A Sparse Measure of Liquidity, and the Impact of Monetary Policy

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\textit{Extended Abstract - (Very) Preliminary Draft}

We incorporate techniques from recent advances in the macroeconomic forecasting literature to acquire a sparse representation of liquidity for the US stock market. This common market liquidity measure allows us to examine the interaction between market and funding liquidity, and to gauge the impact of monetary policy while taking into account these macro-financial linkages. Most importantly, we examine the effects of the Federal Reserve's recent episodes of unconventional policy, with its large scale asset purchases, using a large-scale Bayesian VAR. We find a differential impact of each quantitative easing period on market liquidity, with a strong positive impact found during the first period, a detrimental effect on liquidity during QE2. Finally, we also incorporate a Bayesian factor-augmented framework, in order to track back the effect of monetary policy on each individual liquidity measure. Pushing this exercise to the limit, gives us a more detailed insight on underlying importance of each dimension of liquidity.

Keywords: market liquidity; funding liquidity; monetary policy; transaction costs; price impact; trading volume; macro-finance linkages; financial stability; Bayesian VAR.

JEL Classification: G01, G12, G14, E44

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We would like to thank Haroun Mumtaz and Micheal Flemming for helpful suggestions in setting up the methodological framework. Moreover, we greatly benefited from discussions with Marco Pagano and Domenico Giannone. All remaining errors are our own. The views expressed in this paper are those of the author and do not necessarily represent those of the Central Bank of Ireland.
1 Introduction

Despite the popularity of the Large Asset Purchase Programs (LSAPs), their effectiveness has been heavily debated (Kuttner, 2018). More specifically, there are questions on how the effect of these measures on financial markets trickled through to the wider economy (Boneva et al., 2016). One recent strand of literature, examines the impact of such unconventional monetary policy measures on Treasury market liquidity, and find mixed effects. For example, the IMF (2015) highlights that a central bank can take on the role of a large and reliable market maker, but at the same time warn that LSAPs can have adverse effects on liquidity. Christensen and Gillan (2018) point at the possibility for QE programs to reduce priced frictions to trading. While Bonner et al. (2018) warn that the effects of QE are mixed. Moreover, there is increased attention for a liquidity channel in order to explain monetary policy transmission during recent years. We find both empirical and theoretical evidence of this in Gagnon et al. (2011), Adrian et al (2017), and Lagos and Zhang (2018).

The goal of our paper is threefold. Firstly, we construct a sparse measure of market liquidity that incorporates the full set of information on liquidity. This cloud of information is shown in figure 1. We start at the stock level, by constructing market liquidity measures for the S&P 500, tracking additions and deletions, since 1962. As common in the literature, the data is standardized.1 We repeat this exercise for each of the thirty-six individual liquidity measures that we incorporate in our analysis. We then distill the common factor from these separate market liquidity measures. Figure 2 depicts our common market liquidity measure together with historical periods of stress and recessions. In this context, we can also investigate the link between market liquidity and funding liquidity, highlighted through multiple measure in figure 3.

Secondly, we gauge the effect of unconventional monetary policy on liquidity in the setting of a large Bayesian VAR which allows us to incorporate the important macro-financial linkages. We apply conditional forecasting techniques (Waggoner and Zha, 1999) to create a no policy world. This setting also allows for a more structural analysis where we can incorporate impulse response function using a mix of zero and sign restrictions (following D’Amico et al., 2017) in order to analyze the size of the liquidity channel.

1However, given several institutional changes (tick size, regulatory adjustments) over the sample we are confronted with breakpoints and we can detect shifts the mean of many variables over time. This irregularity in the observations can be overcome by standardizing over a smaller moving window of 60 observations, equivalent to 5 years.
Finally, relaxing the idea of sparsity, and incorporating the full set of liquidity in a Factor-augmented Bayesian VAR, allows us to pinpoint which dimensions of liquidity get affected most by changes in monetary policy over time. This allows us to better understand the different dimensions of liquidity.

2 Methodology

2.1 Common Liquidity Measure

When thinking about the concept of liquidity, we encounter several dimensions each explaining a specific aspect of liquidity (Kyle, 1985, Pastor and Stambaugh, 2003, Amihud et al., 2005, Fong et al, 2013). This multi-layered nature has inspired empirical economists to construct many different methods to measure this latent concept. However, there is very little unanimity in the literature on how to model this large array of information in a sparse or comprehensive way. We apply state of the art statistical techniques developed over the last decades, and more specifically within the (Bayesian) forecasting literature, in order to extract the most useful signals contained in the information set, without throwing away any useful information. Our approach is similar in spirit to Hallin et al. (2011), but we extend the information set to include most of the existing market liquidity measures in the literature.

Dynamic factor models have often been applied in macro-econometrics for both forecasting and structural analysis (Stock and Watson, 2016), and provide a reliable statistical framework for the estimation of synthetic indices, for example of business cycle conditions (Bok et al., 2017). A dynamic factor model assumes that many observed variables \((y_{1,t}, \ldots, y_{n,t})\) are driven by a few unobserved dynamic factors \((f_{1,t}, \ldots, f_{r,t})\), while the features that are specific to individual series, such as measurement errors, are captured by idiosyncratic errors \((e_{1,t}, \ldots, e_{n,t})\). We can depict the model through the following equation:

\[
y_{i,t} = \lambda_{i,1}f_{1,t} + \ldots + \lambda_{i,r}f_{r,t} + e_{i,t} \text{ for } i = 1, \ldots, n
\]  

Commonly applied techniques as ridge and lasso analysis, exploring the literature on sparsity and model selection.

We incorporate thirty-six liquidity proxies, as similarly explored by Garabedian and Inghelbrecht (2017) representing eight different spheres of liquidity, based on spread measures, Roll measures, zero returns measures, Fong measures, effective Tick measures, Amihud measures, volume measures, order flow measure. All measures are expressed as such to denote illiquidity, and all measures are constructed on a monthly frequency. For this purpose, we use daily data from the CRSP database, starting from 1962 until the end of 2013.
which links the data \( y_{i,t} \) to the \( r \) latent common factors \( f_{1,t}, \ldots f_{r,t} \), through the factor loadings \( \lambda_{1,t}, \ldots \lambda_{r,t} \).

Our methodology, where we extract a latent variable from an observable dataset, is close in spirit to the current literature on the natural rate of interest (Holston, Laubach and Williams, 2016), and on shadowrates (Lombardi and Zhu, 2014, Wu and Xia, 2016).

2.2 Large Scale Bayesian VAR

When applying a vector autoregressive (VAR) framework\(^5\), even systems of a moderate size yield a large number of coefficients that have to be estimated, hence the risk of over-parametrisation. Let \( Y_t = (y_{1,t}, y_{2,t}, \ldots, y_{n,t}) \) be a \( n \times 1 \) vector of endogenous data,

\[
Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \ldots + A_p Y_{t-p} + C X_t + \varepsilon_t
\]

where \( A_1, A_2, \ldots, A_p \) are \( p \) matrices of dimension \( n \times n \) containing the parameters for the endogenous variables. Similarly, \( C \) is an \( n \times m \) matrix with the coefficients for the exogenous regressors, which are captured by an \( m \times 1 \) vector \( X_t \) (featuring the constant terms, the time trends, and other exogenous data series). Lastly, \( \varepsilon_t = (\varepsilon_{1,t}, \varepsilon_{2,t}, \ldots, \varepsilon_{n,t}) \) is a vector of residuals following a multivariate normal distribution: \( \varepsilon_t \sim N(0, \Psi) \).

In a Bayesian framework, the technique of applying shrinkage offers a practical solution to overcome the curse of dimensionality, through the incorporation of prior beliefs on the coefficients.\(^6\) The standard approach is to utilize a Minnesota prior, as suggested by Litterman (1986).\(^7\) Practically, this boils down to shrinking the diagonal elements of \( A_1 \) to one, and the remaining coefficients in \( A_1, \ldots, A_p \) to zero. This prior specification embodies the belief that more recent lags carry more reliable info in comparison with more distant ones, and that the own lags should be able to explain more of the variation of a given variable than the lags of other variables. Appendix A1 provides a technical background.

Banbura et al. (2010) convincingly demonstrate that a Bayesian VAR (BVAR) is the appropriate tool for forecasting and structural analysis when one conditions

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\(^5\)Every variable in a VAR is explicitly modeled as a function of the other variables, thus capturing all the mutual interlinkages.

\(^6\)More technically, shrinkage resolves the problem of inverting an otherwise unstable large covariance matrix.

\(^7\)Useful modifications were later proposed by Kadiyala and Karlsson (1997) and Sims and Zha (1998).
on a large information set, allowing researchers to incorporate a large set of macroeconomic and financial variables. The theoretical motivation for working with large BVARs can be found with De Mol et al. (2008), who argue that for time series with strong collinearity, as present in macroeconomic data, Bayesian forecasts converge to the optimal value as long as the tightness of the prior is increased with the number of variables. The practical use of a large Bayesian structure is motivated by several reasons. Firstly, several empirical papers determine that adding information helps to improve forecasts (Beauchemin and Zaman, 2011; Bloor and Matheson, 2011), with medium-sized (twenty variable) BVARs already obtaining significant gains (Giannone et al, 2015). Similarly, Christiano et al. (1999) warn of the omission of variables having negative effects on forecasting and structural analysis. Secondly, central banks need to incorporate information from a large number of variables when performing monetary policy decisions (Iversen et al., 2014). Specifically, when targeting inflation, it is important to track a large cross-section of data series closely in order to understand the future developments. Finally, Gambacorta et al. (2014) highlight the added value of including variables capturing uncertainty, financial turmoil and economic risk to unravel exogenous changes in the central bank balance sheet from endogenous interventions. Hence, the inclusion of many macroeconomic and financial variables is necessary to capture possible spillovers between the real and financial economy.

2.2.1 Dummy observation prior

The above mentioned prior structure by Litterman (1986) has three important shortcomings.\(^8\) Firstly, to get the posteriors, the matrix inversion leads to problems, with a \(q \times 1\) matrix (for the mean) and a \(q \times q\) matrix (for the variance). Therefore, dealing with this step in the Gibbs sampling algorithm increases the computational burden significantly.\(^9\) Secondly, with the Minnesota prior, no prior covariance is assumed among VAR coefficients. Finally, the Minnesota prior does not allow us to incorporate priors about the combination of coefficients in each equation or across equations. This is particularly useful when working with unit root processes (the sum of coefficients on lags of the dependent variable in each equation equals one) or cointegrated processes.

Hence, we use a variation of the Minnesota prior, referred to as a dummy obser-

\(^8\)A comprehensive overview of the technical aspects with respect to the Bayesian estimation can be found in Diepe et al. (2015) and Mumtaz and Blake (2014).

\(^9\)With \(q = n (np + m)\), \(n\) being the number of endogenous variables, \(p\) the number of lags, and \(m\) the number of exogenous variables.
vation prior.\textsuperscript{10} Intuitively, this prior would involve generating artificial data from the model assumed under the prior and mixing this with the actual data. Moreover, the weight placed on the artificial data determines the tightness of the prior. The biggest advantage of this methodology is that it can match the Minnesota moments (without having matrix inversion issues), while simultaneously being consistent with unit root or cointegration processes. Appendix A2 contains the technical details.

Following Banbura et al (2010), we implement this prior\textsuperscript{11} by appending $T_d$ dummy observations, as expressed in $Y_d$ and $X_d$, to the system:

$$Y_d = \begin{pmatrix} diag(\delta_1 \sigma_1, \ldots, \delta_n \sigma_n)/\lambda \\ 0_{n(p-1) \times n} \\ \vdots \\ diag(\sigma_1, \ldots, \sigma_n) \\ 0_{1 \times n} \end{pmatrix}, \quad X_d = \begin{pmatrix} J_p \otimes diag(\delta_1 \sigma_1, \ldots, \delta_n \sigma_n)/\lambda & 0_{np \times 1} \\ \vdots & \vdots \\ 0_{n \times np} & 0_{n \times 1} \\ 0_{1 \times np} & c \end{pmatrix}$$ (3)

with $J_p = diag(1, 2 \ldots, p)$. The first block of dummies represents the prior beliefs on the AR coefficients. The second block summarizes the prior for the covariance matrix. Finally, the third block describes the uninformative prior for the intercept. We retrieve the required structures $Y^*$ and $X^*$ by adding dummies $Y_d$ and $X_d$ to the original data:

$$Y^* = [Y, Y_d], \quad X^* = [X, X_d] \quad (4)$$

Using this appended data, the conditional distributions can be integrated in the Gibbs sampling algorithm. The results are based on 15000 draws from the Gibbs sampler, with a burn-in of 10000.

### 2.2.2 Conditional Forecasting

In this analysis, we focus on conditional forecasting\textsuperscript{12} techniques, popularized by Waggoner and Zha (1999), which allow us to examine exogenous paths or to apply shocks to any variable in the system, in a model-consistent manner (Bloor and Matheson, 2011). The inclusion of prior knowledge, even when not perfect, on the future path of certain variables may carry useful information for forecasting. This technique is commonly used by central banks and other institutions (Banbura et al.,

\textsuperscript{10}We modify the code provided by Blake and Muntaz (2014) to handle the large dataset. Moreover, we include multiple restrictions simultaneously and apply a longer forecasting horizon. We thank Muntaz for valuable feedback while incorporating these extensions.

\textsuperscript{11}Where the coefficients have a normal prior and the covariance matrix has a normal inverted Wishart prior: $vec(B) | \Psi \sim N(vec(B_0), \Psi \otimes \Omega_0), \Psi \sim iW(S_0, \alpha_0)$.

\textsuperscript{12}Popular applications of this methodology can be found in Jarocinski and Smets (2008), Stock and Watson (2012).
Appendix A3 describes the restrictions necessary to produce this strand of forecasts.

Consider a VAR(1) model (Blake and Mumtaz, 2014):

$$Y_t = c + BY_t + A_0 \varepsilon_t \quad (5)$$

with $Y_t$ representing a $T \times N$ matrix of endogenous variables, and $\varepsilon_t$ denoting the uncorrelated structural shocks and $A_0A'_0 = \Sigma$.\textsuperscript{13} When we iterate equation (5) $K$ times forward, we retrieve

$$Y_{t+K} = c \sum_{j=0}^{K} B^j + B^j Y_{t-1} + A_0 \sum_{j=0}^{K} B^j \varepsilon_{t+K-j} \quad (6)$$

Hence, when we place restrictions on the future path of the $J^{th}$ variable in $Y_t$, this also induces restrictions on the other variables in the system. If we re-structure equation (6) this becomes more visible:

$$Y_{t+K} - c \sum_{j=0}^{K} B^j - B^j Y_{t-1} = A_0 \sum_{j=0}^{K} B^j \varepsilon_{t+K-j} \quad (7)$$

When we constrain some of the variables in our dataset to a fixed path, this means that the future innovations on the right hand side of the equation will have restrictions as well. These constraints on future innovations are defined in Waggoner and Zha (1999) as:

$$R \varepsilon = r \quad (8)$$

The elements of $r$ contain the path for the constrained variables minus the unconditional forecasts of the constrained variables. The elements of the matrix $R$ are the impulse responses of the constrained variables to the structural shocks $\varepsilon$ over the desired forecasting horizon. A least square solution for the constrained shocks in (8) is given by Doan et al. (1983):

$$\varepsilon = R' (R'R)^{-1} r \quad (9)$$

Inserting these constrained innovations in equation (5) allow us to calculate the conditional forecasts.

### 3 Analysis

Our sparse representation of liquidity allows us to examine the impact of monetary policy shocks on liquidity, thus unraveling important macro-financial linkages that change over time. Our analysis is related to Christensen and Gillan (2018), which looks at the impact of the recent asset purchase programs on a measure of liquidity premiums in TIPS yields and inflation swap rates.\textsuperscript{14} Practically, we use a seventeen

\textsuperscript{13}$\Sigma$ represents the variance of the reduced VAR residuals.

\textsuperscript{14}Theoretical aspects of this transmission have recently been explored by Lagos and Zhang (2018)
variable Bayesian vector autoregressive system and incorporate dummy observation prior (Banbura et al, 2015). The data is summarized in Table 1. We look at the impact of both conventional and unconventional monetary policy on liquidity across the whole sample period, from mid-60s up to the end of 2013. This allows us to capture multiple historical crisis episodes (oil crisis, the 1987 crisis, the Asian crisis, etc.), and moreover allows us to inspect the impact of the recent unconventional monetary policy (UMP) on market liquidity. The policy measures we incorporate in our analysis are depicted in figure 4 and 5.

More specifically, we differentiate between the three main asset purchase programs by the Federal Reserve. Table 2 highlights the different periods and its main characteristics. We construct conditional forecasts for each subperiod based on the assumption that both the Federal Funds rate and the Federal Reserve total assets would remain unchanged, thus depicting a counterfactual no policy world. Our analysis shows that QE1 has a strong positive impact on market liquidity, similar to what Gertler and Karadi (2012) find for a more general macroeconomic impact. In contrast, during QE2 monetary policy seems to have a detrimental effect on monetary policy. Finally, the least round of easing has no significant effect on market liquidity in our analysis. Our results are summarized in figure 6.

We can further extend our analysis by moving away from our sparse representation, and now explicitly incorporating all individual thirty six variables in a Bayesian factor augmented VAR setting, where we can look at the impact of monetary policy on each specific liquidity measure separately. Computationally, the estimation is very similar to the above-mentioned methodology. However, in this framework, we can distill the impact of monetary policy on every individual aspect of liquidity. This analysis gives us a better understanding of the importance of each liquidity measure in driving the results above.

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15 Measured by the policy rate and variation in the Federal Reserve total assets (Benati and Baumeister, 2013; Kapetanios et al., 2012)

16 QE1 starting from November 2008, QE2 in November 2010, and QE3 from September 2013, to then taper from January 2014 onward, with the program ending in October 2014. Hence, we only look up to the end of 2013 in our analysis of monetary policy shocks.
References


Bok, Brandyn, Daniele Caratelli, Domenico Giannone, Argia Sbordone, and Andrea Tambalotti, 2017, “Macroeconomic Nowcasting and Forecasting with Big Data”, Federal Reserve Bank of New York Staff Reports, no. 830.


Table 1: List of the variables included in the BVAR

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<tbody>
<tr>
<td><strong>Prices</strong></td>
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<tr>
<td>1</td>
<td>Oil Prices</td>
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<td>2</td>
<td>Consumer Price Index</td>
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<td>3</td>
<td>Core PCE</td>
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<td><strong>Labour market</strong></td>
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<td>4</td>
<td>Civil Unemployment</td>
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<td></td>
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<tr>
<td><strong>Cyclical</strong></td>
<td></td>
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<td>5</td>
<td>Real Gross Domestic Product (RGDP)</td>
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<td>6</td>
<td>Industrial Production</td>
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<td><strong>Monetary Policy</strong></td>
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<td>7</td>
<td>Effective FFR</td>
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<tr>
<td>8</td>
<td>Federal Reserve Total Assets</td>
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<tr>
<td>9</td>
<td>Interbank IR Spread</td>
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<td>10</td>
<td>Non-financial private sector credit</td>
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<tr>
<td><strong>Volatility, Uncertainty and Risk (Financial)</strong></td>
<td></td>
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<tr>
<td>11</td>
<td>Common Market Liquidity Measures</td>
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<tr>
<td>12</td>
<td>VIX</td>
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<tr>
<td>13</td>
<td>TED Spread</td>
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<td>14</td>
<td>Excess Bond Premium</td>
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<td>15</td>
<td>Option Adjusted Spread</td>
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<tr>
<td><strong>Asset Prices</strong></td>
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<tr>
<td>16</td>
<td>Russel 2000 Price Index</td>
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<tr>
<td>17</td>
<td>Real Broad Effective ER</td>
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Table 2: Scenario analysis over different subperiods

<table>
<thead>
<tr>
<th>Program</th>
<th>Start (announcement)</th>
<th>Purchases</th>
<th>Length</th>
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<tbody>
<tr>
<td>1</td>
<td>QE1 2008-12 (2008-11, 2009-03\uparrow)</td>
<td>MBS</td>
<td>8Q</td>
</tr>
<tr>
<td>2</td>
<td>QE2 2010-11 (2010-08)</td>
<td>Treasury Securities</td>
<td>8Q</td>
</tr>
<tr>
<td>3</td>
<td>QE3 2012-09 (12)</td>
<td>Bond Purchases</td>
<td>5Q</td>
</tr>
<tr>
<td>4</td>
<td>Taper 2014-01 (2013-06)</td>
<td>Purchases ↓</td>
<td>3Q</td>
</tr>
<tr>
<td>5</td>
<td>QE End 2014-09</td>
<td>-</td>
<td>4Q</td>
</tr>
</tbody>
</table>
Figure 1: Multiple Dimensions of liquidity

The figure depicts the information cloud with all the measures of liquidity incorporated in the analysis.

Figure 2: Market Liquidity and Financial Stress

The figure shows the evolution of our constructed common liquidity measures, together with recessions (highlighted by bars) and important financial stress episodes (depicted by the dotted vertical lines).
Figure 3: Market Liquidity and Funding Liquidity

These figures show the link between our market liquidity measure and multiple measures of funding liquidity. The upper panel examines the relationship with the excess bond premium (Gilchrist and Zakrajšek, 2012) and the Ted rate in the figure on the left, and the noise measure (Hu et al, 2013) in the figure on the right. The lower panel highlights the link with the MOVE measure on the left, and with the
Figure 4: Federal Reserve Balance Sheet Assets
The figure shows the evolution of the Federal Reserve’s Total Assets

Figure 5: Federal Funds Rate
The figure shows the evolution of the Federal Funds rate over time.
The figure summarizes the conditional forecasts for the subsamples, depicting the different periods of quantitative easing. The blue line depicts the actual historical path of the macroeconomic variable under investigation. The solid grey represents the median conditional forecast, approximating a world with no Fed purchase program. Finally, the dotted gray line portrays the sixty-eight percent credible set of outcomes.