Market efficiency of the bitcoin exchange rate: applications to U.S. dollar and Euro

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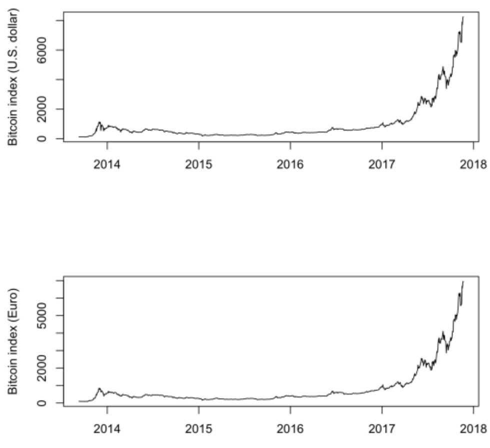
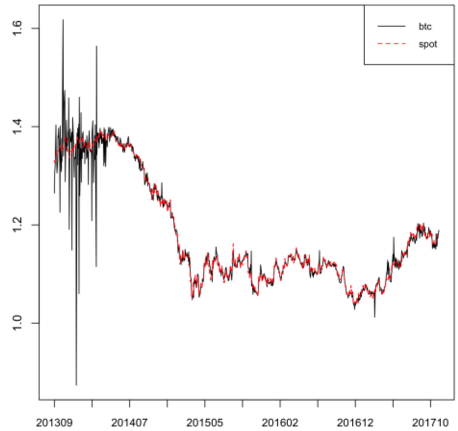
1 Introduction

Bitcoin, a decentralized cryptocurrency system in peer-to-peer network, has long been called digital gold by its fans. On just May 25, 2017, the Economist argued that this analogue sounds like an insult, because, in that week, the price of a bitcoin had touched $2,600 U.S. dollar, over twice as much as an ounce of gold. Similarly, the Euro price of bitcoin exceeded €2,300 (Economist.com). Within six months, the boom and bust bitcoin prices stood at new highs of $8245 and €6961, respectively (Bloomberg, accessed on November 20, 2017). Although, the bitcoin prices are found comprising a substantial speculative bubble component and the fundamental price of bitcoin is zero (Cheah & Fry, 2015), the prices both in U.S. dollar and Euro are still pushed sky-high by markets. Their suddenly exponentially increasing patterns (see figure 1) are questioning bitcoin market's efficiency.



However, after combine these two bitcoin prices to form the U.S. dollar exchange rate against Euro, this indirect exchange rate associated with bitcoin, named the bitcoin exchange rate, presents a feature of intertwining with the its spot exchange rate in foreign market as Figure 2 plotted. This phenomenon has intrigued our great interest to investigate bitcoin market efficiency in terms of the bitcoin exchange rate.



The motivation originates from the fact that there are several bitcoin markets support multi-currency transactions. In these markets, the price of bitcoin in each currency they support is listed. Relying on these bitcoin trading prices, speculators can buy bitcoins with a currency, say U.S. dollar (USD), and then sell these bitcoins for some amount of the other currency, say Euro (EUR). Therefore, the bitcoin exchange rate between USD and EUR is formed. Even for other markets who support only one currency for trading, due to the features bitcoin bearing such like low transaction fee and peer-to-peer payment, speculators could still employ currencies transactions by withdrawing their bitcoin from one market and send them to the account in the other market for trading. Such market behaviors are captured by the bitcoin exchange rate. On November 21, 2017, the estimated bitcoin market capitalization attained \$134.55 billion (USD) and the total USD value of daily trading volume on major bitcoin exchanges reached \$1 billion (blockchain.info). Such active markets attract the speculators from the world which also nurture exchanging currencies in bitcoin markets.

Early studies have employed various cointegration test on foreign exchange market efficiency (MacDonald & Taylor, 1989; Hakkio & Rush, 1989; Baillie & Bollerslev, 1989). The inferences on cointegrating vector based on Johansen procedure clarify the statistic meaning of cointegration with respect to market efficiency ((Johansen & Juselius, 1990; Lai & Lai, 1991). So far, the literature about bitcoin market efficiency is still limited. A multivariate linear regression model with first differenced data shows the price of bitcoin immediately reacts on publicly announced information so that it concludes the bitcoin markets is efficient (Bartos, 2015).

2 Theory

The bitcoin exchange rate is defined as the exchange rate of two currencies using the bitcoin as the medium. In our study, the bitcoin exchange rate of USD against EUR, denoted $xbt_{\$/€}$, is expressed as

$$xbt_{\$/€} = \frac{USD/BTC}{EUR/BTC} = USD/EUR \qquad (1)$$

where BTC denotes the bitcoin. USD/BTC and EUR/BTC are bitcoin prices of USD and EUR listed in a bitcoin exchange. For simplicity, we omitted the subscript of $xbt_{\$/€}$in the rest of paper.

Market efficiency emphasize that prices always ‘fully reflect’ available information (Malkiel & Fama, 1970) and assume two joint hypotheses that (i) agents are risk neutral so that the risk premium is zero and (ii) agents can use available information rationally so that the expected returns to speculators have a zero mean (Hakkio & Rush, 1989). For assumption (i), we have

$$E(xbt_t|I_{t-1}) = spo_t \quad (2)$$

and for 3assumption (ii), we have

$$E(xbt_t - spo_t|I_{t-1}) = 0 \quad (3)$$

where spo_t denotes the time series of the spot exchange rate of USD/EUR and I_{t-1} denotes the information set known at time $(t-1)$.

Given a linear constraint model,

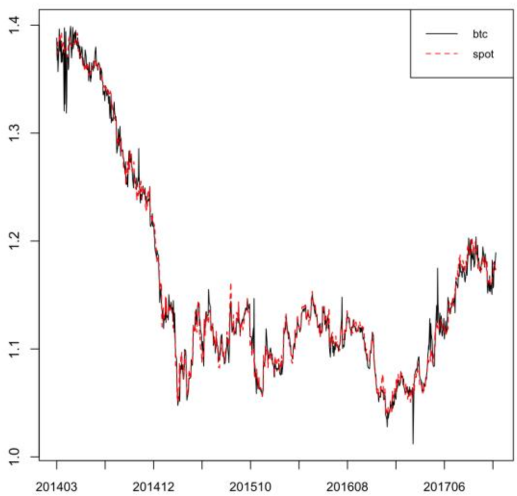
$$xbt_t = a + b \cdot spo_t + \varepsilon_t \quad (4)$$

where ε_t is a white-noise error term, a and b are coefficients of interest, market efficiency necessitates $a = 0$ and $b = 1$. However, since most financial series are found to be nonstationary the regression in (4) is dismissed as 'spurious regression' (Granger & Newbold, 1974), hence t- and F-statistics on $a = 0$ and $b = 1$ tends to bias toward incorrectly rejecting market efficiency (Elam & Dixon, 1988).

Co-integration theory considering equilibrium relationship of a set non-stationary variables is popular in testing market efficiency with non-stationary variables. Let $\boldsymbol{x}_t$ be a vector of economic variables, they are said to be in equilibrium if $\boldsymbol{\beta}'\boldsymbol{x}_t = 0$. In such circumstance, any deviation from equilibrium tends to be pushed back toward equilibrium (Engle & Granger, 1987).

3 Data and methodology

The data consists of daily closing prices for bitcoin index of USD, bitcoin index of EUR (see Figure 1) and spot exchange rate of USD/EUR from September 10, 2013 (as the earliest data available for bitcoin index of EUR) to November 21, 2017 and are collected from Bloomberg. We construct the USD/EUR bitcoin exchange rate using (1) and plot it with the spot exchange rate in Figure 2. After striped the initially volatile phase, the data starts from March 3rd, 2014 which corresponds to a total of 965 observations (see Figure 3). The data are taking the natural logarithm for all statistical tests.



Unit root tests are employed to ensure each variable in the vector $\boldsymbol{x}_t' = (xbt_t, spo_t)$ has a unit root or integrated of order 1, denoted $I(1)$ which is the precondition for cointegration of two variables.

Johansen procedure is applied to test for cointegration and employ inferences on parameters of interest (Johansen, 1988; Johansen & Juselius, 1990). We follow the notation that Johansen and Juselius used to illustrate the procedure. The procedure starts with autoregressive model (VAR)

$$H_0: \boldsymbol{x}_t = \boldsymbol{\Pi}_1 \boldsymbol{x}_{t-1} + \cdots + \boldsymbol{\Pi}_{t-k} \boldsymbol{x}_k + \boldsymbol{\varepsilon}_t \qquad (5)$$

where $\boldsymbol{x}_t$ denotes a p-dimensional vector of economic variables and $\boldsymbol{\varepsilon}_t$ denotes a p-dimensional vector of error term with i.i.d. Gaussian distribution. The $p \times p$ metices $\boldsymbol{\Pi}_1, \ldots \boldsymbol{\Pi}_{t-k}$ are coefficients of k-lags of $\boldsymbol{x}_t$ and $t = 1, \ldots, T$.

Using $\Delta \boldsymbol{x}_t = \boldsymbol{x}_t - \boldsymbol{x}_{t-1}$, rewrite (5) as

$$\Delta \boldsymbol{x}_t = \boldsymbol{\Gamma}_1 \Delta \boldsymbol{x}_{t-1} + \cdots + \boldsymbol{\Gamma}_{\mathrm{k}} \Delta \boldsymbol{x}_{t-k} + \boldsymbol{\Pi} \boldsymbol{x}_{t-1} + \boldsymbol{\varepsilon}_t \quad (6)$$

where

$$\boldsymbol{\Gamma}_{\mathrm{i}} = -(\boldsymbol{\Pi}_{i+1} + \cdots + \boldsymbol{\Pi}_k), \qquad (i = 1, \ldots, k-1),$$

and

$$\boldsymbol{\Pi} = -(\mathbf{I} - \boldsymbol{\Pi}_1 - \cdots - \boldsymbol{\Pi}_k)$$

In this specification, the p-dimensional matrix $\boldsymbol{\Pi}$ is called transitory impact matrix. There are three cases: (i) $rank(\boldsymbol{\Pi}) = p$ impies all variables in $\boldsymbol{x}_t$ are stationary, because there are p linear restrictions on p variables making $\boldsymbol{\Pi x}_{t-1}$ become stationary, (ii) $rank(\boldsymbol{\Pi}) = 0$ suggests all p are non-stationary and there is no linear restriction becoming stationary, (iii) $rank(\boldsymbol{\Pi}) = r$ where $r < p$ indicates r restrictions could make $\boldsymbol{\Pi x}_{t-1}$ become stationary, hence the number of possible co-integration is r. Let $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ be $p \times r$ matrices,

$$H_1: \boldsymbol{\Pi} = \boldsymbol{\alpha\beta}'$$

where $\boldsymbol{\beta}$ denotes the co-integrating vector and $\boldsymbol{\alpha}$ denote the weights or speed-of-adjustment coefficients (Enders, 2014). Hence the hypothesis of cointegration is equivalent to $\boldsymbol{\Pi}$ having a reduced rank form and being able to be decomposed into $\boldsymbol{\alpha\beta}'$.

The $rank(\boldsymbol{\Pi})$ could be calculated through the number of non-zero eigenvalues that are modified by Johansen to be non-negative. The calculation of the estimators of eigenvalues is based on maximum likelihood method and Johansen finds that likelihood ratio tests of these estimators approximately follow χ^2 distribution with the degrees of freedom equal to the number of restrictions being tested.

Similarly, on the presence of co-integration expressed by H_1, the linear restrictions could be put on either $\boldsymbol{\alpha}$ or $\boldsymbol{\beta}$ for employing inferences:

$$H_2: \boldsymbol{\beta} = \boldsymbol{H\varphi}$$

$$H_3: \boldsymbol{\alpha} = \boldsymbol{A\psi}$$

$$H_4: \boldsymbol{\alpha} = \boldsymbol{A\psi} \text{ and } \boldsymbol{\beta} = \boldsymbol{H\varphi}$$

where $p \times s$ matrix $\boldsymbol{H}$ and $p \times m$ matrix $\boldsymbol{A}$ are linear restrictions on $p \times m$ matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$. The restrictions reduce the parameters from $p \times m$ to $\boldsymbol{\varphi}(s \times r)$ and $\boldsymbol{\psi}(m \times r)$. Therefore, the likelihood ratio tests are performed regarding to H_i against H_1 where $i = 2, 3, 4$.

Granger-causality and impulse response function are employed for checking adequacy.

4 Results

Table 1 shows the results from both Augment Dicken-Fuller (ADF) and Phillips-Perron (PP) unit-root tests. For the level series of bitcoin exchange rate and spot exchange rate, both tests fail to reject the presence of a unit root, however, after take first difference of two series, unit-root tests tend to accept the alternative of stationarity. Therefore, both series are $I(1)$.

Table 1 Unit root test statistics				
	xbt_t	spo_t	Δxbt_t	Δspo_t
ADF	-2.145	-2.171	-27.237***	-22.357***
PP	-2.305	-2.159	-44.085***	-32.424***
Note: * significant at 10% level; ** significant at 5% level; ** *significant at 1% level.				

Before Johansen test, the unrestricted VAR model in (5) to employ lag selection. All four types of information criteria Akaike Information Criterion (AIC), Schward-Bayes Criterion (SBC), Hannan-Quinn (HQ) and Akaike's Final Prediction Error (FPE) Criterion suggest using 2 lags ($k = 2$) as shown in Table 2.

Table 2 The lag selection for Johansen approach				
	AIC	HQ	SBC	FPE
$\boldsymbol{x}_t$	2	2	2	2

Table 3 represents the results of Johansen test for co-integration. Both $trace$ and λ_{max} statistics tend to reject $H_0: r = 0$ or no co-integration at 1% significance level. Since $H_1: r \leq 1$ is failed to reject, we could not accept the alternative of the full rank of Π, i.e. both series are stationary. Therefore, only one co-integration exists and the first column of $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$ in Table 3 give the restrictions on matrix $\boldsymbol{\Pi}$. The vector $\widehat{\boldsymbol{\beta}}$ presents the estimate equilibrium relation between the bitcoin exchange rate and the spot exchange rate given by

$$xbt_t = 0.991spo_t - 0.0004$$

and $\widehat{\boldsymbol{\alpha}}$ gives the speed of adjustment towards the estimate equilibrium state, more specifically, if the deviation from the equilibrium of two series happened, the changes in the bitcoin exchange rate approximately decreases by 0.481 whereas the spot exchange rate increases 0.006.

Table 3 Inference on co-integration				
H_1	*trace*	5%(*trace*)	λ_{max}	5%(λ_{max})
$r \leq 1$	5.56	9.24	5.56	9.24
$r = 0$	190.27***	19.96	184.71***	15.67
Eigenvalues (λ_i)				
0.175	0.006	0.000		
Eigenvectors (β)				
β_1 (xbt_t)	1	1	1	
β_2 (spo_t)	-0.991	-7.196	2.896	
β_3 (1)	-0.000	0.686	-1.282	
Weights (α)				
α_1 (xbt_t)	-0.481	0.001	0.000	
α_2 (spo_t)	0.006	0.001	0.000	

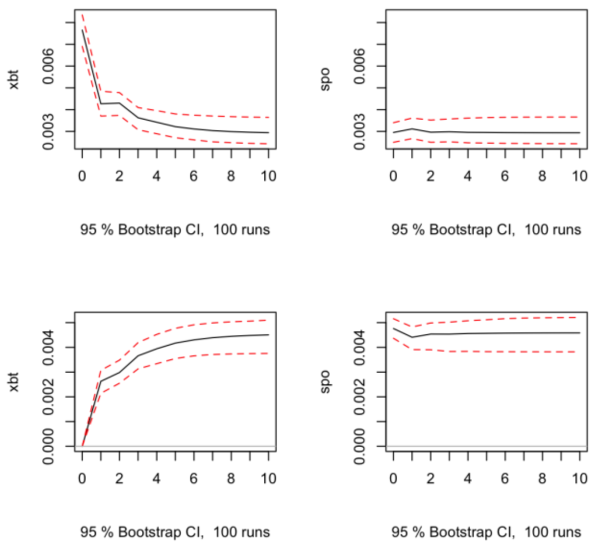
The linear constraints $\mathrm{a} = 0$ and $\mathrm{b} = 1$ on market efficiency is equivalent to $\beta_3 = 0$ and $\beta_1 = -\beta_2$ and we also want to know whether one element of $\widehat{\boldsymbol{\alpha}}$ is equal to zero. Table 4 illustrates the combination of inferences on α and β. The likelihood ratio test statistics and their p-values associated with χ^2 distribution indicate that $H_{2.1}: \beta_1 = -\beta_2$ could be rejected at 10% significance level, but not rejected at 5%. The rejection of joint hypotheses $\mathrm{H}_{2.3}: \beta_1 = -\beta_2$ and $\beta_3 = 0$ implies the market inefficiency i.e. either the assumption of perfect linear relationship between both exchange rate series or the assumption of no initially proportional deviation between two markets could not be hold simultaneously. The rejected $H_{3.1}$ indicates the speed of adjustment on changes of xbt_t is significantly different from zero while the failure of rejecting $H_{3.2}$ suggests the speed of Δspo_t adjusting the deviation could be zero.

Table 4 Inferences on restrictions on α and β								
	H_1	$H_{2.1}$	$H_{2.2}$	$H_{2.3}$	$H_{3.1}$	$H_{3.2}$	$H_{4.1}$	$H_{4.2}$
β-restrictions	-	$\beta_1 = -\beta_2$	$\beta_3 = 0$	$\beta_1 = -\beta_2$ and $\beta_3 = 0$	-	-	$\beta_1 = -\beta_2$ and	$\beta_3 = 0$ and
α-restrictions	-	-	-	-	$\alpha_1 = 0$	$\alpha_2 = 0$	$\alpha_2 = 0$	$\alpha_2 = 0$
test statistics		2.73*	0.17	7.26**	130.61***	0.04	2.95*	0.23
p-values		0.1 (χ_1^2)	0.68 (χ_1^2)	0.03 (χ_2^2)	0.00(χ_1^2)	0.84 (χ_1^2)	0.09 (χ_2^2)	0.68 (χ_2^2)
β_1	1	1	1	1	1	1	1	1
β_2	-0.991	-1	-0.993	-1	-1.009	-0.991	-1	-0.993
β_3	-0.000	0.001	0	0	0.002	-0.000	0.001	0
α_1	-0.481	-0.470	-0.480	-0.457	0	-0.486	-0.480	-0.485
α_2	0.006	0.014	0.007	0.017	0.192	0	0	0

The one-way Granger-causality from $\Delta \mathrm{spo}_t$ onto $\Delta \mathrm{xbt}_t$ found in Table 4 indicates the historical information of the spot exchange rate affects the bitcoin exchange rate significantly. This result also supports the existence of co-integration, because co-integration implies at least one-way Granger-causality presence.

Table 4 Granger-causality		
	F-statistic	P-value ($F_{1,2}$)
Δxbt_t does not Granger-cause Δspo_t	2.27	0.104
Δspo_t does not Granger-cause Δxbt_t	44.90***	0.000

Figure 4 plots impulse response functions of Vector Error Correction Model (VECM). The upper two panels show given an orthogonal impulse to the bitcoin market, the bitcoin exchange rate decrease towards to the equilibrium whereas the spot exchange rate seems increase itself in a tiny amount, however, from the inferences on $\hat{\boldsymbol{\alpha}}$ we know this amount is not significantly different from zero. The lower two panels depict the orthogonal impulse to the spot exchange rate market. The adjustment process might takes 6 days.



5 Conclusion

As most of financial variables, the bitcoin exchange rate and the spot exchange rate both have a unit root. Originally, the linear model of these two non-stationary leads to biased inferences, however, due to the presence of co-integration, the linear combination of two variables become stationary where the equilibrium state happens. Firstly, through Johansen procedure, the existence of only one co-integration is indicated by the maximum likelihood test statistic. Meanwhile, the co-integrating vector gives the linear restriction on variables, and the speed of adjustment is decided by the weights vector. Therefore, any deviation from the equilibrium may be corrected by the error correct mechanism. Secondly, the inferences on the co-integrating vector and the weights vector indicate (i) the linear constraints $a = 0$ and $b = 1$ on market efficiency cannot be held simultaneously, suggestive of inefficiency of the bitcoin exchange market; (ii) the weigh on the spot exchange rate not different from zero suggests that only the bitcoin exchange rate contribute to the adjustment. Thirdly, one-way Granger-causality from returns of the spot exchange rate to returns of the bitcoin exchange rate is found.

In short, the inefficiency tell us the story of always happening deviations and the presence of co-integration provides the information to correct them.

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