What is the utility function of the Brazilian investor? Área de Submissão: Investimentos

Juliana Tessari (IPE/USP)*
Bruno Cara Giovannetti(EESP/FGV)†

2018

Abstract

We analyze which utility function would best represent the Brazilian representative investor with a one-month investment horizon who has to allocate his wealth across three main asset classes (bonds, equities, and risk free). To do this, we compute the optimal portfolio weights by considering four different specifications for the utility function: (i) mean-variance, (ii) constant relative risk aversion (expected utility functions), (iii) ambiguity aversion, and (iv) loss aversion (non-expected utility functions). We compare the optimal portfolio weights to the empirical portfolio - computed by considering the market value of all the assets in our sample - using the Mahalanobis distance. Our results indicate that the traditional utility function, the mean-variance utility, should not be used to represent the behavior of the Brazilian investor. All other utilities are statistically equal and could be used to compute optimal portfolios for the Brazilian investor. However, the constant relative risk aversion (CRRA) and the ambiguity aversion functions are only justified for extremely high levels of risk aversion. As the loss averse function showed the lowest Mahalanobis distance, we propose that the Brazilian investor is best represented by a utility function that incorporates aversion to losses, in which the decrease of utility caused by a loss is much greater than the increase caused by a gain of equal magnitude. Moreover, this different impact of gains and losses on the investor's utility leads individuals to behave as investors with high risk aversion and justifies the fact that loss-aversion preferences have also been widely used to explain why the high risk premium might be consistent with high levels of risk aversion.

Keywords: portfolio. utility function. Brazilian investor.
JEL Classification: G11. G12.

*corresponding author, tessari.juliana@gmail.com
†bruno.giovannetti@fgv.br

Introduction

One of the most important ingredients in any portfolio selection and optimization problem is the investor's objective function, that is, the utility function that represents his preferences and contains the characteristics the investor takes into account in determining the weights of risky assets in the his or her optimal portfolio. In economics and finance, the most commonly used utility function is the mean-variance function proposed by Markowitz (1952). In this approach, the investor has a mean-variance utility type in which the optimal allocation is proportional to the ratio between the conditional mean and the conditional variance of returns.

The reasons why the mean-variance utility proposed by Markowitz (1952) is very successful can be boiled down to two fundamental aspects of portfolio selection: diversification and trade-off between risk and expected return. However, although the mean-variance utility is the standard utility function in finance problems, this function has several limitations. First, the mean-variance problem is an expected utility maximization problem in which the investor's utility function is quadratic[1]. This specification for the utility function is problematic because the utility is not monotonically increasing in wealth. Second, the optimization problem with a quadratic utility function ignores any investor's preferences for higher order moments of returns, in particular asymmetry and kurtosis. Finally, this optimization problem is inherently a single-period problem, whereas most investors think of the portfolio selection problem as a long horizon problem with portfolios being rebalanced in interim periods.

Due to recurrent criticisms of the mean-variance utility function, the literature has historically focused on the time-separable expected utility with hyperbolic absolute risk aversion (HARA), which includes as special cases the logarithmic utility and the constant relative risk aversion utility function (CRRA). However, similarly to the mean-variance utility function, the CRRA utility is also subject to several criticisms. In the context of portfolio optimization, the main criticism with respect to the CRRA utility function is that the elasticity of intertemporal substitution is directly linked to the level of relative risk aversion (one is the reciprocal of another), which creates an unnatural link between very different aspects of investor preferences, namely the willingness to substitute consumption over time versus the willingness to take risks (AÏT-SAHALIA; HANSEN, 2009). Besides that, experiments also showed that several anomalies are related to the use of these utility functions. For example, individuals making decisions under uncertainty tend to systematically violate the axioms of the Expected Utility theory (KAHNEMAN; TVERSKY, 1979; NEUMANN; MORGENSTERN, 2007).

[1] If the returns have a multivariate normal distribution, then the portfolio return will likewise be normal. Thus, portfolio return distributions will differ only by means and variances. Also, in this case the mean-variance criterion is also equivalent to the expect utility approach for any risk averse utility function.

To capture these behavioral anomalies in the portfolio optimization problem, several alternative formulations were proposed using non-expected utility functions, such as the loss aversion and the ambiguity aversion utility functions Kahneman e Tversky (1979), among others. However, there is no consensus among researchers regarding what utility function should be used as an objective function in the portfolio optimization problem, since every utility function best represents the characteristics of a particular investor, and different utility functions emphasize different characteristics of the conditional distribution of returns. For example, an investor with a utility function of the mean-variance type is concerned about mean and variances, whereas a risk-averse investor may be more concerned about predicting the left tail of the distribution of returns.

In Brazil, optimal portfolios are usually computed using the mean-variance approach or the minimum-variance approach (see, for instance, Pereira et al. (2015); Demos, Pires e Moura (2015); Júnior, Campani e Leal (2017); Santos et al. (2010); Rubesam e Beltrame (2013); Santos e Tessari (2012); Naibert e Caldeira (2015); Noda, Martelanc e Securato (2014)). However, up to our knowledge, there is no study to verify whether these functions actually represent preferences of the the Brazilian representative investor and whether or not they can be used in portfolio optimization problems.

In this paper we estimate the optimal weights of a portfolio of risky assets assuming that the preferences of investors can be represented by either expected utility functions (mean-variance and constant relative risk aversion (CRRA)) and non-expected utility functions (ambiguity aversion and loss aversion). Therefore, we will use four different parameterizations of the utility function to compute an optimal portfolio that will be compared to the empirical portfolio. To our knowledge, this is the first paper to investigate what is the utility function of the Brazilian representative investor, considering the most relevant asset classes: equities, bonds and risk-free. Our methodology compares an empirical portfolio, computed using market values of the asset classes aforementioned and an optimal portfolio using four alternative parameterizations of the utility function: (i) mean-variance investors, (ii) constant relative risk averse investors, (iii) ambiguity averse investors, and (iv) loss averse investors.

The first step of this study is to compute the Brazilian empirical portfolio. The empirical portfolio is a theoretical portfolio consisting of all investable assets, in which the proportion invested in each asset corresponds to its market value divided by the sum of the market value of all assets. It represents the view of the investing community with respect to the price, expected return, variance, and correlation characteristics of each Brazilian asset, and can therefore be used as a benchmark for investor's strategic asset allocation and ass a starting point for portfolio construction.

The term "empirical portfolio" is in line with the theory of a global market portfolio, but we decide call it "empirical" because we compute it using only the asset classes with highest market value. This portfolio is the aggregate portfolio of all investors, in which portfolio weights indicate the constitution of the average portfolio. To do that, we use three main asset classes: bonds, equities, and risk-free. In line with the work of Doeswijk, Lam e Swinkels (2014), we focused on the invested market portfolio, which contains all assets in which financial investors have actually invested.

Our results clearly indicate that the traditional mean-variance utility function should not be used to represent the behavior of the Brazilian investor. All other specifications of the utility function could be used to compute optimal portfolios for the Brazilian investor, since the optimal portfolios obtained through them are statistically the same. However, CRRA and ambiguity aversion functions are justified only with extremely high levels of risk aversion. As the loss averse optimal portfolio showed the lowest Mahalanobis distance in relation to the empirical portfolio, we consider that the preferences of the Brazilian representative investor should be represented by a utility function displaying loss aversion. In this context, agents treat gains and losses differently, so that a decrease in utility caused by a marginal loss is always larger than an increase in utility resulting from a marginal gain. In other words, the Brazilian representative investor is more concerned about losses than equally large gains. Also, according to Amonlirdviman e Carvalho (2010), the loss aversion utility function has been widely used to explain why a high equity premium might be consistent with plausible levels of risk aversion, result that we found for the other utility functions used in this paper.

The rest of this paper is organized as follows. In Chapter 2, we describe the data and motivate the variable selection problem with individual moment regressions. We also discuss the four parametrizations of the investor's preferences and propose a methodology to compare the empirical and optimal portfolios to decide which utility function could represent the behavior of Brazilian investor. In Chapter 3, we report and discuss the empirical results. Finally, chapter 4 concludes.

1 Methodology and Data

1.1 Predicting Individual Moments

1.1.1 Data

To estimate the Brazilian empirical portfolio, we first define three asset classes: bonds, equities, and risk-free[2]. Our sample consists of monthly, quarterly, semiannual, and annual time series of market values of all the previously mentioned asset categories from January 2005 to June 2016. For stocks, we use market capitalization data from BM&FBovespa. We considered all the securities that trade on the stock exchange segment of BM&FBovespa during the

[2] Our risk-free portfolio is an equally weighted portfolio in savings, LFTs and CDBs.

sample period. Government bonds[3] data were obtained from Brazilian National Treasury (STN) dataset[4]. For CDBs[5], we used market capitalization data from Cetip[6] and BM&FBovespa. When a specific asset was traded either at Cetip and at BM&FBovespa (*e.g.*, CDBs), we summed up the market capitalization of this asset reported by these two companies. Finally, for savings, we use market capitalization data obtained from the Bacen Time Series Management System.

To estimate the Brazilian optimal portfolio we collect proxies for monthly returns on securities, bonds, and the risk-free portfolio. For equities, we used the market factor from the Nefin[7] database as a proxy. For bonds, we used an index of federal public securities IMA-G[8] from Bacen Time Series Management System. Finally, for the risk-free portfolio we used data from the Nefin database[9]. Our sample consists of monthly returns from January 2005 to June 2016. We report all estimates in terms of Brazilian reais (BRL).

The predictability literature has shown that a growing set of economic variables can partly predict the means, variances, and covariances of returns. Before that, however, the literature on market efficiency argued that stock prices appear to follow a random walk over time (see, for instance, Kendall e Hill (1953)). The literature on predictability initially used past returns to predict price changes and subsequently broaden the analysis by including economic variables such as interest rates, default spreads, dividend-yield, book-to-market, among others. [10]

---

[3] In this category, we used a wide range of securities, including: the Financial Treasury Bill (LFT - Letra Financeira do Tesouro), the National Treasury Bill (LTN – Letra do Tesouro Nacional), the National Treasury Notes Series-F (NTN-F - Notas do Tesouro Nacional série F), the NTN-B and NTN-C. Overall, the securities used represent 99% of the total domestic marketable debt of the federal government.

[4] The Brazilian National Treasury dataset is publicly available at `http://www3.tesouro.fazenda.gov.br/series_temporais/principal.aspx#ancora_consulta`.

[5] Certificates of Deposits (Certificados de Depósito Bancário).

[6] Cetip is the Latin America's largest depositary of private fixed-income securities with a vast over-the-counter, fixed-income derivatives operation. It is the Brazil's largest private-asset clearing house, according to the company's website.

[7] Nefin (Brazilian Center for Research in Financial Economics) was created by researchers from the Department of Economics of the University of São Paulo. It makes available data sets and variables such as Brazilian risk factors and stock portfolios.

[8] The IMA (Market Index ANBIMA) is a family of fixed income indexes that represents the public debt through the market prices of a portfolio of federal public securities. The theoretical portfolio of the IMA-General consists of all eligible securities, representing the evolution of the market as a whole.

[9] The risk-free rate from Nefin is computed from the 30-day DI Swap.

[10] The following is a briefly list of academic papers that document various degrees of mean predictability and the variables used: Campbell e Shiller (1998), dividend yield; Cochrane (1991), investment-to-capital ratio; Fama e Schwert (1977), Treasury bill yield; Fama e French (1989), default spread, dividend yield, term spread; Ferson e Harvey (1991), default spread, dividend yield, lagged returns, term spread, Treasury bill yield; and Pontiff e Schall (1998), book-to-market ratio. Studies on variance predictability include: Campbell (1987), term spread; French, Schwert e Stambaugh (1987), lagged squared return, lagged variance; Harvey (2001), default spread, dividend yield, lagged squared return, lagged

Lewellen (2002) argues that financial ratios, such as dividend-yield (DY) and book-to-market (BtM), share common features. First, each of these ratios measures price relative to 'fundamentals'. As price is high when expected returns are low, and vice-versa, the ratios should be positively related to expected returns. Moreover, the rational-pricing theory claims that the ratios track time-variation in discount rates: the ratios are low when the discount rates are low and high when the discount rates are high (They predict returns because they capture information about the risk premium). Second, they all share similar time series characteristics. At monthly frequency, they have autocorrelations close to one and most of their variation is generated by price changes in the denominator.

For these reasons, we chose to collect monthly data on four popular predictors: dividend-yield, book-to-market ratio, debt-to-market ratio, and the Ibovespa index[11] trend - or momentum variable. The dividend-yield for the Brazilian stock market is calculated as the ratio of total dividend payments in the last 12 months and the market value of equity. The book-to-market is the ratio between the book value and the market value. The debt-to-equity ratio is calculated by dividing the total liabilities by the stockholder's equity[12]. Finally, the trend is the difference between the logarithm of the current Ibovespa index level and the logarithm of the average index level over the previous 12 months. We use the dividend-yield computed by Nefin and all the remaining predictive variables were computed by using data from Economatica.

Table 1 and Figure 1 describe the data. In Table 1, Panel A presents univariate descriptive statistics of the monthly returns, annual returns, and predictors, Panel B reports descriptive statistics of the market capitalizations of equities, bonds, and risk-free portfolio, and finally Panel C shows pairwise correlations of the predictors with each other and with excess stock and bond returns. Figure 1 plots the time series and autocorrelations of the predictors.

variance, term spread, Treasury bill yield; Schwert (1989), debt-to-equity ratio, default spread, lagged variance, volume; and Whitelaw (1994), default spread, lagged variance, paper spread, Treasury bill yield. Finally, representative papers on predicting covariances are: Bollerslev, Engle e Wooldridge (1988), lagged covariances, lagged cross products of returns; and Harvey (1989), default spread, dividend yield, term spread.

11 The Ibovespa index is a total return index comprising the most representative companies in the Brazilian market, both by market capitalization and traded volume. It is the benchmark index of the São Paulo Stock Exchange.

12 We use only stocks selected following the Nefin Eligibility Criteria: a stock traded in BM&FBovespa is considered "eligible" for year t if it meets three criteria: (a) the stock is the most traded stock of the firm (the one with the highest traded volume during last year); (b) the stock was traded in more than 80% of the days in year $t-1$ with volume greater than R$500.000,00 per day. If the stock was listed in year $t-1$, the period considered goes from the listing day to the last day of the year; (c) the stock was initially listed prior to December of year $t-1$.

Table 1: Description of Returns and Predictors

Panel A: Descriptive Statistics

	Mean	Median	Std. Dev.	Skew	Kurtosis	Min	Max	Autocorrelations ρ_1	ρ_3	ρ_6
One-month horizon										
Bond	0.010	0.011	0.009	-0.570	4.501	-0.019	0.034	0.122	0.277	0.132
Equity	0.010	0.008	0.060	-0.208	4.211	-0.235	0.166	0.124	0.016	-0.125
Risk-Free	0.009	0.009	0.002	0.671	3.106	0.006	0.015	0.980	0.908	0.740
One-year horizon										
Bond	0.125	0.123	0.049	-0.736	2.999	0.018	0.186	0.009	0.285	-0.068
Equity	0.144	0.093	0.304	0.230	2.647	-0.389	0.742	-0.387	0.106	-0.170
Risk-Free	0.119	0.115	0.029	0.267	2.841	0.070	0.177	0.283	0.066	-0.243
Predictors										
BtM	0.577	0.597	0.131	-0.059	2.067	0.334	0.887	0.948	0.841	0.689
Trend	0.023	0.028	0.139	-0.761	4.501	-0.474	0.286	0.892	0.589	0.132
DY	0.027	0.027	0.005	0.424	3.192	0.016	0.041	0.780	0.447	0.075
DtE	0.593	0.556	0.128	1.454	4.576	0.454	0.967	0.964	0.893	0.737
Panel B: Descriptive Statistics										
Empirical Bond	1.062	0.985	0.507	0.168	-0.908	0.151	2.039			
Empirical Equity	2.031	2.258	0.501	-0.994	-1.120	0.847	2.702			
Empirical Risk-Free	1.429	1.590	0.327	-0.691	-0.272	0.776	1.920			

Panel C: Correlations

	BtM	Trend	DY	DtE	One-Month Horizon r^b	r^e
BtM	1.000	-0.656	0.348	0.658	-0.034	-0.211
Trend		1.000	-0.392	-0.253	0.088	0.450
DY			1.000	0.264	0.251	-0.054
DtE				1.000	0.037	-0.068

Note: In this table, Panel A reports descriptive statistics of monthly returns on bonds, equities, and risk-free. Panel A also presents descriptive statistics of four predictors: the book-to-market ratio, the Ibovespa index momentum variable trend, the dividend-yield, and the debt-to-equity ratio. Panel B presents descriptive statistics of the monthly market capitalization on bonds, equities, and risk-free (all statistics, except skewness and kurtosis, are reported in BRL trillion). Finally, Panel C reports correlations of the predictors with equity excess returns r^e and bond excess returns r^b. We use monthly data from January 2005 to June 2016, consisting of 138 observations. All variables are expressed in absolute terms.

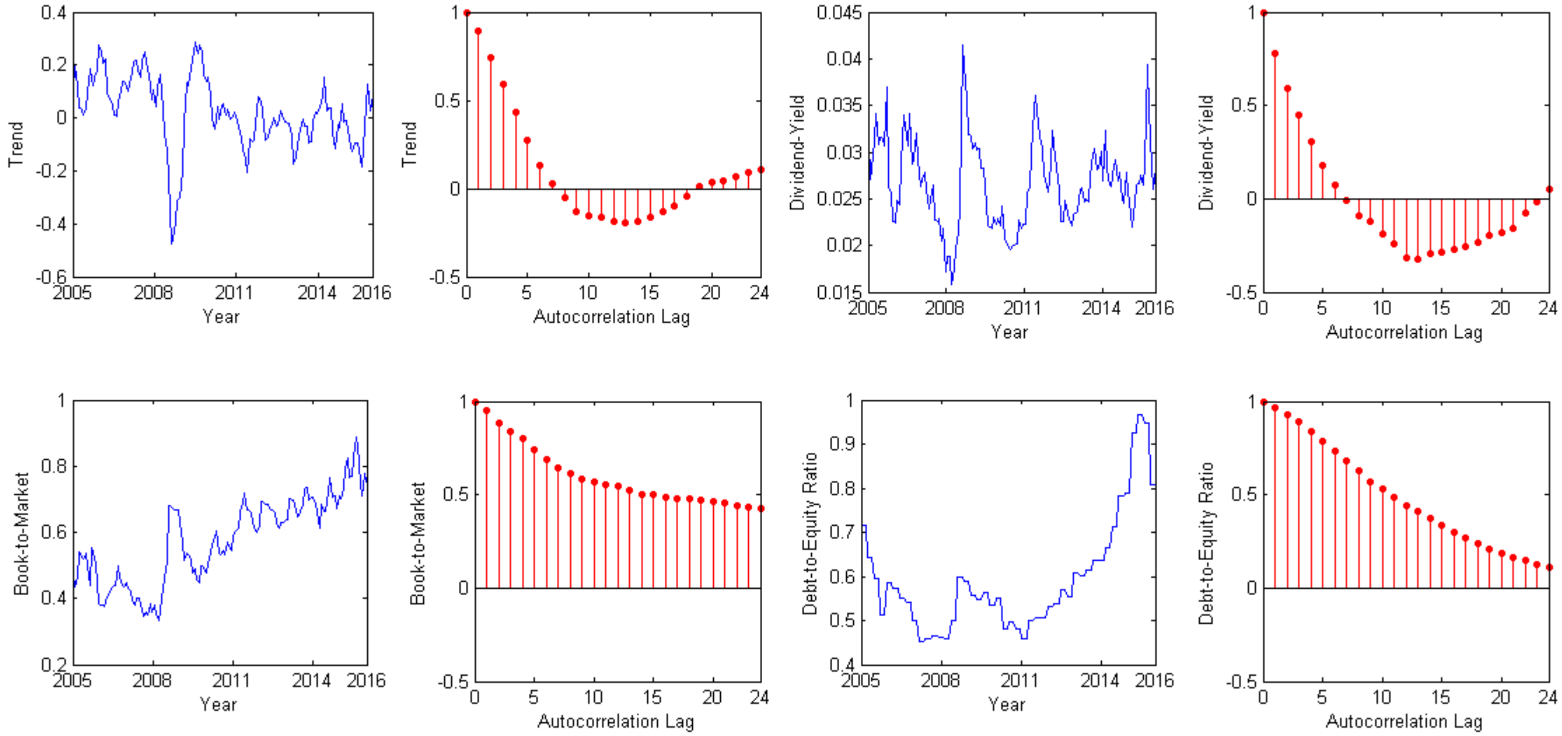

Figure 1: Predictors

This figure shows time series plots and autocorrellograms for the four predictors: the Ibovespa index momentum variable trend, the dividend yield, the book-to-market ratio, and the debt-to-equity ratio. We use monthly data from January 2005 to June 2016, consisting of 138 observations.

1.1.2 Predictive Regressions

To check whether the variables we select as potential predictors actually capture the time variation in at least the first and second moments of bond and stock excess returns, we run the following regressions:

$$\mathbb{E}_t \begin{bmatrix} r^b_{t+1} \\ r^e_{t+1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Z}'_t \boldsymbol{\gamma}_b \\ \boldsymbol{Z}'_t \boldsymbol{\gamma}_e \end{bmatrix}, \tag{1}$$

$$\mathrm{Var}_t \begin{bmatrix} r^b_{t+1} \\ r^e_{t+1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Z}'_t \boldsymbol{\delta}_{be} & \boldsymbol{Z}'_t \boldsymbol{\delta}_{bb} \\ \boldsymbol{Z}'_t \boldsymbol{\delta}_{ee} & \end{bmatrix}, \tag{2}$$

where r^b_{t+1} and r^e_{t+1} denote bond and stock excess returns, respectively, and the vector $\boldsymbol{Z}_t$ contains subsets of the four predictors. We estimate $\boldsymbol{\gamma} \equiv (\boldsymbol{\gamma}'_b, \boldsymbol{\gamma}'_e)'$ and $\boldsymbol{\delta} \equiv (\boldsymbol{\delta}'_{be}, \boldsymbol{\delta}'_{bb}, \boldsymbol{\delta}'_{ee})'$ by using least square estimator (LS) for all one-, two-, three-, and four-dimensional subsets of the four predictors. We compute asymptotic standard errors according to the Hansen e Hodrick (1983)'s procedure.

Panel A of Table 2 presents the regression results. For each security (bonds and equities) and moment (mean, variance, and covariance), we present the best one-, two-, three- and four-variable partial regressions. The best partial regressions are chosen according to the Akaike Information Criterion (AIC).

1.2 Predicting Optimal Portfolio Weights

Let $\boldsymbol{Z}_t$ be a vector of state variables at time t, W_{t+1} the next-period's wealth, $\upsilon(\cdot)$ the utility function, and $\boldsymbol{\alpha}_t$ the portfolio weights at time t. The investor's optimization problem at time t is given by[13]

$$\begin{aligned} \underset{\boldsymbol{\alpha}_t}{\text{Maximize}} \quad & \mathbb{E}[\upsilon(W_t(\boldsymbol{\alpha}'_t \boldsymbol{R}_{t+1}))|\boldsymbol{Z}_t] \\ \text{subject to} \quad & W_{t+1} = W_t(\boldsymbol{\alpha}'_t \boldsymbol{R}_{t+1}), & (3) \\ & \boldsymbol{\alpha}'_t \boldsymbol{\iota} = 1, & (4) \\ & \alpha_{i,t} \in [0,1], \quad i = 1, \ldots, N, & (5) \end{aligned}$$

where $\boldsymbol{R}_{t+1}$ is a vector of gross returns on the assets available to the investor, $\boldsymbol{\iota}$ denotes a vector of ones, and N is the total number of assets. The first restriction represents the budget constraint, the second constraint ensures that all wealth is invested in the portfolio with no leverage, and the last restriction is the short-sale constraint. The optimal portfolio weights $\boldsymbol{\alpha}^*_t$ of each investor is a mapping from the state variables $\boldsymbol{Z}_t$ to $[0,1]^N$.

The relationship between the portfolio weights and the predictability of the individual moments of the returns $\boldsymbol{R}_{t+1}$ given the predictors $\boldsymbol{Z}_t$ depends on the type of the objective function $\upsilon(W_{t+1})$. To analyze how the solution to the portfolio-choice problem varies among investors with different preferences,

[13] We describe our econometric methodology for a single-period problem in order to keep the notation simpler. However, we also computed optimal portfolios over time.

Table 2: Individual Moment Predictability

Panel A of this table presents predictive regressions for expected bond excess returns $\mathbb{E}_t[r^b_{t+1}]$, expected stock excess returns $\mathbb{E}_t[r^e_{t+1}]$, the variance of bond excess returns $\text{Var}_t[r^b_{t+1}]$, the variance of stock excess returns $\text{Var}_t[r^e_{t+1}]$, and the covariance between bond and stock excess returns $\text{Cov}_t[r^b_{t+1}, r^e_{t+1}]$. The predictors are the book-to-market, the Ibovespa index momentum variable, the dividend-yield, and the debt-to-equity ratio. The return horizon is one month. For each moment, we present the best one-, two-, three and four-variable partial regressions, selected based on the Akaike information criterion. Panel B summarizes the choice of the best one or two predictors for each moment, based on the Akaike information criterion, using the 1-month horizon.

Panel A: Predictive Regressions						
One-month horizon						
c	BtM	Trend	DY	DtE	R^2	AIC
$\mathbb{E}_t[r^b_{t+1}]$						
0.0009	0.0061***	0.0013**	-0.0544**	-0.0039***	0.5954	-15.3219
0.0015	0.0050***		-0.0619***	-0.0035***	0.5589	-15.2650
-0.0004		0.0019***	-0.0061***	-0.0042***	0.4759	-15.1002
0.0008	0.0028**		-0.0657**		0.3218	-14.8700
$\mathbb{E}_t[r^e_{t+1}]$						
0.0209	-0.0159	0.0099**		-0.0114	0.3919	-10.4369
0.0226	-0.0159	0.0090**	-0.0752	-0.0109	0.3973	-10.4261
0.0244**	-0.0250**			-0.0082	0.3673	-10.4174
0.0268	-0.0236		-0.1275	-0.0079	0.3774	-10.4142
$\text{Var}_t[r^b_{t+1}]$						
0.0000**	0.0001***	0.0000	-0.0013***	0.0000*	0.7785	-22.8051
0.0000**	0.0001***		-0.0014***	0.0000**	0.7670	-22.7976
0.0000*	0.0002***	0.0000	-0.0012**		0.7524	-22.7417
0.0000	0.0002***		-0.0013***		0.7304	-22.6973
$\text{Var}_t[r^e_{t+1}]$						
0.0034**	-0.0072*		0.1173*	0.0036	0.2110	-12.9268
0.0039*	-0.0082*	-0.0011*	0.1106	0.0040	0.2150	-12.9175
0.0041***	-0.0049*		0.1212*		0.1690	-12.8899
0.0040***	-0.0049**	0.0001	0.1217*		0.1690	-12.8753
$\text{Cov}_t[r^b_{t+1}, r^e_{t+1}]$						
0.0001*	0.0004**	-0.0001	-0.0051*	-0.0001		
0.0001	0.0004***		-0.0046	-0.0002	0.3490	-19.3546
0.0001**	0.0002***	-0.0001	-0.0055*		0.3340	-19.3312
0.0001	0.0003***		-0.0048		0.3030	-19.3010

Panel B: Variable Selection	
One-month horizon	
$\mathbb{E}_t[r^b_{t+1}]$	DY, DtE
$\mathbb{E}_t[r^e_{t+1}]$	Trend
$\text{Var}_t[r^b_{t+1}]$	BtM, DY
$\text{Var}_t[r^e_{t+1}]$	BtM, DY
$\text{Cov}_t[r^b_{t+1}, r^e_{t+1}]$	BtM

***, **, and * denote statistical significance at the 1, 5, and 10 percent levels, respectively.

we consider four alternative specifications for the objective function. The first two, mean-variance and constant relative risk aversion (CRRA), are standard expected utility functions, whereas the last two, ambiguity aversion and loss aversion, are non-expected utility functions.

1.3 Utility Functions

1.3.1 Expected Utility

(A) Mean-Variance

The mean-variance portfolio optimization approach proposed by Markowitz (1952) is one of the most used models in financial portfolio selection in either industry or academia, because it depends exclusively on the first two moments of the returns distribution. The mean-variance preference can be represented by the following objective function of the investor:

$$\mathbb{E}[\upsilon(W_{t+1})|\boldsymbol{Z}_t] = \mathbb{E}[W_{t+1}|\boldsymbol{Z}_t] - \frac{\gamma}{2}\text{Var}(W_{t+1}^2|\boldsymbol{Z}_t), \quad (6)$$

where the absolute risk aversion coefficient is given by $\gamma \geq 0$.

The idea behind Markowitz (1952)'s work is that investors will choose portfolios based on the basic trade-off between expect return (which they like) and risk (which they dislike). When computing optimal mean-variance portfolios, the selection of the desired risk premium depends on the investor's tolerance to risk. Risk-loving investors might be willing to accept a higher volatility in their portfolios in order to achieve a higher risk premium, while risk-averse investors will prefer less volatile portfolios, therefore penalizing performance.

(B) Power Utility (CRRA)

We also consider an investor with constant relative risk aversion (CRRA) or power utility:

$$\upsilon(W_{t+1}) = \begin{cases} \frac{W_{t+1}^{1-\gamma}}{1-\gamma}, & \text{if} \quad \gamma > 1 \\ \ln(W_{t+1}), & \text{if} \quad \gamma = 1 \end{cases} \quad (7)$$

where the parameter γ represents the coefficient of relative risk aversion[14].

The CRRA utility function is widely used in the finance literature of portfolio selection. A very attractive property of the CRRA function, which is a particular case of the utility functions that exhibit linear risk-tolerance, is the wealth homogeneity, that is, the fact that the optimal portfolio weights do not depend on the wealth: it does not matter whether the investor is managing \$10 million or \$100 million, he or she will always

[14] Many studies have suggested that the admissible range of γ should be between one and two CITE. However, Mehra e Prescott (1985) argue that the equity premium puzzle empirically observed can only be solved when the coefficient γ is of the order of 30.

invest the same proportion of the wealth in every asset. In addition, under the CRRA utility we also have that the risk aversion is the same for all levels of wealth. In this case, the advantage of this utility function is also an disadvantage, because investors generally tend to be more risk averse over losses than over gains.

We can estimate the risk aversion indirectly. Suppose we gave you a lottery that pays $1000 with 50% probability or $500 for sure. How much would you be willing to pay for this opportunity? We can map what an investor would pay against the investor's risk aversion, γ. The following table allow us to read off risk aversion levels as a function of the amount you would pay to enter the gamble:

Table 3: The amount of money you should pay to avoid a gamble

Risk Aversion γ	Amount you would pay
0	750
0.5	729
1	707
5	586
10	540
50	507

This table also can be used to represent the absolute risk aversion coefficient of this study, because we are assuming that $W_t = 1$. Therefore, in this case, both coefficients are exactly the same.

1.3.2 Non-Expected Utility

(A) Ambiguity Aversion

The expected utility theory assumes that the investor is able to compute expectations of returns, which requires that the agent knows the probability distribution of returns or that he is able to form beliefs about it. However, Knight (2012) and Ellsberg (1961) argue that the investor may not have all the necessary information to form such expectations. In this context, the investor faces additional "ambiguity" that is not captured in the traditional expected utility framework, because he is not sure about what is the "true" probability distribution. The ambiguity aversion preferences formalize the idea that the investor dislikes the ambiguity about the world.

Consider again an investor with power utility, except that now the investor is uncertain about whether the return distribution is $\overline{p}$ (the empirical distribution, for example) or some other distribution $p \in \mathcal{P}$ in the neighborhood $\overline{p}$. Following Gilboa e Schmeidler (1989) and Dow e

Werlang (1992), the investor's portfolio choice problem is given by:

$$\max_{\boldsymbol{\alpha}_t} \left\{ \min_{p \in \mathcal{P}} \mathbb{E}[\upsilon(W_t(\boldsymbol{\alpha}_t' \boldsymbol{R}_{t+1})|\boldsymbol{Z}_t] \right\}, \tag{8}$$

where $\upsilon(\cdot)$ is the CRRA utility function. Given the ambiguity about the return distribution, the investor considers the worst case outcome (in the neighborhood of $\overline{p}$) through the interior minimization. The exterior maximization then achieves the usual trade-off between risk and expected reward.

To implement this form of ambiguity aversion, we need to characterize the set of possible return distributions $\mathcal{P}$. Following Ellsberg (1961), we adopt the following ϵ-contamination parameterization:

$$\mathcal{P} = \{(1-\epsilon)\overline{p} + \epsilon p : p \in \wp\}, \tag{9}$$

where $\wp$ denotes the σ-algebra generated by the support of the return distribution. The parameter ϵ[15] represents the investor's degree of ambiguity. With $\epsilon = 0$, the investor's objective function becomes the standard CRRA utility and the return distribution is given by $\overline{p}$.

The advantage of this parameterization is that the investor's problem simplifies to:

$$\max_{\boldsymbol{\alpha}_t} \quad (1-\epsilon)\mathbb{E}_{\overline{p}}[\upsilon(W_t(\boldsymbol{\alpha}_t' \boldsymbol{R}_{t+1}))|\boldsymbol{Z}_t] + \epsilon \inf_{\wp} \upsilon(W_t(\boldsymbol{\alpha}_t' \boldsymbol{R}_{t+1})), \tag{10}$$

where we assume that the support of the return distribution is independent of the predictors. Thus, to implement the notion of ambiguity aversion we only need to choose a value for the parameter ϵ and specify the support of the return distribution to evaluate the infimum.

(B) Prospect Theory and Loss Aversion

Kahneman e Tversky (1979) found that people treat gains and losses differently: people are much more distressed by prospective losses than they are happy with equivalent gains. In other words, investors are risk averse when facing gains (a small certain gain is preferred to a probable risk gain) and risk seeking when facing losses (a probable risky loss is preferred to a small certain loss). In this context, people will usually take more risks to avoid losses than to realize gains.

These findings are formalized in the loss averse utility function, an S-shaped value function that is concave for gains, convex for losses, and steeper for losses than for gains. This function is characterized by three properties. First, wealth is measured relative to a given reference point.

[15] Alternatively, one can interpret the portfolio choice as the investor playing a two-stage game against the nature, the non-strategic player. In the first stage, nature replaces with probability ϵ the return distribution $\overline{p}$ with an arbitrary distribution $p \in \wp$. In the second stage, nature draws a set of returns from the return distribution.

Second, the decrement in utility by a marginal loss is always larger (in absolute value) than the increment in utility resulting from a marginal gain. Third, agents are risk averse in the domain of gains and risk loving in the domain of losses.

To capture the differential risk preferences over gains and losses generated by the certainty effect, Tversky e Kahneman (1992) propose the following objective function for the choice stage:

$$v(W_{t+1}) = \begin{cases} (W_{t+1} - \overline{W})^{b_1}, & \text{if} \quad (W_{t+1} - \overline{W}) \geq 0, \\ -l(\overline{W} - W_{t+1})^{b_2}, & \text{if} \quad (W_{t+1} - \overline{W}) < 0, \end{cases} \tag{11}$$

where $(W_{t+1} - \overline{W})$ determines gains or losses, $\overline{W}$ is the reference wealth level determined in the editing stage, l measures the investor's loss aversion, the parameter b_2 captures the degree of risk seeking over losses, and b_1 captures the degree of risk aversion over gains. The kink at the origin introduced by $l > l$ makes losses (relatively) more painful than gains.

The properties of loss averse utility depend on the selection of different parameter values, but there seems to be a few theoretical results that suggest appropriate values for b_1, b_2, and l. Tversky e Kahneman (1992) suggest a set of parameter values for loss averse utility using experiments. Benartzi e Thaler (1995), Barberis e Thaler (2003), and Barberis e Xiong (2009) use similar values in their studies[16]. Other studies such as Wu e Gonzalez (1996) estimate that the values of the two curvature parameters are identical but significantly smaller than those of Tversky e Kahneman (1992). In general, decision makers are loss averse and there is more utility curvature for gains than for losses[17]. For example, when $b_1 > 1$ and $b_2 > 1$, the investor is risk loving with regard to gains since $u''((W_{t+1} - \overline{W})) = (b_1 - 1)(W_{t+1} - \overline{W})^{b_1 - 2} > 0$, while she is risk averse with regard to losses since $u''((W_{t+1} - \overline{W})) = -(b_2 - 1)(-(W_{t+1} - \overline{W}))^{b_2 - 2} < 0$. One simple method to avoid the parameter choice problem that has been used academically and commercially is to assume $b_1 = b_2 = 1$, such that $u(\cdot)$ would be risk neutral with regard to gains or losses (see Benartzi e Thaler (1995)).

1.3.3 Conditional Mean and Conditional Variance

The conditional mean of W_{t+1} given the state variables $\boldsymbol{Z}_t$ is defined as:

$$E[W_{t+1}|\boldsymbol{Z}_t] = \pi + \beta \boldsymbol{Z}_t. \tag{12}$$

16 Tversky e Kahneman (1992) propose that $b_1 = b_2 = 0.88$ and $l = 2.25$, and Barberis e Thaler (2003) used these suggested values in their studies.

17 See Abdellaoui, Klibanoff e Placido (2015) and Wakker, Timmermans e Machielse (2007) for further discussion of the shape of loss averse utility.

Analogously, the conditional covariance of W_{t+1} given the state variables $\boldsymbol{Z}_t$ is given by the following equation:

$$E[(W_{t+1} - E(W_{t+1}|\boldsymbol{Z}_t)^2)|\boldsymbol{Z}_t] = \pi + \eta \boldsymbol{Z}_t. \tag{13}$$

1.3.4 Distance Measures

To decide which combination of parameters, and consequently, which utility function best represents the behavior of the Brazilian representative investor, we compare the empirical portfolio with the optimal portfolio using two distance measures[18]. To compare the unconditional portfolios we use the Euclidean distance, and to compare the portfolios over time we use the Mahalanobis distance.

Assume we have ω_{ik}, the unconditional empirical weights, and α_{jk}, the unconditional optimal weights. In this case, we choose the utility function solving the following optimization problem:

$$\min_{\theta} \quad \left(\sum_{k=1}^{2} (\omega_{ik} - \alpha_{jk})^2 \right)^{1/2}, \tag{14}$$

for every $k = \{returns, bonds\}$ and where $\theta = \{\gamma, \epsilon, b, l\}$ are the optimal parameters. The Euclidean distance can be interpreted as the straight line distance from one point to the other.

The Euclidean distance is a suitable multivariate measure of distance in situations in which all variables are expected to have equal variance. However, since we cannot assume this with respect to the calculated optimal and empirical portfolios over time, we will use a measure of distance that takes into account the variance and covariance between the variables. The Mahalanobis distance accounts for the variance of each variable and the covariance between variables. Geometrically, it does this by transforming the data into standardized uncorrelated data and computing the ordinary Euclidean distance for the transformed data. In this case, the Mahalanobis distance is similar to a univariate z-score: it provides a way to measure distances that takes into account the scale of the data. So, by assuming that ω_{ik} is a vector of unconditional empirical weights over time, and α_{jk} is a vector of the unconditional optimal weights over time, we choose the utility function solving the following optimization problem:

$$\min_{\theta} \quad \left(\sum_{k=1}^{2} |\omega_{ik} - \alpha_{jk}| A^{-1} |\omega_{ik} - \alpha_{jk}| \right), \tag{15}$$

for every $k \in \{returns, bonds\}$, where $\theta = \{\gamma, \epsilon, b, l\}$ are the optimal parameters of each different parametrization of the utility function.

[18] A distance measure gives a score describing how much two items differ.

2 Numerical Results

In this chapter, we briefly describe the numerical results of our analysis. First, we present the results related to the dynamics of the empirical portfolio. Second, we report the results related to the unconditional and conditional optimal portfolio for each of our four different parameterizations of the utility function. Finally, we compare the empirical and optimal portfolios to decide which utility function best represents the Brazilian representative investor.

2.1 Empirical Portfolio

In Figure 2, we depict the Brazilian empirical portfolio over the 2005-2016 period in order to examine the historical dynamics of the empirical portfolio weights for the three asset classes considered. In Table 4, Panel A presents the estimates for the relative empirical portfolio weights for the three asset classes during the 2005-2016 period, and Panel B reports data characteristics for the three main asset classes.

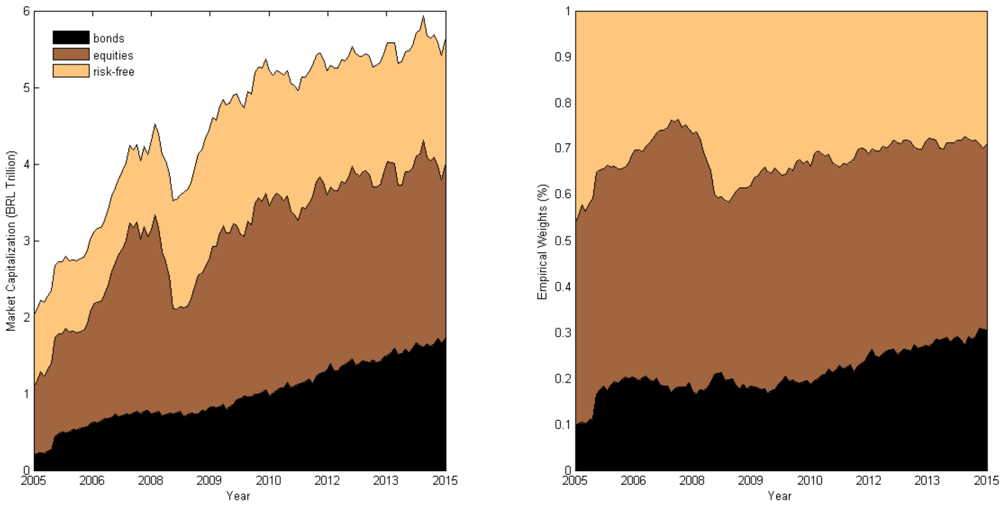

Figure 2: Estimated market value and weights in the Brazilian Empirical Portfolio

The left-hand side figure shows the estimated market values in the Brazilian empirical portfolio over the 2005-2016 period in absolute numbers (BRL trillion). The portfolio in 2005 amounts to approximately BRL 2.44 trillion; in 2010, 5.98 trillion; in 2015, 6.89 trillion. The right-hand side figure presents the empirical weights of Brazilian representative investor over the 2005-2015 period.

In Figure 2, notice that the 2008–09 crises led to a large reduction in the market value of equity and risk-free. As can be seen in Table 4, the corresponding weight of equities in 2015 is equal to the record low of 34.90%. In 2016, the amount invested in equities is almost the same as the amount invested in government bonds. The maximum weight of equities is 54.17%, and it was obtained in 2007. The average weight for equities during the entire sample period is 45.03%. Risk-free are subject to a smaller change in portfolio weights than are equities and bonds over the sample period. The maximum weight for bonds weight of 33.031% of bonds in 2016 is equal to its maximum over the sample period.

The general picture of the Brazilian empirical portfolio is of a declining weight for equities from 2008 on to the benefit of government bonds and risk-free assets. Equities fall from 46.85% in 2005 to 34.90% in the middle of 2016. Government bonds rise from 9.95% to 33.31%. And, finally, our risk-free portfolio falls from 43.20% in 2005 to 31.79% in 2016. Today, the empirical portfolio is almost an equally weighted portfolio in bonds, equities, and risk free.

Table 4: Estimated Weights in the Empirical Portfolio

In this table, Panel A reports the empirical weights of bond, equity, and risk-free over the 2005-2016 period. Panel B presents descriptive statistics for each asset class. All values are reported in percentage terms (%).

Panel A: Empirical Weights			
Year	Bond	Equity	Risk-Free
2005	9.945	46.85	43.20
2006	18.91	47.46	33.62
2007	18.94	54.17	26.89
2008	18.70	49.95	31.35
2009	18.44	43.43	38.13
2010	18.87	46.86	34.27
2011	21.28	46.25	32.47
2012	24.67	45.18	30.14
2013	26.69	44.58	28.73
2014	28.73	42.64	28.63
2015	32.41	38.05	29.54
2016	33.31	34.90	31.79
Panel B: Descriptive Statistics			
Mean	22.57	45.03	32.40
Standard Deviation	6.774	5.061	4.547
Median	20.11	45.72	31.57
Minimum	9.945	34.90	26.89
Maximum	33.31	54.17	43.20

2.2 Optimal Portfolio

In this section we characterize the unconditional portfolio choice of investors with expected and non-expected utility functions. The unconditional portfolio choices serve as a benchmark for the conditional asset allocations over time.

(A) Mean-Variance

Panel A of Table 5 presents estimates of the unconditional portfolio choice of investors with mean-variance utility function and absolute risk aversion (and also for relative risk aversion since $W_t = 1$) of $\gamma \in \{1, 10, 20, 30, 40\}$. The investment horizon is one month, three months, or one year. We omit the results for the six-month horizon investment to save space. We impose the short-sale constraints $0 \geq \alpha_i \geq 1$, $i = 1, \ldots, N$ to prohibit short selling. In brackets, we report asymptotic standard errors computed using the block bootstrap procedure developed separately by Hall (1985) and Carlstain (1986) and Künsch (1989)[19].

Many familiar, but nevertheless interesting, features of the mean-variance optimal portfolios emerge. Except when $\gamma = 1$, in which case the short-sale constraints are binding, all mean-variance investors hold the same risky positions of 100, 95, or 91 percent stocks, and 0, 5 or 9 percent bonds, depending on the horizon but regardless of the risk aversion. Risk aversion only determines the amount of wealth the investor allocates to risky assets rather than the risk-free portfolio. On the monthly horizon, this allocation ranges from 100 percent for $\gamma = 1$ to about 30 percent for $\gamma = 40$.

Graphically, the fact that all mean-variance investors allocate the same proportion of wealth to risky assets, but invest different amounts of wealth in risky assets implies that optimal portfolios are arranged on a straight line in the expect return versus standard deviation space. Also, notice that the optimal standard deviation of wealth is inversely proportional to γ, because the decision of how much wealth to invest in the risky portfolio is inversely proportional to the investors' absolute risk aversion (see equation ?).

(B) Constant Relative Risk Aversion

Panel B of Table 5 reports estimates of the unconditional portfolio choice of investors with CRRA utility functions and relative risk aversion equal to $\gamma \in \{1, 10, 20, 30, 40\}$. The results are very similar to those for the mean-variance preferences, except that power utility investors hold less stocks and more bonds relative to mean-variance investors. However, Ait-Sahalia and Brandt (2001) argue that this is an empirical result, not a theoretical one. In theory, the risky position of a CRRA investor rely on relative risk aversion, since the investor's preferences for higher order

[19] They all started from the criteria of creating blocks of consecutive data. The procedure divides the original time series into blocks of individual observation units or estimated residuals, where the bootstrap data inside each block are created using the classical i.i.d. bootstrap.

Table 5: Unconditional Portfolio Choice with Expected Utility Functions

This table reports parameter estimates of the unconditional portfolio choice of investors with single-period objectives:

$$\text{Panel A: } \max_{\boldsymbol{\alpha}} \left(\mathbb{E}\left[W_{t+1}\right] - \frac{\gamma}{2} \text{Var}\left[W_{t+1}\right] \right),$$

$$\text{Panel B: } \max_{\boldsymbol{\alpha}} \left(\mathbb{E}\left[\frac{W_{t+1}^{1-\gamma}}{1-\gamma} \right] \right),$$

where W_{t+1} is next period's wealth and $\boldsymbol{\alpha} = (\alpha^b, \alpha^e, \alpha^{rf})$ are the fractions of current wealth W_t invested in bonds, stocks, and risk-free, respectively. The investment horizon is one month, six months, or one year. The optimization is subject to the short-sales constraints $0 \geq \alpha_i \geq 1$, $i = 1, \ldots, N$.

	α^e	α^b	α^{rf}	α^e	α^b	α^{rf}	α^e	α^b	α^{rf}
	One-Month Horizon			Six-Month Horizon			One-Year Horizon		
Panel A: Mean Variance Investors									
$\gamma = 1$	0.000	1.000	0.000	0.023	0.977	0.000	0.218	0.782	0.000
$\gamma = 10$	0.000	1.000	0.000	0.035	0.708	0.257	0.026	0.271	0.703
$\gamma = 20$	0.000	0.575	0.425	0.017	0.354	0.629	0.013	0.135	0.852
$\gamma = 30$	0.000	0.383	0.617	0.012	0.236	0.752	0.009	0.090	0.901
$\gamma = 40$	0.000	0.288	0.712	0.009	0.177	0.814	0.006	0.068	0.926
Panel B: Constant Relative Risk-Averse Investors									
$\gamma = 1$	0.000	1.000	0.000	0.019	0.981	0.000	0.268	0.732	0.000
$\gamma = 10$	0.000	1.000	0.000	0.031	0.557	0.412	0.037	0.291	0.673
$\gamma = 20$	0.000	0.569	0.431	0.016	0.280	0.705	0.018	0.146	0.836
$\gamma = 30$	0.000	0.379	0.621	0.011	0.187	0.803	0.012	0.098	0.889
$\gamma = 40$	0.000	0.286	0.714	0.008	0.140	0.852	0.009	0.003	0.989

moments, which differentiate a CRRA investor from a mean-variance investor, are a function of relative risk aversion. However, in the data, the effect of the higher order moments is generally not strong enough to be noticeable in CRRA investor's holdings with different degrees of risk aversion, albeit it is also a factor explaining the different stock holdings of equally risk-averse CRRA and mean-variance investors.

(C) Ambiguity Aversion

We present the results for investors with ambiguity aversion preferences in Panel A of Table 6. We consider the cases in which $\gamma \in \{15, 30\}$ and $\epsilon \in \{0.0001, 0.0005, 0.001, 0.005, 0.01\}$[20]. The case in which $\epsilon = 0$ corresponds to the CRRA portfolio choices in Panel B of Table 5. We parameterize the worst-case returns on stocks and bonds, which we need in order to evaluate the infimum in the objective function 10 as the empirical univariate minimums from Panel A of Table 1.

Ambiguity aversion has two consequences on the optimal portfolio choices. First, an increase in ambiguity aversion leads investors to substitute risk-free portfolio for bonds or risk-free for stocks[21]. Second, the investor does not hold equities regardless the value of ϵ for the monthly horizon. This tendency to not hold a position in some securities happens because the expected returns are not sufficiently positive to justify taking on the associated ambiguity.

(D) Loss Aversion

Panel A of Table 6 presents estimates of the unconditional portfolio choice of prospect theory investors. Guided by the experimental calibrations of Tversky e Kahneman (1992), we consider the parameter values of $b = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\%\}$ and $l = \{2\}$[22]. We set the wealth reference level $\overline{W}$ equal to the initial wealth $W_t = 1$.

In general, the most striking feature of the prospect theory results are the strong horizon effects of the loss averse portfolio. Usually, the composition of the optimal portfolio completely changes with different time horizons. Benartzi e Thaler (1995) explain that the more often a loss-averse investor evaluates his or her portfolio, the less attractive are high expected returns but high variance investments because losses of these investments are realized more often at short horizons than at long horizons. In most cases, loss aversion causes short-term investors to be risk averse, since the return distribution straddles the kink of the utility function, but long term investors to be almost risk neutral, as the mass of the return

[20] The choice of ϵ is *ad hoc*. Camerer e Lovallo (1999) cites attempts to calibrate ambiguity aversion preferences to gambling experiments. Since it is unclear, however, how these experimental results relate to ambiguity aversion in financial markets, we present estimates for a large range of ϵ.

[21] This empirical equivalence was formalized by Liu et al. (1999)

[22] We omit the results for different values of l because the optimal weights did not change when we use a wide range of values for this parameter.

Table 6: Unconditional Portfolio Choice with Non-expected Utility Preferences

This table shows estimates of the unconditional portfolio choice of investors for a single-period problem:

$$\text{Panel A: } \max_{\alpha} \left((1-\epsilon) E\left[\frac{W_{t+1}^{1-\gamma}}{1-\gamma}\right] + \epsilon \inf_{R} \left\{ \frac{W_{t+1}^{1-\gamma}}{1-\gamma} \right) \right\},$$

where W_{t+1} is next period's wealth and $\boldsymbol{\alpha} = (\alpha^b, \alpha^e, \alpha^{rf})'$ are the fractions of current wealth W_t invested in bonds, stocks, and risk-free, respectively. The investment horizon is one month, six months, or one year. The optimization is subject to the short-sales constraints $0 \geq \alpha_i \geq 1$, $i = 1, \ldots, N$.

		α^e	α^b	α^{rf}	α^e	α^b	α^{rf}	α^e	α^b	α^{rf}
		One-Month Horizon			Six-Month Horizon			One-Year Horizon		
		Panel A: Ambiguity Averse Investors								
γ=15	ϵ=0.01%	0.000	0.760	0.240	0.020	0.371	0.609	0.024	0.193	0.783
	ϵ=0.05%	0.000	0.749	0.251	0.019	0.367	0.614	0.024	0.194	0.782
	ϵ=0.1%	0.000	0.735	0.265	0.018	0.362	0.619	0.024	0.192	0.785
	ϵ=0.5%	0.000	0.629	0.371	0.011	0.329	0.660	0.021	0.172	0.807
	ϵ=1%	0.000	0.500	0.500	0.002	0.292	0.705	0.017	0.146	0.837
γ=30	ϵ=0.01%	0.000	0.380	0.620	0.010	0.186	0.804	0.012	0.095	0.893
	ϵ=0.05%	0.000	0.375	0.625	0.010	0.182	0.808	0.012	0.094	0.894
	ϵ=0.1%	0.000	0.368	0.632	0.009	0.182	0.809	0.013	0.091	0.896
	ϵ=0.5%	0.000	0.315	0.686	0.005	0.165	0.830	0.011	0.083	0.907
	ϵ=1%	0.000	0.253	0.747	0.001	0.145	0.854	0.009	0.073	0.918

Table 7: Unconditional Portfolio Choice with Non-expected Utility Preferences

This table shows estimates of the unconditional portfolio choice of investors with single-period objectives:

$$\text{Panel A: } \max_{\alpha} \begin{cases} -\mathbb{E}[l(\overline{W} - W_{t+1})], & \text{if} \quad W_{t+1} < \overline{W} \\ \mathbb{E}[W_{t+1} - \overline{W}], & \text{if} \quad W_{t+1} \geq \overline{W}, \end{cases}$$

where W_{t+1} is next period's wealth, $\overline{W} = 1$ is a subjective wealth level, and $\boldsymbol{\alpha} = (\alpha^b, \alpha^e, \alpha^{rf})'$ are the fractions of current wealth W_t invested in bonds, stocks, and risk-free, respectively. The investment horizon is one month, six months or one year. The optimization is subject to the short-sales constraints $0 \geq \alpha_i \geq 1$, $i = 1, \ldots, N$.

		α^e	α^b	α^{rf}	α^e	α^b	α^{rf}	α^e	α^b	α^{rf}
		One-Month Horizon			Six-Month Horizon			One-Year Horizon		
		Panel A: Loss Averse Investors								
l=2	b=0.1	0.043	0.317	0.640	0.014	0.303	0.683	0.043	0.317	0.640
	b=0.2	0.048	0.352	0.599	0.016	0.333	0.651	0.048	0.352	0.599
	b=0.3	0.055	0.395	0.550	0.017	0.371	0.612	0.055	0.395	0.550
	b=0.4	0.064	0.449	0.487	0.018	0.418	0.565	0.064	0.449	0.487
	b=0.5	0.076	0.517	0.407	0.018	0.475	0.506	0.076	0.517	0.407
	b=0.6	0.092	0.607	0.301	0.018	0.546	0.437	0.092	0.607	0.301
	b=0.7	0.118	0.717	0.164	0.013	0.629	0.359	0.118	0.717	0.164
	b=0.8	0.160	0.829	0.012	0.000	0.714	0.286	0.160	0.829	0.012
	b=0.9	0.236	0.764	0.000	0.000	0.731	0.269	0.236	0.764	0.000
	b=1	0.264	0.736	0.000	0.000	1.000	0.000	0.264	0.736	0.000

distribution moves away from the kink. However, we did not find this empirical evidence when analyzing the investor's unconditional loss averse portfolio. Contrariwise, the choices between the assets practically do not change when we alter the investment horizon. Besides that, the investor is almost risk averse in the shortest investment horizon considered - on month. One possible explanation for this contradictory result is that the asset returns used have an enormous variance in both the short and the long term, so that the investor always chooses the same combination of weights to invest.

2.3 Comparison Between the Empirical Portfolio and the Optimal Portfolio

Table 7 presents the optimal choice of parameters for the unconditional portfolio. We estimate the parameters for each different parameterization of the utility function and investment horizons. We compare the empirical weights shown in Table 3 to the optimal weights obtained through portfolio optimization models. We choose the one that best represents the behavior of the Brazilian investor's portfolio based on a distance measure.

On the monthly horizon, all the utility functions could represent the Brazilian investor, since the Euclidean distances are statistically equal. At the one-month horizon, the Brazilian investor is represented by a mean-variance utility function with absolute risk aversion $\gamma = 25.3$, a constant risk aversion function with $\gamma = 25$, an ambiguity averse function with relative risk aversion $\gamma = 25$, and an ambiguity averse function with $\epsilon = 0.07\%$ or a loss averse function with loss aversion parameter equal to $l = 2$ and risk aversion $b = 0.995$. At quarterly horizons, investors are better represented by a mean-variance function with $\gamma = 17.95$. At the semiannual horizons, the functions chosen based on the Euclidean distance are the ambiguity averse function or the CRRA function, which is a special case of the ambiguity averse preferences. Finally, at annual horizons, the utility function chosen is the loss averse function. Our results indicate that the investor becomes less risk averse when the investment horizon increases.

Table 8 depicts the optimal choice of parameters for the conditional multivariate portfolio. We compare the empirical weights to the optimal weights over time and choose the combinations of weights with the smallest Mahalanobis distance. Clearly, the investor should not be represented by a mean-variance utility function. On the other hand, all other utilities are statistically equal and could be used to compute optimal portfolios for the Brazilian investor. As the loss averse utility function showed the lowest Mahalanobis distance, we consider it to be the most adequate function to the characteristics of the Brazilian investor. Therefore, at the multivariate problem, the investor should be represented by a loss averse utility function with loss aversion parameter equal to $l = 2$ and risk aversion equal to $b = 0.103$. That is, the investors treat gains and losses differently, so that a decrement in utility caused by a marginal loss is always larger than an increment in utility resulting from a marginal

Table 8: Optimal Parameter Estimates for the Unconditional Portfolio

Each Panel of this table presents the optimal parameters for each unconditional portfolio of the Brazilian representative investor considering one month, three months, six months, and one year investment horizons. We also present the minimum Euclidean distance between the unconditional empirical portfolio and the optimal portfolio. Standard errors are computed using a block bootstrap method and are reported in brackets.

	Optimal Parameter		Optimal Weights			
			α^e	α^b	α^{rf}	Min Euclidean Distance
			Panel A: Mean Variance Investors			
	γ					
monthly	25.300		0.000	0.455	0.545	0.049
						[0.126]
quarterly	17.950		0.016	0.461	0.523	0.042
						[0.128]
semiannual	15.250		0.023	0.464	0.513	0.039
						[0.130]
annual	5.750		0.045	0.471	0.484	0.031
						[0.129]
			Panel B: Constant Relative Risk Averse Investors			
	γ					
monthly	25		0.000	0.455	0.545	0.049
						[0.125]
quarterly	15		0.015	0.461	0.525	0.043
						[0.143]
semiannual	11.95		0.026	0.467	0.507	0.038
						[0.141]
annual	6.1		0.060	0.475	0.465	0.026
						[0.128]
			Panel C: Ambiguity Averse Investors			
	ϵ	γ				
monthly	0.07%	24.5	0.000	0.454	0.546	0.049
						[0.126]
quarterly	0.01%	15	0.014	0.459	0.526	0.043
						[0.143]
semiannual	0.01%	12	0.025	0.463	0.511	0.038
						[0.141]
annual	0.03%	6	0.061	0.477	0.462	0.026
						[0.128]
			Panel D: Loss Averse Investors			
	l	b				
monthly	2	0.995	0.000	0.458	0.542	0.049
						[0.151]
quarterly	2	0.681	0.012	0.456	0.531	0.044
						[0.088]
semiannual	2	0.471	0.018	0.457	0.525	0.041
						[0.083]
annual	2	0.446	0.069	0.479	0.451	0.024
						[0.098]

Table 9: Optimal Parameter Estimates for Conditional Portfolio

This table presents the optimal parameter estimates for the conditional porfolio using four different specifications for the utility function. We also present the minimum Mahalanobis distance that yields the optimal parameter estimates reported. Standard errors are computed using a block bootstrap method and are reported in brackets.

Panel A: Optimal Parameters for Portfolio over time		
		Min Mahalanobis Distance
Constant Relative Risk Averse Investors		
	γ	
	40	119.3
		[9.66]
Ambiguity Averse Investors		
γ	ϵ	
40	1%	121.1
		[10.33]
Loss Averse Investors		
l	b	
2	0.103	116.6
		[13.18]
Mean-Variance Investors		
	γ	
	1	148.4
		[8.32]

gain. Therefore, the Brazilian investor is more concerned about losses than about equally large gains.

The results regarding the unconditional and conditional portfolios for the Brazilian investor clearly show that the risk aversion parameter is extremely high. This parameter is supposed to be high in order to explain the historical high risk premium observed on stock markets, which constitutes an empirical phenomenon known as the equity risk premium puzzle[23] (see discussion in Fama (1991) and Cochrane et al. (2005)).

Mehra e Prescott (1985) claim that high risk aversion is a robust and unavoidable feature of any method for matching the model to data. These authors were the first to name and discuss the equity premium puzzle. They point out that the mean of the excess return on stocks is too low unless risk aversion is raised to apparently implausible values (55, in their model). We have several preferences consistent with equity premium and risk-free rates, including habits and Epstein–Zin preferences. These preferences, break the link

[23] Basically, the equity risk premium puzzle is the empirically observed difference between returns on equities and (relatively) riskless Treasury Bills.

between risk aversion and intertemporal substitution, so there is no connection to a “risk-free rate” puzzle anymore, and we can coherently describe the data with high risk aversion. No model has yet been able to account for the equity premium with low risk aversion, so we may have to accept high risk aversion, at least for reconciling aggregate consumption with market returns.

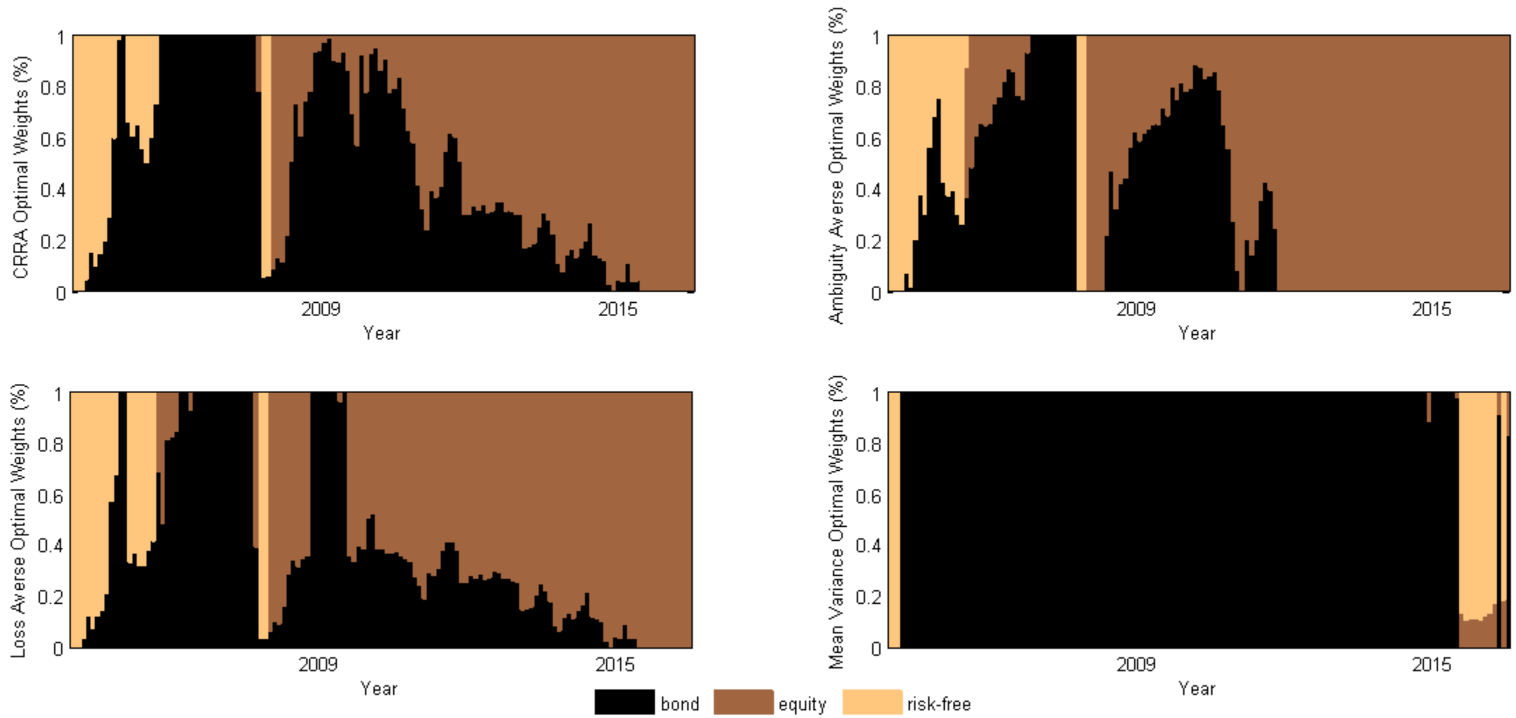

Figure 3: Optimal Portfolios chosen by using the Mahalanobis distance

This figure shows the optimal portfolios for each different specification of the utility function: CRRA, ambiguity averse, loss averse, and mean-variance.

Figure 3 presents the optimal portfolios chosen by using the Mahalanobis distance for the four alternative specification of the utility function considered in this paper. Observe that the composition of equity, bond, and risk-free weights does not change drastically among CRRA, ambiguity averse, and loss averse functions. However, the mean-variance investor portfolio is composed basically of bonds.

According to Amonlirdviman e Carvalho (2010), the loss aversion utility function has been used to explain why a high equity premium might be consistent with plausible levels of risk aversion. Loss averse agents have preferences in which wealth is measured relative to a reference point, with the slope of the utility function over losses being steeper than the utility function over gains. For a given loss or gain, this implies that the decrement in utility by a marginal loss is always larger (in absolute value) than the increment in utility resulting from a marginal gain. Therefore, this non-differentiability of the utility function at the reference point is loosely analogous to locally high risk aversion. This type of loss aversion utility also provides a possible explanation for the reason why investors may prefer safer bonds with low returns to riskier equities with high returns[24].

3 Concluding Remarks

The purpose of this paper is to test which preference generates results that best represent the behavior of the Brazilian investor's portfolio. In order to do that, we use (i) two traditional specifications for the utility function, namely the mean-variance and the constant relative risk-aversion utility, (ii) a utility function that incorporates asymmetric aversion between gains and losses, as the loss aversion utility function and, finally, (ii) an ambiguity aversion utility function in which the investor takes into account the ambiguity about the world. The expected utility functions, besides being unable to incorporate the behavior predicted in the Allais' Paradox Allais (1990), also cannot accommodate investment decisions in which the investor chooses to allocate the most part of his wealth in bonds instead of the stock market (ANG; BEKAERT; LIU, 2005). Our investor can choose between three investment alternatives (bonds, equities, and risk-free) and seeks to maximize the utility generated by its investment portfolio through optimal asset composition. Our multivariate model also assumes that the investor re-evaluates his investment once a month and make decisions based on his expectations of risk and return of the assets previously mentioned. After computing the optimal portfolios of each one of the four different specifications of the utility function, we compare the optimal weights with the empirical weights and decide which utility function best represents the Brazilian investor based on a distance measure.

[24] Benartzi e Thaler (1995) argue that the behaviour of loss aversion can account for the equity premium in a partial equilibrium static model with myopic loss-averse investors, while Barberis, Huang e Santos (2001) incorporate loss aversion into a dynamic general equilibrium pricing model

The greater malleability of the loss averse preference indicates that this function is the most adequate to settle the different types of behavior present in the Brazilian market. This result corroborates with several other works from behavioral theory, from which it is clear the necessity to improve traditional utility functions to incorporate behaviors that are not consistent with the expected utility theory axioms. More specifically, the presence of loss aversion implies that investor's sensitivity is not tied only to variance aversion, but also to the rejection of scenarios with unsatisfactory returns. Thus, assets that reduce the possibilities of bad scenarios, even if this implies a decrease of gains in good scenarios, attract Brazilian investors who have loss aversion.

Although our analysis does not allow us to understand in greater details the decision-making process of each individual from aggregate data, it allows us to anticipate that, once rejected the adequacy of the traditional utility function for the aggregated data, it is possible to reject it as an appropriate model for the individual decision level, at least for most investors (otherwise it would not be possible to reject aggregate behavior, since this represents the average behavior of isolated individuals).

Bibliography

ABDELLAOUI, M.; KLIBANOFF, P.; PLACIDO, L. Experiments on compound risk in relation to simple risk and to ambiguity. *Management Science*, INFORMS, v. 61, n. 6, p. 1306–1322, 2015. 14

AÏT-SAHALIA, Y.; HANSEN, L. P. *Handbook of Financial Econometrics: Tools and Techniques*. [S.l.]: Elsevier, 2009. 2

ALLAIS, M. Allais paradox. In: *Utility and probability*. [S.l.]: Springer, 1990. p. 3–9. 28

AMONLIRDVIMAN, K.; CARVALHO, C. Loss aversion, asymmetric market comovements, and the home bias. *Journal of International Money and Finance*, Elsevier, v. 29, n. 7, p. 1303–1320, 2010. 4, 28

ANG, A.; BEKAERT, G.; LIU, J. Why stocks may disappoint. *Journal of Financial Economics*, Elsevier, v. 76, n. 3, p. 471–508, 2005. 28

BARBERIS, N.; HUANG, M.; SANTOS, T. Prospect theory and asset prices. *The quarterly journal of economics*, Oxford University Press, v. 116, n. 1, p. 1–53, 2001. 28

BARBERIS, N.; THALER, R. A survey of behavioral finance. *Handbook of the Economics of Finance*, Elsevier, v. 1, p. 1053–1128, 2003. 14

BARBERIS, N.; XIONG, W. What drives the disposition effect? an analysis of a long-standing preference-based explanation. *the Journal of Finance*, Wiley Online Library, v. 64, n. 2, p. 751–784, 2009. 14

BENARTZI, S.; THALER, R. H. Myopic loss aversion and the equity premium puzzle. *The quarterly journal of Economics*, Oxford University Press, v. 110, n. 1, p. 73–92, 1995. 14, 20, 28

BOLLERSLEV, T.; ENGLE, R. F.; WOOLDRIDGE, J. M. A capital asset pricing model with time-varying covariances. *Journal of political Economy*, The University of Chicago Press, v. 96, n. 1, p. 116–131, 1988. 6

CAMERER, C.; LOVALLO, D. Overconfidence and excess entry: An experimental approach. *The American Economic Review*, JSTOR, v. 89, n. 1, p. 306–318, 1999. 20

CAMPBELL, J. Y. Stock returns and the term structure. *Journal of financial economics*, Elsevier, v. 18, n. 2, p. 373–399, 1987. 5

CAMPBELL, J. Y.; SHILLER, R. J. Valuation ratios and the long-run stock market outlook. *The Journal of Portfolio Management*, Institutional Investor Journals, v. 24, n. 2, p. 11–26, 1998. 5

COCHRANE, J. H. Production-based asset pricing and the link between stock returns and economic fluctuations. *The Journal of Finance*, Wiley Online Library, v. 46, n. 1, p. 209–237, 1991. 5

COCHRANE, J. H. et al. Financial markets and the real economy. *Foundations and Trends® in Finance*, Now Publishers, Inc., v. 1, n. 1, p. 1–101, 2005. 25

DEMOS, G. do L.; PIRES, T. H. S.; MOURA, G. V. Rebalanceamento endógeno para portfólios de variância mínima (portfolio optimisation and endogenous rebalancing methods). *Revista Brasileira de Finanças*, Sociedade Brasileira de Finanças, v. 13, n. 4, p. 544, 2015. 3

DOESWIJK, R.; LAM, T.; SWINKELS, L. The global multi-asset market portfolio, 1959–2012. *Financial Analysts Journal*, CFA Institute, v. 70, n. 2, p. 26–41, 2014. 4

DOW, J.; WERLANG, S. R. da C. Uncertainty aversion, risk aversion, and the optimal choice of portfolio. *Econometrica: Journal of the Econometric Society*, JSTOR, p. 197–204, 1992. 13

ELLSBERG, D. Risk, ambiguity, and the savage axioms. *The quarterly journal of economics*, JSTOR, p. 643–669, 1961. 12, 13

FAMA, E. F. Efficient capital markets: Ii. *The journal of finance*, Wiley Online Library, v. 46, n. 5, p. 1575–1617, 1991. 25

FAMA, E. F.; FRENCH, K. R. Business conditions and expected returns on stocks and bonds. *Journal of financial economics*, Elsevier, v. 25, n. 1, p. 23–49, 1989. 5

FAMA, E. F.; SCHWERT, G. W. Asset returns and inflation. *Journal of financial economics*, Elsevier, v. 5, n. 2, p. 115–146, 1977. 5

FERSON, W. E.; HARVEY, C. R. Sources of predictability in portfolio returns. *Financial Analysts Journal*, JSTOR, p. 49–56, 1991. 5

FRENCH, K. R.; SCHWERT, G. W.; STAMBAUGH, R. F. Expected stock returns and volatility. *Journal of financial Economics*, Elsevier, v. 19, n. 1, p. 3–29, 1987. 5

GILBOA, I.; SCHMEIDLER, D. Maxmin expected utility with non-unique prior. *Journal of mathematical economics*, Elsevier, v. 18, n. 2, p. 141–153, 1989. 12

HANSEN, L. P.; HODRICK, R. J. Risk averse speculation in the forward foreign exchange market: An econometric analysis of linear models. University of Chicago Press, p. 113–152, 1983. 9

HARVEY, C. R. Time-varying conditional covariances in tests of asset pricing models. *Journal of Financial Economics*, Elsevier, v. 24, n. 2, p. 289–317, 1989. 6

HARVEY, C. R. The specification of conditional expectations. *Journal of Empirical Finance*, Elsevier, v. 8, n. 5, p. 573–637, 2001. 5

JÚNIOR, J. A. d. M.; CAMPANI, C. H.; LEAL, R. P. C. Stock fund selection and the individual investor. *Revista de Administração Contemporânea*, SciELO Brasil, v. 21, n. SPE, p. 41–62, 2017. 3

KAHNEMAN, D.; TVERSKY, A. Prospect theory: An analysis of decision under risk. *Econometrica: Journal of the econometric society*, JSTOR, p. 263–291, 1979. 2, 3, 13

KENDALL, M. G.; HILL, A. B. The analysis of economic time-series. *Journal of the Royal Statistical Society. Series A (General)*, JSTOR, v. 116, n. 1, p. 11–34, 1953. 5

KNIGHT, F. H. *Risk, uncertainty and profit*. [S.l.]: Courier Corporation, 2012. 12

LEWELLEN, J. Momentum and autocorrelation in stock returns. *Review of Financial Studies*, Soc Financial Studies, v. 15, n. 2, p. 533–564, 2002. 6

LIU, L. et al. A new approach to gps ambiguity decorrelation. *Journal of Geodesy*, Springer, v. 73, n. 9, p. 478–490, 1999. 20

MARKOWITZ, H. Portfolio selection. *The journal of finance*, Wiley Online Library, v. 7, n. 1, p. 77–91, 1952. 2, 11

MEHRA, R.; PRESCOTT, E. C. The equity premium: A puzzle. *Journal of monetary Economics*, Elsevier, v. 15, n. 2, p. 145–161, 1985. 11, 25

NAIBERT, P. F.; CALDEIRA, J. F. Seleção de carteiras ótimas sob restrições nas normas dos vetores de alocação: uma avaliação empírica com dados da bm&fbovespa (selection of optimal portfolios under norm constraints in the allocation vectors: an empirical evaluation with data from bm&f bovespa). *Revista Brasileira de Finanças*, Sociedade Brasileira de Finanças, v. 13, n. 3, p. 504, 2015. 3

NEUMANN, J. V.; MORGENSTERN, O. *Theory of games and economic behavior*. [S.l.]: Princeton university press, 2007. 2

NODA, R. F.; MARTELANC, R.; SECURATO, J. R. Eficiência da carteira de mercado no plano média-variância (mean-variance efficiency of the market portfolio). *Revista Brasileira de Finanças*, Sociedade Brasileira de Finanças, v. 12, n. 1, p. 67, 2014. 3

PEREIRA, G. M. et al. Restrição de liquidez para modelos de seleção de carteiras (liquidity constraint for portfolio selection models). *Revista Brasileira de Finanças*, Sociedade Brasileira de Finanças, v. 13, n. 2, p. 288, 2015. 3

PONTIFF, J.; SCHALL, L. D. Book-to-market ratios as predictors of market returns. *Journal of Financial Economics*, Elsevier, v. 49, n. 2, p. 141–160, 1998. 5

RUBESAM, A.; BELTRAME, A. L. Carteiras de variância mínima no brasil (minimum variance portfolios in the brazilian equity market). *Revista Brasileira de Finanças*, Sociedade Brasileira de Finanças, v. 11, n. 1, p. 81, 2013. 3

SANTOS, A. A. P. et al. The out-of-sample performance of robust portfolio optimization. *Brazilian Review of Finance*, Brazilian Society of Finance, v. 8, n. 2, p. 141–166, 2010. 3

SANTOS, A. A. P.; TESSARI, C. Técnicas quantitativas de otimização de carteiras aplicadas ao mercado de ações brasileiro (quantitative portfolio optimization techniques applied to the brazilian stock market). *Revista Brasileira de Finanças*, Sociedade Brasileira de Finanças, v. 10, n. 3, p. 369, 2012. 3

SCHWERT, G. W. Why does stock market volatility change over time? *The journal of finance*, Wiley Online Library, v. 44, n. 5, p. 1115–1153, 1989. 6

TVERSKY, A.; KAHNEMAN, D. Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and uncertainty*, Springer, v. 5, n. 4, p. 297–323, 1992. 14, 20

WAKKER, P. P.; TIMMERMANS, D. R.; MACHIELSE, I. The effects of statistical information on risk and ambiguity attitudes, and on rational insurance decisions. *Management Science*, INFORMS, v. 53, n. 11, p. 1770–1784, 2007. 14

WHITELAW, R. F. Time variations and covariations in the expectation and volatility of stock market returns. *The Journal of Finance*, Wiley Online Library, v. 49, n. 2, p. 515–541, 1994. 6

WU, G.; GONZALEZ, R. Curvature of the probability weighting function. *Management science*, INFORMS, v. 42, n. 12, p. 1676–1690, 1996. 14