = \arg \max_{m} \{ (1 - p + m) \left[ R(L) - B(L) \right] - c(m) \}. \tag{4}
\]

The first-order condition that characterizes an interior solution to this problem is

\[
R(L) - B(L) = \gamma m(L). \tag{5}
\]

Thus, the banks’ monitoring intensity \( m(L) \) will be proportional to the intermediation margin \( R(L) - B(L) \).\(^3\)

\(^2\)Since \( R \) is a monotonic function of \( L \), we may write \( B(R) \) instead of \( B(L) \), that is the banks’ borrowing rate as a function of their lending rate.

\(^3\)We implicitly assume that the cost of monitoring is sufficiently high, so that \( m(L) < p \).
The investors’ participation constraint is given by

$$[1 - p + m(L)]B(L) = R_0. \quad (6)$$

Solving for $B(L)$ in the participation constraint (6), substituting it into the first-order condition (5), and rearranging gives the key equation that characterizes the banks’ intensity of monitoring

$$\gamma m(L) + \frac{R_0}{1 - p + m} = R(L). \quad (7)$$

Let us define

$$R = \min_{m \in [0, p]} \left( \gamma m + \frac{R_0}{1 - p + m} \right). \quad (8)$$

The following result shows the condition under which banks will be able to raise the required funds from investors.

**Proposition 1** Banks will be able to fund their lending $L$ if $R(L) \geq R$, in which case the optimal contract between the bank and the investors is given by

$$m(L) = \max \left\{ m \in [0, p] \mid \gamma m + \frac{R_0}{1 - p + m} \leq R(L) \right\} \quad \text{and} \quad B(L) = \frac{R_0}{1 - p + m(L)}. \quad (9)$$

Whenever monitoring is interior one can show that

$$m(L) = \frac{1}{2\gamma} \left[ R(L) - \gamma(1 - p) + \sqrt{(R(L) + \gamma(1 - p))^2 - 4\gamma R_0} \right]. \quad (10)$$

From here it follows that $R'(L) = -b < 0$ implies $m'(L) < 0$. Thus, higher total lending $L$ (which translates into a lower loan rate $R(L)$) implies less incentives to monitor. Also, an increase in the expected return $R_0$ required by investors reduces banks’ monitoring intensity (for a given value of $L$).

Banks’ profits per unit of loans are

$$\pi(L) = [1 - p + m(L)]R(L) - R_0 - c(m(L)), \quad (11)$$

where we have used the fact that $[1 - p + m(L)]B(L) = R_0$. From here it follows that a symmetric Cournot equilibrium $l^*$ is defined by

$$l^* = \arg \max_l [l\pi(l + (n - 1)l^*)]. \quad (12)$$
Assuming that $\pi(L)$ satisfies $\pi'(L) < 0$ and $\pi''(L) < 0$, the symmetric Cournot equilibrium $l^*$ is characterized by the first-order condition

$$L^* \pi'(L^*) + n \pi(L^*) = 0,$$  

(13)

where $L^* = nl^*$. The equilibrium probability of loan default is then given by $PD = p - m(L^*)$. We are interested in analyzing the effect on $PD$ of changes in two parameter values, namely the expected return $R_0$ required by investors, and the number $n$ of banks in the market, which proxies banks’ market power.

The effect of changes in the number of banks $n$ is straightforward. Differentiating the first-order condition (13) gives

$$\frac{dL^*}{dn} = -\frac{\pi(L^*)}{L^* \pi''(L^*) + (n + 1) \pi'(L^*)} > 0,$$  

(14)

where we have used the assumptions $\pi'(L) < 0$ and $\pi''(L) < 0$. But since $m'(L) < 0$, it follows that increasing the number of banks increases equilibrium total lending, which in turn lowers the monitoring intensity of the banks and hence the probability of loan default.

However, as illustrated in Figure 1, the effect of changes in the safe rate $R_0$ depends on the number of banks $n$. The horizontal axis in this figure represents the safe rate $R_0$, and the vertical axis represents the probability of default $PD$. The different lines show the relationship between $PD$ and $R_0$ for different values of $n$. For fairly competitive markets (high $n$), the relationship is negative, that is higher safe rates translate into lower bank risk-taking. This is essentially the same result in Martinez-Miera and Repullo (2017a), who consider the limit case of perfect competition. The novel result obtains for fairly monopolistic markets (low $n$), where the relationship reverses sign, that is higher safe rates translate into higher bank risk-taking.
The intuition for these results is as follows. In monopolistic markets a reduction in the safe rate reduces banks’ funding cost which translates into lower loan rates. However, in monopolistic markets this pass-through from financing costs to loan rates is not very intense and results in higher intermediation margins. This limited pass-through is crucial as banks’ monitoring (and risk-taking) decisions are determined by intermediation margins; see equation (5). In competitive markets the pass-through is more intense and results in lower intermediation margins and lower monitoring. Figure 2 illustrates the effect of changes in the safe rate $R_0$ on equilibrium intermediation margins $R - B$ for different values of the number of banks $n$. 
Figure 2. Effect of the safe rate on intermediation margins

This figure shows the relationship between the safe rate and the equilibrium intermediation margin for loan markets with 1 (bold line), 2, 5, 7, and 10 (light line) banks.

3 Market Structure

This section reviews our previous results on the relationship between interest rates and banks’ risk-taking when we incorporate three relevant aspects of competition in the loan market. First, we consider the effect of the presence of competitive market lenders that do not monitor borrowers but limit the amount of rents that banks can capture. Second, we look at the effect of banks’ heterogeneity in monitoring costs. Finally, we discuss the longer run effects that obtain when we allow for entry (or exit) of banks from the loan market.

3.1 Market Finance

Consider a variation of our model in which entrepreneurs can obtain funding for their projects from banks and also directly from investors. It is assumed that investors are not able to monitor entrepreneurs’s projects (because they may be dispersed and subject to a free rider problem). They are also assumed to be competitive in the sense that they are willing to lend
at a rate $\overline{R}$ that satisfies the participation constraint

$$(1-p)\overline{R} = R_0. \quad (15)$$

The presence of market lenders imposes a constraint on banks’ lending, since the loan rate $R(L)$ cannot exceed the market rate $\overline{R}$. This means that the inverse loan demand function (2) now becomes

$$R(L) = \min\{a - bL, \overline{R}\}. \quad (16)$$

Clearly, the upper bound will be binding whenever the equilibrium in the absence of the bound is such that $R(L^*) > \overline{R}$. In such case the candidate equilibrium lending will be $\overline{L}$ such that $R(L) = \overline{R}$. By our previous results the banks’ borrowing rate and monitoring intensity will be given by $B(\overline{L})$ and $m(\overline{L})$, respectively. The question is: will a bank $j$ want to deviate when the other $n-1$ banks choose $\overline{l} = \overline{L}/n$?

There are two cases to consider. First, note that setting $l_j < \overline{l}$ is not profitable, since given the upper bound in loan rates the profits per unit of loans would not change from $\pi = \pi(\overline{L})$. Second, setting $l_j > \overline{l}$ is not profitable either since $\pi'(L) < 0$ and $\pi''(L) < 0$ imply

$$\overline{l}\pi'(\overline{L}) + \pi(\overline{L}) < \overline{l}\pi'(\overline{l} + (n-1)l^*) + \pi(\overline{l} + (n-1)l^*) < l^*\pi'(L^*) + \pi(L^*) = 0,$$

where the first inequality follows from the fact that $\overline{l} > l^*$ and

$$\overline{l}\pi''(\overline{l} + (n-1)l) + \pi'(\overline{l} + (n-1)l) < 0,$$

the second from the fact that

$$l\pi''(l + (n-1)l^*) + \pi'(l + (n-1)l^*) < 0,$$

and the equality is just the equilibrium condition in the absence of market finance.

Hence, we conclude that whenever the upper bound $\overline{R}$ is binding, the equilibrium amount of loans will be $\overline{L}$. Figure 3 shows the effect of introducing market finance on equilibrium interest rates for different values of the safe rate $R_0$ and the number of banks $n$. The horizontal axis represents the safe rate $R_0$, and the vertical axis represents the equilibrium loan rate.
The different lines show the relationship between $R$ and $R_0$ for different values of $n$. The upper bound is binding for fairly monopolistic markets (low $n$) and for low values of the safe rate $R_0$.

![Figure 3. Effect of the safe rate on loan rates in the presence of market finance](image)

This figure shows the relationship between the safe rate and the equilibrium loan rate for markets with 1 (bold line), 2, 5, 7, and 10 (light line) banks. The dashed line represents the loan rate under direct market finance.

Figure 4 shows the effect of introducing market finance on the equilibrium probability of loan default $PD$ for different values of the safe rate $R_0$ and the number of banks $n$. The horizontal axis represents the safe rate $R_0$, and the vertical axis represents the probability of loan default $PD$. The different lines show the relationship between $PD$ and $R_0$ for different values of $n$. For fairly competitive markets (high $n$), the relationship is still negative, that is higher safe rates translate into lower bank risk-taking. However, in contrast with the result in Section 2, in fairly monopolistic markets (low $n$) the effect is U-shaped: higher safe rates initially reduce banks’ risk-taking, but beyond certain point they increase banks’ risk taking.
Figure 4. Effect of the safe rate on the probability of loan default in the presence of market finance

This figure shows the relationship between the safe rate and the probability of default for loan markets with 1 (bold line), 2, 5, 7, and 10 (light line) banks and competition from market lenders.

3.2 Heterogenous Monitoring Costs

Suppose now that there are two types of banks that differ in the parameter $\gamma$ of their monitoring cost function (3): $n_H$ banks have high monitoring costs, characterized by parameter $\gamma_H$, while $n_L = n - n_H$ banks have low monitoring costs, characterized by parameter $\gamma_L < \gamma_H$. It is assumed that a bank’s type is observable to investors, so they can adjust their lending rate accordingly.

To characterize the equilibrium of the model with heterogeneous banks, note first that the critical values $R_L$ and $R_H$ defined in (8) by setting $\gamma$ equal to $\gamma_L$ and $\gamma_H$, respectively, satisfy $R_L < R_H$, except in the corner case where $R_L = R_H = R_0/(1 - p)$.\(^4\) From here it

\(^4\)This case obtains when $R_0 \leq \gamma_L(1 - p)^2$
follows that whenever the total supply of loans $L$ is such $R_L < R(L) < R_H$, only the low monitoring banks will operate.

By our results in Section 2, if $R(L) \geq R_j$, the monitoring intensity chosen by bank $j = L, H$ is

$$m_j(L) = \frac{1}{2\gamma_j} \left[ R(L) - \gamma_j(1 - p) + \sqrt{[R(L) + \gamma_j(1 - p)]^2 - 4\gamma_j R_0} \right],$$

(17)

and the corresponding borrowing rate is

$$B_j(L) = \frac{R_0}{1 - p + m_j(L)}.$$  

(18)

One can show that $m_L(L) > m_H(L)$, which implies $B_L(L) < B_H(L)$. That is, low cost banks will choose a higher monitoring intensity, and consequently will be able to borrow at lower rates.

Banks’ profits per unit of loans for $j = L, H$ are then

$$\pi_j(L) = [1 - p + m_j(L)]R(L) - R_0 - c_j(m_j(L)).$$

(19)

Clearly, we have $\pi_L(L) > \pi_H(L)$.

A Cournot equilibrium is defined by a pair of strategies $(l^*_L, l^*_H)$ that satisfy

$$l^*_L = \arg \max_l [l\pi_L(l + (n_L - 1)l^*_L + n_H l^*_H)],$$

(20)

$$l^*_H = \arg \max_l [l\pi_H(l + (n_H - 1)l^*_H + n_L l^*_L)].$$

(21)

From here it follows that the Cournot equilibrium will be characterized by the first-order conditions

$$L^*_L \pi'_L(L^*) + n_L \pi_L(L^*) = 0,$$

(22)

$$L^*_H \pi'_H(L^*) + n_H \pi_H(L^*) = 0,$$

(23)


Figure 5 shows the effect of changes in the safe rate $R_0$ on equilibrium lending by low and high cost banks, $L^*_L$ and $L^*_H$, and equilibrium total lending $L^*$. Increases in the safe rate
$R_0$ reduce lending by both types of banks, but the effect is more significant for high cost banks. In particular, the market share of low cost banks, denoted $s = L^*_L/L^*$, increases with the safe rate, reaching 100% for high values of $R_0$.

Figure 5. Effect of the safe rate on loan supply with heterogeneous monitoring costs

This figure shows the relationship between the safe rate and the aggregate supply of loans (bold line), and the supply of loans by banks with low (dashed line) and high monitoring costs (dotted line).

Since low cost banks choose a higher monitoring intensity, their loans have a lower probability of loan default. But since the market share of these banks increases with the safe rate, it follows that the average probability of loan default will get closer to that of the low cost banks. Figure 6 illustrates the effect of changes in the safe rate $R_0$ on the probability of loan default of low and high cost banks, $PD_L = p - m_L(L^*)$ and $PD_H = p - m_H(L^*)$, as well as on the average probability of default defined by

$$PD = sPD_L + (1 - s)PD_H.$$ 

(24)
Increases in the safe rate $R_0$ translate into increases in the probability of default of the loans granted by high cost banks, and decreases in the probability of default of the loans granted by low cost banks. But due to the effect of increases in $R_0$ on the market share of the latter, the average probability of loan default $PD$ goes down, approaching $PD_L$ for large values of $R_0$.

![Figure 6. Effect of the safe rate on the probability of loan default with heterogeneous monitoring costs](image)

This figure shows the relationship between the safe rate and the average probability of default (bold line), and the probability of default of loans by banks with low (dashed line) and high monitoring costs (dotted line).

### 3.3 Bank Entry

We next consider the longer run effects of changes in the safe rate when we allow for entry (and exit) of banks into (out of) the loan market. In this manner, we intend to shed light on the widespread view that interest rates that are “too low for too long” are detrimental to financial stability.
In order to endogenize the number of banks, we assume that each bank incurs a fixed cost to operate. Banks may have different fixed costs. In particular, let $f_j$ denote the fixed cost of bank $j$, and assume that $f_{j+1} = f_j + z$, for all $j$, with $z \geq 0$. We consider two possible cases: one where all banks have the same fixed cost ($z = 0$), and another one in which the fixed cost is increasing in the number of banks ($z > 0$).

Let $\pi^*_n$ denote the equilibrium level of profits (before subtracting the fixed costs) in a market in which $n$ otherwise identical banks operate. Ignoring integer constraints,\(^5\) the free entry equilibrium is characterized by a number $n$ of banks that satisfy a zero net profit condition for the marginal bank, namely $\pi^*_n - f_n = 0$.

In what follows we analyze the effect of introducing either constant or increasing fixed costs on the relationship between the safe rate $R_0$ and the probability of loan default $PD$. The benchmark for this analysis will be the monopoly case ($n = 1$), in which as shown in Section 2 lower rates translate into lower probabilities of default.

Figure 7 shows the effect of introducing fixed costs on the equilibrium number of banks $n$ for different values of the safe rate $R_0$. The horizontal axis represents the safe rate $R_0$, and the vertical axis represents the number of banks $n$. The horizontal solid line represents the monopoly benchmark case, the dashed line is the constant fixed cost case, and the dotted line is the increasing fixed cost case. As expected, with lower rates there will be entry which will be more pronounced for constant fixed costs.

\(^5\)This implies that the fixed cost for arbitrary $n > 1$ is $f_n = f_1 + (n-1)z$. 
Figure 7. Effect of the safe rate on the intensity of competition

This figure shows the relationship between the safe rate and the equilibrium number of banks for a constant fixed cost (dashed line) and an increasing fixed cost of entry (dotted line). The bold line represents the benchmark with a fixed number of banks.

We have shown that increasing the number of banks increases equilibrium total lending, lowers the monitoring intensity of the banks and hence the probability of loan default. Since there will be more entry with lower rates, we have

\[
\frac{\partial PD}{\partial R_0} + \frac{\partial PD}{\partial n} \frac{dn}{\partial R_0} < \frac{\partial PD}{\partial R_0},
\]

where the first term in the left-hand side shows the direct effect for a fixed number of banks, and the second term the indirect effect through bank entry. It follows that entry will tend to strengthen our previous results on the negative relationship between safe rates and bank risk-taking in fairly competitive markets, and possibly reverse our previous results on the positive relationship between safe rates and bank risk-taking in fairly monopolistic markets.

Figure 8 illustrates the latter results. The horizontal axis represents the safe rate \( R_0 \), and the vertical axis represents the probability of loan default \( PD \). The solid line represents the monopoly benchmark case, the dashed line is the constant fixed cost case, and the dotted
line is the increasing fixed cost case. The effect of entry (the second term in the left-hand side of (25)) is clearly more pronounced for the constant than for the increasing fixed costs.

![Figure 8. Effect of the safe rate on the probability of loan default with endogenous entry](image)

This figure shows the relationship between the safe rate and the probability of default for a constant fixed cost (dashed line) and an increasing fixed cost of entry (dotted line). The bold line represents the benchmark with a fixed number of banks.

### 4 Banks’ Funding Sources

This section analyzes the robustness of our results to incorporating two relevant aspects of banks’ funding costs. First, we consider the effect on the relationship between interest rates and banks’ risk-taking of banks competing à la Cournot in the deposit market. Second, we introduce bank capital, and analyze whether endogeneizing banks’ leverage decision changes the relationship between interest rates and banks’ risk-taking.
4.1 Endogenous Deposit Rates

TBC

4.2 Bank Leverage

We next consider the effects of changes in the safe rate when financial intermediaries can adjust their leverage. As highlighted by Dell Ariccia et al (2014) leverage decisions are an important driver of the risk taking effects of monetary policy. It is important to highlight that in our model equity should be seen as internal equity, i.e. funds provided by individuals that either (i) make the unobservable risk taking decisions or (ii) have no conflict of interest with those that take them.

In order to endogenize banks’ leverage decisions we assume that each bank operates with some amount of inside equity $K_j$ that is costly to raise. It should be noted that in a setup like ours if equity would not be costly to raise banks would be totally funded with equity as the moral hazard problem would disappear. In particular let us assume that in order to raise $K$ amount of inside equity the banker has to exert a cost $G(K)$. We assume that $G'(K) > 0$ and that $G''(K) \geq 0$. We define as $k_j = K_j/l_j$ the (inside) equity ratio of bank $j$, where, given the balance sheet constraint, higher equity ratios result in lower leverage.

In line with our previous line of argumentation we first solve the model for a fixed $K$ and then we allow for $K$ to be endogenously determined. We do so as we think that internal equity might not be easy to raise in the short run. Interestingly, even when $K$ is fixed, we find that banks’ leverage reacts to safe rates in the same qualitative manner that in Dell’Ariccia et al (2014). Lower safe rates result in an increase in banks supply of loans which, given the fixed inside capital, results in higher bank leverage. However, our results regarding loans’ probability of failure differ from those of Dell’Ariccia et al (2014) as we find that for, fixed aggregate capital, lower safe rates result in higher leverage and lower (higher) risk taking in a concentrated (competitive) financial sector. Figure 9 shows the relationship between safe rates and banks’ (inside) equity for a competitive (duopolistic) market where the solid

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6Dell’Ariccia et al (2014) would be a limit case of this setup in which $G'(K) = R_0 + \delta$. 

(dashed) line represents the monopolistic market. The horizontal axis are different values of the safe rate and the vertical axis represents banks’ (inside) equity ratio.

![Graph showing the effect of the safe rate on bank leverage with fixed equity.](image)

**Figure 9. Effect of the safe rate on bank leverage with fixed equity**

Figure 10 represents the relationship between safe rates and PD in a monopolistic and duopolistic market. We can observe that when the (inside) equity is fixed the relationship between safe rates and banks’ risk taking decisions is increasing (decreasing) in a monopolistic (duopolistic) banking market.
We then resort to analyze a setup in which the aggregate amount of (inside) equity that each bank has, $K$, can adjust to the safe rates. We do so by assuming that $G'(K) > 0$ and $G''(K) > 0$. We show how the exact functional form of the cost of raising equity is a crucial driver of the relationship between safe rates and PD. For any given cost of equity we find that leverage goes up when safe rates decrease, but when $G''(K)$ is high enough, the sign of the relationship between safe rates and PD depends on banks market structure. It is also relevant to highlight that we obtain that the "cost of equity premium", $G(K) - R_0$, depends on the safe rate. For higher safe rates banks are more willing to raise equity which increases the (inside) equity premium resulting in a positive relationship between the "cost of equity premium" and safe rates. This result is in line with recent research analyzing how risk premiums evolve with the safe rate that show how risk premia vary positively with safe rates.

Figure 11 shows the relationship between safe rates and leverage for a monopolistic bank with high (solid line) and low (dashed line) equity adjustment costs (high and low $G''(K)$). We can observe how in both cases the banks’ leverage decreases with higher safe rates, but it does so more aggressively when the incremental cost of raising equity are lower (low $G''(K)$).
Figure 11. Effect of the safe rate on a monopolistic bank’s leverage with costly equity

Figure 12 shows the relationship between safe rates and PD for a monopolistic banks with bank with high (solid line) and low (dashed line) incremental cost of raising equity costs. We can observe how in the case of high (low) incremental cost of raising equity, lower safer rates results in lower (higher) risk taking by banks. This results point to the fact that understanding how (inside) equity acumulates in the banking sector and how it interacts with market structure is a crucial determinant of the relationship between bank risk taking incentives and safe rates.
5 Conclusion

TBC
Appendix

**Proof of Proposition 1**

To simplify the notation, let $R$ denote $R(L)$. If $R < R$, for any $m \in (0, p]$ we have

$$R - \frac{R_0}{1 - p + m} - \gamma m < 0,$$

which implies that the bank has an incentive to reduce $m$. But for $m = 0$ we have

$$R - \frac{R_0}{1 - p} < 0,$$

which violates the banks’ participation constraint $B \leq R$.

If $R \geq R$, by the convexity of the function in the right-hand side of (8) there exist an interval $[m^-, m^*] \subset [0, p]$ such that

$$R - \frac{R_0}{1 - p + m} - \gamma m \geq 0 \quad \text{if and only if} \quad m \in [m^-, m^*].$$

By our previous argument, for any $m \in (0, p]$ for which

$$R - \frac{R_0}{1 - p + m} - \gamma m < 0,$$

the bank has an incentive to reduce $m$. Similarly, for any $m \in [0, p)$ for which

$$R - \frac{R_0}{1 - p + m} - \gamma m > 0,$$

the bank has an incentive to increase $m$. Hence, there are three possible values of monitoring in the optimal contract: $m = m^*$, $m = m^-$, and $m = 0$ (when $m^- > 0$).

To prove that the bank prefers $m = m^*$, notice that our assumptions on the monitoring cost function together with the definition of $m^*$ imply

$$\frac{d}{dm} [(1 - p + m)R - c(m)] = R - \gamma m > R - \gamma m^* \geq \frac{R_0}{1 - p + m^*} > 0,$$

for $m < m^*$. Hence, we have

$$(1 - p + m^*)R - R_0 - c(m^*) > (1 - p + m)R - R_0 - c(m),$$

for either $m = m^-$ or $m = 0$ (when $m^- > 0$), which proves the result. □
References


