Trading the Option Implied Volatility Smirk Using Firm Fundamentals

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Abstract

We examine the explanatory power of firm fundamentals to the cross-sectional variation in the shape of the implied volatility (IV) function by constructing a valuation of IV curve using firm fundamentals and market measures via a Bayesian shrinkage method. In-sample cross-sectional fitting and out-sample investment exercise show that fundamental-based valuation is both statistically informative and economically contributing in modelling of the IV levels, slopes, and curvature. These findings highlight the importance of incorporating firm fundamentals in modelling of the IV smirk, the participants of a firm’s stock and stock options markets can improve their trading performance by using firm fundamentals to form their expectations.

JEL classification: C11, C12, C13, G11, G12.

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1. Introduction

Implied volatility (IV) can smile, smirk, or skew depending on the characteristics of the option’s underlying. These IV shapes are well observed\(^1\) and extensively studied in the literature.\(^2\) As an alternative dimensionless forward-looking measure to option price, IV is informative in several ways. On one hand, IV not only reveals the market price of risk on the underlying asset return, it also captures investors’ demand for compensation for uncertainties related to other higher moments of the return and volatility process such as volatility risk premium in Bakshi and Kapadia (2003) and Carr and Wu (2009), skewness risk premium in Chang et al. (2013), and variance-of-variance risk premium in Kaeck (2017). These risk premiums are found to be important risk factors in option and its underlying asset price. On the other hand, the shape of IV function is predictive in future return of the underlying asset. Kozhan et al. (2013) look at skew risk premium corresponding to the slope of the IV function, they find skew risk is closely related to variance risk premium. Xing et al. (2010) and Yan (2011) find a negative predictive relationship between the slope of IV curve and equity return. Fu et al. (2016) look at the shape of IV smirk in terms of call-put IV spread, IV skew, “above-minus-below” and “out-minus-at” of call options, “out-minus-at” of put options, and realised-implied volatility spread, they find significant non-zero risk-adjusted returns on arbitrage portfolios formed

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\(^{1}\)After the October 1987 crash, researchers realize the IV computed from options on US stock indexes shows difference across strike prices in contrary to the constant volatility assumption in Black and Scholes (1973, BS henceforth). This phenomenon is so-called ”implied volatility smile” and becomes one of the most charming anomalies in the derivatives literature (Rubinstein (1994), Jackwerth and Rubinstein (1996), and Das and Sundaram (1997)). The volatility smile suggests that those deep in-the-money (ITM) and deep out-of-the-money (OTM) options tend to be mispriced by the BS model.

\(^{2}\)Various attempts to explain the shape of IV relax the BS assumption of constant volatility by allowing the volatility of underlying security returns to be deterministic or stochastic over time. For example, Cox and Ross (1976) propose a constant elasticity of variance model assuming the volatility to depend functionally on the underlying security price; Hull and White (1987) show the price of a European option is consistent with the BS price, if volatility is stochastic and uncorrelated with the underlying security price. However, those models are questioned by either their poor out-of-sample performance (Emanuel and MacBeth, 1982), or unstable parameters (Dumas et al., 1998). Other influential work includes Black (1976), Stein and Stein (1991), Heston (1993), Dupire (1994), Derman et al. (1994), Bates (1996), Kou (2002), and Chernov et al. (2003), etc. Other advances fit the shape of IV function with an additional jump factor, for example, Bakshi et al. (1997) value index options using a stochastic volatility model with jumps; Pan (2002) argues the inclusion of a jump-risk premium in a stochastic volatility model can capture the steep IV function. Nevertheless, the models’ parameters are unreasonable (Bates, 2000).

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on these variables.

Despite the development of theoretical models and studies of various risk premiums in option and asset pricing, we know little about the determinants of IV function, particularly, the mechanism by which the shape of IV smirk varies from firm to firm. In this study, we examine the extend to which the firm fundamentals affect the shape of IV curve.

In the seminal works of Black and Scholes (1973) and Merton (1974), a firm’s debt and equity can be thought as contingent claims written on the value of the firm. At the maturity of the debt, the debt holders have claim on the firm’s value up to the value of the debt, while equity holders have claim on the residue of the firm’s value. It is therefore clear that leverage influences the price of the equity and its derivatives. Using a compound option model, Geske et al. (2016) demonstrate that incorporating leverage can significantly reduce option pricing error. Apart from the capital structure, other firm fundamentals such as profitability, liquidity, and financing cost are also informative to a firm’s status such as ability to repay its debt, prospect of growth, and future profit generating ability, and in turn should also affect the price of the equity and its derivatives. Intuitively, investors will learn and form their future expectations on a firm based on all the available information, within which firm fundamentals form an important subset.

To gauge the cross-sectional explanatory power of firm fundamentals to the shape of the IV curve, we use 20 years of data on 415 U.S. S&P 500 constituent firms, and perform two sets of estimations on put options IV curve characteristics approximated by a set of simple measures capturing level, slope, and curvature of the IV function. We first use a realised moments and jumps(RMJ) based local linear regression model as the benchmark for the valuation of the IV curve levels, slopes, and curvature. Additional contributions of the firm fundamental factors to these IV curve characteristics are then calculated using stacked regression and Bayesian shrinkage method. On average, the RMJ-based models explain the cross-sectional variations of IV levels well, with an average $R^2$ of 0.61, 0.74, and 0.67 respectively for put options at -0.75, -0.5, and -0.25 delta levels, corresponding
to an in-the-money (ITM), an at-the-money (ATM), and a out-the-money (OTM) put option. However, the RV-based models provide little explanatory power to the cross-sectional variations of slopes and curvature of the put IV function, with an average $R^2$ of 0.07, 0.06, and 0.07 for ITM-minus-OTM, ITM-minus-ATM, and OTM-minus-ATM IV slopes, and 0.06 for IV curvature. The model incorporating fundamentals from the options underlying firm can greatly increase the explanatory power, the corresponding average $R^2$ are 0.73, 0.82, and 0.78 for IV levels, 0.23, 0.26, and 0.29 for IV slopes, and 0.25 for IV curvature.

To examine whether the inclusion of firm fundamentals brings additional economic value, we construct a set of trading strategies according to the fundamental-based model estimates, the returns are then benchmarked by the returns on portfolios constructed using RV-based model estimates. We form delta-neutral covered put, delta-gamma-neutral put spreads and butterfly strategies using market quotes of options and stocks to gauge the economic significance of the fundamental-based modelling of IV levels, slopes, and curvature. When the market observed IV curve characteristics are higher/lower than the model estimated IV curve characteristics, we anticipate these characteristics will revert to their model implied values, and sell/buy the corresponding option strategies accordingly. The fundamental-based investment strategies outperform the RV-based portfolios for ITM and OTM IV level, ITM-minus-ATM and ITM-minus-OTM implied volatility slopes, and IV curvature, generating significantly higher annualised returns and Sharpe ratios. Together with the similar results for call options, our findings suggest that firm fundamentals contain rich information on the determinants of IV function both statistically and economically, and are therefore important set of factors that should be incorporated in IV modelling.

In order to explain why the firm fundamentals help to improve model’s performance, we investigate the weights of each fundamental variables in a cross-sectional stacked regression. We find the weights for each firm fundamental characteristics are generally stable, positive, and significantly differ from zero, indicating all the fundamental vari-
ables on our list bring additional contribution in explaining the cross-sectional variations of IV function characteristics. Specifically, leverage ratio has the largest weight in most cases, consistent with its crucial role in determining the IV function. While the weight for liquidity is relatively small, only becomes large around 2001, after dot-com bubble, and 2010, after the credit crunch, indicating that liquidity doesn’t play a central role in determining IV function under normal market condition, but become a more important determinant when the market wide credit condition deteriorates. Our marginal $R^2$ analysis further shows that leverage ratio and size are the two most important factors in explaining the IV function, and liquidity has the smallest contribution among all fundamental factors. The different marginal explanatory power of each factor improves our understanding of the varied shape of IV smirk across firms.

Our study contributes to the literature in several ways. Our findings shed light on equity IV function modelling. Other researchers examine the determinants of the shape of IV function, for example, Pena et al. (1999) report simple regressions and Granger causality tests and find transaction costs, time to expiration, market uncertainty and momentum are important factors in explaining the IV smile; Bollen and Whaley (2004) examine the relation between net buying pressure and the shape of the IV function; Garleanu et al. (2009) model the demand-pressure effects on option prices and find demand helps explain the skew patterns of options; Han (2007) finds changes in investor sentiment help explain the variation in the slope of IV smile; Cao et al. (2017) investigate the predictability of delta-hedged option return using firm fundamental and market measures; Vedolin (2012) look at time-varying volatility of firm fundamental growth, and its effect on volatility risk premium. Different from those studies, our paper investigates specifically the role of firm fundamentals on IV curve. Our findings provide empirical evidence that firm fundamentals are beneficial for IV modelling.

Options, especially those deep OTM options, are often found to be mispriced. Constantinides et al. (2008) find substantial violations of BS model by post-crash OTM calls; Chambers et al. (2014) find it is generally possible to reject the hypothesis that put
returns are consistent with option pricing models, OTM put includes a significant pre-
mium. We document both statistically and economic significance of IV curve using the
information of firm fundamentals and provide insights to interpret the option mispricing.

Others investigate the determinant of IV curve of FX derivatives. Deuskar et al. (2008) find yield curve variables affect the shape of the IV function derived from Euro
interest rate caps and floors. Han et al. (2016) find economic variables can explain the
IV smiles for three major European currency options. These studies and ours are closest
in spirit, but differ in methodology and perspective. Our research is most closely related
to Bai and Wu (2016) in methodology, in which they focus on credit default swap (CDS)
spreads, and find cross-sectional explanatory power of firm fundamentals on CDS spreads.
To the authors’ best knowledge, our study is the first to investigate firm fundamentals as
determinants of the shape of the IV curve at individual firm level.

The rest of the paper is structured as follows. Section 2 constructs a compound
option model for equity option pricing. Section 3 explains data and empirical procedures.
Section 4 discusses the role of firm fundamentals in explaining cross-sectional variation of
IV function. Section 5 provides trading strategies and performance. Section 5 concludes
the paper.

2. Model

A firm’s equity and debt can be viewed as contingent claims on the firm value (Black
and Scholes (1973) and Merton (1974)), individual equity options of the firm can there-
fore be viewed as compound options written on the underlying firm value as in Geske
et al. (2016). If firm fundamentals are informative to the financial status of the firm
and ultimately the evolution of .the firm value process, they would also be important
determinants of the characteristics of the firm’s equity options. We demonstrate this link
between firm’s fundamental and equity option characteristics through the construction
of a structural model in this section.
We start out from Merton model with the usual assumptions of no tax, transaction cost, divisibility, liquidity, and borrowing related restrictions, and assuming that the value of the firm, \( V \), satisfies the stochastic differential equation (SDE):

\[
dV = \alpha V dt + \sigma V dW - c V dt,
\]

(1)

where \( \alpha \) is the expected rate of return of the firm per unit time, \( c \) is the financing cost of the firm per unit time, \( \sigma \) is the volatility of the firm value process, and \( W \) is a standard Brownian motion. The financing cost of a firm consist of two components: dividend payments to its shareholders and coupon/interest payments to the debt holders,

\[
eV = e_e + e_d.
\]

(2)

Equation (1) and (2) relate the evolution of the firm value to its size, stability, expected profitability, dividend policy, and interest obligations.

If a marketable security, \( F \), whose value is a function of firm value and time, \( F = f(V,t) \), is governed by:

\[
dF = \alpha_F F dt + \sigma_F F dW_F - c_F dt,
\]

(3)

where \( \alpha_F \) is the expected rate of return of the security per unit time, \( c_F \) is payout of the security per unit time, and \( \sigma_F \) is the volatility of the security value process. Merton (1974) show that the value of such security must satisfy:

\[
\frac{1}{2} \sigma^2 V^2 F_{VV} + (r - c)V F_V - r F + F_t + c_F = 0.
\]

(4)

A firm’s equity and equity options belong to this class of instruments, satisfying the same partial differential equation (4), but subjecting to different boundary conditions. Note that we specify different payout dynamics in Equation (1) and (3), it is reasonable to
assume that the firm pay out a certain portion of its return to its investors, but the
same argument cannot hold for, for example, firm’s debt in which case a constant coupon
payment implies a time varying rate with respect to the debt value.

We depart from the Merton (1974) framework, and assume that firms operate on the
basis of continuing business under a stable capital structure. In another word, a firm
would not cease to exist at the maturity of its debt, but rather repay its existing debt
obligation via issuance of new debt of similar structure such that a firm’s equity only
matures when it can no longer secure new debt to cover its existing debt. Typically,
the firm’s potential debt investor will look at the firm’s default informative measures
during the investment decision making process, it is therefore natural to assume time
to maturity of the firm’s equity is a function of the firm’s default sensitive fundamental
measures vector, \( d \):

\[
T_e = T(d)
\]  

In this study, we focus on the cross-sectional characteristics differentiating firms with
different default sensitive informations, we therefore ignore the time variation of \( d \) within
each firm, and keep the model reasonably parsimonious.

The value of the equity \( S(V, t) \) is therefore governed by Equation (4),

\[
\frac{1}{2} \sigma^2 V^2 S_{VV} + (r - c) V S_V - rS + S_t + c_e = 0
\]  

subject to the boundary condition:

\[
S(V, T_e) = (V_{T_e} - B)^+
\]  

where \( B \) denotes the principal of the firm’s overall debt obligation. Similarly, the value
of the equity put option $P(V, t)$ that matures at $T_p$ with strike price $K$ satisfies

$$\frac{1}{2} \sigma^2 V^2 P_{VV} + (r - c)V P_V - rP + P_t = 0$$

subject to the boundary condition:

$$P(V, T_p) = (K - S(V, T_p))^+.$$  \hspace{1cm} (8)

We assume $T_e > T_p$ throughout this paper. Under this model, the stock price at time $t$ can be expressed as:

$$S_t = \int_0^{T_e - t} e^{-ru} e_{t} du + e^{-r(T-t)} \int_{B}^{\infty} (V_{T_e} - B)^+ f(V_{T_e} | t) dV_{T_e}$$

$$= (1 - e^{-r(T_e - t)} \frac{C_e}{r} + e^{-r(T-t)} \int_{B}^{\infty} (V_{T_e} - B)^+ f(V_{T_e} | t) dV_{T_e}$$

while the put option can be viewed as a compound option written on the value of the firm whose value can be determined by a modified Geske (1977) formula:

$$P = \left( e^{-r T_p} K - (e^{-r T_p} - e^{-r T_e}) \frac{C_e}{r} \right) \int_0^{\tilde{V}} f(V_{T_p} | V_0) dV_{T_p}$$

$$\quad - e^{-r T_e} \int_0^{\tilde{V}} \int_{B}^{\infty} (V_{T_e} - B) f(V_{T_e} | V_{T_p}) f(V_{T_p} | V_0) dV_{T_e} dV_{T_p}$$

where $\tilde{V}$ denote $V_{T_p}$ that is the solution to

$$(1 - e^{-r(T_e - t)} \frac{C_e}{r} + e^{-r(T-t)} \int_{B}^{\infty} (V_{T_e} - B)^+ f(V_{T_e} | t) dV_{T_e} = K$$

and $f(V_{t_1} | V_{t_2})$ denote the transition density of $V$ from $t_2$ to $t_1 > t_2$:

$$f(V_{t_1} | V_{t_2}) = \frac{1}{V_{t_1} \sigma \sqrt{2\pi (t_1 - t_2)}} \exp \left( -\frac{1}{2} \left( \frac{\ln(V_{t_1} / V_{t_2}) - (r - c - \frac{1}{2} \sigma^2)(t_1 - t_2)}{\sigma \sqrt{(t_1 - t_2)}} \right)^2 \right)$$

(13)
The price of the call option can be derived in a similar fashion.

Note that the equity price under this model can be interpreted as the combination of the Gordon growth model up to the firm’s default, and an option premium that give the equity holder the right to default at time \( T_e \). Given the solution for equity put option pricing in Equation (11), one may then calculate put option prices across different strikes \( K \) provided that \( T_e \) is reasonably modelled, these option prices can then be ported back to the Black-Scholes formula to calculate the IV at each strike price.

We conduct simulation based on this compound option model specification, paying particular attention to the change in model induced IV curve with respect to the change in face value of the debt, financing cost, and the time to default. We simulate the IV curve for a put option that matures in one year. Figure (1) plots IV against log moneyness defined as \( m = \log(\frac{K}{S_t}) \) shows the change in IV curve with respect to change in dividend payout, time to default, and face value of the debt.

Our compound option model for option pricing is capable of generating IV smirks, and is scale invariant if the dividend payout is kept at a constant proportion to the current asset value. In addition, the IV curve tilt to the left, especially in the in-the-money region, as the dividend payment each year increases. IV curve flattens and shifts upward as the time to default increase if the firm’s financing cost exceeds the risk-free interest rate, and shift downwards in the out-of-the-money region as the time to default decrease if the financing cost is less than the risk-free interest rate. The IV curve shifts upward and flattens as the face value of the debt increase. In the appendix, we also plot the first and second order derivative of IV with respect to \( M \). Note that panel (a), (c), and (e) represent a more realistic IV curve dynamic as we should always anticipate the required rate of return of a firm’s investor is always greater than the risk-free interest rate.

This indicates, from a theoretical point of view, that the fundamental characteristics of a firm have significant influences on the implied volatility of the firm’s equity options.
3. Data and Procedure

We analyse all the S&P 500 constituent firms that have options actively traded between 10 January 1996 and 27 April 2016. Firm fundamentals are collected from Compustat Capital IQ; stock market price and volume data are collected from the Center for Research in Security Prices (CRSP); Data on stock option implied volatility and stock price historical volatility are collected from Ivy DB Option Metrics. During our sample period, we apply a set of data filters in preparation for the fundamental-based cross-sectional modelling of the IV curve on every Wednesday. On any given day, a company is included if: i) the 30-day implied volatility curve is available, ii) the book-value of debt is not missing from the last quarterly financial report, iii) stock price and total of shares outstanding is available to calculate the market capitalisation of the firm, iv) the stock has been traded for at least 365 calendar days. The sample contains 1050 active trading weeks, 415 companies, and a total of 356,809 company-week observations.

Adapting the standard accounting literature practice, we assume that companies disclose quarterly balance sheet data 45-days after the last day of each quarter, these data are then matched with market data on every Wednesday until next disclosure date. Table 1 shows the data matching between quarterly fundamental data and the market data on every Wednesday throughout our sampling period.

[ Insert Table 1 near here ]

We gauge the firm fundamental and market price characteristics by analysing the data items disclosed in the firm’s quarterly financial report and its market price data. Specifically, we measure firm characteristics according to thirteen measures documented in Table 2, capturing firm’s financial leverage, interest coverage, liquidity, profitability, investment, size, equity market momentum, earnings, dividend, and book-to-market ratio. On one hand, these fundamental measures are closely related to the default probability

\footnote{There are numerous firm fundamental measures, we follow Bai and Wu (2016) choosing the first eight measures on a firm’s survival ability, and Welch and Goyal (2007) for the last five focusing on a firm’s profit generating ability for its investors}
of the underlying firm, and in turn, the skew of its option IV curve. On the other hand, they can also reveal firm’s profit generating capability and likelihood of future good news, and therefore influencing the upper section of IV curve.

[ Insert Table 2 near here ]

We first analyse 30-day IV curve produced by put options, considering only options with delta equal to -0.75, -0.5, and -0.25 to capture its shape. We use the IV levels at option delta equal to -0.75, -0.5, and -0.25, and the slopes and curvature of the IV curve between these delta points. The slopes of the IV curve are approximated by the implied volatilities of put spread option strategies that simultaneously buy and sell put options at different delta level. The curvature of the implied volatility curve is approximated by a butterfly spread option strategy that buys out-of-the-money (OTM) and in-the-money (ITM) options, and sells at-the-money (ATM) options. Denoting the implied volatilities at delta equal to -0.75, -0.5, and -0.25 by $IV^+$, $IV^\circ$, and $IV^-$, the slope of the IV function is captured by:

\[
IV^{(1)} = IV^+ - IV^-;
\]

\[
IV^{(2)} = IV^+ - IV^\circ;
\]

\[
IV^{(3)} = IV^- - IV^\circ;
\]

while the curvature of the IV function is captured by:

\[
IV^{(4)} = \frac{IV^+ + IV^-}{2} - IV^\circ.
\]

The main advantage of the measures demonstrated in Equation (14) - (17) is simplicity\(^4\). As we wish to examine the cross-sectional variation of the IV curve, we don’t need

\(^4\)More sophisticated procedures to capturing the shape of the IV function are available. For example, one may wish to specify a particular functional form on the implied volatility curve and surface, such that once fitted to the market observed implied volatility, the set of parameters obtained can be served as a storage for the information on the shape of implied volatility. Dumas et al. (1998), Goncalves and
to capture the finer details of a particular section of the curve, but rather its general form. In a sense, these IV characteristics are variations of the functional form proposed in \textit{Bedendo and Hodges (2009)} around three discrete delta points. As we are dealing with a large sample size, this significantly reduces the computation burden in preparing data for the proceeding analyses. We also avoid the imperfect capturing of the IV curve generally associated with calibrated stylised models such as stochastic volatility models with jumps, and potential temporal overfitting of a local volatility model. In contrast to the non-parametric approaches such as principle component analysis, any statistical findings governing the measurement Equations (14) - (17) directly result in executable trading strategies.

After settling for the measurement of the shape of the IV curve, we now describe the firm fundamental-based modelling procedures for these IV curve characteristics. We use the same procedure developed in \textit{Bai and Wu (2016)} which we briefly outline here.

We first establish a benchmark model to which addition contributions of firm fundamental characteristics can be added at a later stage. The realised moments of the return distribution of the underlying equity price are natural choices in fulfilling this role, we include the realised variance (RV) and realised skewness (RS) of daily equity returns in forming the benchmark model. In addition, in light of the ample evidence from the index options that IV skew is closely related to the occurrence of jumps in the underlying return process documented in, for example, \textit{Pan (2002)}, it is natural to consider whether a market-based jump measure could improve the cross-sectional fitting of the IV curve characteristics, especially for IV slopes and curvature. $BV_t$ denotes the realised bipower

\textit{Guidolin (2006), and Bedendo and Hodges (2009)} estimate the shape of the IV function using polynomial fitting to market observed IVs with respect to time to maturity and strike price/option moneyness to various degree. Alternatively, one may specify diffusive processes for the underlying stock return and its instantaneous volatility complemented by jump processes of various type, the implied volatility can then be approximated either direct calibration from market observed BS implied volatility in \textit{Jacquier and Lorig (2015)} and \textit{Ait-Sahalia et al. (2017)}, or via the calibration from option prices in \textit{Bates (2000)} and \textit{Christoffersen et al. (2009)}. Non-parametric models such as principle component analysis and Karhunen-Loève decomposition are also widely used in construction of measures for the shape of IV curve, for example in \textit{Alexander (2001), Cont et al. (2002), and Christoffersen et al. (2017)}. 

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variation introduced in Barndorff-Nielsen and Shephard (2004) that takes form of

\[ BV_t = \frac{\pi}{2} \frac{m}{m - 1} \sum_{j=2}^{m} |r_j r_{j-1}| \]  \hspace{1cm} (18)

where \( r_j \) denotes log return of the asset at time period \( j \). \( RJ_t \) denotes the ratio representing the portion of quadratic variation of the log return process that is due to jumps:

\[ RJ_t = \frac{(RV_t - BV_t)^+}{RV_t}. \]  \hspace{1cm} (19)

This measure is mainly used in detecting intraday jumps from high frequency data, but with the limited access to high frequency data, we argue that this may also be used as a tool to extract the overall amount of realised quadratic variation due to jumps, and is also informative in the future expected jumps.

On each Wednesday, we start with fitting the IV levels, slopes, and curvature cross-sectionally by non-parametric regression in RV, RJ, and RS:

\[ IV^i_t = f^i(RV_t, RJ_t, RS_t) + \epsilon^i_t, \quad i = 1, \ldots, 4 \]  \hspace{1cm} (20)

where \( f^i(.) \) denotes local linear transformation.

We then use this as the benchmark model, and examine the additional explanatory power of each firm fundamental factors to the IV shape characteristics. These can then be combined by a stacking regression with Bayesian update.

Denoting the benchmark model by \( RIV^i_t = \hat{f}^i(RV_t, RJ_t, RS_t) \) and let \( F_t \) to denote \((N \times K)\) matrix of \( K \) firm fundamental characteristics on \( N \) companies at time \( t \). On each day, we orthogonalise the additional contribution of fundamental characteristics to the IV characteristics from the benchmark model by applying local linear regression on each firm characteristic against \( RIV^i_t \):

\[ F^k_t = f^{i,k}_t(RIV^i_t) + x^{i,k}_t, \quad i = \circ, +, -, 1 \cdots 4, \quad k = 1 \cdots K. \]  \hspace{1cm} (21)
Next, we regress residues of the benchmark model $\epsilon_i^t$ against these orthogonalised fundamental characteristics $x_{i,k}^{t,k}$ by another local linear regression:

$$
\epsilon_i^t = f_i^{i,k}(x_{i,k}^{t,k}) + \epsilon_{i,k}^{i,k}, \quad i = o, +, -, 1 \cdots 4, \quad k = 1 \cdots K.
$$

(22)

We then stack the predictions on the residues of the benchmark model from each orthogonalised firm characteristics into matrix form:

$$
X_i^t = \begin{bmatrix}
\hat{\epsilon}_i^{i,1} \\
\hat{\epsilon}_i^{i,K}
\end{bmatrix},
$$

(23)

such that their weighting of contribution to the overall residue $W_i^t$ can be estimated via

$$
\epsilon_i^t = X_i^tW_i^t + \epsilon_i^t.
$$

(24)

Given that a firm’s fundamental characteristics are not always available in full, the predictions to the residues of the benchmark model from the missing firm characteristics at certain time $t$ is approximated by the average of the predictions from other fundamental characteristics weighted according to $R^2$ from the regression described in Equation (22). For $j^{th}$ firm at time $t$, if $l^{th}$ firm characteristic is missing, its residue prediction on $i^{th}$ IV characteristic is approximated by the residue predictions from $\bar{K}$ available firm characteristics:

$$
\epsilon_i^{i,j,l} = \frac{1}{\bar{K}} \sum_{k=1}^{\bar{K}} w_i^{i,k} \epsilon_i^{i,k},
$$

(25)

$$
w = \epsilon^\top(\epsilon \epsilon^\top + \text{diag} \langle 1 - R^2 \rangle)^{-1}.
$$

(26)

After completing the residue prediction matrix $X_i^t$, we estimate the weighting vector
$W_t^i$ at each time via a Bayesian regression update:

$$\hat{W}_t^i = \left( X_t^i \top X_t^i + P_{t-1}^i \right)^{-1} \left( X_t^i \top \epsilon_t^i + P_{t-1}^i \hat{W}_{t-1}^i \right) ,$$  \hspace{1cm} (27)

$$P_t^i = \text{diag} \left( \left( X_t^i \top X_t^i + P_{t-1}^i \right) \phi \right) .$$  \hspace{1cm} (28)

Starting with equal weights, we take the weights estimated last Wednesday as prior, and update our beliefs on the weights according to the residue predictions of each firm characteristics. We diagonalise the precision matrix $P_t^i$ to reduce any potential multicollinearity issue, and choose $\phi = 0.98$ to control the relative weight of the prior during the updates. Once the time weights $W_t^i$ is estimated, we can then add the weighted average of the residue prediction in Equation (24) back into Equation (20) such that a new set of estimations of IV characteristics is calculated:

$$\text{FIV}_t^i = \text{RIV}_t^i + X_t^i \hat{W}_t^i .$$  \hspace{1cm} (29)

Table 3 lists the summary statistics of firm and put option IV curve characteristics. The second column reports the sample means of IV curve and firm fundamental characteristics calculated on the pooled data of 356,809 firm-week observations. We then group these observations according to the five quintiles of the ATM IV level, and report sample mean of each characteristics within each group in column three to seven. We also report the sample standard deviations of these characteristics on pooled data, cross-sectional sample standard deviations at each week averaged over 1050 active trading weeks (XS), time-series sample standard deviations for each firm averaged over 415 firms (TS), and time-series sample standard deviations of the weekly changes in these characteristics for each firm averaged over all firms in our sample (TSC). The average ATM put option volatilities have a pooled average of 34.02%, the 5 quintiles have an average of 17.63%, 23.80%, 29.73%, 37.78%, and 61.17%, respectively, suggesting the distribution of the ATM put volatilities is positively skewed. All the pooled average OTM-minus-
ATM, ITM-minus-ATM and butterfly volatilities are positive, indicating the put option IV function indeed smiles. The negative average ITM-minus-OTM volatility shows the smile is asymmetric around the ATM IV. This asymmetric IV smile pattern motivates our analyses of firm fundamentals on the whole shape of IV curve.

[ Insert Table 3 near here ]

$IV^+,$ $IV^\circ,$ and $IV^-$ are highly correlated, with pairwise correlation coefficients higher than 0.90, and each of them is highly correlated with RV. For other pairwise relations, monotonic increasing patterns are observed across grouped sample means for liquidity, leverage 1, and book-to-market ratio, while monotonic decreasing patterns are observed for $IV^{(1)},$ $IV^{(2)},$ leverage 2, interest coverage, investment, size, RS, earning-price ratio, and dividend-earning ratio. $IV^{(3)}$ decreases in the first three ATM IV quintile groups, and increases in the last two quintiles. $IV^{(4)}$ decreases in all but the last ATM IV quintile groups. RJ increases in the first three quintiles and decreases in the last two. Profitability measure increases in the first three group, and decreases thereafter. Momentum measure increases in the first two groups, and decreases in the remaining groups. Dividend-price ratio and dividend yield decrease in the first four quintile groups, and only increase in the last quintile group. We observe IV smile in the first three ATM IV quintile groups and IV skew in the last two, suggesting the smile flattens and the skew deepens as ATM IV increases. The cross-sectional variations of IV curve characteristics and realised volatility are slightly smaller than its time-series counterpart. Apart from the profitability measure, cross-sectional variations of firm fundamental characteristics are much greater than the time-series variations.
4. Cross-Sectional Variation of IV Characteristics and Firm Fundamentals

The explanatory power of firm fundamental characteristics on its equity option IV curve characteristics are assessed by two sets of cross-sectional regressions on each sample date where we regress market-observed IV curve characteristics against its model-generated counterparts:

\[ IV_{t}^{i,k} = RIV_{t}^{i,k} + \varepsilon_{t}^{i,k}, \quad (30) \]
\[ IV_{t}^{i,k} = FIV_{t}^{i,k} + \varepsilon_{t}^{i,k}. \quad (31) \]

We use the regressions in Equation (30) as benchmark, and check whether there are significant improvements in model fitting using Equation (31). Figure 1 plots the time series of \( R^2 \) from these two sets of cross-sectional regressions. Summary statistics for \( R^2 \) are reported in table 4. The RV-based models capture the IV levels relatively well with average cross-sectional \( R^2 \) of 74%, 61%, and 67% for ATM, ITM, OTM put options. However, the explanatory powers exhibit strong time variations, for example, the \( R^2 \) for ATM IV level varies from 16% to 91% with a standard deviation of 0.12. The RV-based models performs less well in explaining the cross-sectional variations of other IV characteristics with average \( R^2 \) of 7%, 6%, 7%and for ITM-minus-OTM, ITM-minus-ATM, and OTM-minus-ATM IV slope measures respectively, and 6% for IV curvature.

Regressing market observed IV characteristics against FIV improves the \( R^2 \) across all the IV curve characteristics. For IV level, fundamental-based model improves the average \( R^2 \) to 82%, 73%, and 78% while reduces the time variation to standard deviations of 0.08, 0.14, and 0.11 for ATM, ITM, and OTM options respectively. The average cross-sectional \( R^2 \) are 19%, 21%, and 23% for ITM-minus-OTM, ITM-minus-ATM, and OTM-minus-ATM IV slopes, and 25% for IV curvature. The average increases in \( R^2 \) are 0.09, 0.11, and 0.11 for IV levels, 0.15, 0.19, and 0.22 for IV slopes, and 0.20 for IV curvature,
suggesting a significant improvement in incorporating firm fundamental characteristics in explaining the cross-sectional variation of the IV curve characteristics over naïve RMJ-based measures.

[ Insert Figure 2 near here ]

The $R^2$ time series from FIV on IV levels are much more stable than their RIV counterpart, FIV model performs particularly well when the explanatory power of RIV model is low, for example, between August 1997 and March 1998, between January 2003 and February 2004, and between August 2009 and March 2010, three crises are identified during these periods as the Asian crisis, accounting scandal, and the credit crunch, indicating investors may pay more attention to firms’ fundamental when the market is in turmoil. There are negative jumps in the time series of $R^2$, it is unlikely that firms simultaneously release news regarding to their fundamentals, these jumps are likely to be caused by significant information updates in macro-economic conditions, followed by, for example, FOMC monetary decisions and the US Census Bureau economic indicator releases. If we ignore these jumps, the $R^2$ fluctuates in a narrow range roughly between 0.75 and 0.95 from 1996 to the first half of 2010, it then fluctuates in a wider range between 0.6 and 0.9 from the second half of 2010 to the end of 2014, and raises back up to the 0.7 – 0.9 range in 2015 and the beginning of 2016. For ITM-minus-OTM and OTM-minus-ATM implied volatility slope, the explanatory power of RIV and FIV peaks at the height of the financial crisis between the end of 2008 and the first half of 2009, while fluctuate around their respective means otherwise. For ITM-minus-ATM IV slope and IV curvature, the $R^2$ from FIV fluctuates in the range between 0.1 and 0.4, while the same $R^2$ from RIV are generally less than 0.1. Overall, the FIV models capture the cross-sectional variations of the IV characteristics much better than their RIV counterparts, there are only eleven weekly cross-sectional regressions where at least one FIV estimates produce $R^2$ less than the RIV estimates in fitting market-observed IV function characteristics, corresponding to around 1% of the overall cross-sectional fitting we perform.
For robustness, we perform a set of identical estimations and statistical tests for IV characteristics obtained from call options. The results are presented in the appendix and are qualitatively similar to the ones obtained from the put options.

Considering the additional benefits of incorporating firm fundamentals, two natural questions are how those factors perform over time and why firms exhibit different IV shapes. To answer the first question, Figure 3 plots the time series of relative weights of each firm fundamental characteristics to the put option IV curvature, other IV characteristic weights are plotted in the appendix. We find the weights for each firm fundamental characteristics are generally stable, positive, and significantly differ from zero, indicating all the fundamental variables bring additional contribution in explaining the cross-sectional variations of IV function characteristics. Specifically, leverage ratio has the largest weight in most cases, consistent with its crucial role in determining the IV function. While the weight for liquidity is relatively small and only becomes large around 2009 to 2011, after the credit crunch period, indicating that liquidity doesn’t play a central role in determining IV function under normal market condition, but become a more important determinant when the market wide credit condition deteriorates. On the other hand, the weight for interest coverage decreases during the same period, indicating investors care less about the firm’s ability to cover its interest payment when everyone else are facing similar credit problems. Our second measure of leverage has the least weighting among all the firm fundamental measures indicating investors tend to care more about the firms’ current liability level rather than their longer term debt obligations. Weightings for other fundamental measures generally stay above 10% over our sample period.

In order to answer the second question on the possible reason of varied IV shapes across firms, we conduct a marginal $R^2$ analysis by removing fundamental factors from our model one at a time, re-perform the estimation procedures described in Section 2,
and regress the market-observed IV on the resulting FIV estimates to obtain a new set of $R^2$. We then take the difference between the $R^2$ from the full FIV models and $R^2$ from the model disregarding one of the fundamental factors to demonstrate its marginal contribution. This exercise provides us an insight into which factor shows stronger power in explaining the IV function characteristics, among all firm fundamentals in our model. It distinguishes the role of each fundamental factor that causes the IV shapes of firms to vary. For example, if a firm has a better leverage ratio and another firm maintains a higher degree of liquidity, the IV shapes of these two firms can exhibit a rather different pattern, due to that the leverage ratio and liquidity factors are not equally informative across different time periods. Table 5 lists the mean and median of marginal $R^2$ for each firm fundamentals on each IV curve characteristics, figure 4 presents the marginal $R^2$ of each factor for the IV curvature. A smaller marginal $R^2$ indicates a smaller additional contribution of the factor.

[Insert Table 5 near here]

Overall, the size and leverage factors have the largest average marginal $R^2$ across all the IV curve characteristics, removing them from model decreases the explanatory power for IV curvature by 2.92% and 1.58% respectively, while the dividend-earning ratio and book-to-market ratio have the smallest non-negative marginal contribution. The interest coverage and profitability are less important determinants in explaining IV levels, but their explanatory power improves for IV slopes and curvature. This finding enables us to explain the varied shapes of IV curves across firms by their mixed fundamental characteristics. Note that the marginal $R^2$ from dividend-price ratio and dividend yield are only positive for OTM-minus-ATM IV slope, indicating that investors pay more attention to a firm’s dividend policy when deciding to invest in a OTM-ATM bull spread strategy.

[Insert Figure 4 near here]

$^5$Results for other IV characteristics are similar, and are available upon request.
5. Investment Exercise

As shown in our regression analyses, the FIV models are able to capture some of the cross-sectional variations of IV curve characteristics well. In this section, we explore whether these statistical results are useful in predicting the change in the IV curve characteristics by using a set of delta-neutral option trading strategies. If the deviations of market observed IV characteristics from the fitted RIV and FIV models are due to temporal changes of other influencing factors, such as supply and demand shocks, and changes in trading frequency, these deviations are likely to be short lived. The model-based IV characteristic valuation is then informative in forecasting future changes in IV characteristics.

On the other hand, if the deviations are caused by either i) the updates of investors’ belief on firm fundamental characteristics due to firms’ additional disclosure between quarter reports, or ii) change in any other non-transitory factors that are not captured in RIV and FIV models, the model-based IV characteristic valuation will not be informative in future changes in IV characteristics.

We examine this forecasting power by constructing delta-neutral option trading strategies using actual market observed option price data matching the one used in the previous section. Specifically, options data are obtained from Option Metrics Option Prices file, following Goyal and Saretto (2009), we apply several data filters removing i) options that violate no-arbitrage conditions, ii) options with bid price higher than the ask price, iii) options with bid price equals to zero, iv) options with bid-ask spread lower than the minimum tick size ($0.05 for options traded below $3, and $0.1 otherwise), and v) options with open interest equal to zero.

Equity options are not quoted and traded in terms of standard deltas and maturity format appearing in the Option Metrics IV file, we therefore reformat the option price data to approximate the IV curve fundamental valuation discussed in the previous section. On each day we select companies with options maturing next month, that have at least
one option traded in each delta range of 0.175 to 0.325, 0.425 to 0.575, and 0.675 to 0.825 for call options; and −0.175 to −0.325, −0.425 to −0.575, and −0.675 to −0.825 for put options. We select one option from each of these delta ranges with the highest trading volume, then the highest open interest, and then the lowest delta difference to 0.25, 0.5, and 0.75 for call options; and −0.25, −0.5, and −0.75 for put options. In order to utilising the cross-sectional statistical findings effectively and negate model-fitting errors, additional filter is applied to remove weekly observations where less than 10 firms have all the options at three delta points from either call options or put options, we end up with 85,401 firm-week observations over 998 trading weeks.

Since our primary focus is to examine the profitability due to change in implied volatility, we wish to remove as much as possible the changes in option prices that are due to changes in underlying price, we therefore perform delta-hedging to the options by taking delta amount of opposite position in underlying stock. At time $t$, the delta-neutral long position of these put options are:

$$\widetilde{IV}_t^{(+,0,-)} = P_t^{(+,0,-)} - \delta_t^{(+,0,-)} S_t$$  \hspace{1cm} (32)$$

where $P_t^{(+,0,-)}$ and $\delta_t^{(+,0,-)}$ denote the put option prices and their corresponding deltas from the options selected to approximate options with delta of −0.75, −0.5, and −0.25 respectively. Since delta-neutral hedging does not remove completely the changes in values of these option strategies with respect to the change in underlying price, and the first order derivatives of the value of these delta-neutral option strategies with respect to the underlying price are approximately $-\gamma_t^{(+,0,-)} S_t$, these delta-hedged put position are used as building blocks to construct delta-gamma-neutral option strategies approximating
other IV curve characteristics described in Equation (14) to (17):

\[
\widetilde{IV}_t^{(1)} = \frac{\gamma_t^-}{\gamma_t} \tilde{IV}_t^+ - \tilde{IV}_t^-, \tag{33}
\]

\[
\widetilde{IV}_t^{(2)} = \frac{\gamma_t^o}{\gamma_t} \tilde{IV}_t^+ - \tilde{IV}_t^o, \tag{34}
\]

\[
\widetilde{IV}_t^{(3)} = \frac{\gamma_t^o}{\gamma_t} \tilde{IV}_t^- - \tilde{IV}_t^o, \tag{35}
\]

\[
\widetilde{IV}_t^{(4)} = \frac{\gamma_t^o}{\gamma_t^+ + \gamma_t^-} \left( \tilde{IV}_t^+ + \tilde{IV}_t^- \right) - \tilde{IV}_t^o, \tag{36}
\]

where \(\gamma^{(+, o, -)}\) denote the option gammas for the selected options. The option strategy Equations (33) - (35) correspond to ITM-minus-OTM, ITM-minus-ATM, OTM-minus-ATM put spread, while option strategy (36) corresponds to an ATM butterfly symmetric in delta. By taking long and short position of delta-hedged put options according to appropriate gamma scales, these option strategies have delta and gamma equal to zero at inception.

We build portfolios of option strategies from each firm based on the difference between model-based and market observed IV curve characteristics. For any particular IV curve characteristic, if its model-based value is greater than the market-observed value, we invest in a long position in its corresponding option strategy with a notional amount proportional to the difference between model-based and market-observed value, and vice versa if model-based characteristic is less than the market-observed value:

\[
\Pi_t^{i} = \sum_{k=1}^{K} w_{t}^{i,k} \tilde{IV}_t^{i,k}, \quad i = o, +, -, 1 \cdots 4, \tag{37}
\]

On each day, we normalise the weights \(w_{t}^{i,k}\) for \(K\) companies that have IV characteristic estimates available to construct option strategies (32) - (36) such that the overall investment is $1, with a simple assumption that taking a short position requires margin equals
to the amount needed for holding the equivalent long position:

\[
w_{t}^{i,k} = \frac{\tilde{IV}_{t}^{i,k} - IV_{t}^{i,k}}{\sum_{k=1}^{K} |\tilde{IV}_{t}^{i,k} - IV_{t}^{i,k}|}.
\]

(38)

We hold these portfolios for one week, two weeks, and four weeks, and analyse the portfolio performance by calculating portfolio return as weighted average of returns from option strategies from each company scaled by their underlying stock prices:

\[
\Delta \Pi_{t}^{i,q} = \sum_{k=1}^{K} w_{t}^{i,k} \frac{\tilde{IV}_{t}^{i,k} - \tilde{IV}_{t-q}^{i,k}}{S_{t-q}^{k}}, \quad i = +, o, -, 1 \cdots 4, \quad q = 1, 2, 4.
\]

(39)

Scaling returns using underlying stock price is motivated by the margin requirement of taking short positions on options and underlying stock, this also provides a more stable measure of portfolio returns especially in the case of butterfly option strategy, similar scaling method is used in Cao and Han (2013).  

The portfolios are self-financing, we rebalance the option portfolio on a weekly basis. For one week investment horizon, we invest $1 to the portfolio at week zero, and rebalance the portfolio accordingly in the proceeding time periods. For two week investment horizon, we invest $0.5 to the portfolio at week zero and week one, these portfolio holdings are then rebalanced sequentially on and after week two. Similarly, we split our portfolio investment for four week investment horizon, and rebalance the portfolio holdings sequentially every week. Table 6 reports the summary statistics of the investment performance of portfolios constructed according to RIV and FIV models for each IV shape characteristics.

Panel A reports the summary statistics for the returns from 1-week investment horizon.

6A more realistic scaling taking into account of margin requirements of overall short position in options and underlying stock is possible, but this does not provide any additional benefit for the comparison between forecasting power of RIV and FIV models for any given IV shape characteristics, and is therefore omitted. It is important to clarify that the returns generated for different IV shape characteristics are not directly comparable since their overall margin requirements are not the same.
zon. FIV-based strategies provide superior performance, measured by information ratio (IR) of annualised mean return over annualised return standard deviation, compared to the RIV-based strategies apart from the ATM IV level and ITM-minus-ATM IV slope strategies. As discussed earlier, the RIV and FIV models provide excellent cross-sectional explanatory power for OTM, ATM, and ITM IV levels, fitting imperfections are likely to be transient, driven by temporary liquidity shocks of the option trading. The performance of these delta-hedged and delta-gamma-hedged option portfolios demonstrates this predictive power of FIV and RIV models. For example, the FIV portfolio generates a 11.36% annualised mean return with an information ratio of 2.17, compared with the 9.46% return and 1.83 information ratio by the RIV portfolio for the ITM option. Significantly larger returns are generated from four out of seven portfolios using the FIV model over the RIV-based portfolios at the 5% confidence level, with t-statistics equalling to 1.79, 2.00, 5.90 and 5.52, respectively. The FIV model significantly underperforms the RIV model for only one portfolio. The return differences of the remaining two portfolios between the FIV- and RIV-based estimates are insignificant.

Panel B and Panel C reports the summary statistics of the portfolio return for two week and four week holding periods respectively. In general, the information ratios decrease as investment horizon increases, suggesting the convergence of market observed IV shape characteristics to their fundamental-based values. The performance differences between RIV and FIV models for most IV shape characteristics are consistent with the results from one week holding period apart from ATM IV level.

Fundamental-based valuation provide additional predictive power for OTM and ITM IV level across all the investment horizons, producing higher return information ratio. However, it does not provide significant economics gains for ATM IV level for one week investment horizon. The trading volume for ATM option are much higher than the ITM and OTM options, it is therefore reasonable to believe that ATM option prices are less susceptible to temporary supply and demand shock than ITM and OTM options. At two weeks investment horizon, the IR ratio from FIV portfolio is moderately higher than
the IR ration from RIV portfolio, albeit insignificant in terms of $t$-statistics, indicating that ATM IV level deviates from its fundamental value temporarily due to another factor in addition to supply and demand shocks. While RIV is moderately better in exploiting the temporary changes at one week investment horizon, FIV is better in exploiting the temporary deviations of ATM IV level due to changes in factors reverting at a lower speed.

Although the RIV models don’t perform well on average in explaining the cross-sectional variations of IV slopes and curvature, it is still informative in identifying transient states of IV slopes and curvature due to temporary change in non-fundamental factors, yielding average annualised return of 5.43%, 4.41%, and 4.79% for ITM-minus-OTM, ITM-minus-ATM, and OTM-minus-ATM IV slope portfolios, and 2.85% for IV curvature portfolio with one-week investment horizon. Additional portfolio performance can be achieved using FIV models for ITM-minus-OTM and ITM-minus-ATM IV slopes, and IV curvature. The information ratio increases from 2.24 to 2.63 for ITM-minus-OTM IV slope, from 1.87 to 3.55 for ITM-minus-ATM IV slope, and from 3.50 to 4.60 for IV curvature, suggesting that fundamental-based valuation improves predictive power in these cases in addition to RV-based measures.

However, fundamental-based valuation performs worse than the RV-based valuation for OTM-minus-ATM IV slope, IR decreases from 3.63 to 2.46, the predictive power of RV is distorted by in-sample fitting from the firm fundamentals. As discussed earlier, it has been found that investors need to pay a significant positive premium for OTM put options compare to other option contract, this premium can be interpreted as the insurance premium that investors pay for lower-tail risk protection, for instance, Chambers et al. (2014) find the OTM put includes a significant premium. On the other hand, it has been shown both theoretically and empirically that firm fundamentals we choose in this study are closely related to the low-tail risk of an underlying firm. Therefore intuitively, incorporating firm fundamentals in modelling IV characteristics should be more predictive than the RV-based modelling. Our finding contradicts this intuition, suggesting a
large part of the OTM put option premium relative to other options cannot be explained by the firm fundamental characteristics. An interest question arises from this empirical finding to identify the determinant of the additional premium for OTM put option contracts, but is beyond the scope of this study. Nevertheless, the in-sample fitting and out-sample investment performance of OTM put option are better off by incorporating firm fundamentals, these findings improve our understanding of the pricing puzzle of OTM option.

The cumulative values of the option portfolios with one week holding period are plotted in figure 4. Consistent with the values in the Table 8, all the ITM, OTM, ITM-minus-OTM, ITM-minus-ATM and butterfly portfolios generate larger returns for the FIV model than those for the RIV model, while the ATM portfolios are comparable.

As robustness test, we construct equally weighted portfolios where weighting equation 38 is adjusted to only capture the sign of \( \hat{IV}_{i,k}^t - IV_{i,k}^t \) but not its magnitude. The results are documented in table ?? in the appendix. Each equal-weighted portfolios perform less well than its distance-weighted counterparts, generating less annualized returns and information ratios. The performance difference between FIV- and RIV-based portfolios are less significant, none the less, higher IR can be achieve by FIV-based portfolios in most of the cases, the performance relations are qualitatively similar to the one obtained from the distance weighted portfolios discussed above.

Overall, the economics benefit for incorporating firm fundamentals in the modelling of the shape of the IV curve is significant from most of our out-sample investment exercise. Even for IV levels where RV-based measures provide excellent predictive power, firm fundamentals are able to contribute further in separating the fundamental value of the IV levels and its transitory deviations due to trading liquidity shocks or other temporary factors. Firm fundamentals are also particularly informative in identifying transitory states of OTM-minus-ATM IV slope and IV curvature.
6. Conclusion

This paper examines the information content of firm fundamentals in explaining the cross-sectional variation of IV shape characteristics. Using a set of simple IV characteristic measures, we have demonstrated the importance of incorporating firm fundamentals in the modelling of the equity option IV curve. Although RV-based measure provide excellent explanatory power in cross-sectional variation of the IV levels, it is less informative in explaining the cross-sectional variations of other IV shape measures. On the other hand, fundamental-based modelling provides much better cross-sectional explanatory power across all the IV characteristic measures in the scope. Demonstrated in an out-sample investment exercise, fundamental-based modelling is also superior in the predictive power of the IV shape characteristics, providing significant economic benefit compared to RV-based measures. The findings of this paper shed light on equity IV function modelling, a proper model should consider to incorporate more firm fundamental factors, especially when the market is in turmoil. This direction of research may help us explain the mispricing of OTM options and will be the focus of our future study.
Table 1: Data Matching

This table lists the matching rules between firm fundamental and market price data.

<table>
<thead>
<tr>
<th>Firm Fundamental</th>
<th>Market Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quarter 1</td>
<td>15th May to 14th Aug</td>
</tr>
<tr>
<td>Quarter 2</td>
<td>15th Aug to 14th Nov</td>
</tr>
<tr>
<td>Quarter 3</td>
<td>15th Nov to 14th Feb</td>
</tr>
<tr>
<td>Quarter 4</td>
<td>15th Feb to 14th May</td>
</tr>
</tbody>
</table>

Table 2: Firm Fundamental Characteristics

This table lists the measures on firm characteristics. The stock price, number of shares outstanding, and stock return over the past year are obtained from the market data through CRSP, while other data items are obtained from the quarterly financial statements via Compustat Capital IQ.

<table>
<thead>
<tr>
<th>Firm Fundamentals</th>
<th>Measurement Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leverage 1 (LV1)</td>
<td>Current Liability + Long Term Liability / 2</td>
</tr>
<tr>
<td></td>
<td>Share Price × Number of Shares Outstanding</td>
</tr>
<tr>
<td>Leverage 2 (LV2)</td>
<td>Total Liability / Total Asset</td>
</tr>
<tr>
<td>Interest Coverage (INT)</td>
<td>Gross Income / Interest and Related Expense</td>
</tr>
<tr>
<td>Liquidity (LIQ)</td>
<td>Working Capital / Total Asset</td>
</tr>
<tr>
<td>Profitability (PF)</td>
<td>Gross Income / Total Asset</td>
</tr>
<tr>
<td>Investment (INV)</td>
<td>Retained Earning / Total Asset</td>
</tr>
<tr>
<td>Size</td>
<td>log (Share Price × Number of Shares Outstanding / 1000)</td>
</tr>
<tr>
<td>Momentum (MOM)</td>
<td>Stock return over the past year</td>
</tr>
<tr>
<td>Dividend Price Ratio (DP)</td>
<td>log (Dividend / Share Price)</td>
</tr>
<tr>
<td>Dividend Yield (DY)</td>
<td>log (Dividend / Lagged Share Price)</td>
</tr>
<tr>
<td>Earning Price Ratio (EP)</td>
<td>log (Earning / Share Price)</td>
</tr>
<tr>
<td>Dividend Payout Ratio (DE)</td>
<td>log (Dividend / Earning)</td>
</tr>
<tr>
<td>Book-to-Market Ratio (BV)</td>
<td>Book Value / Market Value</td>
</tr>
</tbody>
</table>
Table 3: Summary Statistics of Firm Fundamental and Put Option IV Curve Characteristics

This table reports the sample statistics of the firm fundamental and IV curve characteristics from put options of 415 S&P 500 constituent firms that have options actively traded between 10 January 1996 and 27 April 2016. Panel A reports the average of each firm and IV characteristics both on the pooled sample and at each ATM IV level quintiles. Panel B reports the standard deviations on the pooled sample, cross-sectional standard deviation on each day averaged over time (XS), time-series standard deviation for each firm average over all firms (TS), and time-series standard deviation of weekly changes of characteristics for each firm averaged over all firms (TSC).

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Panel A: Mean at ATM IV Quintiles</th>
<th>Panel B: Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pooled 1 2 3 4 5</td>
<td>Pooled XS TS TSC</td>
</tr>
<tr>
<td>IV(^{(\circ)}) (%)</td>
<td>34.02 17.63 23.80 29.73 37.78 61.17</td>
<td>17.54 12.75 14.20 5.09</td>
</tr>
<tr>
<td>IV(^{+}) (%)</td>
<td>34.46 19.26 24.83 30.17 37.73 60.29</td>
<td>17.74 13.70 14.79 8.58</td>
</tr>
<tr>
<td>IV(^{-}) (%)</td>
<td>37.40 21.08 26.79 32.66 40.92 65.57</td>
<td>18.71 13.54 15.49 5.94</td>
</tr>
<tr>
<td>IV(^{(1)}) (%)</td>
<td>-2.95 -1.82 -1.97 -2.49 -3.20 -5.28</td>
<td>7.82 6.65 7.35 8.08</td>
</tr>
<tr>
<td>IV(^{(2)}) (%)</td>
<td>0.43 1.63 1.03 0.44 -0.06 -0.88</td>
<td>6.25 5.25 5.84 7.10</td>
</tr>
<tr>
<td>IV(^{(3)}) (%)</td>
<td>3.38 3.45 2.99 2.93 3.14 4.40</td>
<td>4.61 4.08 3.96 3.93</td>
</tr>
<tr>
<td>IV(^{(4)}) (%)</td>
<td>1.91 2.54 2.01 1.68 1.54 1.76</td>
<td>3.86 3.43 3.46 4.19</td>
</tr>
<tr>
<td>RV (%)</td>
<td>34.38 18.57 24.73 30.67 38.58 59.36</td>
<td>19.34 13.98 14.90 0.92</td>
</tr>
<tr>
<td>RJ (%)</td>
<td>10.98 10.66 11.45 11.51 11.08 10.20</td>
<td>15.62 15.36 15.14 10.03</td>
</tr>
<tr>
<td>RS</td>
<td>0.09 0.19 0.16 0.10 0.05 -0.06</td>
<td>1.23 1.18 1.20 0.74</td>
</tr>
<tr>
<td>LV1 (%)</td>
<td>41.77 32.66 35.83 37.29 40.92 59.32</td>
<td>65.30 54.29 24.70 5.02</td>
</tr>
<tr>
<td>LV2 (%)</td>
<td>60.14 64.68 61.08 59.15 58.23 57.54</td>
<td>20.91 20.89 7.72 0.86</td>
</tr>
<tr>
<td>INT</td>
<td>2.44 2.63 2.49 2.34 2.18 1.57</td>
<td>1.59 1.35 1.31 0.48</td>
</tr>
<tr>
<td>LIQ (%)</td>
<td>14.56 8.01 12.41 14.64 16.03 21.38</td>
<td>17.94 18.07 6.89 1.11</td>
</tr>
<tr>
<td>PF (%)</td>
<td>1.27 1.25 1.35 1.37 1.28 0.90</td>
<td>0.89 0.68 0.74 0.27</td>
</tr>
<tr>
<td>INV (%)</td>
<td>22.68 30.09 29.62 27.90 23.67 1.97</td>
<td>58.28 54.89 14.71 1.25</td>
</tr>
<tr>
<td>Size</td>
<td>9.28 9.93 9.53 9.35 9.09 8.50</td>
<td>1.26 1.21 0.58 0.05</td>
</tr>
<tr>
<td>MOM (%)</td>
<td>11.52 13.56 14.82 14.11 13.44 1.65</td>
<td>55.08 46.73 41.49 10.42</td>
</tr>
<tr>
<td>DP</td>
<td>-4.70 -4.39 -4.68 -4.85 -4.92 -4.80</td>
<td>1.11 0.98 0.76 0.23</td>
</tr>
<tr>
<td>DY</td>
<td>-4.70 -4.38 -4.68 -4.85 -4.92 -4.81</td>
<td>1.11 0.98 0.75 0.23</td>
</tr>
<tr>
<td>EP</td>
<td>-3.01 -2.93 -2.95 -3.01 -3.06 -3.10</td>
<td>0.67 0.66 0.51 0.10</td>
</tr>
<tr>
<td>DE</td>
<td>-1.78 -1.47 -1.76 -1.90 -1.97 -2.00</td>
<td>1.16 1.05 0.81 0.24</td>
</tr>
<tr>
<td>BV (%)</td>
<td>45.26 42.35 42.76 42.57 44.14 54.53</td>
<td>39.93 34.93 21.28 4.38</td>
</tr>
</tbody>
</table>

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Table 4: Summary Statistics of Cross-Sectional $R^2$ for Put Options

This table reports the sample statistics of the cross-sectional $R^2$ of fitting RIV and FIV models to the market observed put option IV characteristics. Panel A reports fitting performance from RIV models, panel B reports fitting performance from FIV models, panel C reports difference in model fitting $R^2$ between FIV and RIV models.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Max</th>
<th>Min</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: RIV model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I^o$</td>
<td>0.74</td>
<td>0.12</td>
<td>-1.42</td>
<td>2.52</td>
<td>0.91</td>
<td>0.16</td>
<td>198.33</td>
</tr>
<tr>
<td>$I^+$</td>
<td>0.61</td>
<td>0.18</td>
<td>-0.70</td>
<td>-0.49</td>
<td>0.89</td>
<td>0.12</td>
<td>108.08</td>
</tr>
<tr>
<td>$I^-$</td>
<td>0.67</td>
<td>0.16</td>
<td>-1.18</td>
<td>0.91</td>
<td>0.90</td>
<td>0.13</td>
<td>135.71</td>
</tr>
<tr>
<td>$I^{(1)}$</td>
<td>0.07</td>
<td>0.05</td>
<td>1.57</td>
<td>3.50</td>
<td>0.33</td>
<td>0.01</td>
<td>48.86</td>
</tr>
<tr>
<td>$I^{(2)}$</td>
<td>0.06</td>
<td>0.04</td>
<td>1.71</td>
<td>6.54</td>
<td>0.37</td>
<td>0.01</td>
<td>55.23</td>
</tr>
<tr>
<td>$I^{(3)}$</td>
<td>0.07</td>
<td>0.05</td>
<td>2.49</td>
<td>9.22</td>
<td>0.51</td>
<td>0.01</td>
<td>39.72</td>
</tr>
<tr>
<td>$I^{(4)}$</td>
<td>0.06</td>
<td>0.03</td>
<td>1.98</td>
<td>8.97</td>
<td>0.33</td>
<td>0.01</td>
<td>57.69</td>
</tr>
<tr>
<td><strong>Panel B: FIV model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I^o$</td>
<td>0.82</td>
<td>0.08</td>
<td>-1.77</td>
<td>4.94</td>
<td>0.94</td>
<td>0.28</td>
<td>317.99</td>
</tr>
<tr>
<td>$I^+$</td>
<td>0.73</td>
<td>0.14</td>
<td>-0.81</td>
<td>-0.30</td>
<td>0.92</td>
<td>0.29</td>
<td>168.86</td>
</tr>
<tr>
<td>$I^-$</td>
<td>0.78</td>
<td>0.11</td>
<td>-1.30</td>
<td>1.45</td>
<td>0.93</td>
<td>0.31</td>
<td>229.18</td>
</tr>
<tr>
<td>$I^{(1)}$</td>
<td>0.23</td>
<td>0.08</td>
<td>0.95</td>
<td>1.85</td>
<td>0.64</td>
<td>0.01</td>
<td>83.93</td>
</tr>
<tr>
<td>$I^{(2)}$</td>
<td>0.26</td>
<td>0.09</td>
<td>0.96</td>
<td>2.30</td>
<td>0.78</td>
<td>0.01</td>
<td>88.22</td>
</tr>
<tr>
<td>$I^{(3)}$</td>
<td>0.29</td>
<td>0.10</td>
<td>0.50</td>
<td>0.31</td>
<td>0.70</td>
<td>0.00</td>
<td>91.09</td>
</tr>
<tr>
<td>$I^{(4)}$</td>
<td>0.25</td>
<td>0.08</td>
<td>1.01</td>
<td>3.54</td>
<td>0.83</td>
<td>0.04</td>
<td>97.73</td>
</tr>
<tr>
<td><strong>Panel C: FIV - RIV model difference</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I^o$</td>
<td>0.09</td>
<td>0.07</td>
<td>3.10</td>
<td>13.85</td>
<td>0.54</td>
<td>-0.04</td>
<td>41.00</td>
</tr>
<tr>
<td>$I^+$</td>
<td>0.11</td>
<td>0.07</td>
<td>1.74</td>
<td>5.24</td>
<td>0.52</td>
<td>-0.03</td>
<td>48.44</td>
</tr>
<tr>
<td>$I^-$</td>
<td>0.11</td>
<td>0.07</td>
<td>1.85</td>
<td>4.89</td>
<td>0.49</td>
<td>-0.02</td>
<td>46.91</td>
</tr>
<tr>
<td>$I^{(1)}$</td>
<td>0.15</td>
<td>0.07</td>
<td>1.17</td>
<td>3.54</td>
<td>0.58</td>
<td>-0.08</td>
<td>66.62</td>
</tr>
<tr>
<td>$I^{(2)}$</td>
<td>0.19</td>
<td>0.09</td>
<td>1.06</td>
<td>2.93</td>
<td>0.72</td>
<td>-0.10</td>
<td>67.71</td>
</tr>
<tr>
<td>$I^{(3)}$</td>
<td>0.22</td>
<td>0.09</td>
<td>0.69</td>
<td>1.21</td>
<td>0.63</td>
<td>-0.06</td>
<td>81.18</td>
</tr>
<tr>
<td>$I^{(4)}$</td>
<td>0.20</td>
<td>0.08</td>
<td>1.17</td>
<td>4.42</td>
<td>0.77</td>
<td>-0.03</td>
<td>79.28</td>
</tr>
</tbody>
</table>
Table 5: Marginal $R^2$ of Fundamental Characteristics

This table reports the mean and media of marginal $R^2$ of each firm fundamental characteristics on IV shape characteristics, representing the reduction in $R^2$ when the corresponding firm fundamental is excluded from the FIV model.

<table>
<thead>
<tr>
<th>Levels</th>
<th>Slopes</th>
<th>Curvature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>IV°</td>
<td>IV⁺</td>
</tr>
<tr>
<td>LV1</td>
<td>Mean</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.004</td>
</tr>
<tr>
<td>LV2</td>
<td>Mean</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.000</td>
</tr>
<tr>
<td>INT</td>
<td>Mean</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.000</td>
</tr>
<tr>
<td>LIQ</td>
<td>Mean</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.002</td>
</tr>
<tr>
<td>PF</td>
<td>Mean</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.001</td>
</tr>
<tr>
<td>INV</td>
<td>Mean</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.003</td>
</tr>
<tr>
<td>Size</td>
<td>Mean</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.007</td>
</tr>
<tr>
<td>MOM</td>
<td>Mean</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.004</td>
</tr>
<tr>
<td>DP</td>
<td>Mean</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.000</td>
</tr>
<tr>
<td>DY</td>
<td>Mean</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.000</td>
</tr>
<tr>
<td>EP</td>
<td>Mean</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.001</td>
</tr>
<tr>
<td>DE</td>
<td>Mean</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.000</td>
</tr>
<tr>
<td>BV</td>
<td>Mean</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>0.001</td>
</tr>
</tbody>
</table>
Table 6: Summary Statistics of Returns from the Distance-Weighted Put Option Portfolios

This table reports the annualised mean, annualised standard deviation, skewness, and excess kurtosis of the returns generated from the distance-weighted delta-neutral and delta-gamma-neutral put option portfolios described in Equation (39) with investment horizons 1-week, 2-weeks, and 4-weeks in panel A, panel B, and panel C respectively. Student’s $t$-test statistics are also reported on the difference between FIV and RIV generated portfolio returns.

<table>
<thead>
<tr>
<th>Portfolios</th>
<th>Mean(%)</th>
<th>Std. Dev.(%)</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>IR</th>
<th>$t$-stats</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Holding Period 1-Week</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta\Pi^+$</td>
<td>9.46</td>
<td>11.36</td>
<td>5.18</td>
<td>5.24</td>
<td>4.27</td>
<td>50.38</td>
</tr>
<tr>
<td>$\Delta\Pi^+_g$</td>
<td>7.65</td>
<td>7.02</td>
<td>3.89</td>
<td>3.73</td>
<td>2.16</td>
<td>2.03</td>
</tr>
<tr>
<td>$\Delta\Pi^-_g$</td>
<td>4.69</td>
<td>4.93</td>
<td>2.50</td>
<td>2.32</td>
<td>0.13</td>
<td>0.20</td>
</tr>
<tr>
<td>$\Delta\Pi^{(1)}$</td>
<td>5.43</td>
<td>6.42</td>
<td>2.43</td>
<td>2.44</td>
<td>-0.19</td>
<td>2.25</td>
</tr>
<tr>
<td>$\Delta\Pi^{(2)}$</td>
<td>4.41</td>
<td>8.35</td>
<td>2.36</td>
<td>2.35</td>
<td>-2.60</td>
<td>3.22</td>
</tr>
<tr>
<td>$\Delta\Pi^{(3)}$</td>
<td>4.79</td>
<td>2.99</td>
<td>1.32</td>
<td>1.21</td>
<td>1.76</td>
<td>-1.17</td>
</tr>
<tr>
<td>$\Delta\Pi^{(4)}$</td>
<td>2.85</td>
<td>4.20</td>
<td>0.82</td>
<td>1.81</td>
<td>-1.18</td>
<td>3.58</td>
</tr>
<tr>
<td>Panel B: Holding Period 2-Weeks</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta\Pi^+$</td>
<td>6.36</td>
<td>9.00</td>
<td>3.59</td>
<td>3.62</td>
<td>2.30</td>
<td>2.90</td>
</tr>
<tr>
<td>$\Delta\Pi^+_g$</td>
<td>6.95</td>
<td>6.62</td>
<td>3.00</td>
<td>2.75</td>
<td>0.55</td>
<td>0.47</td>
</tr>
<tr>
<td>$\Delta\Pi^-_g$</td>
<td>4.74</td>
<td>4.58</td>
<td>1.90</td>
<td>1.75</td>
<td>-0.25</td>
<td>0.31</td>
</tr>
<tr>
<td>$\Delta\Pi^{(1)}$</td>
<td>2.88</td>
<td>3.88</td>
<td>2.00</td>
<td>1.83</td>
<td>-1.06</td>
<td>0.28</td>
</tr>
<tr>
<td>$\Delta\Pi^{(2)}$</td>
<td>1.13</td>
<td>4.60</td>
<td>2.02</td>
<td>1.82</td>
<td>-2.43</td>
<td>0.59</td>
</tr>
<tr>
<td>$\Delta\Pi^{(3)}$</td>
<td>3.61</td>
<td>2.12</td>
<td>1.13</td>
<td>1.04</td>
<td>0.92</td>
<td>-0.27</td>
</tr>
<tr>
<td>$\Delta\Pi^{(4)}$</td>
<td>1.25</td>
<td>2.55</td>
<td>0.72</td>
<td>0.65</td>
<td>-1.24</td>
<td>2.52</td>
</tr>
<tr>
<td>Panel C: Holding Period 4-Weeks</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta\Pi^+$</td>
<td>2.88</td>
<td>5.34</td>
<td>2.39</td>
<td>2.66</td>
<td>0.40</td>
<td>0.61</td>
</tr>
<tr>
<td>$\Delta\Pi^+_g$</td>
<td>4.13</td>
<td>3.52</td>
<td>2.15</td>
<td>2.20</td>
<td>-0.41</td>
<td>-1.95</td>
</tr>
<tr>
<td>$\Delta\Pi^-_g$</td>
<td>2.85</td>
<td>2.85</td>
<td>1.54</td>
<td>1.52</td>
<td>-0.80</td>
<td>-1.77</td>
</tr>
<tr>
<td>$\Delta\Pi^{(1)}$</td>
<td>1.59</td>
<td>2.51</td>
<td>1.45</td>
<td>1.35</td>
<td>-0.81</td>
<td>-0.20</td>
</tr>
<tr>
<td>$\Delta\Pi^{(2)}$</td>
<td>0.11</td>
<td>2.89</td>
<td>1.41</td>
<td>1.27</td>
<td>-2.22</td>
<td>0.58</td>
</tr>
<tr>
<td>$\Delta\Pi^{(3)}$</td>
<td>2.27</td>
<td>0.87</td>
<td>0.95</td>
<td>0.88</td>
<td>-0.71</td>
<td>0.62</td>
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<tr>
<td>$\Delta\Pi^{(4)}$</td>
<td>0.51</td>
<td>1.56</td>
<td>0.55</td>
<td>0.54</td>
<td>-0.43</td>
<td>0.27</td>
</tr>
</tbody>
</table>

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Figure 1: Change in IV curve due to change in firm characteristics

Figure 1 plots the equivalent Black’s IV curve simulated by the compound option model for the equity option pricing, and the shift of IV curves with respect to change in dividend payout, time to default, and face value of the debt of a firm. We use $V_t = 100$, $B = 40$, $r = 0.02$, $\sigma = 0.1$, $T_e = 5$, $T_p = 0.25$, $c_e = 0.5$, $c = 0.04$ for panel a, c, and e, and $c = 0.01$ for panel b, d, and f as the basis of the parameters used in the simulation, only changing $c_e$ in panel a and b, $T_e$ in panel c and d, and $B$ in panel f and g respectively.
Figure 2: Time Series of Cross-Sectional $R^2$ for Fitting Put Option IV Characteristics from RIV and FIV Models

Panel A plots the time series of $R^2$ estimates from the cross-sectional regression of IV levels from put options against the model estimates from RIV and FIV models.
Figure 2: Time Series of Cross-Sectional $R^2$ for Fitting Put Option IV Characteristics from RIV and FIV Models

Panel B plots the time series of $R^2$ estimates from the cross-sectional regression of IV slopes from put options against the model estimates from RIV and FIV models.
Panel C plots the time series of $R^2$ estimates from the cross-sectional regression of IV curvature from put options against the model estimates from RIV and FIV models.
Figure 3: Relative Weights on Firm Fundamental Characteristics

This figure plots the relative weights of fundamental characteristics inferred via Bayesian regression update for the put option IV curvature.
Figure 4: Marginal $R^2$ of Firm Fundamental Characteristics

This figure plots the marginal $R^2$ of each firm fundamental characteristics in explaining cross-sectional variation of the put option IV curvature between 1997 to 2016.

(a) Leverage 1  (b) Leverage 2  (c) Interest Coverage
(d) Liquidity  (e) Profitability  (f) Investment
(g) Size  (h) Momentum  (i) Dividend-Price Ratio
(j) Dividend Yield  (k) Earning-Price Ratio  (l) Dividend-Earning Ratio
(m) Book-to-Market Ratio
Figure 5: Time Series of One-Week Investment Horizon Portfolio Values

Panel A plots the time series of portfolio values from the IV level portfolios constructed according to RIV and FIV models.
Figure 5: Time Series of One-Week Investment Horizon Portfolio Values (Continued)

Panel B plots the time series of portfolio values from the IV slope portfolios constructed according to RIV and FIV models.

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Panel B: Time Series of Portfolio Values

- **RIV** (Red, solid line)
- **FIV** (Blue, dashed line)

**ITM-minus-OTM IV Slope Portfolios**

**ITM-minus-ATM IV Slope Portfolios**

**OTM-minus-ATM IV Slope Portfolios**

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Panel C plots the time series of portfolio values from the IV curvature portfolio constructed according to RIV and FIV models.
References


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