Fiscal Policy at the Lower Bound
with Labor Search and Matching Frictions

Dunpei Gan *

September 2017

Abstract

This article challenges the conventional wisdom that government spending is effective and labor tax cuts is not effective, when interest rate is at the lower bound. I use a dynamic stochastic general equilibrium New Keynesian model with labor search and matching frictions to evaluate the effectiveness of fiscal policy. Two cases of monetary policy are considered, the non-binding situation when the interest rate can be set freely and the lower bound situation when the interest rate is fixed. Labor tax cuts have similar effect as government spending at the non-binding case. The results are strikingly different compared to Eggertsson (2010) at the lower bound case. Labor tax cuts is expansionary instead of contractionary, and slightly more effective than government spending. Other fiscal policies have negligible effect on the economy overall. Fundamentally, labor search and matching frictions generate a steep aggregate supply curve, which changes the comparative curvature to the aggregate demand curve at the lower bound. From my decomposition analysis, the efficacy of fiscal policy is largely determined by stimulating aggregate supply instead of aggregate demand.

*Department Of Economics, Texas A&M University; Address: Department of Economics, Texas A&M University, 3035 Robert H. and Judy Ley Allen Building; Email: dunpeig@tamu.edu
1 Introduction

The history of a zero lower bound on the interest rate can be dated back to the Great Depression, when the U.S. economy went through a catastrophic setback. For a recent example, most economies around the world experienced a lower bound\(^1\) on the interest rate since the 2007 financial crisis. There are tons of questions concerning economists and policy makers during economic recessions. What kind of monetary and fiscal policy should be implemented? Should we stimulate aggregate demand or stimulate aggregate supply, or even restrict aggregate supply? When the U.S. and Europe faced a liquidity trap as the interest rate fell to the lower bound (LB)\(^2\), they adopted unconventional monetary policy like quantitative easing to provide extra liquidity. At the same time, governments implemented fiscal policy like massive government spending.

What is the effect of government spending on the economy, especially at the lower bound? What are the potential effects of other fiscal policy? This paper attempts to answer these questions.

In this paper I analyze the efficiency of fiscal policy in the framework of the New Keynesian model \(^3\). For a relevant example, Eggertsson (2010) finds that government spending generates an output multiplier more than three times than labor tax cuts at normal times; at the lower bound, government spending is expansionary and effective while labor tax cuts is contractionary. In this paper, I adopt the same New Keynesian framework, and extend it with labor search and matching frictions. I find labor tax cuts to have similar effect as

---

\(^1\)The lower bound instead of the zero lower bound is used in this article. The reasoning is that an economy can commit to slice the interest rate below zero. For example Bank of Japan, European Central Bank in Sweden, Switzerland and Denmark have adopted negative interest rate in 2016.

\(^2\)Another example is for countries in a currency union or states in a country, they can lose the Independence of monetary policy for the obvious reason. (see Farhi and Werning 2016 for a discussion)

\(^3\)See Christiano, Eichenbaum and Rebelo (2009), Eggertsson (2010), Woodford (2011) and Eggertsson and Krugman (2012) for illustrations
government spending at the normal time; at the lower bound, labor tax cuts is
e xpansionary and inflationary, and more effective than government spending.

Labor search and matching frictions is shown in the literature to have important
applications in business cycle models. Christiano, Eichenbaum and Trabandt (2016)
develop a general equilibrium labor search and matching model without wage inertia,
they show it outperforms the standard New Keynesian Calvo sticky wage model to account for key business cycle properties of macroeconomic aggregates. In this paper I show that including labor search and matching can shed new lights on analyzing the effectiveness of fiscal policy at the lower bound. The labor search and matching frictions is established by Diamond (1982), Mortensen (1982) and Pissarides (1985) to account for the labor flows and unemployment dynamics. It is also used in Shimer (2005) and Hall (2005) to show that incorporating it with wage rigidity can help explain business cycle properties of unemployment. In this paper, by extending Eggertsson (2010) with a Diamond-Mortensen-Pissarides (DMP) labor search and match process, I show that labor marketing frictions play a key role in analyzing fiscal policy effectiveness in the New Keynesian model. This labor market friction represents a strong channel of employment stickiness, firms adjust labor much less than price when facing an exogenous shock. This results in a much steeper aggregate supply (AS) curve, so that the elasticity of prices is much bigger than the elasticity of labor. At the lower bound, when aggregate demand (AD) becomes upward sloping, a steeper AS curve gives contradictory results than Eggertsson’s (2010) case.

Not only do I find a novel performance of government spending and labor tax
cuts, but also it suggests a new direction of fiscal policy at the lower bound. An
increase in government spending stimulates AD greatly, but its effect on output
is largely mitigated by the steep slope of the AS curve. Instead, I find in this
paper that most of the increase in output comes from stimulating aggregate
supply. Either a cut in labor tax rate that directly reduces the marginal cost,
or the crowding out of private consumption by government spending which
decreases the marginal rate of substitution and increases labor supply.
This paper is not the only one finding that labor tax cuts is more effective than government spending at the lower bound. Mertens and Ravn (2015) assumes a simple New Keynesian model but with an exogenous confidence shock to the households. They show that this exogenous confidence shock can push the aggregate demand curve into a much flatter slope. As a result of this, government spending has deflationary effects that reduce the output, while a cut in marginal labor tax rate is expansionary at the case of lower bound. Even though for a coincidence, I have similar result as their paper. The driving force here is labor search and matching frictions, and its effect rests on aggregate supply instead of aggregate demand.

There is a mixture of results for the effectiveness of fiscal policy from the empirical literature. Barro (1981) argues that the output multiplier of government spending is around 0.8, while Ramey (2008) estimates the multiplier to be close to 1.2. There are few empirical studies on government spending multiplier for the zero lower bound period. For the U.S., Ramey and Zubairy (2017) find the multiplier to be below unity irrespective of the amount of slackness of the economy. For the zero lower bound, the results are more mixed with a few specifications implying multipliers as high as 1.5. For the zero lower bound period in Japan, Miyamoto et. al (2016) find on-impact output multiplier to be 1.5 and 0.6 out of the recession state. Mertens and Ravn (2013) use a narrative approach and structural vector autoregressions, and find that a 1 percentage point cut in the average personal income tax rate raises real GDP per capita by 1.4 percent on impact and by up to 1.8 percent after three quarters.

The contribution of this paper is to study all four components together: i) the New Keynesian Model ii) the labor search and matching frictions iii) the lower bound on interest rate iv) the fiscal policy. To my knowledge, this is the first time some paper find inefficient government spending but not focus on labor tax cuts: see Baxter and King (1993) for a Real Business Cycle model with distortionary government tax on households; see Monacelli, Perotti and Trigari (2010) for a similar labor search and matching setting, they find that it is hard to reconcile government spending multiplier above unity even with other components.

4Some paper find inefficient government spending but not focus on labor tax cuts: see Baxter and King (1993) for a Real Business Cycle model with distortionary government tax on households; see Monacelli, Perotti and Trigari (2010) for a similar labor search and matching setting, they find that it is hard to reconcile government spending multiplier above unity even with other components.
in the literature that a study considers all those components together. I follow closely to Eggertsson (2010) to reduce the model into a two-dimensional diagram with output and inflation. The dynamics of an exogenous fiscal policy shock can be seen from the shift of the AS and AD curves. One advantage of this approach as comparing to the normal DSGE model with data application is that it gives additional channel of AS/AD to analyze the economy. Also taking the model to the data can be challenging because quantitative easing is often implemented with fiscal policy when the interest rate fell to zero after the financial crisis. By extending labor search and matching frictions, I can accompany the employment stickiness in this model and show its effect on fiscal policy. This paper also adds another story to the conventional New Keynesian model, where researchers often find stimulating aggregate demand from government spending can be very effective.

In addition to the striking difference in the results of labor tax cuts and government spending at the LB, as compared to Eggertsson’s (2010) work, in my decomposition analysis, the shift of the AD curve leads to a much smaller output change than the shift of the AS curve. In this framework, stimulating aggregates supply is much more important than stimulating aggregate demand. This contrasts the well-known belief that aggregate demand is vital in recession.

Interestingly, in the sensitivity test, varying the labor market parameters, either to the U.S. or European standards, does not change the results much. The only sensitive parameter is the expected probability of how long (likely) the recession would last. If the economy is more confident for a short recession, government spending is more effective like Eggertsson (2010) suggests. However, for a pessimistic economy, the labor tax cuts would be more efficient as shown in the benchmark case in this paper. This finding coincides with the results of Mertens and Ravn (2015), in which the expectation parameter plays a similar role in determining the effectiveness of the two fiscal polices. It points out the importance of expectation formation for agents at the LB. Therefore, fiscal authorities should base their choice of fiscal policy on the confidence of agents.

The remainder of this paper is outlined as follows: Section 2 gives out the
model of this paper, section 3 defines the equilibrium and solution of the model, section 4 shows how the model is calibrated, section 5 discusses the main results of the paper, section 6 presents the sensitivity test and section 7 concludes.

2 Model

This paper extends the backbone New Keynesian model with price stickiness to include the labor search and match frictions, which establish distinction from Eggertsson (2010). I consider varieties of fiscal policies like distortionary taxes on labor, consumption, capital and profits, as well as government spending. The discussion is composed of two cases, when the nominal interest rate can adjust freely or is fixed at the lower bound.

2.1 Consumer’s Problem

I assume this economy is populated by a large number of identical households. Each household is made up of a continuum of members, each specialized in a different labor service indexed by $j \in [0,1]$. Income is pooled within each household, which acts as a risk sharing mechanism. A typical household seeks to maximize

$$E_0 \sum_{t=1}^{\infty} \beta^t \xi_t [u(C_t) + g(G_t) - \int_0^1 v(N_t(j))dj] \tag{1}$$

Where $C_t$ is the consumption choice of the household, $G_t$ is the amount of government spending exogenously determined by the government and $N_t$ is the labor supplied by the household. Specifically I assume a constant relative risk aversion (CRRA)$^5$ utility function for $u$ and $g$: $u(C_t) = \frac{(C_t)^{1-\sigma}-1}{1-\sigma}$, $g(G_t) = \frac{(G_t)^{1+\sigma}-1}{1+\sigma}$, $v(N_t(j)) = \frac{N_t(j)^{1+\phi}}{1+\phi}$ is the disutility function for labor.

$^5$I assume this utility function as it is simple to solve, and also to be comparable to Eggertsson(2010)
I assume that in each period the household is subject to a periodic preference shock $\xi_t$, which can be interpreted as a shock to the discount factor $\beta$. Apart from consumption, the government spending $G_t$ affects consumer’s utilities directly, but in a separable form. I disavow the substitutable government spending as in Eggertsson (2010) because it would be irrelevant for the household’s problem eventually in this setting. 6

The period budget constraint of the household is as follows, it is subject to consumption tax $\tau_s t$, capital tax $\tau_A t$, profit tax $\tau_p t$, and payroll tax $\tau_w t$. I assume the government uses a lump-sum tax $T_t$ to clear the deficits and surplus in government budget in every period. $Z_t(i)$ is the profit earned by firm $i$.

$$(1 + \tau_s t)P_tC_t + B_t = (1 - \tau^{A}_{t-1})(1 + i_{t-1})B_{t-1} + (1 - \tau^{p}_{t}) \int_0^1 Z_t(i)di + (1 - \tau^{w}_{t})P_t \int_0^1 W_t(j)N_t(j) dj - T_t$$

Since consumers are choosing consumption level and labor decision in every period, I have the following F.O.C.s with respect to consumption and labor:

$$C_t^{-\sigma} = (1 + i_t)(1 - \tau^{A}_{t})\beta E_t \frac{\xi_{t+1} P_t}{\xi_t} \frac{1 + \tau^{p}_{t}}{1 + \tau^{s}_{t+1}}$$

$$1 - \tau^{w}_{t} \frac{W_t(j) = N_t(j) C_t^{-\sigma}}{1 + \tau^{s}_{t}} \frac{1}{1 + \tau^{p}_{t}}$$

I can combine the resource constraint $Y_t = C_t + G_t$ with the Dynamic Euler equation (3) to solve for the consumer’s side “IS” curve or the aggregate demand(AD) equation. The AD equation from Eggertsson (2010), is in the following log-linearized version:

$$-\sigma \frac{Y}{Y} G (\bar{Y} - \bar{G}) = i_t - E_t \pi_{t+1} - \tau^{s}_{t} + \sigma \frac{Y}{Y} G E_t (Y_{t+1} - G_{t+1})$$

$$-\chi A \tau^{A}_{t} + \chi A E_t (\bar{\tau}^{A}_{t} - \tau^{s}_{t+1})$$

6 The author is fully aware of the importance of this kind of government spending. As shown in Oh and Reis (2012), during the Obama administration, a large amount of government spending is transferable and targeted.

7 Assume a lump-sum tax can simplify the computation of this model. As shown in Baxter and King (1997) and known in the literature, a distortionary tax would lead to smaller output multiplier for government spending.
where the letter without subscript denotes the steady state value of a variable. Following the same definition for parameters and log-linearized variables, I arrive at the same 'AD' equation as Eggertsson (2010).

2.2 Firm’s Problem

As for a conventional New Keynesian model, I assume a continuum of firms indexed by \( i \in [0, 1] \). Each monopolistically competitive firm produces a differentiated good with a technology represented by the following production function:

\[
Y_t(i) = N_t(i) \tag{6}
\]

The production function is assumed to be constant returns to scale, and the only input is labor.

I assume labor is unique in a continuum between 0 and 1. Each household is endowed with one unit of labor and can supply in any labor market. Thus the firm \( i \) has an index of labor input \( N_t(i) \) defined as follows:

\[
N_t(i) = \left( \int_0^1 N_t(i, j)^{1 - \frac{\epsilon_w}{\epsilon_w - \tau}} dj \right)^{-\frac{\epsilon_w}{\epsilon_w - \tau}} \tag{7}
\]

where \( N_t(i, j) \) denotes the quantity of type-\( j \) labor employed by firm \( i \) in period \( t \). The parameter \( \epsilon_w \) represents the elasticity of substitution among labor varieties. The above equation just states that firm \( i \)'s labor demand is a summation of labor over the space of labor type.

I can define the aggregate wage as an indexation of sectoral wage as follows:

\[
W_t \equiv \left( \int_0^1 W_t(j)^{1 - \epsilon_w} dj \right)^{\frac{1}{1 - \epsilon_w}} \tag{8}
\]

Further, I can define the wage bill of a firm as the product of the wage index times that firm’s employment index:

\[
\int_0^1 W_t(j)N_t(i, j) dj = W_tN_t(i) \tag{9}
\]

\(^8\)See Appendix in Eggertsson (2010) for detailed definitions.
Given the cost minimization problem of a firm, I have the following demand schedules for each firm:

\[ N_t(i, j) = (\frac{W_t(j)}{W_t})^{-\epsilon_w} N_t(i) \]  \hspace{1cm} (10)

Similarly, I can define the aggregate price index as a summation of different varieties of prices for goods for different firms:

\[ P_t = \left( \frac{1}{P_t(i)^{1-\epsilon_p} di} \right)^{1-\epsilon_p} \]  \hspace{1cm} (11)

The consumption indexation problem can be written as:

\[ C_t = \left( \frac{1}{C_t(i)^{1-\epsilon_p} di} \right)^{1-\epsilon_p} \]  \hspace{1cm} (12)

where \( \epsilon_p \) governs the elasticity of substitution between different varieties of consumption goods.

So the consumption maximization problem gives consumption demand schedules:

\[ C_t(i) = (\frac{P_t(i)}{P_t})^{-\epsilon_p} C_t \]  \hspace{1cm} (13)

### 2.2.1 Firms and the labor market: Nash Bargaining

In this section I describe the firm’s interactions with the labor market. The assumptions mainly follow from Blanchard and Gali (2006) and Abbritti, Boitani and Damiani (2007).

In these studies the employment dynamics are determined by a job separation rate \( \delta \), where \( \delta \in (0, 1) \) defines the proportion of employers from the previous period who get separated from their jobs. Also, a pool of workers \( h_t^i \) is restored to a representative firm \( i^\prime s \) employment pool. Thus:

\[ N_t^i = (1 - \delta) N_{t-1}^i + h_t^i \]  \hspace{1cm} (14)

As an aggregation, the total employment \( N_t = \sum_{i} N_t^i di \) evolves according to the following:

\[ N_t = (1 - \delta) N_{t-1} + H_t \]  \hspace{1cm} (15)
where $H_t = \sum_i h_i^t$ denotes the aggregate hiring level.

Labor is assumed to be involuntarily supplied from the household. Those who are currently not occupied with a job form the pool of jobless. As labor force is normalized to 1, the jobless pool $U_t$ is defined as follows:

$$U_t = 1 - (1 - \delta)N_{t-1}$$

(16)

This is also referred to as the beginning-of period unemployment according to Blanchard and Gali (2006). An alternative measure of unemployment is given by:

$$u_t = 1 - N_t$$

(17)

The hiring cost of firm $i$ is determined by the following:

$$F_i = F \frac{H_t}{U_t} h_i^t$$

(18)

where $F$ is a positive scaling constant, and the ratio $x_t = \frac{H_t}{U_t}$ governs the tightness of the labor market. The searching cost function is directly proportional to the number of workers recruited $H_t$ from the jobless pool and is inversely proportional to the overall level of the jobless pool $U_t$. The former makes the firm search with higher intensity, and the latter smooths the matching process between firms and potential workers.

The searching cost is incurred to firms as a prerequisite for production. It can be thought of as rent charged from an outside matching institution that has superior information than firms in the labor market.

In addition, I assume all the searching profits are spent by this institution in the goods market. In sum, the output is consumed by either the public sector, the government, or two private sectors, the household and the labor matching institution. In other words:

$$Y_t = C_t + G_t + \frac{FH_t}{U_t} H_t$$

(19)

Note that in the above equation, the hiring cost is assumed to be small$^9$

$^9$In the calibration, the hiring cost is calibrated to be one percent of total output, which is small enough to be ignored.
compared to other components in the output. Thus I can ignore it, which is also assumed in Blanchard and Gali (2006).

Next I discuss the real wage determination under Nash Bargaining, which follows the standard literature and my base papers.

I assume that the worker and the intermediate firm bargain in each period to maximize the weighted product of both parties’ surpluses:

$$\max \omega_W = (V_t^E - V_t^U)^s S_t^{1-s}$$

where $V_t^E$ is the surplus of workers associated with being employed and $V_t^U$ is the surplus of workers associated with being unemployed. Both are expressed in terms of consumption units. $s$ and $1 - s$ are below 1 unit and represent workers’ and the firm’s bargaining powers, respectively. The intuition is that the associated surplus $\omega_W$ is split between the bargaining parties. The surplus $S_t$ is the firm’s surplus from the matching, which represents the opportunity cost of searching again in the labor market. The amount equals our unit hiring cost:

$$S_t = \frac{FH_t}{U_t}$$

I can use recursive formulation to solve for $V_t^E$ and $V_t^U$. The workers’ marginal value from employment is the current after-tax real wage minus the disutility of labor in terms of marginal consumption (marginal rate of substitution), plus discounted next period marginal employment and unemployment values.

The probability of being employed next period is the sum of the probability of remaining employed and getting into the jobless pool (but finding a job immediately): $[(1 - \delta) + (\delta \frac{H_{t+1}}{U_{t+1}})]$. The expression for employment value is given below:

$$V_t^E = (1 - \tau_w)W_t^{Nash} - (C_t + G_t)\sigma N_t^s + \beta E_t \left\{ \left( \frac{C_{t+1} + G_{t+1}}{C_t + G_t} \right)^{-\sigma}[1 - \delta(1 - \frac{H_{t+1}}{U_{t+1}})]V_{t+1}^E + \delta(1 - \frac{H_{t+1}}{U_{t+1}})V_{t+1}^U \right\}$$

(22)

The value of being unemployed is similarly written. The unemployed worker receives the discounted benefits of next period employment and unemployment
values, where $\frac{H_{t+1}}{U_{t+1}}$ is the probability of finding a job next period in the jobless pool:

$$V_t^U = \beta E_t \left\{ \left( \frac{C_{t+1} + G_{t+1}}{C_t + G_t} \right)^{-\sigma} \left[ \frac{H_{t+1}}{U_{t+1}} V_{t+1}^E + (1 - \frac{H_{t+1}}{U_{t+1}}) V_{t+1}^U \right] \right\}$$  \hspace{1cm} (23)

Combining both conditions, we get the net value of being employed:

$$V_t^E - V_t^U = (1 - \tau_t^w) W_t^{Nash} - (C_t + G_t) \theta N_t^\phi + \beta (1 - \delta) E_t \left\{ \left( \frac{C_{t+1} + G_{t+1}}{C_t + G_t} \right)^{-\sigma} \left[ (1 - \frac{H_{t+1}}{U_{t+1}})(V_{t+1}^E - V_{t+1}^U) \right] \right\}$$  \hspace{1cm} (24)

The Nash solution would give the following condition, where $\eta \equiv (s/(1 - s))$ denotes the workers’ relative bargaining power:

$$V_t^E - V_t^U = \eta S_t$$  \hspace{1cm} (25)

Thus, I can solve for the Nash wage as follows using the above two equations:

$$(1 - \tau_t^w) W_t^{Nash} = (C_t + G_t) \theta N_t^\phi + \eta S_t - \beta (1 - \delta) E_t \left\{ \left( \frac{C_{t+1} + G_{t+1}}{C_t + G_t} \right)^{-\sigma} \left[ (1 - \frac{H_{t+1}}{U_{t+1}}) \eta S_{t+1} \right] \right\}$$  \hspace{1cm} (26)

Note that in this paper I log-linearize the marginal rate of substitution (MRS) terms in the above equation directly. Blanchard and Gali (2010) deviate from this approach by assuming a form of real wage rigidity right away. In their paper the MRS term is a combination of a constant term and the exponent form of technology. In this paper I prefer not to consider technology, as I focus on reduced form solution.\textsuperscript{10}

\textbf{2.2.2 Firm’s optimising problem under price stickiness}

In this paper I consider price rigidity of setting prices by intermediate firms. Following the Calvo sticky price assumption, each intermediate firm has a random probability of $1 - \theta$ to adjust its price in every period.

\textsuperscript{10}Another way to model wage rigidity is to assume a linear relationship between MRS and the current level of real wage, but the parameter of wage rigidity plays a key role in determining the size of real wage. It seems not to be a perfect way of modeling real wage rigidity either and is subject to debate.
The firm’s price decision in every period is to maximize the expected discounted profits. Following the derivation for firm’s problem in Blanchard and Gali (2006), I have the following F.O.C. from the firm’s choice of optimal price:

\[ 0 = E_t \sum_{k=0}^{\infty} \theta^k A_{t+k} Y_{t+k|t} (1 - \tau^p_t)(P^*_t - M P_{t+k} M C_{t+k}) \]  

(27)

where \(\tau_p\) is a tax on profits, \(MC_{t+k}\) is the real marginal cost for the firm which we will describe very soon, \(A_{t+k} \equiv \beta^k U_{c,t+k}/U_{c,t}\) is the stochastic discount factor, \(M \equiv \frac{\tau_p}{\tau_p - 1}\) and \(Y_{t+k|t}\) is the time \(t+k\) demand schedule for an intermediate firm setting the time \(t\) optimal price \(P^*\):

\[ Y_{t+k|t} = \left( \frac{P^*_{t+k}}{P^*_t} \right)^{-\epsilon_p} (C_{t+k} + G_{t+k} + F \frac{H_{t+k}}{U_{t+k}} H_{t+k}) \]  

(28)

In addition, I can express the price equation as:

\[ P_t = \left[ (1 - \theta_p)(p^*_t)^{1-\epsilon_p} + \theta_p P^*_{t-1} \right]^{1/(1-\epsilon_p)} \]  

(29)

Using equations (27) and (29), I can derive the firm’s AS equation, where \(\lambda \equiv \frac{(1-\theta_p)(1-\theta_p)}{\theta_p}\):

\[ \pi_t = \beta E_t \pi_{t+1} + \lambda m \hat{c}_t \]  

(30)

Now I want to give an expression for the real marginal cost of the intermediate firm. It is a sum of the real wage and the cost of hiring one additional worker, minus the discounted next period cost of hiring, given that the worker is not separated from this employment. For simplicity, I also define a labor market tightness measure \(x_t = \frac{H_t}{U_t}\). The equation for real marginal cost is as follows:

\[ MC_t = W_t + Fx_t - \beta (1 - \delta) E_t [C_{t+1} + G_{t+1}]^{-\sigma} F \hat{x}_{t+1} \]  

(31)

To solve additionally for the firm’s AD equation, I need equations (26) and (31).
to get the log-linearized expression for real marginal cost as:

\[ \hat{mc}_t = M \frac{W}{1 - \tau_w} \hat{\tau}_w + M(1 + \eta)F \hat{x}_t - M \beta(1 - \delta)F \hat{x}(1 + \eta(1 - 2x))x_{t+1} + M \beta(1 - \delta)F x_{\frac{1}{\sigma}}[1 + \eta(1 - x)](E_t G_{t+1} - E_t Y_{t+1}) - M\left(\beta(1 - \delta)F x_{\frac{1}{\sigma}}[1 + \eta(1 - x)] + \frac{N^\phi}{1 - \tau_w} Y\right) \hat{G}_t + M\left(\beta(1 - \delta)F x_{\frac{1}{\sigma}}[1 + \eta(1 - x)] + \frac{N^\phi}{1 - \tau_w}(\phi(Y - G) + Y)\right) \hat{Y}_t \]

where \( W \) is the steady state value of real wage, \( x \) is the steady state labor market tightness measure, \( \tau_w \) is the steady state income tax rate, \( Y \) is the steady state output level, and \( G \) is the steady state government spending.

From equations (15) and (16), I can describe \( x_t \) as a relationship between the current employment \( N_t \) and the previous employment rate \( N_{t-1} \). Also I use the strict relationship between employment and output in my model such that \( Y_t = N_t \). Then I arrive at the following log-linearized relationship between labor market tightness and the output gap:

\[ \delta \hat{x}_t = \hat{Y}_t - (1 - \delta)(1 - x)Y_{t-1} \]

Substituting equation (33) into the real marginal cost equation (32), I get the
following 11:

\[ n\hat{c}_t = A^w\hat{r}_t^w - \omega_1 Y_{t-1} + \omega_2 \hat{Y}_t - \omega_3 E_t Y_{t+1} + A^G_t E_t \hat{G}_{t+1} - A^G_t \hat{G}_t \] (34)

From the above equation, the marginal cost of the firm depends not only on the current output gap and expected next period output gap and but also on the previous output gap. This is because the hiring choice made in the previous period would determine the labor demand this period, thus affecting the hiring cost incurred in this period.

Finally, I can express the New Keynesian Phillips Curve (NKPC) as follows. This is also the final AS equation I have:

\[ \pi_t = \beta E_t \pi_{t+1} + \lambda A^w \hat{r}_t^w - \lambda \omega_1 Y_{t-1} + \lambda \omega_2 \hat{Y}_t - \lambda \omega_3 E_t Y_{t+1} - \lambda A^G_t E_t \hat{G}_{t+1} - \lambda A^G_t \hat{G}_t \] (35)

2.3 Monetary Policy

A Taylor type monetary policy by the Central bank is assumed. The nominal interest rate would react to the output gap and inflation in addition to the real interest rate of the economy. An approximation of the interest rate would be given in the following equation:

\[ i_t = \max(LB, r_t^* + \phi_\pi \pi_t + \phi_y \hat{Y}_t) \] (36)

where \( \phi_\pi \) is the coefficient on inflation and is larger than 1, and \( \phi_y \) is the corresponding weight on output gap and is larger than 0. Note that the nominal interest rate cannot take a value less than the lower bound given great deflation or negative output gap. When the interest rate takes the LB, it is one scenario where I have

\[ A^w_t = M (1 + \frac{\eta (1-x)}{1+\gamma} Y) \omega_1 = M (1 + \frac{\eta (1-x)}{1+\gamma}) F x_t \frac{1}{1+\gamma} \] 
\[ w_2 = M \left( 1 + \frac{\eta (1-x)}{1+\gamma} \right) F x_t \frac{1}{1+\gamma} + \beta (1 - \delta) F x_t \frac{(1 + \frac{\eta (1-x)}{1+\gamma}) (1-x)}{1+\gamma} + \beta (1 - \delta) F x_t \frac{(1 + \frac{\eta (1-x)}{1+\gamma}) (1-x)}{1+\gamma} + \frac{\eta (1-x)}{1+\gamma} (\phi - G + Y) \right), \omega_3 = M \beta (1 - \delta) F x_t \left( 1 + \frac{\eta (1-x)}{1+\gamma} \right) \frac{1}{1+\gamma} + \frac{1}{1+\gamma} + \frac{\eta (1-x)}{1+\gamma} \) as the calibrated parameters which take positive values.

11 Where I have \( A^w = M \frac{W}{1+\gamma} \), \( A_1^G = M \beta (1-\delta) F x_t \frac{1}{1+\gamma} + \frac{\eta (1-x)}{1+\gamma} \) \( A_2^G = M \left( \beta (1-\delta) F x_t \frac{1}{1+\gamma} + \frac{\eta (1-x)}{1+\gamma} \right) \omega_1 = M (1 + \frac{\eta (1-x)}{1+\gamma}) F x_t \frac{1}{1+\gamma} + \beta (1 - \delta) F x_t \frac{(1 + \frac{\eta (1-x)}{1+\gamma}) (1-x)}{1+\gamma} + \beta (1 - \delta) F x_t \frac{(1 + \frac{\eta (1-x)}{1+\gamma}) (1-x)}{1+\gamma} + \frac{\eta (1-x)}{1+\gamma} (\phi - G + Y) \right), \omega_3 = M \beta (1 - \delta) F x_t \left( 1 + \frac{\eta (1-x)}{1+\gamma} \right) \frac{1}{1+\gamma} + \frac{1}{1+\gamma} + \frac{\eta (1-x)}{1+\gamma} \) as the calibrated parameters which take positive values.
I study intensively later. Overall, the monetary policy has implications on the aggregate demand (AD) of the economy.

3 Approximated Equilibrium

For a given policy rule for taxes and government spending, equations 5, 35, and 36 close the model. An approximate equilibrium can now be defined as a collection of stochastic processes for \( \{\tilde{Y}_t, \pi_t\} \) that satisfy equations 5, 35 and 36 given an exogenous path for \( \{r_e^t\} \), a monetary policy specifying the process \( \{i_t\} \) that satisfies equation 36, and fiscal rules that determine the path for \( \{\tilde{\tau}_w^t, \tilde{\tau}_s^t, \tilde{\tau}_A^t, \tilde{G}_t\} \), which would be more specific in upcoming policy experiment.

In the long run, it is assumed that \( r_e^t \) has gone to the steady state, while the short run is the period in which the economy is subject to temporary disturbance. More precisely, in the short run the interest rate would have a negative equilibrium value such that \( r_e^t = r_e^{S,T} \). The shock to the interest rate reverts back to long run steady state, \( \bar{r} \), with probability \( 1 - \mu \) in each period. So let \( T^c \) be the period the shock is back to the steady state. Then \( t < T^c \) is the short run and \( t > T^c \) is the long run. I assume the monetary policy follows (36), and the fiscal policy is perfectly correlated with the shock. So that \( (\tilde{\tau}_w^t, \tilde{\tau}_s^t, \tilde{\tau}_A^t, \tilde{G}_t) = (\tilde{\tau}_w^S, \tilde{\tau}_s^S, \tilde{\tau}_A^S, \tilde{G}_S) \) in the short run and \( (\tilde{\tau}_w^t, \tilde{\tau}_s^t, \tilde{\tau}_A^t, \tilde{G}_t) = (0, 0, 0, 0) \) in the long run.

3.1 Short run and long run equilibrium analysis

The above definition of long run and short run equilibrium follows Eggertsson (2010). This type of analysis is very convenient to compute and solve analytically, and it also has a straightforward interpretations of the effects of fiscal policy changes in different monetary policy circumstances.

I also impose a short run equilibrium when the shock to the real natural
interest rate is big enough that pushes it under some negative threshold. The nominal interest rate is at the lower bound \( i_t = \bar{i}^* = 0 \), the output deviates from steady state at some fixed level \( \hat{Y}_1 = \hat{Y}_s^2 \quad \forall t < T^c \), and price inflation is at some short run level \( \pi_t = \pi^s \).

Following Eggertsson (2010), I want to focus on the special circumstances when output collapse is associated with interest rate bounding at the LB, because I can derive some fiscal policy effects from this case. I consider the model when there is no fiscal intervention initially; that is, each of the fiscal variables is at the long run steady state. Then the shock to the real natural interest rate \( r^e_t \) generates a recession that pushes the nominal interest rate to the LB. As Eggertsson (2010) explains, the source of this shock can be interpreted as a preference shock or a banking crisis. I also assume this shock follows a Markov chain process with probability \( \mu \) to stay the same and probability \( 1 - \mu \) to revert back to long-run equilibrium. Once the economy reverts back to the long run equilibrium, it would stay there forever.

In the short run, \( t < T^c \), I consider two cases:

1. The nominal interest rate is above the LB in the short run. I assume there is a locally unique bounded equilibrium such that
   \[
   \pi_t = \pi_t^p \quad \forall t < T^c; \quad \hat{Y}_t = \hat{Y}_s^p \quad t < T^c; \quad i_t = \bar{i}_s^p = r^e_s + \phi_x \pi_s + \phi_y \hat{Y}_s > LB.
   \]

2. The interest rate is at the LB in the short run. I assume there is locally unique bounded equilibrium such that
   \[
   \pi_t = \pi_t^s \quad \forall t < T^c; \quad \hat{Y}_t = \hat{Y}_s^2 \quad t < T^c; \quad i_t = \bar{i}_s^* = LB.
   \]

### 4 Calibration

For convenience of notation and comparison to Eggertsson (2010), I define \( \hat{\sigma}^{-1} = -(\hat{u}_c \hat{Y}/\hat{u}_c) = \sigma \frac{\hat{Y}^2}{\hat{Y}^2 - u_s^2} \) and let \( \sigma = 1 \). \( G_{ss} \) is computed by solving for its steady state value in this model, which equals to 15.99 percent of the output level. This \(^{12}\) is seen as an increase in the probability of default by borrowers.
results in the defined $\hat{\sigma} = 0.9744$.

The following table shows all the parameters used in this paper. Eggertsson (2010) uses the empirical findings of Dene and Eggertsson (2009)$^{13}$, which applies a Bayesian estimation in a DSGE model. The target is to match only one data point: the trough of the Great Depression. During that time, output collapses for 30 percent, and there is 10 percent deflation. The calibrated Markov Chain probability of staying in the short-run recession is $\mu = 0.9030$, this paper adopts the same value for that parameter.

Following Eggertsson (2010), I have the price stickiness parameter equal to 0.7747 and an elasticity of substitution between goods of 12.7721. This implies an equilibrium markup charged by the intermediate firm equal to 1.085. The long-run equilibrium labor tax rate is equal to 20 percent, the sales tax rate is 5 percent and the profit tax on the firm is 0 percent. All of which are following Eggertsson (2010).

I follow Blanchard and Gali (2010), and calibrate the labor market as the case of the U.S.$^{14}$. I assume a Nash Bargaining worker’s relative bargaining power $\eta$ equal to 0.5, which is standard in the literature$^{15}$. Other parameters like those summarized before output and fiscal policy variables in the AS equation can be computed using equations I find in the model; I do not elaborate on each one here.

Finally, I need the Central bank’s response to price inflation and the output gap. Eggertsson (2010) assumes central bank responses to inflation by 1.5 ($\phi_\infty$) which is a common parameter for the determinace of monetary policy. The response to output gap is 0.125.

$^{13}$If I want to focus on the recent recession in 2007, I need to replicate the Dene and Eggertsson (2009) exercise for 2007 data set. This could be of interest to future research, to study the fiscal policy implication in 2007 and have a comparison to other empirical findings. $^{14}$The steady state unemployment rate is 5 percent, the job market tightness parameter is 0.7, job separation rate $\delta$ is computed from these two in the steady state as 0.1228, the steady state labor supply amount $N$ is 0.95 out of 1, and the labor search cost parameter $F$ is equal to 0.11 to have an equilibrium search cost approximately equal to 1 percent of GDP. $^{15}$The Hosios condition is satisfied. And the interpretation of this parameter is that the worker has half of the bargaining power of the firm in determining wage.
<table>
<thead>
<tr>
<th>Table 1 Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4Parameters (Mode)</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>20Parameters (calibrated)</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td><strong>2Shocks</strong></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

I define \( \phi \equiv \bar{v}_n \bar{N}/\bar{v}_n \), \( \chi^A \equiv (1 - \beta)/(1 - \tau^A) = 0.003 \), and \( \chi^A \equiv 1/(1 + \tau^A) = 0.9524 \). Note that \( G_t^{N^*} \) is the log deviation of government spending from the steady state of output. And since the tax rate is in percentage, the deviation terms are corresponding level changes from their steady state percentage levels.

One advantage of solving the model by hand is I can decompose the parameters into different forces to have a better understanding of labor search and match frictions. In the AS equation, the parameters \( \omega_2, \omega_3 \) are before current and expected output respectively. Since those are key to pin down the slope of the AS curve, the larger values correspond to a steeper AS curve. \( \omega_2 \) can be decomposed into labor search and match (1.4127), the marginal rate of substitution (2.8513) and other terms (-0.1040) out of the total (4.1601). \( \omega_3 \) is composed of labor search and match (0.9990) and others (-0.1040) as a total of
0.8951, therefore the effects of labor search and match frictions contribute to a steeper AS curve.

5 Fiscal Policy analysis in the recession

5.1 Output collapse in the absence of Fiscal Intervention

In periods $t \geq T^c$, the solution is $\pi_t = \bar{Y}_t = 0$. In periods $t < T^c$, the assumption about the shock implies that inflation next period is either zero (with probability $1 - \mu$) or the same as at time $t$, that is, $\pi_t = \pi_s$ (with probability $\mu$). Hence the solution of a lower bound on the interest rate in $t < T^c$ satisfies the following AD and AS equations:

$$AD: \quad \bar{Y}_s = \mu \bar{Y}_s + \sigma \mu \pi_s + \sigma r^e_s$$

$$AS: \quad \pi_s = \beta \mu \pi_s + \lambda (\omega_2 - \mu \omega_3) \bar{Y}_s$$

The AD equation just states that the current output depends on expected future output and inflation. The New Keynesian Phillips (AS) curve also has inflation increasing in the output gap, given that the parameters $\lambda (\omega_2 - \mu \omega_3)$ takes a positive value.

From the above equations both curves are positively sloped, the aggregate demand becomes upward sloping because when there is persistent deflationary pressure. When the nominal interest rate is set at the lower bound, the Central bank is unable to decrease the interest rate further to react to deflation or a negative output gap, and the real interest rate falls further which worsens the output collapse. This phenomenon is commonly referred to in the literature as the 'Liquidity Trap', in the sense that monetary policy cannot provide additional liquidity to the economy to push it out of a recession.

It is helpful to plot the AD and the AS curves in the output gap and inflation diagram. Supposing the probability of getting out of recession equals to 1 ($\mu = 0$), the AD curve would be a vertical line and the AS curve would be positively
Figure 1: An illustrating short run equilibrium for the lower bound sloped. Then the output level would solely be determined by the AD curve. For \( \mu \) to take a positive value, which means that there is some probability that the recession lasts for more than one quarter, then both the AS and the AD curve have positive slope. More precisely the AD curve has a slope equal to \( \frac{1-\mu}{\hat{\mu}} \), and the AS curve has the slope equal to \( \frac{\lambda(\omega_2-\omega_3)}{1-\mu} \).  

Given my calibrated parameters, for \( 0.642 < \mu < 1 \) the AS curve would have larger slope than the AD curve as shown in the graph below. Both take the same slope if \( \mu = 0.642 \). A smaller slope for the AS curve as in Eggertsson (2010), if \( \mu < 0.642 \).
5.2 The case of labor tax cut

I want to ask the same question as in Eggertsson(2010): Can government implement fiscal policy that help resolve the issue of output collapse in a crisis? In particular, can a labor tax cut generate positive and larger than 1 output multiplier?

First I want to analyze the labor tax cut under normal circumstances when the interest rate is above the lower bound. I assume a temporary labor tax cut \( \bar{\tau}_w^t = \bar{\tau}_s^w < 0 \), which is reversed with probability \( 1 - \rho \) in each period to the steady state \( \bar{\tau}_s^w = 0 \). And I let all other fiscal variables be silent such that \( \bar{G}_t^N = \bar{\tau}_s^A = 0 \). The model has forward looking parts in both the AD and the AS equations, and I can divide the expectations into two states: the long run equilibrium state such that the output gap, inflation and labor tax rate would take long run equilibrium values of 0; and the other state when all variables take corresponding short run equilibrium values. For the backward looking part in the AD curve, I assume that the previous period output gap is just 0. For monetary policy, I assume the central bank can respond to output gap and inflation by setting the nominal interest rate as a standard Taylor rule.

So I have the following new AD and AS equations relating output gap, inflation and labor tax cut:

\[
AD : \quad \tilde{Y}_s = -\bar{\sigma} \frac{\phi_{\pi} - \rho}{1 - \rho + \sigma \phi_{\pi}} \pi_s \tag{39}
\]

\[
AS : \quad (1 - \beta \rho) \pi_s = \lambda (\omega_2 - \rho \omega_3) \tilde{Y}_s + \lambda A \bar{\tau}_s^w \tag{40}
\]

By substitution of the two equations I get the following expression:

\[
\tilde{Y}_s = \frac{\lambda A \bar{\tau}_s^w}{\lambda (\phi_{\pi} - \rho)(\omega_2 - \rho \omega_3) \bar{\sigma} + (1 - \beta \rho)(1 - \rho + \sigma \phi_{\pi})} \pi_s \tag{41}
\]

I follow Eggertsson (2010) to assume \( \rho = \mu \). The multiplier is computed as 0.3414. If the government cuts the labor tax by 1 percentage point in a given period, the output increases by 0.3414 percent. The interpretation is if labor tax rate decreases by one percentage point, output would increase by about 0.34 percent. Compared to the multiplier of 0.0816 in Eggertsson (2010), the
Figure 2: Labor tax cuts: the non-binding case
calculated labor tax cut multiplier above is bigger. Given a less than unit labor tax cut multiplier, government should not necessarily implement labor tax cut in normal circumstances.

The only difference between Eggertsson (2010) and this paper is the labor search and match frictions. The AD curve is exactly the same in both papers, however in this paper the AS curve takes a much steeper slope. Thus, for a rightward shift in the AS curve, the variation in output is bigger than Eggertsson’s (2010) case, which results in a larger output multiplier for labor tax cuts. The intuition is that due to labor market frictions, firms tend to vary prices more often than labor decisions. In this case, a direct stimulus in labor market incentive as represented as a labor tax cut, pushes firms to hire more workers than the friction-less Eggertsson’s (2010) case.

Then I show the effects of labor tax cuts when the lower bound is binding for the nominal interest rate. The goal of implementing this fiscal policy is to push the economy out of a recession when further change to monetary policy is impossible. When the Central bank cannot respond to deflationary pressure caused by a decrease in price inflation, the real interest rate drops further, which generates deflationary pressure to the economy. In this case, both the AD curve and the AS curve become upward sloping. I have the AD and AS equations as follows:

\[
AD : \quad (1 - \rho)\tilde{Y}_s = \tilde{\sigma} \mu \pi_s + \tilde{\sigma} r^s_s
\]

\[
AS : \quad (1 - \beta \rho)\pi_s = \lambda(\omega_2 - \rho \omega_3)\tilde{Y}_s + \lambda A^w \tilde{\tau}^w
\]

Given I set \( \rho = 0.9033 \), the AS curve would have a steeper slope than the AD curve, as shown in the graph below:

As labor tax cuts shift out the AS curve, given a steeper slope of the AS curve, this results in an increase in output combined with a higher inflation rate. This result is contrary to what Eggertsson (2010) finds in the case of labor tax cut at the LB, he has an ineffective and negative multiplier (-1.0301). As shown below, in my case the multiplier is positive:

\[
\frac{\Delta \tilde{Y}_s}{-\Delta \tilde{\tau}^w} = \frac{\lambda A^w \tilde{\sigma} \rho}{\lambda(\omega_2 - \rho \omega_3)\tilde{\sigma} \rho - (1 - \beta \rho)(1 - \rho)} = 0.4295
\]
Figure 3: Labor tax cuts: the lower bound case
Compared to the non-LB interest rate case the multiplier is bigger but still less than 1. The interpretation is for an 1 percentage point cut in labor tax rate, output increases by 0.43 percent.

The positive labor tax cut multiplier at the LB comes from a steeper AS curve, so that a rightward shift of the AS curve leads to an increase in output. The economic intuition behind this is that due to the labor market frictions, the price elasticity of supply is much bigger than the labor elasticity of supply. After an increase in aggregate supply caused by labor tax cuts, an inflationary steady state is expected, and the economy is stimulated. The multiplier at the LB is slightly bigger than the non-LB case, but still less than 1. An effective labor tax cut is not found, even by varying the values of the parameters.\textsuperscript{17}

5.3 The case of government spending

Similar to the case before, I first consider an increase in $G_t$ when the nominal interest rate is not binding. For a probability of $\rho$, government spending stays at the short-run equilibrium level; otherwise, it reverts back to the long-run equilibrium level. The AD and AS equations are as follows:

\begin{equation}
AD: \quad \bar{Y}_s = -\bar{\sigma}\frac{\phi_\pi - \rho}{1 - \rho + \sigma \phi_y} \pi_s + \frac{1 - \rho}{1 - \rho + \sigma \phi_y} \bar{G}_s, \quad (45)
\end{equation}

\begin{equation}
AS: \quad (1 - \beta \rho) \pi_s = (\lambda \omega_2 - \lambda \rho \omega_2) \bar{Y}_s + (\lambda A_s^G \rho - \lambda A_s^G) \bar{G}_s \quad (46)
\end{equation}

From these two equations, an increase in government spending would shift both the AS and AD curves to the right. This would generate a higher level of output. The dynamics are shown in the graph below:

\begin{equation}
\bar{Y}_s = \frac{(1 - \rho)(1 - \beta \rho) + \bar{\sigma}(\phi_\pi - \rho)(\lambda A_s^G \rho + \lambda A_s^G)}{(\lambda \omega_2 - \lambda \rho \omega_2) \bar{\sigma}(\phi_\pi - \rho) + (1 - \beta \rho)(1 - \rho + \sigma \phi_y)} \bar{G}_s \quad (47)
\end{equation}

The multiplier is computed to be 0.3765. Compared to Eggertsson’s (2010) case of 0.46, this multiplier is slightly smaller. The interpretation is that for one dollar of government spending, output increases by 37 cents. When at

\textsuperscript{17}See the sensitivity test part for more information.
Figure 4: Government spending: the non-binding case
the non-LB interest rate, an increase of government spending would stimulate aggregate demand because of the effect of crowding out private consumption. A lower marginal utility of consumption would lead to a higher demand from consumers. This is represented as the rightward shift of the AD curve. This crowding out also induces workers to provide their labor at lower wages, and it leads to a lower marginal cost for firms, which is seen as a rightward shift of the AS curve.

However, the real expansionary effect on output depends largely on the slope of the two curves. In this case, the AS curve is much steeper than the AD curve, thus the increase in output from the AD shift is much smaller than the AS shift. In my decomposition analysis, the AD shift accounts for 0.0740 out of the 0.3765 multiplier, and the AS shift accounts for the rest 0.3025 out of the total. Because of labor search and match frictions, stimulating aggregate demand is not so efficient because firms face huge cost of adjusting labor input, so they are less likely to adjust production. In this case stimulating aggregate supply can directly affect the marginal cost of firms, and thus the production decision as well.

Consider now the effect of increasing government spending at the lower bound. Under this specification, I have the following AD and AS equations:

\[
AD : \quad (1 - \mu)Y_s = \hat{\sigma} \mu \pi_s + \hat{\sigma} r^e_s + (1 - \mu)\tilde{G}_s
\]

\[
AS : \quad (1 - \beta \rho)\pi_s = (\lambda \omega_2 - \lambda \rho \omega_3)\bar{Y}_s - (\lambda A^G_1 \rho + \lambda A^G_2)\tilde{G}_s
\]

Again, the aggregate demand curve has an upward slope for the same reason analyzed before at the lower bound of interest rate. From equation 48, government spending is expansionary and moves the AD curve rightward. The AS curve is also pushed rightward, which can be seen from equation 49. The graph is shown below:

\[
\frac{\Delta Y_s}{-\Delta \tilde{G}_s} = \frac{- (1 - \rho)(1 - \beta \rho) + \hat{\sigma} \rho (\lambda A^G_1 \rho + \lambda A^G_2)}{(\lambda \omega_2 - \lambda \rho \omega_3) \hat{\sigma} - (1 - \beta \rho)(1 - \rho)} = 0.3190
\]

\[18\]The decomposition comes from keeping one shift silent and set the other one to be existed.
Figure 5: Government spending: the lower bound case
The multiplier is computed to be 0.3190, which exemplifies that each dollar of government spending increases output by 31 cents. Eggertsson (2010) finds a very big multiplier of 2.2931, while my case here is much smaller. The main reason is that the steeper slope of the AS curve largely mitigates the effect of stimulating aggregate demand. For a steeper AS curve, a rightward shift of the AD curve leads to a decrease in output.

In my analysis, the rightward shift of the AD curve leads to a decrease in output by 0.0616. The rightward shift of the AS curve contributes to an increase of output by 0.3806. A combination of the two effects gives a final positive output multiplier, although the AS curve shift plays a bigger role. It means that stimulating aggregate supply is much more important in this case. Government spending cannot stimulate the economy through changing aggregate demand. Overall, I cannot find multipliers larger than 1 in both cases for government spending. It is much less efficient at the LB case. From the decomposition analysis, most of the stimulation in output comes from stimulating aggregate supply, by giving direct incentive for firms.

5.4 The case of consumption tax cut

The effect of consumption tax cuts only affects the aggregate demand of the consumers, thus only the AD curve would shift to the right. Given a much steeper AS curve, its effect is much more mitigated. This reinforces previous intuition that stimulating AD alone would be very inefficient.

For the interest rate above LB, the multiplier is computed as follows:

\[
\frac{\Delta Y_s}{-\Delta \tau^*_c} = \frac{\chi^*(1 - \rho)(1 - \beta \rho)\hat{\sigma}}{(1 - \rho + \phi_x\hat{\sigma})(1 - \beta \rho) + \hat{\sigma}(\phi_x - \rho)(\lambda \omega_2 - \lambda \omega_3 \rho)} = 0.0601
\]

This generates a slightly positive multiplier, the interpretation is for a one percentage point cut in consumer tax rate, output would increase by 0.0601 percent.

For the case with the LB, the AS curve is steeper than the AD curve, thus shifting the AD curve to the right would lead to a negative multiplier. But since
the slope of the AS curve is so steep, after all, the multiplier is only a small negative number as shown below:

$$\frac{\Delta \bar{Y}}{-\Delta \tau^*_s} = \frac{-\chi(1 - \rho)(1 - \beta\rho)\hat{\sigma}}{\sigma\rho(\lambda_2 - \lambda_3\rho) - (1 - \rho)(1 - \beta\rho)} = -0.0520 \quad (52)$$

To sum up, consumption tax cuts have small impact on the economies in both cases given a steeper AS curve.

### 5.5 The case of dividend tax and capital tax cuts

The dividend tax $\tau^p_t$ would drop out during the calculation of the equilibrium. But capital tax $\tau^A_t$ still remains. The capital tax cuts have very similar effects on output and inflation as the case of sales tax cuts; the multiplier is negligible due to a steeper AS curve.

The following tables summarized all fiscal policy multipliers found in this paper, with comparison to Eggertsson’s (2010) results.

<table>
<thead>
<tr>
<th>Type of Fiscal policy</th>
<th>This paper</th>
<th>Eggertsson’s(2010) result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labor (non-binding)</td>
<td>0.3414</td>
<td>0.16</td>
</tr>
<tr>
<td>Labor (LB)</td>
<td>0.4295</td>
<td>-1.02</td>
</tr>
<tr>
<td>Government Spending (non-binding)</td>
<td>0.3765</td>
<td>0.46</td>
</tr>
<tr>
<td>Government Spending (LB)</td>
<td>0.3190</td>
<td>2.2931</td>
</tr>
<tr>
<td>Sales Tax cuts (non-binding)</td>
<td>0.0601</td>
<td>larger than 0.46</td>
</tr>
<tr>
<td>Sales Tax cuts (LB)</td>
<td>-0.0520</td>
<td>larger than 2.2931</td>
</tr>
<tr>
<td>Capital Tax cuts (non-binding)</td>
<td>negligible</td>
<td>-0.0013</td>
</tr>
<tr>
<td>Capital Tax cuts (LB)</td>
<td>negligible</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

See the footnote for explanations.  

19 Labor(non-binding) refers to labor tax cuts multiplier with non-binding interest rate. Labor(LB) refers to the case of labor tax cuts multiplier with LB at the interest rate. Government Spending(non-binding) refers to government spending multiplier at the non-binding interest rate. Government Spending(LB) refers to government spending multiplier at the LB case. Sales Tax cuts (non-binding) refers to sales tax cuts multiplier at the non-binding case.
6 Sensitivity Test

It is curious to know the changes in the computed multipliers when changing some parameters of this model. Since the labor tax cuts and government spending are the most obvious and important ones, this section would only focus on these two.

When I change the preference parameters, there is not much change in the values of the multipliers. These results are not sensitive to how I set the preference parameters, like the degree of price stickiness.

In this paper, the chosen labor market parameters: low unemployment rate of 5 percent and high labor market tightness of 0.7 (big amount of new hires) are calibrated to be close to the U.S. labor market. When I change the labor market parameters to a combination of high unemployment and low labor market tightness, to resemble economies like the Europe, the multipliers do not change too much.

The only sensitive parameter in this model is the expectation parameter $\mu$. It governs how long (likely) the recession would last, I also interpret it as the confidence of the agents. This parameter is calibrated by following Eggertsson (2010), which uses the Great Depression as the benchmark. The interpretation is that the agents expect the recession to still exist for next period with a probability of 0.9030.

When I change the expectation parameter to a smaller value, the results would change a lot. This is because the AS curvature would get flatter and the AD curvature would get steeper. So by changing the expectation parameter I can get the model into a situation similar to Eggertsson (2010): an efficient and larger than one government spending multiplier, and a negative and inefficient labor tax cuts at the LB. The computed multipliers for labor tax cuts and government Sales Tax cuts (LB) refers to sales tax cuts multiplier at the LB case. Capital Tax cuts (non-binding) refers to capital tax multiplier at the non-binding case. Capital Tax cuts (LB) refers to capital tax cuts multiplier at the LB case.
spending with different expectation values can be found at the following table:

<table>
<thead>
<tr>
<th>Expectation Value</th>
<th>Labor (non-B)</th>
<th>Labor (LB)</th>
<th>GS (non-B)</th>
<th>GS (LB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9030 (Benchmark)</td>
<td>0.3414</td>
<td>0.4295</td>
<td>0.3765</td>
<td>0.3190</td>
</tr>
<tr>
<td>0.8</td>
<td>0.2335</td>
<td>0.4044</td>
<td>0.3801</td>
<td>0.0899</td>
</tr>
<tr>
<td>0.7</td>
<td>0.1962</td>
<td>0.6949</td>
<td>0.4560</td>
<td>-0.5415</td>
</tr>
<tr>
<td>0.6</td>
<td>0.1665</td>
<td>-3.2770</td>
<td>0.5231</td>
<td>8.3753</td>
</tr>
<tr>
<td>0.5</td>
<td>0.1432</td>
<td>-0.3054</td>
<td>0.5793</td>
<td>1.6874</td>
</tr>
<tr>
<td>0.4</td>
<td>0.1249</td>
<td>-0.1198</td>
<td>0.6256</td>
<td>1.2695</td>
</tr>
</tbody>
</table>

For the above table, The expectation value refers to the parameter $\mu$, which represents how likely the economy would stay in recession. Labor (non-B) refers to labor tax cuts multiplier with non-binding interest rate. Labor (LB) refers to the case of labor tax cuts multiplier with LB at the interest rate. GS (non-B) refers to government spending multiplier at the non-binding interest rate. GS (LB) refers to government spending multiplier at the LB case.

7 Conclusion

This paper has extended Eggertsson (2010) to consider the effectiveness of fiscal policies with labor search and matching. To do that, I add a Nash Bargaining process every period between the consumers and the firms. This greatly change the Aggregate Supply of the economy, which is the root of the difference in the results compared to Eggertsson (2010).

It is surprising to find a much smaller government spending multiplier at the LB. From my decomposition analysis, this difference mainly comes from the labor search and match frictions. In addition, stimulating aggregate demand is not effective in my model. Instead, stimulating aggregate supply contributes to most of the increase in output. Lastly, as shown in the sensitivity test section, the likely future state of the economy affects the multipliers of different fiscal
policies. This points out the importance to the fiscal authority that to know the expectation of agents.

Overall, this paper contributes to combining the study of fiscal and monetary policies with the DMP model of labor search and matching. For future research, I think it is necessary to put data into a similar DSGE framework to re-estimate the parameters. Then analyzing impulse response functions of different variables to an exogenous shock would be interesting.

8 References

References


9 Appendix

This Appendix explains the main derivation and log-linearization of this article.

The AD equation in this paper is exactly the same as Eggertsson’s (2010) case by solving the consumer’s problem. First I maximize the consumer’s utility equation (1) by the budget constraint equation (2), to get the first order condition of consumption equation (3). Then I use the resource constraint among consumption, government spending and aggregate output $C_t = Y_t - G_t$; substitute it into equation (3) and log-linearize it to get the AD equation (5). Overall there is no difference from what Eggertsson (2010) derives.

The AS equation is derived by solving the firm’s problem. It is standard to solve a sticky price firm’s problem by combing equation (27) and the price equation (29), to have a standard firm’s AS equation (30) which relates inflation to expected inflation and the firm’s marginal cost.

Given the labor search and match assumption of this paper, the above marginal cost is related to the Nash Bargained wage, this period and next period search costs as described by equation (31). To reduce the marginal cost variables into only output and exogenous fiscal policy variables, I need to go extra miles to derive the simplest forms.

Start by log-linearize equation (26) the wage equation, I get the following equations:

\[
(1 - \tau^w_i)WW_t - W\tau^w_t = (Y - G)^\sigma \phi Y^\phi Y_t + (Y - G)^\sigma \sigma Y^\phi \frac{Y_t}{Y} \tilde{Y}_t \\
- (Y - G)^\sigma Y^\phi \sigma \frac{Y}{Y - \sigma} \tilde{G}_t + \eta Fx\tilde{x}_t - \beta(1 - \delta)(1 - x)Fxx_{t+1} \\
+ \beta(1 - \delta) \eta Fx^2_{t+1} - \beta(1 - \delta)(1 - x)\eta Fx[-(1 - \sigma)\frac{Y}{Y - \sigma}Y_{t+1} - \frac{Y}{Y - \sigma}G_{t+1})] \\
\]


Which simplifies into the following:

\[
\tilde{W}_t = \frac{\tilde{W}}{1-\tau_{W}} + \frac{(Y-G)^{\phi}Y^\phi}{(1-\tau_{W} W)} \tilde{Y}_t - \frac{(Y-G)^{\psi}Y^\psi}{(1-\tau_{W} W)} (\sigma \frac{Y_{Y^G}}{Y_{Y^G}}) \tilde{G}_t + \frac{\eta_{F_X}}{(1-\tau_{W} W)} \tilde{x}_t - \frac{\beta(1-\delta) \eta_{F_X}(1-2x)}{(1-\tau_{W} W)} \tilde{x}_{t+1} - \frac{\beta(1-\delta)(1-x) n_{F_X}}{(1-\tau_{W} W)} (\sigma \frac{Y_{Y^G}}{Y_{Y^G}}) (\tilde{Y}_t - \tilde{Y}_{t+1}) - \sigma \frac{Y_{Y^G}}{Y_{Y^G}} (\tilde{G}_t - \tilde{G}_{t+1})
\]

Log-linearize the marginal cost equation (31) I get the following:

\[
m\tilde{c}_t = M \tilde{W}_t + \frac{F_x}{mcc} \tilde{x}_t - \frac{\beta(1-\delta) F_x}{mcc} \tilde{x}_{t+1} + \frac{\beta(1-\delta) F_x \sigma}{mcc} [\frac{Y_{Y^G}}{Y_{Y^G}} (\tilde{Y}_{t+1} - \tilde{Y}_t) + \frac{Y_{Y^G}}{Y_{Y^G}} (\tilde{G}_t - \tilde{G}_{t+1})]
\]

The next step is to substitute the wage equation into the marginal cost equation, and use the labor market tightness equation that \(x_t = (\tilde{Y}_t - (1-\delta)(1-x) \tilde{Y}_{t-1})/\delta\).

We can solve the marginal cost equation into the simplest form of only output and fiscal policy terms. Lastly substitute into the New Keynesian Phillips curve I arrive at the AS equation.