Social Choice for Anaphora Resolution

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Abstract
Disambiguation of pronoun reference has been an important issue for both theoretical and computational linguists. While linguistic theories on binding conditions eliminate impossible readings to a certain extent, many inter-sentential anaphora remain ambiguous between bound variable reading and coreference with salient discourse entities, as in *John said he broke his leg* where *he* can refer either to *John* or someone else previously mentioned [10]. This paper addresses such issues by application of computational social choice [2] and considers pronoun resolution as a social choice among discourse participants, where the speaker dictates the decision—the social welfare function returns the identical preference for any profile.

1 Binding Conditions, Proximity and Saliency

Pronouns are known for their referential ambiguities. In a dialogue taken from a speech corpus in (1), the italicized pronoun *she* has multiple candidates for its antecedent—Emma, Lisa and Lisa’s mom. Proximity and saliency of antecedents have been considered to be key factors to decide [8]. In (1), the most proximate antecedent *her (Lisa)’s mom* is identified to be the antecedent for *she*.

(1) Frances: ...Not while Emma’s not here. You know Emma
Billy: Mm.

1
Frances: she’s, she was walking with Lisa and I weren’t there and her Mum sh– jus–, like she muc–, she mucks about a lot and she told Leigh that if he don’t serve her he’s gonna die, she’s gonna punch him right! Cos she’s quite big, you know....

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However, proximity does not always resolve referential ambiguity of pronouns.

(2) a. John$i$ likes him$_{i/j\neq i}$.

b. John$_i$ said he$_{i/j}$ likes himself$_{i/j}$.

*Him* in (2a) unambiguously means someone other than the closest *John*—some discourse-salient entity, as indicated by the indices. When *he* is embedded under the matrix clause as in (2b), the pronoun becomes ambiguous between *John* and someone else salient in the discourse.

Linguistic binding theory [3, 10] well explains the phenomena. Antecedents are called *binders*, which bind bindees that are anaphoric pronouns, such as *him* or *himself*.

- Condition B: pronouns must be free in their local domain.

Pronouns are *free* when they are not bound by the antecedent by means of coindexing and c-commanding relation. *C-command* is roughly equivalent to precedence, with some restrictions.

However, binding conditions are not by themselves sufficient. Consider another example:

(3) Anna: Bill$_j$ is a good goalkeeper.

Kim: John$_i$ said he$_{i/j}$ broke his$_{i/j}$ leg recently.

(3) is ambiguous in four ways and can have either one of the following interpretations:

(4) a. John broke John’s leg

b. John broke Bill’s leg.
c. Bill broke Bill’s leg.

d. Bill broke John’s leg.

He and his can be bound by either John or another salient discourse entity Bill. The binding theories have no way of disambiguating these pronouns since there is no way of knowing speaker intention. Proximity does not predict the readings in (4b-d) either.

2 Social Choice Theory

2.1 Collective Decision Making

When multiple interpretations are available for pronouns, speaker’s intention decides the reading. Although Social Choice Theory [1, 9, 11, 6] has not yet been studied from linguistic perspective in my knowledge, it enables theoretical incorporation of speaker’s intention to pronoun resolution. This section will overview social choice theory which has been studied in economics and political science and explains decision making by a group of people. When each member has different views and preferences, aggregation of every individual’s view results in a single view. Computational social choice theory [2, 5] is a fairly new theory developed from the classical social choice theory.

Typically, social choice theory explains collective decision making in case of voting and has solved the problems with majority decision. Elections may follow the majority rule, which ranks one candidate $x$ above another candidate $y$ if and only if a majority of the individuals do. When people vote for their preferred candidate according to their own ranking among the candidates, aggregation of people’s preferences helps in selecting a certain candidate. Note that “$\alpha \succ \beta$” denotes $\alpha$ is preferred to $\beta$. For example, the dominating ranking in the situation in (5) would be “Obama (o) $\succ$ Clinton (c)” and “Clinton $\succ$ Mcain (m),” that is, “Obama $\succ$ Clinton $\succ$ Mcain,” which is a preferred ranking by the majority if the preference relation is transitive. However, this conflicts with “Mcain $\succ$ Obama,” which is also preferred by three people (Condorcet Paradox [4]).

(5) a. Anna (a): Obama $\succ$ Clinton $\succ$ Mcain

b. Kim (k): Clinton $\succ$ Mcain $\succ$ Obama
c. Heather (h): Obama $\succ$ Clinton $\succ$ Mcain

d. George (g): Mcain $\succ$ Obama $\succ$ Clinton

e. Nathan (n): Mcain $\succ$ Clinton $\succ$ Obama

f. Social preference: oRc $\land$ cRm $\land$ mRo

2.2 Arrow’s Social Welfare Function

Arrow’s axiomatic methods of social choice theory have tackled the above-mentioned problems of aggregation [1]. Preferences are ordering between alternatives and should satisfy the following two axioms. When $R$ stands for a knowledge of all pairs and $x, y$ and $z$ for alternatives.

**Axiom 1** For all $x$ and $y$, either $xRy$ or $yRx$.

**Axiom 2** For all $x, y,$ and $z$, $xRy$ and $yRz$ imply $xRz$.

Axiom I states that the relation $R$ is connected—every candidate is related to each other. Relations that satisfy Axiom II are transitive. In (5), $N$, a finite set of individuals or voters, consists of five individuals and $\chi$, a nonempty set of alternatives or candidates, has three members. Let $L(\chi)$ denote the set of all linear orders on $\chi$. A profile $R$ is a vector of linear orders, or preferences. $R_i$ is a vector of preferences of an individual $i$. $N_{x>y}^R$ denotes the set of individuals that prefers the candidate $x$ to $y$. Supposing $R$ the profile given in this model, $N_{o>c}^R$ is a set of people who prefers Obama to Clinton, that are, Anna, Heather and George.

(6) a. $N = \{a, k, h, g, n\}$

b. $\chi = \{o, c, m\}$

c. $R = (R_a, R_k, R_h, \ldots) \in L(\chi)^N$

d. $N_{o>c}^R = \{a, h, g\}$

Individual’s preferences are aggregated and returns a single preference order, that results in collective decisions. A *social welfare function* (SWF) $F$ is a function which takes individual’s preferences and returns collective preference which is supposed to represent people.
(7) SWF F: \( L(\chi)^{|N|} \rightarrow L(\chi) \)

Arrow demonstrated that any SWF for three or more alternatives that satisfies the Pareto condition and IIA must be a dictatorship.

**Theorem 1** *(General Possibility Theorem)* If there are at least three alternatives which the members of the society are free to order in any way, then every social welfare function satisfying Conditions 1 and 2 and yielding a social ordering satisfying Axioms I and II must be either imposed or dictatorial.

**Condition 1** *(Pareto condition)* A SWF \( F \) satisfies the Pareto condition if, whenever all individuals rank \( x \) above \( y \), then so does society: \( N_{z>y}^R = N \implies xF(R)y \)

Condition 2 states that the relative ranking of two candidates remain unchanged regardless no matter how other candidates are ranked.

**Condition 2** *(Independence of irrelevant alternatives (IIA))* A SWF \( F \) satisfies (IIA) if the relative social ranking of two alternatives only depends on their relative individual rankings: \( N_{z>y}^R = N_{z>y}^{R'} \implies xF(R)y \Leftrightarrow xF(R')y \)

When an individual’s preference dominates the collective preference, that individual is called a dictator. *Dictatorship* is a SWF that maps any member’s profile to a single individual profile.

### 3 Application of Social Choice Theory to Pronoun Resolution

#### 3.1 Anaphora Resolution as Social Choice

This section will show that social welfare function for pronoun resolution satisfies Arrow’s General Possibility Theorem by satisfying Axioms I, II, Pareto Condition and IIA but demonstrating dictatorship.

Since the referents of pronouns can be ambiguous as discussed in section 1, pronoun resolution can be compared with voting by multiple voters—in this case, discourse participants. The candidates or choices would be different interpretation of the sentence.
For example, the first sentence in the following dialogue in (8) is ambiguous between two interpretations: (i) the pronoun *him* refers to *John*, or (ii) *him* means someone else, Bob, who is salient in the discourse.

(8) Chris: John likes *him*. I mean, John likes Bob.
Naomi: I thought you meant John liked himself.

Such ambiguity actually does not exist at all since the first reading is impossible, as Binding Condition B properly eliminates the first reading. *Him* should be free in the locality.

In the following dialogues (9) and (10), the referent of the pronouns *he* in both (9) and (10) is ambiguous between *John*, the binder, and some other discourse referent. Suppose that the speaker meant the referent of *he* to be *Bob* who appeared in their previous discourse, while the hearer interpreted *him* to be *John*.

(9) Chris: John said he broke his leg.
Naomi: Did he? John looked fine when I saw him this morning.
Chris: It is Bob who broke his leg.
Naomi: I thought you were talking about John.

(10) Naomi: I saw Bob’s car dented.
Chris: John said he met with an accident.
Naomi: John, too?
Chris: I mean, it was Bob who met an accident.
Naomi: I thought you said John was hit.

*He* in the embedded clause has been considered as a bound variable bound by *John* [7]. Since both variable binding and coreference with salient discourse entities are available, ambiguities remain. Binding Conditions alone do not eliminate such ambiguity.

Therefore, I argue that identifying the antecedent of pronouns is a social choice and SWF decides the antecedent.

(11) a. Individuals I = {c, n}
b. Candidates \( \chi = \{j, b\} \)
c. Ordering \( jR_n b \land bR_n j \)
d. Denote the set of linear orders on $\chi$ by $L(\chi)$. Preferences (or ballots) are taken to be elements of $L(\chi)$.

e. A profile $R = (R_c, R_n) \in L(\chi)^{|I|}$ is a vector of preferences.

f. A social choice function (SCF) or voting rule is a function $F : L(\chi)^{|I|} \rightarrow 2^{\chi} \setminus \{\emptyset\}$ mapping any given profile to a nonempty set of winners.

g. A social welfare function (SWF) is a function $F : L(\chi)^{|I|} \rightarrow L(\chi)$ mapping any given profile to a (single) collective preference order.

h. $R_c = F(R_c) = F(R_n) = R$

In dialogues (9) and (10), there are two voters, Chris and Naomi who are discourse participants. The decision is made regarding the referent of the pronoun $he$. The preferences between the candidates vary between the individuals. The social welfare function returns a single preference order for the references since ambiguities are resolved during the conversation.

### 3.2 Axioms I and II

There are three possible antecedents for $she$ in (1)—Emma, Lisa and Lisa’s mother. Let us say that Billy (b) prefers Emma (e) to Lisa (l), and also Lisa to Lisa’s mother (m) to be the antecedent. On the other hand, the speaker Francis (f) prefers Lisa’s mother to the other two, Lisa’s mother to Lisa, and Lisa to Emma according to the proximity. All three candidates are ordered in accordance with Axiom I.

\begin{align*}
(12) \quad & a. \quad eR_b l \land lR_bm \\
& b. \quad mR_f l \land lR_fe 
\end{align*}

Transitivity also holds for pronoun antecedent preferences. (12a) and (12b) each implies (13a) and (13b).

\begin{align*}
(13) \quad & a. \quad eR_b lR_bm \\
& b. \quad mR_f lR_fe 
\end{align*}
3.3 Pareto Condition

Pareto condition states if, whenever all individuals rank $x$ above $y$, then so does the society. Suppose the situation when Billy, Francis and another discourse participant Heather, who stayed quiet during the conversation, all prefer the same ranking in (1).

\[(14) \quad mR_b l \land lR_e l \land mR_f l \land lR_f e \land mR_h l \land lR_h e \to mR_l \land lR_e\]

Therefore, $N_{m \succ l \succ e} = N$ implies $mF(R)lF(R)e$

When everyone understands that Francis referred to Lisa’s mom with *her*, the society which consists of discourse participants also refers Lisa’s mom to be the antecedent. Therefore, Pareto condition is met.

By means of reduction, I will show that the assumption that, all individuals rank $x$ above $y$ but society does not rank $x$ above $y$, leads to contradiction.

**Proof** Let us prove the contrary does not hold.

Assume,

$N_{x \succ y} = N$ implies $yF(R)x$ —(A)

Suppose,

$jR_c b \land jR_n b \to jRb$

Therefore, (A) does not hold.

QED.

3.4 IIA

A SWF $F$ satisfies IIA if the relative social ranking of two alternatives only depends on their relative individual rankings. Suppose the dialogue in (9) is modified into (15) and (16). The preference relations are denoted by $R$ for (15) and $R'$ for (16). As shown in (17), the relative rankings between Bob and John remain unaffected by irrelevant candidate Victor’s ranking; therefore IIA is satisfied.

(15) Chris: Bob is a good skier. But John said he broke his leg.

Naomi: Did he? Poor Bob!

(16) Chris: Victor is a good skier and so is Bob. But John said he broke

his leg.

Naomi: Did he? Poor Bob!
Then, $N_{b > j}^R = N_{b > j}^{R'}$ implies $bF(R)j \Leftrightarrow bF(R')j$

Below, by means of reduction, I will show that it is contradictory to say that $N_x^R > y$ implies $xF(R)y \Leftrightarrow xF(R')y$.

**Proof** Let us assume the following:

$bR_{c,j} \wedge bR_{n,j} \wedge bR'_{v,j} \wedge vR'_{c,j} \wedge bR'_{n,j} \wedge jR'_{v,j}$

Then, $N_{b > j}^R = N_{b > j}^{R'}$

Suppose,

$bF(R)j \Leftrightarrow bF(R')j \quad -(B)$

However,

$bF(R)j \Leftrightarrow bF(R')j$

Therefore, (B) is contradictory.

QED.

### 3.5 Dictatorship

Corrections made by the speaker in dialogues suggest that the speaker dictates the decision. The different preference rankings are aggregated to a single choice made by the speaker. The hearers are forced to interpret the speaker utterance as he meant.

What is called the plurality rule does not apply to pronoun resolution. The plurality rule is one to elect the candidate ranked first most often. Although voting usually satisfies the plurality rule, the selection of possible interpretation of pronouns does not depend on the sum of the discourse participants who share the same interpretation. Rather, pronoun resolution is dominated, or dictated, by the speaker’s meaning.

(18) Dictator: speaker $c \in I$

The speaker dominates the choice of referents and reserves right to correct the hearer’s interpretation.

$F(R) = R_c$ for any profile $R$, that is, the outcome is always identical to the preference supplied by the dictator.

**Proof** Suppose

$F(R) \neq R_c$ where $c$ is a speaker

Suppose a following dialogue:
(19) 11 Chris: Bob is a good skier. But John said he broke his leg.
12 Naomi: Did he? Poor Bob!
13 Chris: No. I mean John broke his leg.

\[ F(R) = R_n \text{ until the line 2. However, } F(R) = R_c \text{ after all at line 3.} \]
Therefore, the premise leads to contradiction.
QED.

**Lemma 1** The choice of antecedent for anaphora is a social welfare function which satisfies conditions 1 and 2.

### 4 Conclusion

This paper is the first attempt to apply social choice theory to linguistic problems in my knowledge. While anaphora resolution has been a problem for natural language processing and theoretical linguistics, it is considered to be a social choice function dictated by a dictator, the speaker.

**References**


