Supervision, Loophole Innovation and Bank Leverage Cycles

Jianxing Wei*  Tong Xu†
UPF and Barcelona GSE  Emory University
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Preliminary

Abstract

This paper develops a model of financial intermediation in which the dynamic interaction of regulator supervision and loophole innovation generates bank leverage cycles. In the model, banks’ leverages are constrained due to risk-shifting problem. The regulator supervises the banks to ease this moral hazard problem, and its expertise in supervision improves gradually through learning by doing. At the same time, banks can engage in loophole innovation to circumvent supervision, which acts as an endogenous opposing force that demolishes the value of regulator’s accumulated expertise. In equilibrium, banks’ leverage and loophole innovation move together with the regulator’s supervision ability. Our model captures that bank leverage are pro-cyclical and asymmetric over the credit cycle. We show that a crisis is more likely to occur and the consequences are more severe after a longer boom. In addition, we investigate the welfare implications of maximum leverage ratio in the environment of loophole innovation.

Keywords: risk shifting, supervision, loophole innovation, regulation, endogenous business cycles.

*Email: jianxing.wei@upf.edu.
†Email: tong.xu@emory.edu.
The more effective regulation is, the greater the incentive to find ways around it. With time and considerable money at stake, those within the regulatory boundary will find ways around any new regulation. The obvious danger is that the resultant dialectic between the regulator and the regulated will lead to increasing complexity, as the regulated find loopholes which the regulators then move (slowly) to close.

– Goodhart and Lastra (2010)

1 Introduction

It is now widely accepted that excessive risk-taking by banks contributed to the financial crisis of 2007-2009. While the causes of excessive risk-taking remain subject to debate, many observers and policymakers believe that supervision failure is one of the key contributing factors (Acharya and Richardson, 2009; Acharya et al., 2011). Freixas et al. (2015) also argues that microprudential-oriented regulation prior to the crisis was unable to tame systemic risk in the financial system. Indeed, several countries have made great efforts to improve their supervision of banks in the aftermath of crisis.

Why may the supervision on banks fail? Among various factors, financial innovation is mentioned as one key factor that can undermine the effectiveness of the regulator’s supervision, (see, e.g., Kane (1988); Miller (1986); Silber (1983); Tufano (2003)). Undoubtedly, good financial innovations provide numerous benefits to the economy. However, there also exist bad financial innovations that can create new ways for financial institution to go around current supervision and take excessive risks. For instance, Stein (2013) argues that the second-generation securitization like subprime CDOs is a bad financial innovation that evolved in response to flaws in prevailing models and incentive schemes. Another related example is Credit Default Swap (CDS). CDS was widely used to free up regulatory capital in the banks’ balance sheets prior to the crisis. However, when the risky assets of banks and the insurer are correlated, banks can use CDS to engage in regulatory arbitrage and take excessive risk under the Basel regulatory framework (Yorulmazer 2013). As is illustrated in the recent financial crisis, the regulator didn’t fully and timely understand the danger of bad financial innovation in some instances, facilitating the excessive risk-taking of banks. In this paper, we focus the effect of bad financial innovation on regulator’s supervision and banks’

1 Other mentioned factors include, for instance, shortcomings in financial institutions’ incentive structures and risk management practices, misplaced reliance of credit rating agencies, etc.
2 For instance, financial innovations help improve risk sharing, complete the market, reduce trade costs, see Beck et al. (2016) for an excellent survey of the debate on the ‘bright’ and ‘dark’ sides of financial innovation.
risk-taking, and investigate its macroeconomic implications over the credit cycles. To the best of our knowledge, this paper is the first one to explicitly model the dynamic interaction between the ‘dark’ side of financial innovation and the regulator’s supervision.

We build a dynamic model of financial intermediation with bank risk-taking and financial loophole innovation. The novelty of the model is that financial sector itself is the source for the adverse shocks, which generates real economic fluctuations. In particular, the longer the boom, more likely there is a crisis and the consequences are more severe, which corresponds to Minsky (1986)’s hypothesis that good times sow the seeds of the next financial crisis. Different from the existing literature focusing on amplifying and propagating roles for the financial sector, this paper emphasizes the role of financial sector as the source of economic fluctuations. Moreover, the model’s predictions reconcile well with some empirical facts of the credit cycles. For instance, our model predicts pro-cyclical banks’ leverage over the business cycles, consistent with the findings in Adrian and Shin (2010, 2011). The model also generates asymmetric credit cycles, i.e. long periods of credit booms are followed by sudden and sharp busts while the recovery is slow and gradual, as documented in Reinhart and Reinhart (2010).

In the model, there are a continuum of banks, competitive depositors, and a regulator. Banks borrow from depositors in the form of debt to finance their investment opportunities. The investment opportunities could be safe or risky projects. Following the theoretical banking literature, banks in our model are subject to risk shifting moral hazard due to limited liability. Banks have incentives to take inefficient risky projects, in which they enjoy the upside of payoff if projects succeed but depositors bear loss if projects fail. One solution to the moral hazard problem is market discipline: depositors impose a leverage constraint on the banks. If banks have enough “skin in the game”, they will behave properly. However, market discipline is costly in the sense that it limits the investment size of banks.

Another complementary solution to the moral hazard problem is supervision by the regulator. In this paper, we formally distinguish regulation from supervision from the perspective of verifiability of bank information and actions, following Eisenbach et al. (2016). Through actively monitoring banks’ activities, the regulator can promote safety and soundness of the banks. The leverage constraint and supervision from regulator work together to address banks’ risk shifting problem. As a result, the size of the banks depends on depositors’ belief

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3Regulation is written into law and enforced through courts, so it can only be contingent on verifiable information. In contrast, supervision involves the assessment of the safety and soundness of banks through monitoring by the regulator, and corrective actions in response to the assessment. Supervision can be contingent on non-verifiable information.
about the regulator’s monitoring ability. When depositor’s confidence in regulator’s competence is high, depositors will permit banks to take high leverage without worrying about the risk-shifting problem. If regulator’s ability is perceived to be low, depositors have to tighten the leverage constraint to make sure banks behave properly.

Even though regulator’s supervision helps banks increase their leverage from an ex ante perspective, banks always have incentives to find loopholes to circumvent the regulator supervision ex post. In our paper, we model loophole innovation as discovering a new type of risky projects which are not currently supervised by the regulator, and thus it provides banks with opportunities to take risky projects without being monitored by the regulator. This acts as an endogenous opposite force to demolish the regulator’s expertise in supervision. Banks need to exert costly effort to conduct this loophole innovation in each period. Moreover, as banks’ leverage goes up, they have a stronger incentive to discover new loopholes. This is because the profit from exploiting the loophole is higher when banks are more indebted. Eventually, when the loophole innovation succeeds at some time, the regulator’s supervision becomes less effective. However, the regulator and depositors are not aware of the new loopholes immediately, so banks will take the inefficient risky projects, which leads to massive defaults and severe decline for output. After the bust, depositors realize that regulator’s expertise has become obsolete and they lose confidence in the financial system. In response, they constrain banks from taking high leverage to prevent their risk-taking activities, which implies a sharp contraction in the financial sector’s balance sheet and credit.

We incorporate the regulator’s supervision and bank’s loophole innovation into a dynamic model. We assume that regulator’s expertise to supervise banks on known risky projects gradually improves through a learning by supervising process. This assumption is supported by some recent studies on how prudential supervision works in practice, (see, e.g., Dahlgren (2011), Dudley (2014), Eisenbach et al. (2016)). As the regulator’s expertise grows, it has two effects on the moral hazard problem of banks. On the one hand, by reducing the moral hazard problem in the current period, it allows for banks to take a higher leverage. On the other hand, a higher leverage encourages the banks to engage in loophole innovation.

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4 For instance, according to Eisenbach et al. (2016), “The current structure and organization of FRBNY FISG supervisory staff dates from a significant reorganization that took place in 2011. That reorganization drew on lessons learned during the financial crisis to reshape the internal structure of the group and the way that staff interacts with one another to enhance communication and facilitate identification of emerging risks through a greater emphasis on cross-firm perspectives. The reorganization was designed to foster enhanced and more frequent engagement between senior supervisory staff and senior managers and members of the board of directors at supervised firms”.
more actively. Therefore, as the regulator’s supervision ability increases, banks have a higher leverage and larger investment size, so the total output goes up in normal times. In this way, the economy experiences a boom accompanied with rising leverage in the financial sector. However, at the same time, banks’ efforts for loophole innovation are larger, and banks are more likely to discover a loophole. If the loophole innovation realizes, there is a crisis in the economy.

Our main result for the dynamic model is that, the interaction between the regulator’s supervision and banks’ attempt to circumvent supervision can lead to regime changes in the banks’ moral hazard problems and generates macroeconomic fluctuations. In our economy, the sources for the economic downturn endogenously come from banks’ loophole innovation. Since the loophole innovation probability determines the evolution rules among the states in the economy, we also characterize the properties of the stationary distribution of states in the economy in the long run. Moreover, the business cycles are asymmetric in our economy: periods of gradual expansions in banks’ leverage, investment, and aggregate output are followed by sudden and sharp contractions, and then the economy starts the gradual growth again. This result arises from the asymmetric nature of loophole innovation. Although the regulator takes time to gradually improve its supervision ability through a learning-by-doing process, its expertise can be total obsolete the moment that new loopholes are discovered.

The 2007-2009 financial crisis is a good example to illustrate our mechanism. Before the crisis, banks discovered vulnerabilities in the rules of regulation and supervision, and by exploiting these loopholes, they took excessive risk due to oversight by the regulator. When the massive failures occurred and the crisis unfolded, regulators and investors realized that there had been so many cracks in the financial system. As Timothy F. Geithner (Geithner, 2010) recognized, “Our regulatory framework was built in a different era for a long extinct form of finance. It long ago fell behind the curve of market developments. Parts of the system were crawling with regulators but parts of the system were without any meaningful oversight. This permitted and even encouraged arbitrage and evasion on an appalling scale.” In response to the vulnerabilities in the financial system, investors cut their lending to the banks and there was a sharp deleveraging process for the financial sector.

We also investigate the regulation implications for this model. We consider the regulation with the maximum leverage ratio. The regulator’s supervision ability can be seen as the states for the economy, and loophole innovation probability in each state determines the evolution rules for the supervision states, which characterizes the stationary distribution of the regulator’s supervision skills in the long run. Sometimes the regulator would set a
maximum leverage ratio to restrict the upper-bound leverage for the banks. This regulation has two effects. First, it may reduce the banks’ leverage and can potentially decrease the output in the boom periods. Second, this regulation decreases the innovation probability in those states affected. The decrease in innovation probability shifts the stationary distribution towards high states, which improves the average output in the long run. The regulator will trade off these two effects to set the optimal maximum leverage ratio.

The model’s empirical implications are broadly consistent with the stylized facts found in many empirical studies. First, Schularick and Taylor (2012) study 14 developed countries over 140 years, concluding that long periods of credit growth is the best single predictor of financial crises. Second, Reinhart and Reinhart (2010) find that credit cycles are asymmetric: long period of credit expansion are followed by sudden stops, and then gradual recovery. Third, Adrian and Shin (2010, 2011) find that financial intermediaries’ leverages are procyclical over the business cycles. Fourth, Dell’Ariccia et al. (2014) find that during a boom, financial intermediaries’ lending standards decrease and loan default rates increase, which is accompanied by massive failures in the financial sector. Our model’s results are consistent with these facts in a unified framework.

This paper contributes to the existing literature in several ways. First, different from most regulation and supervision literature which focus on static models, this paper studies the regulator’s supervision and the financial sector’s reaction in a dynamic framework. Second, complementary to a small but growing literature on endogenous business cycles, which focus on the non-financial firms, this paper provides a novel mechanism to generate endogenous credit cycles originated from the financial sector. By analyzing the dynamics of financial intermediary’s moral hazard problem, this paper is able to rationalize some of the key features of the credit cycles that are not explained by the existing literature. Third, this paper provides a new rationale for the maximum leverage ratio when there exists an interaction between regulator’s supervision and financial sector loophole innovation, which complements the existing literature on regulation. We show that tightening intermediaries’ leverage ratio involves a systemic risk and current output trade-off, and the regulator can lower the likelihood of systemic crises at the cost of decreasing output in normal times.

The paper’s structure is as follows. Section 2 discusses the related literature. Section 3 presents the static model about bank risk-shifting, supervision, and loophole innovation. Section 4 nests the static model in a dynamic model, analyzing the macroeconomic implications of the interaction between banks’ loophole innovation and regulator’s supervision evolution, and also studying long-run stationary distribution properties. Sector 5 investigates
the optimal regulation for maximum leverage ratio and its welfare implications. Section 6 adds learning about unknown loophole and the regulator’s investigation choice in the model. Section 7 discusses several setup in the model. Section 8 is the conclusion.

2 Literature

This paper is linked to different strands of the literature on bank’s risk-taking, financial innovation, financial crises, and credit cycles.

Our paper follows the literature on bank’s risk-taking and financial stability, (see, e.g., Keeley (1990), Suarez (1994), Matutes and Vives (1996), Boyd and Nicoló (2005) and Martínez-Miera and Repullo (2017)). Different from most literature assuming exogenous capital structure, in our paper bank’s leverage is endogenously chosen by the bank as a commitment device to reduce moral hazard. In this respect, our paper is mostly related to a recent paper by Dell’Ariccia et al. (2014), in which they show how interest rate affects bank’s risk taking when bank can choose its leverage optimally. However, all of these papers don’t consider the role of regulator supervision in alleviating bank’s moral hazard. Also, our model departs from this literature by focusing on the dynamic macroeconomic implications of bank’s risk taking behaviors on the cycles.

Our work is related to the literature on regulator supervision, (see, e.g., Bhattacharya et al. (2002); Dewatripont et al. (1994); Marshall and Prescott (2006); Prescott (2004); Rochet (2008)). More recently, Eisenbach et al. (2015, 2016) formally distinguish bank supervision and regulation and develop a static framework to to explain the relationship between supervisory efforts and bank characteristics observed in the data. We depart from this literature by focusing on the connection between regulator’s competence and credit cycles. In this respect, our paper is closely related to Morrison and White (2005, 2013). They show that crisis will only occur when public confidence in the regulator’s ability to detect bad banks through screening is low. While regulator’s ability is constant in the static model in Morrison and White (2005, 2013), we study the dynamic interaction of regulator supervision and banks’ loophole innovation and its rich macroeconomic implications. In this regard, we consider our model a first attempt to formalize Kane (1988)’s influential idea of “regulatory dialectic”.

Our interest in endogenous business cycle relates to Suarez and Sussman (1997), Martin (2008), Favara (2012), Myerson (2012), and Gu et al. (2013). Among these papers, our paper is mostly related to Myerson (2012), who shows how boom-bust credit cycles can be sustained in economies with moral hazard in financial intermediation. Different from Myerson
our model focus on the role of the regulator supervision in curbing moral hazard in financial intermediation, and more importantly, our paper generates richer macroeconomic implications consistent with stylized facts found in the empirical literature.

Our work is also linked to the literature on asymmetric business cycles. Some papers, including Veldkamp (2005), Ordoñez (2013), and Kurlat (2015), study the asymmetric nature of the credit cycles from the perspective of the asymmetric information flow over the cycles. A recent paper by Asriyan and Vanasco (2014) studies the role of financial intermediaries learning in generating and amplifying the informational cycles. Our paper also features a regulator whose expertise grows through learning by doing. The key difference is that, in our paper, the shock to the fundamental is endogenously generated by the financial sector itself rather than exogenously. And our paper also stress the role of banks’ leverage over the cycle.

There are an emerging literature studying the close relationship between boom and bust in the business cycles. In Gorton and Ordoñez (2014, 2016), booms are associated with loss of information while crises happen when the economy transits from information-insensitive states to information-sensitive states. Boz and Mendoza (2014) and Biais et al. (2015) emphasize the role of beliefs related to innovation on boom and bust patterns. Good belief builds up in the boom periods, but adverse realization of the fundamental decreases the belief dramatically and leads to the bust. Boissay et al. (2016) build a model featuring an interbank market with moral hazard and adverse selection problems. Increased savings during expansions drive down the return on loans, and when fundamental becomes weak, interbank market freezes due to agency problem, which leads to bank crisis. Different from these papers, we build a model focusing on the interaction between supervision and loophole innovation.

3 Static Model

Consider an economy with a mass-one continuum of risk-neutral banks with limited liability. Banks can use own endowment $\omega$ and absorbed deposit $x$ from risk-neutral households to make investment in either safe or risky projects. The safe project’s payoff is $R/\eta^s$ with probability $\eta^s$ and zero with probability $1 - \eta^s$. The expected payoff for the risky project is $\bar{\lambda}R$, while banks can choose the riskiness of the risky project, i.e. success probability. When the success probability is $\eta \in [\bar{\eta}, \tilde{\eta}]$, the payoff conditional on project success is $\bar{\lambda}R/\eta$ for the risky project. Let $\bar{\eta} < \eta^s$ and $\bar{\lambda}/\bar{\eta} > 1/\eta^s$, which means that the risky project is more likely

5 We assume that there is no deposit insurance
to fail but pay more when they succeed than the safe project. Banks’ project choices are not observed by depositors and are not contractable. At the beginning of each period, banks offer a menu of leverage and deposit rate to households, and households with large endowment choose to deposit their endowment in banks or invest in a storage technology with a fixed return $r_0$. Here, we assume that $R > r_0 > \lambda R$. Thus, safe project has the highest expected return, and risky project has a negative NPV.

There exists a benevolent regulator who can supervise the banks. The regulator can control the riskiness for banks’ risky projects through monitoring. To model the regulator’s supervision, we assume that the regulator can prevent banks from choosing high riskiness when taking risky projects. More specifically, when the regulator’s supervision ability or expertise is $\eta^*$, the risky projects cannot have a success probability lower than $\eta^* \in \left[\eta^*, \bar{\eta}^*\right]$, so banks only can choose the success probability for risky projects within the interval $[\eta^*, \bar{\eta}]$. The setup for supervision is similar to [Eisenbach et al.] (2016), where the regulator can take corrective actions to reduce the variance of final payoff for the projects.

However, the regulator’s supervision is not always perfect. Sometimes banks may discover a new type of risky project, which is off the radar for current supervision. We call this discovery a loophole innovation. With loophole innovation, banks are able to take the new risky projects with any riskiness levels. Borrowing the setup from technology innovation literature, such as [Aghion and Howitt] (2009) and [Laeven et al.] (2015), we assume that in each period only one bank has a capable idea for a loophole, and it can discover a loophole or produce a successful innovation with some probability, which depends on its effort. We call this bank the capable bank. The bank knows it is capable after all the banks have already absorbed the deposits. When the capable bank’s effort level is $e \in [0, 1]$, innovation occurs with probability $e$. The utility cost for the capable bank is $\frac{1}{2} ce^2 \cdot (\omega + x)$, where $c$ is the coefficient governing the convex cost of innovation effort, and the cost is linear in the scale of the bank. If innovation succeeds in this period, all banks learn about this loophole, and new risky projects immune from supervision are available to them. Thus, the regulator’s supervision is ineffective when loophole innovation occurs.

The timing for the static model is as follows: at the beginning of each period, banks offer menus of leverage and deposit rate to households. Households decide whether or not to make deposits in the banks. After banks receive deposits, one of the banks knows it is the capable one and exerts loophole innovation effort. If the innovation realizes, a new type of risky project emerges, and all other banks learn about it. Otherwise, only safe and old

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6 We can generalize this assumption for $N$ banks, as long as $N$ is finite. Otherwise, innovations occur every period.
risky project are available. Banks make project choices and choose riskiness levels under the regulator’s supervision if they invest in risky projects. At the end of each period, projects pay off. Banks pay back the depositors if their projects succeed. Otherwise, they default and go bankruptcy.

At the beginning of the period, banks offer a menu of total deposits and interest rate to households, and households decide whether they deposit their endowment into the banks. Denote the leverage \( L \equiv \frac{\omega + x}{\omega} \), and the menu of total deposit and interest rate is equivalent to the menu of leverage and interest rate, \( \{L, r\} \). Since the problem is linear with banks’ endowment, banks’ endowment does not affect banks’ choices, and we will omit it in the following part. Due to banks’ limited liability, banks would like to choose the highest riskiness if they invest in risky projects. Banks choose the menu to maximize their expected profits, and the problem is

\[
\max_{L, r} (1 - p) \max \{RL - \eta^s r(L-1), \lambda RL - \eta^s r(L-1)\} + p \max \{RL - \eta^\ast r(L-1), \lambda RL - \eta^\ast r(L-1)\}
\]

where \( p \) is the probability that loophole innovation occurs. Banks always choose between the safe projects and the risky ones. When innovation does not occur, banks are monitored by the regulator, and the highest riskiness available to them are risky projects with success probability \( \eta^s \). When innovation occurs, banks can evade the regulator’s supervision so they can take the largest risk for the new risky projects, which has a success probability \( \eta^\ast \).

Here banks can only have finite leverage due to the risk-shifting problem. The following assumption is the sufficient condition that the existence of risk-shifting problem always constrains the banks’ leverage.

**Assumption 1.** \( \frac{(1 - \lambda)R}{(\eta^s - \eta^\ast)r_0} < 1 \).

This assumption implies two things. First, the risky projects are sufficiently attractive so that banks with too high leverage will choose the risky projects. Second, the maximum supervision ability is not high enough to eliminate the risk-shifting problem.

From now on, this paper focuses on the case where the innovation probability \( p \) is small, and later we will discuss the parameter space that guarantees a small innovation probability.

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7In this paper, we endogenize bank’s capital structure. This treatment is supported by two observations under existing bank regulations. First, bank’s true leverage may be higher than the regulatory limit because banks can overstate capital by not recognizing losses. Second, banks can save on capital by engaging in regulatory arbitrage of capital requirements.

8Since there are a continuum of banks and only one of them is capable, each bank expects itself to be the capable one with a probability of measure zero. Thus, all the banks do not consider the disutility from innovation effort when they decide the menus of leverage and interest rate.
We think this case is relevant in reality. In the model, loophole innovation is associated with crises, and in the real economy crises are rare events. Thus, small innovation probability is reasonable.

To absorb deposits from depositors, banks have to offer interest rate which at least makes depositors break even between deposit and storage technology. We define 

\[ L_0 \equiv \frac{1}{(1 - (1 - \bar{\lambda}) R \eta^s / ((\eta^s - \eta) r_0))} \] 

and 

\[ L^* \equiv 1 / (1 - (1 - \bar{\lambda}) R ((1 - p) \eta^s + p\eta) / ((\eta^s - \eta^s) r_0)). \]

\( L_0 \) is the highest leverage under which banks will always choose the safe projects no matter whether or not there is innovation. \( L^* \) is the highest leverage under which banks will choose the safe project when there is no innovation and choose the risky project when innovation occurs. It is easy to see that for small innovation probability \( p \), \( L^* \) is larger than \( L_0 \). The interest rate to make depositors indifferent between deposit in the bank and storage technology is

\[
r = \begin{cases} 
\frac{r_0}{\eta^s}, & \text{if } L \leq L_0 \\
\frac{r_0}{(1-p)\eta^s + p\eta}, & \text{if } L_0 < L \leq L^* \\
\frac{r_0}{\eta^s}, & \text{if } L > L^* 
\end{cases}
\]

For each bank, the depositors treat the probability of loophole innovation as given, since the probability for a certain bank to be the capable one is nearly zero. If a bank’s leverage is lower than or equal to \( L_0 \), safe project is more attractive to the bank, and it will never invest in the risky project with or without innovation. The interest rate for depositors to break even is \( r_0 / \eta^s \). If bank’s leverage is higher than \( L_0 \) but lower than or equal to \( L^* \), the bank will take the safe project without innovation and invest in the risky project with innovation as long as the interest rate is lower than \( R / \eta^s \). With probability \( 1 - p \), there is no innovation, and banks choose the safe projects. With probability \( p \), innovation occurs, and banks choose the risky projects with highest riskiness, which will succeed with probability \( \bar{\eta} \). In this case, the interest rate that makes the depositors break even ex ante is \( r_0 / ((1 - p) \eta^s + p\eta) \). If a bank’s leverage is higher than \( L^* \), it will always take the risky project, so the interest rate need to be as high as \( r_0 / \eta^* \). Since the risky project has a negative NPV, banks will never choose leverage higher than \( L^* \) since \( r_0 > \bar{\lambda} R \). Because the profit is a linear function of leverage, banks will choose either \( L_0 \) or \( L^* \). In the case that \( p \) is small, the profit with menu \( \{L^*, r_0 / ((1 - p) \eta^s + p\eta)\} \) is higher than that with menu \( \{L_0, r_0\} \), so all banks will choose the leverage level \( L^* \). Next we will solve the model in this case.

When there is no innovation, we can write the incentive compatible constraint for banks to take safe projects as

\[
RL - \eta^s r(L - 1) \geq \bar{\lambda} RL - \eta^* r(L - 1)
\]
The left hand is the expected profit for banks to take the safe projects. The right hand is the profit for banks to take the risky projects without innovation, where the regulator limits the lowest success probability for risky projects to $\eta^*$. If the incentive compatible constraint is not satisfied, the bank will always choose the risky project with or without innovation. Since the risky project has a negative NPV, banks will never offer any menu not satisfying equation (3).

After all the banks absorb deposits, one bank knows that it is the capable one, and it can exert efforts to make the loophole innovation trial. Given the leverage level and deposit rate, the innovation effort problem for the capable bank is

$$\max_e (1 - e)[RL - \eta^*r(L - 1)] + e[\lambda R - \eta r(L - 1)] - \frac{1}{2}ce^2L$$

(4)

If there is no innovation, the capable bank will choose the safe project. If innovation occurs, it will choose the new risky project.

The first-order condition can be written as

$$-[R - \eta^*r(1 - 1/L)] + [\lambda R - \eta r(1 - 1/L)] = ce$$

(5)

We can see that given leverage and interest rate, higher innovation cost coefficient decreases innovation effort. Given interest rate, higher leverage leads to higher innovation effort, because more debt gives the capable bank more incentive to innovate so it can take risk with highest riskiness. Given leverage, higher interest rate results in higher innovation, since loophole innovation provides an opportunity to escape paying interest rate.

In equilibrium, the ex ante innovation probability equals to the innovation effort of the capable bank, i.e. $p = e$, and the break-even condition for interest rate is

$$r_0 = [(1 - e)\eta^* + e\eta]r$$

(6)

Above we consider the case where all banks’ leverages are higher than $L_0$. It may be possible that some banks would choose a menu of $\{L_0, r_0\}$. However, when innovation cost is high, the innovation probability $p$ is low. For small $p$, banks’ size is larger with higher leverage, and the interest rate is not too high as long as incentive compatible constraint (3) is satisfied. Thus, the optimal leverage makes the incentive compatible constraint binding, and the profit is higher than that with leverage $L_0$ due to the large scale.

**Lemma 1.** Under Assumption 1 and large innovation cost coefficient $c$, the incentive constraint equation (3) is always binding for each bank.
Here we need large innovation cost coefficient for two reasons. First, large innovation cost coefficient decreases innovation probability, which makes the interest rate in the equilibrium lower than the safe project payoff. Second, large innovation cost coefficient makes the interest rate low enough if the incentive compatible constraint is satisfied, so all banks would prefer a larger scale even though the interest rate is a little bit higher.

**Definition 1.** An equilibrium in the static model consists of the loophole innovation probability and decision rules \( \{L(\eta), r(\eta), e(\eta)\} \) such that (i) the deposit contract \( \{L(\eta), r(\eta)\} \) solves the banks’ problem (1) given (2); (ii) \( e(\eta) \) solves the capable bank’s problem (4); (iii) the loophole innovation probability is consistent with the capable bank’s innovation effort.

We can solve the bank’s problem in an explicit form. From equations (3), (6), and (5), we can solve the innovation effort, deposit rate, and leverage in equilibrium given the supervision ability \( \eta^* \),

\[
e = \frac{\eta^* - \eta}{\eta^* - \eta^*} \frac{(1 - \bar{\lambda})R}{c} \quad (7)
\]

\[
r = \frac{r_0}{(1 - e)\eta^* + e\eta} \quad (8)
\]

\[
L = \frac{1}{1 - \frac{(1 - \bar{\lambda})R}{(\eta^* - \eta^*)r}} \quad (9)
\]

From the above equations, we can get the following proposition.

**Proposition 1.** Under Assumption \([1]\) and large innovation cost coefficient \( c \), as regulator’s supervision ability increases,

(I) innovation effort (probability) is increasing;

(II) deposit rate is increasing;

(III) banks’ leverage is increasing.

The output depends on whether there is loophole innovation or not. If there is no innovation, all banks take the safe projects, and the output is \( (R - r_0)\omega L \). If there is innovation, all banks take the risky projects, and the output is \( (\bar{\lambda}R - r_0)\omega L \). Thus, the expected output at the beginning of this period is \( \omega \cdot [(1 - e)(R - r_0) + e(\bar{\lambda}R - r_0)]L \). We plot these results in the figure \([1]\).

Next we study the comparative statics. We focus on how the cost coefficient for loophole innovation \( c \), the payoff for the safe projects \( R \), and the relative payoff for risky projects \( \lambda \) will affect innovation probability \( e \), deposit rate \( r \), and leverage \( L \).
Lemma 2. Under Assumption 1 and large innovation cost coefficient $c$,

(I) $e$ is decreasing in $c$, increasing in $R$, and decreasing in $\bar{\lambda}$;

(II) $r$ is decreasing in $c$, increasing in $R$, and decreasing in $\bar{\lambda}$;

(III) $L$ is increasing in $c$, increasing in $R$, and decreasing in $\bar{\lambda}$;

It is easy to see that loophole innovation is less likely to occur with a larger cost coefficient. This decreases the deposit rate, and lower deposit rate increases leverage through the incentive compatible constraint. The effects of the expected payoff for the safe project, $R$, are more complicated. Larger $R$ makes safe projects more attractive. On the one hand, this directly dampens the incentive of loophole innovation. On the other hand, this increases the leverage for banks, which indirectly gives the capable bank stronger incentive to innovate. The latter effect dominates the former one, so innovation effort increases. Following similar logic, larger relative payoff for the risky project, $\bar{\lambda}$, increases the attractiveness for the risky projects, but the low leverage associated with it decreases the capable bank’s incentive to innovate. Overall, the innovation probability decreases with a larger $\bar{\lambda}$.

4 Dynamic Model

4.1 Setup

In this section, we extend the static model into a dynamic model. All banks live only one period. Each bank endows with $\omega$ goods at the beginning of each period and offers menu of leverage and deposit rate to household to absorb deposit. In each period, only one bank has a capable idea, and it can produce a successful loophole innovation with probability depending on its effort. The loophole innovation is spreading in two dimensions. First, if innovation succeeds, all banks in the current period learn about it and are able to exploit this loophole. Second, all the banks in the following periods after the innovation will learn about it. Households choose between deposit in the banks and storage technology.

One key element in the dynamic model is the evolution of the regulator’s supervision ability. We assume that after each loophole innovation, the regulator can investigate this innovation and improve its monitoring skill on this specific loophole each period through (passive) learning by doing. The regulator’s supervision ability increases in normal times.

\footnote{Since there are infinite number of banks in the economy, the public can infer the occurrence of loophole innovation from the share of bank failures at the end of the period.}
However, each innovation discovers a new type of risky project that is off the radar for old monitoring skills, and regulator has to accumulate its supervision expertise from the beginning for these new risky projects. After each innovation, the regulator recognizes the existence of new risky projects and starts to monitor whether banks take advantage of the new loophole. As time goes, the regulator learns more and more about this new loophole, so the regulator can reduce the riskiness of the specific type of risky projects gradually. In the period after a loophole innovation, the regulator can limit the riskiness of risky projects related to this loophole to the success probability $\eta_k^*$. $\eta_k^*$ increases with $k$, which is a reduced form for the learning by doing process for the regulator’s supervision ability. It is easy to see that the regulator’s supervision ability about a specific type of risky project is increasing each period. We assume that there is an upper-bound for the regulator’s supervision ability so the risk-shifting problems always exist for the banks.

If an innovation occurs in the current period, the capable bank discovers a new type of risky project. All banks can take the new risky projects, which the regulator cannot monitor. From next period, the regulator starts to supervise banks on these new risky projects and limit their riskiness. The regulator’s supervision ability is lowest for the risky projects discovered in the latest loophole innovation, so these risky projects are the most attractive to banks due to the limited liability for banks. It is easy to see that if banks do not take the risky projects discovered in the latest innovation, they would not take risky projects associated with earlier innovations. Thus, the regulator’s supervision ability related to the latest innovation is sufficient to describe the state for the economy.

The regulator’s supervision ability space is $\{\eta_1^*, \eta_2^*, \ldots, \eta_k^*, \ldots \eta_K^*\}$, where $k$ denotes the periods from the last known innovation. This space is the regulator’s supervision ability if there will not be any loophole innovation, so it reflects the perceived supervision ability or confidence in the supervision at the beginning of the period rather than the real supervision ability, which depends on whether the innovation will succeed or not later in the current period. The perceived supervision ability or supervision ability for the last known risky projects (which we will call the supervision ability hereafter for simplicity) is a state variable for the economy, and it determines the deposit contracts between banks and depositors. If innovation occurs in the current period, supervision is ineffective for the new risky projects. From the next period on, the regulator’s supervision ability becomes $\eta_1^*$ and start to evolve along the supervision ability space gradually. After $K$ periods, it will stay constant unless loophole innovation occurs. Thus, the evolution law for the regulator’s supervision ability, i.e. the regulator’s supervision ability $\eta_k$ in the period $t$ since the last innovation occurred in
the period \( \hat{t} \) is

\[
\eta_t = \begin{cases} 
\eta^*_t, & \text{if } t - \hat{t} = s < K \\
\eta^K_t, & \text{if } t - \hat{t} = s \geq K
\end{cases}
\] (10)

The timing for the dynamic model is as follows: at the beginning of each period, the regulator’s supervision ability is known to the public. Banks offer menus of leverage and deposit rate to household. Household decide whether or not to make deposits in the banks. After banks receive deposits, one of the banks knows it is the capable bank, and it makes loophole innovation effort. If the innovation realizes, all other banks can learn about it. Banks make project choices and choose riskiness levels under the regulator’s supervision if they invest in risky projects. At the end of each period, projects pay off. Banks pay back the depositors if their projects succeed. Otherwise, they default. In the next period, the regulator improves its supervision ability on known risky projects. If innovation realizes in this period, all banks living later learn about it, and the regulator starts to accumulate supervision ability on this loophole from next period on.

### 4.2 Dynamics

Within each period, the problem is the same to the static model. As shown in the static model, in normal times without loophole innovation, all banks choose safe projects. However, if loophole innovation realizes, all banks will choose risky projects. Given the regulator’s supervision ability \( \eta_t \) in the period \( t \), we have the following results

\[
e_t = \frac{\eta_t - \eta_t^*}{\eta^* - \eta_t^*} \frac{(1 - \bar{\lambda})R}{c} \quad (11)
\]

\[
r_t = r_0 \frac{(1 - e_t)\eta^* + e_t\eta_t}{(1 - e_t)\eta^* + e_t\eta_t} \quad (12)
\]

\[
L_t = \frac{1}{1 - \frac{(1 - \bar{\lambda})R}{(\eta^* - \eta_t)r_t}} \quad (13)
\]

If there is no innovation in the period, all banks choose safe projects. The output is \( y^n_t = \omega \cdot (R - r_0)L_t \), and \( 1 - \eta^* \) share of banks fail at the end of the period. As the supervision ability \( \eta_t \) becomes larger, the banks have a higher leverage, and the output in the economy becomes larger without loophole innovation. We say that the economy is in the boom. However, if innovation occurs in the period, all banks choose risky projects, and the output is \( y^i_t = \omega \cdot (\bar{\lambda}R - r_0)L_t \). Since \( \bar{\lambda}R < r_0 \), the larger the leverage, the drop in the output is larger. Also, \( 1 - \eta \) share of banks fail in the risky projects and default. Due to the massive defaults and declining output, we call that there is a crisis in the economy when a loophole innovation realizes.
Proposition 2. Under Assumption 1 and large innovation cost coefficient $c$, the longer the boom,

(I) the leverage is higher;

(II) a crisis is more likely to occur;

(III) conditional on a crisis occurring, the decline in the output is larger.

Proof. Since the regulator improves its supervision ability for known risky projects each period through learning by supervision, the regulator’s supervision ability is higher when the boom is longer. From Proposition 2, we know that banks’ leverage and the capable bank’s innovation effort are increasing with supervision ability. Thus, leverage is higher for a longer boom, and at the same time innovation probability is higher, so crises are more likely to happen. Conditional on innovation realizing, the output is $y_t^i = \omega \cdot (\bar{\lambda}R - r_0)L_t$. Since $\bar{\lambda}R < r_0$, the larger the leverage, the drop in the output is larger.

To illustrate Proposition 2, we simulate a certain path of innovation realizations in the economy, and the results are in the figure 2. Although loophole innovation is possible to realize in every period, we set two innovations occurring in the period 15 and 25, so there are crises in those two periods. And the boom period before the first crisis is longer than that before the second one. Both leverage and output increase in the boom periods, and the longer the boom, the higher the leverage and output. At the same time, the innovation effort increases as well, which means a higher probability that there will be a crisis. When innovation does realize after a long boom, banks with a high leverage choose the risky projects and cause a more severe consequence, as is shown in the figure 2 that the drop in the output is larger in the first crisis.

4.3 Long-run Distribution Properties

Next, we investigate the long-run distribution for the economy. The supervision ability for known risky projects characterizes the states of this dynamic economy. Given the regulator’s supervision ability state $\eta^*_t$, banks offer the same contracts to household, which determines the same leverage, deposit contract, and loophole innovation probability. However, the evolution of the supervision ability state depends on whether the loophole innovation realization or not. To make a more general case, we let the supervision ability for known risky projects grow with probability $q$ without loophole innovation and stays the same with probability $1 - q$, and it is publicly known. When $q = 1$, this comes back to the previous case where
supervision ability grows in each period without innovation. If the current supervision ability is \( \eta_i^* \), i.e. \( \eta_i = \eta_i^* \), we can write down the general rule for supervision ability evolution for the case \( i < K \),

\[
\eta_{i+1} = \begin{cases} 
\eta_i^* + 1, & \text{with prob. } q \text{ in case of no innovation;} \\
\eta_i^*, & \text{with prob. } 1 - q \text{ in case of no innovation;} \\
\eta_1^*, & \text{in case of innovation.}
\end{cases}
\] (14)

For the case \( i = K \),

\[
\eta_{i+1} = \begin{cases} 
\eta_K^*, & \text{in case of no innovation;} \\
\eta_1^*, & \text{in case of innovation.}
\end{cases}
\] (15)

For supervision ability \( \eta_i^* \), with probability \( 1 - e_i \), loophole innovation does not occur in the period, and next period the economy will move to the next level of supervision ability \( \eta_{i+1}^* \) if \( i < K \) or stay \( \eta_K^* \) if \( i = K \) with probability \( q \), and stay the same level of supervision ability \( \eta_i^* \) with probability \( 1 - q \). With probability \( e_i \), loophole innovation occurs in the current period, and next period the economy will reset to the supervision ability level \( \eta_1^* \). Thus, the supervision ability states follow a Markov process. With the innovation probability, we can write the transition matrix for the Markov process as

\[
P = \begin{bmatrix}
e_1 + (1 - q)(1 - e_1) & q(1 - e_1) & 0 & \ldots & 0 \\
e_2 & (1 - q)(1 - e_2) & q(1 - e_2) & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
e_{K-1} & 0 & 0 & \ldots & q(1 - e_{K-1}) \\
e_K & 0 & 0 & \ldots & 1 - e_K
\end{bmatrix}
\] (16)

The element \( P_{ij} \) denotes the probability from current period state \( i \) to the next period state \( j \). If current state is \( i \), the regulator’s supervision ability is \( \eta_i^* \), and the innovation effort is \( e_i \). For states \( 1 \leq i < K \), with probability \( e_i \) innovation occurs, and the economy will evolve to the state 1 in the next period. With probability \( 1 - e_i \) innovation does not occur, and the economy will evolve to the next state \( i + 1 \) with probability \( q \) and stay the same state \( i \) with probability \( 1 - q \) in the next period. For the state \( K \), the difference is that the economy will stay the same state next period if there is no innovation in the current period.

Since there are only finite number of recurrent states, which follows a Markov process, we can get the following lemma.

**Lemma 3.** Under Assumption 7 and large innovation cost coefficient \( c \), there exists a stationary distribution \( \pi \) for the supervision ability Markov process, i.e. \( \pi = \pi P \).
The stationary distribution $\pi$ is a $1 \times K$ row vector, where the $i$th element $\pi_i$ is the stationary distribution probability of the economy related to the supervision ability $\eta_i^*$. Since the first state occurs only after a loophole innovation, the first element $\pi_1$ equals to the probability of crises in the long run.

As is shown in the Lemma 2 when the values of parameter such as $c$, $R$, and $\bar{\lambda}$ change, the innovation probability changes. This leads to changes in the transition matrix and the stationary distribution. We can get the following lemma.

**Lemma 4.** Under Assumption 1 and large innovation cost coefficient $c$, if $c$ increases, $R$ decreases, or $\bar{\lambda}$ increases, the probability of the lowest state decreases, and the probability of highest state increases.

The intuition is that if $c$ increases, $R$ decreases, or $\bar{\lambda}$ increases, the innovation probability in each state decreases. With less innovation, there are fewer crises, and the economy is less likely to return to the lowest state. With lower innovation probability, the economy is more likely to evolve into the next state and finally end up in the highest state.

We can further characterize the property for the whole distribution in the following proposition.

**Proposition 3.** Under Assumption 1 and large innovation cost coefficient $c$, if $c$ increases, $R$ decreases, or $\bar{\lambda}$ increases, the new stationary distribution will first-order stochastic dominate the original one.

First-order stochastic dominance means that the cumulative density function of the new stationary distribution is lower than that of the original one, so the whole distribution shifts to the higher states on average. The probability that regulator has a high supervision ability is higher in the long run. In the Figure 3, we plot the innovation probability and stationary distribution with different innovation cost coefficients. As shown in the figure, we can see that with small $c$ the stationary distribution has a higher probability for low supervision ability, i.e. the economy is more likely to stay with low supervision ability in the long run.

5 Regulation: Maximum Leverage Ratio

In this section, we discuss the case that regulator uses maximum leverage ratio as regulation tool. When banks offer deposit menu to depositors, they do not consider that they may be the capable bank and there are the adverse external consequences of loophole innovations. There are negative externalities for loophole innovation on two dimensions. First, innovation
realization will reduce the output in the current period since all banks will invest in inefficient risky projects. Second, after innovation realization, the regulator has to learn about it and improve its supervision ability gradually from the start. This leads to a low leverage for the banks in the following periods. These externalities provide the regulator with justification for setting the maximum leverage ratio to curb loophole innovation probability. As shown before, when regulator has a high supervision ability, banks have a high leverage. But at the same time, this high leverage results in a high probability of innovation. To curb the high probability of innovation, the regulator may want to restrict the leverage level for all the banks. To achieve this goal, the regulator can set a maximum leverage ratio for the economy. Since leverage is increasing with regulator’ supervision ability, it is easy to see that this maximum leverage ratio does not play a role when supervision ability is very low. Market discipline constrains the leverage banks can take in those states. In that case, banks offer deposit contracts with a leverage level lower than the maximum leverage ratio to satisfy the incentive compatible constraint. Only when the regulator has a high supervision ability, the maximum leverage ratio is potentially effective. Originally banks offer deposit contracts with higher leverage, but under regulation, they can only offer the regulated leverage now.

Let us consider the case where the regulator sets the maximum leverage ratio as $\bar{L}$. Under regulator’s supervision ability $\eta^*_i$, we denote the leverage without regulation or only with market as $L^m_i$. If $L^m_i \leq \bar{L}$, banks will still offer the original leverage rate without violating the regulation. There is no effect for maximum leverage ratio, and the leverage under regulation $L^*_i$ is the same as the one without regulation $L^m_i$. However, if $L^m_i > \bar{L}$, regulation has effects on leverage, which also affects innovation efforts. Banks cannot offer the menu with leverage $L^m_i$ due to the regulation, so instead they can only offer a leverage of $\bar{L}$. From Proposition 1 we know that $L^m_i$ is increasing with the regulator’s supervision ability, so regulation is more likely to be effective when supervision ability is high.

When the leverage regulation is effective, the incentive compatible constraint is slack. The deposit contracts banks offer to depositors contain the regulated maximum leverage and a deposit rate to make the depositors break even. The first order condition for the capable bank’s innovation effort is

$$- [R - \eta^* r (1 - 1/\bar{L})] + [\bar{\lambda} R - \eta r (1 - 1/\bar{L})] = ce \quad (17)$$

and the interest rate in the equilibrium is

$$r = \frac{r_0}{(1 - c) \eta^* + c \eta} \quad (18)$$

By solving the above two equations, we can get the innovation probability $\bar{e}$ when banks’
leverages are restricted by the regulation. Thus, the innovation probability under regulation is

\[ e^r_i = \begin{cases} 
  e^m_i, & \text{if } L^m_i \leq \bar{L} \\
  \bar{e}, & \text{if } L^m_i > \bar{L}
\end{cases} \quad (19) \]

Since banks’ leverage is constrained with a maximum leverage ratio, the innovation probability under regulation is always smaller than or equal to that without regulation, i.e. \( e^r_i \leq e^m_i \).

With the above innovation probability, we can write the transition matrix under regulation as

\[
P^r = \begin{bmatrix}
  e^r_1 + (1 - q)(1 - e^r_1) & q(1 - e^r_1) & 0 & \ldots & 0 \\
  e^r_2 & (1 - q)(1 - e^r_2) & q(1 - e^r_2) & \ldots & 0 \\
  \vdots & \vdots & \vdots & \ddots & \vdots \\
  e^r_{K-1} & 0 & 0 & \ldots & q(1 - e^r_{K-1}) \\
  e^r_K & 0 & 0 & \ldots & 1 - e^r_K
\end{bmatrix} \quad (20)\]

where the element \( P^r(i, j) \) is the probability of moving from the state \( i \) to the state \( j \) in the next period. With this transition matrix, we can get the stationary distribution under regulation, \( \pi^r \). Since we know that certain states’ innovation probabilities are smaller with regulation if there are some \( L^m_i > \bar{L} \), we can compare the two stationary distributions with and without regulation in the following proposition.

**Proposition 4.** Under Assumption 1 and large innovation cost coefficient \( c \), if the regulator sets a maximum leverage lower than the highest one determined by the market, the stationary distribution under regulation will first-order stochastic dominate the one without regulation.

The results are shown in the Figure 4. The figure includes three cases: without regulation, lenient regulation (high maximum leverage), and strict regulation (low maximum leverage). The case without regulation can be seen as a very slack regulation that has a so high maximum leverage that it never has effect. As we can see in the graphs, as regulation becomes stricter, leverage and innovation probability under more states are deviated from the case with only market discipline. Also, the leverage and innovation probability under those affected states are lower under stricter regulation. The changes in innovation probability affect the transition matrix and also stationary distribution. As we see in the graph, the stationary distribution shifts more to the high states under stricter regulation.

We assume that the regulator sets the maximum leverage ratio to maximize the average output in the long run. The expected output in the state \( i \) is

\[
y^r_i = \omega \cdot [(1 - e^r_i)(R - r_0) + e^r_i(\bar{\lambda}R - r_0)]L^r_i \quad (21)\]
and the average output in the stationary distribution is

\[ EY = \sum_{i=1}^{K} \pi_{i}^{r} y_{i}^{r} \]  

The maximum leverage ratio has effects on the average output in two ways. First, it can directly affect the expected output \( y_{i}^{r} \) in certain states through its effects on leverage and innovation probability. Effective regulation decreases the leverages in the affected states, which has a negative effect on the output in the affected states given the expected output per unit investment. But in the same time, regulation reduces the innovation probability, which increases the expected output per unit investment. The overall effect of regulation on expected output in affected states depends on which effect dominates. Usually when supervision ability is low, the former effect dominates, so the expected output in the affected states will decrease with strict regulation. Second, it can affect the stationary distribution \( \pi_{i}^{r} \) through its effect on innovation probability. Strict regulation will shift the distribution towards high states, which usually have higher output. If regulation decreases the expected output in the affected states, there exists a trade-off for the regulator between expected output in affected states and probability of staying high states in the stationary distribution.

Under certain parameter space, the regulator optimally choose a maximum leverage level that the incentive compatible constraint is not binding when the regulator has a high supervision ability. The results are shown in the Figure 5. As we can see, the optimal regulation sets a maximum leverage ratio which is effective in some states. The expected output in those affected states becomes lower under regulation. However, the loophole innovation probability is also reduced for those high supervision ability states, because the capable bank has less incentive to innovate under regulation. The change of innovation probability shifts the stationary distribution. Compared to no regulation case, the economy has a higher probability to stay in high supervision ability states, as shown in the fourth graph.

## 6 Learning about Loophole Innovation

In the previous sections, the regulator and investors know exactly about the existence about a loophole innovation at the end of the period when loophole innovation realizes. Next we study the dynamics where there exists some uncertainty about whether there has been an unknown loophole in the economy.

We assume that there are \( N \) banks endowed with \( \omega \) goods in the economy in each period. Finite number of banks can prevent perfectly revealing the existence of loophole innovation.
through the share of banks failing. Other setup related to banks and household is the same as before. Banks offer menu of leverage and interest rate to household to absorb deposit. In each period, one of the $N$ banks is a capable one and can discover a loophole with probability by exerting effort. The banks can choose to invest in safe projects or risky projects. Household can deposit in the banks or invest in the storage technology.

Regarding the uncertainty on unknown loophole, we assume that at the end of each period, the public only observe the number of bank failures and update their belief about unknown loophole with it. If the public can observe the payoff of banks, they know whether banks invest in risky projects, and they can infer the existence of loophole perfectly. When observing bank failures, the public need to infer whether these failures come from the safe projects or risky ones.

The supervision part of the regulator is the same as previous setup. It can supervise the banks, its supervision ability for known risky projects is gradually increasing. Different from previous parts, we add an investigation role for the regulator. Since there is uncertainty about the existence of loophole innovation, the regulator can pay a fixed cost $\chi/r_0$ to investigate into the banking sector at the beginning of each period, and the investigation result is publicly observed. We assume that the investigation cost comes from the lump tax from household. If there exists a loophole, the public know it, and the regulator’s supervision ability for it starts to grow gradually from the lowest level. If there is no unknown loophole, it is revealed to the public, and the supervision ability evolves. Thus, investigation plays two roles in the model. First, it eliminates the uncertainty on unknown loophole. Second, it is the starting point for the gradual growth of supervision ability for a certain type of risky project. Eisenbach et al. (2015) discusses that one of the supervisory job for the central bank is “discovery examination”, which focuses on understanding a specific business activity and filling the knowledge gap. In our model, investigation from the regulator serves the similar role.

The timing is as follows: at the beginning of each period, the regulator decides whether or not to investigate. If it investigates and finds a loophole, its supervision ability resets to the lowest level. The investigation result is publicly observed, and the public update belief about unknown loophole. Banks offer menus of leverage and deposit rate to household. Household decide whether or not to make deposits in the banks. After banks receive deposits, banks know whether there is a loophole unknown to the regulator, and they learn about the loophole if there is one. One of the banks knows it is the capable bank, and it makes loophole innovation effort. If the innovation realizes, all other banks can learn about it. Then banks make project choices under the regulator’s supervision. At the end of each period, projects
pay off. Banks pay back the depositors if their projects succeed. Otherwise, they default. The public updates the belief about the existence of unknown loophole in the economy. In the next period, the regulator’s supervision ability on known risky projects evolves.

Consider the deposit menu banks offer to depositors. As in previous sections, we focus on the case where innovation cost coefficient is large so the innovation probability is small. It is easy to see that there are at most two contracts offered, one with low leverage and the other with high leverage. The first one is that banks offer a leverage and deposit rate menu \(\{L_0, r_0/\eta^s\}\), where \(L_0 = 1/(1 - (1 - \bar{\lambda})R\eta^*/((\eta^* - \eta)r_0))\). Under this menu, the bank will never invest in any risky project even if there is unknown loophole, so they only need to pay a low interest rate \(r_0/\eta^s\) to make the depositors break even. Also, if the capable bank offers this menu, it will not exert any effort for loophole innovation. Thus, the expected profit or utility for the banks choosing this menu is

\[
\pi_0 = RL_0 - r_0(L_0 - 1)
\]

The second one is a menu with leverage higher than \(L_0\) and a deposit rate making depositors break even. For large innovation cost coefficient, the incentive compatible constraint is binding, i.e. \(R - \eta^*r(1 - 1/L) = \bar{\lambda}R - \eta^*r(1 - 1/L)\). Since there is uncertainty about unknown loophole, the belief about the probability that there exist a unknown loophole, \(\theta\), plays a role in the deposit contract. The capable bank will make innovation efforts only if it offers the high-leverage menu, so the number of banks choosing high leverage menu affects the probability of loophole innovation, which determines the utility of banks with higher leverage menu. For belief \(\theta\), supervision ability for known risky projects \(\eta^*\), and \(n\) out of \(N\) banks choose high leverage menu, we have the following results related to banks choosing high leverage menu

\[
e = \frac{\eta^* - \eta}{\eta^* - \eta^*} (1 - \bar{\lambda})R
\]

\[
r = \frac{r_0 (1 - \theta)(1 - \frac{n}{N}e)\eta^s + [\theta + (1 - \theta)\frac{n}{N}e]\eta}{(1 - \theta)(1 - \frac{n}{N}e)\eta^s + [\theta + (1 - \theta)\frac{n}{N}e]\eta}
\]

\[
L = \frac{1}{1 - (1-\bar{\lambda})R\eta\eta^*r}
\]

This menu is only feasible if the interest rate \(r\) is not higher than \(R/\eta^*\), which implies that the belief \(\theta\) cannot be too large.

The utility for banks choosing a high leverage menu is

\[
\pi(n, \theta, \eta^*) = (1-\theta) \left(1 - \frac{n}{N}e\right) [RL-\eta^*r(L-1)] + \left[\theta + (1-\theta)\frac{n}{N}e\right] \left[\bar{\lambda}RL - \eta^*r(L-1)\right] - \frac{1 - \theta}{N} \frac{1}{2} ce^2 L
\]
With probability \((1 - \theta)(1 - \frac{n}{N} e)\), no unknown loophole was discovered before, and no new loophole is discovered in this period, so high-leverage banks will choose the safe projects. With probability \(\theta + (1 - \theta) \frac{n}{N} e\), either there exists unknown loophole or new loophole is discovered in this period, high-leverage banks invest in the risky projects evading the regulator’s supervision. The probability that one high-leverage bank is a capable one is \(\frac{1}{N}\), and it will exert innovation effort when there is no unknown loophole. We can see that \(\pi^*(n, \theta, \eta^*)\) is decreasing function for \(n\), decreasing function for \(\theta\), and increasing function for \(\eta^*\) for large innovation cost coefficient. The number of banks choosing high leverage, \(n\), is endogenously determined in the equilibrium, where no bank has the incentive to switch to the other menu. Let \(n^*\) denote the number of banks choosing high leverage menu, then

\[
    n^* = \begin{cases} 
    0, & \text{if } \pi(1, \theta, \eta^*) < \pi_0 \\
    n, & \text{if } \pi(n, \theta, \eta^*) \geq \pi_0 > \pi(n+1, \theta, \eta^*) \\
    N, & \text{if } \pi(N, \theta, \eta^*) \geq \pi_0 
    \end{cases} \tag{26}
\]

If the low-leverage utility is higher than the high-leverage utility even if only one bank choose high-leverage menu, all banks offer the low-leverage deposit contracts. This case occurs when the belief is very pessimistic, i.e. \(\theta\) is large. If high-leverage utility is higher than the low-leverage utility even if all banks choose high-leverage menu, all banks offer the high-leverage deposit contracts. This case is related to a small \(\theta\). When \(\theta\) is in the medium range, some banks may choose high-leverage menu while others choose the low-leverage one. The number of banks choosing high-leverage menu is determined in such a way that high-leverage utility is higher than or equal to the low-leverage utility while one extra bank switching to high-leverage menu will make banks prefer low-leverage menu. Let \(e^*, r^*, L^*, n^*, \text{ and } \pi^*\) to denote the innovation effort, interest rate, leverage, number of banks, and utility associated with high leverage menu in the equilibrium.

Next we consider the belief updating problem. At the end of each period, the public can update their belief about the existence of unknown belief from the performance of banks in the current period. For the banks choosing low leverage contract, there is no information about the existence of unknown loophole since they never choose risky projects. Thus, all the information related to belief update comes from those banks choosing high leverage menu. If the public observe \(m\) banks failing out of \(n^*\) banks choosing high leverage menu, the updated belief is

\[
    \tilde{\theta}(m, \theta, \eta^*) = \frac{\left[\theta + (1 - \theta) \frac{n^*}{N} e^*\right] \eta^m (1 - \eta)^{n^* - m} + \theta + (1 - \theta) \frac{n}{N} e^*}{(1 - \theta) \left(1 - \frac{n}{N} e^*\right) (\eta^*)^m (1 - \eta^*)^{n^* - m} + \theta + (1 - \theta) \frac{n}{N} e^*} \eta^n (1 - \eta)^{n - m} \tag{27}
\]

For a certain belief \(\theta\) and supervision ability \(\eta^*\), the updated belief after observing banks’
performance can only have \( n^* + 1 \) possible values. Denote \( \mathcal{M}(\theta, \eta^*) \) the set for all possible updated belief,

\[
\mathcal{M}(\theta, \eta^*) = \{ \tilde{\theta}(m, \theta, \eta^*) | m = 0, 1, \ldots, n^* \}
\]

For a belief \( \tilde{\theta}(m) \) in the set \( \mathcal{M}(\theta, \eta^*) \), the probability that the public will have that belief after observing banks’ performance is

\[
\Gamma(\tilde{\theta}(m) | \theta, \eta^*) = (n^*)^m (1 - \eta^*)^m (1 - \eta^*)^{n^* - m} \left( \theta + (1 - \theta) \frac{n^* \eta^*}{N} \right) \left( 1 - \eta^* \right)^{n^* - m}
\]

In each period, if the public belief is too pessimistic before deposit contracts, all banks will offer low leverage menus, and there will be no belief update in this period. Let \( \bar{\theta}(\eta^*) \) denote the threshold belief where at least one bank will choose high leverage menu given the supervision ability \( \eta^* \), and it satisfies the following condition

\[
\pi(1, \bar{\theta}(\eta^*), \eta^*) = \pi_0
\]

Since \( \pi(n, \theta, \eta^*) \) is a decreasing function in \( \theta \) and increasing function in \( \eta^* \) for large \( c \), we get

**Lemma 5.** Under Assumption 1 and large innovation cost coefficient \( c \), there exists a unique belief threshold before deposit contract for any supervision ability, above which no bank will choose high-leverage menu, and thus the belief on unknown loophole is not updated in the period. And the belief threshold is increasing with supervision ability.

When the belief \( \theta \) is higher than \( \bar{\theta}(\eta^*) \), no bank chooses the high-leverage menu, so there is no update about unknown loophole from the banks’ performance. The belief at the end of the period will be the same as \( \theta \). We call \( \bar{\theta}(\eta^*) \) the belief-update threshold because there is update on belief only if the belief is lower than \( \bar{\theta}(\eta^*) \) for supervision ability \( \eta^* \).

Next we discuss the effects of changes in belief and supervision ability on the economy. The analysis is complicated by the fact that the number of banks choosing high-leverage menu is also changing with them. We use \( n^* e^* / N, [(N-n^*)r_0/\eta^* + n^* r^*] / N \), and \( [(N-n^*)L_0 + n^* L^*] / N \) to denote expected innovation effort, average interest rate, and average leverage respectively. We have the following proposition

**Proposition 5.** Under Assumption 1 and large innovation cost coefficient \( c \),

(I) if \( \theta \) increases, \( n^* \) stays the same or decreases.

(i) If \( n^* \) stays the same, expected innovation effort stays the same, average interest rate increases, average leverage decreases;
(ii) If \( n^* \) decreases, expected innovation effort decreases, interest rate may increase, decrease or stay the same, average leverage decreases.

(II) if supervision ability \( \eta^* \) increases, \( n^* \) stays the same or increases. Expected innovation effort increases, average interest rate increases, average leverage increases.

For the first part of Proposition 5, the effects of belief \( \theta \) mainly come from its effect on interest rate. When it is large, depositors worry about the unknown loophole, so banks choosing high leverage menu have to pay a high interest rate. Higher interest rate lowers the leverage through incentive compatible constraint. Its effect on innovation efforts comes from the extensive margin, i.e. banks switch to low-leverage menu. For the second part of Proposition 5, the effects of supervision ability could come from both intensive and extensive margin. For the intensive margin, banks choosing high-leverage menu can offer a higher leverage, which also leads to a higher innovation effort if the capable bank is among the high-leverage banks. If more banks choose high-leverage menu with increasing supervision ability, this increases the average leverage and expected innovation probability from the extensive margin. This shows that the results in the Proposition 1 are robust even including learning in the model.

Different from previous sections, the regulator faces a investigation problem now, i.e. when to pay a fixed cost to investigate whether there is any unknown loophole. The regulator uses lump sum tax from household to fund the investigation cost, and we abstract from tax distortion here. The regulator has a discount factor \( \beta \), and its aim is to maximize the discounted expected output including the loss from investigation cost. The expected output given belief \( \theta \) and supervision ability \( \eta^* \) is

\[
y(\theta, \eta^*) = n^* \left[ (1 - \theta) \left( 1 - \frac{n^*}{N} e^* \right) (R - r_0) + \left( \theta + (1 - \theta) \frac{n^*}{N} e^* \right) (\bar{\lambda}R - r_0) \right] L^* + (N - n^*)(R - r_0)L_0
\]

The regulator makes decision on investigation based on the belief at the beginning of each period, which is the same to updated belief based on banks performance in the last period. If the regulator does not investigate, the belief stays the same, and banks offer menus based on it. Otherwise, the belief will reset to zero after investigation, since the investigation eliminates the uncertainty about unknown loophole for the economy. If the regulator finds a loophole through investigation, the regulator has to accumulate supervision ability from the beginning for the new type of risky projects. If the investigation does not find a loophole, the regulator’s supervision ability continues to evolve from last period. Let \( \tilde{\theta} \) be the belief before investigation in current period, and \( \tilde{\theta}' \) be the belief before investigation in the next
We can write down the regulator’s problem in the recursive form

\[
V(\tilde{\theta}, \eta_i^*) = \max_{d \in \{0, 1\}} \left[ (1 - d) \left[ y(\tilde{\theta}, \eta_i^*) + \beta \sum_{\tilde{\theta} \in M(\tilde{\theta}, \eta_i^*)} \Gamma(\tilde{\theta}|\tilde{\theta}, \eta_i^*) (q \ast V(\tilde{\theta}, \eta_{i+1}^*) + (1 - q) \ast V(\tilde{\theta}, \eta_i^*)) \right] \\
+ d \left\{ -\chi + \tilde{\theta} \left[ y(0, \eta_i^*) + \beta \sum_{\tilde{\theta} \in M(0, \eta_i^*)} \Gamma(\tilde{\theta}|0, \eta_i^*) (q \ast V(\tilde{\theta}, \eta_i^*) + (1 - q) \ast V(\tilde{\theta}, \eta_i^*)) \right] \\
+ (1 - \tilde{\theta}) \left[ y(0, \eta_i^*) + \beta \sum_{\tilde{\theta} \in M(0, \eta_i^*)} \Gamma(\tilde{\theta}|0, \eta_i^*) (q \ast V(\tilde{\theta}, \eta_{i+1}^*) + (1 - q) \ast V(\tilde{\theta}, \eta_i^*)) \right] \right\} \right]
\]

(29)

If the regulator chooses not to investigation, i.e. \(d = 0\), the expected output is \(y(\tilde{\theta}, \eta_i^*)\), the belief in the next period \(\tilde{\theta}'\) is updated from \(\tilde{\theta}\) through the banks’ performance, and the supervision ability evolves to \(\eta_{i+1}^*\) with probability \(q\) and stays the same with probability \(1 - q\) in the next period. If the regulator chooses to investigation, i.e. \(d = 1\), it needs to collect the tax from the household and pay the fixed cost at the beginning of the period, and the related loss in the output is \(\chi\). If the regulator finds a loophole through investigate, the regulator has to accumulate its expertise for this new type of risky projects from the beginning, and its supervision ability resets to the lowest level. The expected output is \(y(0, \eta_i^*)\) in the current period, and the supervision ability and belief evolve following the rules. If the regulator does not find a loophole, the expected output is \(y(0, \eta_i^*)\), and the supervision ability and belief evolve. In this economy, the belief \(\tilde{\theta}\) and the supervision ability for known loophole \(\eta_i^*\) are important states characterizing the evolution of the economy.

We can see that if the investigation cost is zero, the regulator will choose to investigate each period, because eliminating uncertainty can increase the banks’ leverage and reduce the risk related to unknown loophole. Thus, there is no uncertainty about unknown loophole when banks offer menus to household. The results will be the same to those in the Section 4. If the investigation cost is too large, there exist some absorbing states with positive probability, where the economy will stay there forever once it enters. Since the supervision ability for known loophole increases with positive probability, the absorbing states can only include the highest supervision ability \(\eta_K^*\). If the belief at the beginning of the period is higher than the belief-update threshold \(\tilde{\theta}(\eta_K^*)\), banks will always choose low leverage menu if the regulator does not investigate. For sufficiently large investigation cost, the regulator would not choose to investigate. In this case, there is no update for the belief and no evolution for the supervision ability, and the economy stuck there.

The case with a medium investigation cost is more interesting. We plot the belief thresholds in the Figure 6 for a certain parameter space where investigation cost is not too large or too small. The black dashed line denotes the belief-update threshold. If the belief \(\theta\) is higher than this threshold, all banks will choose low leverage menu, and there will be no update about unknown loophole. As is shown in the Lemma 5, the threshold is higher for high supervision ability, and the black dashed line is higher on the right side. The blue solid
line denotes the belief threshold for investigation. If the belief is higher than the threshold, the regulator will investigate whether there exists an unknown loophole. We can see that the relationship between the investigation threshold and the supervision ability is not monotone. Within the belief-update region, on the one hand, higher supervision ability leads to higher leverage, and the drop in the expected output will be larger if there exists an unknown loophole. This force makes the belief threshold decreases with the supervision ability. In the extreme case where $\beta$ is zero, it is easy to show that the belief threshold for investigation is a decreasing function for supervision ability. On the other hand, the investigation cost is irreversible, so investigation is like an option for the regulator, there could exist wait-and-see effect. The regulator may need more information before paying the fixed investigate cost. This force makes the regulator willing to delay investigation. Within the no-belief-update region, the regulator has an incentive to investigate to eliminate the uncertainty so that banks can have higher leverage. Also, since the belief-update threshold is increasing with supervision ability, the regulator may withhold investigation so that the belief could fall below the threshold in the higher supervision ability. Thus, the relationship between belief threshold for investigation and supervision ability may not be monotone.

7 Discussion

In this section, we discuss several parts of the setup in the model regarding the regulator’s supervision, loophole innovation, and sources for the business cycles.

In this paper, we do not explicitly model the regulator’s supervision, and instead we take it relatively as a passive and reduced form. First, we treat the regulator’s supervision as passive in the model, so we can focus on the decisions on the bank side especially the loophole innovation in this paper. Second, we model the evolution of the regulator’s supervision ability through passive learning by doing to keep it in a simplistic way. But in the Section 6, we add investigation role for the regulator, and it is related to the evolution of the supervision ability. The regulator’s supervision is important in reality, but how the regulator supervises banks is not the focus of the paper.

Regarding loophole innovation, we assume that there is only one capable bank in each period. The essence of this assumption is that loophole innovation should not be very frequent in the model, since crises are rare events in reality. We can easily extend one capable bank to any finite capable ones. In this case, the choice of innovation effort for one capable bank depends on the choices for other banks, which makes the model more complicated, but all
the main results would still hold. In the model, we assume that all current and following banks can learn about the loophole innovation if one capable bank discovers it. Since the loophole innovation provides banks with the opportunities to take the risky projects, which the regulator tries to forbid, the capable bank cannot rely on any legal patent system to protect this kind of innovation. Also, there is no competition among the banks in the model, so the capable bank has no incentive to prevent other banks from taking advantage of the loophole innovation. For the innovation spreading timing, we assume that all other current banks can learn about the innovation. In this way, we can link loophole innovation to crises. Otherwise, only the capable bank takes the risky project in the innovation period, and the decline in the output is associated with restricted leverage in the next period rather the adverse consequences from risky projects in the current period. However, the timing is not essential for the interaction between loophole innovation and supervision, and endogenous cycles still exist even if other banks learn about loophole only in the following periods.

In the paper, the source for business cycles comes from the interaction between the regulator’s supervision ability and loophole innovation in the financial sector. There are other important sources for business cycles, which have been widely discussed in the literature, and we can potentially incorporate some common shocks in the business cycle literature into our model. We consider our mechanism as a complementary one to those in the previous literature, and we omit other sources in the model only to illustrate our mechanism.

8 Conclusion

In this paper we develop a model on the dynamic interaction between regulator’s supervision and loophole innovation from the banking sector and study its implications on the banks leverage cycles. In the model, banks’ leverages are constrained due to risk-shifting problem. The regulator supervises the banks to ease this moral hazard problem, and its expertise in supervision improves gradually through learning by doing. At the same time, banks can engage in loophole innovation to circumvent supervision, which acts as an endogenous opposing force that demolishes the value of regulator’s accumulated expertise. In equilibrium, banks’ leverage and loophole innovation move together with the regulator’s supervision ability. The model shows that long periods of gradual expansion in banks’ leverage, investment, and aggregate output, are followed by sudden and sharp recessions. In our model, even in the absence of exogenous perturbations, financial intermediaries themselves can become the sources of adverse shocks to the real economy. We show that the longer the boom, the
more likely there follows a crisis and the consequences are more severe, which corresponds to Minsky’s hypothesis that good times sow the seeds of the next financial crisis. The model’s empirical implications are broadly consistent with the stylized facts from empirical studies on the credit cycles. Based on this model, we also discuss the regulation implications in the environment of loophole innovation.
A Proofs

Proof for Proposition 1

From equation (7), we can get
\[
\frac{\partial e}{\partial \eta^*} = \frac{\eta^s - \eta}{(\eta^s - \eta^*)^2} \frac{(1 - \bar{\lambda})R}{c} > 0
\] (30)

Thus, innovation effort (probability) is increasing with supervision ability.

From equation (8), we can get
\[
\frac{\partial r}{\partial \delta} = \frac{(\eta^s - \eta)}{r_0} \cdot \frac{\partial e}{\partial \eta^*} > 0
\] (31)

Thus, deposit rate is increasing with supervision ability.

From equation (9), we can get
\[
\frac{\partial L}{\partial \eta^*} = \frac{(1 - \bar{\lambda})R}{(\eta^s - \eta^*)^3 c L^2 r_0} [\eta^s (\eta^s - \eta^*) c - (1 - \bar{\lambda})(\eta^s - \eta)(\eta^s + \eta^* - 2\eta) R] (32)
\]

From the above equation we can see that, as long as \( c \geq \frac{(1 - \bar{\lambda})(\eta^s - \eta)(\eta^s + \eta^* - 2\eta) R}{\eta^s(\eta^s - \eta^*)} \) holds, leverage is always increasing with supervision ability. Q.E.D.

Proof for Lemma 2

From equation (7), we can get
\[
\frac{\partial e}{\partial c} = -\frac{\eta^* - \eta}{\eta^s - \eta^*} \frac{(1 - \bar{\lambda})R}{c^2} < 0
\] (33)
\[
\frac{\partial e}{\partial R} = \frac{\eta^* - \eta}{\eta^s - \eta^*} \frac{1 - \bar{\lambda}}{c} > 0
\] (34)
\[
\frac{\partial e}{\partial \bar{\lambda}} = -\frac{\eta^* - \eta}{\eta^s - \eta^*} \frac{R}{c} < 0
\] (35)

From equation (8), we can get
\[
\frac{\partial r}{\partial c} = \frac{(\eta^s - \eta)}{r_0} \cdot \frac{\partial e}{\partial c} < 0
\] (36)
\[
\frac{\partial r}{\partial R} = \frac{(\eta^s - \eta)}{r_0} \cdot \frac{\partial e}{\partial R} > 0
\] (37)
\[
\frac{\partial r}{\partial \bar{\lambda}} = \frac{(\eta^s - \eta)}{r_0} \cdot \frac{\partial e}{\partial \bar{\lambda}} < 0
\] (38)
From equation (9) and under large innovation cost coefficient $c$, we can get

\[
\frac{\partial L}{\partial c} = -\frac{(1 - \bar{\lambda})R}{(\eta^* - \eta^*)L}\frac{\partial r}{\partial c} > 0
\]

(39)

\[
\frac{\partial L}{\partial R} = \frac{1 - \bar{\lambda}}{(\eta^* - \eta^*)^2cL^2r_0}[\eta^*(\eta^* - \eta^*)c - 2(1 - \bar{\lambda})(\eta^* - \eta)(\eta^* - \eta)R]
\]

(40)

\[
\frac{\partial L}{\partial \lambda} = -\frac{1}{(\eta^* - \eta^*)^2cL^2r_0}[\eta^*(\eta^* - \eta^*)c - 2(1 - \bar{\lambda})(\eta^* - \eta)(\eta^* - \eta)R]
\]

(41)

It is easy to see that for sufficiently large $c$, $\frac{\partial L}{\partial R} > 0$ and $\frac{\partial L}{\partial \lambda} < 0$. In fact, as long as $\frac{\partial L}{\partial \eta^*} > 0$, $\frac{\partial L}{\partial R} > 0$ and $\frac{\partial L}{\partial \lambda} < 0$. Q.E.D.

\textbf{Proof for Lemma} [4]

From Lemma 2, we know that when $c$ increases, $R$ decreases, or $\bar{\lambda}$ increases, the innovation probabilities are lower for each state. If we can show that lower innovation probabilities lead to a lower probability in the lowest state and a higher probability in the highest state, we can prove this lemma. We use superscripts $o$ to denote the old states and $n$ to denote the new states.

From $\pi = \pi P$, we can get $\pi(I - P) = 0$. We can write down the relationship between the probabilities of two nearby states as follows

\[
\pi_{j+1} = \begin{cases} \frac{q(1-e_j)}{1-(1-q)(1-e_{j+1})}\pi_j, & \text{if } 1 < j < K - 1 \\ \frac{q(1-e_K)}{e_K}\pi_{K-1}, & \text{if } j = K - 1 \end{cases}
\]

(42)

With this relationship, we can write down the relationship between the probability of the lowest state and that of others

\[
\pi_j = \begin{cases} \prod_{i=1}^{j-1} \frac{q(1-e_j)}{1-(1-q)(1-e_{j+1})} \cdot \pi_1, & \text{if } 1 < j < K \\ \frac{q(1-e_K)}{e_K}\prod_{i=1}^{K-1} \frac{q(1-e_j)}{1-(1-q)(1-e_{j+1})} \cdot \pi_1, & \text{if } j = K \end{cases}
\]

(43)

We can define $\Delta_j$ as

\[
\Delta_j = \begin{cases} \prod_{i=1}^{j-1} \frac{q(1-e_j)}{1-(1-q)(1-e_{j+1})}, & \text{if } 1 < j < K \\ \frac{q(1-e_K)}{e_K}\prod_{i=1}^{K-1} \frac{q(1-e_j)}{1-(1-q)(1-e_{j+1})}, & \text{if } j = K \end{cases}
\]

(44)

So $\pi_j = \Delta_j \cdot \pi_1$ for any $j \geq 2$. It is easy to see that as $c$ increases, $R$ decreases, or $\bar{\lambda}$ increases, all $e_j$’s decrease, so all $\Delta_j$’s increase. Substitute $\pi_j$ into $\sum_{j=1}^{K} \pi_j = 1$, we can get $\pi_1 = 1/(\sum_{j=1}^{K} \Delta_j)$, so $\pi_1$ decreases.

We will prove $\pi_n^K > \pi_o^K$ by contradiction. If $\pi_n^K \leq \pi_o^K$, since $e_j$ becomes smaller, from equation (42), we can get $\pi_j^n < \pi_j^o$ for all $1 < j < K$. And from above, we know $\pi_1^n < \pi_1^o$. So
\[ \sum_{j=1}^{K} \pi_{j}^{n} < \sum_{j=1}^{K} \pi_{j}^{0} = 1, \] and there is contradiction. Thus, \( \pi_{K}^{n} > \pi_{K}^{0} \). Q.E.D.

**Proof for Proposition 3**

To prove the new stationary distribution first-order stochastic dominates the original one, we just need to show that the cumulative probability \( \sum_{j=1}^{k} \pi_{j}^{n} \) is smaller or equal to \( \sum_{j=1}^{k} \pi_{j}^{0} \) for all \( k \) and with strict inequality for some \( k \) following the definition of first-order stochastic dominance.

If \( c \) increases, \( R \) decreases, or \( \bar{\lambda} \) increases, \( e_{j}^{n} < e_{j}^{0} \) for all \( j \). From equation \((42)\), it is easy to see that (1) if \( \pi_{j}^{n} > \pi_{j}^{0} \) for some \( j \), this inequality holds for all \( k \) larger than \( j \); (2) if \( \pi_{j}^{n} < \pi_{j}^{0} \) for some \( j \), this inequality holds for all \( k \) smaller than \( j \). From Lemma 4, there must exist a 1 < \( k < K \), where \( \pi_{k}^{n} \leq \pi_{k}^{0} \) and \( \pi_{k+1}^{n} > \pi_{k+1}^{0} \). For \( j > k \), \( \pi_{j}^{n} > \pi_{j}^{0} \), so \( \sum_{i=1}^{j} \pi_{i}^{n} < \sum_{i=1}^{j} \pi_{i}^{0} \). For \( j > k \), \( \pi_{j}^{n} > \pi_{j}^{0} \), so \( \sum_{i=j}^{N} \pi_{i}^{n} > \sum_{i=j}^{N} \pi_{i}^{0} \). For \( k < j < N \), \( \sum_{i=1}^{j} \pi_{i}^{n} = 1 - \sum_{i=j}^{N} \pi_{i}^{n} < 1 - \sum_{i=j}^{N} \pi_{i}^{0} = \sum_{i=j}^{N} \pi_{i}^{0} \). Thus, we can show that \( \sum_{i=1}^{j} \pi_{i}^{n} \leq \sum_{i=1}^{j} \pi_{i}^{0} \) for all \( j \) and with strict inequality for \( j < K \). Q.E.D.

**Proof for Proposition 4**

With innovation probability under regulation, we can write down the relationship between the probabilities of two nearby states as follows

\[
\pi_{j+1}^{r} = \begin{cases} 
\frac{q(1-e_{j}^{r})}{1-(1-q)(1-e_{j+1}^{r})} \pi_{j}^{r}, & \text{if } 1 < j < K-1 \\
\frac{q(1-e_{K-1}^{r})}{e_{K}} \pi_{K-1}^{r}, & \text{if } j = K-1
\end{cases}
\]  \quad (45)

With this relationship, we can write down the relationship between the probability of the lowest state and that of others

\[
\pi_{j}^{r} = \begin{cases} 
\prod_{i=1}^{j-1} \frac{q(1-e_{i}^{r})}{1-(1-q)(1-e_{i+1}^{r})} \cdot \pi_{1}^{r}, & \text{if } 1 < j < K \\
\frac{q(1-e_{K-1}^{r})}{e_{K}} \prod_{i=1}^{K-1} \frac{q(1-e_{i}^{r})}{1-(1-q)(1-e_{i+1}^{r})} \cdot \pi_{1}^{r}, & \text{if } j = K
\end{cases}
\]  \quad (46)

We can define \( \Delta_{j}^{r} \) as

\[
\Delta_{j}^{r} = \begin{cases} 
\prod_{i=1}^{j-1} \frac{q(1-e_{i}^{r})}{1-(1-q)(1-e_{i+1}^{r})}, & \text{if } 1 < j < K \\
\frac{q(1-e_{K-1}^{r})}{e_{K}} \prod_{i=1}^{K-1} \frac{q(1-e_{i}^{r})}{1-(1-q)(1-e_{i+1}^{r})}, & \text{if } j = K
\end{cases}
\]  \quad (47)

From \( \sum_{j=1}^{K} \pi_{j}^{r} = 1 \), we get \( \pi_{1}^{r} = 1/(\sum_{j=1}^{K} \Delta_{j}^{r}) \). If the regulator sets a maximum leverage lower than the highest one determined by the market, there exists at least one \( e_{j}^{r} \) which is smaller than that without regulation. We can get that all \( \Delta_{j}^{r} \)'s are larger than or equal to the correspondent without regulation, \( \Delta_{j}^{m} \)'s, and some are strictly larger. Then, \( \sum_{j=1}^{K} \Delta_{j}^{r} \) is
larger, so $\pi^r_1$ is lower that that without regulation, $\pi^m_1$. It is easy to see that $\pi^r_K$ is higher than the case without regulation, $\pi^m_K$.

Since the regulator sets a maximum leverage lower than the highest one determined by the market, there must exist a $1 \leq \bar{k} \leq K$, where all states lower than or equal to $\bar{k}$ are not affected by the regulation, while all states higher than $\bar{k}$ are affected by the regulation. For $j \leq \bar{k}$, $e^r_j = e^m_j$, and for $j > \bar{k}$, $e^r_j < e^m_j$. For equations (42) and (45), we can see that $\pi^r_j < \pi^m_j$ for $j \leq \bar{k}$. And if $\pi^r_j > \pi^m_j$ for some $j$, this inequality holds for all $k$ larger than $j$. Since $\pi^r_K > \pi^m_K$, there must exist one $\bar{k} < \hat{k} \leq K$, where $\pi^r_j \leq \pi^m_j$ for $j < \hat{k}$ and $\pi^r_j > \pi^m_j$ for $j \geq \hat{k}$. For $j < \hat{k}$, $\pi^r_j \leq \pi^m_j$ with some strict inequality, so $\sum_{i=1}^{j} \pi^r_i < \sum_{i=1}^{j} \pi^m_i$. For $j \geq \hat{k}$, $\pi^r_j > \pi^m_j$, so $\sum_{i=j}^{K} \pi^r_i > \sum_{i=j}^{K} \pi^m_i$. For $\hat{k} \leq j < K$, $\sum_{i=1}^{j} \pi^r_i = 1 - \sum_{i=j}^{K} \pi^r_i < 1 - \sum_{i=j}^{K} \pi^m_i = \sum_{i=1}^{j} \pi^m_i$. Thus, we can show that $\sum_{i=1}^{j} \pi^r_i \leq \sum_{i=1}^{j} \pi^m_i$ for all $j$ and with strict inequality for $j < K$. Q.E.D.
References


Figure 1: Relationship with Supervision Ability

- **Leverage**
- **Innovation Probability**
- **Deposit Rate**
- **Expected Output**
Figure 2: Dynamics

Note: The dotted vertical lines indicate the periods when loophole innovation occurs.
Figure 3: Long-run Stationary Distribution
Figure 4: Maximum Leverage Ratio

Note: Black dot-dashed line: without regulation; Blue dashed line: lenient regulation; Red solid line: strict regulation.
Figure 5: Optimal Regulation

Note: Black dashed line: without regulation; Blue solid line: optimal regulation.
Figure 6: Belief Threshold

Note: Black dashed line: belief threshold for no belief update; Blue solid line: belief threshold for investigation.