Job Search over the Life Cycle*

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Preliminary

Abstract

This paper studies the worker's job search behavior over the life-cycle. We first document that job search intensity, defined as average time spent on job hunting in a typical day, displays a hump-shaped age profile. In order to explain the patter of the worker's search behavior, we build a life-cycle job search model with three key features: endogenous labor supply, age-specific labor productivity, and wealth accumulation. We then calibrate the model and quantitatively evaluate the effect of age on worker's search intensity through the above channels.

JEL classification: D91, E24, J22, J64

Key words: job search, life-cycle, time use

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1 Introduction

This paper studies the worker's job search behavior over the life-cycle. When a worker becomes unemployed, instead of enjoying leisure, he can choose to invest his time in searching for a new job. Data shows that an unemployed worker in the U.S. on average spends more than half an hour looking for job on a daily basis, and this number varies according to worker's age. In this paper, we document a hump-shaped age profile of time spent on job search.

We build a life-cycle job search model to explain the change of worker's search intensity at different ages. There are three main features of the model. First, unemployed workers can invest labor effort in the job hunting process and his effort is positively correlated with the probability of receiving a wage offer. Second, workers at different ages have different labor productivity, which affects their actual wage income. Finally, both unemployed and employed workers can access the asset market and accumulate wealth.

When a worker is at his early age, his labor productivity is low, and his opportunity cost of waiting for a better wage offer is low as well. So he may choose to search harder and wait longer for a better job. On the other hand, the worker does not possess much wealth and tends to accept a job as soon as possible. The direction of these effect will reverse as the worker becomes older and more productive. After he has accumulated much wealth, the worker can use his savings to compensate for the low income of unemployment benefit if being laid off, and enjoy leisure time while waiting for a new wage offer to arrive. On the other hand, his high labor productivity makes leisure time expensive and encourages him to find a new job faster. We call the effect of labor productivity on job search the productivity channel and the effect through wealth accumulation the wealth channel. It is the interaction of the two channels that generates a hump-shaped age profile of search intensity.

In order to quantitatively evaluate the contributions through each channel, we numerically solve the model and calibrate the parameters to match the stylized facts. We also solve for alternative equilibrium by shutting down one channel at a time and isolate the effects through each individual channel. We also conduct counterfactual experiments on labor market policy.

The rest of the paper is organized as the following. In Section 2, we document the stylized facts regarding job search intensity, unemployment rate, and wage earnings. Then in Section 3, we use a simple infinite-horizon model with search intensity to present the economic intuition regarding how search effort responds to the change of labor productivity efficiency. A life-cycle model with asset and endogenous labor supply is presented in Section 4. We solve the model numerically, calibrate the parameters, and evaluate the quantitative performance of the model. Section 5 concludes.

2 Stylized Facts

The facts we document in this section are based on data from the American Time Use Survey (ATUS). This data set provides nationally representative estimates of how, where, and with whom people divide their time among life's activities. Each ATUS respondent provides detailed information in an interview on all activities during a designated 24-hour period, including the time spent in searching for job. In this paper, we focus on the time period from 2003 to 2011.

First, we define job search intensity as the average number of minutes that an unemployed worker spends in searching for a new job in a typical day. We pool the data from nine years into a single cross-section sample, calculate the average daily job search time at each age for all cohorts, and construct the age profile of average job search intensity in Figure 1. We find that the age profile

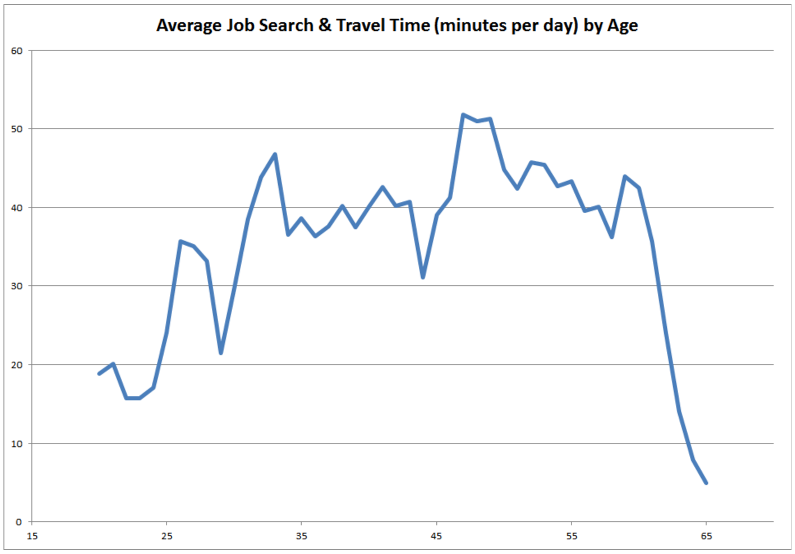


Figure 1: Age Profile of Job Search Intensity

displays a hump shape with a peak lying between age 40 and 45.

We also follow Aguiar, Hurst, and Karabarbounis (2013) to estimate life-cycle profiles by splitting the population in five-year age bins. Given the complex sample design, we use the weighting methodology introduced in 2006 to produce a representative sample, ensuring that the sums of the respondent weights add up to the appropriate number of weekday and weekend person-days over the quarter. Working on the weighted sample instead of the raw data, we generate the age profile of job search time in Figure 2, similar to the findings in Aguiar, Hurst, and Karabarbounis (2013). We also include orthogonalized age polynomials in the regression and find a smoothed age profile in Figure 3.

We also document two other relevant facts using data from ATUS. First, the age profile of average unemployment rate is roughly U-shaped with a sharp decline around age 20 and a slight pick up around 60, as shown in Figure 2. Second, in Figure 2, the age profile of average weekly earnings of the employed workers displays a hump shape with a peak between age 40 and 50. The same

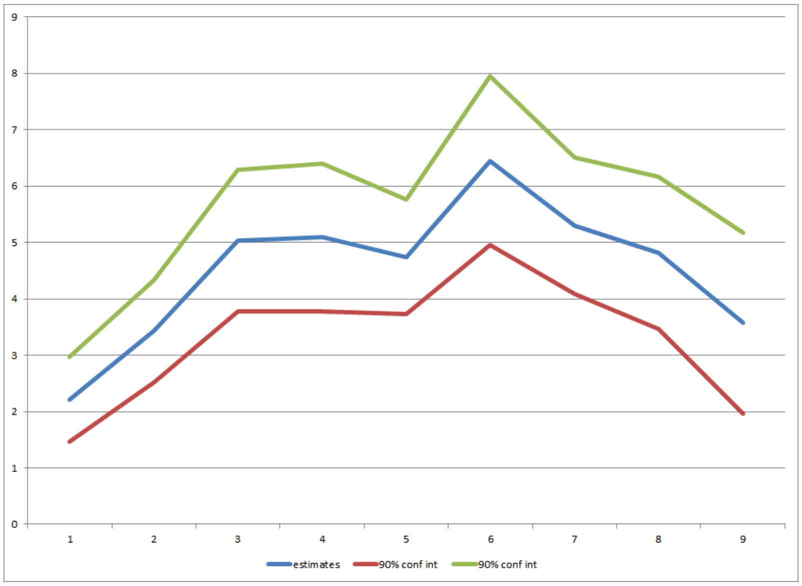


Figure 2: Age Profile of Job Search Intensity

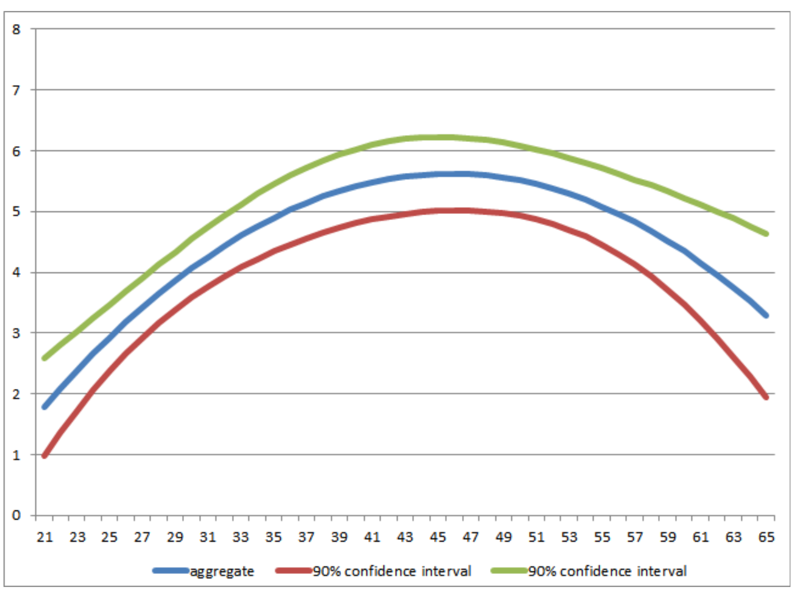
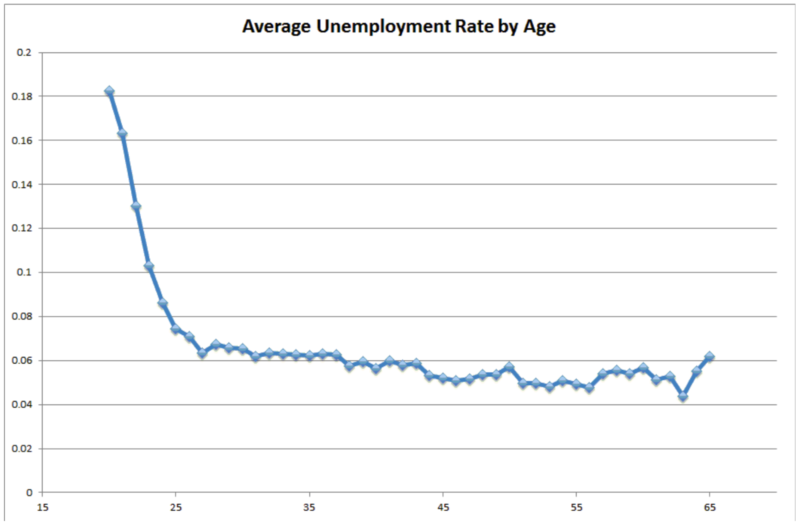


Figure 3: Age Profile of Job Search Intensity

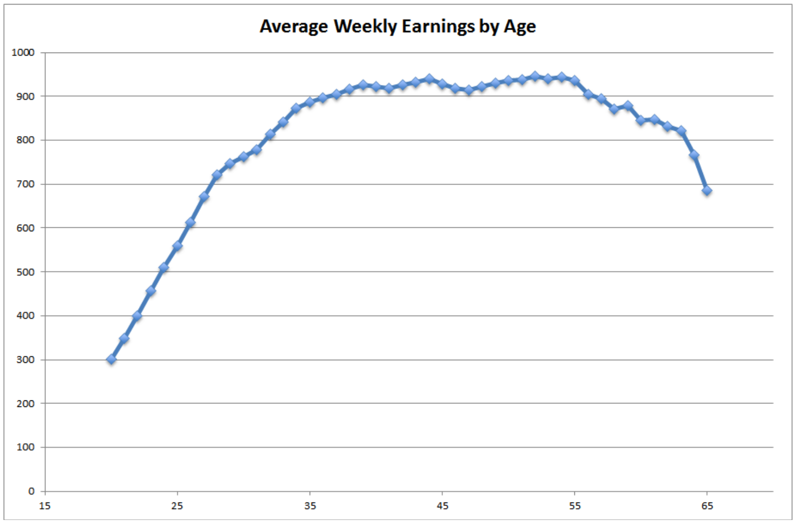


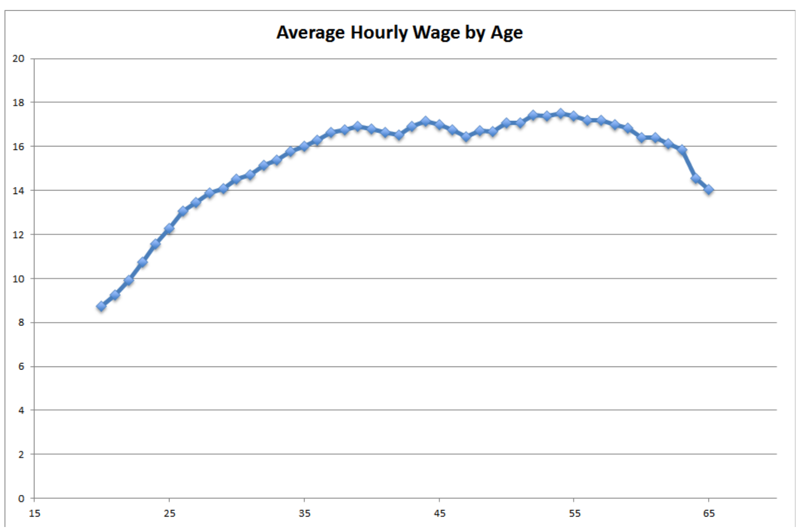
pattern can be observed in the age profile of average hourly earnings as in Figure 2. In the quantitative exercise of this paper, we establish a life-cycle model which not only generates the pattern of job search intensity, but also reproduces the above facts.

3 An Infinite-Horizon Model with Search Intensity

In this section, we start with an infinite-horizon job search model with endogenous search intensity, in order to get some intuition on the relationship between the among of time that workers spend on job search and their productivity efficiency.

We start with an unemployed worker. In each period, he receives unemployment compensation b and he can draw an independently and identically distributed wage offer w from wage distribution $F(w)$ with support $[0, \bar{w}]$ and we assume $\bar{w} > b$. If the unemployed worker chooses to accepts the offer, he becomes employed and is actually paid εw, where ε represents the productivity


Average Weekly Earnings by Age
1000
900
800
700
600
500
400
300
200
100
0
15
25
35
45
55
65


Average Hourly Wage by Age
20
18
16
14
12
10
8
6
4
2
0
15
25
35
45
55
65

efficiency of labor supplied. If the worker does not accept the wage offer, he remains unemployed. At the end of each period, an employee faces probability $\lambda \in (0,1)$ of being separated from the job and λ is fixed over time and independent of wage offers. Once unemployed, a worker will sit out one period of unemployment before receiving new job offers.

The worker derives a decision rule that maximizes $\mathbb{E}\Sigma_{t=0}^{\infty}\beta^t u(c_t)$, where $c_t = b$ or $c_t = w$, depending on whether the worker is employed or not. Now for simplicity, we assume that $u(c_t) = c_t$ and the worker does not save, while these assumptions will be relaxed later. One way to interpret this is to say that the individual is risk neutral, so that he does not care about smoothing consumption. $\beta = \frac{1}{1+r}$ is the discount factor.

We allow an unemployed worker to choose search intensity s. In each period, the worker now can draw an i.i.d. wage offer w from $F(w)$ with probability $\alpha(s) \in (0,1)$. The search cost occurred is denoted as $v(s)$. We assume $\alpha(s)$ is increasing, twice differentiable, and concave, while $v(s)$ is also increasing and twice differentiable but convex.

We use V^u to denote the value function of being unemployed for the current period and $V^e(w)$ of being employed with wage w. The Bellman equations are

$$\begin{aligned} V^u &= \max_s \left\{ b - v(s) + \alpha(s)\beta \int_0^{\bar{w}} \max\left[V^u, V^e(w)\right] dF(w) \right\} \\ V^e(w) &= \varepsilon w + \delta\beta V^u + (1-\delta)\beta V^e(w) \end{aligned}$$

We can simplify the above equations into

$$rV^u = \max_s \left\{ [b - v(s)](1+r) + \alpha(s) \int_0^{\bar{w}} \max\left[V^e(w) - V^u, 0\right] dF(w) \right\} \quad (1)$$
$$rV^e(w) = \varepsilon w (1+r) + \delta\left[V^u - V^e(w)\right]$$

The second value function can be further simplified into

$$V^e(w) = \frac{\varepsilon w(1+r) + \delta V^u}{r+\delta}.$$

Since $\frac{\partial V^e(w)}{\partial w} = \frac{\varepsilon(1+r)}{r+\delta} > 0$ and $\frac{\partial V^u}{\partial w} = 0$, $V^e(0) < V^u$, and $\lim_{w\to\infty} V^e(w) > V^u$, there exists w^* such that $V^e(w^*) = V^u$. For $w \le w^*$, $V^e(w) < V^u$, and for $w \ge w^*$, $V^e(w) > V^u$. We call w^* the reservation wage. Then,

$$V^u = \frac{\varepsilon w^* (1+r)}{r}.$$

Plug V^u and $V^e(w)$ back into (1) and we can get

$$w^* = \max_s \left\{ \frac{b - v(s)}{\varepsilon} + \frac{\alpha(s)}{r+\delta} \int_{w^*}^{\bar{w}} [1 - F(w)]\, dw \right\}$$

Therefore, the optimal search intensity s^* and reservation wage w^* satisfy the following two conditions.

$$w^* = \frac{b - v(s^*)}{\varepsilon} + \frac{\alpha(s^*)}{r+\delta} \int_{w^*}^{\bar{w}} [1 - F(w)]\, dw \tag{2}$$

$$\frac{v'(s^*)}{\varepsilon} = \frac{\alpha'(s^*)}{r+\delta} \int_{w^*}^{\bar{w}} [1 - F(w)]\, dw. \tag{3}$$

We call the first equation "reservation wage" (RW) condition and the second "search intensity" (SI) condition. In order to understand how productivity efficiency affects reservation wage and search intensity, we need to first solve for the stationary equilibrium (s^*, w^*) characterized by the two equations.

Start with the SI condition. Taking derivatives of both sides of the equation w.r.t. s^* yields

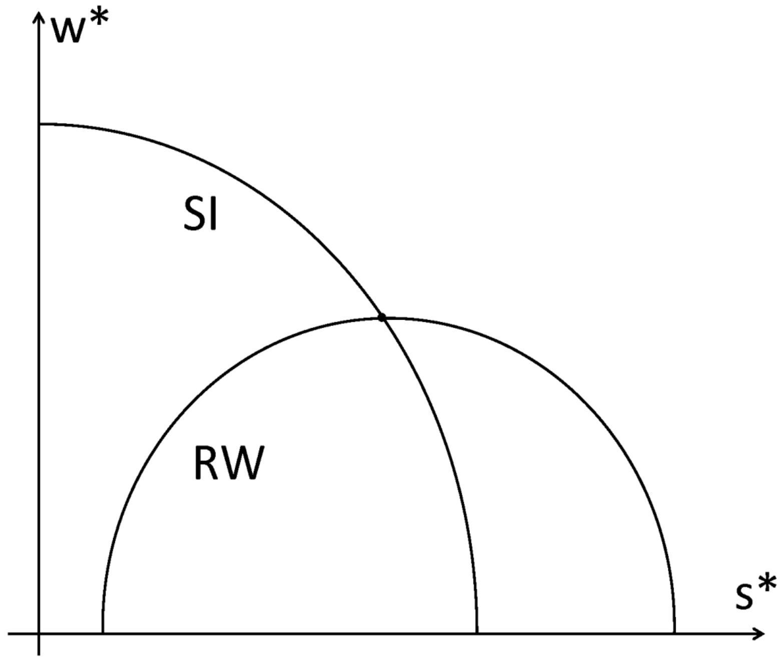
$$\frac{v''(s^*)}{\varepsilon} = \frac{\alpha''(s^*)}{r+\delta} \int_{w^*}^{\bar{w}} [1 - F(w)]\, dw - \frac{\alpha'(s^*)}{r+\delta} [1 - F(w^*)] \frac{\partial w^*}{\partial s^*}.$$

After rearranging terms, we find the SI condition implies

$$\frac{\partial w^*}{\partial s^*} = \frac{\frac{\alpha''(s^*)}{r+\delta} \int_{w^*}^{\bar{w}} [1 - F(w)]\, dw - \frac{v''(s^*)}{\varepsilon}}{\frac{\alpha'(s^*)}{r+\delta} [1 - F(w^*)]} < 0.$$

Then, we turn to the RW condition. Similar algebra gives

$$\frac{\partial w^*}{\partial s^*} = \frac{\frac{\alpha'(s^*)}{r+\delta} \int_{w^*}^{\bar{w}} [1 - F(w)]\, dw - \frac{v'(s^*)}{\varepsilon}}{1 + \frac{\alpha(s^*)}{r+\delta} [1 - F(w^*)]}. \tag{4}$$



Notice that the numerator of the right-hand side is equal to zero at the intersection of the two curves.

We plot the two curves in Figure 3. Note that in the (s^*, w^*) plane, to the left of the SI curve, the RW curve implies w^* increases with s^* since the numerator of the fraction in (4) is positive, and vice versa. We can conclude that there exists a unique equilibrium with (s^*, w^*) characterized by equations (2) and (3).

3.1 Reservation Wage and Search Intensity

Over the life-cycle of a worker, his efficiency productivity increases and decreases at different age. In order to help us understand the different forces at play in the life-cycle model when productivity changes, we want to examine, in the infinite horizon model first, the effects of ε on reservation wage w^*, wage income εw^*, and search intensity s^*.

First, we focus on the change of reservation wage. We start with the RW condition and take derivatives w.r.t. ε. After rearranging terms, we have

$$\begin{aligned}\frac{\partial w^*}{\partial \varepsilon} &= \frac{-v'(s^*)\frac{\partial s^*}{\partial \varepsilon}\varepsilon - [b - v(s^*)]}{\varepsilon^2} + \frac{\alpha'(s^*)\frac{\partial s^*}{\partial \varepsilon}}{r+\delta}\int_{w^*}^{\bar{w}}[1-F(w)]\,dw \\ &\quad -\frac{\alpha(s^*)}{r+\delta}[1-F(w^*)]\frac{\partial w^*}{\partial \varepsilon}. \end{aligned} \tag{5}$$

The RHS of (5) can be rewritten as

$$\left\{-\frac{v'(s^*)}{\varepsilon} + \frac{\alpha'(s^*)}{r+\delta}\int_{w^*}^{\bar{w}}[1-F(w)]\,dw\right\}\frac{\partial s^*}{\partial \varepsilon} - \frac{b-v(s^*)}{\varepsilon^2} - \frac{\alpha(s^*)}{r+\delta}[1-F(w^*)]\frac{\partial w^*}{\partial \varepsilon}.$$

Plug in (3) and we can simplify (5) to

$$\frac{\partial w^*}{\partial \varepsilon} = -\frac{b-v(s^*)}{\varepsilon^2} - \frac{\alpha(s^*)}{r+\delta}[1-F(w^*)]\frac{\partial w^*}{\partial \varepsilon},$$

which implies

$$\frac{\partial w^*}{\partial \varepsilon} = -\frac{r+\delta}{\varepsilon^2}\frac{b-v(s^*)}{r+\delta+\alpha(s^*)[1-F(w^*)]} < 0, \tag{6}$$

as long as $b - v(s^*) > 0$, i.e. an unemployed worker has positive consumption.[1]

Next, we want to understand the effect of ε on wage income εw^*. Note that

$$\frac{\partial(\varepsilon w^*)}{\partial \varepsilon} = w^* + \varepsilon\frac{\partial w^*}{\partial \varepsilon}$$

Then, $\frac{\partial(\varepsilon w^*)}{\partial \varepsilon} > 0$ is equivalent to $w^* + \varepsilon\frac{\partial w^*}{\partial \varepsilon} > 0$ and $-\frac{\partial w^*/w^*}{\partial \varepsilon/\varepsilon} < 1$, i.e. the elasticity of wage w.r.t. productivity change is less than one.

Plug (6) into $-\frac{\partial w^*/w^*}{\partial \varepsilon/\varepsilon}$, and we get

$$-\frac{\partial w^*/w^*}{\partial \varepsilon/\varepsilon} = \frac{r+\delta}{\varepsilon w^*}\frac{b-v(s^*)}{r+\delta+\alpha(s^*)[1-F(w^*)]}. \tag{7}$$

Rearranging terms in (2) yields

$$\frac{(r+\delta)[b-v(s^*)]}{\varepsilon w^*} = r+\delta-\alpha(s^*)\int_{w^*}^{\bar{w}}\frac{1-F(w)}{w^*}dw.$$

Plug the above equation into (7) and we get

$$-\frac{\partial w^*/w^*}{\partial \varepsilon/\varepsilon} = \frac{r+\delta-\alpha(s^*)\int_{w^*}^{\bar{w}}\frac{1-F(w)}{w^*}dw}{r+\delta+\alpha(s^*)[1-F(w^*)]} < 1.$$

[1] Is this always true in equilibrium?

Therefore, we have $\frac{\partial(\varepsilon w^*)}{\partial\varepsilon} > 0$ and worker's efficiency wage income increases with productivity.

Lastly in this section, we want to understand how labor efficiency influences search intensity. We take derivatives w.r.t. ε on both sides of the SI equation (3).

$$\left[\frac{v''(s^*)}{\varepsilon} - \frac{\alpha''(s^*)}{r+\delta}\int_{w^*}^{\bar{w}}[1-F(w)]\,dw\right]\frac{\partial s^*}{\partial\varepsilon} = \frac{v'(s^*)}{\varepsilon^2} - \frac{\alpha'(s^*)}{r+\delta}[1-F(w^*)]\frac{\partial w^*}{\partial\varepsilon.} \tag{8}$$

We define the first term of the product on the LHS of the above equation as Δ. From (3) we can substitute out $\int_{w^*}^{\bar{w}}[1-F(w)]\,dw$ and simplify Δ as

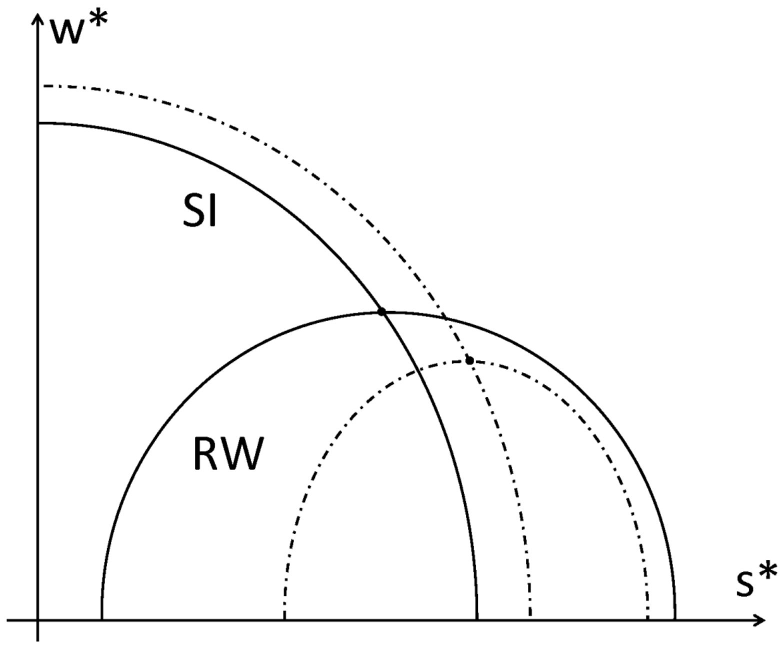
$$\begin{aligned}\Delta &= \frac{v''(s^*)}{\varepsilon} - \frac{\alpha''(s^*)}{r+\delta}\int_{w^*}^{\bar{w}}[1-F(w)]\,dw \\ &= \frac{v''(s^*)}{\varepsilon} - \frac{\alpha''(s^*)}{r+\delta}\frac{v'(s^*)(r+\delta)}{\varepsilon\alpha'(s^*)} \\ &= \frac{1}{\varepsilon}\left[v''(s^*) - \frac{v'(s^*)\alpha''(s^*)}{\alpha'(s^*)}\right] > 0\end{aligned}$$

Then, we plug Δ and (6) into (8) and solve for $\frac{\partial s^*}{\partial\varepsilon}$ as

$$\begin{aligned}\frac{\partial s^*}{\partial\varepsilon} &= \frac{v'(s^*)}{\varepsilon^2\Delta} - \frac{\alpha'(s^*)[1-F(w^*)]}{(r+\delta)\Delta}\left[-\frac{r+\delta}{\varepsilon^2}\frac{b-v(s^*)}{r+\delta+\alpha(s^*)[1-F(w^*)]}\right] \\ &= \frac{1}{\varepsilon^2\Delta}\left\{v'(s^*) + \frac{\alpha'(s^*)[1-F(w^*)][b-v(s^*)]}{r+\delta+\alpha(s^*)[1-F(w^*)]}\right\} > 0,\end{aligned}$$

as long as $b - v(s^*) > 0$ holds.

We can use a graph to illustrate the above results. When ε increases, the SI curve shifts to the right while the RW curve shifts down, as shown in Figure 3.1. We can see that as efficiency wage ε increases, the new equilibrium is at the lower-right corner of the original equilibrium. Therefore, the equilibrium related to a higher labor productivity displays a smaller reservation wage and a larger search intensity, i.e. $\frac{\partial w^*}{\partial\varepsilon} < 0$ and $\frac{\partial s^*}{\partial\varepsilon} > 0$, but we cannot determine the change of wage income using the graphical analysis.



3.2 Unemployment Rate

Assume the total measure of workers is 1 and we use u_t to denote the total measure of unemployed workers in period t. Unemployment rate u_t must follow the law of motion

$$\begin{aligned} u_{t+1} &= u_t - u_t \alpha(s^*)[1 - F(w^*)] + (1 - u_t)\delta \\ &= u_t[1 - \alpha(s^*) + \alpha(s^*) F(w^*) - \delta] + \delta. \end{aligned}$$

Since $|1 - \alpha(s^*) + \alpha(s^*) F(w^*) - \delta| < 1$, u_t converges to a constant u, which is determined by $u_{t+1} = u_t = u$. So we can solve the above equation to get

$$u = \frac{\delta}{\delta + \alpha(s^*)[1 - F(w^*)]}.$$

This is the unemployment rate in the stationary equilibrium.

We already know that $\frac{\partial w^*}{\partial \varepsilon} < 0$ and $\frac{\partial s^*}{\partial \varepsilon} > 0$. As ε increases, both $\alpha(s^*)$ and $1 - F(w^*)$ increase. The denominator gets larger and the unemployment rate u

gets smaller. Unemployment rate drops due to two effects: unemployed workers search harder and they are more likely to accept arrived job offers. Intuitively, if higher labor efficiency can increase search intensity while decrease reservation wage, both higher search intensity and lower reservation wage will lower the unemployment rate.

4 A Life-cycle Model with Asset and Endogenous Labor Supply

In order to study worker's job search intensity when they are unemployed and account for the data of time spent in job search when unemployed, we need a model with endogenous search intensity.

We modify the benchmark toy model by allowing an unemployed worker to choose search intensity s. In each period, the worker now can draw an i.i.d. wage offer w from $F(w)$ with probability $\alpha(s) \in (0,1)$. The search cost occurred is denoted as $v(s)$. We assume $\alpha(s)$ is increasing, twice differentiable, and concave, while $v(s)$ is also increasing and twice differentiable but convex.

Now we present a parsimonious life-cycle model with asset and endogenous labor decision. The value function for an unemployed household is

$$V_j^u = \max_{c_j, a_{j+1}, s_j} \Big\{ u\left(c_j, 1 - s_j\right) + \alpha\left(s_j\right)\beta \int_{\underline{w}}^{\bar{w}} \max\left\{V_{j+1}^u, V_{j+1}^e\left(w\right)\right\} dF\left(w\right) + \left[1 - \alpha\left(s_j\right)\right] V_{j+1}^u \Big\} \quad (9)$$

$$s.t.\ c_j + a_{j+1} = b + (1+r)\, a_j$$

The unemployed household collects unemployment benefit b, chooses consumption c, asset holding a for the next period, and job search intensity s. The marginal gain of search intensity is a higher probability of finding a job, while the marginal cost is foregone leisure. The value function for an employed household is

$$V_j^e(w_j) = \max_{c_j, a_{j+1}, n_j} u(c_j, 1-n_j) + \delta\beta V_{j+1}^u + (1-\delta)\beta V_{j+1}^e(w_{j+1}) \tag{10}$$
$$s.t.\ c_j + a_{j+1} = \varepsilon_j w_j n_j + (1+r) a_j$$

If the household has a job, he chooses his labor supply, consumption, and asset holding for the next period. With probability of δ, he is separated from the job, and he can keep it at the same wage with probability $1-\delta$. ε_j represents efficiency wage at age j.

4.1 Computation

State variables are a_j and w_j, and control variables are c_j, a_{j+1}, s_j, and n_j. The transition matrix is characterized as

	u_{j+1}	e_{j+1}
u_j	$1-\alpha(s_j^*) + \alpha(s_j^*) F(w_j^*)$	$\alpha(s_j^*)\left[1-F(w_j^*)\right]$
e_j	δ	$1-\delta$

We set $J = 45$ and $r = 4\%$.

We first need to discretize the support of $F(w)$, i.e. discretize $[\underline{w}, \bar{w}]$. Starting from the last period J, if the household is unemployed, we need to solve for V_J^u, and we know $V_{J+1}^u = V_{J+1}^e = 0$. Then, the household chooses search intensity $s_J^* = 0$ and asset holding $a_{J+1} = 0$. If the household is employed, we need to solve for $V_J^e(w)$ for each $w \in [w^R, \bar{w}]$. Since $V_{J+1}^u = V_{J+1}^e = 0$, the household chooses c_J and n_J to maximize $V_J^e(w)$ subject to constraint

$$c_J = \varepsilon_J w_J n_J + (1+r) a_J. \tag{11}$$

Notice that we need to save a vector of values for $V_J^e(w)$ and later on we need the vector to solve for V_{J-1}^u.

Now we go to period $J-1$. It is straightforward to solve for $V_{J-1}^e(w)$ by optimizing c_{J-1}, n_{J-1}, and a_J. Regarding V_{J-1}^u, we first need to solve for the reservation wage in period $J-1$, w_{J-1}^* by equating $V_J^e(w_{J-1}^*) = V_J^u$. Then, the integral in V_{J-1}^{u} can be simplified into two intervals, $[\underline{w}, w_{J-1}^{*}]$ and $[w_{J-1}^{*}, \bar{w}]$ and in the first interval the household chooses V_J^u and in the second interval $V_J^e(w_J^*)$. After the integral is calculated, everything else is simple.

We need to calculate the asset distribution of households as follows

$$\lambda_{j+1}^{e}\left(a_{j+1}, w_{j+1}\right) = \sum_{w} \sum_{a: a_{j+1}=g^{e}(a_j, w)} \sigma_{ee} \lambda_{j}^{e}\left(a_j, w\right) + \sum_{a: a_{j+1}=g^{u}(a_j, b)} \sigma_{eu} \lambda_{j}^{u}\left(a_j, b\right) \quad (12)$$

$$\lambda_{j+1}^{u}\left(a_{j+1}, b\right) = \sum_{w} \sum_{a: a_{j+1}=g^{e}(a_j, w)} \sigma_{ue} \lambda_{j}^{e}\left(a_j, w\right) + \sum_{a: a_{j+1}=g^{u}(a_j, b)} \sigma_{uu} \lambda_{j}^{u}\left(a_j, b\right) \quad (13)$$

where $\lambda_{j+1}^{e}(a_{j+1}, w_{j+1})$ represents the measure of households who are employed with wage w_{j+1} and hold a_{j+1}, and $\lambda_{j+1}^{u}(a_{j+1}, b)$ represents those who are unemployed and still hold a_{j+1}.

Finally, we need to use the following equations to calculate the model implied job search intensity. The average search intensity for age j is

$$\bar{s}_j = \sum_{a} \lambda_{j}^{u}\left(a_j, b\right) s_j\left(a_j, b\right), \quad (14)$$

and the unemployment rate for age-j cohort is

$$u_j = \sum_{a} \lambda_{j}^{u}\left(a_j, b\right). \quad (15)$$

4.2 Calibration

We need to calibrate the following objects to match data: offer arrival rate or matching function $\alpha(s)$, search cost $v(s)$, wage distribution $F(w)$, unemployment benefit or replacement ratio b, job separation rate δ.

First, let us discuss about the separation rate. In Shimer (2005), he tries to use a textbook MP model to generate the observed business-cycle-frequency fluctuations in unemployment and job vacancies in response to shocks of average labor productivity or separation rate of a plausible magnitude. In section I-D, Shimer documents the monthly job separation rate of the US, which has an average of 0.034 from 1951 to 2003. As shown in the calibration exercise in section II-D of the paper, it suggests a quarterly separation rate of 0.10, so jobs last for about 2.5 years on average. Hagedorn and Manovskii (2008) and Hornstein, Krusell, and Violante (2005) discuss alternative calibration targets regarding job arrival rate, job separation rate, and leisure value.

Second is about the job arrival rate. Based on Burdett, Shi, and Wright (2001) and following Lagos and Rocheteau (2005), we consider the following functional form

$$\alpha(s) = 1 - e^{-\eta s}, \tag{16}$$

where η is a parameter. If we assume the measures of workers and firms are σ_w and σ_f, respectively. Define average search intensity to be

$$\bar{s} = \frac{\int s_i di}{\sigma_b}.$$

Then, according to Burdett, Shi, and Wright (2001), the number of matches generated in such an environment is

$$m(\sigma_s, \bar{s}\sigma_b) = \sigma_s \left[1 - \left(1 - \frac{1}{\sigma_s}\right)^{\bar{s}\sigma_b}\right].$$

Then, the job arrival rate for an individual worker is

$$\alpha(s) = s\frac{m(\sigma_s, \sigma_b)}{\bar{s}\sigma_b}.$$

We assume $\sigma_s = \sigma_b = 1$ and in equilibrium $s = \bar{s}$. The above equation can be simplified to

$$\begin{aligned} \alpha(s) &= s\frac{1 - e^{-\frac{\bar{s}\sigma_b}{\sigma_s}}}{\frac{\bar{s}\sigma_b}{\sigma_s}} \\ &= \frac{s(1 - e^{-s})}{s} \\ &= 1 - e^{-s}. \end{aligned}$$

To make it more general, we add a parameter η and now the job arrival rate function is (16).

An alternative specification is Cobb–Douglas matching function

$$m(\sigma_s, \bar{s}\sigma_b) = \eta (\bar{s}\sigma_b)^a \sigma_s^{1-a},$$

which implies the job arrival rate for a worker is

$$\begin{aligned}\alpha(s) &= s\frac{\eta (\bar{s}\sigma_b)^a \sigma_s^{1-a}}{\bar{s}\sigma_b} \\ &= s\eta\bar{s}^{a-1}\left(\frac{\sigma_s}{\sigma_b}\right)^{1-a} \\ &= \eta s^a.\end{aligned}$$

Note that job offer arrival rate cannot be observed in the data. Instead, we can have job-finding rate, which is defined as the probability of landing on a job for an unemployed worker. In a standard MP model, each worker who meets with a firm gets employed in the end, which is not the case in our model. Here, the job-finding rate is $\alpha(s)[1-F(w^R)]$, i.e. an unemployed receives a wage offer higher than his reservation wage. We need to match this equation to the estimated average monthly job finding rate 0.45 from 1951 to 2003, as in Shimer (2005).

We follow Nakajima (2012) to calibrate the level of unemployment benefits b as being fraction $\rho_b = 0.435$ of the average labor income in the steady state. The value is the mean replacement ratio across states, computed by Gruber (1998).

Then, about the function of search cost, we have several options. The first one is the common linear form

$$v(s) = As.$$

Option two is the common convex cost function

$$v(s) = As^{\xi},$$

where $\xi > 1$. Finally, we may also follow Lagos and Rocheteau (2005) and use

$$v(s) = As^{\xi}(1-s)^{-\xi}.$$

We try different functional forms and verify that the results are robust to the specification of search cost function.

Finally, we need to estimate the wage distribution. Based on different literatures that I have read, there are generally three different ways to do this. First, we can build up a general equilibrium model with firms and workers. Given certain wage determining mechanism, wage distribution will emerge as an equilibrium object. Then, we just do either calibration or structural estimation with the wage distribution. Second, we use wage data from ATUS to non-parametrically estimate the wage distribution, as in Postel-Vinay and Robin (2002), Joliveta, Postel-Vinay, and Robin (2006). Third, we make parametric assumptions on the form of wage distribution and then calibrate or estimate the parameters, as in Flinn and Heckman (1982), Eckstein and Wolpin (1995), Flabbi and Leonardi (2010). This approach as well as structural empirical work on labor search models has been surveyed in Eckstein and van den Berg (2007). Basically, we need the wage sampling distribution to satisfy a recoverability condition, as proved in Flinn and Heckman (1982). Different functional forms that have been proved to satisfy this condition include normal distribution, exponential distribution, and log normal distribution. Notice that these functional forms are for the wage sampling distribution, i.e. the distribution of posted wages, not actual wage distribution.

In this paper, we consider two options. We can assume that $F(w)$ is a log normal probability distribution denoted by two parameters, (μ, ϵ) and characterized by the following probability density function

$$f\left(w;\mu,\epsilon\right)=\frac{1}{\epsilon w\sqrt{2\pi}}\exp\left(-\frac{\left(\ln w-\mu\right)^{2}}{2\epsilon^{2}}\right),$$

which is also supported by Figure 3 in Joliveta, Postel-Vinay, and Robin (2006). Alternatively, we can assume that $F(w)$ follows an exponential distribution

$$f\left(w;\mu,\epsilon\right)=\begin{cases}\frac{1}{\epsilon}\exp\left(-(w-\mu)/\epsilon\right), & w\geq\mu;\\ 0, & w<\mu.\end{cases}$$

5 Conclusion

to be added.

References

[1] Aguiar, Mark, Erik Hurst, and Loukas Karabarbounis (2013) "The Life-Cycle Profile of Time Spent on Job Search," *American Economic Review: Papers and Proceedings* 103(3), 111-16.

[2] Gruber, Jonathan (1998) "Unemployment Insurance, Consumption Smoothing, and Private Insurance: Evidence from the PSID and CEX," *Research in Employment Policy* 1, 3-32.

[3] Menzio, Guido, Irina Telyukova, and Ludo Visschers (2012) "Directed Search over the Life Cycle," *NBER Working Paper* No. 17746

[4] Mukoyama, Toshihiko, Christina Patterson, and Aysegul Sahin (2013) "Job Search Behavior over the Business Cycle," *working paper.*

[5] Pissarides, Christopher A., *Equilibrium Unemployment Theory*, 2nd ed., 2000, The MIT Press.

[6] Sandra L. Hofferth, Sarah M. Flood, and Matthew Sobek. 2013. *American Time Use Survey Data Extract System: Version 2.4* [Machine-readable database]. Maryland Population Research Center, University of Maryland, College Park, Maryland, and Minnesota Population Center, University of Minnesota, Minneapolis, Minnesota. http://www.atusdata.org**??**