Macroprudential Policy Implementation in a Heterogeneous Monetary Union

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Abstract

This paper studies the conduct of macroprudential policies in a monetary union with heterogeneous members, both from a positive and a normative perspective. I develop a two-country new Keynesian general equilibrium model with housing and collateral constraints to explore this issue. I consider three types of cross-country heterogeneity: asymmetric shocks, different proportion of borrowers, and mortgage contract heterogeneity. As a macroprudential tool, I propose a Taylor-type rule for the loan-to-value ratio which responds to deviations in output and house prices. This policy can be implemented at a national or union level. Results show that the welfare-maximizing rule responds relatively more aggressively to house prices, especially in the case of the mortgage contract asymmetry. However, depending on the source of heterogeneity, the rule should be implemented at a national or a union level.

Keywords: Macroprudential, Housing market, LTV, monetary union

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"Looking ahead, I am convinced that the complementarity of the ECB’s monetary policy strategy to the new EU framework for macro-prudential oversight will contribute to enhancing crisis prevention and to strengthening the resilience of the European financial system, in an environment of price stability. We should not forget – and the crisis will not allow us to forget at least for some time – that prevention is always better than cure". Lucas Papademos, 3 May 2010.

1 Introduction

The severe crisis we have experienced has taught us that we need to use policies to avoid such episodes to happen again. Scholars and policy makers agree that macroprudential measures could help avoid systemic risks and ensure a more stable financial system. Although the empirical evidence is still scarce, some central banks have successfully already implemented policies of this type. We can find some examples in emerging markets, especially in Asia. These macroprudential measures include countercyclical capital buffers linked to credit growth, countercyclical provisioning, loan-to-value (LTV) limits or direct controls on lending to specific sector.

Research on this topic is still at an early stage, however there is consensus that macroprudential policies deliver financial stability and that this is welfare enhancing. A common way to introduce this kind of policies in general equilibrium models is through automatic rules that cut credit when the economy is expanding. These rules serve as a proxy for the instruments that have been used by some institutions. We find in the literature Taylor-type rules that restrict credit whenever the economy experiences a boom. LTV rules have become particularly popular. See for instance, Gruss and Sgherri (2009) which analyses the welfare effects of procyclical loan-to-value ratios in a real business cycle model with borrowing constraints. Funke and Paetz (2012) uses a non-linear rule on the LTV and finds that it can help reduce the transmission of house price cycles to the real economy. In a similar way, Kannan, Rabanal and Scott (2012) examines a monetary policy rule that reacts to prices, output and changes in collateral values with a macroprudential instrument based on the LTV.

When applying macroprudential policies, it has to be taken into account the fact that these measures need to coexist with other policies such as monetary policy. Monetary policy aims at ensuring price stability while macroprudential measures focus on maintaining financial stability. Those two goals, although they could reinforce each other, could also be in conflict in some occasions. In principle, a stable financial system may help the monetary policy transmission mechanism work in a more effective
However, it could be that monetary policy reactions to the effects of macroprudential measures make house prices move in a non-desirable direction (See Rubio and Carrasco-Gallego (2013)). The literature also shows that welfare benefits of introducing macroprudential policies that interact with monetary policy depend on the source of the shocks and on the specification of the Taylor rule (See Kannan et al (2012) and Rubio and Carrasco-Gallego (2013)).

Nevertheless, the implementation of these macroprudential tools becomes more complex if countries are not able to manage their own monetary policy and rely on a single central bank that acts in favor of majority. Optimal currency areas has been a much-discussed topic, especially in relation to Europe's Economic and Monetary Union (EMU). Cross-country asymmetries or country-specific shocks have been an issue of concern and skepticism for the well-functioning of EMU. Countries in Europe clearly differ in their housing markets. There is evidence of different loan-to-value ratios (LTVs), different proportions of residential debt relative to GDP across countries, and heterogeneous mortgage contracts. Also, house-price movements do not follow the same pattern in every country. In its study "Housing Finance in the Euro Area," the European Central Bank (ECB 2009) also remarks on the importance of such differences for the EMU. Table 1 in the Appendix shows that countries in Europe have different LTVs, as well as different residential-debt-to-GDP ratios. LTVs are as low as 50% in Italy and as high as 90% in the Netherlands, where the debt-to-GDP ratio exceeds 100%. In countries with a high LTV or a high proportion of indebted consumers, housing collateral effects are stronger. Differences in mortgage contracts across countries are another important source of heterogeneity in Europe. In countries such as Germany or France, the majority of mortgages are fixed rate. Conversely, the predominant type of mortgages in such countries as the United Kingdom, Spain, and Greece is variable rate. Calza et al (2009) and Rubio (2011) showed that the mortgage structure of an economy is an important factor in the transmission of shocks.

There is an extensive literature that shows that institutional, consumption, financial or housing market heterogeneity can endanger the optimality of EMU as a currency area (See Maclellan et al., 1998, ECB, 2009, Rubio, 2013). However, if an extra set of policies, namely macroprudential, are to be introduced in the European context, researchers and policy-makers also have to question themselves what the optimal design for such policies in such specific framework is. Macroprudential regulation could be implemented at a union level, as monetary policy, and respond to the average performance of the whole area. The alternative would be to have a decentralized system of national regulators which would take into account the economic conditions of their specific region. This question is irrelevant
in an homogenous union. However, given the single monetary policy restriction, if we find important cross-country differences or asymmetric shocks, we need to assess if the best option is having centralized or decentralized macroprudential policies.

In this paper, I analyze the implementation of macroprudential policies, in particular a rule on the LTV, in the context of a monetary union with heterogeneous members.¹ I develop a two-country new Keynesian general equilibrium model with housing and collateral constraints, allowing for cross-country differences in mortgage and housing markets as well as asymmetric technology shocks. In particular, I allow for differences in borrower’s labor-income shares across countries, as a proxy for different strengths of the financial accelerator. I also consider differences in the structure of mortgage contracts (fixed versus variable rate). I provide a quantitative study on how monetary and macroprudential measures should be conducted in the euro area and pay special attention at how cross-country structural differences in housing markets affect the implementation of these policies.

In this paper, I propose an implementation of the macroprudential policy which is analogous to how monetary policy is conducted. In particular, I assume that the same way that the central bank follows a Taylor rule for monetary policy, the macroprudential authority also follows a linear rule to carry out the macroprudential policy. The monetary policy literature has extensively shown that simple rules result in a good performance; therefore it seems sensible to apply this kind of rules to macroprudential supervision (See Yellen, 2010).

There is consensus that the objectives of the monetary and the macroprudential authority should include output, inflation and financial stability. In order to achieve these objectives, monetary policy uses the interest rate as an instrument while the macroprudential authority uses the LTV. I consider a rule on the LTV ratio which responds to output and house prices. The macroprudential regulator aims at avoiding systemic risk and excessive credit growth. However, in practical terms these objectives are not so clear. The IMF (2013) states that a macroeconomic environment which gives rise to credit growth will contribute to the build-up of systemic risk. Therefore, booms that lead to increase in borrowing should be moderated. They also consider that a rise in house prices can act as a leading indicator of excessive credit growth since they lead to wealth effects that permit the increase in borrowing. Then, following this lines, I propose that the macroprudential regulator follows a Taylor-type rule in which the LTV responds to house prices and output.

¹Rules on capital requirement ratios are also common in the literature, however they require the introduction of a financial intermediary that sets different interest rates for savers and borrowers. Given that in this paper I am already considering two different rates for borrowers (fixed and variable), for simplicity, I restrict to rules on the LTV.
The basic modelling framework follows closely Rubio (2013), to which I add macroprudential measures. In each country, there is a group of individuals that are credit constrained and need housing collateral to obtain loans. Countries trade goods, and savers in each country have access to foreign assets. I study how dynamics change if we introduce centralized and decentralized macroprudential measures. Finally, from a normative point of view, I numerically calculate consumption equivalent welfare improvements when macroprudential policies are introduced, comparing both the national and the union level implementation. Then, I obtain the optimal combination of LTV rule reaction parameters that maximizes welfare for each source of asymmetry, given monetary policy.

This paper relates to different strands of the literature. The model constitutes a two-country version of the seminal paper Iacoviello (2005), that introduces a financial accelerator that works through the housing sector, in the flavor of Aspachs and Rabanal (2010). However, it introduces cross-country housing-market heterogeneity as in Rubio (2013). This paper is also related to the recent literature on macroprudential and monetary policies in Iacoviello-type models such in the aforementioned Kannan, Rabanal and Scott (2012) or Rubio and Carrasco-Gallego (2013). However, it explores the issue in a two-country setting as in Quint and Rabanal (2013) and Brzoza-Brzezina et al (2012). However, these two latter papers only consider country size and asymmetric shocks as the only source of heterogeneity, they are silent about the effects of institutional or housing market asymmetries on the implementation of macroprudential measures. The novelty of this paper is that I introduce structural differences across countries, namely differences in the financial accelerator strength and different mortgage structures, and I find that they matter for the optimal conduction of macroprudential policies. It is not the focus of this paper to study the coordination problem between the two policies as in Quint and Rabanal (2013) and Angelini et al. (2012). In the present paper, I abstract from this issue by restricting the problem to the special case in which the macroprudential regulator takes monetary policy as given and study if it should conducted at a national or at a union level depending on the structure of the economy.

Results show that, for the case of symmetry across countries, which we take as a benchmark, introducing an LTV rule is unambiguously welfare enhancing for the economy. Although at the expense of the savers, borrowers benefit from a more stable financial system that help them smooth consumption. Given monetary policy, the combination of parameters that maximizes welfare is one in which the LTV rule reacts relatively more aggressively to house prices rather than to deviations in output. However, when there is an asymmetric shock, welfare gains appear especially in the case of a centralized policy.

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2 In this paper I abstract from differences in country size and I focus on structural asymmetries.
because, since the shock is transmitted to the other country, the whole union can benefit from a more stable financial system. For the case of different strengths of the financial accelerator, the union also benefits if there are macroprudential policies, however, there is no difference between the centralized and the decentralized case, since the shock causes redistributinal effects but not differences in aggregate volatility. The optimal combination of parameters does not differ whether we are in a centralized or decentralized setting. Finally, when the asymmetry comes from different mortgage contracts, the same shock delivers different volatilities in each country. In this case, the optimal rule should respond even more aggressively to house prices than with the other asymmetries. Furthermore, if the rule is implemented at the national level, the fixed-rate country should implement a rule which fights more strongly to house price movements, as compared with the variable-rate country. This policy combination would deliver a higher level of welfare than the decentralized case. In conclusion, asymmetries in a monetary union matter for the conduct of macroprudential policies, especially when heterogeneity results in differences in aggregate volatility. In this case, a decentralized macroprudential policy is preferable. However, when the only source of heterogeneity is an asymmetric shock, a centralized macroprudential regulation can help homogenize the effects of the shock and therefore facilitate the conduct of the single monetary policy. In the case of different proportion of borrowers across countries, the national or union-level implementation of policies is irrelevant.

The paper is organized as follows. Section 2 describes the model. Section 3 presents the model dynamics. Section 4 analyzes welfare and the optimal combination of policy parameters. Section 5 concludes. Tables, steady-state relationships, and the linearized model are shown in the Appendix.

2 Model Setup

I consider an infinite-horizon, two-country economy inside a monetary union. The home country is denoted by A and the rest of the union by B. Households consume, work, and demand real estate. There is a financial intermediary in each country that provides mortgages and accepts deposits from consumers. Each country produces one differentiated intermediate good, but households consume goods from both countries. For simplicity, housing is a non-traded good. I assume that labor is immobile across the countries. Firms follow a standard Calvo problem. In this economy, both final and intermediate goods are produced. Prices are sticky in the intermediate-goods sector. Monetary policy is conducted by a single central bank that responds to a weighted average of inflation in both countries. There is
a rule to the LTV which serves as a macroprudential measure. I explore two scenarios; one in which macroprudential policies are centralized at the union level and a second one in which each country can conduct its own macroprudential policy. I allow for housing-market heterogeneity across the countries.

2.1 The Consumer’s Problem

There are three types of consumers in each country: unconstrained consumers, constrained consumers who borrow at a variable rate, and constrained consumers who borrow at a fixed rate. The proportion of each type of borrower is fixed and exogenous. Consumers can be constrained or unconstrained in the sense that constrained individuals need to collateralize their debt repayments in order to borrow from the financial intermediary. Interest payments in the next period cannot exceed a proportion of the future value of the current house stock. In this way, the financial intermediary ensures that borrowers are going to be able to fulfill their debt obligations in the next period. As in Iacoviello (2005), I assume that constrained consumers are more impatient than unconstrained ones. There is a financial intermediary in each country. The financial intermediary in Country A accepts deposits from domestic savers, and it extends both fixed- and variable-rate loans to domestic borrowers.

2.1.1 The Financial Intermediary

I assume a competitive framework, and thus the intermediary takes the variable interest rate as given. The profits of the financial intermediary are defined as:

\[ F_t = \alpha_A R_{A,t-1} b_{t-1}^{cv} + (1 - \alpha_A) R_{A,t-1} b_{t-1}^{cf} - R_{A,t-1} b_{t-1}^u. \]  

(1)

In equilibrium, aggregate borrowing and saving must be equal, that is,

\[ \alpha_A b_t^{cv} + (1 - \alpha_A) b_t^{cf} = b_t^u. \]  

(2)

Substituting (2) into (1), we obtain,
\[ F_t = (1 - \alpha_A) b^c_{t-1} (R_{At-1} - R_{At-1}) . \] (3)

For the two types of mortgage to be offered, the fixed-interest rate has to be such that the intermediary is indifferent between lending at a variable or fixed rate. Hence, the expected discounted profits that the intermediary obtains by lending new debt in a given period at a fixed-interest rate must be equal to the expected discounted profits the intermediary would obtain by lending it at a variable rate:

\[ E_{\tau} \sum_{i=\tau+1}^{\infty} \beta^{i-\tau} \Lambda_{\tau,i} R^{OPT}_{At} = \frac{E_{\tau} \sum_{i=\tau+1}^{\infty} \beta^{i-\tau} \Lambda_{\tau,i} R_{Ai-1}}{E_{\tau} \sum_{i=\tau+1}^{\infty} \beta^{i-\tau} \Lambda_{\tau,i}} , \] (4)

where \( \Lambda_{t,i} = \frac{C_{At}}{C_{At+i}} \) is the unconstrained-consumer relevant discount factor. Since the financial intermediary is owned by the savers, their stochastic discount factor is applied to the financial intermediary’s problem. Notice that, as stated before, variable-rate debt is in one period, but the portion of new debt acquired at a fixed rate is associated with a long-term contract. Since the agent is infinitely lived, I assume here that the maturity of fixed-rate mortgages is also infinity.

We can obtain the equilibrium value of the fixed rate in period \( \tau \) from expression (4):

\[ R^{OPT}_{At} = \frac{E_{\tau} \sum_{i=\tau+1}^{\infty} \beta^{i-\tau} \Lambda_{\tau,i} R_{Ai-1}}{E_{\tau} \sum_{i=\tau+1}^{\infty} \beta^{i-\tau} \Lambda_{\tau,i}} . \] (5)

Equation (5) states that for every new debt issued at date \( \tau \), there is a different fixed-interest rate that has to be equal to a discounted average of future variable-interest rates. Notice that this is not a condition on the stock of debt, but on the new amount obtained in a given period. New debt at a given point in time is associated with a different fixed-interest rate. Both the fixed-interest rate in period \( \tau \) and the new amount of debt in period \( \tau \) are fixed for all future periods. However, the fixed-interest rate varies with the date the debt was issued, so that in every period there is a new fixed-interest rate associated with new debt in this period. If we consider fixed-rate loans to be long term, the financial intermediary obtains interest payments every period from the whole stock of debt, not only from the new ones. Hence, we can define an aggregate fixed-interest rate as the one the financial intermediary effectively charges every period for the whole stock of mortgages. This aggregate fixed-interest rate is composed of all past fixed-interest rates and past debt, together with the current-period equilibrium fixed-interest rate and new amount of debt. Therefore, the effective fixed-interest rate that the financial
intermediary charges for the stock of fixed-rate debt every period is as follows:

\[
\bar{R}_{At} = \begin{cases} 
\frac{\bar{R}_{At-1}b_{t-1}^{cf}+\bar{R}_{At}^{OPT}(b_{t}^{cf}-b_{t-1}^{cf})}{b_{t}^{cf}} & \text{if } b_{t}^{cf} > b_{t-1}^{cf} \\
\bar{R}_{At-1} & \text{if } b_{t}^{cf} \leq b_{t-1}^{cf}
\end{cases}
\]  
(6)

Equation (6) states that the fixed-interest rate that the financial intermediary charges today is an average of what it charged the previous period for the previous stock of mortgages and what it charges in the current period for the new amount. If there is no new debt, the fixed-interest rate will be equal to that of the previous period. Then, in the same way that variable rates are revised every period, fixed-rates are revised by including the new optimal fixed-interest rate for the new debt originating in this period. Importantly, this assumption is not crucial for results. Both \(\bar{R}_{At}^{OPT}\) and \(\bar{R}_{At}\) are practically unaffected by interest rate shocks.\(^7\) This assumption is a way to make the model compatible with the fact that fixed-rate loans are not one-period assets but longer-term ones.

As noted above any profits from financial intermediation are rebated to the unconstrained consumers every period. Even if the financial intermediary is competitive and does not make profits in the absence of shocks, should a shock occur, the fact that only the variable-interest rate is directly affected can generate non-zero profits.\(^8\)

The financial intermediary problem for Country B is symmetrical.

2.1.2 Unconstrained Consumers (Savers)

Unconstrained consumers in Country A maximize as follows:

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln C_t^u + j \ln H_t^u - \frac{(L_t^u)^{\eta}}{\eta} \right),
\]  
(7)

Here, \(E_0\) is the expectation operator, \(\beta \in (0, 1)\) is the discount factor, and \(C_t^u\), \(H_t^u\), and \(L_t^u\) are consumption at \(t\), the stock of housing, and hours worked, respectively.\(^9\) \(j\) represents the weight of housing in the utility function. \(1/ (\eta - 1)\) is the aggregate labor-supply elasticity.

Consumption is a bundle of domestically and foreign-produced goods, defined as: \(C_t^u = (C_{At}^u)^n (C_{Bt}^u)^{1-n}\), where \(n\) is the size of Country A.

The budget constraint for Country A is as follows:

\(^7\)In log-linearized terms, the new fixed interest rate is always equal to the past fixed interest rate, therefore, equation (6) does not introduce a kink.

\(^8\)This modelling of the fixed interest rate follows Rubio (2011) and Rubio (2013).

\(^9\)It is assumed that housing services are proportional to the housing stock.
\[ P_{At}C_{At}^u + P_{Bt}C_{Bt}^u + Q_{At}H_{t}^u + R_{At-1}B_{t-1}^u + R_{t-1}D_{t-1} + \frac{\psi}{2}D_{t}^2 \leq Q_tH_{t-1}^u + \\
W_{t}^uL_{t}^u + B_{t}^u + D_{t} + P_{At}F_{t} + P_{At}S_{t}, \quad (8) \]

where \( P_{At} \) and \( P_{Bt} \) are the prices of the goods produced in Countries A and B, respectively, \( Q_t \) is the housing price in Country A, and \( W_{t}^u \) is the wage for unconstrained consumers. \( B_{t}^u \) represents domestic bonds denominated in the common currency. \( R_{At} \) is the nominal interest rate in Country A. Positive bond holdings signify borrowing, and negative signify savings. However, as we will see, this group will choose not to borrow at all: they are the savers in this economy. \( D_{t} \) are foreign-bond holdings by savers in Country A.\(^{10} \) \( R_{t} \) is the nominal rate of foreign bonds, which are denominated in euros. As is common in the literature, to ensure stationarity of net foreign assets I introduced a small quadratic cost of deviating from zero foreign borrowing, \( \frac{\psi}{2}D_{t}^2. \)\(^{11} \) Savers obtain interest on their savings. \( S_{t} \) and \( F_{t} \) are lump-sum profits received from the firms and the financial intermediary in Country A, respectively.

Dividing by \( P_{At} \), we can rewrite the budget constraint in terms of goods A:

\[ C_{At}^u + \frac{P_{Bt}}{P_{At}}C_{Bt}^u + q_{At}H_{t}^u + \frac{R_{At-1}B_{t-1}^u}{\pi_{At}} + \frac{R_{t-1}D_{t-1}}{P_{At}} + \frac{\psi}{2}D_{t}^2 \leq Q_tH_{t-1}^u + w_{t}^uL_{t}^u + b_{t}^u + d_{t} + F_{t} + S_{t}, \quad (9) \]

where \( \pi_{At} \) denotes inflation for the goods produced in Country A, defined as \( P_{At}/P_{At-1}. \)

Maximizing (7) subject to (9), we obtain the first-order conditions for the unconstrained group:

\[ \frac{C_{At}^u}{C_{Bt}^u} = \frac{nP_{Bt}}{(1 - n)P_{At}} \quad (10) \]

\[ \frac{1}{C_{At}^u} = \beta E_{t} \left( \frac{R_{At}}{\pi_{At+1}C_{At+1}^u} \right), \quad (11) \]

\[ \frac{1 - \psi d_{t}}{C_{At}^u} = \beta E_{t} \left( \frac{R_{t}}{\pi_{At+1}C_{At+1}^u} \right), \quad (12) \]

\(^{10} \)Savers have access to international financial markets.

\(^{11} \)See Iacoviello and Smets (2006) for a similar specification of the budget constraint.
\[ w_t^u = (L_t^u)^{n-1} \frac{C_{At}^u}{n}. \]  

(13)

\[ \frac{j}{H_t^u} = \frac{n}{C_{At}^u}q_{At} - \beta E_t \frac{n}{C_{At+1}^u}q_{At+1}. \]  

(14)

Equation (10) equates the marginal rate of substitution between goods to the relative price. Equation (11) is the Euler equation for consumption. Equation (12) is the first-order condition for net foreign assets. Equation (13) is the labor-supply condition. These equations are standard. Equation (14) is the Euler equation for housing and states that at the margin the benefits from consuming housing have to be equal to the costs.

Combining (11) and (12) we obtain a non-arbitrage condition between home and foreign bonds:\(^{12}\)

\[ R_{At} = \frac{R_t}{(1 - \psi d_t)}. \]  

(15)

Since all consumption goods are traded and there are no barriers to trade, I assume in this paper that the law of one price holds:

\[ P_{At} = P_{At}^*, \]  

(16)

where variables with a star denote foreign variables.

2.1.3 Constrained Consumers (Borrowers)

Constrained consumers in Country A are of two types: those who borrow at a variable rate and those who do so at a fixed rate. The difference between them is the interest rate they are charged. The variable-rate constrained consumer faces \( R_{At} \), which will coincide with the rate set by the central bank. The fixed-rate borrower pays \( \overline{R}_{At} \), derived from the financial intermediary’s problem. The proportion of variable-rate consumers in Country A is constant and exogenous and is equal to \( \alpha_A \in [0, 1] \).

Constrained consumers are more impatient than unconstrained ones, that is \( \tilde{\beta} < \beta \). Constrained consumers face a collateral constraint: the expected debt repayment in the next period cannot exceed a proportion of the expectation of tomorrow’s value of today’s stock of housing:

\(^{12}\)The log-linearized version of this equation could be interpreted as the uncovered interest-rate parity.
where equations (17) and (18) represent the collateral constraint for the variable- and fixed-rate borrower, respectively. $k_{At}$ can be interpreted as the loan-to-value ratio in Country A. Notice that such models with collateral constraints, the LTV is typically considered exogenous. At the macroeconomic level, LTVs partly depend on exogenous factors such as regulation. This parameter is usually calibrated to match the average LTV in the country analyzed. However, in this model, it can vary depending on economic conditions, as a macroprudential policy variable. As I pointed out when I introduced the problem of the financial intermediary, $R_{At}$ is an aggregate interest rate that contains information on all the past fixed-interest rates associated with past debt. Each period, this aggregate interest rate is updated with a new interest rate linked to the new amount of debt originating in that period.

Without loss of generality, I present the problem for the variable-rate borrower since that for the fixed rate is symmetrical. Variable-rate borrowers maximize their lifetime utility function:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln C_{at}^{cv} + j \ln H^{cv}_t - \frac{(L_t^{cv})^\eta}{\eta} \right),$$

where $C_t^{cv} = (C_{At}^{cv})^n (C_{Bt}^{cv})^{1-n}$, subject to the budget constraint (in terms of good A):

$$C_{At}^{cv} + \frac{P_{Bt}}{P_{At}} C_{Bt}^{cv} + q_{At} H^{cv}_t + \frac{R_{At-1} b^{cv}_{t-1}}{\pi_{At}} \leq q_{At} H^{cv}_{t-1} + w^{cv}_t l^{cv}_t + b^{cv}_t,$$

and subject to the collateral constraint (17). Notice that variable-rate borrowers repay all debt every period and acquire new debt at the current new interest rate. This assumption implies that the interest rate on variable-rate mortgages is revised every period for the whole stock of debt and changed according to the policy rate.\(^{13}\) To make the problem for fixed-rate borrowers symmetrical and analogous to existing models with borrowing constraints, I assume the same debt-repayment structure for this type of borrower. Obviously, fixed-rate contracts are not revised every period. However, to make the model more realistic,

\(^{13}\)This assumption is consistent with reality, in which variable-interest rates are revised very frequently and changed according to an interest-rate index tied to the interest rate set by the central bank.
but still tractable, the fixed-interest rate will be such that a revised fixed rate will be applied only on new
debt, keeping constant the interest rate applied to existing debt. In this way, I reconcile the structure
of the model with the fact that fixed-rate contracts are long term.\footnote{Another option would be to have an overlapping generation model in which we are able to keep track of the debt issued
each period. However, the model would become more complex and less comparable with the standard collateral constraint
DSGE models, such as that of Iacoviello (2005).}

The first-order conditions for these consumers are as follows:

\[
\frac{C_{At}^{cv}}{C_{Bt}^{cv}} = \frac{n P_{Bt}}{(1 - n) P_{At}} \quad (21)
\]

\[
\frac{n}{C_{At}^{cv}} = \beta E_t \left( \frac{n R_{At}}{\pi_{At+1} C_{At+1}^{cv}} \right) + \lambda_{At}^{cv} R_{At}, \quad (22)
\]

\[
w_{it}^{cv} = (L_{it}^{cv})^{-1} \frac{C_{At}^{cv}}{n}, \quad (23)
\]

\[
\frac{J}{H_{it}^{cv}} = \frac{n}{C_{At}^{cv}} q_{At} - \beta E_t \frac{n}{C_{At+1}^{cv}} q_{At+1} - \lambda_{At}^{cv} E_t q_{At+1} \pi_{At+1}. \quad (24)
\]

These first-order conditions differ from those of unconstrained individuals. In the case of constrained
consumers, the Lagrange multiplier on the borrowing constraint ($\lambda_{At}^{cv}$) appears in equations (22) and (24). As in Iacoviello (2005), the borrowing constraint is always binding, so that constrained individuals
borrow the maximum amount they are allowed, and their saving is zero.\footnote{From the Euler equations for consumption of the unconstrained consumers, we know that $R_A = 1/\beta$, where variables
without a time subscript denote steady-state variables. If we combine this result with the Euler equation for consumption
for the constrained individual, we have $\lambda^{cv} = n \left( \beta - \beta \right) / C_{At}^{cv} > 0$. Given that $\beta > \beta$, the borrowing constraint holds with
equality in steady state. Since the model is log-linearized around the steady state and low uncertainty is assumed, this
result can be generalized to off-steady-state dynamics.}

The problem for consumers is analogous in Country B.

\section{2.2 Firms}

\subsection{2.2.1 Final-Goods Producers}

In Country A, there is a continuum of final-goods producers that aggregate intermediate goods according
to the production function:

\[
Y_{1t}^{k} = \left[ \int_{0}^{1} Y_{1t}^{k} (z) \frac{z-1}{z} dz \right]^\frac{1}{\tau-1}, \quad (25)
\]
where $\varepsilon > 1$ is the elasticity of substitution among intermediate goods.

The total demand of intermediate-good $z$ is given by $Y_{At}(z) = \left(\frac{P_{At}(z)}{P_{At}}\right)^{-\varepsilon} Y_{At}$, and the price index is $P_{At} = \left[\int_0^1 P_{At}(z)^{1-\varepsilon} \, dz\right]^{1/\varepsilon}.$

### 2.2.2 Intermediate-Goods Producers

The intermediate-goods market is monopolistically competitive. Following Iacoviello (2005), intermediate goods are produced according to the following production function:

$$Y_{At}(z) = \xi_t (L^u_t(z))^{\gamma_A} (L^c_t(z))^{(1-\gamma_A)},$$

where $\xi_t$ represents technology. I assume that $\log \xi_t = \rho_\xi \log \xi_{t-1} + u_{\xi_t}$, where $\rho_\xi$ is the autoregressive coefficient and $u_{\xi_t}$ is a normally distributed shock to technology. $\gamma_A \in [0, 1]$ measures the relative size of each group in terms of labor. We make this parameter country specific, as a proxy for the different debt-to-GDP ratios we observe across countries. $L^c_t$ is labor supplied by constrained consumers, defined as $\alpha_A L^c_t + (1 - \alpha_A) L^c_f$.

The first-order conditions for labor demand are the following:

$$w^u_t = \frac{\xi_t}{X_t} \frac{Y_{At}}{L^u_t},$$
$$w^c_t = \frac{\xi_t}{X_t} (1 - \gamma_A) \frac{Y_{At}}{L^c_t},$$

where $X_t$ is the markup, or the inverse of marginal cost.

The price-setting problem for the intermediate-goods producers is a standard Calvo-Yun case. An intermediate-goods producer sells goods at price $P_{At}(z)$, and $1 - \theta$ is the probability of being able to change the sale price in every period. The optimal reset price $P^\text{OPT}_{At}(z)$ solves the following:

$$\sum_{k=0}^{\infty} (\theta \beta)^k E_t \left\{ \Lambda_{t,k} \left[ \frac{P^\text{OPT}_{At+k}(z)}{P_{At+k}} - \frac{\varepsilon}{(\varepsilon - 1)} \right] Y^\text{OPT}_{At+k}(z) \right\} = 0.$$ (29)

The aggregate price level is given as follows:

$$P_{At} = \left[ \theta P_{At-1}^{1-\varepsilon} + (1 - \theta) P^\text{OPT}_{At} \right]^{1/(1-\varepsilon)}.$$ (30)

---

16Symmetry across firms allows avoiding index $z$. 

Using (29) and (30) and log-linearizing, we can obtain the standard forward-looking Phillips curve (see equation (80) in the Appendix).\textsuperscript{17}

The firm problem is similar in Country B.

### 2.3 Aggregate Variables and Market Clearing

Given $\alpha_A$, the fraction of variable-rate borrowers in Country A, we can define aggregates across constrained consumers as the sum of variable-rate and fixed-rate aggregates, so that $C_t^c = \alpha_A C_t^{cv} + (1 - \alpha_A) C_t^{cf}$, $H_t^c = \alpha_A H_t^{cv} + (1 - \alpha_A) H_t^{cf}$ and $b_t^c = \alpha_A b_t^{cv} + (1 - \alpha_A) b_t^{cf}$.

Therefore, economy-wide aggregates in Country A are $C_t = C_t^u + C_t^c$, $L_t = L_t^u + L_t^c$. The aggregate supply of housing is fixed, so that market clearing requires $H_t = H_t^u + H_t^c = H$.\textsuperscript{18}

The market clearing condition for the final good in Country A is $nY_t = nC_t^u + (1 - n) C_t^v + n \psi d_t^2$. Domestic financial markets clear: $b_t^c = b_t^u$. The world bond market clearing condition is $nd_t + \frac{P_B t}{P_A t} d_t^* = 0$, where $d_t$ denotes the foreign bonds in real terms. The net foreign asset position follows $d_t = \frac{R_{t-1}}{(1-\psi d_t)} d_{t-1} + Y_t - C_t^u - \frac{P_B t}{P_A t} C_B t$. Everything is similar in Country B.

### 2.4 Monetary Policy

The model closes with a Taylor rule, with interest-rate smoothing for interest-rate setting by a single central bank.\textsuperscript{19}

$$R_t = (R_{t-1})^\rho \left( \left( \frac{n}{(1-n)} \psi \right) (1+\phi) \right)^{1-\rho} \varepsilon_{R,t}, \quad (31)$$

$0 \leq \rho \leq 1$ is the parameter associated with interest-rate inertia. $(1 + \phi)$ measures the sensitivity of interest rates to current inflation. $\varepsilon_{R,t}$ is a white noise shock process with zero mean and variance $\sigma^2$.

This rule is consistent with the primary objective of the ECB being price stability.

\textsuperscript{17}This Phillips curve is consistent with other two-country models with financial accelerator. See for instance Gilchrist et al (2002) or Iacoviello and Smets (2006).

\textsuperscript{18}An endogenous supply of housing could be easily introduced in a two-sector version of this model. However, the qualitative results would not change for the demand side of the model which is the focus of this paper. For two-sector models, see, for example, Iacoviello and Smets (2006) or Iacoviello and Neri (2010).

\textsuperscript{19}This type of rule is also used in other monetary-union models. See Iacoviello and Smets (2006) or Aspachs and Rabanal (2008). Furthermore, as shown in Iacoviello (2005) and Rubio and Carrasco-Gallego (2013), a rule that only responds to inflation enhances the financial accelerator.
2.5 Macroprudential Policy

As an approximation for a realistic macroprudential policy, I consider a Taylor-type rule for the loan-to-value ratio. In standard models, the LTV ratio is a fixed parameter which is not affected by economic conditions. However, we can think of regulations of LTV ratios as a way to moderate credit booms. When the LTV ratio is high, the collateral constraint is less tight. And, since the constraint is binding, borrowers will borrow as much as they are allowed to. Lowering the LTV tightens the constraint and therefore restricts the loans that borrowers can obtain. Recent research on macroprudential policies has proposed Taylor-type rules for the LTV ratio so that it reacts inversely to variables such that the growth rates of GDP, credit, the credit-to-GDP ratio or house prices. These rules can be a simple illustration of how a macroprudential policy could work in practice. Here, I assume that there exists a macroprudential Taylor-type rule for the LTV ratio, so that it responds to output and house prices.\footnote{I have also experimented with rules that react directly to credit growth and results for the dynamics of the model are similar.} The first variable would correspond to the objective of the macroprudential regulator to moderate booms in the economy that could lead to an excessive credit growth. As for the house prices, given collateral constraints, they are the key causal variable for the dynamics of loans to households, and it appears to correspond to the actual behavior of policymakers.\footnote{See Angelini et al. (2012) for further discussion.}

We consider first a case in which the macroprudential policy is centralized, that is, as monetary policy, is implemented by a simple regulator that takes into account an average of the credit growth in both countries:

\[
k_t = k_{SS} \left[ \left( \frac{Y_{At}}{Y_A} \right)^n \left( \frac{Y_{Bt}}{Y_B} \right)^{1-n} \right]^{-\phi_y^k} \left[ \left( \frac{q_{At}}{q_A} \right)^n \left( \frac{q_{Bt}}{q_B} \right)^{1-n} \right]^{-\phi_q^k},
\]

where \(k_{SS}, Y_A,\) and \(q_A\) are the steady-state values for the loan-to-value ratio, output and house prices in country A. \(\phi_y^k \geq 0, \phi_q^k \geq 0\) measure the response of the loan-to-to value to output and house prices, respectively. This kind of rule would deliver a lower LTV ratio in booms, when output and house prices are high, therefore restricting the credit in the economy and avoiding a credit boom derived from good economic conditions.

The second case is the decentralized macroprudential policy in which each country can implement its own rule:

\[
k_{At} = k_{SSA} \left( \frac{Y_{At}}{Y_A} \right)^{-\phi_y^k} \left( \frac{q_{At}}{q_A} \right)^{-\phi_q^k},
\]
\[ k_{Bt} = k_{SSB} \left( \frac{Y_{Bt}}{Y_{B}} \right)^{-\phi_{B}^k} \left( \frac{q_{Bt}}{q_{B}} \right)^{-\phi_{B}^q}. \]  

(34)

3 Dynamics

3.1 Parameter Values

The discount factor for savers, \( \beta \), is set to 0.99 so that the annual interest rate is 4% in steady state. The discount factor for borrowers, \( \tilde{\beta} \), is set to 0.98.\(^{22}\) The steady-state weight of housing in the utility function, \( j \), is set to 0.1 in order for the ratio of housing wealth to GDP to be approximately 1.40 in the steady state.\(^{23}\) I set \( \eta = 2 \), implying a value of the labor supply elasticity of 1.\(^{24}\) For the loan-to-value ratio I considered a steady-state value of 0.9, as in Iacoviello, 2013, in order to emphasize the financial accelerator mechanism. The labor-income share of unconstrained consumers, \( \gamma_A = \gamma_B \), was set to 0.7 for the symmetric case.\(^{25}\) I picked a value of 6 for \( \varepsilon \), the elasticity of substitution among intermediate goods. This value implies a steady-state markup of 1.2. The probability of not changing prices, \( \theta \), is set to 0.75, implying that prices change every four quarters on average. For the Taylor Rule parameters, I used \( \rho = 0.8 \), \( \phi_n = 0.5 \). The first value reflects a realistic degree of interest-rate smoothing.\(^{26}\) \( \phi_n \) is consistent with the original parameters proposed by Taylor in 1993. For the baseline model, I considered \( \alpha_A = \alpha_B = 1 \), that is, all mortgages are variable rate.\(^{27}\) However, I also considered the case of fixed-rate mortgages. In order to focus on the rest of asymmetries, I consider that the two countries are equal in size. For the macroprudential policy rule, \( \phi^k_y = \phi^k_q = 0.05 \) as a benchmark, although I will do welfare experiments with a continuum of values of these parameters and I find the optimal values of these parameters under different settings. As a first step and to gain some insight, I will restrict the analysis to equal values. For the policy optimality analysis I will allow for these parameters to be different.

A technology shock was a 1% positive technology with 0.9 persistence.\(^{28}\) Table 2 in the Appendix

---

\(^{22}\)Lawrance (1991) estimated discount factors for poor consumers at between 0.95 and 0.98 at quarterly frequency.

\(^{23}\)This value corresponds to the US. I assume here that the ratio is similar across most industrialized countries, given the lack of housing wealth data for European countries. See Aspachs and Rabanal (2008).

\(^{24}\)Microeconomic estimates usually suggest values in the range of 0 and 0.5 (for males). Domeij and Flodén (2006) showed that in the presence of borrowing constraints this estimate could have a downward bias of 50%.

\(^{25}\)This value is in the range of the estimates of Iacoviello (2005) and Iacoviello and Neri (2010) for the US, and Campbell and Mankiw (1991) for the US, Canada, France, and Sweden. Table 4 in the Appendix shows robustness checks for welfare considering different values of this parameter.

\(^{26}\)See McCallum (2001).

\(^{27}\)This value makes the model comparable with the standard models, where fixed-rate mortgages are not considered.

\(^{28}\)This high persistence value for technology shocks is consistent with what is commonly reported in the literature. Smets and Wouters (2002) estimated a value of 0.822 for this parameter in Europe; Iacoviello and Neri (2010) estimated it as 0.93 for the US.
Figure 1: Impulse Responses to a Technology Shock. Symmetric Countries. Baseline versus Macroprudential

presents a summary of the parameter values.

3.2 The Symmetric Case

In order to gain some insight about the mechanisms of the model, I present impulse-response functions for the symmetric case. Here, countries do not differ in their housing markets, are equal in size and suffer the same shocks. Figure 1 displays responses of the main variables to a technology shock, comparing the baseline case in which there is no macroprudential policy with the case in which the loan-to-value rule is active. Since countries are symmetric, it is irrelevant if the rule is the centralized or the decentralized one. Given a technology shock, output increases and inflation decreases in both cases. However, if a macroprudential policy is implemented, output does not increase as much as in the baseline case. The reason for that is that the macroprudential rule cuts borrowing (see lower-right panel). This depresses housing and goods demand for borrowers, which in turn slows down aggregate activity making the increase in output less pronounced. Inflation decreases slightly more in the macroprudential case, due to the fall in demand by borrowers. Monetary policy reacts to this change in inflation, reducing interest rates slightly by more in the macroprudential case. Asset prices move inversely with the interest rate and produce a slight larger increase in the macroprudential case due to the interest-rate response.29

Figure 2 displays the evolution of the loan-to-value ratio, the instrument of the macroprudential regulator, after the shock hits the economy. We see that, since output and house prices are increasing and this could potentially generate a situation of excessive credit growth, the regulator cuts the LTV.

29 Here, I present a selection of impulse responses. The whole set is available upon request.
3.3 Asymmetric Technology Shock

In this section I present the first case of asymmetry. Let us imagine that there is a technology shock only in one of the countries (Country A), everything else equal.

In order to understand how the shock that comes to Country A is transmitted through Country B, I display 3 in which we abstract from the macroprudential policy and present impulses responses for the baseline case (no macroprudential) comparing the two countries. Here, I am comparing the effects of the shock both for Country A and B, to understand how the shock is transmitted from country to country. In such a setting in which countries share the same monetary policy and are linked through trade and financial markets, even if the shock happens in one of the countries, it is rapidly transmitted to the other one. We see that output in Country A increases because of the effects of the shock and, since producing is more efficient, inflation in that country decreases. However, Country B wants to benefit from the shock and labor and borrowing in that country increase, increasing the demand in consumption. Furthermore, monetary policy reacts to inflation and the common interest rate goes down. This expansionary monetary policy measure makes production in B also increase. As a result, they import more goods from Country A but they also are able to produce more increasing their labor supply. However, in Country B, the production expansion comes from the demand side of the economy, thus increasing inflation. Therefore, in real terms, the interest rate is decreasing even further, giving an important impulse to borrowing. House prices are increasing because they move inversely with the interest rate. Then, since the collateral is worth more, borrowing is increasing even further and by more than in Country A. Inflation in that country increases, since it is demand driven. House prices increase in both countries but they do by a larger amount in Country A, the country that receives the shock. The interest rate, which is common to both countries, slightly decrease because, on average inflation goes down. This decrease in the interest rate represents an extra impulse for borrowing, especially for Country B given that inflation is increasing.
in that country and the real interest rate is decreasing by more than in Country B. Borrowing in Country B is increasing more strongly on impact but, given that house prices are not increasing as much in this country, it decreases rapidly, showing less persistence as the increase in Country A. We see that this type of shock, even though is happening just in one of the countries, it is affecting both of them through different mechanisms.

Now, let us see what happens if we introduce the macroprudential rule, considering the centralized and the decentralized setting for its implementation. We see from figure 4 that the LTV policy is different depending on the setting we are in. In all cases, since we are considering a situation in which borrowing is increasing, the LTV ratio is cut, however it matters if the policy is centralized or decentralized. If the policy is decentralized, the LTV is reduced by more in the country in which the shock is happening, notice that in that country output and house prices increase by more. In Country B the LTV is also reduced but not so much, in comparison with the country that is hit by the shock. The LTV pattern in the centralized case, since it is considering the average of variables in both countries lies in between the LTV from the two countries in the decentralized case.

Now, I compare the impulse responses of the baseline case with the ones in which there is a centralized and a decentralized macroprudential rule. Figure 5 shows this impulse responses. We can see that even if at the aggregate level the differences between the LTVs in the three cases are not noticeable, they are for the borrowing, and this has implications for financial stability. For Country A, borrowing is decreasing with the macroprudential rule, regardless of whether it is centralized or decentralized. However, it
Figure 4: LTV Response to a Technology shock in Country A. Centralized versus decentralized macro-prudential rule.

Figure 5: Impulse Responses to a Technology Shock in Country A. Baseline versus Centralized and Decentralized Macroprudential

decreases by more in the decentralized case because the LTV ratio is decreasing even further, as we saw in figure 4. For Country B, borrowing is decreasing by less in the decentralized case because under this setting its LTV is not decreasing as much as in the centralized case (see figure 4).

3.4 Different LTVs

Figure 6 presents impulse responses to a common technology shock when countries have different steady-state LTV ratios. In particular, Country A has a high LTV and Country B has a low LTV, namely 0.9 and 0.5, respectively. The LTV ratio dictates the strength of the financial accelerator since it is directly related with the tightness of the collateral constraint. In a country in which the LTV is higher, the financial accelerator effects will be stronger. We can see that these differences in LTVs have an impact on borrowing. In Country A, the country with a higher LTV, borrowing increases by more than in the other country. Also consumption increases by more, however in aggregate terms differences are not as noticeable.
In figure 7 we can see the response of the LTV to the increase in the shock. We see that, since this is an expansionary shock that increases borrowing, the LTV decreases. However, the fact that we do not find aggregate differences between the two countries leads to very similar responses of the LTV for both the centralized and the decentralized cases.

Figure 8 compares the impulse responses of the baseline scenario with the one in which the macroprudential policy is introduced, that is, the LTV is cut. We see that borrowing is cut in both countries, however, as figure 7 indicated, there are no differences between the centralized and the decentralized case.

3.5 Different proportion of borrowers

In this section I consider that the two countries differ in their debt-to-GDP ratio. Although this is not an explicit parameter in the model, the borrower labor-income share can serve as a proxy. In a
country where this share is high, the proportion of borrowers is also high and so it is the debt-to-GDP in consequence. I consider a common technology shock for two countries in which the proportion of borrowers is 0.7 and 0.3, respectively.

Figure 9 shows how a common technology shock is transmitted differently across countries. Consumption in Country A, the country with high borrowers share increases by more than in the other country, given the high proportion of borrowers. However, this makes inflation decrease less than in Country B. As a consequence, given the common interest rate, real rates in Country B are lower. This makes borrowing in Country B increase by more. Thus, house prices and output increase slightly by more in Country B, the country with a low proportion of borrowers.
Figure 10: LTV response to a common technology shock. High proportion of borrowers in Country A, low proportion in Country B. Centralized versus decentralized macroprudential rule.

Figure 11: Impulse Responses to a common Technology Shock. Country A: High debt, Country B: Low debt. Baseline versus Centralized and Decentralized Macroprudential

In figure 10, we see how the LTV response follows the a consequent pattern. Given that house prices, output and borrowing increase by more in Country B, this is the country that cuts the LTV by more in the decentralized case. The centralized case lies in between the decentralized ones.

Figure 11 displays the impulse responses for the baseline and the macroprudential case. We see how the macroprudential rule cuts borrowing in both countries. However, there is not much difference between the centralized and the decentralized rule since, even if there are differences in the LTV response, they are not very large. As in the case of the previous asymmetry, the effects seem to be more distributional than aggregate and this is why the centralized and the decentralized case do not display large differences.

3.6 Different mortgage contracts

Here, I consider that borrowers in Country A take mortgages at a variable interest rate, while borrowers in Country B do it at a fixed rate. Figure 12 presents this case. Given a common technology shock, the union interest rate goes down. This affects more strongly borrowers in Country A, since their mortgage
Figure 12: Impulse responses to a common technology shock. Variable rates in Country A, fixed rates in Country B.

Figure 13: LTV response to a common interest rate shock. Variable rates in Country A, fixed rates in Country B. Centralized versus decentralized macroprudential rule.

rates vary one for one with the policy rate. However, in Country B the nominal interest rate is fixed. Since inflation is decreasing in both countries, in real terms, the interest rate in Country B decreases by a larger amount. House prices are increasing in both countries. That makes borrowers in Country A take out more loans. However, the fact that house prices are not increasing as much combined with the decline in real rates makes borrowing in Country B decrease.

Figure 13 shows the LTV response in the case of different mortgage contracts across countries. When the rule is decentralized and countries can implement their own macroprudential policy, the LTV does not decrease as much in the fixed rate country, since borrowing there is not increasing. The centralized case lies in between the decentralized cases.

Figure 14 shows that when the macroprudential rule is introduced, borrowing does not increase as much as in the baseline case. If the rule is decentralized, borrowing is further reduced for country A.
Figure 14: Impulse Responses to a common Technology Shock. Country A: Variable rate, Country B: Fixed rate. Baseline versus Centralized and Decentralized Macroprudential

However, in Country B, the cut in borrowing is stronger if the rule is centralized although differences between the two cases are not so noticeable.

4 Welfare Analysis

4.1 Welfare Measure

I numerically evaluated how cross-country asymmetries affect welfare for a given policy rule and for technology shocks. As discussed in Benigno and Woodford (2008), the two approaches that have recently been used for welfare analysis in DSGE models include either characterizing the optimal Ramsey policy, or solving the model using a second-order approximation to the structural equations for given policy and then evaluating welfare using this solution. As in Mendicino and Pescatori (2007), I take this latter approach to be able to evaluate the welfare of the three types of agents separately. The individual welfare for savers and borrowers in Country A is defined, respectively, as follows:

\[ V_{u,t} = E_t \sum_{m=0}^{\infty} \beta^m \left( \ln C_{t+m}^u + j_t \ln H_{t+m}^u - \frac{(L_{t+m}^u)^{\eta}}{\eta} \right) , \]

\[ V_{cv,t} = E_t \sum_{m=0}^{\infty} \beta^m \left( \ln C_{t+m}^{cv} + j_t \ln H_{t+m}^{cv} - \frac{(L_{t+m}^{cv})^{\eta}}{\eta} \right) , \]

I used the software Dynare to obtain a solution for the equilibrium implied by a given policy by solving a second-order approximation to the constraints, then evaluating welfare under the policy using this approximate solution, as in Schmitt-Grohe and Uribe (2004). See Monacelli (2006) for an example of the Ramsey approach in a model with heterogeneous consumers.
Following Mendicino and Pescatori (2007), I define social welfare in Country A as a weighted sum of
the individual welfare for the different types of households:

\[ V_t = (1 - \beta) V_{u,t} + \left( 1 - \tilde{\beta} \right) \left[ \alpha_A V_{cv,t} + (1 - \alpha_A) V_{cf,t} \right]. \]  (37)

Borrowers and savers’ welfare are weighted by \( 1 - \tilde{\beta} \) and \( (1 - \beta) \), respectively, so that the two groups
receive the same level of utility from a constant consumption stream. Everything is symmetrical for
Country B.

Total welfare is defined as a weighted sum of the welfare in the two countries:

\[ W_t = n V_t + (1 - n) V_t^*. \]  (38)

In order to make the results more intuitive, I present welfare changes in terms of consumption
equivalents. I use as a benchmark the welfare evaluated when the macroprudential policy is not active
and compare it with the welfare obtained when such policy is implemented. Since we are interested in
calculating the welfare benefit of introducing a macroprudential policy, I convert the difference between
those values in consumption equivalent units to obtain an understandable measure. The consumption
equivalent measure defines the constant fraction of consumption that households should give away in
order to obtain the benefits of the macroprudential policy. Then, when there is a welfare gain, households
would be willing to pay, in consumption units, for the measure to be implemented because it is welfare
improving. I present welfare results as the equivalent in consumption units of this welfare improvement.
I multiply results by -1, so that a positive value means a welfare gain, that is, how much the consumer
would be willing to pay to obtain the welfare improvement. I evaluate welfare at the steady state when
the macroprudential policy is not active and at the steady state when it is, the derivation of the welfare
benefits in terms of consumption equivalent units is as follows:

\[ CE_u = 1 - \exp \left[ (1 - \beta) \left( V_u^{MP} - V_u^* \right) \right], \]  (39)

\[ CE_{cv} = 1 - \exp \left[ \left( 1 - \tilde{\beta} \right) \left( V_{cv}^{MP} - V_{cv}^* \right) \right], \]  (40)

\[ CE_A = 1 - \exp \left[ \left( V_A^{MP} - V_A^* \right) \right], \]  (41)
\[ CE_W = 1 - \exp \left( (V_W^{MP} - V_W^*) \right), \]  

(42)

where the superscripts in the welfare values denote the benchmark case when macroprudential policies are not introduced and the case in which they are, respectively.\textsuperscript{31}

4.2 The Symmetric Case

Here, to gain some insight about the normative aspects of the model, I analyze how introducing a macroprudential rule affects welfare. I consider a symmetric case in which there is a common technology shock affecting two countries that are equal in all aspects. All values are presented in consumption equivalents with respect to the benchmark case in which there is no macroprudential policy. Therefore, a positive value means that when the macroprudential rule is active, there is a welfare improvement with respect to the benchmark. Here I consider different values of the reaction parameters in the LTV rule to see how welfare is affected by the rule aggressiveness. As a first step and to give a graphical view of the results, I restrict to the case in which the two parameters are equal in value. Then, I drop this restriction and I search for the optimal combination of parameters that maximizes welfare.

Figures (15) and (16) show that for low values of the reaction parameters in the LTV rule, introducing a macroprudential policy unambiguously increases total welfare. We can see that borrowers in both countries are better off with the macroprudential rule because this is introducing financial stability. Welfare, for these agents, increases unambiguously for low values of the reaction parameters, reaches its maximum at around 0.2 and starts to decrease until it becomes negative when the LTV rule becomes too aggressive for them (at around 0.4). The intuition for that is the following; since borrowers are constrained they do not have the possibility to smooth consumption, however, if policies reduce aggregate volatility by ensuring a more stable financial system, they will be able to enjoy a more stable consumption path. However, a too aggressive LTV rule limits their ability to access financial markets to a great extent and this is not welfare enhancing anymore. These gains are made of course at the expense of savers that reduce their capacity to save, generating a trade-off between the welfare of the two agents. In the aggregate, the welfare improvement of borrowers compensates the loss of the savers and we find an overall welfare improvement. However, results depend on the reaction parameters of the LTV rule. We see that when we start from low values of this variables, increasing the aggressiveness of the rule...\textsuperscript{31} We follow Ascari and Ropele (2009).
If we look at the behavior of the whole economy we see that welfare mimics the pattern of borrowers, softened by the negative effects produced on the welfare of savers. The point at which welfare is maximized is when $\phi_y^k = \phi_q^k = 0.19$, reflecting the fact that, at least in this benchmark case, the economy is better off with macroprudential rules that are not too aggressive and do not cut credit excessively, allowing borrowers to enjoy an acceptable consumption level.

If we allow for different values of the reaction parameters, results show that the optimal macroprudential policy is one in which the LTV responds little to changes in output while relatively more aggressively to changes in house prices. This result is intuitive since house prices are a variable that enters directly in the collateral constraint. However, output affects borrowing in an indirect way. Therefore, in order to decrease the negative effects that the collateral constraint may have for financial stability, house prices are the variable to target. These results are in line with Rubio and Carrasco-Gallego (2013) which show
Figure 16: Welfare gains from Macroprudential Policy. Different values of reaction parameters. Symmetric case

similar values in a closed economy setting.

<table>
<thead>
<tr>
<th>Country A/Country B</th>
<th>$\phi^y$</th>
<th>$\phi^k$</th>
<th>Welfare gain</th>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\phi^k$</td>
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<td></td>
<td>1.7929</td>
</tr>
</tbody>
</table>

4.3 Asymmetric Techno Shock

The literature on currency unions has focused on the analysis of the optimality of a single monetary policy when there are non-synchronized business cycles across members. Here, I present an analogous experiment to explore the optimality of macroprudential policies in the context of asymmetric technology shocks. Figure (17) presents disaggregated welfare, that is, for all agents and both countries, when there is a shock in Country A. Here, since now there are asymmetries in the individual country response to the shock, I consider two scenarios; one in which the macroprudential rule is implemented at a national level (decentralized) and a second one at a union level (centralized). Recall that under this scenario, borrowing was increasing by more in Country B, the country that did not directly receive the shock, because of the real interest rate decreasing in this country.

Results show that for Country A, the country where the shock takes place, welfare for borrowers slightly increases, but only for low values of the reaction parameters, reaching a maximum for the
centralized case at around $\phi^k_y = \phi^k_q = 0.2$. It seems that it is not optimal to have a too aggressive policy (remember that borrowing was not increasing as much in this country). However, borrowers in Country A definitely prefer a centralized policy because, taking into account the increase in borrowing in the other country, the same policy is more effective to increase financial stability. We find the same trade-off with savers as in the benchmark symmetric case. Savers are worse off with the macroprudential policy. At a country level, we see that Country A’s behavior mimics borrowers welfare pattern. However, welfare decreases because the gain experimented by borrowers does not sufficiently compensate the loss of the savers.

In Country B, borrowers are better off with the macroprudential policy, reaching higher welfare gains than borrowers in Country A. If the policy is centralized, low values of the reaction parameters are prefered. If it is decentralized higher values are. They prefer more aggressive policies than their counterpart in Country A, finding a maximum close to the value of 0.6 of the reaction parameters, for the centralized case. Borrowers, with this policy, are able to enjoy the spillovers of the shock in Country A, a more stable financial system and, if the policy is centralized, it helps equalizing the effects of the shock. Interestingly, savers are also better off with this policy, the equalizing effects of the macroprudential policy affect them positively. However, they would prefer a decentralized policy. The whole Country B benefits from the introduction of the LTV rule, given that all the agents in the economy are better off under this policy. Given the discrepancy of agents with respect to the centralization of policies, for low values of the reaction parameters, Country B prefers a centralized policy while a decentralized one is more desirable with a more aggressive policy.

If we look at welfare of the whole monetary union in figure (18), we see that there are welfare gains in the centralized macroprudential policy case, reaching a maximum when $\phi^k_y = \phi^k_q = 0.35$. For the decentralized case, this is only welfare enhancing for small values of these parameters. For the case of an asymmetric technology shock happening only in Country A, we find that a centralized macroprudential policy delivers an unambiguously higher welfare.

However, if we allow for the parameters to be different among each other, we can find which combination of them maximizes welfare. Optimal parameters in the centralized case are the same as in the symmetric scenario. However, allowing for decentralized policies, the optimal rule is different across countries. For Country A, the country that receives the shock, it is optimal not to respond too aggressively to any of the variables. However, Country B should respond more strongly to house price deviations. Interestingly, what we observe is that if the policy is centralized, it is optimal not have an
aggressive macroprudential policy in the country that is producing more efficiently. Having a more stable financial system is desirable, but not at the expense of efficiency.

<table>
<thead>
<tr>
<th>Table 2: Optimal Macroprudential Policy, given TR</th>
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<tbody>
<tr>
<td>Centralized</td>
</tr>
<tr>
<td>Country A</td>
</tr>
<tr>
<td>$\phi^k_y$</td>
</tr>
<tr>
<td>$\phi^k_q$</td>
</tr>
<tr>
<td>Welfare Gain</td>
</tr>
</tbody>
</table>

4.4 Different proportion of borrowers

Here, I introduce another source of asymmetry, a different proportion of borrowers in each country. Country A has a high proportion of borrowers while in Country B borrowers are less numerous. This creates a stronger financial accelerator effect in Country A. In figure (19) see that the effects on welfare of the macroprudential policies are similar in both countries in qualitative terms, however they are quantitative stronger in the country in which the proportion of borrowers is lower. In both countries, welfare for borrowers unambiguously increases, reaching its maximum at a value of the reaction parameters of approximately 0.25. As in the benchmark case, we can also observe the trade-off between borrowers and savers welfare, who are worse off when the macroprudential measure is introduced. Interestingly, for
Figure 18: Welfare gains from Macroprudential Policy. Different values of reaction parameters. Technology shock in Country A

this asymmetry, the fact of whether the policy is implemented at a national or a union level seems to be irrelevant in terms of welfare. This asymmetry does not imply volatility asymmetry but redistributional effects between borrowers and savers and implementing the macroprudential measure at a centralized or decentralized level does not have effects on aggregate volatility.

For the monetary union (figure (20)) welfare behavior is similar to the symmetric case although the aggregate welfare gains are larger. Welfare unambiguously improves for low values of the reaction parameters and it reaches its maximum at around 0.15.

With this case of asymmetry, as we saw in the graph, decentralized and centralized policies show similar results in terms of welfare. When we search for the optimal combination of parameters, this pattern is maintained. The optimal rule is similar for all cases, one that responds slightly to deviations in output and more strongly to deviations in house prices. Welfare gains are very similar for policies implemented at national or union levels.

<table>
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<th></th>
<th>Centralized</th>
<th>Decentralized</th>
</tr>
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<tbody>
<tr>
<td></td>
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<td>Country B</td>
</tr>
<tr>
<td>$\phi_{y}^{k*}$</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>$\phi_{q}^{k*}$</td>
<td>0.29</td>
<td>0.36</td>
</tr>
<tr>
<td>Welfare Gain</td>
<td>6.1936</td>
<td>6.074</td>
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Finally, I consider another case of asymmetry: different mortgage contracts across countries. Figure (21) shows that, in this case, we are in a situation of asymmetry in the volatility that the same shock produces. Here, it matters again if the policy is centralized or decentralized. For borrowers in Country A, the variable-rate country, the introduction of the macroprudential measure improves welfare, especially in the centralized case. These borrowers, as compared to country B, suffer from a higher volatility in their borrowing after the shock and the macroprudential measure helps reducing it. The effect is stronger for the centralized case, since they can live in a more stable world, not only in their country. We find the trade-off for savers, as in the benchmark case. The effects for Country A mimic the pattern of borrowers, softened by the existence of savers. In Country B, aggregate volatility is lower, given the fixed rates that are prevalent there. For low values of the reaction parameters, the effects on welfare for borrowers is very small because they are already living in a stable world. However, for larger values they would prefer to have a decentralized policy because otherwise, their borrowings are cut too much. Savers behave as in the standard case and there is no difference between the centralized and the decentralized policy. For the whole country, the macroprudential policy is welfare enhancing but for larger values of the reaction parameters, it is preferable a decentralized setting.

In figure (22) we see that, for the monetary union, welfare gains are strong with the macroprudential policy, reaching a maximum at around 0.4. The centralized case is preferred given that borrowers in Country A...
Country A strongly prefer this setting.

In this case, when we search for the optimal parameters, we are able to find a combination of parameters which improves welfare a great deal and that finds larger improvements for the decentralized case, contrary to the graphical intuition that we found before. In the centralized case, it is optimal to respond to house prices in a very aggressive fashion. For the decentralized case, the Country B, the one with fixed rates is the one that should respond relatively more strongly to house prices.

| Table 5: Optimal Macroprudential Policy, given TR |
|-----------------------------------|-----------------------------------|-------------------|
|                                   | Centralized | Decentralized |
|                                   | Country A   | Country B     |
| $\phi_y^{k*}$                    | 0.01        | 0.02           | 0.03             |
| $\phi_q^{k*}$                    | 1.13        | 0.48           | 1.45             |
| Welfare Gain                     | 156.77      | 1425.24        |

With fixed-rate mortgages, there is less aggregate volatility but monetary policy is less efficient to fight against inflation. An aggressive macroprudential policy compensates the lack of effectiveness of monetary policy. The optimal macroprudential policy responds more strongly to house prices than in the previous cases. In the centralized case because there are fixed-rates in half of the union. In the decentralized, for the country that has fixed rates. The decentralized case is preferred.
Figure 21: Welfare gains from Macroprudential Policy. Different values of reaction parameters. Variable rates in Country A

5 Concluding Remarks

In this paper, I build a two-country DSGE model, with housing, and collateral constraints in order to explore the effects of macroprudential policies. Countries are within a monetary union in which monetary policy is set by a single central bank. For the case of macroprudential policies, I experiment with two settings; one in which it is implemented at a national level and a second one in which is set at a union level.

As a benchmark, I consider a monetary union in which members are symmetric and shocks are synchronized. Then, I consider three sources of asymmetries within the monetary union: the first one comes from non-synchronized business cycles, in the spirit of studies that analyzed the optimality of currency areas. The second one comes from asymmetries on the strength of financial accelerator effects, proxied by differences in labor income shares of borrowers. Finally, I consider mortgage contract asymmetries, in the sense that in one of the countries borrowers own variable-rate mortgages while fixed-rate in the other one.

Results show that, for the benchmark case, introducing an LTV rule is unambiguously welfare enhancing for the economy. Although at the expense of the savers, borrowers benefit from a more stable financial system that help them smooth consumption. Furthermore, the optimal rule is one that responds more strongly to house prices than to output deviations. However, when there is an asymmetric shock, welfare gains appear especially in the case of a centralized policy because, since the shock is transmitted
to the other country, the whole union can benefit from a more stable financial system. The optimal policy is one in which macroprudential regulation is more aggressive in the country that does not receive the shock. For the case of different strengths of the financial accelerator, the union also benefits if there are macroprudential policies, however, there is no difference between the centralized and the decentralized case, since the shock causes redistributional effects but not differences in aggregate volatility. Finally, when the asymmetry comes from different mortgage contracts, the same shock delivers different volatilities in each country. Then, welfare improvements are important when the LTV rule responds strongly to house prices, especially in the decentralized case.

As a summary, we could say that asymmetries in a monetary union matter for the conduct of macroprudential policies, especially when heterogeneity results in differences in aggregate volatility, as in the case of mortgage contract heterogeneity. When there is an asymmetric shock, centralized policies are preferred. For asymmetries that cause distributional effects (debt-to-GDP), it does not matter if the policy is centralized or decentralized. For asymmetries that cause different aggregate volatilities (mortgage contracts), the decentralized policy is better, being more aggressive for the country that has less aggregate volatility but in which monetary policy is less efficient.

I have restricted the analysis to the case in which monetary policy is given. Further research could go in the direction of optimizing monetary policy depending on the kind of macroprudential policy that is being implemented.
Appendix

Steady-state relationships

Relative prices in the steady state are derived from equations (10), (21) and their counterparts for Country B:

\[
\frac{n}{1 - n} \frac{P_B}{P_A} = \frac{C^u_A}{C^u_B} = \frac{C^c_A}{C^c_B} = \frac{C^{us}_A}{C^{us}_B} = \frac{C^{cs}_A}{C^{cs}_B}
\]  

(43)

Interest rates:

\[
R_A = R = R_B = \bar{R} = \bar{R}^* = 1/\beta
\]  

(44)

We can find the consumption-to-housing ratio for savers and borrowers in Country A by using the first-order conditions for housing:

\[
\frac{C^u_A}{qH^u} = \frac{n}{j} (1 - \beta)
\]  

(45)

\[
\frac{C^c_A}{qH^c} = \frac{n}{j} \left[ (1 - \beta) - k_A (\beta - \bar{\beta}) \right] = \frac{n}{j} \zeta
\]  

(46)

Similarly, for Country B:

\[
\frac{C^{us}_B}{q^uH^{us}} = \frac{(1 - n)}{j^*} (1 - \beta)
\]  

(47)

\[
\frac{C^{cs}_B}{q^cH^{cs}} = \frac{(1 - n)}{j^*} \left[ (1 - \beta) - k_B (\beta - \bar{\beta}) \right] = \frac{(1 - n)}{j^*} \xi^*
\]  

(48)

Borrowing in the steady state is as follows:

\[
b^c = \beta k_A qH^c.
\]  

(49)

\[
b^u + b^c = 0
\]
\[ b^{c*} = \beta k_B q^* H^{c*}. \quad (50) \]

\[ b^{u*} + b^{c*} = 0 \]

From the firm problem, we have that in the steady state:

\[ w^u = \frac{1}{X^\gamma} \frac{Y_A}{L^u}, \quad (51) \]

\[ w^c = \frac{1}{X^\gamma} \frac{Y_A}{L^c}, \quad (52) \]

\[ w^{u*} = \frac{1}{X^\gamma} \frac{Y_B}{L^{u*}}, \quad (53) \]

\[ w^{c*} = \frac{1}{X^\gamma} \frac{Y_B}{L^{c*}}. \quad (54) \]

where \( X = X^* = \frac{\varepsilon - 1}{\varepsilon} . \)

Combining the steady-state budget constraint for unconstrained consumers in Country A with (45) and (51) we obtain:

\[ \frac{C^u_A}{Y_A} = n \frac{(\gamma + X - 1)}{X (1 - j k_A)} \quad (55) \]

Similarly, for constrained consumers:

\[ \frac{C^c_A}{Y_A} = \frac{1 - \gamma}{X} \frac{\zeta n}{\zeta + j k_A (1 - \beta)} \quad (56) \]

The market-clearing conditions for goods produced in Country A imply:

\[ \frac{C^*_A}{Y_A} = \frac{n}{1 - n} \left( 1 - \frac{C^u_A}{Y_A} - \frac{C^c_A}{Y_A} \right) \]

Using (45) and (55) we can find the housing-to-output ratio for savers in Country A:
Analogously, using (46) and (56) we can find the housing-to-output ratio for constrained consumers in Country A:

\[
\frac{H^u}{Y_A} = \frac{j(\gamma + X - 1)}{Xq(1 - jk_A)(1 - \beta)}
\] (57)

Similarly, for Country B:

\[
\frac{H^c}{Y_A} = \frac{(1 - \gamma)j}{Xq\zeta + jk_A(1 - \beta)}
\] (58)

\[
\frac{C^u^c}{Y_B} = \frac{(1 - \gamma)(\gamma + X^* - 1)}{X^*(1 - j^*k_B)}
\] (59)

\[
\frac{C^c^c}{Y_B} = \frac{1 - \gamma}{X^*\zeta + j^*k_B(1 - \beta)}
\] (60)

\[
\frac{H^u^*}{Y_B} = \frac{j^*(\gamma + X^* - 1)}{X^*q^*(1 - j^*k_B)(1 - \beta)}
\] (61)

\[
\frac{H^c^*}{Y_B} = \frac{(1 - \gamma)j^*}{X^*q^*\zeta + j^*k_B(1 - \beta)}
\] (62)

Log-linearized equations

Variables in deviations from the steady state are expressed in lower-case and with a hat.

Interest rates

\[
\hat{r}_{At} = \hat{r}_{Bt} + E_t(\hat{e}_{t+1} - \hat{e}_t) + \psi,
\] (63)

\[
\hat{r}_{At} = \hat{r}_{Bt} = 0.
\] (64)

Aggregate demand

\[
\hat{c}_{At} = E_t\hat{c}_{At+1} - (\hat{r}_{At} - E_t\hat{r}_{At+1}),
\] (65)
\[ \hat{c}_{Bt}^{u*} = E_t \hat{c}_{Bt+1}^{u*} - (\hat{\tau}_{Bt} - E_t \hat{\pi}_{Bt+1}) , \] 

\[ \hat{c}_{At}^c = \left( \frac{\zeta + j k_A (1 - \beta)}{\zeta} \right) (\hat{y}_{At} + \hat{\xi}_t - \hat{x}_t) - \frac{j}{\zeta} (\hat{h}_t^c - \hat{h}_{t-1}^c) + \frac{k_A j}{\zeta} (\beta \hat{h}_t^c - \hat{h}_{t-1}^c) - k_A j (\alpha_A \hat{r}_{At-1} - \hat{\pi}_{At}) , \] 

\[ \hat{b}_t^c = E_t \hat{y}_{Bt+1} + \hat{h}_t^c - (\alpha_A \hat{r}_{At} - E_t \hat{\pi}_{At+1}) , \]

\[ \hat{c}_{Bt}^c = \left( \frac{\zeta^* + j^* k_B (1 - \beta)}{\zeta^*} \right) (\hat{y}_{Bt} + \hat{\xi}_t - \hat{x}_t) - \frac{j^*}{\zeta^*} (\hat{h}_t^{c*} - \hat{h}_{t-1}^{c*}) + \frac{k_B j^*}{\zeta^*} (\beta \hat{h}_t^{c*} - \hat{h}_{t-1}^{c*}) - k_B j^* (\alpha_B \hat{r}_{Bt-1} - \hat{\pi}_{Bt}) , \]

\[ \hat{h}_t^{c*} = E_t \hat{y}_{Bt+1}^* + \hat{h}_t^{c*} - (\alpha_B \hat{r}_{Bt} - E_t \hat{\pi}_{Bt+1}) , \]

\[ \hat{c}_{At}^c - \hat{c}_{Bt}^c = \hat{c}_{At}^c - \hat{c}_{Bt}^c \]

**Housing equations**

\[ \hat{h}_t^u = \frac{1}{1 - \beta} (\hat{c}_{At}^u - \hat{q}_t) - \frac{\beta}{1 - \beta} E_t (\hat{c}_{At+1}^u - \hat{q}_{t+1}) , \] 

\[ \hat{h}_t^{u*} = \frac{1}{1 - \beta} (\hat{c}_{Bt}^{u*} - \hat{q}_t^*) - \frac{\beta}{1 - \beta} E_t (\hat{c}_{Bt+1}^{u*} - \hat{q}_{t+1}^*) , \]

\[ \hat{h}_t^c = \frac{1 - k_A \beta}{\zeta} \hat{c}_t^c - \frac{1}{\zeta} \hat{q}_t - \frac{k_A \beta}{\zeta} (\alpha_A \hat{r}_{At} - E_t \hat{\pi}_{At+1}) + \frac{\beta}{\zeta} E_t \hat{q}_{t+1} + \frac{\beta (1 - k_A)}{\zeta} E_t \hat{c}_{t+1}^c . \]

\[ \hat{h}_t^{c*} = \frac{1 - k_B \beta}{\zeta^*} \hat{c}_t^{c*} - \frac{1}{\zeta^*} \hat{q}_t^* - \frac{k_B \beta}{\zeta^*} (\alpha_B \hat{r}_{Bt} - E_t \hat{\pi}_{Bt+1}) + \frac{\beta}{\zeta^*} E_t \hat{q}_{t+1}^* + \frac{\beta (1 - k_B)}{\zeta^*} E_t \hat{c}_{t+1}^{c*} . \]
Aggregate supply

\[ \hat{y}_{At} = \frac{\eta + 1}{\eta - 1} \hat{\xi}_t - \frac{1}{\eta - 1} (\gamma \hat{c}_{At}^u + (1 - \gamma) \hat{c}_{At}^c + \hat{x}_t), \]  

(76)

\[ \hat{y}_{At} = \left( \frac{C_A^u}{Y_A} + \frac{C_A^c}{Y_A} \right) \hat{c}_{At} + \left( 1 - \frac{C_A^u}{Y_A} - \frac{C_A^c}{Y_A} \right) \hat{c}_{At}^*, \]  

(77)

\[ \hat{y}_{Bt} = \frac{\eta + 1}{\eta - 1} \hat{\xi}_t^* - \frac{1}{\eta - 1} (\gamma \hat{c}_{Bt}^u + (1 - \gamma) \hat{c}_{Bt}^c + \hat{x}_t^*), \]  

(78)

\[ \hat{y}_{Bt} = \left( \frac{C_B^u}{Y_B} + \frac{C_B^c}{Y_B} \right) \hat{c}_{Bt}^* + \left( 1 - \frac{C_B^u}{Y_B} - \frac{C_B^c}{Y_B} \right) \hat{c}_{Bt}, \]  

(79)

\[ \hat{\pi}_{At} = \beta \hat{\pi}_{At+1} - \bar{k} \hat{x}_t + u_{At}, \]  

(80)

\[ \hat{\pi}_{Bt} = \beta \hat{\pi}_{Bt+1} - \bar{k} \hat{x}_t^* + u_{Bt}, \]  

(81)

where \( \bar{k} = \frac{(1-\theta)(1-\beta \theta)}{\eta} \) and \( u_{At} \) and \( u_{Bt} \) are cost-push shocks.

Monetary policy

\[ \hat{r}_t = \rho \hat{r}_{t-1} + (1 - \rho) \{(1 + \phi_y) [n \hat{\pi}_{At} + (1 - n) \hat{\pi}_{Bt}] + \phi_y [n \hat{y}_{At} + (1 - n) \hat{y}_{Bt}]\} + \epsilon_{R,t} \]  

(82)

Macroprudential policy

Centralized

\[ \hat{k}_t = \log(k_{SS}) - \phi_y^k [n \hat{y}_{At} + (1 - n) \hat{y}_{Bt}] - \phi_q^k [n \hat{q}_{At} + (1 - n) \hat{q}_{Bt}] \]  

(83)

Decentralized

\[ \hat{k}_{At} = \log(k_{SSA}) - \phi_y^k \hat{y}_{At} - \phi_q^k \hat{q}_{At} \]  

(84)

\[ \hat{k}_{Bt} = \log(k_{SSB}) - \phi_y^k \hat{y}_{Bt} - \phi_q^k \hat{q}_{Bt} \]  

(85)
### Tables and Figures

<table>
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<th>Country</th>
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<th>Rate</th>
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<td>83</td>
<td>43,3</td>
<td>F</td>
</tr>
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<td>FINLAND</td>
<td>75</td>
<td>58</td>
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Table 1: Characteristics of mortgage markets. Source: IMF (2008)

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Table 2: Parameter Values
References


