Endogenous Time-Variation in Vector Autoregressions*

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[VERY PRELIMINARY AND INCOMPLETE]

Abstract

This paper proposes a new econometric framework to provide robust inference on the origins of instabilities in the relationship between key macroeconomic variables. We introduce a class of Time-Varying Parameter Vector Autoregression (TVP-VAR) models where the set of underlying structural shocks are allowed to potentially influence the dynamics of the autoregressive coefficients. The proposed Endogenous TVP-VAR framework is applied to study the sources of instabilities in the relationship between the unemployment, inflation and interest rates of the U.S. economy. The results show that the main sources of instabilities are the demand and monetary policy shocks, while the cost-push shocks play a negligible role.

Keywords: Vector Autoregression, Time-varying Parameter, Bayesian, Endogenous, Precision.

JEL Classification Code: C11, C31, C32.

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1 Introduction

The relationship between key macroeconomic variables often experience substantial instabilities over time. These instabilities have important implications for policy makers and investors when either performing structural analysis or constructing forecasts. Initial efforts to account for such a feature in a multivariate macroeconomic modelling context are provided by Canova (2009), Sims (1993), Stock and Watson (1996) and Cogley and Sargent (2001), who proposed the use of vector autoregressions (VAR) models with coefficients that are allowed to evolve over time. Time instabilities are of special interest when assessing potential changes in the monetary policy reaction function or in the propagation of monetary policy shocks to the economy (Clarida et al. (2000), Boivin (2005)). In particular, several works, such as Primiceri (2005), Koop et al. (2009) and Canova and Gambetti (2009), have employed structural VAR models with drifting parameters to assess the reaction and consequences of monetary policy, finding strong evidence of substantial variation in the propagation shocks.

Even though inferring changes in the relationship between key macroeconomic variables is crucial for policy makers, it is also important understanding why those changes occurred in the first place. Typically, the autoregressive coefficients in TVP-VAR models are assumed to evolve according to independent random walks, each driven by its own exogenous innovations. Consequently, this modelling framework remains silent about the underlying structural sources of such instabilities. Parameter instabilities can be caused by a variety of reasons, which can be broadly grouped into two types. First, actions taken by agents, which at the aggregate level can, for example, be manifested as demand, cost-push or productivity shocks, and that in the medium or long run could induce structural changes to the economy. Second, actions taken by policy makers, such as monetary, fiscal or macroprudential decisions, which could reveal unexpected information about the economy to the agents. Therefore, knowledge about the underlying sources of parameter instabilities in VAR models can be useful to conduct economic policies that might also help to promote macroeconomic and financial stability.

In order to investigate the origins of parameter instabilities in VAR models, we rely on the most exogenous source of variation defined in the macroeconometric modelling practice, which are the structural shocks. Accordingly, we provide a modelling framework that helps to identify the structural shocks that are mainly responsible for inducing parameter instabilities in VAR models. In particular, we propose a class of TVP-VAR models which do not only account for changes over time in the propagation of the underlying structural shocks to the economy, but that also are informative about what structural shocks are mainly driving such instabilities, yielding an endogenous, or self-exciting, TVP-VAR framework. We also propose algorithms that are suitable to estimate this class of models based on simple Gibbs sampling procedures.

The proposed approach is applied to the study the sources of instabilities in the relationship between the unemployment, inflation and interest rates of the U.S. economy, potentially occurred since the 1940s until the present time. In this simple three-variable VAR model, the autoregressive coefficients are allow to be not only a function of its past values plus exogenous innovations, but also a function of the structural demand, cost-push and monetary policy shocks, identified from the same VAR model. The results show strong statistical evidence that favors modelling instabilities as a function of the structural innovations against modelling them as exogenous processes. This is obtained based on the Bayes factors computed with the Savage-Dickey Density Ratio. In particular, our results indicate that instabilities in the U.S. economy have been significantly affected by demand and monetary policy shocks, but not by cost-push shocks. This implies that instabilities found in structural VAR models that are used to assess the effects of the Fed's actions, are partly a product of the Fed's actions itself, and its feedback to real activity.

The rest of the paper is organized as follows. Section 2 present the endogenous TVP-VAR framework with two alternative modelling strategies. Section 3 applies the proposed framework to the study of the U.S. economy. Section 4 concludes.

2 Modelling Endogenous Time-variation

This section presents two alternative strategies for modelling endogenous variation in TVP-VARs. The first one considers that autoregressive coefficients depend on past realizations of structural shocks, while the second strategy assumes that coefficients are a function of current realizations of shocks. Each alternative has different interpretations, and therefore, may have different implications for policy makers. We evaluate from a statistical standpoint the most appropriate modelling strategy.

2.1 Lagged Innovations

Let $y_t = (y_{1,t}, y_{2,t}, ..., y_{n,t})'$ be a vector of key variables for a given economy, and ϕ_t be the vector that measures the contemporaneous relationship between the variables in y_t, that is, the time-varying autoregressive coefficients Our first scenario considers that at period t, the state of the economy is influenced by a set of underlying structural shocks e_t. These shock are supposed to influence macroeconomic relations between the variables in the next period, which are measured by ϕ_{t+1}. Also, there are other factors that will affect the dynamics of ϕ_{t+1} that are not captured by the past structural innovations, and that are collected in η_{t+1}. Accordingly, consider the following structural TVP-VAR model,

$$
\begin{aligned}
y_t &= X_t \phi_t + A e_t, \quad e_t \sim N(0, I) & (1) \\
\phi_t &= \phi_{t-1} + H_\lambda e_{t-1} + \eta_t, \quad \eta_t \sim N(0, \Sigma_\eta), & (2)
\end{aligned}
$$

where $X_t = (y_{t-1}, ..., y_{t-p})'$, A denotes the impact multiplier matrix, $H_\lambda = \lambda' \otimes \iota$ such that ι is a column vector of ones and row dimension equivalent to ϕ_t and $\lambda = (\lambda_1, ..., \lambda_n)'$ is the vector containing the sensitivity of the autoregressive coefficients to the structural shocks. The vector λ represents the key piece of information for the remainder of the paper, since it provides useful information to identify the main structural shocks driving instabilities in the multivariate system.

In terms of estimation, we can treat both the autoregressive coefficients, ϕ_t, and the structural shocks, e_t, as latent variables. Then, we can express the model in equations

(1)-(2) in the following state-space form,

$$
\begin{aligned}
y_t &= \begin{bmatrix} X_t & A \end{bmatrix} \begin{bmatrix} \phi_t \\ e_t \end{bmatrix}, \\
\begin{bmatrix} \phi_t \\ e_t \end{bmatrix} &= \begin{bmatrix} I & H_\lambda \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \phi_{t-1} \\ e_{t-1} \end{bmatrix} + \begin{bmatrix} \eta_t \\ v_t \end{bmatrix}, \quad \begin{bmatrix} \eta_t \\ v_t \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_\eta & 0 \\ 0 & I \end{bmatrix} \right).
\end{aligned}
$$

Once the model is casted in a linear and Gaussian state-space, any appropriate posterior simulation algorithm can be used to generate draws for $\boldsymbol{\phi} = (\phi_1, \cdots, \phi_T)'$ conditional on $\mathbf{y} = (y_1, \cdots, y_T)'$ and parameters, A, H_λ and Σ_η. Next, conditional on $\mathbf{y}$ and $\boldsymbol{\phi}$, one can use independent normal and inverse Wishart priors to derive well-known full conditional posteriors for sampling parameters.

2.2 Contemporaneous Innovations

Our second scenario considers that structural shocks at time t, can influence changes in the relationship between the variables at the same time period. This might be the case in financial applications, where variables usually tend to experience abrupt changes rather than gradual ones. For example, uncertainty shocks due to unexpected political outcomes could almost instantaneously influence the relationship between financial variables, such as stock exchange indexes and expectations of agents about the future path of policy rates. Therefore, consider the following TVP-VAR model,

$$
y_t = X_t \phi_t + u_t, \tag{3}
$$

where the error term is defined as $u_t = Ae_t + e_t^*$. That is, reduced form innovations are given by a linear combination involving orthogonalised innovations Ae_t plus a measurement error term e_t^*. The latter is simply introduced to aid the construction of the likelihood function and facilitate computation.[1]

In order to estimate the above endogenous TVP-VAR specification we express the sys-

[1] A similar expanded form for the reduced form innovations in the context of VARMA models can be found in Metaxoglou and Smith (2007) and Chan et al. (2016).

tem in the following state-space representation,

$$y_t = X_t\phi_t + Ae_t + e_t^*, \tag{4}$$

$$\phi_t = \phi_{t-1} + H_\lambda e_t + \eta_t, \tag{5}$$

$$\begin{bmatrix} e_t^* \\ e_t \\ \eta_t \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_{e^*} & 0 & 0 \\ 0 & \Sigma_e & 0 \\ 0 & 0 & \Sigma_\eta \end{bmatrix}\right). \tag{6}$$

Note that e_t can now be interpreted as static factor that enters both measurement and state equations – in (4) and (5) respectively – linearly. Therefore, since $e_t \sim \mathcal{N}(0,\ \Sigma_e)$, its full conditional posterior is also Gaussian. Draws for e_t can thus be obtained by simply introducing an additional Gibbs step to the estimation algorithm for the lagged innovations case. Moreover, as discussed in Bai and Wang (2015), the usual lower triangular with positive diagonal entries structure for A is a sufficient condition to avoid rotational indeterminacy issues for e_t. Lastly, sampling the states in $\boldsymbol{\phi}$ and remaining VAR parameters is carried out in a similar fashion to the lagged innovations variant.

3 Sources of the U.S. Macroeconomic Instability

We apply the proposed approach to assess the main sources of instabilities between three key variables of the U.S. macroeconomy. In particular, we employ a parsimonious VAR model that contains information on unemployment, inflation and interest rates to identify three underlying structural shocks, which are demand, cost-push and monetary policy shocks. The sample spans from 1963Q1 until 2018Q2. To identify the nonsystematic component of the Fed's policy actions, we follow the line of Cogley and Sargent (2001) and Primiceri (2005) who rely on the assumption that monetary policy actions affect inflation and unemployment with at least one period of lag. Therefore, inflation rate is ordered first, followed by the unemployment rate, and interest rates are placed at the end. However, alternative identification schemes, such as the inclusion of sign restrictions, could be alternatively used.

Another important source of instability in VAR models concerns to changes in the size of the structural shocks, which are measured by variations in their variance-covariance matrix, as it is shown in Primiceri (2005). This is particularly the case for the U.S. economy, where a structural decline in the volatility of output fluctuations occurred around the mid-1980s was documented by, for example, Pérez-Quirós and McConnell (2000) and Stock and Watson (2003), and referred to as the "Great Moderation". In order to account for those changes in volatility of the innovations but without loosing the focus of our discussion, which is on the instabilities of the autoregressive coefficients, we propose alternative specifications that are able to capture such breaks in volatility.

We estimate four alternative TVP-VAR models with endogenous parameter instability, which are listed in Table 1. *Model 1* allows for parameter instability as a function of lagged structural innovations. *Model 1-Break* includes the same features that Model 1 has, but additionally, it allows for two breaks in the variance-covariance matrix of the innovations, which are placed in 1984Q1, due to the "Great Moderation", and in 2007Q3, due to the "Great Recession". *Model 2* allows for parameter instability as a function of contemporaneous structural innovations. Finally, *Model 2-Break* includes the same features as Model 2, but additionally, it allows for the two breaks in the variance-covariance matrix.

To first evaluate the need for endogenizing the time-variation in the autoregressive coefficients of a VAR model, we compute the Bayes factors associated to the TVP-VARs subject to endogenous and exogenous parameter instability. That is, with exogenous parameter instability, it holds that $\lambda_i = 0$, for $i = 1, 2, 3$. Table 2 shows Bayes factor estimates based on computation of the Savage-Dickey Density Ratio for the corresponding coefficients associated to each model under consideration. Values greater than 10 should be interpreted as strong evidence in favor of TVP-VAR models exhibiting coefficients with endogenous time variation. The results indicate strong evidence in favor of endogenous time-variation, this is the case for all the coefficients associated to all the four different specifications.

Next, we proceed to evaluate the sensitivity of the autoregressive coefficients to monetary policy shocks, which is measured by λ_3. Figure 1 shows the estimated posterior density of λ_3 obtained with the four different models, showing that parameter instabilities are significantly sensitive to lagged, but not to contemporaneous, monetary policy shocks. This is the case whether breaks in the variance-covariance matrix are accounted for or not. These results imply that instabilities found in structural VAR models, which are used to assess the effects of the Fed's actions, are partly a product of the Fed's actions itself. Moreover, the proposed framework could be useful to infer potential instabilities in the relationship between economic fundamentals generated by adopting different paths of monetary policy normalization, not just for the U.S., but for other economies that have also reached the zero lower bound to face the Great Recession.

Lastly, we evaluate the sensitivity of the autoregressive coefficients to the remaining two structural innovations. These are the cost-push and demand shocks, whose sensitivities are measured by λ_1 and λ_2, respectively. The first two columns of Figure 2 show the estimated posterior density of λ_1 and λ_2, while the last column shows also the density of λ_3, for comparison purposes. When assuming that parameter instability depends on lagged structural shocks, the results indicate that the influence of demand shocks on the time-varying coefficients is statistically significant and of similar magnitude to the influence of monetary policy shocks. However, the influence of cost-push shocks, despite being statistically significant, is negligible in economic terms. Also, when assuming that parameter instability depends on contemporaneous structural innovations, most of the mass of the λ's the posterior densities are either centered to, or close to be centered to, zero, implying that parameter instabilities depend on lagged, rather than on contemporaneous, shocks for the present macroeconomic application.

4 Conclusions

[TO BE COMPLETED]

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Tables and Figures

Table 1: List of Models

Identifier	Model Features
Model 1	Endogenous TVP-VAR. Endogenous parameter changes are driven by lagged SVAR shocks
Model 1-Break	Endogenous Heteroskedastic TVP-VAR. Endogenous parameter changes are driven by lagged SVAR shocks. Covariance breaks are allowed in 1984Q1 and 2007Q3
Model 2	Endogenous TVP-VAR. Endogenous parameter changes are driven by contemporaneous SVAR shocks
Model 2-Break	Endogenous Heteroskedastic TVP-VAR. Endogenous parameter changes are driven by contemporaneous SVAR shocks. Covariance breaks are allowed in 1984Q1 and 2007Q3

Table 2: Model comparisons

	Model 1	Model 1-Break	Model 2	Model 2-Break
λ_1	15.46	328.93	1.03	0.68
λ_2	340.76	83.31	104.11	68.23
λ_3	406.16	380.40	0.21	0.47

Entries are based on computing $2\log(Bayes\ Factor)$ between TVP-VARs with and without endogenous time-variation. TVP-VARs with exogenous parameter dynamics assume $\lambda_i = 0$ for $i = 1, 2, 3$. Values greater than ten should be interpreted as strong evidence in favor of TVP-VAR models exhibiting coefficients with endogenous time variation. See Kass and Raftery (1995) for details on using the Bayes Factor as a metric for model comparison. Bayes factor estimates are based on computation of the Savage-Dickey Density Ratio (see Verdinelli and Wasserman (1995))

Figure 1: Posterior densities for λ_3: loading coefficient associated to monetary policy shocks underlying endogenous dynamics of TVPs

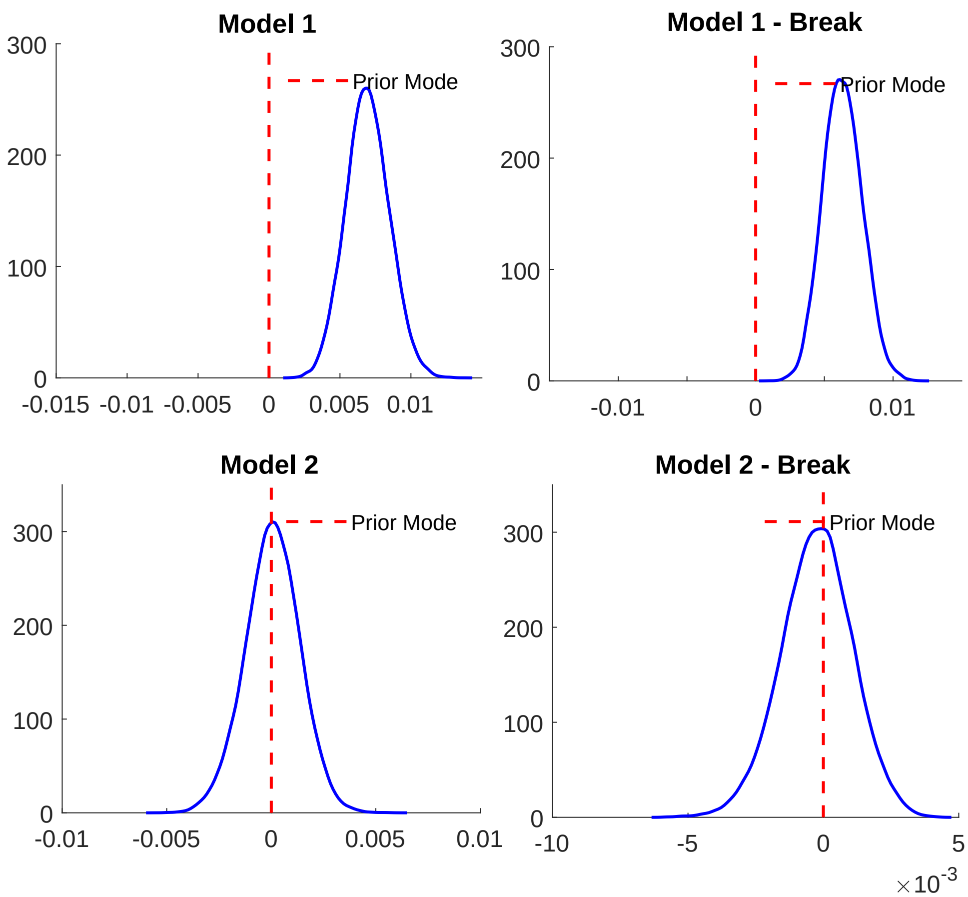
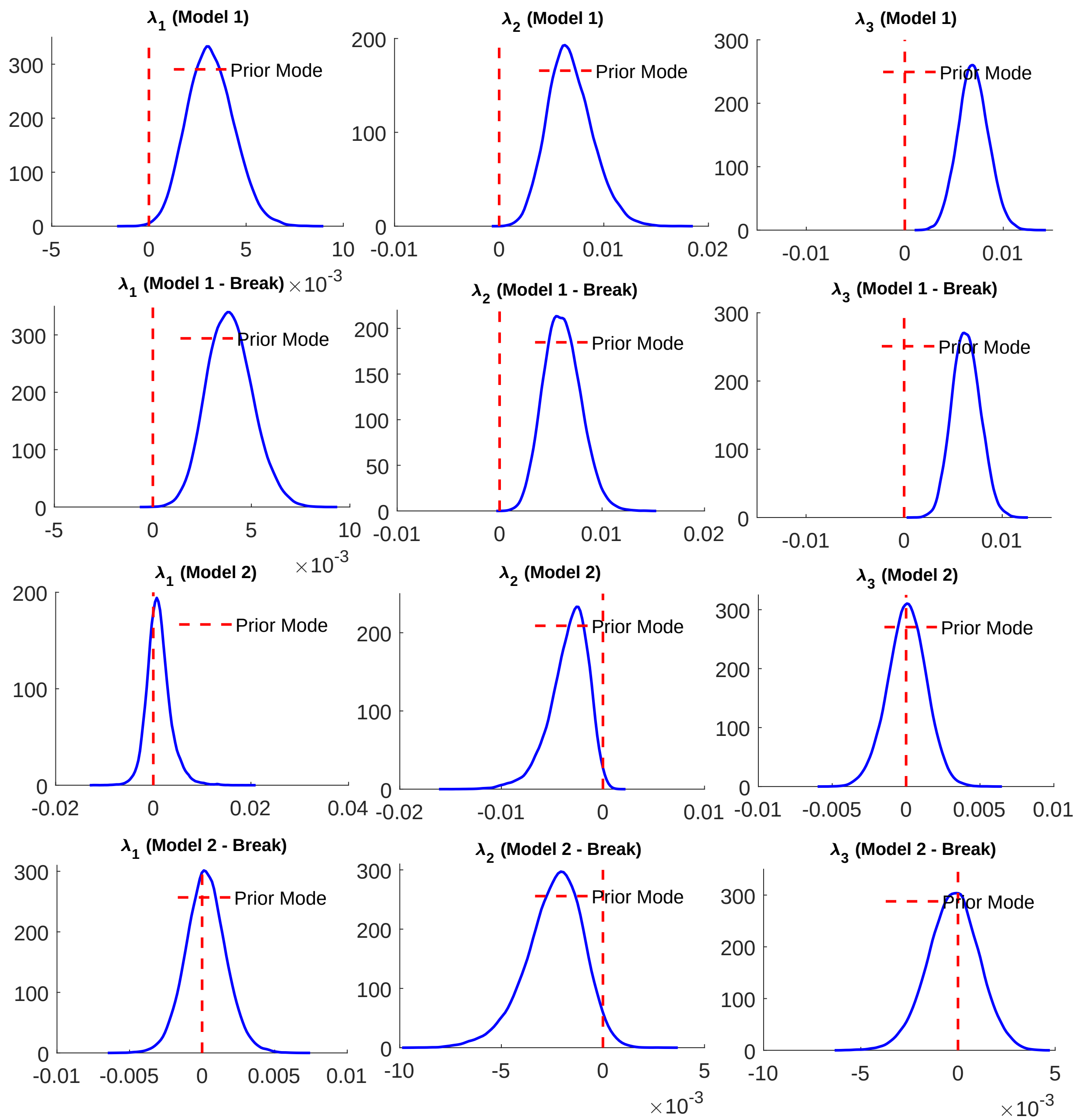


Figure 2: Posterior densities for all loading coefficients associated with the structural shocks underlying endogenous dynamics in TVPs. λ_1 corresponds to inflation; λ_2 corresponds to unemployment; and λ_3 corresponds to monetary policy



A Technical Appendix

A.1 Estimation Algorithm

[TO BE COMPLETED]