Random Dictatorial Domains

Shurojit Chatterji *, Arunava Sen † and Huaxia Zeng ‡

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Abstract

We consider strategy-proof social choice functions in a pure voting model where the set of alternatives has at least three elements and preference orderings over these alternatives are strict. The classic Gibbard-Satterthwaite Theorem establishes that the complete domain of preferences is a dictatorial domain in that every unanimous and strategy-proof social choice function defined on it is necessarily dictatorial. Gibbard (1977) proves that the dictatorial finding extends to random social choice functions that satisfy unanimity, when the strategy-proofness requirement is modelled as requiring that the lottery over alternatives resulting from truth-telling stochastically dominates lotteries resulting from misrepresentations of preferences.

The complete domain is thus an instance of a domain that is dictatorial for both deterministic social choice functions (SCF’s) and random social choice functions (RSCF’s). This paper addresses the conjecture “are domains that are dictatorial for SCF’s necessarily dictatorial for RSCF’s?” We show that this conjecture is false by showing that a Linked Domain of preferences, which is known to be dictatorial for SCF’s (Aswal, Chatterji and Sen (2003)), need not be dictatorial for RSCF’s. Specifically, we provide conditions under which an abstract domain of preferences admits non-dictatorial RSCF’s that are strategy-proof and unanimous and show that particular Linked Domains satisfy these conditions.

*Singapore Management University, Singapore.
†Indian Statistical Institute, New Delhi, India.
‡Singapore Management University, Singapore.
Next we identify two alternative conditions which when added to Linked Domains renders them dictatorial for RSCF’s. The first of these conditions, Condition $\phi$, requires the existence of an alternative that is connected (in the sense of Aswal et al (2003)) to every other alternative. The second condition, Condition $\psi$, uses a stronger notion of connectivity (in the sense of Chatterji, Sanver and Sen (2010)) but weakens the requirement that there exists an alternative that is connected to every other alternative.

The aforementioned results are obtained for the case of two voters. We provide an induction argument that extends the dictatorial finding for RSCF’s to the case of $N > 2$ voters for Linked Domains that satisfy additionally Condition $\psi$. 