Cost-Push Shocks, Trend Inflation, and the Great Moderation: An Alternative Interpretation*

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Abstract

This paper re-assesses the factors behind the Great Moderation and it makes two contributions, one methodological and one applied. First, we develop and analyze the dynamics of a model with trend inflation, real wage rigidity and cost-push shocks. Then, we estimate this artificial economy using a sequential Monte Carlo algorithm which builds a particle approximation to the posterior distribution iteratively through likelihood tempering while allowing for indeterminacy. Unlike the literature's preponderant conception, this analysis does not find evidence for indeterminacy for the 1970s, thus, it makes the case against structural changes having led to the Great Moderation.

1 Introduction

There are two polar views about the causes of the transition from the excessive instability of the 1970s U.S. macroeconomy to the relative calm times that followed after 1982. On the one hand, there is the notion that changes of the way monetary policy is conducted have occasioned the Great Moderation. From this viewpoint, the 1970s were a period in which the Federal Reserve only passively responded to inflation. Such weak response to inflation leads to aggregate instability (i.e. indeterminacy), a property found, for example, by Lubik and Schorfheide (2004) in an estimated sticky price model. For the post-1982 period, however, monetary policy has become more active and indeterminacy can no longer be established. Thus, the argument goes, it is this about-face of the economy's dynamics that has generated a less volatile environment which only ended with the onset of the Great Recession.[1] On the other hand, according to the bad luck theory, smaller shocks have hit the economy during

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[1] Blanchard and Galí (2009) put forward a further structural factor that reduced volatility: the increased flexibility in the workings of labor markets.

the Great Moderation than the large, adverse disturbances of the 1960s and 1970s.[2] The large oil price shocks of the earlier period – in response to production cuts by the OPEC nations – are the most obvious examples.

The current paper re-visits this discussion but it incorporates several important aspects that have so far either been largely left untouched or only been discussed in isolation. While doing this, we make two contributions, one methodological and one applied. We first develop and analyze a model with trend inflation, real wage rigidity and cost-push shocks. We then estimate this artificial economy, which encompasses elements of both articulated explanations of the Great Moderation, to explore its macroeconomic implications. Unlike the literature's preponderant conception, we do not find evidence for indeterminacy for the 1970s. Thus, we provide evidence that tosses the bad-policy theory aside.

Trend inflation adds an important element of the 1970s to the model economy as the period was plagued by high levels of inflation and by certainly higher levels than the zero inflation steady state that is considered in standard New Keynesian models including the one chosen by Lubik and Schorfheide (2004). This addition is of prime relevance in light of Ascari and Ropele (2009) who demonstrate that trend inflation induces significant changes to the dynamical properties of the New Keynesian model. Coibion and Gorodnichenko (2011) take a lead from Ascari and Ropele and they show that the shift from indeterminacy to determinacy also arises in a trend inflation model. However, Coibion and Gorodnichenko's study does not include an estimation of their full model. Such joint estimation of the model economy as well as the monetary policy parameters is conducted by Hirose, Kurozumi and Van Zandweghe (2017) who, again, re-affirm Lubik and Schorfheide's (2004) storyline.[3]

We expand on these works in two important and for trend inflation models novel directions. First, we introduce cost-push shocks into the model economy. Second, we augment the labor market in the form of real wage rigidity. While doing this, we adopt the two key factors for the moderation asserted by Blanchard and Galí (2009). Cost-push shocks enter in the form of oil that acts as inputs for both production and consumption. Wage rigidity is formulated as in Blanchard and Galí (2007), that is, in a parsimonious nature such to include various possible pathways for a slow wage adjustment to changing labor market conditions. The artificial economy is estimated using a sequential Monte Carlo algorithm which builds a particle approximation to the posterior distribution iteratively through likelihood tempering while allowing for indeterminacy. The macroeconomic observable variables include core and headline inflation as well as wage data to put discipline on the identification of the inefficient technology shocks as well as the magnitude of wage rigidity. Thus our estimation is well equipped to evaluate the two interpretations of the Great Moderation. Most importantly, our results question the habituated belief that indeterminacy prevailed during the 1970s. Once the effects of oil price shocks and rigid wages are taken into account, the data favours a version of the artificial economy that is characterized by

[2] Discussions of structual changes that led to the Great Moderation include Clarida, Galí and Gertler (2000), Lubik and Schorfheide (2004), Galí and Gambetti (2009), Blanchard and Galí (2009), Coibion and Gorodnichenko (2011). The bad luck theory has been put forward by amongst others Arias, Hansen and Ohanian (2007), Sims and Zha (2006), Benati and Surico (2009), Primiceri (2005) and Canova and Gambetti (2010).

[3] To our knowledge, Arias, Ascari, Branzoli and Castelnuovo (2017) is the only other paper with trend inflation and cost-push shocks. However, they do not estimate the policy rule and they do not use data to sharpen the identification of cost-push shock.

determinacy.

2 Model economy

The model is a New Keynesian economy with trend inflation and a commodity product which we interpret as oil, thus, this model's micro-founded setup naturally features various rates of inflation. The economy consists of monopolistically competitive wholesale firms who produce differentiated goods using labor and oil. These goods are bought by perfectly competitive retail firms that weld them together into the final good. People consume this final good and oil and they also rent out their labor services on competitive markets which are afflicted by sluggish wage adjustments. Firms and people are price takers on the market for oil. The economy boils down to a variant of the model in Blanchard and Galí (2009) when approximated around a zero inflation steady state. Our primary contribution in this section is to allow for a positive trend inflation in concert with costs-push shocks and wage rigidity.

2.1 People

People are represented by an agent whose preferences depend on consumption C_t and hours worked H_t. The preferences are summarized by the expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t d_t u(C_t, N_t) \qquad 0 < \beta < 1$$

which the agent aims to maximize. Here, E_t denotes the expectations operator and the term d_t stands for a shock to the discount factor β. This shock evolves as a first-order autoregressive process

$$\ln d_t = \rho_d \ln d_{t-1} + \epsilon_{d,t}$$

in which $\epsilon_{d,t}$ is a zero-mean, serially uncorrelated innovation that is normally distributed with standard deviation σ_d. Period utility is additively separable in consumption and hours worked

$$u(C_t, N_t) = \ln C_t - \nu_t \frac{N_t^{1+\varphi}}{1+\varphi} \qquad\qquad \upsilon > 0,\ \varphi \geq 0.$$

This functional form of utility ensures that the economy is consistent with balanced growth. The parameter φ measures the inverse Frisch elasticity of substitution for labor supply and ν_t denotes the stochastic disutility of working. This preference shock follows the autoregressive process

$$\ln \nu_t = \rho_\nu \ln \nu_{t-1} + \epsilon_{\nu,t}$$

in which $\epsilon_{\nu,t}$ is $\mathsf{N}(0, \sigma_\nu^2)$. Consumption is formed by collecting the domestically produced goods and the imported oil $C_{m,t}$ as in the Cobb-Douglas aggregator

$$C_t = \chi^{-\chi}(1-\chi)^{-(1-\chi)} C_{m,t}^{\chi} C_{q,t}^{1-\chi} \qquad\qquad 0 < \chi < 1$$

where χ equals the share of energy in total consumption. $C_{q,t}$ is a consumption index of the domestic output given by

$$C_{q,t} = \left(\int_0^1 C_{q,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

in which ε is the elasticity of substitution between different goods. The agent sells her labor services to the wholesale firms at the nominal wage W_t and has access to a market for one-period riskless bonds B_t at the interest rate R_t. Any generated profits Π_t flow back to her and, thus, the period budget is constrained by

$$W_t N_t + R_t B_{t-1} + \Pi_t \geq P_{q,t} C_{q,t} + P_{m,t} C_{m,t} + B_t.$$

$P_{q,t}$ stands for the domestic output price index. Denote by $P_{c,t}$ the price of the overall consumption basket, then we obtain the following expression that links consumption across any two periods

$$\frac{d_t}{P_{c,t} C_t} = \beta E_t \frac{R_t d_{t+1}}{P_{c,t+1} C_{t+1}}.$$

Intratemporal optimality condition is described by

$$\frac{W_t}{P_{c,t}} = (\nu_t N_t^{\varphi} C_t)^{1-\gamma} \left(\frac{W_{t-1}}{P_{c,t-1}} \right)^{\gamma}$$

in which γ quantifies the degree of real wage stickiness. This *ad hoc* installation of slow-moving wage adjustments follows Hall (2005a, b) and Blanchard and Galí (2007). Blanchard and Galí (2007), for example, promote such parsimonious formulation on the grounds that it entails many micro-founded makeups without the need to confine itself to a particular one. Lastly, in the optimal allocation

$$P_{q,t} C_{q,t} = (1-\chi) P_{c,t} C_t$$

and

$$P_{m,t} C_{m,t} = \chi P_{c,t} C_t$$

hold. Here $P_{c,t} \equiv P_{m,t}^{\chi} P_{q,t}^{1-\chi}$ and $P_{m,t}$ stands for the nominal price of oil. Furthermore, $P_{c,t} = P_{q,t} s_t^{\chi} \equiv P_{q,t}^{1-\chi} P_{m,t}^{\chi}$ with s_t the real price of oil which follows the process

$$\ln s_t = \rho_s \ln s_{t-1} + \epsilon_{s,t}.$$

2.2 Firms

Firms come in two forms. Final goods firms produce output that can be consumed. This output is made up from the range of differentiated goods that are supplied by intermediate goods firms. These firms have market power.

2.2.1 Final good firm

The representative final good firm produces a homogenous good Q_t by choosing a combination of intermediate inputs $Q_t(i)$ to maximize profit. Specifically, the problem of the final good firm involves solving

$$\max_{Q_t(i)} P_{q,t} Q_t - \int_0^1 P_{q,t}(i) Q_t(i) di$$

subject to the CES-technology

$$Q_t = \left[\int_0^1 Q_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Here, $P_{q,t}(i)$ is the price of the intermediate good i and $\varepsilon > 0$ is the price elasticity of demand for each intermediate good. Then the final good firm's demand for intermediate good i is given by

$$Q_t(i) = Q_t \left(\frac{P_{q,t}(i)}{P_{q,t}} \right)^{-\varepsilon}.$$

Substituting this demand for retail good i into the CES-bundler function gives

$$P_{q,t} = \left[\int_0^1 P_{q,t}(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}.$$

2.2.2 Intermediate good firm

Intermediate good i is produced using labor $N_t(i)$ and oil $M_t(i)$ according to the technology

$$Q_t(i) = [A_t N_t(i)]^{1-\alpha} M_t(i)^{\alpha} \qquad 0 < \alpha < 1.$$

Both input factors are traded on perfectly competitive markets. The share of oil in production is α and A_t stands for labor augmenting technological progress whose growth rate $g_t \equiv \frac{A_t}{A_{t-1}}$ follows the exogenous process

$$\ln g_t = \ln g + \epsilon_{g,t}.$$

g is the steady state gross rate of technological change and $\epsilon_{g,t}$ is $N(0, \sigma_g^2)$. Each firm's marginal cost is given by

$$\psi_t(i) = \frac{W_t}{(1-\alpha) Q_t(i)/N_t(i)} = \frac{P_{m,t}}{\alpha Q_t(i)/M_t(i)}$$

while the markup $\mathcal{M}_t^P(i)$ equals

$$\mathcal{M}_t^P(i) = \frac{P_{q,t}(i)}{\psi_t(i)}.$$

Cost minimization implies that the firms' demand for oil is given by:

$$M_t(i) = \frac{\alpha}{\mathcal{M}_t^P(i)} \frac{Q_t(i)}{s_t} \frac{P_{q,t}(i)}{P_{q,t}}.$$

Let $Q_t \equiv (\int_0^1 Q_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di)^{\frac{\varepsilon}{\varepsilon-1}}$ denote aggregate gross output and define $\Delta_t \equiv \int_0^1 (\frac{P_{q,t}(i)}{P_{q,t}})^{-\varepsilon} di$ as the relative price dispersion measure, then it follows that

$$M_t = \frac{\alpha}{\mathcal{M}_t^P} \frac{Q_t}{s_t} \Delta_t^{\frac{\varepsilon-1}{\varepsilon}}$$

where we have used the demand schedule faced by intermediate good firm i and defined $\mathcal{M}_t^P$ as the average gross markup weighted by firms' input shares. Next combining the cost minimization conditions for oil and for labor with the aggregate production function yields the factor-price frontier:

$$\left(\frac{W_t}{P_{c,t}} \right)^{1-\alpha} \mathcal{M}_t^P = \mathcal{C} A_t^{1-\alpha} s_t^{-\alpha-\chi(1-\alpha)} \Delta_t^{-\frac{1}{\varepsilon}}.$$

Price Setting The intermediate goods producers face a constant probability $0 < \theta < 1$ of being able to adjust prices to a new optimal price $P^*_{q,t}(i)$ in order to maximize expected discounted profits

$$E_t \sum_{j=0}^{\infty} \theta^j \beta^j \frac{\lambda_{t+j}}{\lambda_0} \left[\begin{array}{c} \frac{P^*_{q,t}(i)\left(\overline{\pi}^{\omega j}\right)^{1-\mu}\left(\pi_{q,t-1|t+j-1}\right)^{\mu}}{P_{q,t+j}} Q_{t+j}(i) - \\ \frac{W_{t+j}}{(1-\alpha)P_{q,t+j}A^{1-\alpha}_{t+j}} \left\{\frac{(1-\alpha)P_{m,t+j}}{\alpha W_{t+j}}\right\}^{\alpha} Q_{t+j}(i) \end{array} \right].$$

This choice is constrained by

$$Q_{t+j}(i) = \left[\frac{P^*_{q,t}(i) \left(\overline{\pi}^{\omega j}\right)^{1-\mu} \left(\pi_{q,t-1|t+j-1}\right)^{\mu}}{P_{q,t+j}} \right]^{\varepsilon} Q_{t+j}$$

and

$$\begin{aligned} \pi_{q,t|t+j} &= \frac{P_{q,t+1}}{P_{q,t}} \times \frac{P_{q,t+2}}{P_{q,t+1}} \times ... \times \frac{P_{q,t+j}}{P_{q,t+j-1}} \qquad \text{for } j \geqslant 1 \\ &= 1 \qquad \text{for } j = 0 \end{aligned}$$

where $\overline{\pi}$ denotes the central bank's inflation target which is equal to the level of trend inflation. $\pi_{q,t}$ denotes domestic output price inflation and $\Lambda_{t|t+j} = \beta^j \frac{\lambda_{t+j}}{\lambda_0}$ is the stochastic discount factor. This formulation is general as $\omega \in [0, 1]$ allows for any degree of price indexation and $\mu \in [0, 1]$ allows for any degree of geometric combination of the two types of indexation usually employed in the literature: to steady state inflation or to past inflation rates. The first-order condition for the optimal relative price $p^*_{q,t}(i)(= \frac{P^*_{q,t}(i)}{P_{q,t}})$ is given by

$$p^*_{q,t}(i) = \frac{\varepsilon}{(\varepsilon-1)(1-\alpha)} \frac{E_t \sum_{j=0}^{\infty} (\theta\beta)^j \lambda_{t+j} \frac{W_{t+j}}{P_{q,t+j}A^{1-\alpha}_{t+j}} \left[\frac{(1-\alpha)P_{m,t+j}}{\alpha W_{t+j}}\right]^{\alpha} \left[\frac{\left(\overline{\pi}^{\omega j}\right)^{1-\mu}\left(\pi_{q,t-1|t+j-1}\right)^{\mu}}{\pi_{q,t|t+j}}\right]^{-\varepsilon} Q_{t+j}}{E_t \sum_{j=0}^{\infty} (\theta\beta)^j \lambda_{t+j} \left[\frac{\left(\overline{\pi}^{\omega j}\right)^{1-\mu}\left(\pi_{q,t-1|t+j-1}\right)^{\mu}}{\pi_{q,t|t+j}}\right]^{1-\varepsilon} Q_{t+j}}.$$

The joint dynamics of the optimal reset price and inflation can be compactly rewriting recursively as

$$p^*_{q,t}(i) = \frac{\varepsilon}{(\varepsilon-1)(1-\alpha)} \frac{\kappa_t}{\phi_t}$$

with κ_t and ϕ_t two auxiliary variables that allow to rewrite the infinite sums that appear in the numerator and denominator of the above equation in recursive form

$$\kappa_t = \mathcal{C} \left(\frac{W_t}{P_{c,t}}\right)^{1-\alpha} s_t^{\chi(1-\alpha)+\alpha} A_t^{\alpha-1} Q_t \widetilde{\Xi}_t + \theta\beta\overline{\pi}^{-\varepsilon(1-\mu)\omega} \pi_{q,t}^{-\mu\omega\varepsilon} E_t \left\{\pi^{\varepsilon}_{q,t+1} \kappa_{t+1}\right\}$$

and

$$\phi_t = Q_t \widetilde{\Xi}_t + \theta\beta\overline{\pi}^{(1-\mu)(1-\varepsilon)\omega} \pi_{q,t}^{\omega\mu(1-\varepsilon)} E_t\{\pi^{\varepsilon-1}_{q,t+1}\phi_{t+1}\}$$

where $\mathcal{C}$ is a constant and where we have used the definition $\Lambda_{t|t}\lambda_0 = \frac{d_t}{C_t} = \Xi_t P_{c,t} = \widetilde{\Xi}_t$. Note that κ_t and ϕ_t can be interpreted as the present discounted value of marginal costs and marginal revenues respectively. Moreover, the aggregate price level evolves according to:

$$
\begin{aligned}
P_{q,t} &= \left[\int_0^1 P_{q,t}(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \Rightarrow \\
1 &= \theta \overline{\pi}^{(1-\varepsilon)(1-\mu)\omega} \pi_{q,t-1}^{(1-\varepsilon)\mu\omega} \pi_{q,t}^{\varepsilon-1} + (1-\theta) p_{q,t}^{*}(i)^{1-\varepsilon} \\
p_{q,t}^{*}(i) &= \left[\frac{1-\theta\overline{\pi}^{(1-\varepsilon)(1-\mu)\omega} \pi_{q,t-1}^{(1-\varepsilon)\mu\omega} \pi_{q,t}^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}}.
\end{aligned}
$$

This leaves us with deriving a number of equilibrium relationships before setting up the central bank's policy. Gross output is characterized by

$$Q_t \Delta_t = M_t^{\alpha} (A_t N_t)^{1-\alpha}.$$

The condition that trade be balanced gives us a relation between consumption and gross output

$$P_{c,t} C_t = \left(1 - \frac{\alpha}{\mathcal{M}_t^P} \Delta_t^{\frac{\varepsilon-1}{\varepsilon}}\right) P_{q,t} Q_t.$$

The GDP deflator $P_{y,t}$ is implicitly given via

$$P_{q,t} \equiv (P_{y,t})^{1-\alpha} (P_{m,t})^{\alpha}$$

and value added (or GDP) is

$$P_{y,t} Y_t = \left(1 - \frac{\alpha}{\mathcal{M}_t^P} \Delta_t^{\frac{\varepsilon-1}{\varepsilon}}\right) P_{q,t} Q_t.$$

Lastly, define price dispersion as $\Delta_t \equiv \int_0^1 (\frac{P_{q,t}(i)}{P_{q,t}})^{-\varepsilon} di$, then under the Calvo price mechanism, the above expression can be rewritten recursively as

$$\Delta_t = (1-\theta) p_{q,t}^{*}(i)^{-\varepsilon} + \theta \overline{\pi}^{-\varepsilon\omega(1-\mu)} \pi_{q,t-1}^{-\varepsilon\omega\mu} \pi_{q,t}^{\varepsilon} \Delta_{t-1}$$

2.3 Monetary policy

The central bank conducts monetary policy using the nominal interest rate as its instrument. It implements the following rule

$$\log R_t = \rho_R \log R_{t-1} + (1-\rho_R) \left[\begin{array}{c} \log R + \psi_\pi (\log \pi_{c,t} - \log \overline{\pi}) + \psi_x \log x_t^f + \\ \psi_g \left(\log \frac{Y_t}{Y_{t-1}} - \log g\right) \end{array} \right] + \epsilon_{R,t}$$

for which $0 \leq \rho_R < 1$, x_t the output gap, $\epsilon_{R,t}$ an i.i.d. monetary policy shock, and $R \geq 1$ the steady state gross policy rate. The parameters ψ_π, ψ_x and ψ_g govern the central bank's responses to inflation, the output gap and output growth respectively.

2.4 Indeterminacy zones

Figures 1 and 2 show the determinacy regions for combinations of ψ_π with the other policy parameters as well as with the degree of real wage rigidity gamma and the Calvo probability theta. Figure 1 portrays these regions in the zero inflation steady state case where the model is a variant of Blanchard and Gali (2010) and Blanchard and Riggi (2013). As can be seen from Figure 1, the Taylor Principle continues to hold in this micro-founded model with a role for oil – a result that has been documented in Doko-Tchatoka, Groshenny, Haque and Weder (2017).

The case for positive trend inflation is portrayed in Figure 2 where steady state inflation is set to 4 percent. In line with the results of Ascari and Ropele (2009) and Ascari and Sbordone (2014), higher trend inflation tends to destabilize inflation expectations and monetary policy should respond more to deviations of inflation from target and less to output gap deviations. Furthermore, as stressed by Coibion and Gorodnichenko (2011), inertial Taylor-type rules and rules that respond to output growth, rather than to output gap, lower the minimum level of inflation response needed to determinacy. In fact, determinacy appears to be guaranteed when the central bank responds to inflation by more than one-for-one and either policy inertia or response to output growth (or both) is above a certain threshold.

Finally, real wage rigidity and the degree of price stickiness also affect the boundary. The higher the real wage rigidity the lower the minimum response to inflation that is required to generate determinacy. Hence, real wage rigidity tend to dampen the effect of trend inflation on the dynamics. On the other hand, higher degree of price stickiness (above a certain threshold) widens the indeterminacy region.

3 Solution under indeterminacy

This section describes the model solution, the data as well as the estimation strategy. We solve the model following Sims (2002) and Lubik and Schorfheide (2003). Let us denote by η_t the vector of one-step ahead expectational errors, define ϱ_t as the vector of endogenous variables and call ε_t as vector of fundamental shocks. Then, the linear rational expectation system can be compactly written as

$$\Gamma_0(\theta)\varrho_t = \Gamma_1(\theta)\varrho_{t-1} + \Psi(\theta)\varepsilon_t + \Pi(\theta)\eta_t$$

where $\Gamma_0(\theta)$, $\Gamma_1(\theta)$, $\Psi(\theta)$, and $\Pi(\theta)$ are appropriately defined coefficient matrices. Sims' (2002) solution algorithm has the advantage of being general and explicit in dealing with expectation errors since it makes the solution suitable for solving and estimating models which feature multiple equilibria. In particular, under indeterminacy η_t will be a linear function of the fundamental shocks and the purely extrinsic sunspot disturbances, ζ_t. Hence, the full set of solutions to the linear rational expectations model entails

$$\varrho_t = \Phi(\theta)\varrho_{t-1} + \Phi_\varepsilon(\theta, \widetilde{M})\varepsilon_t + \Phi_\zeta(\theta)\zeta_t$$

where $\Phi(\theta)$, $\Phi_\varepsilon(\theta, \widetilde{M})$ and $\Phi_\zeta(\theta)$[4] are the coefficient matrices.[5] The sunspot shock satisfies $\zeta_t \sim i.i.d.\mathsf{N}(0, \sigma_\zeta^2)$. Accordingly, indeterminacy can manifest itself in one of two ways: (i) pure extrinsic non-fundamental disturbances can affect model dynamics through endogenous expectation errors and (ii) the propagation of fundamental shocks cannot be uniquely pinned down and the multiplicity of equilibria affecting this propagation mechanism is captured by the arbitrary matrix $\widetilde{M}$.

Following Lubik and Schorfheide (2004) we replace $\widetilde{M}$ with $M^*(\theta) + M$ and in the subsequent empirical analysis set the prior mean for M equal to zero. The particular

[4] Lubik and Schorfheide (2003) express this term as $\Phi_\zeta(\theta, M_\zeta)$, where M_ζ is an arbitrary matrix. For identification purpose, they impose the normalization such that $M_\zeta = I$.

[5] Under determinacy, the solution boils down to $\varrho_t = \Phi^D(\theta)\varrho_{t-1} + \Phi_\varepsilon^D(\theta)\varepsilon_t$.

solution employed in their paper selects $M^*(\theta)$ by using a least squares criterion to minimize the behavior of the model under determinacy and indeterminacy by assuming that it remains unchanged across the boundary. "Behavior" needs be described in some meaningful way and we follow them by choosing $M^*(\theta)$ such that the response of the endogenous variables to fundamental shocks, $\partial \varrho_t / \partial \varepsilon_t'$, are continuous at the boundary between the determinacy and the indeterminacy region. Analytical solution for the boundary in this model is unavailable and hence, following Hirose (2014), we resort to a numerical procedure for the model to find the boundary by perturbing the parameter ψ_π in the monetary policy rule.

4 Econometric Strategy

This section sets up the estimation procedure, lists the data and discusses the calibration.

4.1 Bayesian estimation with sequential Monte Carlo algorithm

Bayesian techniques estimate the parameters of the model and test for indeterminacy using posterior model probabilities. In our estimation, we employ the Sequential Monte Carlo (SMC) algorithm proposed by Herbst and Schorfheide (2014, 2015) which is particularly suitable for irregular and non-elliptical posterior distributions. Another practical advantage of using an importance sampling algorithm like SMC is that the process does not require to find the mode of the posterior distribution, a task that can prove to be difficult particularly under indeterminacy. First, the priors are described by a density function of the form

$$p(\theta_S|S).$$

Here $S \in \{D, I\}$ where D and I stand for determinacy and indeterminacy respectively, θ_S represents the parameter of the model S, and $p(.)$ stands for probability density function. Next, the likelihood function

$$\mathcal{L}(\theta_S|X_T, S) \equiv p(X_T|\theta_S, S)$$

describes the density of the observed data where X_T are the observations until period T. By using Bayes' Theorem we can combine the prior density and the likelihood function to obtain the posterior density

$$p(\theta_S, X_T, S) = \frac{p(X_T|\theta_S, S)p(\theta_S|S)}{p(X_T, S)}$$

in which $p(X_T|S)$ denotes the marginal marginal density of the data conditional on the model which is given by

$$p(X_T|S) = \int_{\theta_S} p(X_T|\theta_S, S)p(\theta_S|S)d\theta_S.$$

We employ the SMC algorithm of Herbst and Schorfheide (2014, 2015) to build a particle approximation of the posterior distribution through tempering the likelihood. A sequence of tempered posteriors is defined as

$$\pi_n(\theta_S) = \frac{[p(X_T|\theta_S, S)]^{\phi_n} p(\theta_S|S)}{\int_{\theta_S} [p(X_T|\theta_S, S)]^{\phi_n} p(\theta_S|S)d\theta_S},$$

where ϕ_n is the tempering schedule that slowly increases from zero to one. The algorithm generates weighted draws from the sequence of posteriors $\{\pi_n(\theta)\}_{n=1}^{N_\phi}$, where N_ϕ is the number of stages. At any stage, the posterior distribution is represented by a swarm of particles $\left\{\theta_n^i, W_n^i\right\}_{i=1}^{N}$ where W_n^i is the weight associated with θ_n^i and where N denotes the number of particles. The algorithm has three main steps. First, in the *correction* step, the particles are re-weighted to reflect the density in iteration n. Next, in the *selection* step, any particle degeneracy is eliminated by re-sampling the particles. Finally, in the *mutation* step, the particles are propagated forward using a Markov transition kernel to adapt to the current bridge density. Note that in the first stage, i.e. when $n = 1$, ϕ_1 is close to zero. Hence, the prior density serves as an efficient proposal density for $\pi_1(\theta)$. That is, the algorithm is initialized by drawing the initial particles from the prior. Likewise, the idea is that the density of $\pi_n(\theta)$ may be a good proposal density for $\pi_{n+1}(\theta)$. The tempering schedule is a sequence that slowly increases from zero to one and is determined by $\phi_n = \left(\frac{n-1}{N_\phi-1}\right)^\tau$ where τ controls the shape of the schedule. In our estimation, the tuning parameters N, N_ϕ and τ are fixed *ex ante*. We use $N = 10,000$ and $N_\phi = 200$ stages. Also, τ, which is the parameter that controls the tempering schedule, is set at 2.1 following Herbst and Schorfheide (2015). Re-sampling is necessary to avoid particle degeneracy. A rule-of-thumb measure of this degeneracy, proposed by Herbst and Schorfheide (2014, 2015), is given by the reciprocal of the uncented variance of the particles and is called the effective sample size ESS which is defined as:

$$\widehat{ESS_n} = \frac{N}{\frac{1}{N}\sum_{i=1}^{N}\left(\widetilde{W_i^n}\right)^2},$$

where $\widetilde{W_i^n}$ is the normalized particle weight. Following Herbst and Schorfheide (2014, 2015) we use systematic re-sampling whenever $\widehat{ESS_n} < \frac{N}{2}$. Finally, we use one step of a single-block Random Walk Metropolis Hastings (RWMH) algorithm to propagate the particles forward. To assess the quality of the model's fit to the data we use log marginal data densities and the posterior model probabilities for both parametric regions (Table 2). The SMC algorithm-based approximation of the marginal data density is given by

$$p^{SMC}(X_T|S) = \prod_{n=1}^{N_\phi}\left(\frac{1}{N}\sum_{i=1}^{N}\widetilde{w}_n^i W_{n-1}^i\right),$$

where $\widetilde{w}_n^i$ is the incremental weight defined by

$$\widetilde{w}_n^i = [p(X|\theta_{n-1}^i, S)]^{\phi_n-\phi_{n-1}}.$$

4.2 Data

The set of observables x_t contains the quarterly growth rate of real per-capita GDP, the annualized inflation rate of the consumer price index (CPI), the annualized inflation rate of the core consumer price index (Core CPI), the annualized Federal Funds rate and the quarterly growth rate of real wages. Wage data are taken from the BLS (hourly compensation for the NFB sector, all persons). Hourly compensation is deflated by the CPI in order to obtain a consumption real wage variable. The measurement equation is

$$x_t = \begin{bmatrix} \gamma^* \\ \pi^* \\ \pi^* \\ r^* \\ \gamma^* \end{bmatrix} + \begin{bmatrix} \widehat{g}_{y,t} \\ 4\widehat{\pi}_{c,t} \\ 4\widehat{\pi}_{q,t} \\ 4\widehat{R}_t \\ \widehat{g}_{w,t} \end{bmatrix}.$$

The equation's parameters and variables are: the quarterly steady state output growth rate γ^*, the annualized steady state inflation rate π^*, the annualized nominal steady state interest rate r^*, the growth rate of output $\widehat{g}_{y,t}$, the consumer price inflation $\widehat{\pi}_{c,t}$, the core consumer price inflation $\widehat{\pi}_{q,t}$, the growth rate of real wages $\widehat{g}_{w,t}$, deviations from the steady state interest rate $\widehat{R}_t$, and the growth rate of real wages (deflated by the consumer price index) $\widehat{g}_{w,t}$. Hatted variables stand for log deviations from the steady state. To test for indeterminacy and estimate the model parameters, we consider two sample periods in our benchmark analysis: 1966:I to 1981:II and 1982:IV to 2007:III. In the first sample, we wish to include the second oil price shock of 1979 and its after-effects. This allows to include data points up to the 1981-82 recession. In doing so, we rope in some of the data points out of the Volcker disinflation period during which monetary policy might not have followed an interest rate rule. However, our results regarding the posterior probabilities of (in)determinacy remain robust even when we exclude the Volcker disinflation period and stop the estimation in 1979:II.

4.3 Calibrated parameters

We calibrate a subset of the model parameters. We set the discount factor β to 0.99, the steady-state markup at ten percent (i.e. $\varepsilon = 11$), and the inverse of the labor-supply elasticity equal to one. Following the computations in Blanchard and Galí (2010), we calibrate the shares of oil in production and consumption to $\alpha = 0.015$ and $\chi = 0.023$ for the first sample and $\alpha = 0.012$ and $\chi = 0.017$ for the second sample. Furthermore, the shocks to the growth rate of technology are i.i.d., i.e. $\rho_z = 0$. Following Blanchard and Riggi (2013), we fix the autoregressive parameter of the commodity shock at $\rho_s = 0.99$ in order to have the commodity price being very close to random walk. We set the indexation parameters ω and μ equal to zero. We estimate all the remaining parameters with Bayesian techniques.

4.4 Prior Distributions

The specification of the prior distribution is summarized in Table 1. The prior for the inflation coefficient ψ_π follows a Gamma distribution centred at 1.50 with a standard deviation of 0.75 while the response coefficient to output gap and output growth are centred at 0.125 with standard deviation 0.10. We use a loose Beta distribution centered at 0.50 to place an agnostic prior on the smoothing coefficient ρ_R and the wage-rigidity parameter γ, while the Calvo probability θ, and the autoregressive parameter for the discount factor shock and the labor supply shock, ρ_d and ρ_ν, are centred at 0.70 with standard deviation of 0.10. The remaining priors are fairly standard and are reported in Table 1. The choice of the prior leads to a prior probability of determinacy of 0.53, which is quite even and suggests no prior bias toward either determinacy or indeterminacy.

5 Results

This section discusses our main results.

Testing for indeterminacy The model is estimated for the two sample periods using the SMC algorithm under the priors listed in Table 1. To assess the quality of the model's fit to the data over the two regions of the parameters space, we present marginal data densities and posterior model probabilities in Table 2. The posterior probabilities reveal striking differences with existing findings. The posterior concentrates all its mass in the determinacy region for both sample periods. Hence, our estimation results suggest that the U.S. economy was likely in the determinacy region before and after 1982. Thus, the generally stated view that the U.S. economy shifted from indeterminacy to determinacy after the Great Inflation does not hold in our estimated economy.

Parameter estimates Table 3 reports posterior estimates of the structural parameters for the two samples conditional on determinacy and sheds more light on our finding. Foremost, the policy response to inflation ψ_π is active even during the pre-1980 period. This result is consistent with that of Hirose, Kurozumi and Van Zandweghe (2017) who also find an active policy response to inflation during the Great Inflation period. Furthermore, as reported in Table 3, four of the estimated parameters changed substantially between the sample periods. First, the policy response to inflation ψ_π increases from 1.34 to 1.93. Second, the policy response to output growth more than doubles. Third, trend inflation falls by about one percent. These results are qualitatively in line with Coibion and Gorodnichenko (2011) and Hirose, Kurozumi and Van Zandweghe (2017). By contrast, the policy response to the output gap is low in both sample periods with the estimate in the second sample being almost zero.

A key parameter in the original Blanchard and Galí (2010) and Blanchard and Riggi (2013) model is the degree of real wage rigidity γ. To sharpen the identification of this feature, we have also used real wage data in our estimation. The parameter estimates of γ suggest a high degree of wage rigidity in both the Great Inflation and Great Moderation periods. In fact, the estimate is slightly higher in the former sample. This result stands in sharp contrast to Blanchard and Riggi (2013) who find that real wages were perfectly flexible during the Great Moderation period. This divergence might be due to the different estimation strategy we employ. While Blanchard and Riggi (2013) adopt a limited information approach that matches the impulse response to a commodity price shock in the DSGE model and in the structural VAR, we use a full-information Bayesian estimation technique with multiple shocks. A high degree of real wage rigidity is a prominent determinant of our results. As mentioned earlier, real wage rigidity substantially dampens the effect of trend inflation on the indeterminacy region and lowers the minimum response to inflation required for determinacy.[6] As such, the possible effects of high trend inflation are likely to have

[6] See also Ascari, Branzoli and Castelnuovo (2011) who find a similar effect arising from nominal wage indexation.

been dampened by the high degree of real wage rigidity in both the Great Inflation and Great Moderation period. Together with a strong response to inflation gap and output growth and a weak response to output gap, it gives support to our main finding that the U.S. economy was likely in the determinacy region of the parameter space both before and after the 1982.

The decline in the volatility of output growth and inflation Table 4 confirms that both output growth and inflation were significantly less volatile during the second sample period. Our estimated model is able to replicate this volatility reduction, even though it overpredicts the variance of output growth and inflation in both periods. The decline in the volatility of output and inflation is driven by two factors.

First, we observe a marked change in the conduct of monetary policy across the two periods. Our estimates emphasize a shift in the Federal Reserve's behavior away from the discretionary erratic "stop and go" policy that was prevalent in the 1970s towards a more systematic approach during the Great Moderation period. In particular, significant difference between the two sample periods concerns the standard error of the monetary policy shock which has been halved. The improved predictability of monetary policy in the second period is also reflected by the higher estimated degree of interest-rate inertia. Finally, the significant increase in the central bank's response coefficients to the inflation gap and especially to output growth across the two periods demonstrate that the Federal Reserve became much more active in stabilizing business cycle fluctuations. This structural change in the conduct of monetary policy can be seen in Figures 3-7 which report the estimated impulse responses of key macroeconomic variables to every shocks for both periods. For each disturbance, the most striking difference between the two periods shows up in the response of the policy rate.

Furthermore, the standard error of discount factor shocks dropped by 50 percent. This finding suggests that more benign demand shocks in the second sample period were a dominant source of the drop in the volatility of output and inflation. Such a view is compatible with Justiniano and Primiceri (2008) who conclude that the main cause of the Great Moderation was a decline in the standard deviation of investment-specific technology shock, a demand-type disturbance. In our model without capital, this factor is captured by a similar decrease in the size of another kind aggregate-demand type shock, namely discount factor disturbances.

6 Conclusion

This paper re-assesses the factors behind the Great Moderation. We make two contributions, one methodological and one applied. We first develop and analyze a model with trend inflation, oil and wage stickiness. We find that while trend inflation makes the economy more susceptible to indeterminacy, real wage rigidity shrinks this parameter zone. We then estimate this artificial economy using a sequential Monte Carlo algorithm which builds a particle approximation to the posterior distribution iteratively through likelihood tempering while allowing for indeterminacy. Unlike the literature's preponderant conception, we do not find evidence for indeterminacy in the 1970s. This result provides some evidence to toss the bad-policy theory aside as it makes the case against structural changes to have led to the Great Moderation.

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7 Appendix

7.1 Final Equations of the Detrended Non-linear System

The non-linear model is described by the following equations. A tilde stands for dentrended variable and η_t denotes the consumption real wage $\frac{W_t}{P_{c,t}}$.

$$\widetilde{\eta}_t = \widetilde{\eta}_{t-1}^{\gamma}(\nu_t N_t^{\varphi}\widetilde{C}_t)^{1-\gamma} \tag{1m}$$

$$\frac{d_t}{\widetilde{C}_t} = \beta E_t \left[\frac{Rt}{\pi_{c,t+1}}\right] \frac{d_{t+1}}{\widetilde{C}_{t+1}} g_{t+1}^{-1} \tag{2m}$$

$$P_{c,t}\widetilde{C}_t = \left(1 - \frac{\alpha}{\mathcal{M}_t^P}\Delta_t^{\frac{\varepsilon-1}{\varepsilon}}\right) P_{q,t}\widetilde{Q}_t \tag{3m}$$

$$\widetilde{Q}_t\Delta_t = \widetilde{M}_t^{\alpha} N_t^{1-\alpha} \tag{4m}$$

$$\widetilde{M}_t = \frac{\alpha}{\mathcal{M}_t^P}\frac{\widetilde{Q}_t}{s_t}\Delta_t^{\frac{\varepsilon-1}{\varepsilon}} \tag{5m}$$

$$\widetilde{\eta}_t^{1-\alpha}\mathcal{M}_t^P = \mathcal{C} s_t^{-\alpha-\chi(1-\alpha)}\Delta_t^{\frac{-1}{\varepsilon}} \tag{6m}$$

$$p_{q,t}^*(i) = \frac{\varepsilon}{(\varepsilon-1)(1-\alpha)}\frac{\kappa_t}{\phi_t} \tag{7m}$$

$$\kappa_t = \mathcal{C}\widetilde{\eta}_t^{1-\alpha} s_t^{\chi(1-\alpha)+\alpha}\widetilde{Q}_t d_t \widetilde{C}_t^{-1} + \theta\beta\overline{\pi}^{-\varepsilon\omega(1-\mu)}\pi_{q,t}^{-\mu\omega\varepsilon} E_t\left[\pi_{q,t+1}^{\varepsilon}\kappa_{t+1}\right] \tag{8m}$$

$$\phi_t = \widetilde{Q}_t d_t \widetilde{C}_t^{-1} + \theta\beta\overline{\pi}^{(1-\mu)(1-\varepsilon)\omega}\pi_{q,t}^{\mu\omega(1-\varepsilon)} E_t\left[\pi_{q,t+1}^{\varepsilon-1}\phi_{t+1}\right] \tag{9m}$$

$$1 = \theta\overline{\pi}^{(1-\mu)(1-\varepsilon)\omega}\pi_{q,t-1}^{\mu\omega(1-\varepsilon)}\pi_{q,t}^{(\varepsilon-1)} + (1-\theta)p_{q,t}^*(i)^{1-\varepsilon} \tag{10m}$$

$$\Delta_t = (1-\theta)p_{q,t}^*(i)^{-\varepsilon} + \theta\overline{\pi}^{-\varepsilon\omega(1-\mu)}\pi_{q,t-1}^{-\mu\omega\varepsilon}\pi_{q,t}^{\varepsilon}\Delta_{t-1} \tag{11m}$$

$$P_{y,t}\widetilde{Y}_t = \left(1 - \frac{\alpha}{\mathcal{M}_t^P}\Delta_t^{\frac{\varepsilon-1}{\varepsilon}}\right) P_{q,t}\widetilde{Q}_t \tag{12m}$$

$$P_{c,t} \equiv P_{q,t} s_t^{\chi} \tag{13m}$$

$$\log R_t = \rho_R \log R_{t-1} + (1-\rho_R)\left[\begin{array}{c} \log R + \psi_\pi(\log\pi_{c,t} - \log\overline{\pi}) + \psi_x \log x_t^f + \\ \psi_g\left(\log\frac{Y_t}{Y_{t-1}} - \log g\right)\end{array}\right] + \epsilon_{R,t} \tag{14m}$$

$$\ln s_t = \rho_s \ln s_{t-1} + \epsilon_{s,t} \tag{15m}$$

$$\ln g_t = \rho_g \ln g_{t-1} + \epsilon_{g,t} \tag{16m}$$

$$\ln d_t = \rho_d \ln d_{t-1} + \epsilon_{d,t} \tag{17m}$$

$$\ln \nu_t = \rho_\nu \ln \nu_{t-1} + \epsilon_{\nu,t} \tag{18m}$$

7.2 The Log-linearized Model

Following Ascari and Sbordone (2014), we take a log-linear approximation around the deterministic steady state defined for a generic steady state trend inflation rate:

$$\widehat{\eta}_t = \gamma\widehat{\eta}_{t-1} + (1-\gamma)\left[\varphi\widehat{N}_t + \widehat{C}_t\right] + \widehat{\nu}_t \tag{1L}$$

$$\widehat{C}_t = E_t\widehat{C}_{t+1} - \widehat{R}_t + E_t\widehat{\pi}_{c,t+1} + \widehat{d}_t - E_t\widehat{d}_{t+1} + E_t\widehat{g}_{t+1} \tag{2L}$$

$$\widehat{C}_t = \widehat{Q}_t - \chi\widehat{s}_t + \xi\widehat{\mu}_t - \xi\left(\frac{\varepsilon-1}{\varepsilon}\right)\widehat{\Delta}_t \tag{3L}$$

$$\widehat{Q}_t = \alpha\widehat{M}_t + (1-\alpha)\widehat{N}_t - \widehat{\Delta}_t \tag{4L}$$

$$\widehat{M}_t = \widehat{Q}_t - \widehat{\mu}_t - \widehat{s}_t + \left(\frac{\varepsilon-1}{\varepsilon}\right)\widehat{\Delta}_t \tag{5L}$$

$$(1-\alpha)\widehat{\eta}_t + \widehat{\mu}_t + \{\alpha + \chi(1-\alpha)\}\widehat{s}_t + \frac{1}{\varepsilon}\widehat{\Delta}_t = 0 \tag{6L}$$

$$\widehat{p^*_{q,t}}(i) = \widehat{\kappa}_t - \widehat{\phi}_t \tag{7L}$$

$$\begin{aligned}\widehat{\kappa}_t \quad = \quad & (1-\theta\beta\overline{\pi}^{\varepsilon(1-\omega)})\left[(1-\alpha)\widehat{\eta}_t + \{\chi(1-\alpha)+\alpha\}\widehat{s}_t + \widehat{Q}_t + \widehat{d}_t - \widehat{C}_t\right] + \\ & \theta\beta\overline{\pi}^{\varepsilon(1-\omega)}\left[\widehat{\kappa}_{t+1} + \varepsilon\widehat{\pi}_{q,t+1} - \mu\omega\varepsilon\widehat{\pi}_{q,t}\right]\end{aligned} \tag{8L}$$

$$\widehat{\phi}_t = (1-\theta\beta\overline{\pi}^{(\varepsilon-1)(1-\omega)})\left[\widehat{Q}_t + \widehat{d}_t - \widehat{C}_t\right] + \theta\beta\overline{\pi}^{(\varepsilon-1)(1-\omega)}\left[\widehat{\phi}_{t+1} + (\varepsilon-1)\widehat{\pi}_{q,t+1} + \mu\omega(1-\varepsilon)\widehat{\pi}_{q,t}\right] \tag{9L}$$

$$\widehat{p^*_{q,t}}(i) = \frac{\theta\overline{\pi}^{(\varepsilon-1)(1-\omega)}}{1-\theta\overline{\pi}^{(\varepsilon-1)(1-\omega)}}\left[\widehat{\pi}_{q,t} - \mu\omega\widehat{\pi}_{q,t-1}\right] \tag{10L}$$

$$\widehat{\Delta}_t = \left[-\varepsilon(1-\theta\overline{\pi}^{\varepsilon(1-\omega)})\right]\widehat{p^*_{q,t}}(i) + \theta\overline{\pi}^{\varepsilon(1-\omega)}\left[-\mu\omega\varepsilon\widehat{\pi}_{q,t-1} + \varepsilon\widehat{\pi}_{q,t} + \widehat{\Delta}_{t-1}\right] \tag{11L}$$

$$\widehat{Y}_t = \widehat{Q}_t + \left(\frac{\alpha}{1-\alpha}\right)\widehat{s}_t + \xi\widehat{\mu}_t - \xi\left(\frac{\varepsilon-1}{\varepsilon}\right)\widehat{\Delta}_t \tag{12L}$$

$$\widehat{\pi}_{c,t} = \widehat{\pi}_{q,t} + \chi(\widehat{s}_t - \widehat{s}_{t-1}) \tag{13L}$$

$$\widehat{R}_t = \rho_R \widehat{R}_{t-1} + (1 - \rho_R) \left[\psi_\pi \widehat{\pi}_{c,t} + \psi_x x_t^f + \psi_g (\widehat{Y}_t - \widehat{Y}_{t-1} + \widehat{g}_t) \right] + \varepsilon_{R,t} \tag{14L}$$

$$\widehat{s}_t = \rho_s \widehat{s}_{t-1} + \varepsilon_{s,t} \tag{15L}$$

$$\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \varepsilon_{g,t} \tag{16L}$$

$$\widehat{d}_t = \rho_d \widehat{d}_{t-1} + \varepsilon_{d,t} \tag{17L}$$

$$\widehat{\nu}_t = \rho_\nu \widehat{\nu}_{t-1} + \varepsilon_{\nu,t} \tag{18L}$$

Table 1 - Prior distributions for parameters.

Name	Range	Density	Prior Mean	St. Dev
ψ_π	$\mathbb{R}^+$	Gamma	1.5	0.75
ψ_x	$\mathbb{R}^+$	Gamma	0.125	0.1
ψ_g	$\mathbb{R}^+$	Gamma	0.125	0.1
ρ_R	[0,1)	Beta	0.5	0.2
π^*	$\mathbb{R}^+$	Gamma	4.0	2.0
r^*	$\mathbb{R}^+$	Gamma	4.0	2.0
γ^*	$\mathbb{R}$	Normal	0.5	0.1
θ	[0,1)	Beta	0.7	0.1
γ	[0,1)	Beta	0.5	0.2
ρ_d	[0,1)	Beta	0.7	0.1
ρ_ν	[0,1)	Beta	0.7	0.1
σ_s	$\mathbb{R}^+$	Inv-Gamma	2.0	∞
σ_g	$\mathbb{R}^+$	Inv-Gamma	0.5	∞
σ_r	$\mathbb{R}^+$	Inv-Gamma	0.5	∞
σ_d	$\mathbb{R}$	Inv-Gamma	0.5	∞
σ_ν	$\mathbb{R}^+$	Inv-Gamma	0.5	∞
σ_ζ	$\mathbb{R}^+$	Inv-Gamma	0.5	∞
$M_{s,\zeta}$	$\mathbb{R}$	Normal	0.0	1.0
$M_{s,\zeta}$	$\mathbb{R}$	Normal	0.0	1.0
$M_{r,\zeta}$	$\mathbb{R}$	Normal	0.0	1.0
$M_{d,\zeta}$	$\mathbb{R}$	Normal	0.0	1.0
$M_{\nu,\zeta}$	$\mathbb{R}$	Normal	0.0	1.0

The inverse gamma priors are of the form $p(\sigma|\upsilon,\varsigma) \propto \sigma^{-\upsilon-1}e^{-\upsilon\varsigma^2/2\sigma^2}$ where $\nu = 2$ and $\varsigma = 0.282$ for all shocks but the commodity price shock. For the commodity price shock, we set $\varsigma = 2.55$. The prior predictive probability of determinacy is 0.53.

Table 2: Determinacy versus Indeterminacy

Model	Log-data density		Probability	
	Determinacy	Indeterminacy	Determinacy	Indeterminacy
1966:I-1981:II	-572.35	-592.44	1	0
1982:IV-2007:III	-698.75	-764.28	1	0

Notes: According to the prior distributions, the probability of determinacy is 0.53.

Table 3 - Posterior distribution of parameters

	1966:I-1981:II		1982:IV-2007:III	
Name	Mean	St. Dev	Mean	St. Dev
ψ_π	1.34	0.12	1.93	0.19
ψ_x	0.04	0.02	0.01	0.01
ψ_g	0.78	0.19	1.70	0.18
ρ_R	0.60	0.06	0.80	0.04
π^*	4.47	1.26	3.52	0.54
r^*	4.93	1.45	5.55	0.71
γ^*	0.48	0.06	0.45	0.06
θ	0.74	0.04	0.85	0.02
γ	0.93	0.03	0.97	0.02
ρ_d	0.91	0.02	0.91	0.02
ρ_ν	0.87	0.04	0.81	0.07
σ_s	18.63	1.66	20.50	1.53
σ_g	0.64	0.07	0.78	0.06
σ_r	0.45	0.08	0.22	0.05
σ_d	2.13	0.50	1.11	0.19
σ_ν	0.36	0.08	0.46	0.11

Results based on 10,000 particles from final stage in SMC algorithm. For both sample periods, the posterior distribution is conditional on determinacy.

Table 4: Actual and Model Standard Deviations of GDP Growth and Inflation

	1960:1-1981:2		1982:4-2007:3		Percentage Change	
	Actual	Model	Actual	Model	Actual	Model
Output Growth	0.98	1.35	0.56	0.88	−43%	−35%
Headline Inflation	3.36	4.32	1.56	2.47	−54%	−43%
Core Inflation	3.03	3.93	1.19	1.97	−61%	−50%

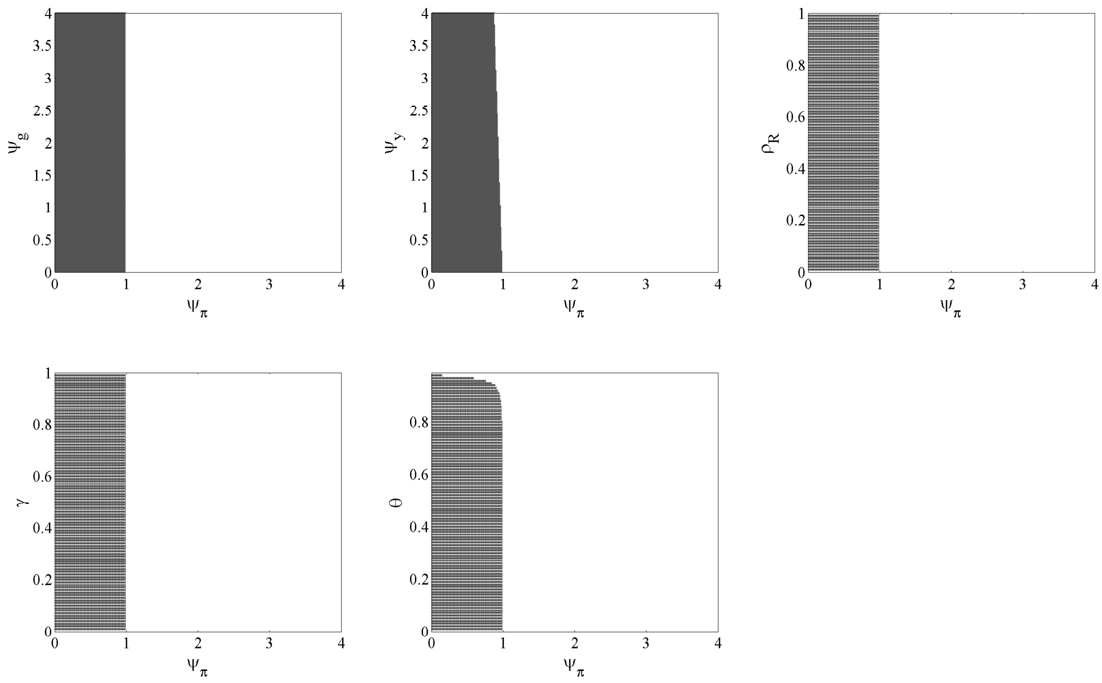

Figure 1: Indeterminacy region when trend inflation is equal to 0 percent.

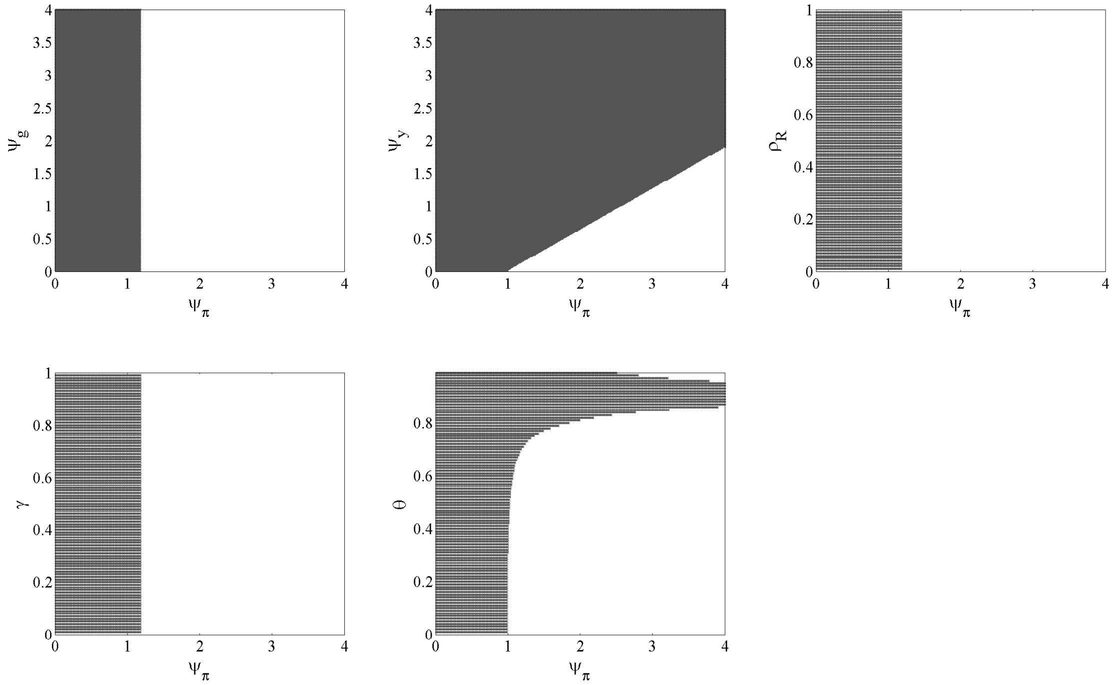

Figure 2: Indeterminacy region when trend inflation is equal to 4 percent.

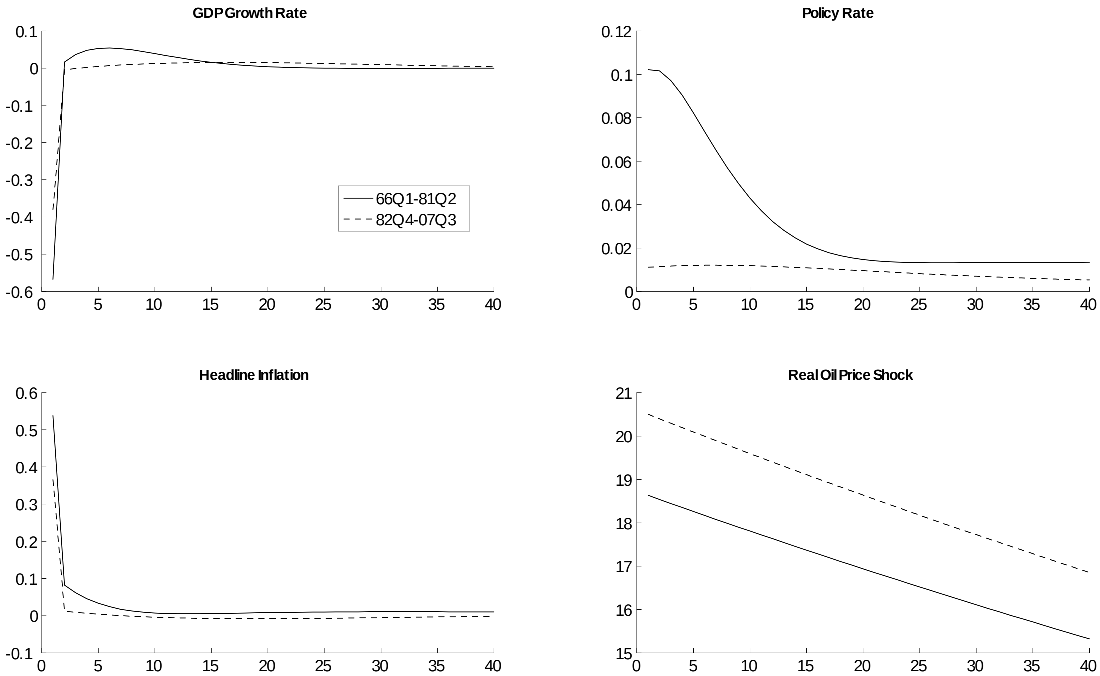

Figure 3: Impulse responses to a one-standard deviation shock to the real price of oil. Responses are computed at the posterior mean. The solid line refers to the model estimated over the sample period 1966:1-1981:2. The dashed line refers to the model estimated over the sample period 1982:4-2007:3.

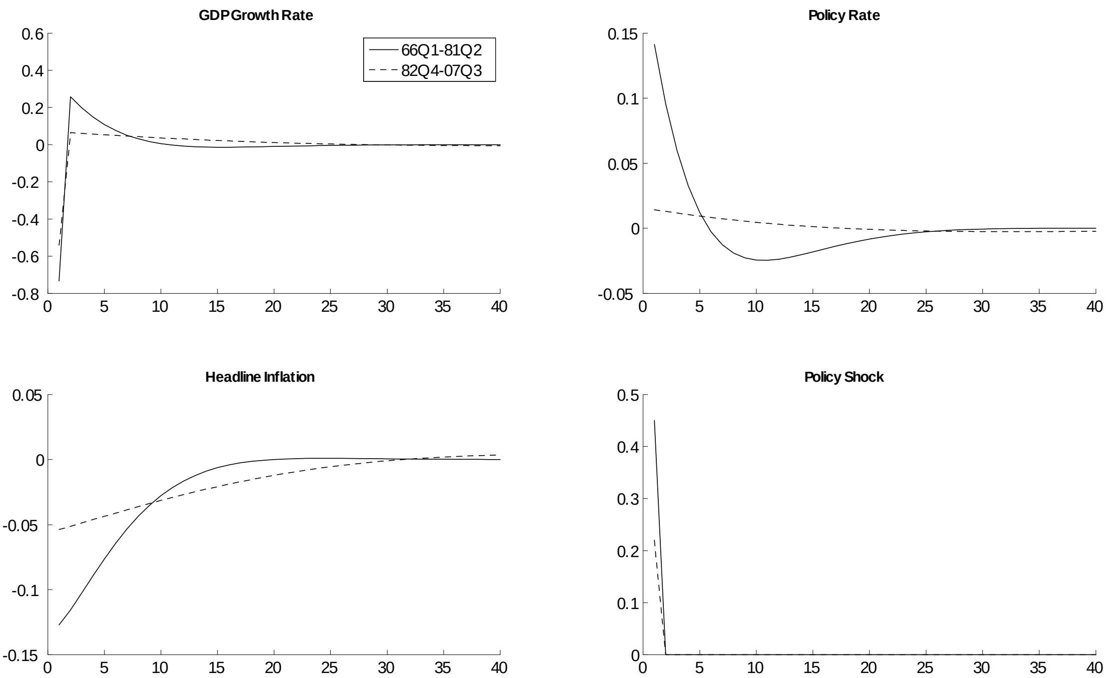

Figure 4: Impulse responses to a one-standard deviation monetary policy shock. Responses are computed at the posterior mean. The solid line refers to the model estimated over the sample period 1966:1-1981:2. The dashed line refers to the model estimated over the sample period 1982:4-2007:3.

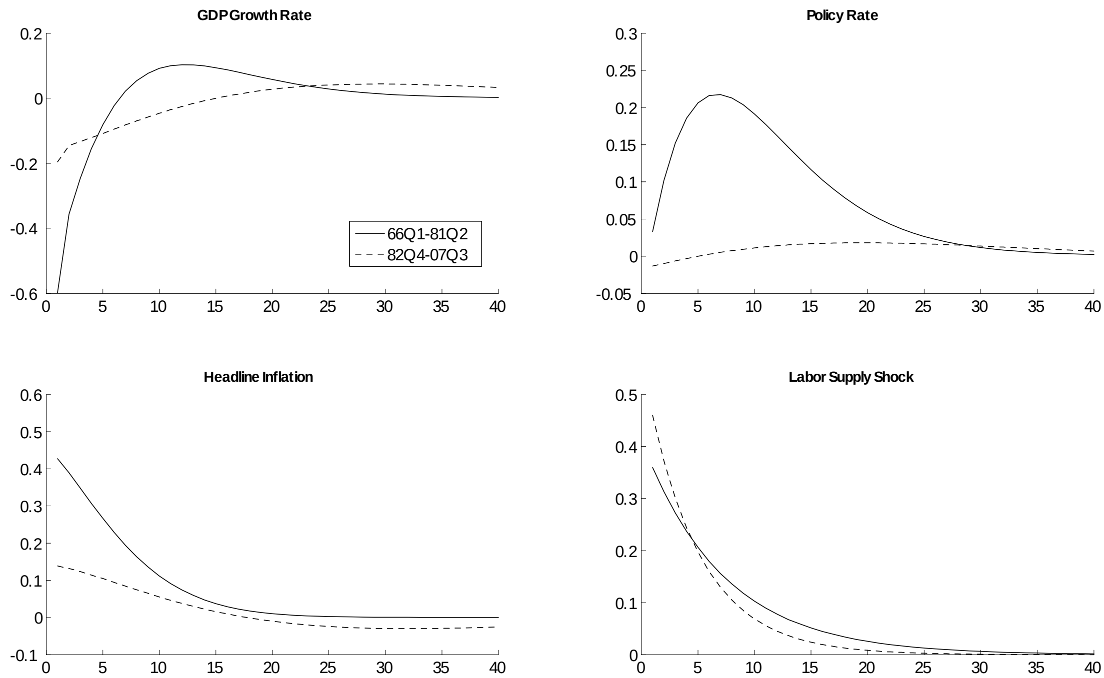

Figure 5: Impulse responses to a one-standard deviation labor supply shock. Responses are computed at the posterior mean. The solid line refers to the model estimated over the sample period 1966:1-1981:2. The dashed line refers to the model estimated over the sample period 1982:4-2007:3.

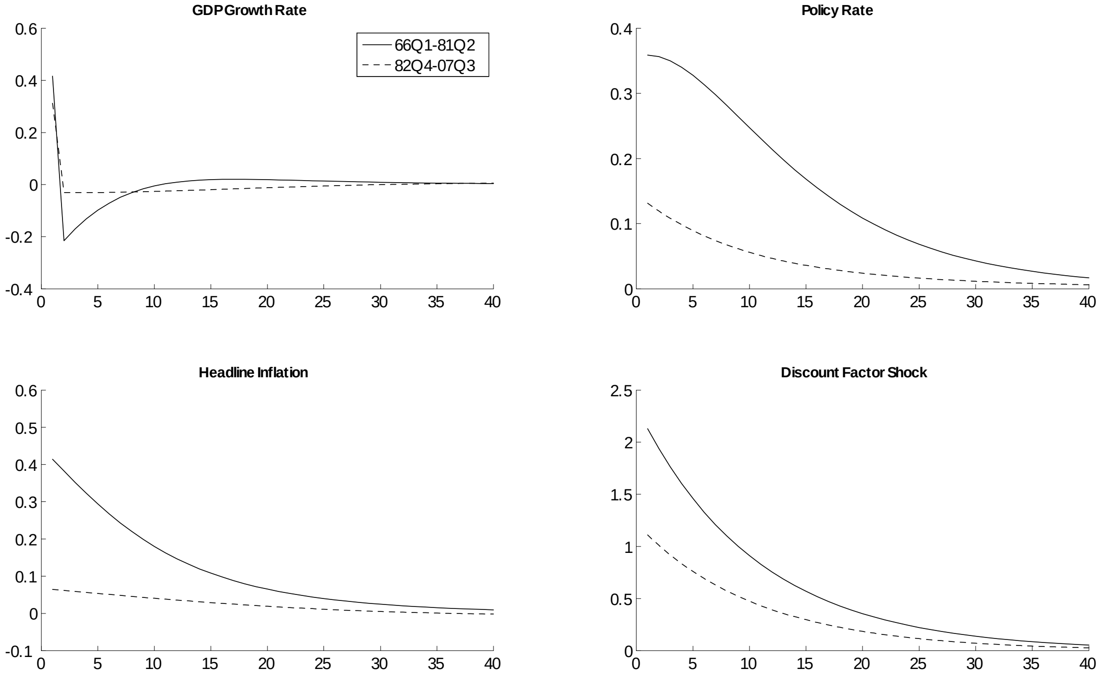

Figure 6: Impulse responses to a one-standard deviation discount factor shock. Responses are computed at the posterior mean. The solid line refers to the model estimated over the sample period 1966:1-1981:2. The dashed line refers to the model estimated over the sample period 1982:4-2007:3.

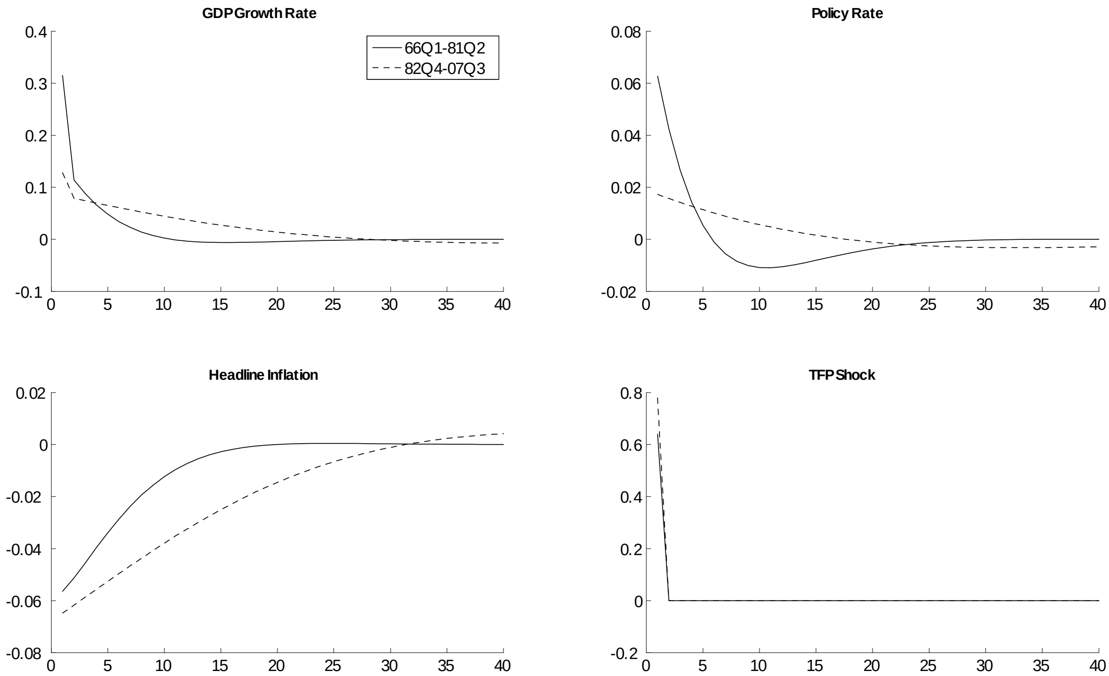

Figure 7: Impulse responses to a one-standard deviation shock to the growth rate of technology. Responses are computed at the posterior mean. The solid line refers to the model estimated over the sample period 1966:1-1981:2. The dashed line refers to the model estimated over the sample period 1982:4-2007:3.