Foreign data are not necessary for identification in a small new open economy model

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First version: 14 April 2016
This version: April 30, 2017

Abstract

The closed economy literature highlights that dynamic stochastic general equilibrium models may be subject to non-trivial identification problems making it difficult to estimate key structural parameters even in the presence of an infinite supply of data. In this paper we explore the extent to which a New Keynesian small open economy model suffers from identification problems, and explore how foreign data affect identification.

1 Introduction

> "What econometricians can usefully do is to clarify what conclusions can and cannot logically be drawn given empirically relevant combinations of assumptions and data."
> Manski (2003, p. 12)

The parameters of business cycle models govern the properties of macroeconomic variables – their moments, impulse responses, forecasts and variance decompositions. Conversely, parameter estimates depend on the variables used to inform the estimation. In this paper we focus on this inverse mapping from data back to parameters for a *small open economy model*. This is a question of identification: can the parameters ever be recovered from the data?[1] We focus on identification in the context of *Bayesian* estimation of a dynamic stochastic general equilibrium (DSGE) model, as simulation methods associated with the Bayesian paradigm have become pre-eminently popular to estimate such models.[2] Our particular focus is on the importance of foreign observables in the estimation of our small open economy model, and on the assumptions that need to be made for these foreign variables to be informative for estimation. Exogenous or pre-determined variables are central to the achievement of identification. Since small open economy models have additional exogenous (foreign) variables, there is the tantalising possibility that openness may facilitate parameter identification, even when identification is infeasible for a similar closed economy model.

*I would like to thank Pedro Gomis-Porqueras, Alfred Guender, Yothin Jinjarak, Christoph Thoenissen, and Shaun Vahey, for helpful feedback. I would also like to acknowledge comments from seminar participants at Deakin University and the New Zealand Econometrics Study Group 2017. This paper also owes a great deal to Carl Christ who taught me identification of simultaneous equations models in the mid-1990s.

[1]See Gabrielsen (1978) for an elegant definition of identifiability.

[2]An and Schorfheide (2007) and Herbst and Schorfheide (2014) discuss these methods.

Conversely, if foreign variables are *not* necessary for estimation that is a boon for researchers modelling small open economies, because the specification of 'rest-of-world' data is difficult in practice.

Identification is an important issue for Bayesians for two distinct reasons. First, identification analysis can determine which data series can be used to refine beliefs about parameters, and thus informs estimation. Second, identification analysis sharpens focus on the prior beliefs governing un-identified parameters. Identification analysis can thus shed light on whether posterior inferences will ultimately be governed by the likelihood or whether prior beliefs will always play an important role in shaping beliefs about parameters.

Bayesian analysis provides a mechanism in which prior beliefs about a model and its parameters are updated given a sample of observed data y. Traditionally, the data y constitute the out-growth of an 'experiment'. But in setting up experiments – or in performing econometric exercises to estimate model parameters – we are often able to make a choice about the series to be included in the data y. In designing our analysis we want to ensure that the data used as observables are actually informative. Otherwise the effort and resources expended in sequentially updating prior beliefs serves little purpose. In short, what data should be observed?

We focus on our ability to infer the parameters of the model with an infinite sample of data. A notionally-infinite supply of data is the most favourable data context possible for estimation. If we cannot infer model parameters with an infinite sample then inference with finite samples cannot be definitive about the parameters of interest. As Christ (1968, p. 300) notes, "there is little point [in] attempting to estimate structural parameters from a finite sample if even an infinite sample could not give the desired information about the parameters." We use this asymptotic perspective to inform a choice about which data series to use as observables.[3]

Although a somewhat contentious issue in earlier decades,[4] Bayesian econometricians now seem to accept that identification presents as the same problem in both classical and Bayesian analysis, and depends on the properties of the likelihood of the data.[5] The emphasis on the likelihood for Bayesian identification dates back to at least Kadane (1975, p. 175) and Drèze (1975b, pp. 165, 167).[6] As the likelihood principle of Bayesian analysis implies that the likelihood summarizes all the information in the data (Berger and Wolpert, 1988), it is unsurprising that the likelihood takes centre stage when considering the mapping from data back to estimated parameters. Because the likelihood depends on the data series that are observed, it is also clear that the choice of observables – if there is a choice – may affect the identifiability of a model.

The importance of the data for parameter estimates has been taken up for closed economy models, but this issue has not been pursued systematically for open economy models. The closed-economy literature demonstrates that inferences can be sensitive to the data used to estimate models. Guerron-Quintana (2010), for example, shows that the choice of observed data matters for parameter estimates, and thus impulse responses and out-of-sample forecasts. In particular, he finds that the parameter estimates of a Taylor rule and estimates of wage and price stickiness are

[3] Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016, p. 652) note that the probability distribution of some processes may have infinite trajectories that do not fully reveal the probability distribution of the process. We explore linear models that are not susceptible to such pathologies.

[4] Hsiao (1983, p. 272) observed that it was 'unresolved' whether Bayesian theory required a different definition of identification to that used in classical analysis. Discussing the history of likelihoods and identification in Bayesian analysis, Aldrich (2002) suggests that there was no agreement as to whether identification applied to priors, posteriors or likelihoods.

[5] See Poirier (1998) and Koop (2003, p. 291) for example.

[6] Hsiao (1983), without attribution, associates this perspective with Savage; see also the introduction in Fienberg and Zellner (1975). Aldrich (2002, p. 192) discusses the influence of Drèze in convincing Bayesians from Louvain that identification was a property of the likelihood.

materially affected by the choice of observables.[7] Qu and Tkachenko (2012) raise similar concerns for the estimation of Taylor rule coefficients and Del Negro and Schorfheide (2008) show that 'standard macro time-series' may not enable one to discriminate between different degrees of wage and price rigidity. Iskrev (2010b) also explores identification of the canonical Smets and Wouters (2007) model of a closed economy. Using the same data set as Smets and Wouters, Iskrev concludes that only 39 of the 41 parameters can be identified. In particular, there are identification issues that arise because it is difficult to distinguish the Calvo price parameter in Smets and Wouters' model from the Kimball curvature parameter for goods aggregation; a similar problem arises with respect to the Calvo wage adjustment parameter and the Kimball curvature parameter for aggregating labour varieties.[8]

Canova and Sala (2009) examine the use of impulse-response matching to estimate parameters for a closed economy model similar in spirit to Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), but find that impulse response matching is problematic for identification because impulses are nonlinear functions of the underlying structural parameters. Fukač and Pagan (2007) also illustrate problems with 'matching' DSGE models to vector autoregressions (VARs) via impulse responses. An issue that arises is the appropriate reduced form model of a subset of variables may be a vector autoregressive moving average (VARMA) process. To approximate such VARMAs with a VAR may require an impractically large number of lags, as discussed by Kapetanios, Pagan, and Scott (2007).

There is also clear empirical evidence that variance decompositions depend on the data used as observables. Schmitt-Grohé and Uribe, for example, estimate a model with and without the relative price of investment and find that investment specific technology shocks "play no role in generating economic fluctuations" if the relative price of investment is included as an observable.[9] Similarly, Christiano, Motto, and Rostagno (2014, p. 29) find that "[o]ur conclusion that the risk shock is the most important shock depends crucially on including the four financial variables in our dataset." The dependence of these inferences on the data reflects the fact that variance decompositions are functions of the underlying structural parameters, and parameter estimates depend on the observed data.

In this paper we focus on identification in the context of a small open economy model. As noted earlier, identification issues in open economy models have not been investigated in any detail. While methodologically similar to iconic representations of closed economies, open economy models can be both simpler and more complex than their closed economy counter-parts. For example, SOE models typically abstract from the frictions that temper adjustment to shocks in closed economy models (such as capacity utilisation and possibly even capital), yet incorporate multiple sectors and multiple assets (domestic and foreign goods and bonds for example). We investigate whether the 'pathologies' that affect identification in medium-scale closed economy DSGE models also plague a model that is largely representative of the new open economy macroeconomics (NOEM).[10]

The rest of the paper proceeds as follows. In section 2 we discuss identification in the context

[7]Strictly speaking, Guerron-Quintana's analysis is one of estimation rather than identification, since he seeks to infer parameters given various finite data samples.

[8]See Smets and Wouters (2007) and Kimball (1995) for details of such aggregation.

[9]Greenwood, Hercowitz, and Krusell (1997, 2000), Fisher (2006), and Justiniano, Primiceri, and Tambalotti (2010) all argue that investment-specific technology (IST) shocks are important drivers of cyclical fluctuations. A conventional IST shock is exactly equal to the inverse price of investment goods in terms of consumption. Kamber, Smith, and Thoenissen (2015) show that IST shocks are not important drivers when a simple loan-to-value ratio friction is introduced into the model – theory matters for our interpretation of the underlying drivers of fluctuations.

[10]Lane (2001) surveys the early NOEM literature.

of Bayesian estimation of a DSGE model. Section 3 briefly describes the small open economy that we use as our test-bed for identification analysis. In section 4 we explore identification in our small open economy dynamic stochastic general equilibrium model using the approach of Iskrev (2010b). Our primary objective is to understand identification issues in the context of this open economy model, focussing particularly on the implications of the data used to inform parameter estimation. We provide a contrast between identification in closed and open economy models to illustrate the role that exogenous foreign variables can play for identification. Lastly, we conclude in section 5.

2 Identification of a Bayesian DSGE model

In this section we provide the mathematical machinery used to describe a DSGE model, illustrate the nature of an identification problem in this context, discuss why a 'data choice' arises in the estimation of DSGE models, and explain why we need to explore identification at multiple points in the parameter space. At its most basic level, we seek to ascertain whether it is possible to map uniquely from the 'data' back to the structural parameters that generated that data.

We begin by noting a conventional definition of model identification.[11] Like Koopmans (1953), we conceive of a 'model' as a system of equations, and a family of probability distributions that govern the stochastic behaviour of any latent error terms.[12] A model parameterised with a specific parameter vector θ will be referred to as a 'structure'. A model is thus a set of structures.

Definition 1. *Let $\mathcal{Y}$ be a sample space and $\mathcal{P} = \{P_\theta : \theta \in \Theta\}$ a family of probability distributions for $\mathcal{Y}$ indexed by the parameter vector θ belonging to the parameter space Θ. This model is* identifiable *if for any $\theta_1, \theta_2 \in \Theta$, $P_{\theta_1}(Y = y) = P_{\theta_2}(Y = y)$ for all $y \in \mathcal{Y}$ implies $\theta_1 = \theta_2$.*

The converse of this definition[13] is that two distinct parameterisations ($\theta_1 \neq \theta_2$) can be distinguished from each other (identified) if there exists some sample $y \in \mathcal{Y}$ in which the probabilities implied by the two parameterisations differ, $P_{\theta_1}(y) \neq P_{\theta_2}(y)$. As Koopmans and Reiersøl (1950) observe, the study of identification depends on hypothetical, exact knowledge of the distribution of observables, as from a notionally infinite sample.[14] Any discrepancy between two distribution functions will become evident as the sample asymptotes towards the population.

We apply this notion of identification to a dynamic stochastic general equilibrium model. Following the notation of Iskrev (2010b), a typical linearized DSGE model can be specified as:

$$\Gamma_0(\theta) z_t = \Gamma_1(\theta) \operatorname*{E}_t z_{t+1} + \Gamma_2(\theta) z_{t-1} + \Gamma_3(\theta) u_t \tag{1}$$

where z_t denotes an $n \times 1$ vector of (possibly unobserved) variables at time t; E_t is a rational expectation formed at time t; and u_t is an $m \times 1$ vector of independent and identically distributed Normal shocks with mean zero and an $m \times m$ covariance matrix corresponding to an identity matrix, I_m. $\Gamma_0(\theta), \Gamma_1(\theta)$ and $\Gamma_2(\theta)$ are $n \times n$ matrices of parameters, and $\Gamma_3(\theta)$ is an $n \times m$ matrix. The

[11] Bauwens, Lubrano, and Richard (1999, p. 41, definition 2.7) develops a definition of model identification from the identification of individual model parameters.

[12] See Florens, Marimoutou, and Péguin-Feissolle (2007, pp. 395-8) for a more elaborate description of a structural model.

[13] For variations on this theme see Koopmans and Reiersøl (1950, p. 169), Bowden (1973, definition 1), Kadane (1975, pp 176-7), Drèze (1975a, p. 161, definition 2.1), Pesaran (1987, ch. 6), Bauwens et al. (1999, p. 41, definition 2.7), Koop (2003, p. 291), Florens et al. (2007, p. 4), Rubio-Ramírez, Waggoner, and Zha (2010, p. 669), or Wechsler, Izbicki, and Esteves (2013).

[14] See also Manski (1993).

elements of these Γ matrices are functions of the underlying structural parameters, denoted by θ, which reflect the $k \times 1$ vector of parameters defining preferences and structural constraints.

If it exists, a unique reduced form solution can be expressed as:[15]

$$z_t = A(\theta)z_{t-1} + B(\theta)u_t \tag{2}$$

where $A(\theta)$ and $B(\theta)$ are parameter matrices that are also functions of the structural parameters in θ. (Later we suppress the dependence on θ for the sake of brevity.) Since some elements in z_t may not be observed, it is common to cast the model in state space form, appending an observation equation such as:

$$x_t = s(\theta) + Cz_t \tag{3}$$

with x_t denoting the observed variables with steady states $s(\theta)$. (We will assume that z_t is demeaned relative to its own steady state.) The parameters A, B and s fully characterise the probability distribution of the variables in x_t. The matrix C will typically be a deterministic selection matrix. In this paper we provide guidance as to what model variables should be selected by matrix C. Florens et al. (2007, p. 399) note that statistical analysis enables us to learn about the parameters of the data generating process (dgp). We can only learn about the parameters of the *structural* model if they are uniquely determined by the dgp. Thus, DSGE models specified as in equations (1)-(3) are only identified if it is possible to uniquely map back from the parameters that define the distribution of observables, A, B and s – or from functions of these parameters – to the structural parameters contained in θ.

The mapping from structural parameters θ to reduced form parameters can be thought of as proceeding in two steps, from $\Theta \rightarrow \mathbf{\Gamma}$ and thence from $\mathbf{\Gamma} \rightarrow \mathbf{A}$, using $\mathbf{\Gamma}$ to represent the space of admissible Γ_i $(i = 0, 1, 2, 3)$ matrices and defining $\mathbf{A}$ to be the space representing admissible reduced form parameters, e.g. $\begin{bmatrix}(\operatorname{vec} A)' & (\operatorname{vech} B)' & (\operatorname{vec} s)'\end{bmatrix}' \in \mathbf{A}$. If each mapping, from the structural parameters to the parameters defined by the Γ's, and then from the Γ's to the reduced form parameters $\begin{bmatrix}s & A & B & C\end{bmatrix}$ is one-to-one and onto (bijective), then there will be a unique inverse mapping in return – but this requirement is stricter than is actually required, for reasons discussed next.

The second mapping, $\mathbf{\Gamma} \rightarrow \mathbf{A}$, is similar to that required for the simultaneous equations models (SEMs) common in the 1940s-1960s, though the equations of a DSGE model differ from a traditional SEM in four major respects: i) some variables are unobserved in a DSGE model; ii) the DSGE contains expectations terms; iii) the DSGE model is dynamic and contains lagged regressors; and iv) the elements within Γ's are functions of the underlying structural parameters, i.e. there are many 'cross-equation restrictions'. Let us make some observations about i) and iv).

If the state variables are unobservable then there may be identification problems even with the reduced form. If the parameters of the state and observation equations both need to be estimated then, as noted by Fernández-Villaverde et al. (2016, p. 654), it may be possible to transform the state variables by an invertible matrix, say G, and transform the parameters by G^{-1}. In our case, we will assume that C is a known matrix, tying down the parameterisation of the reduced form.

With regards to iv), the cross-equation restrictions implied by the DSGE model mean that individual elements in the Γ matrices are determined by the smaller subset of underlying structural parameters, and consequently the elements of the Γ's are not strictly independent of each other. For example the Γ elements may be (positively or negatively) correlated if the underlying structural

[15] For a general exposition of linear rational expectations models and their solutions see Binder and Pesaran (1995).

parameters are perturbed. Consequently, some individual elements in the Γ's – and possibly entire equations – may not be necessary to identify the underlying structural parameters since the structural parameters may feature in multiple Γ elements and hence be identifiable from other equations. We will come back to these two mappings, $\Theta \to \mathbf{\Gamma}$ and $\mathbf{\Gamma} \to \mathbf{A}$, in our empirical analysis below.

The identifiability of any model depends on *both* the variables that are in the sample space $\mathcal{Y}$ and on the theoretical relationships specified between different variables. In the classic supply and demand example of Working (1927) identification is achieved by theoretical knowledge that there are exogenous or predetermined variables that perturb some equations and not others. For example, exogenous changes in supply help to trace out the demand curve with respect to price. Identification is provided by *observing* these exogenous or predetermined variables, in conjunction with a theory that says these variables enter the right-hand side of some equations and not others.

Identification problems are often consanguineous with simultaneity issues. The prototypical solution to resolve simultaneity is to specify exogenous instruments that are correlated with the endogenous variables on the right-hand side of an equation of interest yet uncorrelated with the associated error term. Pesaran (1987, ch. 6) and Koop, Pesaran, and Smith (2013, sn. 2) emphasize the importance of higher order dynamics in exogenous variables to achieve identification in linear rational expectations models and DSGE models respectively. These higher order dynamics serve to provide (lagged) instruments for the endogenous regressors in the model. However, as Sims (1980) has argued, restrictions on lag lengths may be 'incredible', making them a weak foundation for identification. Hatanaka (1975) explores identification of simultaneous equations models when there is uncertainty about lag lengths, and highlights the importance of exogenous variables whose lags are *entirely* excluded from the equation whose parameters we seek to identify. We will set aside this difficulty, by assuming that theory truly is informative about these (relative) lag lengths.

Kadane (1975, p. 180) states that "identification analysis is properly a part of the study of the design of experiments". From a Bayesian perspective an experiment is a triplet of data, parameters, and model, respectively $\{y, \theta_A, p(y|\theta_A, A)\}$, where A is an index distinguishing a given model (Geweke, 2005, p. 98). Having witnessed some particular data $y = y_0$, an experiment provides 'evidence' that enables the experimenter to make an inference regarding the parameter vector θ_A, which in turn determines all other inferences that might be drawn from the model. By studying identification we learn, a priori, which data series we should observe and can develop theoretical insight into interdependencies between different parameters (Drèze, 1975a). In some cases 'observability' is dictated by circumstance, since some variables may be intrinsically unobservable. Manski (1995, 2007), for example, considers identification when data are 'missing', as occurs when data are censored and so forth. Often the econometrician cannot control the censoring or 'missingness' of the data. In other cases, however, the econometrician may be able to *choose* which variables are members of the observed data set, y.

Observability is of especial interest in *DSGE* modelling. As Canova (2007, eg p. 440) and Canova, Ferroni, and Matthes (2014) point out, the number of endogenous variables typically exceeds the number of shocks entering DSGE models. Consequently, as noted by Kim and Pagan (1995, p. 368) for example, the 'extra' observables may be written as deterministic functions of the others, implying that the variance-covariance matrix of errors is singular and the likelihood is hence undefined. While linear models with fewer shocks than observables *must* result in stochastic singularities, Komunjer and Ng (2011, p. 2001) and Christiano (2012, fn. 9) note that singularities are the exception rather than the rule in the data, implying that there should be at least as many shocks as observable variables.

There are four methods commonly used to resolve stochastic singularities:[16]

i. Include additional structural shocks;[17]

ii. Include measurement errors;[18]

iii. Solve out variables from the optimality conditions until the number of shocks equals the number of variables; or

iv. Use a subset of observables less than or equal to the number of shocks entering the model (see Kim and Pagan 1995, p. 368).

Chari, Kehoe, and McGrattan (2009) criticise the first strategy, suggesting that some of the newly-introduced shocks are 'dubiously structural'. Canova et al. (2014) caution that the first and second strategies "may distort parameter estimates and jeopardize inference", while Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramírez (2010) suggest that the introduction of measurement errors may complicate identification. Canova et al. (2014, p. 1099-1100) remark that the third strategy is problematic since "the convenient state-space structure of the decision rules is lost" and "the likelihood is an even more nonlinear function of the structural parameters and cannot necessarily be computed with standard Kalman filter recursions."

The fourth strategy, which we focus on here, requires one to select the subset of observables that is most informative for the estimation of the structural parameters. Komunjer and Ng (2011, p. 1997) comment on this difficulty, saying "[a]lthough we can drop some variables so that the system is full rank, the results will not be robust unless we know which variables are ancillary for the parameters of interest." (See also Canova et al. 2014, p. 1100.) Exactly which data series to retain, to estimate parameters in an open economy model, is the question we take up below.[19]

Methodologically, identification is the study of simple conditions that imply model identifiability. Identification can be established in different ways because it is possible to distinguish random variables ('the data') using different statistics. For example, conditions to ensure that parameterisations are distinct could be expressed in terms of the parameters of the probability distributions, in terms of their moments or moment generating functions, their spectral properties, or even their cumulants (which can be obtained from the logarithm of the moment generating function). Modelling practitioners can then check that their models are identified by establishing that their models have the required characteristics from these various identification methods prior to estimation.

In a classic reference, Rothenberg (1971) demonstrates that, subject to a few technical conditions, a model evaluated at a parameter vector θ_0 will be locally identified provided that the information matrix evaluated at θ_0 is non-singular, where the information matrix is the expectation of the matrix of second derivatives of the log-likelihood with respect to the parameter vector, with the expectation taken across all possible data. (See also Bowden 1973.) Iskrev (2010b) takes a different tack to Rothenberg and Bowden and uses the first and second order moments of observables implied by the model to establish identification, while Qu and Tkachenko (2012) specify conditions in the frequency domain. Mutschler (2015a) extends the conditions of Iskrev

[16] Bierens (2007) discusses a fifth method, which involves convoluting the model distribution with a non-singular distribution to match – by optimizing the structural parameters – an empirical model that has been convoluted similarly. See also Canova et al. (2014). Lai (2008) provides a variation on this theme.

[17] See for example Smets and Wouters (2007).

[18] As per Sargent (1989), Altug (1989) and Ireland (2004).

[19] Komunjer and Ng (2011) develop separate identification conditions for stochastically singular and non-singular models.

to higher order (nonlinear) approximations to the model, using the recursive framework outlined in Andreasen, Fernández-Villaverde, and Rubio-Ramírez (2014). Mutschler provides identification conditions for non-Gaussian models using the first four cumulants. For Gaussian models only the first two cumulants are non-zero and the higher order cumulants are therefore uninformative. Thus for Gaussian processes Mutschler's proposition ensuring local identification reverts to theorem 2 of Iskrev (2010b).

The conditions applied to ensure identifiability are typically 'local' to a given parameter value θ_0, as per definition 2; see also Iskrev (2010b, p. 192). A typical condition ensures that within some local neighbourhood of a candidate parameterisation there is no alternative parameterisation that has the same probabilistic implications for the data. Conditions that ensure global identifiability are more difficult to establish (Komunjer and Ng 2011, Iskrev 2010b, p. 192).

Definition 2. *Let $\mathcal{Y}$ be a sample space and $\mathcal{P} = \{P_\theta : \theta \in \Theta\}$ a family of probability distributions for $\mathcal{Y}$ indexed by the parameter vector θ belonging to the parameter space Θ. The parameter vector $\theta_0 \in \Theta$ is* locally identifiable *if there exists an open neighbourhood* $\mathrm{N}(\theta_0)$ *around θ_0 such that for any $\theta_1 \in \mathrm{N}(\theta_0)$, $P_{\theta_0}(Y = y) = P_{\theta_1}(Y = y)$ for all $y \in \mathcal{Y}$ implies $\theta_0 = \theta_1$.*

A number of the approaches discussed here are implementations of the inverse function theorem (see Rudin 1976, p. 221). To paraphrase, this theorem implies that a vector-valued function will have a well-defined inverse (local to a particular point in its domain) if the matrix of partial derivatives evaluated at that point is invertible. Iskrev (2010b), for example, specifies a function mapping from the parameter space to the data moments and then evaluates the rank of the Jacobian matrix. Provided the Jacobian of this mapping has full rank (and is thus invertible) the inverse mapping from data moments to structural parameters will be bijective in a neighbourhood of the point being evaluated. Rothenberg's emphasis on the rank of the information matrix can be interpreted in a similar light; the vector-valued function for Rothenberg is the score.

The data that are useful for identification may depend on the true data generating process, including its precise parameterisation, and the particular parameters of interest – potentially some sub-vector of the parameter vector θ. We use the first and second moments to characterise the data, adopting the approach of Iskrev (2010b) to explore the identification of a small open economy model across the parameter space. Using the first and second moments is less computationally demanding than computing the information matrix as proposed by Rothenberg (1971). For a Gaussian model the first and second moments define the probability density function and hence implicitly the Fisher information matrix. Like most approaches in this literature, Iskrev's approach relies on properties *local* to a particular parameter vector, e.g. θ_0.

A complete identification analysis across the parameter space poses a substantial numerical challenge because of the curse of dimensionality. Suppose that the structural parameter vector has two elements and that each parameter is evaluated for identification at just five possible values. Then the number of parameter vectors that needs to be checked for identification is $5 \times 5 = 25$. Suppose instead that the structural parameter vector has ten elements, and suppose that each structural parameter is evaluated at eight different values. In this latter case the total number of distinct parameter vectors is $8^{10} \approx 1.07$ billion. Obviously, considering only eight distinct values for any single parameter is quite limited, since most DSGE priors for parameters are continuous rather than discrete.

Rather than focusing on a discrete lattice of points in the parameter space, we use Sobol' sequences, a quasi-Monte Carlo method, to generate variates from the prior distributions.[20] Sobol'

[20] Ratto (2008) makes use of Sobol' sequences in his sensitivity analsysis.

sequences are vector valued sequences whose domain is a unit hyper-cube. Quasi-random sequences produce faster rates of convergence to the integrals of interest than do conventional estimates derived from averaging 'actual' pseudo-random variates. Sobol' sequences can be used directly to sample uniformly from the finite support of random variables or can be used indirectly to sample from prior distributions, using inverse distribution functions (where available). The goal with these sequences is to ensure that the proportion of points 'sampled' in a given subset of the (hyper) unit cube is as close to the volume of the subset as possible. One pitfall is that Sobol' sequences explore the edges of the hypercube, which is inconvenient when trying to map to a parameter space with an infinite support, such as the normal distribution for example. To avoid the extreme tails of the distribution, we truncate the exploration near the edges of the unit hyper-cube, at 0.001 and 0.999.

In the next section, we outline the model which forms the test-bed of our open economy analysis. We then turn to the results of our identification analysis in section 4.

3 Model

In this section we briefly describe the log linearised equations of the New Open Economy Macroeconomics model used in our analysis. As we use the model outlined in Justiniano and Preston (2010), we keep the discussion fairly brief. The model has traditional New Keynesian elements of imperfect competition and sticky prices à la Calvo (1983).[21] The linearized equations are presented in table 1 and 2; variable definitions are provided in table 3; and parameters are defined in tables 4 and 5.

As discussed by Justiniano and Preston, a variety of frictions are introduced from the closed economy literature, including (exogenous) habit in consumption, Calvo pricing and wage-setting, and indexation of prices and wages. These features originated in papers such as Erceg, Henderson, and Levin (2000), Fuhrer (2000), Woodford (2003), Smets and Wouters (2003,0), Christiano et al. (2005), and Monacelli (2005). We allow Calvo pricing for imports, which results in a 'law of one price gap' between between the domestic currency price of imported goods and their foreign price (adjusting for the currency of denomination). Implicitly, we assume that it is possible to segment foreign and domestic markets (one of the three mechanisms considered by Benigno and Thoenissen 2003 to generate deviations from purchasing power parity). Since there are multiple distortions in the model, which are not perfectly correlated, it is not possible to use monetary policy to simultaneously eliminate all distortions. Thus, policy-makers must trade off the various distortions when optimizing policy.

Foreign shocks affect domestic intertemporal substitution, via foreign bonds, and trade in foreign and domestic goods. World prices affect the real exchange rate, which affects uncovered interest parity given incomplete financial markets;[22] foreign interest rates influence the domestic economy through the same channel. The general equilibrium character of the model means that foreign distortions, such as price stickiness for imports for example, affect domestic-foreign relative prices and therefore have domestic welfare implications by affecting domestic consumption and labour supply, which are the arguments of the utility function of domestic agents and thus the proximate drivers of welfare.

This open economy model is 'closed' by assuming a debt elastic interest rate premium (see Schmitt-Grohé and Uribe 2003), ensuring that transitory shocks do not have a permanent effect

[21]Closely related predecessors in the open economy literature include Galí and Monacelli (2005) and Monacelli (2005). Kirsanova, Leith, and Wren-Lewis (2006) is similar in spirit, but is a complete-markets version.

[22]Implicitly, complete risk-sharing is assumed within each country, but not between countries.

on the steady state of the model. For the purpose of this premium, debt is measured relative to the fraction of steady state consumption of the imported good. Foreign output (income) propagates through to demand for domestically-produced goods. The exchange rate and interest rates are the key prices that connect domestic and foreign resource allocations, and their macroeconomic dynamics. Taxes are assumed to match the subsidy needed to offset the markup stemming from imperfect competition, offsetting the steady-state distortion that would otherwise arise. Independent and identically distributed cost push shocks are introduced perturbing both domestic and import prices; these shocks can be thought of as exogenous variations in mark-ups, perhaps generated by stochastic changes in competitive pressures.

The equations governing foreign dynamics are akin to the equations for the domestic economy with a modest number of exceptions. First, consumption of (domestically-produced) imports by foreigners is negligible and is disregarded. Relatedly, there are no import cost push shocks for the foreign economy, and the terms of trade effect on the foreign price level is negligible (and hence dispensed with). Second, the law of one price is assumed to hold continuously. Third, in the foreign monetary policy rule $\theta_{\Delta e^*} = 0$. Fourth, foreigners are assumed to be net creditors, and thus there is no interest elasticity with respect to the level of foreigners' debt. Note that η^* is the foreign counterpart to η, the intratemporal substitution elasticity.[23] We use e_t to denote the nominal exchange rate and s_t to denote the terms of trade, following the notation of Justiniano and Preston (2006) rather than the notation of Justiniano and Preston (2008) or (2010).[24]

Tables 1 and 2 report the linearised equations of the model for the doemstic and foreign parts of the model, respectively. Definitions of the variables of the model can be found in table 3. Definitions of parameters can be found in 4 and 5. The prior distributions of the endogenous and exogenous parameters are reported in tables 6 and 7.

[23] Justiniano and Preston denote the foreign intratemporal substitution elasticity using λ in place of η^*.

[24] We have amended the notation for the linearised risk premium shock from ϕ_t to $\epsilon_{rp,t}$ for greater consistency with the notation of other exogenous shocks. We use θ's with subscripts to denote parameters of monetary policy rules, with asterisks for foreign parameters and variables. The substitution elasticities for goods varieties have been changed from θ to ϵ and the substitution elasticity for labour varieties has been changed from θ_w to ϵ_w. θ with no subscript or a numerical subscript refers to the vector of parameters estimated using Bayesian methods. Exogenous technological progress is denoted $\epsilon_{a,t}$, ensuring greater notational consistency with the remaining exogenous variables.

Table 1: Domestic linearised equations

1. Consumption Euler equation (with external habit)
$c_t - hc_{t-1} = E_t(c_{t+1} - hc_t) - \sigma(1-h)(r_t - E_t\pi_{t+1}) + \sigma(1-h)(\epsilon_{g,t} - E_t\epsilon_{g,t+1})$

2. Goods market clearing
$y_t = (1-\tau)c_t + (\tau\eta^* + \tau\eta(1-\tau))\, s_t + \tau\eta^*\psi_{F,t} + \tau y_t^*$

3. Law of one price (LOOP) gap
$\psi_{F,t} \equiv e_t + p_t^* - p_{F,t}$

4. Terms of trade
$\Delta s_t = \pi_{F,t} - \pi_{H,t}$

5. Real exchange rate†
$q_t = e_t + p_t^* - p_t = \psi_{F,t} + (1-\tau)s_t$

6. Domestic Phillips curve
$\pi_{H,t} - \gamma_H\pi_{H,t-1} = \beta E_t(\pi_{H,t+1} - \gamma_H\pi_{H,t}) + \xi_H(w_t + \tau s_t - \epsilon_{a,t}) + \epsilon_{cH,t}$

7. Import Phillips curve
$\pi_{F,t} - \gamma_F\pi_{F,t-1} = \beta E_t(\pi_{F,t+1} - \gamma_F\pi_{F,t}) + \xi_F\psi_{F,t} + \epsilon_{cF,t}$

8. Wage Phillips curve
$\pi_t^W - \gamma_W\pi_{t-1} = \beta E_t\left(\pi_{t+1}^W - \gamma_W\pi_t\right) + \xi_W(v_t - w_t)$

9. Production function
$y_t = \epsilon_{a,t} + n_t$

10. Uncovered interest parity
$r_t - E_t\pi_{t+1} - \left(r_t^* - E_t\pi_{t+1}^*\right) = E_t\Delta q_{t+1} - \chi B_t - \epsilon_{rp,t}$

11. Flow budget constraint
$c_t + b_t = \beta^{-1}b_{t-1} - \tau(s_t + \psi_{F,t}) + y_t$

12. Monetary policy rule
$r_t = \theta_r r_{t-1} + (1-\theta_r)(\theta_\pi\pi_t + \theta_y y_t + \theta_{\Delta y}\Delta y_t + \theta_{\Delta e_t}\Delta e_t) + \epsilon_{m,t}$

13. Consumer price inflation
$\pi_t = \pi_{H,t} + \tau\Delta s_t$

14-24. Exogenous shocks††
$\epsilon_{\cdot,t} = \rho_\cdot\epsilon_{\cdot,t-1} + \check{\epsilon}_{\cdot,t}$

† Note that e_t can be substituted out of the model using the LOOP definition.
†† All exogenous shocks are stationary AR(1) processes, except monetary policy shocks, domestic cost-push shocks, and cost-push shocks for the rest of the world, all of which are independent and identically distributed (IID). The subscript $\cdot$ is replaced with $a, g, cH, cF, w, m, a^*, g^*, c^*, w^*, m^*$ as appropriate. $\check{\epsilon}_{\cdot,t}$ is an IID innovation.

Table 2: Foreign equations

25. Foreign Euler equation
$y_t^* - h^* y_{t-1}^* = E_t \left(y_{t+1}^* - h^* y_t^* \right) - (1 - h^*)\sigma^*(r_t^* - E_t \pi_{t+1}^*) + (1 - h^*)\sigma^*(E_t \epsilon_{g^*,t+1} - \epsilon_{g^*,t})$

26. Foreign Phillips curve
$\pi_t^* - \gamma^* \pi_{t-1}^* = \beta^* E_t \left(\pi_{t+1}^* - \gamma^* \pi_t^* \right) + \xi_*(w_t^* - \epsilon_{a^*,t}) + \epsilon_{\pi^*,t}$

27. Foreign wage Phillips curve
$\pi_{w,t}^* - \gamma_{w^*} \pi_{w,t-1}^* = \beta^* E_t \left(\pi_{w^*,t+1} - \gamma_{w^*} \pi_t^* \right) + \xi_{w^*}(v_t^* - w_t^*) + \epsilon_{w^*,t}$

28. Foreign marginal rate of substitution
$v_t^* = \varphi^*(y_t^* - \epsilon_{a^*,t}) + \frac{1}{(\sigma^*(1-h^*))}(y_t^* - h^* y_{t-1}^*)$

29. Foreign production
$y_t^* = \epsilon_{a_t^*} + n_t^*$

30. Foreign real wage
$w_t^* = w_{t-1}^* + \pi_{w,t}^* - \pi_t^*$

31. Foreign Monetary Policy
$r_t^* = \theta_r^* r_{t-1}^* + (1 - \theta_r^*) \left(\theta_{\pi^*} \pi_{t+1}^* + \theta_{y^*,t} * y_t^* + \theta_{\Delta y^*}(y_t^* - y_{t-1}^*) \right) + \epsilon_{m^*,t}$

32. Change in exchange rate
$\Delta q_t \equiv q_t - q_{t-1}$

33. Change in terms of trade
$\Delta s_t \equiv s_t - s_{t-1}$

34. Foreign goods market clearing
$y_t^* = c_t^*$

Table 3: Variable definitions

Endogenous variables		
c_t	Consumption	Log dev. from s.s.
b_t	Foreign bonds	Log dev. from s.s
y_t	Output	Log dev. from s.s.
r_t	Interest rate (nominal)	$\approx$ % in decimal (qoq)
q_t	Real exchange rate	Log dev. from s.s.
s_t	Terms of trade	Log dev. from s.s.
π_t	Headline inflation	$\approx$ % in decimal (qoq)
$\pi_{H,t}$	Home inflation	$\approx$ % in decimal (qoq)
$\pi_{F,t}$	Import inflation	$\approx$ % in decimal (qoq)
$\pi_{W,t}$	Wage inflation	$\approx$ % in decimal (qoq)
$\psi_{F,t}$	Deviation from law of one price	Log dev. from s.s.
v_t	$\equiv \varphi\left(y_t - \epsilon_{a,t}\right) + \frac{1}{\sigma(1-h)}\left(c_t - hc_{t-1}\right) + \epsilon_{w,t}$	Log dev. from s.s.
p_t	Domestic price level	Domestic currency units
$p_{F,t}$	Import price level	Domestic currency units
e_t	Nominal exchange rate	Domestic-$ per foreign-$
w_t	Real wages	Domestic currency units (real)
n_t	Labour supply	Log deviation from s.s.
Δy_t	$= y_t - y_{t-1}$	Growth rate of output
Δe_t	$= e_t - e_{t-1}$	Growth rate of nominal exchange rate
Δs_t	$= s_t - s_{t-1}$	Growth rate of terms of trade

Foreign variables		
y_t^*	Foreign income (real)	Foreign currency units (real)
p_t^*	Foreign price level	Foreign currency units
π_t^*	Foreign inflation	$\approx$ % in decimal (qoq)
r_t^*	Foreign interest rate	$\approx$ % in decimal (qoq)
v_t^*	$\varphi^*(y_t^* - \epsilon_{a,t}^*) + \frac{1}{\sigma^*(1-h^*)}(y_t^* - h^* y_{t-1}^*)$	Foreign MRS (log dev. from s.s.)

Exogenous shocks		
$\epsilon_{a,t}$	Technology shock, AR(1)	$\approx$ % in decimal (qoq)
$\epsilon_{g,t}$	Demand shock (e.g. Gov't), AR(1)	$\approx$ % in decimal (qoq)
$\epsilon_{w,t}$	Labour supply shock, AR(1)	$\approx$ % in decimal (qoq)
$\epsilon_{cH,t}$	Domestic cost push shock, AR(1)	$\approx$ % in decimal (qoq)
$\epsilon_{cF,t}$	Import cost push shock, AR(1)	$\approx$ % in decimal (qoq)
$\epsilon_{rp,t}$	Risk premium shock	$\approx$ % in decimal (qoq)
$\epsilon_{m,t}$	Monetary policy shock, IID	$\approx$ % in decimal (qoq)
$\epsilon_{a^*,t}$	Foreign technology shock, AR(1)	$\approx$ % in decimal (qoq)
$\epsilon_{g^*,t}$	Foreign demand shock (e.g. Gov't), AR(1)	$\approx$ % in decimal (qoq)
$\epsilon_{w^*,t}$	Foreign labour supply shock, AR(1)	$\approx$ % in decimal (qoq)
$\epsilon_{c^*,t}$	Foreign cost push shock, AR(1)	$\approx$ % in decimal (qoq)
$\epsilon_{m^*,t}$	Foreign monetary policy shock, IID	$\approx$ % in decimal (qoq)

Note: s.s. = steady state; dev. = deviation. MRS = marginal rate of substitution. AR(1) = autoregressive process of order one. IID = independent and identically distributed.

Table 4: Domestic parameter definitions

Preference & structural parameters	
β	Discount factor
ε	Substitution elasticity between goods varieties
ε_W	Substitution elasticity between labour varieties
φ	Inverse Frisch (labour supply) elasticity
σ	Intertemporal substitution elasticity
α_H	Calvo domestic prices
α_F	Calvo import prices
α_W	Calvo wages
γ_H	Indexation domestic prices
γ_W	Indexation wages
h	Consumption habit
τ	Degree of openness (inverse of home bias)
η	Domestic intratemporal substitution elasticity home & foreign goods
χ	Interest elasticity w.r.t. debt

Domestic policy parameters	
θ_π	Response to inflation
θ_y	Response to output gap
$\theta_{\Delta y}$	Response to output growth
$\theta_{\Delta e}$	Response to exchange rate change
θ_r	Interest rate smoothing

Auxiliary parameter definitions	
ξ_H	$\equiv (1-\alpha_H)(1-\alpha_H\beta)/\alpha_H$
ξ_F	$\equiv (1-\alpha_F)(1-\alpha_F\beta)/\alpha_F$
ξ_W	$\equiv (1-\alpha_W)(1-\alpha_W\beta)/(\alpha_W(1+\varphi\varepsilon_W))$
ξ_*	$\equiv (1-\alpha^*)(1-\beta^*\alpha^*)/\alpha^*$
ξ_{W^*}	$\equiv (1-\alpha_{W^*})(1-\alpha_{W^*}\beta^*)/(\alpha_{W^*}(1+\varphi^*\varepsilon_{W^*}))$

Exogenous shock persistences & standard deviations	
ρ_a	AR(1) persistence parameter technology shock
ρ_g	AR(1) persistence parameter exogenous demand shock
ρ_{cH}	AR(1) persistence parameter domestic cost push shock
ρ_{cF}	AR(1) persistence parameter import cost push shock
ρ_w	AR(1) persistence parameter labour supply shock
ρ_{rp}	AR(1) persistence parameter risk premium shock
σ_a	Standard deviation technology shock
σ_g	Standard deviation consumption preference shock
σ_{cH}	Standard deviation domestic cost push shock
σ_{cF}	Standard deviation import cost push shock
σ_w	Standard deviation labour supply shock
σ_{rp}	Standard deviation risk premium shock
σ_m	Standard deviation monetary policy shock

Table 5: Foreign parameter definitions

Preference & structural parameters	
β^*	Foreign discount factor
ε^*	Foreign substitution elasticity between varieties
ε_{W^*}	Foreign substitution elasticity between labour varieties
φ^*	Foreign inverse Frisch (labour supply) elasticity
σ^*	Foreign intertemporal substitution elasticity
α_{H^*}	Foreign Calvo domestic prices
α_{W^*}	Foreign Calvo import prices
γ_{H^*}	Foreign indexation domestic prices
γ_{w^*}	Foreign indexation wages
h^*	Foreign consumption habit
η^*	Foreign price elasticity of foreign demand for domestic goods

Foreign policy parameters	
θ_{π^*}	Foreign response to inflation
θ_{y^*}	Foreign response to output gap
$\theta_{\Delta y^*}$	Foreign response to output growth
θ_{r^*}	Foreign interest rate smoothing

Foreign exogenous shock persistences & standard deviations	
ρ_{a^*}	AR(1) persistence parameter foreign technology shock
ρ_{g^*}	AR(1) persistence parameter foreign consumption preference shock
ρ_{l^*}	AR(1) persistence parameter foreign labour supply shock
ρ_{π^*}	AR(1) persistence parameter foreign cost push shock
σ_{a^*}	Standard deviation foreign technology shock
σ_{g^*}	Standard deviation foreign consumption preference shock
σ_{w^*}	Standard deviation foreign labour supply shock
σ_{π^*}	Standard deviation foreign cosh push shock
σ_{m^*}	Standard deviation foreign monetary policy shock

Table 6: Prior distributions domestic parameters (Justiniano and Preston, 2010, p. 67)

Parameters	Description	Prior	Mean	Std-Dev.
Preference & structural parameters				
β	Discount factor	C	0.99	–
ε	Substitution elasticity between goods varieties	C	8.00	–
ε_W	Substitution elasticity between labour varieties	C	8.00	–
χ	Interest elasticity to debt	C	0.01	–
φ	Inverse Frisch (labour supply) elasticity	N	1.00	0.30
σ	Intertemporal substitution elasticity	N	1.00	0.40
α_H	Calvo domestic prices	β	0.60	0.10
α_F	Calvo import prices	β	0.50	0.20
α_W	Calvo wages	β	0.60	0.10
γ_H	Indexation domestic prices	β	0.50	0.20
γ_W	Indexation wages	β	0.50	0.20
h	Consumption habit	β	0.50	0.10
$(1-\tau)$	Degree of home bias	β	0.71	0.02
η	Intratemporal substitution elasticity home & foreign goods	N	0.90	0.10
χ	Interest elasticity w.r.t. debt	C	0.01	–
θ_π	Response to inflation	N	1.80	0.30
θ_y	Response to output gap	G	0.25	0.13
$\theta_{\Delta y}$	Response to output growth	N	0.30	0.20
$\theta_{\Delta e}$	Response to exchange rate change	G	0.30	0.20
θ_r	Interest rate smoothing	β	0.60	0.20
Foreign structural parameters				
β^*	Discount factor	C	0.99	–
ε^*	Substitution elasticity between goods varieties	C	8.00	–
ε^*_W	Substitution elasticity between labour varieties	C	8.00	–
φ^*	Inverse Frisch (labour supply) elasticity	N	1	0.3
σ^*	Intertemporal substitution elasticity	N	1	0.3
α^*_H	Calvo prices	β	0.6	0.1
α^*_W	Calvo wages	β	0.6	0.1
γ^*_H	Indexation prices	β	0.5	0.15
γ^*_W	Indexation wages	β	0.5	0.15
h^*	Consumption habit	β	0.50	0.1
η^*	Intratemporal substitution elasticity home & foreign goods	N	1.5	0.50
θ^*_π	Response to inflation	N	1.80	0.30
θ^*_y	Response to output gap	G	0.25	0.13
$\theta^*_{\Delta y}$	Response to output growth	N	0.30	0.20
θ^*_r	Interest rate smoothing	β	0.60	0.20

Note: Prior distributions: C = Calibrated; β = Beta; N = Normal; G = Gamma; G^{-1} = Inverse Gamma. ROW=Rest of world. For symmetry with the domestic economy we have included $\theta^*_{\Delta y}$ in rest-of-world policy rule.

Table 7: Prior distributions exogenous parameters (Justiniano and Preston, 2010, p. 67)

Exogenous shock persistences & standard deviations

Parameters	Description	Prior	Mean	Std-Dev.
ρ_a	AR(1) persistence parameter technology shock	β	0.60	0.20
ρ_g	AR(1) persistence parameter exogenous demand shock	β	0.60	0.20
ρ_w	AR(1) persistence parameter labour supply shock	β	0.60	0.20
ρ_{cH}	AR(1) persistence parameter domestic cost push shock	β	0.60	0.20
ρ_{cF}	AR(1) persistence parameter import cost push shock	β	0.60	0.20
ρ_{rp}	AR(1) persistence parameter risk premium shock	β	0.60	0.20
ρ_m	No persistence in monetary shock	C	0	–
ρ_a^*	AR(1) persistence foreign technology shock	β	0.80	0.15
ρ_g^*	AR(1) persistence foreign preference shock	β	0.80	0.15
ρ_w^*	AR(1) persistence parameter labour supply shock	β	0.80	0.15
ρ_{cH}^*	AR(1) persistence in foreign shock	β	0.8	0.15
ρ_m^*	No persistence in foreign monetary shock	C	0	–
σ_a	Standard deviation technology shock	G^{-1}	0.50	1.00
σ_g	Standard deviation consumption preference shock	G^{-1}	1.00	1.00
σ_w	Standard deviation labour supply shock	G^{-1}	2.00	1.00
σ_{cH}	Standard deviation domestic cost push shock	G^{-1}	0.15	1.00
σ_{cF}	Standard deviation import cost push shock	G^{-1}	1.00	1.00
σ_{rp}	Standard deviation risk premium shock	G^{-1}	1.00	1.00
σ_m	Standard deviation monetary policy shock	G^{-1}	0.15	1.00
σ_a^*	Standard deviation technology shock	G^{-1}	1.00	2.00
σ_g^*	Standard deviation foreign consumption preference shock	G^{-1}	2.00	2.00
σ_w^*	Standard deviation labour supply shock	G^{-1}	4.00	2.00
σ_{cH}^*	Standard deviation foreign cost push shock	G^{-1}	0.25	2.00
σ_m^*	Standard deviation foreign monetary shock	G^{-1}	0.25	2.00

Note: Prior distributions: C = Calibrated; β = Beta; N = Normal; G = Gamma; G^{-1} = Inverse Gamma

4 Results

4.1 Identification of foreign parameters

Before beginning our open economy analysis, we first consider the identification of a closed economy counterpart to this small open economy model. This exercise allows us to ascertain the features of the model – independent of openness – that may be subject to identification problems. Conveniently, the foreign block of the model represents just such a counterpart. We strip off the domestic economy and examine how the choice of foreign (i.e. closed economy) data series influence identification of the foreign parameters.

The linearised equations in section 3 above are a specific model corresponding to the expectational equations (1). As noted in section 2, we can think of the mapping from structural parameters to the reduced form as occurring in two steps – from θ to Γ's and then from Γ's to s, A, B, C. We conjecture that any identification problems are likely to arise in the first mapping.[25] If this conjecture is correct, then a structural parameter that fully specifies an element of Γ should be identifiable if the variables associated with the relevant equation are all observed. Other structural parameters may, however, be difficult to identify because the deep structural parameters map in a non-unique way to the elements in the Γ matrices. For example, the Phillips curve equations for foreign inflation and foreign wage inflation have parameters ξ^* and ξ_{w*} that are functions of Calvo parameters, discount rates, and in the case of wages the foreign Frisch elasticity. The discount factor β^* can be tied down by the steady-state interest rate, and is typically calibrated at ≈ 0.99. Given a calibrated value $\beta*$ the ξ_* parameter is a monotonic function of the structural Calvo parameter α^*, and therefore the Calvo parameter can be recovered if this ξ parameter is identifiable. Since the parameters φ^* and ϵ_{W*} only enter ξ_{W*}, and enter as a product term, these parameters are only separately identifiable because ϵ_{W*} is calibrated. The wage Calvo parameter α_{W*} also should not be separately identifiable from φ^* since their influence on the model arises solely through ξ_{W*}.

We now turn to our computational techniques to see whether they imply the same identification problems. We simulate $10,000$ parameter draws from the parameter space (drawing from the prior distributions reported in tables 6 and 7 using Sobol sequences in conjunction with inverse probability integral transforms). Adapting Matlab code from Schmitt-Grohé and Uribe (2004), which uses the symbolic toolbox of Matlab, we solve the rational expectations model for its reduced form and compute the Jacobian of the variance-covariance matrix and first auto-covariance matrix of different samples of data. Computing the derivatives symbolically mitigates a problem noted by Fernández-Villaverde et al. (2016, p. 654), namely it reduces the risk that numerical errors adversely affect the computation of the rank of the matrix.

If there is no unique solution to the rational expectations model – either because there is no solution at all or there is a multiplicity of solutions – we discard the parameter vector from the analysis. In practice, for the foreign/closed economy model, there are very few parameter draws that do not result in unique solutions.

For each set of observables (which defines a set of moments), and for each parameter vector drawn from our sample of $10,000$, we evaluate the ranks of the Jacobians of the moments with respect to parameters to see whether different observables affect the identification of the model. In

[25] It is clear that the DSGE model is relatively sparsely parameterised, in comparison to a normal SEM, with many of the endogenous and predetermined variables featuring in only a limited number of equations, thus the order conditions of SEM identification look to be satisfied. (To identify a given equation the number of excluded exogenous variables must be greater than or equal to the number of included endogenous variables. Thus, there are are potentially enough instruments available for the included endogenous variables.)

principle, different parameter vectors may have different 'local' identification characteristics even for the same set of observables.

We draw observables from the following set: foreign output y^*; foreign inflation π^*; foreign wage inflation π_W^*; the foreign interest rate r^*; foreign wages w^*; and hours worked n^*, and examine how identification is affected by the choice of observables. There are five foreign/closed-economy shocks (technology, 'demand', wage and price markup, and monetary policy shocks), thus to avoid stochastic singularity we can have no more than five observables.

We check identification using the variance matrix of observables and the first autocovariance matrix. Let k be the number of observables. Since the variance-covariance matrix is symmetric there are $k(k+2)/2$ distinct elements, while the first auto-covariance matrix need not be symmetric and thus has k^2 distinct elements. Given that there are 23 parameters that we wish to estimate, we require at least $k = 4$ observables ($\Rightarrow k^2 + k(k+2)/2 = 28$) to fully identify the model (given that we are not using steady-states to contribute to identification).

Using each of the possible observable samples and each parameter vector drawn from the prior distributions, we compute the Jacobian matrix of the variance-covariance matrix and the first autocorrelation matrix. For each data set, there is a *histogram* of matrix ranks associated with the $10,000$ parameter vectors. Table 8 illustrates the distribution of Jacobian ranks (histograms in tabular form) for each data set with four or more observables (22 different samples). What is evident from this table is that 3 samples of observed data usually identify 18 parameters, while the remaining 19 samples predominantly identify 21 parameters (out of a possible 23). Of the 10,000 points in the parameter space that were examined, 38 parameter vectors yielded non-unique (indeterminate) solutions for the rational expectation model. A further 4 data samples (from the latter 19) have trouble identifying a small fraction of parameters (relative to the 21 parameters that can usually be identified). Looking at these 7 'problem' cases, it appears that the key observables are interest rates in combination with output or hours worked. Exactly how the observable set is rounded out (to the minimum of 4 required for complete identification) does not seem to be important.

Table 8 makes it clear that 21 of 23 parameters can be identified. Which parameters cannot be identified and why? As Iskrev (2010a) notes, a lack of identification may arise in two ways: i) a parameter may not feature in the likelihood; and/or ii) two or more parameters may not be separately identifiable. The first possibility presents itself as a column of zeroes in the Jacobian. If the second alternative arises then two or more columns of the Jacobian are linearly dependent.

The one parameter that is unambiguously unidentified from the foreign equations is η^*. This lack of identification arises because η^* only enters the *domestic* goods market clearing condition.

Drawing out the relationship between the observed data and the parameters that are identifiable via computational methods is difficult because it could in principle depend on the exact parameterisation. While the results suggestively indicate that the parameterisation may be largely unimportant for identification, the following comments should be treated cautiously.

When the interest rate is included as an observable, preliminary analysis indicates that the parameter pair ϵ_{w*} and α_{w*} are not separately identifiable.[26] The columns of the Jacobian matrix associated with these two parameters are perfectly collinear. This relationship was verified by sequentially eliminating parameters (columns) from the Jacobian for a given data set and parameterisation and then examining whether the rank of the Jacobian was reduced by the removal of the parameter. If the rank is unchanged, then that implies the parameter is not separately identifiable.

[26] This contrast was undertaken for just two samples: $\{\pi_w^*, r^*, w^*, n^*\}$, and $\{y^*, \pi^*, r^*, w^*, n^*\}$.

Table 8: Histograms of ranks for different sets of foreign observables

	Frequency of ranks									
Observables	15	16	17	18	19	20	21	22	23	Indet.
$pi_w^* \ r^* \ w^* \ n^*$	0	0	0	0	0	0	9962	0	0	38
$\pi^* \ r^* \ w^* \ n^*$	0	0	0	0	0	0	9962	0	0	38
$\pi^* \ \pi_w^* \ w^* \ n^*$	0	0	0	9962	0	0	0	0	0	38
$\pi^* \ \pi_w^* \ r^* \ n^*$	0	0	0	0	0	0	9962	0	0	38
$\pi^* \ \pi_w^* \ r^* \ w^*$	0	0	0	9962	0	0	0	0	0	38
$y^* \ r^* \ w^* \ n^*$	0	0	0	0	0	0	9962	0	0	38
$y^* \ \pi_w^* \ w^* \ n^*$	0	0	0	0	0	8	9954	0	0	38
$y^* \ \pi_w^* \ r^* \ n^*$	0	0	0	0	0	0	9962	0	0	38
$y^* \ \pi_w^* \ r^* \ w^*$	0	0	0	0	0	0	9962	0	0	38
$y^* \ \pi^* \ w^* \ n^*$	0	0	0	0	0	9	9953	0	0	38
$y^* \ \pi^* \ r^* \ n^*$	0	0	0	0	0	0	9962	0	0	38
$y^* \ \pi^* \ r^* \ w^*$	0	0	0	0	0	0	9962	0	0	38
$y^* \ \pi^* \ \pi_w^* \ n^*$	0	0	0	0	0	11	9951	0	0	38
$y^* \ \pi^* \ \pi_w^* \ w^*$	0	0	0	9962	0	0	0	0	0	38
$y^* \ \pi^* \ \pi_w^* \ r^*$	0	0	0	0	0	0	9962	0	0	38
$\pi^* \ \pi_w^* \ r^* \ w^* \ n^*$	0	0	0	0	0	0	9962	0	0	38
$y^* \ \pi_w^* \ r^* \ w^* \ n^*$	0	0	0	0	0	0	9962	0	0	38
$y^* \ \pi^* \ r^* \ w^* \ n^*$	0	0	0	0	0	0	9962	0	0	38
$y^* \ \pi^* \ \pi_w^* \ w^* \ n^*$	0	0	0	0	0	4	9958	0	0	38
$y^* \ \pi^* \ \pi_w^* \ r^* \ n^*$	0	0	0	0	0	0	9962	0	0	38
$y^* \ \pi^* \ \pi_w^* \ r^* \ w^*$	0	0	0	0	0	0	9962	0	0	38
$y^* \ \pi^* \ \pi_w^* \ r^* \ w^* \ n^*$	0	0	0	0	0	0	9962	0	0	38

Indet.= Indeterminacy: corresponds to the number of parameter vectors that were not uniquely identified. Note: Rows sum to $10,000$, corresponding to the number of points (parameter vectors) in the parameter space that were evaluated.

If the interest rate is not an observable, as for samples $\{\pi^*,\ \pi^*_w,\ w^*,\ n^*\}$ or $\{y^*,\ \pi^*,\ \pi^*_w,\ w^*\}$, then the inter-dependencies between parameters become much more complex. When four variables are included, then there are 26 rows in the Jacobian matrix and up to 8 parameters could be excluded before an independent column was eliminated (resulting in a decline in rank). Again, preliminary analysis indicates many interdependencies between parameters, indicating that different combinations cannot be separately identified.

4.2 Identification of domestic parameters

We turn now to the empirical analysis of the domestic part of the model, to determine whether domestic or foreign variables are more useful to achieve identification of the domestic parameters. Computationally, the analysis faces a similar problem to that of variable selection in Bayesian model averaging of linear regressions. Suppose that that there are k series that can be used as observables, then there $(2^k) - 1$ different ways of sampling from this set of data series (where the -1 means that we ignore the data sample that has *zero* columns). Intuitively, suppose that X is a $T \times k$ matrix of data and specify a $k \times 1$ row vector of ones and zeroes, where a 1 indicates that a column is included as an observable and a 0 indicates that it is not. For example [01011] would indicate that the second, fourth and fifth columns are treated as observables, and the remaining two columns are not. This 5-element vector can take binary values between 0 and 11111, where the latter number corresponds to 32 in base-10 terms. However, the model has 28 variables, all of which could – theoretically – be used as observables. Since 2^{28} corresponds to roughly 2.6 million combinations it is infeasible to consider all such alternatives.

To reduce the dimensionality of the analysis we break up the data in four ways. First, some variables are treated as inherently unobservable and are excluded from the analysis. Second, we include four data series in all sets of observables – output, inflation, interest rates and the level of the real exchange rate. Third, we augment this baseline with i) additional domestic variables; *or* ii) additional foreign variables. Let kf equal the number of foreign observables and kd equal the number of domestic variables. Rather than 2^{kf+kd} permutations we consider only $2^{kf} + 2^{kd}$ by separately including foreign and domestic variables. The additional foreign variables are drawn from:

1. Foreign output
2. Foreign inflation
3. Foreign wages
4. Foreign wage inflation
5. Foreign interest rates, and
6. Foreign hours worked.

We thus consider 64 different samples of observables, with between zero and six foreign variables, where zero corresponds to the baseline model with domestic output, inflation, interest rates, and the exchange rate.

The additional domestic variables are drawn from:

1. domestic consumption

2. domestic consumption of home-produced goods
3. domestic consumption of imports
4. the inflation rate for home-produced goods
5. the inflation rate for imported goods
6. the inflation rate for domestic wages
7. the terms of trade, and
8. domestic hours worked.

Thus, we consider $2^8 = 256$ different samples of domestic variables, with a minimum of four (baseline) variables and a maximum of twelve. Given that there are twelve shocks in the model, all of these samples avoid stochastic singularity. As noted above, we treat some variables as inherently unobservable (or rely on other observables that are close in character). For example, we treat the law-of-one-price gap, marginal cost, and the marginal rate of substitution as unobservable. We also refrain from using the change in the nominal exchange rate, the change in the real exchange rate, and the change in the terms of trade as observables. (But of course the level of the real exchange rate is included as a baseline observable.) Amongst the foreign series we treat the foreign marginal rate of substitution and foreign consumption of domestically-produced goods as unobservables. Perhaps more dubiously, we also refrain from using foreign bond holdings (or foreign debt if negative) as an observable. We exclude calibrated parameters from the identification analysis; see table 6 for the estimated domestic parameters.

Figure 1 depicts histograms of ranks for different samples of domestic observables. The main conclusion to draw from this figure is that the identification of this particular model is largely independent of the sample of observables and the parameterisation of the model: provided that five or more data series are used as observables it is possible to identify all 29 model parameters. (Each color in the chart corresponds to a different set of observables.) It should be noted that the baseline model only suffices to identify 26 parameters because with $k = 4$ observable series there are only 26 moments, which poses in this instance a binding upper bound on the rank of the Jacobian matrix.

Figure 2 illustrates an analogous set of histograms when the baseline set of observables is augmented with additional *foreign* variables. As is apparent, the inclusion of additional foreign variables also facilitates identification of all the parameters under investigation.

Table 9 presents the information contained in figures 2 and 1 in a tabular format. The table illustrates the histogram of the ranks that can be achieved via the baseline and the average histogram of the ranks, where the average is computed across, respectively, the sets of foreign and domestic observables. This table illustrates that including domestic variables generally results in better identification than including foreign variables.[27] Interestingly, the average ranks are fairly indicative of the histograms for *all* sets of observables (see tables 10 and 11 in the appendix). In other words, given the baseline set of observables, it appears that the choice of observables is not particularly material for the local identifiability of the model. At the margin, domestic variables appear to be

[27] Of the $10,000$ parameter vectors examined here, 43 yield no unique solution for the DSGE model – less than $1/2$ a percent of the parameters drawn from the prior distribution of parameters. (Koop et al. (2013) note that changing the distribution by just a few percent can cause posterior plots to differ from priors, and comparison plots of priors and posteriors can suggest, misleadingly, that parameters are identified when in fact they are not, but it is hard to believe that this effect will be very material here.)

Figure 1: Histograms of ranks for models with baseline + domestic variables

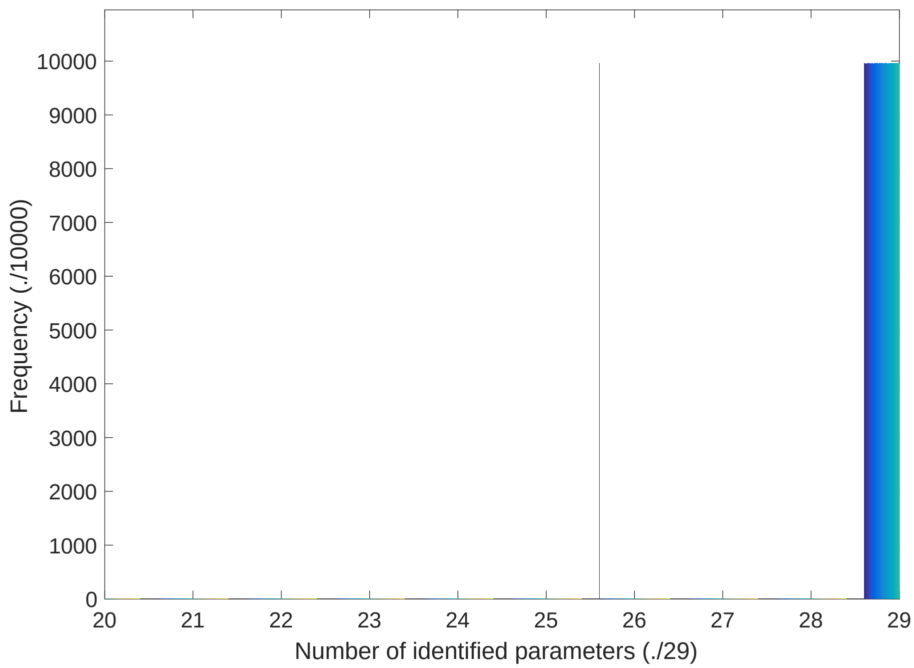

Figure 2: Histograms of ranks for models with baseline + foreign variables

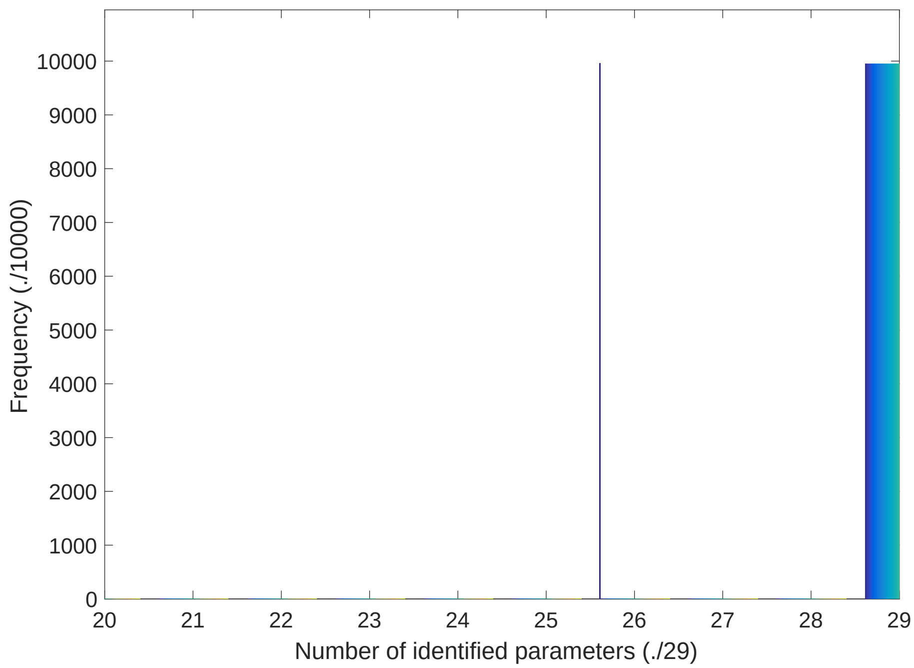

Table 9: Ranks of Jacobians

Rank identified	Frequency y, π, r, q	Average frequency y, π, r, q+Foreign	Average frequency y, π, r, q+Domestic
20	0	0	0
21	0	0	0
22	0	0	0
23	0	0	0
24	0	0	0
25	0	0	0
26	9957	0	0
27	0	0.11	0
28	0	5.87	1.45
29	0	9951	9955.55
Undefined	43	43	43

The column titled 'Rank' indicates the rank of the Jacobian matrix, and thus indicates the number of parameters that can be identified. The Jacobians are computed for 10,000 points in the parameter space. There are 43 parameter vectors that yield indeterminate solutions (a multiplicity of possible solutions). Jacobians cannot be computed for these values.

more useful than foreign variables for identification. For econometricians faced with constructing 'foreign' data sets for small open economies with diverse trading patterns this result will come as something of a relief. It does not appear that these foreign data are required.

5 Conclusion

In this paper we explored whether the parameters of a small open economy DSGE model are identified. That is, can a notionally infinite sample of data enable us to correctly infer which parameterisation of the model actually generated the data? We use the technique of Iskrev (2010b) and examine whether observed data moments change given local perturbations in parameter vectors. In particular, we examine whether changing the set of observables has material implications for local identification. Our particular focus is on the use of foreign data. We examined whether foreign data are materially more useful than domestic variables in achieving model identification. For countries with heavily diversified and/or idiosyncratic trade and financial relationships, constructing foreign data can be a challenging task, fraught with mis-measurement.

Our results indicate that, if anything, *domestic* observable are more useful than foreign variables for achieving identification. For modellers in many small economies this finding will come as a relief, because it implies that one does not need to 'construct' foreign data to estimate small open economy models.

Our results also support two conclusions drawn by Iskrev and Ritto (2016). Namely, identification can be dependent on the precise parameterisation of the model. While diverse subsets of observables have similar identification properties – as represented by the ranks of Jacobian matrices of moments with respect to parameters – there is a small subset of parameter vectors that result in a number of parameters being unidentified. The second conclusion is that the rank of Jacobian matrices does not provide particular sharp guidance as to which data should be used as observables. Thus, like Iskrev and Ritto (2016) and Canova et al. (2014), one needs to turn to alternative criteria to guide the choice of data. Koop et al. (2013) provide a Bayesian method to evaluate weak identification which could be used to guide the choice of data, though computational limitations may make it infeasible to consider many different points in the parameter space. However, our results, which indicate that identification is largely independent of the precise parameterisation of the model, may mean that a representative parameterisation may suffice.

While we focused on the parameters of the model, which directly affect the positive properties of the model, these identification issues are also important for normative assessments of policy. Positive properties, when combined with a preference function, determine an ordinal ranking of different policies. As Hansen and Heckman (1996) observe: "Different models that "fit the facts" may produce conflicting estimates of welfare costs and dissimilar predictions about the response of the economy to changes in resource constraints." We intend to pursue this research agenda to understand what implications identification issues have for monetary policy rules. Our ultimate goal is to understand how sensitive key substitution elasticities (such as the inter- and intra-temporal substitution elasticities) are to the choice of observables.

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A Appendix

Table 10: Jacobian ranks with baseline+domestic variables

Variables	25	26	27	28	29	Indet.
y,r,ppi,q	0	9957	0	0	0	43
y,r,ppi,q,n	0	0	0	1	9956	43
y,r,ppi,q,s	0	0	0	5	9952	43
y,r,ppi,q,piw	0	0	0	1	9956	43
y,r,ppi,q,pif	0	0	0	4	9953	43
y,r,ppi,q,pih	0	0	0	4	9953	43
y,r,ppi,q,cf	0	0	0	4	9953	43
y,r,ppi,q,ch	0	0	0	6	9951	43
y,r,ppi,q,c	0	0	0	4	9953	43
y,r,ppi,q,s,n	0	0	0	1	9956	43
y,r,ppi,q,piw,n	0	0	0	1	9956	43
y,r,ppi,q,piw,s	0	0	0	1	9956	43
y,r,ppi,q,pif,n	0	0	0	1	9956	43
y,r,ppi,q,pif,s	0	0	0	4	9953	43
y,r,ppi,q,pif,piw	0	0	0	1	9956	43
y,r,ppi,q,pih,n	0	0	0	1	9956	43
y,r,ppi,q,pih,s	0	0	0	5	9952	43
y,r,ppi,q,pih,piw	0	0	0	1	9956	43
y,r,ppi,q,pih,pif	0	0	0	4	9953	43
y,r,ppi,q,cf,n	0	0	0	1	9956	43
y,r,ppi,q,cf,s	0	0	0	4	9953	43
y,r,ppi,q,cf,piw	0	0	0	0	9957	43
y,r,ppi,q,cf,pif	0	0	0	4	9953	43
y,r,ppi,q,cf,pih	0	0	0	4	9953	43
y,r,ppi,q,ch,n	0	0	0	1	9956	43
y,r,ppi,q,ch,s	0	0	0	5	9952	43
y,r,ppi,q,ch,piw	0	0	0	0	9957	43
y,r,ppi,q,ch,pif	0	0	0	4	9953	43
y,r,ppi,q,ch,pih	0	0	0	4	9953	43
y,r,ppi,q,ch,cf	0	0	0	4	9953	43
y,r,ppi,q,c,n	0	0	0	1	9956	43
y,r,ppi,q,c,s	0	0	0	5	9952	43
y,r,ppi,q,c,piw	0	0	0	0	9957	43
y,r,ppi,q,c,pif	0	0	0	3	9954	43
y,r,ppi,q,c,pih	0	0	0	4	9953	43
y,r,ppi,q,c,cf	0	0	0	4	9953	43
y,r,ppi,q,c,ch	0	0	0	4	9953	43
y,r,ppi,q,piw,s,n	0	0	0	1	9956	43
y,r,ppi,q,pif,s,n	0	0	0	1	9956	43
y,r,ppi,q,pif,piw,n	0	0	0	1	9956	43
y,r,ppi,q,pif,piw,s	0	0	0	1	9956	43

Table 10: Jacobian ranks with baseline+domestic variables

y,r,ppi,q,pih,s,n	0	0	0	1	9956	43
y,r,ppi,q,pih,piw,n	0	0	0	1	9956	43
y,r,ppi,q,pih,piw,s	0	0	0	1	9956	43
y,r,ppi,q,pih,pif,n	0	0	0	1	9956	43
y,r,ppi,q,pih,pif,s	0	0	0	5	9952	43
y,r,ppi,q,pih,pif,piw	0	0	0	1	9956	43
y,r,ppi,q,cf,s,n	0	0	0	1	9956	43
y,r,ppi,q,cf,piw,n	0	0	0	0	9957	43
y,r,ppi,q,cf,piw,s	0	0	0	0	9957	43
y,r,ppi,q,cf,pif,n	0	0	0	1	9956	43
y,r,ppi,q,cf,pif,s	0	0	0	4	9953	43
y,r,ppi,q,cf,pif,piw	0	0	0	0	9957	43
y,r,ppi,q,cf,pih,n	0	0	0	1	9956	43
y,r,ppi,q,cf,pih,s	0	0	0	5	9952	43
y,r,ppi,q,cf,pih,piw	0	0	0	0	9957	43
y,r,ppi,q,cf,pih,pif	0	0	0	4	9953	43
y,r,ppi,q,ch,s,n	0	0	0	1	9956	43
y,r,ppi,q,ch,piw,n	0	0	0	0	9957	43
y,r,ppi,q,ch,piw,s	0	0	0	0	9957	43
y,r,ppi,q,ch,pif,n	0	0	0	1	9956	43
y,r,ppi,q,ch,pif,s	0	0	0	5	9952	43
y,r,ppi,q,ch,pif,piw	0	0	0	0	9957	43
y,r,ppi,q,ch,pih,n	0	0	0	1	9956	43
y,r,ppi,q,ch,pih,s	0	0	0	5	9952	43
y,r,ppi,q,ch,pih,piw	0	0	0	0	9957	43
y,r,ppi,q,ch,pih,pif	0	0	0	4	9953	43
y,r,ppi,q,ch,cf,n	0	0	0	0	9957	43
y,r,ppi,q,ch,cf,s	0	0	0	5	9952	43
y,r,ppi,q,ch,cf,piw	0	0	0	0	9957	43
y,r,ppi,q,ch,cf,pif	0	0	0	4	9953	43
y,r,ppi,q,ch,cf,pih	0	0	0	4	9953	43
y,r,ppi,q,c,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,piw,n	0	0	0	0	9957	43
y,r,ppi,q,c,piw,s	0	0	0	0	9957	43
y,r,ppi,q,c,pif,n	0	0	0	1	9956	43
y,r,ppi,q,c,pif,s	0	0	0	5	9952	43
y,r,ppi,q,c,pif,piw	0	0	0	0	9957	43
y,r,ppi,q,c,pih,n	0	0	0	1	9956	43
y,r,ppi,q,c,pih,s	0	0	0	5	9952	43
y,r,ppi,q,c,pih,piw	0	0	0	0	9957	43
y,r,ppi,q,c,pih,pif	0	0	0	4	9953	43
y,r,ppi,q,c,cf,n	0	0	0	0	9957	43
y,r,ppi,q,c,cf,s	0	0	0	5	9952	43
y,r,ppi,q,c,cf,piw	0	0	0	0	9957	43

Table 10: Jacobian ranks with baseline+domestic variables

y,r,ppi,q,c,cf,pif	0	0	0	4	9953	43
y,r,ppi,q,c,cf,pih	0	0	0	4	9953	43
y,r,ppi,q,c,ch,n	0	0	0	1	9956	43
y,r,ppi,q,c,ch,s	0	0	0	5	9952	43
y,r,ppi,q,c,ch,piw	0	0	0	0	9957	43
y,r,ppi,q,c,ch,pif	0	0	0	4	9953	43
y,r,ppi,q,c,ch,pih	0	0	0	4	9953	43
y,r,ppi,q,c,ch,cf	0	0	0	4	9953	43
y,r,ppi,q,pif,piw,s,n	0	0	0	1	9956	43
y,r,ppi,q,pih,piw,s,n	0	0	0	1	9956	43
y,r,ppi,q,pih,pif,s,n	0	0	0	1	9956	43
y,r,ppi,q,pih,pif,piw,n	0	0	0	1	9956	43
y,r,ppi,q,pih,pif,piw,s	0	0	0	1	9956	43
y,r,ppi,q,cf,piw,s,n	0	0	0	0	9957	43
y,r,ppi,q,cf,pif,s,n	0	0	0	1	9956	43
y,r,ppi,q,cf,pif,piw,n	0	0	0	1	9956	43
y,r,ppi,q,cf,pif,piw,s	0	0	0	0	9957	43
y,r,ppi,q,cf,pih,s,n	0	0	0	1	9956	43
y,r,ppi,q,cf,pih,piw,n	0	0	0	1	9956	43
y,r,ppi,q,cf,pih,piw,s	0	0	0	0	9957	43
y,r,ppi,q,cf,pih,pif,n	0	0	0	1	9956	43
y,r,ppi,q,cf,pih,pif,s	0	0	0	5	9952	43
y,r,ppi,q,cf,pih,pif,piw	0	0	0	1	9956	43
y,r,ppi,q,ch,piw,s,n	0	0	0	0	9957	43
y,r,ppi,q,ch,pif,s,n	0	0	0	1	9956	43
y,r,ppi,q,ch,pif,piw,n	0	0	0	0	9957	43
y,r,ppi,q,ch,pif,piw,s	0	0	0	0	9957	43
y,r,ppi,q,ch,pih,s,n	0	0	0	1	9956	43
y,r,ppi,q,ch,pih,piw,n	0	0	0	0	9957	43
y,r,ppi,q,ch,pih,piw,s	0	0	0	0	9957	43
y,r,ppi,q,ch,pih,pif,n	0	0	0	1	9956	43
y,r,ppi,q,ch,pih,pif,s	0	0	0	5	9952	43
y,r,ppi,q,ch,pih,pif,piw	0	0	0	0	9957	43
y,r,ppi,q,ch,cf,s,n	0	0	0	1	9956	43
y,r,ppi,q,ch,cf,piw,n	0	0	0	0	9957	43
y,r,ppi,q,ch,cf,piw,s	0	0	0	0	9957	43
y,r,ppi,q,ch,cf,pif,n	0	0	0	0	9957	43
y,r,ppi,q,ch,cf,pif,s	0	0	0	5	9952	43
y,r,ppi,q,ch,cf,pif,piw	0	0	0	0	9957	43
y,r,ppi,q,ch,cf,pih,n	0	0	0	0	9957	43
y,r,ppi,q,ch,cf,pih,s	0	0	0	5	9952	43
y,r,ppi,q,ch,cf,pih,piw	0	0	0	0	9957	43
y,r,ppi,q,ch,cf,pih,pif	0	0	0	4	9953	43
y,r,ppi,q,c,piw,s,n	0	0	0	0	9957	43

Table 10: Jacobian ranks with baseline+domestic variables

y,r,ppi,q,c,pif,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,pif,piw,n	0	0	0	0	9957	43
y,r,ppi,q,c,pif,piw,s	0	0	0	0	9957	43
y,r,ppi,q,c,pih,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,pih,piw,n	0	0	0	0	9957	43
y,r,ppi,q,c,pih,piw,s	0	0	0	0	9957	43
y,r,ppi,q,c,pih,pif,n	0	0	0	1	9956	43
y,r,ppi,q,c,pih,pif,s	0	0	0	5	9952	43
y,r,ppi,q,c,pih,pif,piw	0	0	0	0	9957	43
y,r,ppi,q,c,cf,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,cf,piw,n	0	0	0	0	9957	43
y,r,ppi,q,c,cf,piw,s	0	0	0	0	9957	43
y,r,ppi,q,c,cf,pif,n	0	0	0	0	9957	43
y,r,ppi,q,c,cf,pif,s	0	0	0	5	9952	43
y,r,ppi,q,c,cf,pif,piw	0	0	0	0	9957	43
y,r,ppi,q,c,cf,pih,n	0	0	0	0	9957	43
y,r,ppi,q,c,cf,pih,s	0	0	0	5	9952	43
y,r,ppi,q,c,cf,pih,piw	0	0	0	0	9957	43
y,r,ppi,q,c,cf,pih,pif	0	0	0	4	9953	43
y,r,ppi,q,c,ch,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,ch,piw,n	0	0	0	0	9957	43
y,r,ppi,q,c,ch,piw,s	0	0	0	0	9957	43
y,r,ppi,q,c,ch,pif,n	0	0	0	1	9956	43
y,r,ppi,q,c,ch,pif,s	0	0	0	5	9952	43
y,r,ppi,q,c,ch,pif,piw	0	0	0	0	9957	43
y,r,ppi,q,c,ch,pih,n	0	0	0	1	9956	43
y,r,ppi,q,c,ch,pih,s	0	0	0	5	9952	43
y,r,ppi,q,c,ch,pih,piw	0	0	0	0	9957	43
y,r,ppi,q,c,ch,pih,pif	0	0	0	4	9953	43
y,r,ppi,q,c,ch,cf,n	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,s	0	0	0	5	9952	43
y,r,ppi,q,c,ch,cf,piw	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,pif	0	0	0	4	9953	43
y,r,ppi,q,c,ch,cf,pih	0	0	0	4	9953	43
y,r,ppi,q,pih,pif,piw,s,n	0	0	0	1	9956	43
y,r,ppi,q,cf,pif,piw,s,n	0	0	0	1	9956	43
y,r,ppi,q,cf,pih,piw,s,n	0	0	0	1	9956	43
y,r,ppi,q,cf,pih,pif,s,n	0	0	0	1	9956	43
y,r,ppi,q,cf,pih,pif,piw,n	0	0	0	1	9956	43
y,r,ppi,q,cf,pih,pif,piw,s	0	0	0	1	9956	43
y,r,ppi,q,ch,pif,piw,s,n	0	0	0	1	9956	43
y,r,ppi,q,ch,pih,piw,s,n	0	0	0	1	9956	43
y,r,ppi,q,ch,pih,pif,s,n	0	0	0	1	9956	43
y,r,ppi,q,ch,pih,pif,piw,n	0	0	0	0	9957	43

Table 10: Jacobian ranks with baseline+domestic variables

y,r,ppi,q,ch,pih,pif,piw,s	0	0	0	1	9956	43
y,r,ppi,q,ch,cf,piw,s,n	0	0	0	0	9957	43
y,r,ppi,q,ch,cf,pif,s,n	0	0	0	1	9956	43
y,r,ppi,q,ch,cf,pif,piw,n	0	0	0	0	9957	43
y,r,ppi,q,ch,cf,pif,piw,s	0	0	0	0	9957	43
y,r,ppi,q,ch,cf,pih,s,n	0	0	0	1	9956	43
y,r,ppi,q,ch,cf,pih,piw,n	0	0	0	0	9957	43
y,r,ppi,q,ch,cf,pih,piw,s	0	0	0	0	9957	43
y,r,ppi,q,ch,cf,pih,pif,n	0	0	0	0	9957	43
y,r,ppi,q,ch,cf,pih,pif,s	0	0	0	5	9952	43
y,r,ppi,q,ch,cf,pih,pif,piw	0	0	0	0	9957	43
y,r,ppi,q,c,pif,piw,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,pih,piw,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,pih,pif,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,pih,pif,piw,n	0	0	0	0	9957	43
y,r,ppi,q,c,pih,pif,piw,s	0	0	0	1	9956	43
y,r,ppi,q,c,cf,piw,s,n	0	0	0	0	9957	43
y,r,ppi,q,c,cf,pif,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,cf,pif,piw,n	0	0	0	0	9957	43
y,r,ppi,q,c,cf,pif,piw,s	0	0	0	0	9957	43
y,r,ppi,q,c,cf,pih,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,cf,pih,piw,n	0	0	0	0	9957	43
y,r,ppi,q,c,cf,pih,piw,s	0	0	0	0	9957	43
y,r,ppi,q,c,cf,pih,pif,n	0	0	0	0	9957	43
y,r,ppi,q,c,cf,pih,pif,s	0	0	0	5	9952	43
y,r,ppi,q,c,cf,pih,pif,piw	0	0	0	0	9957	43
y,r,ppi,q,c,ch,piw,s,n	0	0	0	0	9957	43
y,r,ppi,q,c,ch,pif,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,ch,pif,piw,n	0	0	0	0	9957	43
y,r,ppi,q,c,ch,pif,piw,s	0	0	0	0	9957	43
y,r,ppi,q,c,ch,pih,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,ch,pih,piw,n	0	0	0	0	9957	43
y,r,ppi,q,c,ch,pih,piw,s	0	0	0	0	9957	43
y,r,ppi,q,c,ch,pih,pif,n	0	0	0	1	9956	43
y,r,ppi,q,c,ch,pih,pif,s	0	0	0	5	9952	43
y,r,ppi,q,c,ch,pih,pif,piw	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,ch,cf,piw,n	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,piw,s	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,pif,n	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,pif,s	0	0	0	5	9952	43
y,r,ppi,q,c,ch,cf,pif,piw	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,pih,n	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,pih,s	0	0	0	5	9952	43

Table 10: Jacobian ranks with baseline+domestic variables

y,r,ppi,q,c,ch,cf,pih,piw	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,pih,pif	0	0	0	4	9953	43
y,r,ppi,q,cf,pih,pif,piw,s,n	0	0	0	1	9956	43
y,r,ppi,q,ch,pih,pif,piw,s,n	0	0	0	1	9956	43
y,r,ppi,q,ch,cf,pif,piw,s,n	0	0	0	0	9957	43
y,r,ppi,q,ch,cf,pih,piw,s,n	0	0	0	0	9957	43
y,r,ppi,q,ch,cf,pih,pif,s,n	0	0	0	1	9956	43
y,r,ppi,q,ch,cf,pih,pif,piw,n	0	0	0	0	9957	43
y,r,ppi,q,ch,cf,pih,pif,piw,s	0	0	0	0	9957	43
y,r,ppi,q,c,pih,pif,piw,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,cf,pif,piw,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,cf,pih,piw,s,n	0	0	0	0	9957	43
y,r,ppi,q,c,cf,pih,pif,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,cf,pih,pif,piw,n	0	0	0	0	9957	43
y,r,ppi,q,c,cf,pih,pif,piw,s	0	0	0	0	9957	43
y,r,ppi,q,c,ch,pif,piw,s,n	0	0	0	0	9957	43
y,r,ppi,q,c,ch,pih,piw,s,n	0	0	0	0	9957	43
y,r,ppi,q,c,ch,pih,pif,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,ch,pih,pif,piw,n	0	0	0	0	9957	43
y,r,ppi,q,c,ch,pih,pif,piw,s	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,piw,s,n	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,pif,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,ch,cf,pif,piw,n	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,pif,piw,s	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,pih,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,ch,cf,pih,piw,n	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,pih,piw,s	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,pih,pif,n	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,pih,pif,s	0	0	0	5	9952	43
y,r,ppi,q,c,ch,cf,pih,pif,piw	0	0	0	0	9957	43
y,r,ppi,q,ch,cf,pih,pif,piw,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,cf,pih,pif,piw,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,ch,pih,pif,piw,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,ch,cf,pif,piw,s,n	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,pih,piw,s,n	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,pih,pif,s,n	0	0	0	1	9956	43
y,r,ppi,q,c,ch,cf,pih,pif,piw,n	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,pih,pif,piw,s	0	0	0	0	9957	43
y,r,ppi,q,c,ch,cf,pih,pif,piw,s,n	0	0	0	1	9956	43

Indet.= Indeterminacy: corresponds to the number of parameter vectors that were not uniquely identified.
Note: Rows sum to 10,000, corresponding to the number of parameter vectors that were evaluated.

Table 11: Jacobian ranks with baseline+foreign variables

Observed variables	25	26	27	28	29	Indet.
y,r,ppi,q	0	9957	0	0	0	43
y,r,ppi,q,nstar	0	0	0	6	9951	43
y,r,ppi,q,ws	0	0	0	5	9952	43
y,r,ppi,q,rs	0	0	0	6	9951	43
y,r,ppi,q,piws	0	0	0	6	9951	43
y,r,ppi,q,pis	0	0	0	6	9951	43
y,r,ppi,q,ys	0	0	0	6	9951	43
y,r,ppi,q,ws,nstar	0	0	0	6	9951	43
y,r,ppi,q,rs,nstar	0	0	0	6	9951	43
y,r,ppi,q,rs,ws	0	0	0	6	9951	43
y,r,ppi,q,piws,nstar	0	0	0	6	9951	43
y,r,ppi,q,piws,ws	0	0	0	6	9951	43
y,r,ppi,q,piws,rs	0	0	0	6	9951	43
y,r,ppi,q,pis,nstar	0	0	0	6	9951	43
y,r,ppi,q,pis,ws	0	0	0	6	9951	43
y,r,ppi,q,pis,rs	0	0	0	6	9951	43
y,r,ppi,q,pis,piws	0	0	0	6	9951	43
y,r,ppi,q,ys,nstar	0	0	0	6	9951	43
y,r,ppi,q,ys,ws	0	0	0	6	9951	43
y,r,ppi,q,ys,rs	0	0	0	6	9951	43
y,r,ppi,q,ys,piws	0	0	0	6	9951	43
y,r,ppi,q,ys,pis	0	0	0	6	9951	43
y,r,ppi,q,rs,ws,nstar	0	0	0	6	9951	43
y,r,ppi,q,piws,ws,nstar	0	0	0	6	9951	43
y,r,ppi,q,piws,rs,nstar	0	0	0	6	9951	43
y,r,ppi,q,piws,rs,ws	0	0	0	6	9951	43
y,r,ppi,q,pis,ws,nstar	0	0	0	6	9951	43
y,r,ppi,q,pis,rs,nstar	0	0	0	6	9951	43
y,r,ppi,q,pis,rs,ws	0	0	0	6	9951	43
y,r,ppi,q,pis,piws,nstar	0	0	0	6	9951	43
y,r,ppi,q,pis,piws,ws	0	0	0	6	9951	43
y,r,ppi,q,pis,piws,rs	0	0	0	6	9951	43
y,r,ppi,q,ys,ws,nstar	0	0	0	6	9951	43
y,r,ppi,q,ys,rs,nstar	0	0	0	6	9951	43
y,r,ppi,q,ys,rs,ws	0	0	0	6	9951	43
y,r,ppi,q,ys,piws,nstar	0	0	0	6	9951	43
y,r,ppi,q,ys,piws,ws	0	0	0	6	9951	43
y,r,ppi,q,ys,piws,rs	0	0	0	6	9951	43
y,r,ppi,q,ys,pis,nstar	0	0	0	6	9951	43
y,r,ppi,q,ys,pis,ws	0	0	0	6	9951	43
y,r,ppi,q,ys,pis,rs	0	0	0	6	9951	43
y,r,ppi,q,ys,pis,piws	0	0	0	6	9951	43
y,r,ppi,q,piws,rs,ws,nstar	0	0	0	6	9951	43

Table 11: Jacobian ranks with baseline+foreign variables

y,r,ppi,q,pis,rs,ws,nstar	0	0	0	6	9951	43
y,r,ppi,q,pis,piws,ws,nstar	0	0	0	6	9951	43
y,r,ppi,q,pis,piws,rs,nstar	0	0	0	6	9951	43
y,r,ppi,q,pis,piws,rs,ws	0	0	0	6	9951	43
y,r,ppi,q,ys,rs,ws,nstar	0	0	0	6	9951	43
y,r,ppi,q,ys,piws,ws,nstar	0	0	0	6	9951	43
y,r,ppi,q,ys,piws,rs,nstar	0	0	0	6	9951	43
y,r,ppi,q,ys,piws,rs,ws	0	0	0	6	9951	43
y,r,ppi,q,ys,pis,ws,nstar	0	0	0	6	9951	43
y,r,ppi,q,ys,pis,rs,nstar	0	0	0	6	9951	43
y,r,ppi,q,ys,pis,rs,ws	0	0	0	6	9951	43
y,r,ppi,q,ys,pis,piws,nstar	0	0	0	6	9951	43
y,r,ppi,q,ys,pis,piws,ws	0	0	0	6	9951	43
y,r,ppi,q,ys,pis,piws,rs	0	0	0	6	9951	43
y,r,ppi,q,pis,piws,rs,ws,nstar	0	0	1	5	9951	43
y,r,ppi,q,ys,piws,rs,ws,nstar	0	0	1	5	9951	43
y,r,ppi,q,ys,pis,rs,ws,nstar	0	0	1	5	9951	43
y,r,ppi,q,ys,pis,piws,ws,nstar	0	0	1	5	9951	43
y,r,ppi,q,ys,pis,piws,rs,nstar	0	0	1	5	9951	43
y,r,ppi,q,ys,pis,piws,rs,ws	0	0	1	5	9951	43
y,r,ppi,q,ys,pis,piws,rs,ws,nstar	0	0	1	5	9951	43

Indet.= Indeterminacy: corresponds to the number of parameter vectors that were not uniquely identified.
Note: Rows sum to $10,000$, corresponding to the number of parameter vectors that were evaluated.
Variables ending in s or star are foreign variables.