Analysis of Asymmetric GARCH Volatility Models with Applications to Margin Measurement

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Abstract

We explore properties of asymmetric GARCH models in the Threshold GARCH family and propose a more general GTARCH model, which captures both high frequency return volatility and low frequency macroeconomic volatility as well as the special characteristics of negative news. Based on the Monte Carlo Experiment and Maximum Likelihood Estimation method on simulated data, we find that a more general version of asymmetric reaction to past negative news has better fit and predictive power. We evaluate the performance of asymmetric GARCH models and EWMA model in VaR, Expected Shortfall, and risk aversion for the SPX& 500 and S&P/TSX indices. The backtest results shows that initial margins modelled by GTARCH model capture both tail risk and procyclicality mitigation well. Finally, we propose a threshold scheme in setting and evaluating initial margins.

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1 Introduction

Volatility is a well known measurement of financial risk because risk premia on conditionally heteroskedastic assets or portfolios may follow a dynamics depending on their time-varying volatility. The generalized autoregressive conditional heteroscedasticity (GARCH) model is one of the most popular model to estimate the time variation in the volatility of asset and portfolio returns.

There are several extension of GARCH models that resulted in better statistical fit and forecasts. For example, TARCH or GJR-GARCH (Glosten, Jagannathan, & Runkle (1993)[12]) is a GARCH models with an asymmetric term which captures the effect of negative shocks in equity prices on volatility or is referred to as a leverage effect. Nelson (1991)[23] proposed an alternative asymmetric model (EGARCH) of the logarithmic transformation of conditional variance that does not require positivity constraints on parameters. Different volatility regimes can also be captured by Markov Regime Switching ARCH and GARCH models (Cai (1994)[6] and Hamilton and Susmel (1994)[15]) allowing for stochastic time variation in parameters .

The regular GARCH family models has a special property that the long run volatility forecast converges to a constant level. Engle and Rangel (2008)[11] created Spline-GARCH model that allows the unconditional variance to change with time as an exponential spline and the high frequency component to be represented by a unit GARCH process. This model may incorporate macroeconomic and financial variables into the slow moving component and improves long run forecasts of international equity indices. A special feature of this model is that the unconditional volatility coincides with the low-frequency volatility. Rangel and Engle (2012)[26] extended this model to a Factor-Spline-GARCH model developed to estimate high and low frequency components of equity correlations. Their model is a combination of the asymmetric Spline GJR-GARCH (or TARCH) and the DCC (dynamic conditional correlations) models. Another application of the asymmetric Spline GJR-GARCH model for commodity volatilities is done in Carpantier and Dufays (2012)[7].

In this paper we generalize the asymmetric Spline-GARCH models to a Spline-GTARCH model by combining the Spline-GARCH model and a more general threshold GARCH model introduced in Goldman (2012)[13]. The widely used asymmetric GJR-GARCH (TARCH) model has a problem that the unconstrained estimated coefficient of α often has a negative value for equity indices. A typical solution to this problem is setting the coefficient of α to zero in the constrained Maximum Likelihood optimization. Following Goldman (2012)[13] we use a generalized threshold GARCH (GTARCH) model where both coefficients, α and β, in the GARCH model are allowed to change to reflect the asymmetry of volatility due to negative shocks. Different from Rangel and Engel (2012)[26] that macroeconomic variables were modeled against model residuals in the Spline part, we impose the macroeconomic variables and their volatility directly into the Spline, in which the low frequency volatility is captured instead of pass through from residuals with model errors. We show that the Spline-GTARCH model fits better as well as does not have a negative alpha bias for several equity indices and numerical examples.

Since tail risk measures are essential for risk management and volatility is commonly perceived as a measure of risk, estimating the tail risk from volatility models turned important. Engle and Mezrich (1995)[10] introduced a way to estimate value at risk (VaR) using a GARCH model, while Hull and White (1998)[17] proved that a GARCH model has a better performance than a stochastic volatility model in calculation of VaR. The TARCH or GJR-GARCH model was used by Brownlees and Engle (2012)[4] among others for forecasting volatility and measuring tail and systemic risks. Under the condition that the risk distribution has a long tail, expected shortfall (ES) is suggested to capture the extreme loss.Therefore, different GARCH models is evaluated based on their tail risk performance in this paper.

Initial margin are usually determined by risk-based models and VaR is a fundamental base to set up the initial margins (Murphy et.al (2016)[22], Knott and Polenghi (2006)[19]). Due to the strong procyclicality of the risk measure during the stressed market, literature and practitioners try to control the negative impact on the funding liquidity from the overestimated initial margins (Murphy et.al. (2014[21],2016[22]), Glasserman and Wu(2017)[25]). We apply our Spline-GTARCH model and compare the VaR estimation between Spline-GTARCH model and the popular Riskmetric exponential weighted moving average (EWMA) model. We also backtest and compare the performance of margin measurement from the GTARCH type and traditional EWMA models. The application on both US SPX & 500 and Canada TSX data show that the Spline-GTARCH model not only account for the persistence of the asymmetric information from the negative news and capture more risk aversion, but also mitigate the procyclicality of the margin estimates.

Traditionally, CCP market will set up a floor for initial margins such as takeing the 25% quantile of the tail risk distribution. In order to quantify the initial margin ceiling, we propose a 3 regime scheme with two thresholds to evaluate both floor and ceiling of the initial margins.

The paper is organized as follows. Section 2 presents the asymmetric GARCH models, their fitting process and forecasting. Section 3 evaluates the fitting performance of the asymmetric GARCH models through the Monte Carlo Simulation and the predication and forecasts through the asymmetric GARCH model on SPX and TSX data, compares the results from the GARCH type models and EWMA models. Section 4 applies the back test and explores the initial margin requirements and procyclicality properties. Section 5 is the concluding remarks and further development.

2 Asymmetric GARCH Models, Fitting Process and Forecasting

In this section, we propose a new generalized asymmetric GARCH models with Spline enlightened from Engle and Rangel (2008)[11]. For a better understanding, we also investigate some well-known benchmark GARCH and threshold GARCH models.

2.1 The Asymmetric Threshold GARCH or TARCH model

The popular generalized autoregressive conditional heteroscedasticity (GARCH) model described in Appendix B is a symmetric model without considering the special performance of the negative return. In the stock market, a larger negative equity return reduces company's equity value, which throws more risk uncertainties into the market liquidity than the positive return will induce and this may trigger a higher leverage ratio. Under the economic shock, especially during the financial crisis such as the 2008 US housing crisis, a negative return will influence the subsequent conditional variance more than a positive return does, in asset pricing this phenomenon is called leverage effect or asymmetric affect.

The most popular model to account for this asymmetric effect is the threshold ARCH (GJR-TARCH or TARCH) model created by Glosten, Jagannathan, and Runkle's (1993)[12], which is an extension of the GARCH model.

A simple return on asset can be defined as $r_t = \mu + \sigma_t \eta_t$ and η_t follows an i.i.d. $N(0,1)$. Let $\varepsilon_t = \sigma_t \eta_t$ and Engle (1982)[9] proposed the following ARCH(1) model to estimate the variance:

$$r_t = \mu + \varepsilon_t, \tag{1}$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_t^2 + \gamma \varepsilon_t^2 I(r_{t-1} - \mu < 0) + \beta \sigma_{t-1}^2, \tag{2}$$

where $1 - \alpha - \beta - \frac{1}{2}\gamma > 0$. This TARCH model includes an extra γ to exploit negative returns. Since tail risk management measures use forecasts of volatility model, Brownlees and Engle (2012)[4] among others used this TARCH model to forecast volatility and measure tail and systemic risks.

However, this widely used asymmetric GJR-GARCH (TARCH) model has a problem that the unconstrained estimated coefficient of α often has a negative value for equity indices. A typical solution to this problem is to constrain coefficient of α as zero in the constrained Maximum Likelihood optimization. Goldman (2012)[13] proposed a more generalized threshold GARCH (GTARCH) model to allow changes in the asymmetry of volatility due to negative shocks:

$$\sigma_t^2 = \omega + \alpha\varepsilon_t^2 + \gamma\varepsilon_t^2 I(r_{t-1} - \mu < 0) + \beta\sigma_{t-1}^2 + \delta\sigma_{t-1}^2 I(r_{t-1} - \mu < 0), \tag{3}$$

where[1] $1 - \alpha - \beta - \frac{1}{2}\gamma - \frac{1}{2}\delta > 0$. This model not only seems to resolve the problem of a negative bias but also shows different dynamics for GARCH parameters when the news is negative.

2.2 The Spline Generalized Threshold GARCH (SGTARCH) Models

There are very few models incorporating both financial volatility and economic variables. Different methodologies (Officer(1973)[24], Schwert(1989)[27], Roll (1988)[28], Balduzzi et al. (2001)[2], Anderson et al. (2007)[1])with linkage between aggregate volatility and economy show very weak relationships, although volatility is higher during recessions &following stages and lower during normal period. Engle and Rangel (2008)[11] innovated the Spline-GARCH model to combine both high frequency financial data and low frequency macro data. This model realizes the forecasting of the effect of potential macroeconomic events on equity volatility and also can be implemented to forecast the volatility that could be expected in a new market. In addition, this model releases the assumption of the mean revering to a constant level, which is almost a property of all GARCH and SV models, and generates a dynamic long-run unconditional variance path instead of a fixed

[1] The fundamental idea of threshold GTARCH here should be $1 - \alpha - \beta - \theta\gamma - \theta\delta > 0$. In practice the rule of thumb estimation for θ is close to 0.5 for different kinds of data.

constant.

The Spline-GARCH model follows the following setup:

$$\begin{aligned} r_t - E_{t-1}r_t &= \sqrt{\tau_t \sigma_t^2} z_t, & (4)\\ \sigma_t^2 &= (1-\alpha-\beta) + \alpha\left(\frac{(r_{t-1}-E_{t-1}r_t)^2}{\tau_{t-1}}\right) + \beta\sigma_{t-1}^2, & (5)\\ \tau_t &= c exp(w_0 + \sum_{i=1}^{k} w_i((t-t_{i-1})_+)^2 + m_t\gamma), & (6) \end{aligned}$$

where z_t is a standard Gaussian white noise process, σ_t^2 is a non-negative time series such as GARCH with unconditional mean of 1,

$$(t-t_i)_+ = \begin{cases} (t-t_i), & \text{if } t \geq t_i, \\ 0, & \text{otherwise,} \end{cases}$$

where $(t_0 = 0, t_1, t_2, ..., t_k = T)$ is a partition of total number of observation T into k equal subintervals. Engle and Rangel (2008)[11] claim that m_t is the set of weakly exogenous variables (i.e. macroeconomic variables), but they only use the residuals from the spline equation as the response and analyze macro economic impact based on a series of macro economic explanatory variables.

Combining with the idea in equation (3) from Goldman(2012)[13], Goldman and Wang (2012)[14] introduced an asymmetric Spline-Threshold-GARCH model that generalizes the Spline-GARCH and asymmetric TARCH models. Based on their analysis, we propose a new Spline Generalized Threshold GARCH (STGARCH) model as

$$\begin{aligned} r_t &= \mu + \sqrt{\tau_t \sigma_t^2} z_t, & (7)\\ \sigma_t^2 &= \omega + \alpha\left(\frac{(r_{t-1}-\mu)^2}{\tau_{t-1}}\right) + \gamma\left(\frac{(r_{t-1}-\mu)^2}{\tau_{t-1}}\right) I(r_{t-1}-\mu<0) & (8)\\ &\quad + \beta\sigma_{t-1}^2 + \delta\sigma_{t-1}^2 I(r_{t-1}-\mu<0), \\ \tau_t &= c exp(\sum_{i=1}^{k} w_i((t-t_{i-1})_+)^2 + m_t\gamma), & (9) \end{aligned}$$

where ω[2] $= (1-\alpha-\beta-\frac{1}{2}\gamma-\frac{1}{2}\delta) > 0$, and all other notations are consistent with the TGARCH above. Higher number of knots (k) implies more cycles in the low-frequency volatility while parameters $w_1,...,w_k$ represent the sharpness of the cycles.Here we simplify the return process with a generalize μ instead of the time variant conditional mean; and we also drop the w_0 in the quadratic spline, which follows the same spline process as on the NYU Stern V-Lab, because we noticed that w_0 is usually insignificant in most cases. For m_t, in this analysis we inject similar set of macroeconomic variables suggested by Engle and Rangel (2008)[11]. This Spline-GTARCH model accounts for both asymmetric effect and spline effect and it is worth to compare its performance among others.

2.3 Spline-GTARCH Model Fitting, Forecasting, and Tail Risk Estimation

2.3.1 Model Fitting Process and Model Specification Criteria

In this paper we use the Maximum Likelihood Estimation (MLE) method to estimate the jointly parameters in the STGARCH model: $\theta = \{\mu,\alpha,\beta,\gamma,\delta,c,w_1,...,w_k\}$. Assume that $\varepsilon = \sqrt{\tau_t\sigma_t^2}z_t$ follows a normal[3] distribution and the density function is

$$f(r_t;\mu,\sigma_t,\tau_t) = \frac{1}{\sqrt{2\pi\tau_t\sigma_t^2}}e^{-\frac{1}{2\sigma_t^2}\frac{(r_t-\mu)^2}{\tau_t}}. \tag{10}$$

Then the log likelihood function is

$$L(\hat{\theta}) = log(L(r_t|\theta)) = -\frac{T}{2}log2\pi - \frac{1}{2}\sum_{t=1}^{T}(log\sigma_t^2 + log\tau_t + \frac{(r_t-\mu)^2}{\sigma_t^2\tau_t}). \tag{11}$$

The final solution is

$$\hat{\theta} = arg\{max_\theta\{L(r_t|\theta)\}\}, \tag{12}$$

[2] To be notified that ω here is different from the ω in the regular GARCH model

[3] This assumption is not crucial becasue asymptotically a quasi-maximum likelihood approach can be used if returns are not Gaussian

which are consistent[4] and most efficient[5] estimators.

As to the model evaluation, we will use the following two model specification criterion: Bayesian-Schwartz information criterion ($(BIC = -2L(\hat{\theta}) + d \times 2/T)$) has penalty on the degree of freedom (d is the dimension of $\hat{\theta}$); and Akaike information criterion ($AIC = -2L(\hat{\theta}) + d \times ln(T)/T$).

2.3.2 Forecasting

The special feature of the STGARCH model is that the unconditional volatility coincides with the low-frequency volatility (Engle and Rangel(2008)[11]) τ in equation (9), that means

$$E[(r_t - \mu)^2] = \tau_t E(\sigma_t^2) = \tau_t.$$

In the STGARCH model, there are two series of in-sample variance estimation: the conditional variance $\tau_t \sigma_t^2$, which reflect the high frequency volatility; and the unconditional variance τ_t, which reflect the low frequency volatility. The forecast of the l-step out of sample ahead conditional variance is $\tau_{t+l}\sigma_{t+l}^2|t$. Similarly as defined in Appendix B, the forecasts of Spline-GTARCH model are:

$$\begin{aligned}
\tau_{t+1}\sigma_{t+1}^2|t &= \tau_t(1-\alpha-\beta-\frac{1}{2}\gamma-\frac{1}{2}\delta)+\alpha(r_t-\mu)^2)+\gamma((r_t-\mu)^2)I(r_t-\mu<0) \\
&\quad +\tau_t\beta\sigma_t^2+\tau_t\delta\sigma_t^2 I(r_t-\mu<0), \\
&= \tau_t(1-\alpha-\beta-\frac{1}{2}\gamma-\frac{1}{2}\delta)+\tau_t\alpha(\sigma_t)^2+\tau_t\gamma(\sigma_t)^2)I(r_t-\mu<0) \\
&\quad +\beta\sigma_t^2\tau_t+\tau_t\delta\sigma_t^2 I(r_t-\mu<0), \\
&= \tau_t+(\alpha+\beta+\frac{1}{2}\gamma+\frac{1}{2}\delta)(\sigma_t^2-\tau_t)
\end{aligned}$$

[4] As the sample size $T \to \infty$, the probability that the estimator $\hat{\theta}$ shows a large divergence from the true parameter values θ converges to zero.

[5] Show the smallest standard errors among all the unbiased estimators.

$$
\begin{aligned}
\tau_{t+2}\sigma^2_{t+2}|t &= \tau_t + (\alpha+\beta+\frac{1}{2}\gamma+\frac{1}{2}\delta)(\tau_{t+1}\sigma^2_{t+1}|t-\tau_t) \\
&= \tau_t + (\alpha+\beta+\frac{1}{2}\gamma+\frac{1}{2}\delta)^2(\sigma^2_t-\tau_t) \\
\ldots & \\
\tau_{t+l}\sigma^2_{t+l|t} &= \tau_t + (\alpha+\beta+\frac{1}{2}\gamma+\frac{1}{2}\delta)(\tau_{t+l-1}\sigma^2_{t+l-1}|t-\sigma^2) \\
&= \tau_t + (\alpha+\beta+\frac{1}{2}\gamma+\frac{1}{2}\delta)^l(\sigma^2_t-\tau_t)
\end{aligned}
$$

2.3.3 Tail Risk Estimation

One of the most popular statistics to measure the tail risk of the return is value at risk (VaR). VaR of a return is defined as the largest loss return[6] such that the probability of loss return exceeding the VaR is (1-q):

$$P(r_t < -VaR_{1-q}) = 1-q. \tag{13}$$

Both in-sample and out-of-sample daily VaR can be estimated from the SGTARCH model through the volatility estimation and forecast. A simple way to estimate the daily VaR is by using the following equation:

$$VaR_{t+1} = \sigma_{t+1} \times F^{-1}_{(1-q)}, \tag{14}$$

where $F^{-1}_{(1-q)}$is the $(1-q)$ quantile of the distribution of F. A lot of literature apply standard normal distribution for F so that the daily VaR can be estimated based on $F^{-1}_{(1-q)} = 1.282, 1.645, 2.326$ for $q = 90\%, 95\%, 99\%$, respectively.

If we release the normal distribution assumption, we can apply a semi-parametric method or the filtered historical simulation (FHS) based on Hull and Whilte (1998)[17] to estimate the daily VaR through a filtered process by estimating the F's quantile instead of using parametric distribution

[6]When r_t is negative, it is the loss return, which is a negative number. Here the VaR focus on the left tail of the return distribution.

assumption. Let the updated error $e_{t+1} = \frac{r_{t+1}-\mu}{\sigma_{t+1}}$ and order the error series such that

$$e_1 < e_2 < e_3 < e_4 < e_4 < ... < e_n.$$

The estimate of F^{-1}_{1-q} is the $1-q$ quantile of the empirical distribution of the updated error e_t. Apply the equation (14), the FHS daily VaR can be estimated. We will estimate both types of VaR and present their performances.

If the loss distribution has a very long tail, the actual loss (L) may be much greater than the VaR, another popular tail risk measure such as expected shortfall (ES) is recommended. ES is is defined as the expected value of the portfolio loss given a VaR exceedance has occurred. In the asset return case, our focus is the left tail of the return so that the ES is

$$ES_{1-q} = E(L|L < VaR_{1-q}). \tag{15}$$

Same as the VaR, we can apply a parametric method and estimate the expected short fall by

$$ES_{1-q} = \frac{(f(VaR_{1-q}))}{q} \times \sigma_t, \tag{16}$$

where f can be assumed as a standard normal density function.

For L-day cumulative VaR or ES estimation, one could use bootstrap or rolling window forecasting. Since the forecasting precision is out of the scope of this paper and the magnitude of VaR may only be changed at the same level of proportion for each model described in this paper, we will follow the approximation method described in the CDCC operation manual[8] as below

$$CumulativeVaR_{t+l} = \sqrt{l} \times \sigma_{t+1} \times F^{-1}_{(1-q)}, \tag{17}$$

3 Monte Carlo Experiment and Application of the Asymmetric Garch Models

3.1 Monte Carlo Experiment

In order to investigate the robustness of model fitting process, we create a Monte Carlo (MC) experiment and compare the following four sets of models (totally eight models).

1. GTARCH and Spline-GTARCH(1,1,1,1)
2. GTARCH0 and Spline-GTARCH0(1,1,0,1) – no γ
3. GJR-TARCH and Spline-GJR-TARCH(1,1,1,0) – no δ
4. GARCH and Spline-GARCH(1,1,0,0) – no γ and δ

We apply the data generating process (DGP) by equation (7,8 ,9) in the SGTARCH model. We set the MC iteration = 400, and in each iteration we generate 5000 observations and fit the eight models with constrains or without constrains.

The summaries of the MC experiment in Table -2 are means of the 400 samples, and our results show that most estimated parameters of the SGTARCH are very close to the true values and most parameters are statistically significant except for those weight variables. AIC chooses the STGARCH as the best fitting model among the eight models with the smallest AIC value. We also notice that α in the unconstrained STARCH is negative because STARCH does not effectively capture the negative news in the return. It is clear that δ in SGTARCH reflects the volatility change when return is negative.

All models with spline satisfy $\alpha+\beta+\frac{1}{2}\gamma+\frac{1}{2}\delta<1$ so that the stationary condition hold for these models. GTARCH without spline failed the stationarity request due to the lack of spline in adjusting the long-run variance. Because the data were generated by a Spline model, GTARCH without spline does not capture the spline effect.

3.2 Model the TSX and SPX Data

3.2.1 Data

In the real application, we will investigate both US and Canada equity market and it is worthwhile to check our model implementations for both markets. We apply SPX 500 index for the US asset price and TSX index for the Canada asset price level. Figure 1 shows our TSX data sample ranges from January 2 2001 to December 28 2016 with 4017 observation and Figure 2 shows SPX data with 4025 observations during the same data range. Both returns are log difference of the price index.

The time series of returns from both US (SPX) and Canada (TSX) markets are typical high frequency return data, which show insufficient serial correlation along the time. Therefore, the forecast of today's expected return is unaffected by the past information due to the lack of correlations. Because these two markets are high related, the returns follow the same trend: both returns have big fluctuation during the 2008 -2009 financial crisis while the SPX has higher levels in both positive and negative returns. This indicates that the US equity market is much more rewarding as well more risker than the Canada equity market.

The time series of the squared returns show clusters periodically: high (low) volatility tends to be followed by high (low) volatility, indicating a strong persistence. Our asymmetric GARCH models are designed for such kind of clustering volatility path, which follows a stochastic process where today's volatility is positively correlated with the volatility of subsequent returns.

Although the return or the squared return times series followed the similar trends, we noticed that US and Canada market have very different cumulative returns within the same time period.

Canadian market held positive cumulative returns from 2005 to 2008, dropped to negative in 2009, and bounced back to positive immediately after 2010 and have kept the 2008 level or so since then; while US market have held negative cumulative returns before the little rise during 2007 -2008 since the 2001 IT bubble, dropped sharply in 2009, and increased further above the 2007-2008 level by the end of 2016.

We also consider the following macroeconomic variables and most of them are listed in Engle and Rangel(2008):

Table 1: Economic variables in the analysis

Name	description
inflation	Inflation rate
$inflation_V$	Volatility of inflation
overnight	Short-term interest rate
$overnight_V$	Volatility of interest rate
$unemployment_V$	Volatility of unemployment rate
USD_V	Volatility of US dollar value indicator
gdpgrowth	GDP growth rate
$gdpgrowth_V$	Volatility of GDP growth rate

The inflation is calculated by the monthly CPI, overnight rate is posted day-end rate, USD dollar value is the USD value index, gdpgrowth is defined as log difference of real GDP. Here GDP is quarterly base, USD value index and overnight rates are daily base, and other variables are monthly data. Following the idea in Engle and Rangel(2008), we obtain the absolute value of the residuals from an AR(1) model and we compute a moving average of the squared residuals.

For example $inflation = \Delta log(yt)$. Then the AR(1) for inflation is similar to:

$$\Delta log(y(t)) = c + \beta * \Delta log(y(t-1)) + e(t).$$

The volatility of inflation is computed by the following moving average equation and moving average window size N below is set as 25 days for daily data and 250 days for other variables.

$$\sigma^2_{(y,t)} = \frac{1}{N}\sum_{j=1}^{N} e_j^2.$$

3.2.2 Model Results for the TSX and SPX Data

The model fitting results in Table 3 and Table 4 show that both SPX and TSX choose the Spline-GTARCH model instead of the other eight models. In both cases, $\alpha+\beta+\frac{1}{2}\gamma+\frac{1}{2}\delta<1$, indicating the satisfaction of stationarity. Since AIC/BIC will be penalized by the number of variables, Spline model with macro economic variables is supposed to have a relatively larger AIC/BIC, but Spline GTARCH with macro still performs the best comparing to other models with macro economic variables. In general, simple GARCH model has the worst fit based on its largest AIC/BIC.

From Table-3 we can conclude that adding macro economic variables to SPX data drop the knots size from 17 to 8 since macro economic variables capture the volatility dynamics due to the economic cycle. Except inflation and volatility of GDP growth, all other six macro economic variables are significant. Volatilities of inflation and unemployment rate and GDP growth have negative impact on the return volatility, which is consistent with the economic impact on the return: the return volatility in general is small when these economic factor volatilities are larger. On the other hand, interest rates and USD dollar value has positive impact on the return volatility.

Macro economic variables in Table-4 reflect different status for TSX data comparing to the SPX data that interest rate volatility has a negative impact on the return volatility and volatility of unemployment rate has positive impact. We test the Canadian data and notice that there is multicolinearity between the volatility of inflation and the unemployment rate, which may be the reason why the significant sign is not consistent with the US data. In addition, we notice that adding macroeconomic variables into Canadian data did not change the knots of the model, which means that macro economic variables may not pick up the TSX volatility cycle efficiently. Another explanation may be that Canadian macroeconomic data has no obvious long-run economic cycles impacting the return volatility. For the reason of cross comparison between SPX and TSX data, we keep the current macro economic variable lists since the predicted volatility does not change significantly if we drop one variable here.

A good way to evaluate these model in Table-5 is the degree of risk aversion, which is quantified by the correlation of observed return and log of estimated volatility. In theory this correlation should be negative, which means that log volatility is countercyclical to the level of the return. From the market risk aspect, when market has liquidity risk, the volatility is high so that the return is constrained. If the correlation is a higher negative number, the model captures more risk aversion, which means that the model is good to identify the sensitivity of the market change. Tabel-5 clearly shows that asymmetric model has better risk aversion impact: GTARCH type models have the highest level risk aversions, followed by GJR-GARCH model. Traditional GARCH and EWMA have very small risk aversions, which is weaker in estimating sensitive tail risk.

For simplicity, in the following analysis we will report the results from the Spline-GTARCH(1,1,1,1) vs GTARCH(1,1,1,1) model, for comparison we will also report the STARCH(1,1,1,0) vs TARCH(1,1,1,0) model.

Figure 4 and Figure 3 shows daily volatility forecasts[7] of these four models for both US and Canada markets. The volatility paths are consistent with the squared return that US market has higher and much more volatile volatility than the volatility in Canada. Both markets approached relatively high volatility regime during 2011-2012 and 2015-2016 post-crisis, while US also went through a little higher volatility regime during the 2010-2011.

It is clear that the volatility forecasts with spline are smoother: the short-run conditional volatility with Spline (Hvol) is a little higher(lower) than the volatility without Spline (VOL) during the low(high) volatility regime, which means that the Spline mitigates some effects of the too-high

[7] For demonstration and comparison purpose, the volatility forecasts in the graphs are annualized standard deviation of the volatility (i.e., $\sqrt{VOL*252}$). The comparison rank will not be influenced by this transformation.

persistence from the volatility and combine the long-run economic effect from the relative cycles. The red lines are the long-run unconditional variance path, which reflect the economic cycles in the past 16 years and is in fact a unique output from the Spline asymmetric GARCH type models. Generally SGTARCH generate a higher level of short-run volatility path than the GTARCH, but their long-run unconditional volatilities (red lines for reach models) are very close to each other.

3.3 Estimate Tail Risk of the TSX and SPX Data

In this section we will compare the tail risk estimations . Because EWMA[8] model is also a very popular benchmark model for tail risk measure and initial margin estimation, we will also compare our model results with EWMA model results.

A broadly cited benchmark margin model in the literature (Murphy et.al. (2014)[21], Murphy et. al(2016)[22]) is the exponential weighted moving average (EWMA) model, which computes the daily variance of returns using an exponentially weighted moving average with decay factor λ. This decay factor controls how rapidly recent information is incorporated into the volatility estimation so that today's variance equals yesterday's variance (weighted by λ) plus yesterday's squared return (weighed by $(1-\lambda)$). This is also the most papular RiskMatric model (J.P. Morgan (1996)) applied in the risk management with $\lambda = 0.94$. To be more accurate, we fit the EWMA model as described in the Appendix A and obtain $\lambda = 0.9384$ for the SPX and $\lambda = 0.9394$ for the TSX.

To be consistent with CCP market by using the 1-3 days margins, the 1-day to 3-day ahead volatility, cumulative VaR and expected shortfall were reported in Table 6. Generally SPX's volatility forecasts are more than twice of the TSX's forecasts and the VaR estimations follow the same trend at each quantile (90%, 95% and 99%), and this reflects that US market is much more volatile

[8] Appendix A show the set up of the EWMA model and its fitting and forecasting

than the Canada Market. The forecasts were estimated during a low volatility regime so that most forecasts of tail risk are increased along the time. For both SPX and TSX, forecasts derived from model with Spline are smaller than the forecasts from model with macro variables and the simple model without Spline; and forecasts accounting for asymmetric in volatility δ are higher than forecasts without δ. We conclude that δ captured different dynamics of the volatility with negative returns and Spline captured the real economic cycle. For EWMA[9], we only report the 1-day forecast due to its deficiency in long run forecasting described in Appendix A and it is obvious that EWMA has lower volatility estimates and tail risk comparing to other eight models.

Table-7 is the similar set of forecasting and tail risk estimations during the high volatility period. It is interesting to see that volatility forecasts based on Spline and Macro model are smaller than other two types of models for the SPX data and forecast rank order for high volatility data is consistent with results from low volatility data. This findings is consistent with the parameter estimation that US macro economic variables capture the economic cycle better than the Canadian macro economic variables. During this high volatility period in 2008, US market volatility is more than 6 times of the Canadian market volatility, which indicate a more risker market in the US. Since GTARCH type models is more risk aversion and mean reversion, it also has smaller VaR forecast than most models without spline.

4 Initial Margin and Procyclicality, Backtest

4.1 Initial Margin and Procyclicality

Margin is a system of collateralisation to improve the financial stability. Central counterparties (CCPs) require members to place a sum of initial margin in a CCP account in order to protect

[9]One can also implement rolling window forecasting or bootstrap forecasting by using EWMA to reach the long-run forecasting. However, GARCH type model can reach long-run forecasting without extra simulation error

market participants against counter party default, therefore the exchange-traded and some non-exchange traded derivatives contracts are against counter party failure by a CCP.

Risk Management system for the CCPs is based on the default waterfall with the first line of defense composed of margins for its members followed by default fund contributions. The initial margins, in particular, are often set based on Value at Risk (VaR) over the time that the security is cleared and the CCP gets exposure taking opposite positions for the buyer and the seller. Central bank and regulators concerned with the systemic stability so that their focus on CCPs increased in the past few years.

Posting margin in time has implication for liquidity management. A party has to post margin often has to find and fund that margin by either cash or high quality securities in daily base. Initial margin requirements are often estimated by risk based model and normally a 99% VaR over some assumed liquidation horizon from a risk model determines the initial margin.

Risk models are used by both CCP and bilateral counterparties to estimate margin requirements of portfolios of financial instruments. During the stressed market, risk volatilities often increase the initial margins, which lead to a procyclicality : the initial margin tend to increase a lot in the stressed economy, and thus put a greater burden on margin posters under the already less-liquid market. This property pushes the financial stress more in tense and make the recovery more difficult (Kashyap and Stein(2004)[18], Brunnermeier and Pedersen(2009)[5], Heller and Vause (2012)[16], Murphy et.al. (2016)[22]). This phenomenon tends to increase the period of crisis and cause investors to confront funding liquidity risk.

We explore procyclicality of margin requirements based on VaR models. On the one hand there is a need for margins to adjust to changes in the market and be responsive to risk. Thus margins are higher in times of stress and lower when volatility is low. However, such a setting may produce big changes in margins when markets are stressed and in turn may lead to liquidity shocks; while in stable times margins might be too low. CCPs try to reduce procyclicality of models by using various methods including setting floors on margin. Some of methods are discussed in white papers by the Bank of England (Murphy et. al (2016)) . Their study suggests five tools including a floor margin buffer of 25% or greater to be potentially used at the time of stressed conditions. We suggest to use both the floor and the ceiling on margins by using threshold autoregressive model with three regimes as well as expert judgement based on historical margin settings. We illustrate the use of this method below.

We estimated the Riskmetrics EWMA volatility model for the SP500 (SPX) and Toronto Stock Exchange (TSX) indices using daily data between 3/17/2003-3/31/2017 (3500 observations). The smoothing parameter was very close to .94 in both cases which is frequently used in practice. Figures 7 and 8 show the returns in red and negative of 1 day 99% Value at Risk (VaR) for SPX and TSX correspondingly. We generated 1 day 99% VaRs using Hull and White (1998) bootstrap method (the blue line) and Normal Distribution (the green line). The margin requirements with Hull and White method are higher because this method uses the actual returns distribution with fat tails compared to Normal distribution.

Next we estimated a Threshold Autoregressive Model with three regimes and two corresponding thresholds. The results for thresholds are given in Table 9 and presented graphically as horizontal black lines in Figures 7 and 8. The three regime threshold model provides a straightforward method of setting both the floor and the ceiling for the initial margin that is stable and not too procyclical: the margins are bounded between 1.74% and 2.95% for SPX and between 0.71% and 1.15% for TSX. This way when volatility is low the margins are fixed at a conservative floor level that corresponds historically to about 29% quantile of lowest margins for SPX and at the time of market stress they cant go above the upper threshold. For TSX the margin buffer is a higher 32% of observations. On the other hand, at the time of stress the higher regime thresholds correspond to 38% proportions of observations for SPX and TSX correspondingly which might be not too conservative. One could add here historical margins settings by CCPs at the time of stress to see if the upper bound was historically higher and would have resulted in lower percentage of observations for the high regime. In order to guarantee that the margins floor and ceiling would be sufficient at the time of crisis we need to make sure that the time series of VaRs in the regime of high volatility are stationary and revert back inside the bounds. If the margins were allowed to be set within two bounds and the high volatility regime was not persistent margins would be stable. Such policy could be also useful to manage expectations at the times of stressed liquidity.

4.2 Backtest

A balanced risk sensitivity is required during the margin modeling process. We need a model to cover the risk, but not over react into strong procyclicality mentioned above.

Backtest is a very good way to assess the risk sensitivity of the margin models. We would like to backtest the VaR to assess the rates of return breaches. If the breach rate is too low, the VaR underestimate the margin, which will put extra loss for the CCP; if the breach rate is too high, the VaR overestimate the margin, which will lead to the undesired procycliccality. The optimal margin set up will be based on VaR that has reasonable estimates falling within a certain statistical confidence level.

The most popular backtest method is the Kupiec (1995)[20] proportion of failures (POF) tests. Let X be the number of breaches and 'T' is the number of observation. The null hypothesis of the Kupiec test is:

$$H_0 : (1-q) = \frac{x}{T},$$

and the alternative hypothesis is

$$H_0 : (1-q) \neq \frac{x}{T}.$$

The test statistic is

$$LR = \frac{(1-q)^x q^{T-x}}{(1-\frac{x}{T})^x(\frac{x}{T})^{T-X}}, \tag{18}$$

where LR follows a chi-square distribution with degree of freedom as 1 ($X^2(1)$). This is a two-sided test so that we should consider both lower bound and the upper bound of the critical values and breach request.

By applying this Kupiec test, we tested the daily VaR estimated by the historical daily volatility prediction from all models in this paper. Figure 5 and Figure 6 illustrate a visual comparison about the daily VaR from SGTARCH and EWMA models VS the returns. The daily 99% VaR estimates from SGTARCH model for both SPX and TSX data are larger during the high negative returns than the VaR estimates from EWMA because the SGTARCH captured the asymmetric effect of the volatility.

The formal Kupiec test results in Table 8 show that all VaR estimates from GTARCH type model ($q = 90\%, 95\%, 99\%$) passed the Kupeic test at significant level $\alpha = 5\%$ for both SPX and TSX data while EWMA failed the test. The first table reports the upper and lower bound of by using the Basel traffic light technique: the number of observation breach the upper bond means that the margin estimation is underestimated; the number of observations breach the lower bound indicate an overestimation of the margins. Breaches of SGTARCH at each $1-q$ quantile are within the allowed ranges, while breaches of EWMA model falls above the upper bound request, indicating that EWMA underestimates the margins. The result show that GTARCH type models will be preferred to mitigate the procyclicality since they pass all the back test with more risk aversions during the high volatility regime.

5 Conclusion and Further Development

In this paper we explore the properties of the asymmetric GARCH models and generalize the threshold and spline GARCH model to a Spline generalized threshold GARCH model (Spline-GTARCH). Through the Monte Carlo Experiment, we test the robustness of our customized fitting process and evaluate eight types of GARCH models and our results show that Spine-GTARCH model not only has better fit and predictive power but also could capture both short-run and long-run volatility dynamics. We compare the GARCH type models and EWMA model fitting on both US SPX and Canada TSX data, and analyze their performance on the out-of-sample volatility and VaR forecasts as well as risk aversion characteristics. We investigate the impact of the low frequency macroeconomic variables on the high frequency return and conclude that US macroeconomic data capture economic dynamic well in the Spline model and overnight interest rate volatility, unemployment rate volatility and US dollar value are most significant factors in influencing the market return fluctuation. Meanwhile, Canadian macro economic variables weakly impact the market return volatility. We then extend our analysis on the initial margin measurement and VaR related margin modeling issues, and we also set up the floor and ceiling of the initial margins by a three-regime threshold scheme . From the backtest on the predicted historical in-sample daily VaR, we conclude that GTARCH type models perform better than the EWMA model.

A lot of other researches related to this work can be put on the agenda. For example, in the technique area, we can try different bootstrap algorithms with rolling windows to estimate and forecast VaR within different time span, with which to enhance the initial margin set up. We can also compare different fitting techniques such as Bayesian algorithm and apply other back tests to full fill our analysis. In the application area, we can investigate the performance of the Spline-GTARCH model with different macro economic variables and compare their impact during the financial crisis. We can asses the effect of the suggested margin buffer proposed by this paper and measure its influence on the financial stability and funding liquidity under stressed market.

A EWMA Model Fitting and Forecasting

Exponentially Weighted Moving Average (EWMA) is the JP Morgans RiskMetrics model and EWMA can be defined as:

$$\sigma_t^2 = (1-\lambda)r_{t-1}^2 + \lambda\sigma_{t-1}^2; \tag{19}$$

Forecasts of time t variance are obtained as a weighted average of previous variance and squared return, with weights λ and $1(1-\lambda)$.

To fit the λ, we can apply the MLE and the log likelihood function is :

$$L(\hat{\lambda}) = log(L(r_t)|\lambda) = -\frac{T}{2}log2\pi - \frac{1}{2}\sum_{t=1}^{T}(log\sigma_t^2 + \frac{r_t^2}{\sigma_t^2}). \tag{20}$$

There is only one-step ahead forecast for the EWMA model because EWMA volatility representation does not assume a long-run average volatility, and thus, for any forecast horizon beyond one-step, the EWMA returns a constant value:

$$\begin{aligned} E(\sigma_{t+1}^2|t) &= (1-\lambda)E(r_t^2) + \lambda\sigma_t^2; \\ E(\sigma_{t+1}^2|t) &= (1-\lambda)E(\sigma_t^2) + \lambda\sigma_t^2 = \sigma_t^2; \\ E(\sigma_{t+k}^2|t) &= \sigma_t^2. \end{aligned}$$

B The intuition behind the GARCH model

The generalized autoregressive conditional heteroscedasticity (GARCH) model is originated from estimating the time variations in the volatility of asset return, which represent the conditional heteroscedasticity in returns and capture the risk premia on conditionally heteroscedastic assets or portfolios.

A simple return on asset can be defined as $r_t = \mu + \sigma_t \eta_t$ and η_t follows an i.i.d. $N(0,1)$. Let $\varepsilon_t = \sigma_t \eta_t$ and Engle (1982)[9] proposed the following ARCH(1) model to estimate the variance:

$$r_t = \mu + \varepsilon_t, \tag{21}$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2. \tag{22}$$

ARCH model omitted persistence effect after the crisis because variance in time t only depends on previous squared residuals. Bollerslev (1986)[3] developed the following type of GARCH model in sprit of the ARMA model to capture the volatility persistence:

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2. \tag{23}$$

One benefit of this GARCH(1,1) model is that it can be transformed as an ARCH(∞) model as below so that GARCH(1,1) avoids picking the lag in the ARCH(q) model and holds powerful success in empirical practice.

$$
\begin{aligned}
\sigma_t^2 &= \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \\
&= \omega + \alpha\varepsilon_{t-1}^2 + \beta(\omega + \alpha\varepsilon_{t-2}^2 + \beta\sigma_{t-2}^2) \\
\dots \\
&= \frac{\omega}{1-\alpha-\beta} + \alpha\sum_{i=0}^{\infty}\beta^i\varepsilon_{t-1-i}^2 \\
&= \frac{\omega}{1-\alpha-\beta} + ARCH(\infty)
\end{aligned}
$$

This indicates that the conditional variance is the sum of weighted squared residuals with decaying weights along the time plus the the long-run unconditional variance $E(\sigma^2) = \frac{\omega}{1-\alpha-\beta}$.

Another advantage of the GARCH(1,1) model is its forecasting capability. Following the algebra above, variance forecasts can be written as

$$
\begin{aligned}
\sigma_{t+1|t}^2 &= \omega + \alpha\varepsilon_t^2 + \beta\sigma_t^2 \\
&= E(\sigma_2)(1-\alpha-\beta) + \alpha\varepsilon_t^2 + \beta\sigma_t^2 \\
&= \sigma^2 + (\alpha+\beta)(\sigma_t^2 - \sigma^2) \\
\sigma_{t+2|t}^2 &= \sigma^2 + (\alpha+\beta)(\sigma_{t+1|t}^2 - \sigma^2) \\
&= \sigma^2 + (\alpha+\beta)^2(\sigma_t^2 - \sigma^2) \\
\dots \\
\sigma_{t+l|t}^2 &= \sigma^2 + (\alpha+\beta)(\sigma_{t+l-1|t}^2 - \sigma^2) \\
&= \sigma^2 + (\alpha+\beta)^l(\sigma_t^2 - \sigma^2)
\end{aligned}
$$

When $\alpha+\beta < 1$, the data is stationary and $lim(\alpha+\beta)^l \to 0$ if $l \to \infty$, which means that the very long run horizon forecast is its unconditional long-run variance σ^2. This property reflects the mean-reverting characteristic of the GARCH(1,1) model and therefore high $(\alpha+\beta)$ can be treated as a high persistence level. If $(\alpha + \beta)$ is relatively small, the forecasted variance will converge quicker to the unconditional long-run variance. Since any persistent economic shocks will push volatility away from its long-run variance for a long time, analyzing the persistence will help us measure the magnitude of the economic shock.

Table 2: **Monte Carlo Experiment Results**

		SGTARCH				*STGARCH*				*STARCH*				*SGARCH*			
	True	*cons*		*cons*		*uncon*		*cons*		*uncon*		*cons*		*uncon*		*cons*	
	parm	*parm*	*Std*	*parm*	*Std*	*parm*	*Std*	*parm*	*Std*	*parm*	*Std*	*parm*	*Std*	*parm*	*Std*	*parm*	*Std*
μ	0.0065	0.0031	0.0104	0.0003	0.3363	−0.0040	0.0219	0.0003	0.0221	−0.0005	0.0016	0.0005	0.0016	−0.0016	0.0079	0.0004	0.0087
α	0.0031	0.0277	0.0529	0.0052	0.1266	0.0611	0.0244	0.0577	0.0246	−0.0077	0.0064	0.0028	0.0065	0.0914	0.0120	0.0901	0.0131
β	0.8516	0.8553	0.1512	0.8581	0.2629	0.8301	0.0202	0.8202	0.0204	0.9082	0.0084	0.9009	0.0086	0.8780	0.0165	0.8801	0.0182
γ	0.0990	0.0879	0.1558	0.0987	0.2073				0.0000	0.1531	0.0132	0.1431	0.0135				0.0000
δ	0.1315	0.1165	0.0799	0.1278	0.3566	0.1869	0.0779	0.1995	0.0786								0.0000
c	0.3506	0.1979	0.1583	0.0452	0.2270	0.1765	6.8225	0.0389	6.8907	0.0518	0.0106	0.0511	0.0108	0.0459	0.0107	0.0530	0.0118
w_1	−0.3197	0.1181	0.3234	0.1274	0.1618	0.1098	0.1081	0.0283	0.1092	0.1102	0.0693	0.1313	0.0707	0.0568	0.3952	0.0664	0.4347
w_2	0.7751	0.2206	0.3907	0.4429	0.5336	0.3265	1.3754	0.6374	1.3891	0.3525	0.2061	0.3945	0.2103	0.5185	0.6521	0.4910	0.7174
w_3	−0.5593	−1.3276	0.5630	−1.8251	0.9026	−1.5347	1.5205	−2.0534	1.5357	−1.5181	0.2931	−1.7557	0.2990	−1.8405	0.3985	−1.7590	0.4384
w_4	0.4743	1.0500	0.6388	1.4523	1.4921	1.2221	1.0201	1.7567	1.0303	1.1295	0.3030	1.3786	0.3091	1.4629	0.4838	1.3668	0.5321
w_5	−1.3315	0.4537	0.6137	0.4199	2.1252	0.5049	0.8145	0.1805	0.8226	0.5144	0.3254	0.4612	0.3319	0.3504	0.8634	0.4571	0.9498
w_6	1.6216	−0.8487	0.5668	−1.2525	2.0282	−1.4335	1.1449	−1.1121	1.1564	−1.0699	0.3248	−1.2075	0.3313	−1.1228	0.9353	−1.1960	1.0288
w_7	−0.9912	0.6161	0.5297	1.2320	3.6665	1.3881	0.9591	1.0211	0.9687	0.8949	0.3273	1.2248	0.3338	1.1405	0.6883	1.1313	0.7571
w_8	0.8693	−0.0879	0.5527	−0.5241	3.5744	−0.2291	1.4051	−0.3886	1.4192	−0.2553	0.3603	−0.5858	0.3676	−0.5362	0.5941	−0.5435	0.6535
w_9	−1.4078	−1.0345	0.6587	−1.2359	3.2580	−1.2289	1.4352	−0.9556	1.4496	−1.3796	0.5260	−1.1621	0.5365	−1.0486	0.7626	−1.0268	0.8389
$\alpha+\beta+\frac{1}{2}\gamma+\frac{1}{2}\delta$	0.9699	0.9852		0.9765		0.9846		0.9777		0.9771		0.9752		0.9694		0.9701	
BIC		−0.2768		−0.2932		−0.2733		−0.2776		−0.2860		−0.2845		−0.2541		−0.2535	
AIC		−0.2963		−0.3127		−0.2916		−0.2959		−0.3042		−0.3028		−0.2710		−0.2705	

		GTARCH				*TGARCH*				*TARCH*				*GARCH*			
	True	*cons*		*cons*		*uncon*		*cons*		*uncon*		*cons*		*uncon*		*cons*	
	parm	*parm*	*Std*	*parm*	*Std*	*parm*	*Std*	*parm*	*Std*	*parm*	*Std*	*parm*	*Std*	*parm*	*Std*	*parm*	*Std*
μ		−0.0016	0.0016	0.0001	0.0018	−0.0022	0.0016	0.0001	0.0018	−0.0015	0.0018	0.0000	0.0019	−0.0024	0.0019	0.0001	0.0019
α		0.0360	0.0070	0.0365	0.0070	0.0734	0.0063	0.0736	0.0065	0.0736	0.0073	0.0736	0.0066	0.0000	0.0000	0.0001	0.0000
β		0.8883	0.0103	0.8874	0.0100	0.8603	0.0096	0.8583	0.0097	0.1386	0.0144	0.1413	0.0147	0.9148	0.0067	0.9148	0.0068
γ		0.0895	0.0138	0.0859	0.0136					0.8593	0.0093	0.8584	0.0098				
δ		0.0724	0.0179	0.0754	0.0165	−0.0022	0.0016	0.0001	0.0018								
ω		0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0001	0.0000
$\alpha+\beta+\frac{1}{2}\gamma+\frac{1}{2}\delta$		1.0053		1.0045		0.9326		0.9320		0.6418		0.6441		0.9148		0.9148	
BIC		−0.2539		−0.2540		−0.2452		−0.2462		−0.2458		−0.2462		−0.2289		−0.2285	
AIC		−0.2617		−0.2618		−0.2517		−0.2527		−0.2523		−0.2527		−0.2341		−0.2337	

Table 3: **Results of Estimated Models for SPX**

Parm	GTARCH						GTARCH0						GJR-GARCH						GARCH					
	No Spline		Spline		SMacro		No Spline		Spline		SMacro		No Spline		Spline		SMacro		No Spline		Spline		SMacro	
	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std
μ	0	(0)	0.008	(0.015)	0.013	(0.015)	0.001	(0)	0.026	(0.014)	-0.726	(0.154)	0.018	(0.013)	0.025	(0.012)	0.028	(0.013)	0.055	(0.014)	0.058	(0.013)	0.058	(0.013)
ω	0.023	(0.004)					0.022	(0.003)					0.023	(0.003)					0.023	(0.003)				
α	0	(0.013)	0	(0.022)	0	(0.023)	0.078	(0.008)	0.065	(0.006)	0.061	(0.018)	0	(0.019)	0	(0)	0	(0)	0.024	(0.004)	0.097	(0.009)	0.089	(0.010)
β	0.837	(0.019)	0.771	(0.028)	0.781	(0.030)	0.789	(0.015)	0.734	(0.015)	0.729	(0.033)	0.888	(0.019)	0.840	(0.013)	0.841	(0.016)	0.875	(0.012)	0.833	(0.015)	0.826	(0.018)
γ	0.140	(0.020)	0.131	(0.025)	0.122	(0.023)							0.175	(0.021)	0.189	(0.017)	0.177	(0.020)						
δ	0.160	(0.025)	0.243	(0.033)	0.230	(0.035)	0.001	(0)	0.310	(0.025)	0.316	(0.031)												
c			2.718	(0.222)	0.936	(0.289)			1.800	(0.300)	0.846	(0.513)			2.246	(0.418)	0.754	(0.193)			1.873	(0.373)	0.640	(0.158)
w_1			-1.973	(0.236)	-0.711	(0.118)			-1.454	(0.370)	-0.726	(0.154)			-2.215	(0.426)	-0.760	(0.106)			-1.994	(0.462)	-0.753	(0.107)
w_2			4.110	(0.626)	0.854	(0.251)			2.860	(1.030)	0.821	(0.448)			4.587	(1.211)	1.032	(0.241)			4.196	(1.281)	0.891	(0.222)
w_3			-2.430	(0.712)	2.106	(0.567)			-1.300	(1.157)	2.365	(0.838)			-2.706	(1.591)	1.854	(0.618)			-2.589	(1.516)	2.301	(0.566)
w_4			-0.008	(0.700)	-4.443	(0.743)			-0.894	(0.900)	-4.796	(0.936)			0.185	(1.765)	-4.267	(0.804)			0.099	(1.424)	-4.759	(0.759)
w_5			1.669	(0.679)	2.831	(0.455)			2.948	(0.872)	2.918	(0.468)			1.645	(1.704)	2.752	(0.486)			2.109	(1.436)	2.917	(0.464)
w_6			-1.607	(0.687)	-1.002	(0.398)			-3.167	(1.101)	-0.872	(0.569)			-2.343	(1.519)	-0.972	(0.380)			-2.689	(1.521)	-0.964	(0.353)
w_7			1.628	(1.152)	0.736	(0.527)			3.230	(1.157)	0.651	(0.791)			3.429	(1.782)	0.837	(0.405)			3.071	(1.652)	0.911	(0.365)
w_8			-5.859	(1.744)	-0.859	(0.728)			-7.984	(1.153)	-1.153	(1.161)			-9.085	(2.101)	-1.130	(0.559)			-8.526	(2.104)	-1.532	(0.506)
w_9			6.568	(1.585)					8.707	(0.996)					10.785	(1.950)					10.411	(2.108)		
w_{10}			-1.421	(0.907)					-2.060	(0.763)					-5.566	(1.875)					-4.571	(1.650)		
w_{11}			-1.728	(0.815)					-2.658	(0.803)					2.000	(2.214)					0.363	(1.501)		
w_{12}			1.752	(0.815)					2.842	(0.783)					-1.081	(2.132)					-0.282	(1.476)		
w_{13}			-1.128	(0.782)					-1.278	(1.194)					0.239	(1.876)					0.904	(1.412)		
w_{14}			0.175	(0.796)					-0.011	(1.153)					-0.013	(1.834)					-0.960	(1.667)		
w_{15}			2.089	(1.145)					1.689	(1.088)					1.971	(2.014)					2.481	(1.911)		
w_{16}			-3.193	(1.194)					-3.076	(1.212)					-3.218	(2.222)					-4.310	(1.951)		
w_{17}			0.005	(1.105)					0.523	(1.220)					-0.201	(2.488)					1.848	(2.378)		
$Inflation$					0.084	(0.109)					0.081	(0.150)					0.112	(0.103)					0.062	(0.092)
$Inflation_V$					-1.309	(0.829)					-1.983	(0.748)					-1.190	(1.150)					-1.983	(1.122)
$InterestR$					0.675	(0.152)					0.749	(0.165)					0.666	(0.152)					0.768	(0.149)
$InterestR_V$					2.077	(1.166)					2.050	(2.079)					4.326	(2.033)					4.972	(1.541)
$unemp_V$					-1.456	(0.779)					-5.099	(1.911)					-5.132	(4.081)					-6.893	(3.957)
USD_V					0.992	(0.332)					1.176	(0.329)					1.028	(0.315)					1.167	(0.305)
GDP					-0.218	(0.088)					-0.250	(0.154)					-0.245	(0.087)					-0.269	(0.076)
GDP_V					0.224	(0.216)					0.500	(0.247)					0.403	(0.235)					0.601	(0.231)
$Persistence$	0.987		0.958		0.957		0.878		0.953		0.948		0.867		0.935		0.929		0.976		0.930		0.915	
BIC	2.324		2.333		2.330		2.344		2.357		2.350		2.335		2.346		2.343		2.371		2.390		2.382	
AIC	2.313		2.292		2.291		2.336		2.319		2.313		2.335		2.309		2.308		2.326		2.353		2.347	

Table 4: **Results of Estimated Models for TSX**

Parm	GTARCH						GTARCH0						GJR-GARCH						GARCH					
	No Spline		Spline		SMacro		No Spline		Spline		SMacro		No Spline		Spline		SMacro		No Spline		Spline		SMacro	
	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std	parm	Std
μ	0	0	0.010	(0.005)	0.013	(0.006)	0	0	0.016	(0.005)	-1.256	(0.772)	0.014	(0.005)	0.015	(0.005)	0.016	(0.005)	0.025	(0.005)	0.026	(0.005)	0.024	(0.005)
ω	0.002	0					0.002	0					0.003	(0.001)									0.002	(0.001)
α	0.019	(0.009)	0	(0.011)	0	(0.062)	0.058	(0.007)	0.046	(0.005)	0.045	(0.010)	0.010	(0.009)	0	0	0	0	0.099	(0.010)	0.077	(0.008)	0.002	(0.001)
β	0.869	(0.013)	0.841	(0.014)	0.826	(0.058)	0.844	(0.013)	0.808	(0.020)	0.796	(0.023)	0.914	(0.011)	0.872	(0.013)	0.861	(0.015)	0.890	(0.011)	0.838	(0.017)	0.902	(0.012)
γ	0.069	(0.015)	0.076	(0.016)	0.079	(0.047)							0.107	(0.015)	0.130	(0.016)	0.122	(0.013)						
δ	0.137	(0.024)	0.192	(0.022)	0.168	(0.031)	0	0	0.239	(0.026)	0.227	(0.045)												
c			0.871	(0.122)	1.334	(0.362)			0.713	(0.095)	1.314	(1.305)			0.466	(0.063)	0.530	(0.113)	0.962	(0.094)	0.736	(0.038)		
w_1			-0.674	(0.140)	-1.332	(0.218)			-0.632	(0.218)	-1.256	(0.772)			-0.286	(0.252)	-0.805	(0.330)	-0.418	(0.506)	-0.783	(0.241)		
w_2			2.026	(0.314)	3.000	(0.437)			1.938	(0.636)	2.911	(1.450)			0.722	(0.798)	1.466	(0.854)	1.117	(1.509)	1.308	(0.669)		
w_3			-1.882	(0.498)	-0.468	(0.492)			-1.692	(0.681)	-0.408	(1.480)			-0.029	(1.165)	1.060	(1.104)	-0.373	(1.732)	1.662	(0.863)		
w_4			0.370	(0.985)	-4.362	(0.980)			0.235	(0.544)	-4.445	(6.642)			-1.166	(1.343)	-5.151	(1.582)	-0.668	(1.844)	-5.982	(1.057)		
w_5			0.738	(1.138)	5.727	(1.627)			0.605	(0.772)	5.771	(10.502)			1.635	(1.373)	6.368	(1.716)	0.440	(2.174)	6.999	(1.298)		
w_6			0.564	(0.773)	-1.189	(1.090)			0.474	(1.171)	-1.575	(4.702)			0.173	(1.283)	-0.871	(1.125)	1.001	(1.863)	-2.058	(1.531)		
w_7			-5.860	(1.009)	-6.600	(1.753)			-5.225	(1.311)	-5.827	(3.282)			-5.863	(1.389)	-8.453	(2.149)	-5.400	(1.665)	-6.957	(2.570)		
w_8			8.342	(1.637)	8.539	(1.679)			7.426	(1.262)	7.997	(1.943)			8.688	(1.551)	10	(1.689)	7.829	(1.922)	9.785	(2.378)		
w_9			-4.116	(1.457)	-3.104	(0.968)			-3.014	(1.217)	-2.804	(3.835)			-4.278	(1.481)	-3.269	(1.066)	-3.860	(1.928)	-3.972	(1.353)		
w_{10}			-0.486	(1.081)	-1.983	(0.993)			-1.745	(0.998)	-2.405	(4.409)			-1.089	(1.386)	-2.301	(1.086)	-1.057	(1.910)	-1.900	(0.982)		
w_{11}			2.021	(0.868)	3.062	(1.029)			3.212	(1.035)	3.600	(3.912)			2.575	(1.351)	3.018	(1.252)	2.359	(2.149)	3.009	(1.007)		
w_{12}			-1.660	(0.744)	-2.623	(0.880)			-2.686	(1.083)	-2.871	(2.801)			-1.462	(1.340)	-1.785	(1.251)	-1.136	(2.138)	-1.866	(1.072)		
w_{13}			1.716	(0.739)	4.839	(1.106)			3.417	(0.950)	4.603	(3.546)			2.481	(1.643)	3.933	(1.325)	1.975	(2.456)	4.095	(1.268)		
w_{14}			-1.846	(0.908)	-7.824	(1.689)			-5.281	(0.866)	-7.622	(4.738)			-5.155	(1.943)	-7.644	(1.583)	-4.909	(2.715)	-8.343	(1.480)		
w_{15}			0.286	(0.999)	8.409	(2.383)			4.938	(1.715)	8.602	(6.186)			4.835	(2.432)	8.482	(2.208)	5.256	(3.321)	10	(1.696)		
w_{16}																								
w_{17}																								
Inflation					-0.191	(0.145)					-0.281	(0.138)					-0.263	(0.125)			-0.395	(0.105)		
$Inflation_V$					-10	(2.537)					-9.358	(19.123)					-9.413	(3.119)			-10	(1.798)		
InterestR					0.163	(0.140)					0.119	(1.013)					0.332	(0.140)			0.191	(0.091)		
$InterestR_V$					-2.888	(1.116)					-2.279	(6.446)					-0.806	(1.012)			-6.055	(1.211)		
$unemp_V$					1.604	(1.196)					0.853	(2.361)					1.197	(1.172)			5.960	(4.520)		
USD_V					0.412	(0.143)					0.417	(0.148)					0.501	(0.137)			0.578	(0.107)		
GDP					-0.036	(0.054)					-0.036	(0.063)					-0.023	(0.040)			-0.032	(0.034)		
GDP_V					0.057	(0.059)					0.048	(0.240)					0.028	(0.044)			-0.022	(0.060)		
Persistence	0.991		0.974		0.949		0.902		0.974		0.955		0.978		0.938		0.922		0.989		0.915		0.904	
BIC	2.306		2.313		2.326		2.312		2.324		2.337		2.312		2.321		2.332		2.373		2.354		2.327	
AIC	2.296		2.276		2.275		2.303		2.289		2.287		2.303		2.288		2.284		2.340		2.307		2.320	

Table 5: **Degree of Risk Aversion for SPX and TSX**

		GTARCH	GTARCH0	GJR-GARCH	GARCH	EWMA
SPX	Spline+Macro	-0.726	-0.57	-0.658	-0.158	
	Spline	-0.764	-0.6	-0.659	-0.183	
	No Spline	-0.755	-0.544	-0.659	-0.192	-0.146
TSX	Spline+Macro	-0.744	-0.614	-0.628	-0.213	
	Spline	-0.762	-0.638	-0.661	-0.245	
	No Spline	-0.715	-0.584	-0.632	-0.255	-0.19

Table 6: **Forecasts of Volatility and Tail Risk Estimations in Low Volatility Period**

		GTARCH			GTARCH0			GJR-GARCH			GARCH			EWMA
	Forecast	NoSpline	Spline	SMacro	NoSpline	Spline	SMacro	NoSpline	Spline	SMacro	NoSpline	Spline	SMacro	
Panel A:	SPX	t = December 30, 2016												
	t+1day	10.875	9.656	10.841	10.762	8.399	9.409	10.178	9.328	10.383	9.615	8.481	9.116	8.044
σ	t+2day	11.065	9.717	10.949	10.976	8.437	9.468	10.345	9.391	10.516	9.814	8.595	9.256	
	t+3day	11.249	9.775	11.051	11.183	8.473	9.524	10.506	9.450	10.638	10.004	8.699	9.382	
VaR_{1day}	$q = 90\%$	0.854	0.772	0.867	0.849	0.683	0.771	0.820	0.765	0.858	0.787	0.713	0.759	
	$q = 95\%$	1.199	1.065	1.185	1.179	0.930	1.033	1.113	1.032	1.138	1.073	0.947	0.995	
	$q = 99\%$	1.842	1.647	1.822	1.841	1.430	1.601	1.735	1.550	1.741	1.655	1.436	1.540	
VaR_{2day}	$q = 90\%$	1.208	1.092	1.226	1.201	0.966	1.090	1.160	1.082	1.214	1.113	1.008	1.074	0.647
	$q = 95\%$	1.696	1.506	1.675	1.668	1.315	1.461	1.574	1.459	1.610	1.518	1.339	1.407	0.892
	$q = 99\%$	2.605	2.330	2.576	2.604	2.022	2.264	2.454	2.192	2.462	2.341	2.030	2.178	1.414
VaR_{3day}	$q = 90\%$	1.480	1.338	1.501	1.471	1.183	1.335	1.421	1.325	1.486	1.363	1.235	1.315	
	$q = 95\%$	2.077	1.844	2.052	2.043	1.611	1.789	1.928	1.787	1.971	1.859	1.640	1.723	
	$q = 99\%$	3.190	2.853	3.155	3.189	2.476	2.773	3.006	2.685	3.015	2.867	2.487	2.667	
ES_{1day}	$q = 90\%$	0.949	0.858	0.963	0.944	0.759	0.856	0.911	0.850	0.953	0.875	0.792	0.844	0.719
	$q = 95\%$	1.262	1.121	1.247	1.241	0.979	1.087	1.171	1.086	1.198	1.130	0.997	1.047	0.939
	$q = 99\%$	1.861	1.664	1.840	1.860	1.444	1.617	1.753	1.566	1.758	1.672	1.450	1.556	1.429
ES_{2day}	$q = 90\%$	1.342	1.214	1.362	1.335	1.073	1.211	1.289	1.202	1.348	1.237	1.120	1.193	
	$q = 95\%$	1.785	1.585	1.764	1.756	1.384	1.538	1.657	1.536	1.694	1.597	1.410	1.481	
	$q = 99\%$	2.631	2.353	2.602	2.630	2.042	2.287	2.479	2.215	2.487	2.364	2.051	2.200	
ES_{3day}	$q = 90\%$	1.644	1.486	1.668	1.635	1.314	1.483	1.579	1.472	1.651	1.515	1.372	1.461	
	$q = 95\%$	2.186	1.941	2.160	2.150	1.696	1.883	2.029	1.881	2.075	1.956	1.727	1.814	
	$q = 99\%$	3.223	2.882	3.187	3.222	2.501	2.801	3.036	2.712	3.046	2.896	2.512	2.694	
Panel B:	TSX	t = March 31, 2017												
	t+1day	4.273	3.711	4.111	4.132	3.679	4.018	4.262	3.659	4.108	4.011	3.625	4.089	3.627
σ	t+2day	4.325	3.738	4.149	4.191	3.724	4.073	4.290	3.666	4.133	4.052	3.653	4.142	
	t+3day	4.375	3.764	4.184	4.250	3.767	4.126	4.317	3.672	4.157	4.093	3.680	4.190	
VaR_{1day}	$q = 90\%$	0.346	0.307	0.344	0.339	0.307	0.335	0.356	0.308	0.336	0.340	0.310	0.351	
	$q = 95\%$	0.472	0.413	1.185	0.456	0.414	0.454	0.478	0.413	0.461	0.459	0.416	0.474	
	$q = 99\%$	0.724	0.611	1.822	0.722	0.625	0.685	0.743	0.625	0.681	0.728	0.636	0.720	
VaR_{2day}	$q = 90\%$	0.490	0.434	1.226	0.479	0.434	0.473	0.503	0.436	0.475	0.481	0.438	0.496	0.299
	$q = 95\%$	0.668	0.583	1.675	0.644	0.585	0.642	0.677	0.584	0.653	0.648	0.588	0.670	0.410
	$q = 99\%$	1.024	0.865	2.576	1.021	0.883	0.968	1.050	0.884	0.963	1.029	0.900	1.018	0.646
VaR_{3day}	$q = 90\%$	0.600	0.531	1.501	0.587	0.532	0.580	0.616	0.533	0.582	0.589	0.536	0.608	
	$q = 95\%$	0.818	0.715	2.052	0.789	0.717	0.787	0.829	0.715	0.799	0.794	0.720	0.821	
	$q = 99\%$	1.254	1.059	3.155	1.251	1.082	1.186	1.286	1.083	1.180	1.261	1.102	1.247	
ES_{1day}	$q = 90\%$	0.385	0.341	0.382	0.377	0.341	0.372	0.395	0.342	0.379	0.378	0.344	0.390	0.332
	$q = 95\%$	0.497	0.434	0.481	0.480	0.436	0.478	0.504	0.435	0.493	0.483	0.438	0.499	0.432
	$q = 99\%$	0.731	0.618	0.688	0.729	0.631	0.692	0.750	0.632	0.699	0.735	0.643	0.727	0.653
ES_{2day}	$q = 90\%$	0.544	0.482	0.541	0.533	0.482	0.526	0.559	0.484	0.535	0.535	0.487	0.552	
	$q = 95\%$	0.703	0.614	0.680	0.678	0.616	0.676	0.712	0.615	0.697	0.683	0.619	0.705	
	$q = 99\%$	1.034	0.873	0.973	1.032	0.892	0.978	1.061	0.893	0.988	1.040	0.909	1.028	
ES_{3day}	$q = 90\%$	0.667	0.590	0.662	0.652	0.591	0.644	0.685	0.593	0.656	0.655	0.596	0.676	
	$q = 95\%$	0.861	0.752	0.832	0.831	0.755	0.828	0.872	0.753	0.854	0.836	0.758	0.864	
	$q = 99\%$	1.267	1.070	1.192	1.263	1.093	1.198	1.299	1.094	1.210	1.273	1.114	1.259	

Table 7: **Forecasts of Volatility and Tail Risk Estimations in High Volatility Period**

		GTARCH			GTARCH0			GJR-GARCH			GARCH			EWMA
	Forecast	NoSpline	Spline	SMacro	NoSpline	Spline	SMacro	NoSpline	Spline	SMacro	NoSpline	Spline	SMacro	
Panel A:	SPX	t = November 11, 2008												
	t+1day	91.214	88.806	84.678	83.340	79.724	79.610	81.014	80.449	77.198	66.241	64.261	62.656	71.063
σ	t+2day	90.654	87.345	83.405	82.996	78.529	78.561	80.039	78.526	75.390	65.520	63.063	61.794	
	t+3day	90.098	85.924	82.168	82.654	77.374	77.553	79.077	76.685	73.670	64.808	61.927	60.994	
VaR_{1day}	$q = 90\%$	7.385	7.130	6.771	6.787	6.628	6.562	6.611	6.669	6.412	5.560	5.451	5.285	
	$q = 95\%$	10.119	9.726	9.044	9.138	8.683	8.475	8.893	8.845	8.113	7.490	7.205	6.581	
	$q = 99\%$	14.558	14.461	13.050	13.962	12.585	12.886	12.726	12.163	11.703	10.608	9.938	9.714	
VaR_{2day}	$q = 90\%$	10.444	10.083	9.575	9.599	9.373	9.280	9.350	9.431	9.068	7.863	7.709	7.475	5.717
	$q = 95\%$	14.311	13.755	12.791	12.923	12.280	11.986	12.577	12.508	11.473	10.593	10.189	9.307	7.881
	$q = 99\%$	20.589	20.451	18.455	19.746	17.799	18.223	17.997	17.202	16.550	15.001	14.054	13.738	12.494
VaR_{3day}	$q = 90\%$	12.792	12.349	11.727	11.756	11.480	11.365	11.451	11.551	11.106	9.630	9.441	9.155	
	$q = 95\%$	17.527	16.846	15.665	15.828	15.040	14.679	15.404	15.320	14.052	12.973	12.479	11.399	
	$q = 99\%$	25.216	25.048	22.603	24.183	21.799	22.319	22.042	21.068	20.270	18.373	17.213	16.825	
ES_{1day}	$q = 90\%$	8.206	7.922	7.523	7.541	7.364	7.291	7.346	7.410	7.124	6.177	6.057	5.873	6.352
	$q = 95\%$	10.652	10.238	9.520	9.619	9.140	8.921	9.361	9.310	8.540	7.884	7.584	6.928	8.296
	$q = 99\%$	14.705	14.607	13.181	14.103	12.713	13.016	12.855	12.286	11.821	10.715	10.038	9.812	12.620
ES_{2day}	$q = 90\%$	11.605	11.203	10.639	10.665	10.414	10.311	10.388	10.479	10.075	8.736	8.565	8.305	
	$q = 95\%$	15.064	14.479	13.464	13.604	12.926	12.616	13.239	13.167	12.077	11.150	10.725	9.797	
	$q = 99\%$	20.797	20.658	18.641	19.945	17.978	18.407	18.179	17.375	16.717	15.153	14.196	13.877	
ES_{3day}	$q = 90\%$	14.213	13.721	13.030	13.062	12.755	12.628	12.723	12.834	12.340	10.700	10.490	10.172	
	$q = 95\%$	18.449	17.733	16.490	16.661	15.832	15.452	16.214	16.126	14.792	13.656	13.136	11.999	
	$q = 99\%$	25.471	25.301	22.831	24.428	22.019	22.544	22.265	21.280	20.474	18.558	17.387	16.995	
Panel B:	TSX	t = September 16, 2008												
	t+1day	14.694	17.452	16.305	13.449	16.265	15.233	15.724	17.760	17.229	14.554	16.287	15.911	13.363
σ	t+2day	14.649	17.404	16.149	13.459	16.295	15.183	15.570	17.541	16.913	14.477	16.218	15.760	
	t+3day	14.604	17.358	15.999	13.469	16.325	15.135	15.417	17.333	16.617	14.400	16.152	15.621	
VaR_{1day}	$q = 90\%$	1.183	1.366	1.322	1.091	1.319	1.236	1.279	1.415	1.388	1.210	1.330	1.313	
	$q = 95\%$	1.648	1.862	1.185	1.533	1.749	1.657	1.771	1.936	1.880	1.660	1.827	1.785	
	$q = 99\%$	2.573	2.794	1.822	2.377	2.689	2.486	2.810	2.834	2.856	2.648	2.625	2.706	
VaR_{2day}	$q = 90\%$	1.674	1.933	1.226	1.543	1.865	1.748	1.808	2.002	1.963	1.711	1.880	1.857	1.101
	$q = 95\%$	2.331	2.634	1.675	2.169	2.473	2.344	2.504	2.738	2.658	2.347	2.584	2.524	1.511
	$q = 99\%$	3.639	3.951	2.576	3.361	3.803	3.515	3.974	4.008	4.039	3.745	3.713	3.827	2.382
VaR_{3day}	$q = 90\%$	2.050	2.367	1.501	1.890	2.284	2.141	2.215	2.451	2.404	2.096	2.303	2.274	
	$q = 95\%$	2.855	3.225	2.052	2.656	3.029	2.870	3.067	3.354	3.256	2.875	3.165	3.091	
	$q = 99\%$	4.456	4.838	3.155	4.117	4.658	4.306	4.867	4.908	4.947	4.587	4.547	4.687	
ES_{1day}	$q = 90\%$	1.315	1.518	1.469	1.213	1.465	1.373	1.421	1.573	1.393	1.344	1.477	1.459	1.223
	$q = 95\%$	1.735	1.960	1.855	1.614	1.841	1.744	1.864	2.038	1.930	1.747	1.923	1.879	1.590
	$q = 99\%$	2.599	2.822	2.595	2.401	2.717	2.511	2.838	2.862	2.880	2.675	2.652	2.734	2.406
ES_{2day}	$q = 90\%$	1.859	2.147	2.077	1.715	2.072	1.942	2.009	2.224	1.970	1.901	2.089	2.063	
	$q = 95\%$	2.454	2.772	2.624	2.283	2.604	2.467	2.636	2.882	2.729	2.471	2.720	2.657	
	$q = 99\%$	3.675	3.991	3.670	3.395	3.842	3.551	4.014	4.048	4.073	3.783	3.750	3.866	
ES_{3day}	$q = 90\%$	2.277	2.630	2.544	2.100	2.538	2.379	2.461	2.724	2.412	2.329	2.559	2.527	
	$q = 95\%$	3.005	3.395	3.214	2.796	3.189	3.021	3.228	3.530	3.342	3.026	3.331	3.254	
	$q = 99\%$	4.501	4.887	4.495	4.158	4.705	4.349	4.916	4.958	4.988	4.633	4.593	4.735	

Table 8: **Backtests for SPX and TSX**

Upper and lower bound from the Kupiec Test		
Breaches allowed at 95% CI	LB	UB
VaR_{Q90}	310	350
VaR_{Q95}	146	175
VaR_{Q99}	22	35

Breaches for SPX Data

		GTARCH	GTARCH0	GJR-GARCH	GARCH	EWMA
Spline	90% VaR	343	336	336	307	
	95% VaR	169	166	167	160	
	99% VaR	32	33	33	29	
Spline + Macro Variable	90% VaR	336	327	326	308	
	95% VaR	171	167	166	160	
	99% VaR	34	29	32	30	
No Spline	90% VaR	336	327	326	308	354
	95% VaR	171	167	166	160	176
	99% VaR	34	29	32	30	36

Breaches for TSX Data

		GTARCH	GTARCH0	GJR-GARCH	GARCH	EWMA
Spline	90% VaR	341	328	327	321	
	95% VaR	168	163	162	155	
	99% VaR	34	32	31	30	
Spline + Macro Variable	90% VaR	330	329	332	312	
	95% VaR	164	152	162	151	
	99% VaR	33	33	33	27	
No Spline	90% VaR	330	329	332	312	356
	95% VaR	164	152	162	151	182
	99% VaR	33	33	33	27	36

Table 9: **Threshold Regimes for SPX and TSX**

	Model	Thresholds (in %)		Proportion			Unit Root		
				Low regime	Middle regime	High regime	Low regime	Middle regime	High regime
Panel B: SPX Data									
No Spline	GTARCH	1.90	2.44	33%	25%	42%	NO	NO	NO
	GTARCH0	1.91	2.95	30%	41%	28%	NO	NO	NO
	GJR-GARCH	1.88	2.39	31%	25%	44%	NO	NO	NO
	GARCH	1.93	2.47	27%	29%	44%	NO	NO	NO
	EWMA	1.82	2.90	25%	44%	31%	NO	NO	NO
	Average	1.89	2.63	29%	33%	38%			
Spline	GTARCH	1.74	2.66	26%	39%	35%	NO	NO	NO
	GTARCH0	1.92	2.47	33%	26%	41%	NO	NO	NO
	GJR-GARCH	1.87	2.47	36%	26%	38%	NO	NO	NO
	GARCH	1.86	2.29	27%	25%	48%	NO	NO	NO
	Average	1.85	2.48	31%	29%	40%			
Spline + Macro Variables	GTARCH	1.74	2.74	28%	40%	32%	NO	NO	NO
	GTARCH0	1.78	2.74	25%	41%	34%	NO	NO	NO
	GJR-GARCH	1.74	2.18	26%	25%	49%	NO	NO	NO
	GARCH	1.85	2.85	27%	43%	30%	NO	NO	NO
	Average	1.78	2.63	26%	37%	36%			
Overall	**Average**	1.84	2.58	29%	33%	38%			
Panel B: TSX Data									
No Spline	GTARCH	0.74	0.97	29%	29%	42%	NO	NO	NO
	GTARCH0	0.77	1.10	27%	41%	32%	NO	NO	NO
	GJR-GARCH	0.74	1.01	26%	33%	40%	NO	NO	NO
	GARCH	0.83	1.05	32%	29%	40%	NO	NO	NO
	EWMA	0.79	1.02	34%	26%	40%	NO	YES	YES
	Average	0.77	1.03	30%	32%	39%			
Spline	GTARCH	0.73	0.94	33%	26%	41%	NO	YES	YES
	GTARCH0	0.87	1.15	48%	25%	27%	NO	YES	YES
	GJR-GARCH	0.71	0.93	26%	30%	44%	NO	YES	YES
	GARCH	0.74	0.95	25%	27%	48%	NO	YES	YES
	Average	0.76	0.99	33%	27%	40%			
Spline + Macro Variables	GTARCH	0.82	1.12	47%	26%	27%	NO	YES	NO
	GTARCH0	0.82	1.04	41%	26%	33%	NO	YES	NO
	GJR-GARCH	0.70	0.87	25%	27%	47%	NO	NO	NO
	GARCH	0.75	1.05	26%	39%	36%	NO	YES	NO
	Average	0.77	1.02	35%	29%	36%			
Overall	**Average**	0.77	1.01	32%	29%	38%			

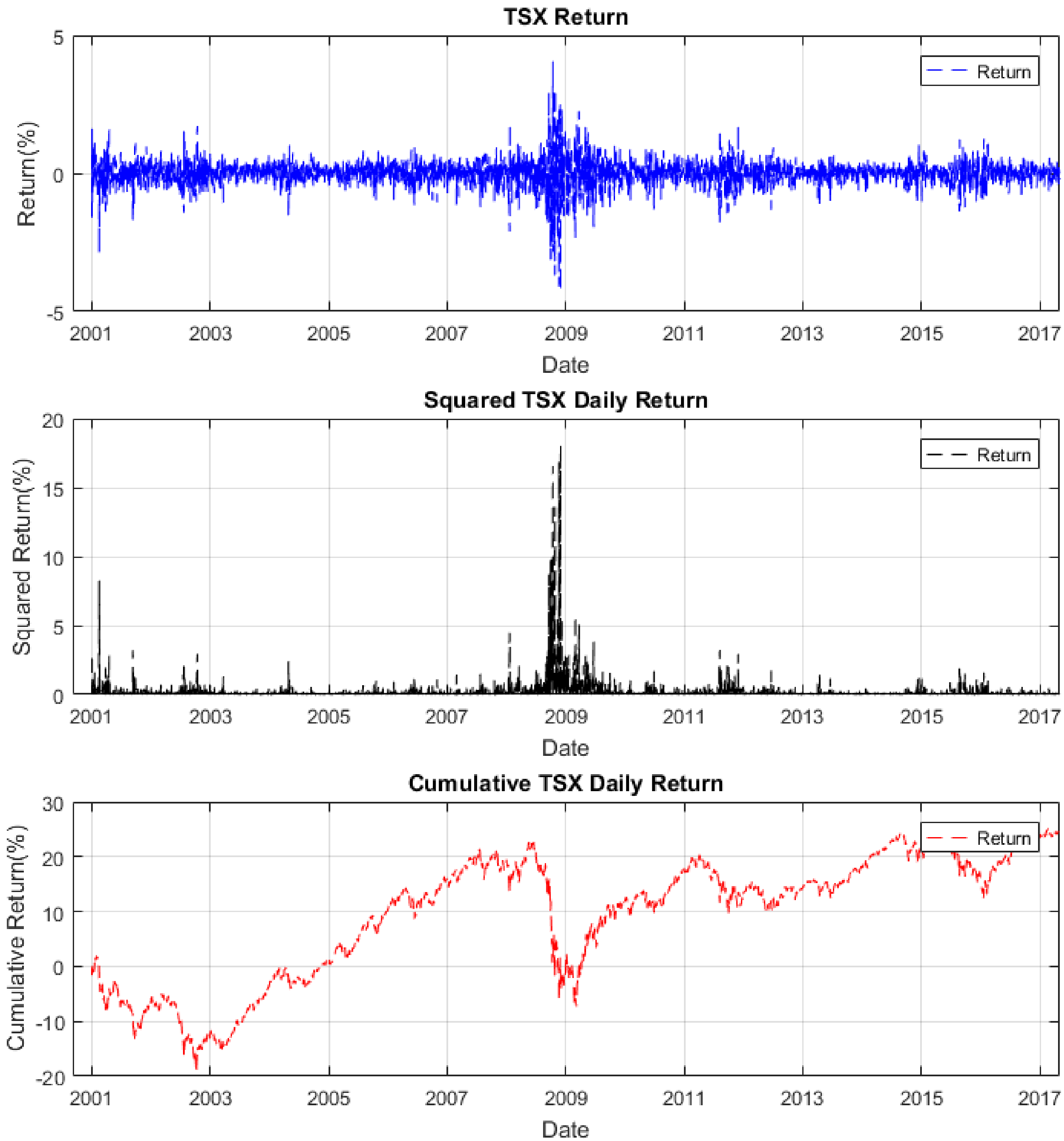

Figure 1: **TSX Data**

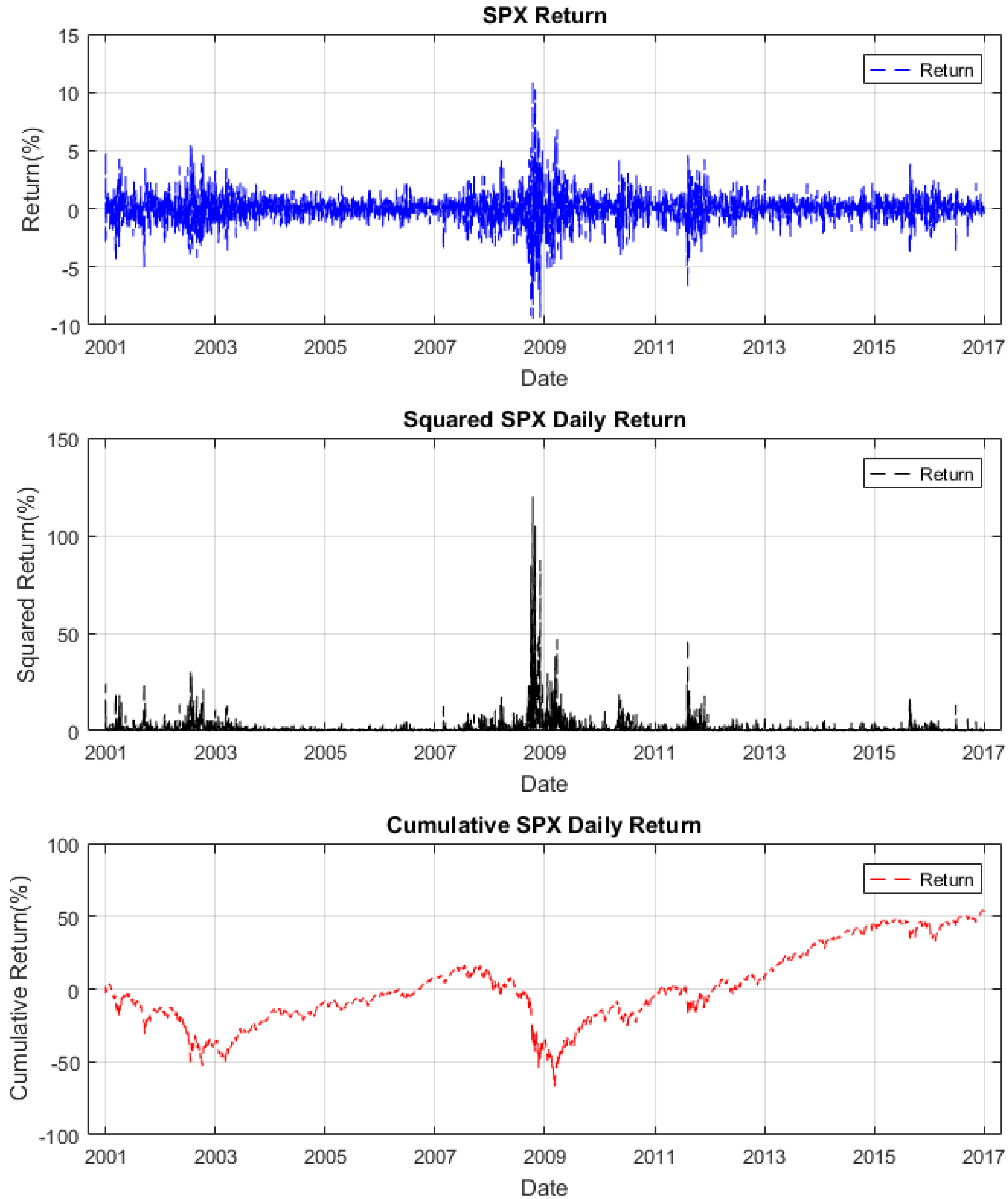

Figure 2: **SPX Data**

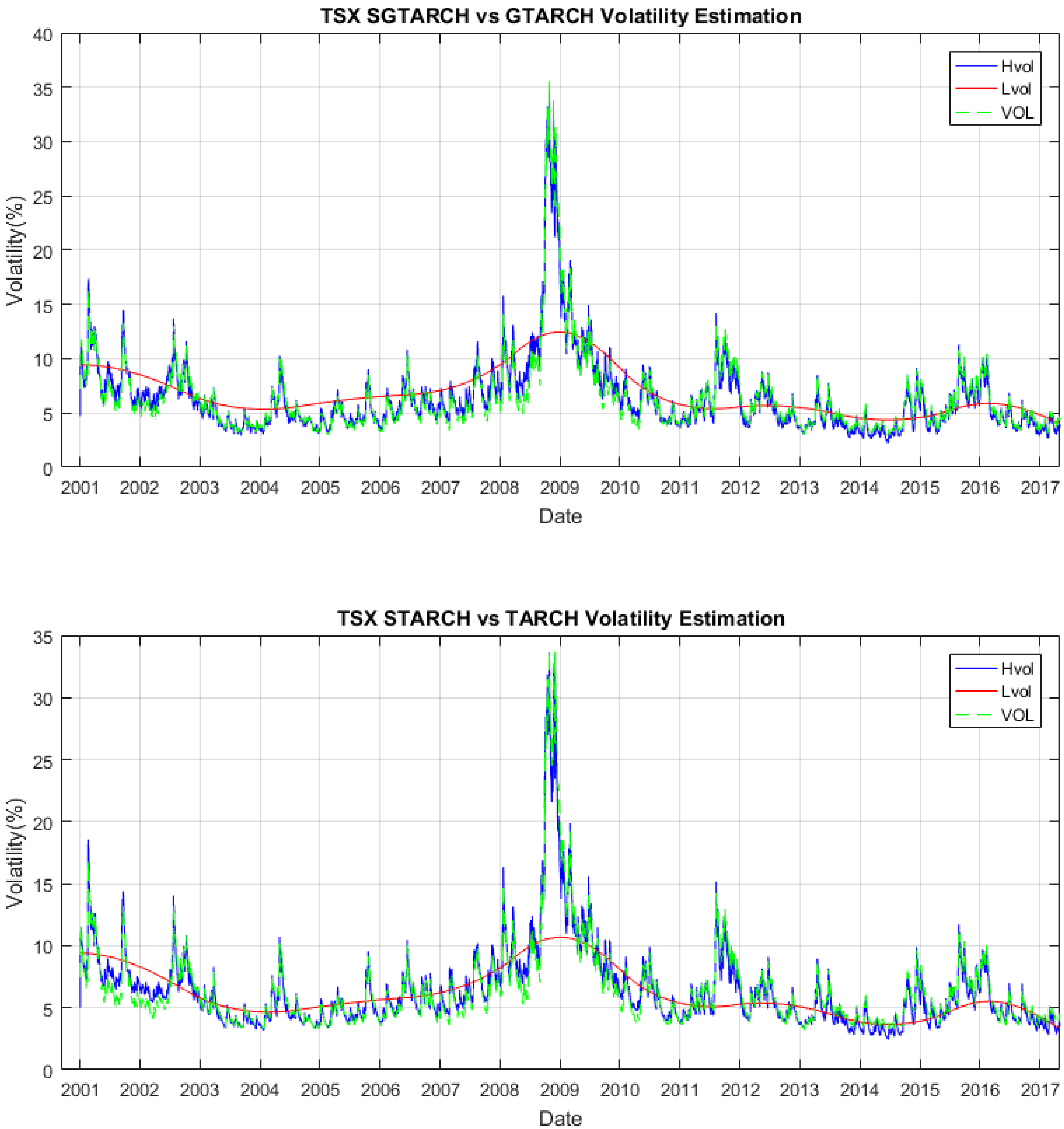

Figure 3: **Volatility Estimation of the TSX Data**

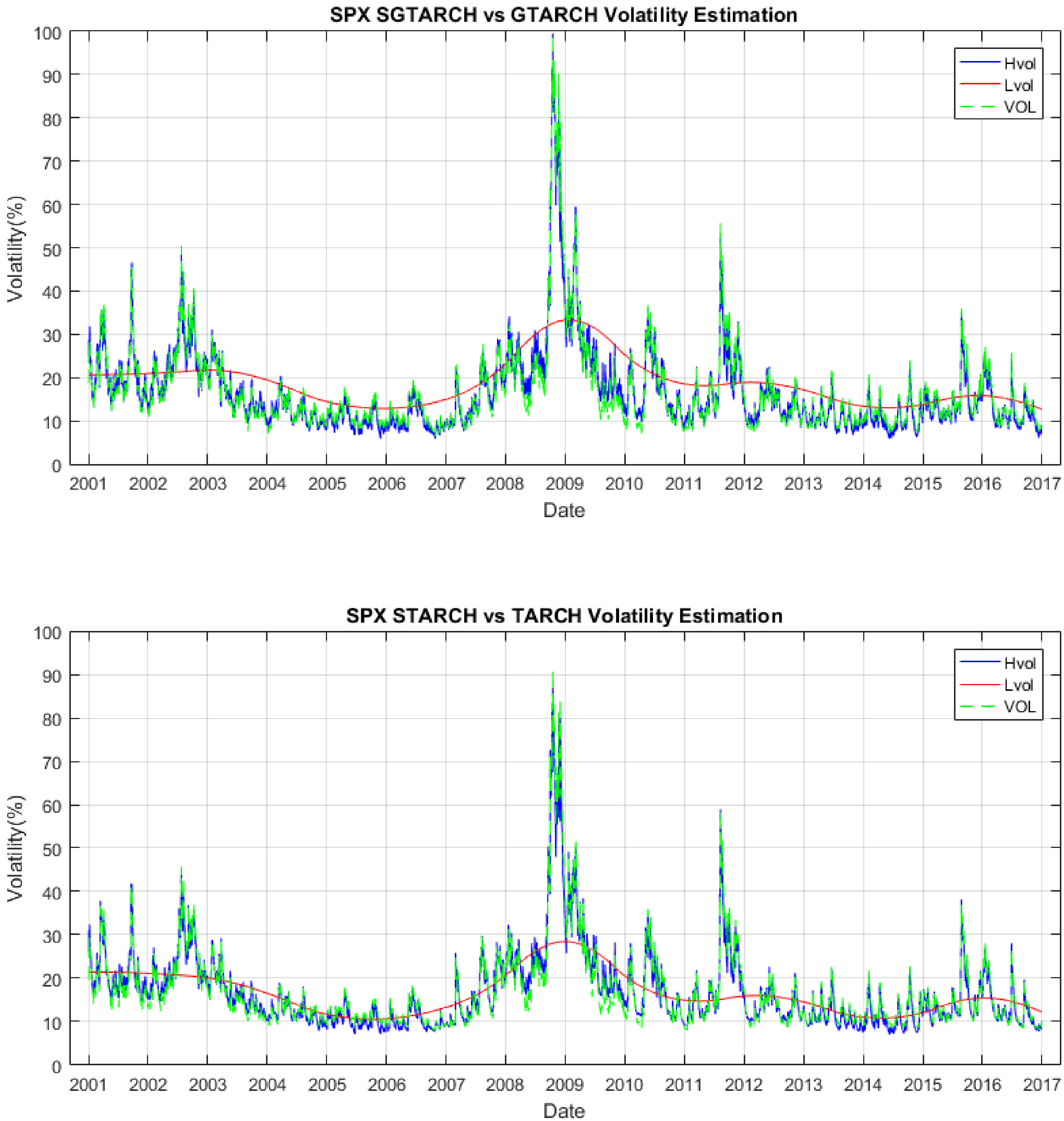

Figure 4: **Volatility Estimation of the SPX Data**

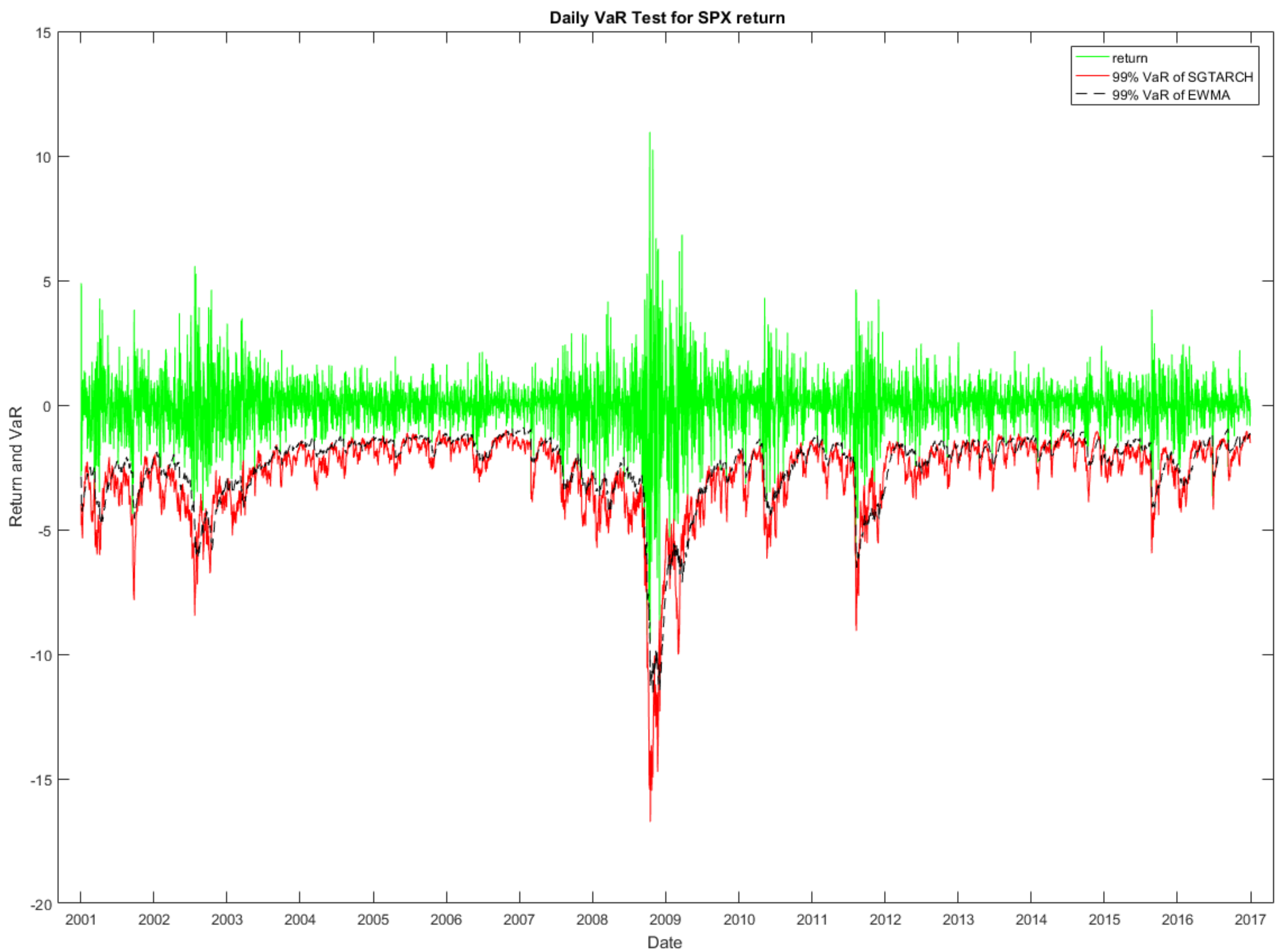

Figure 5: **Backtest the VaR for estimates of SPX**

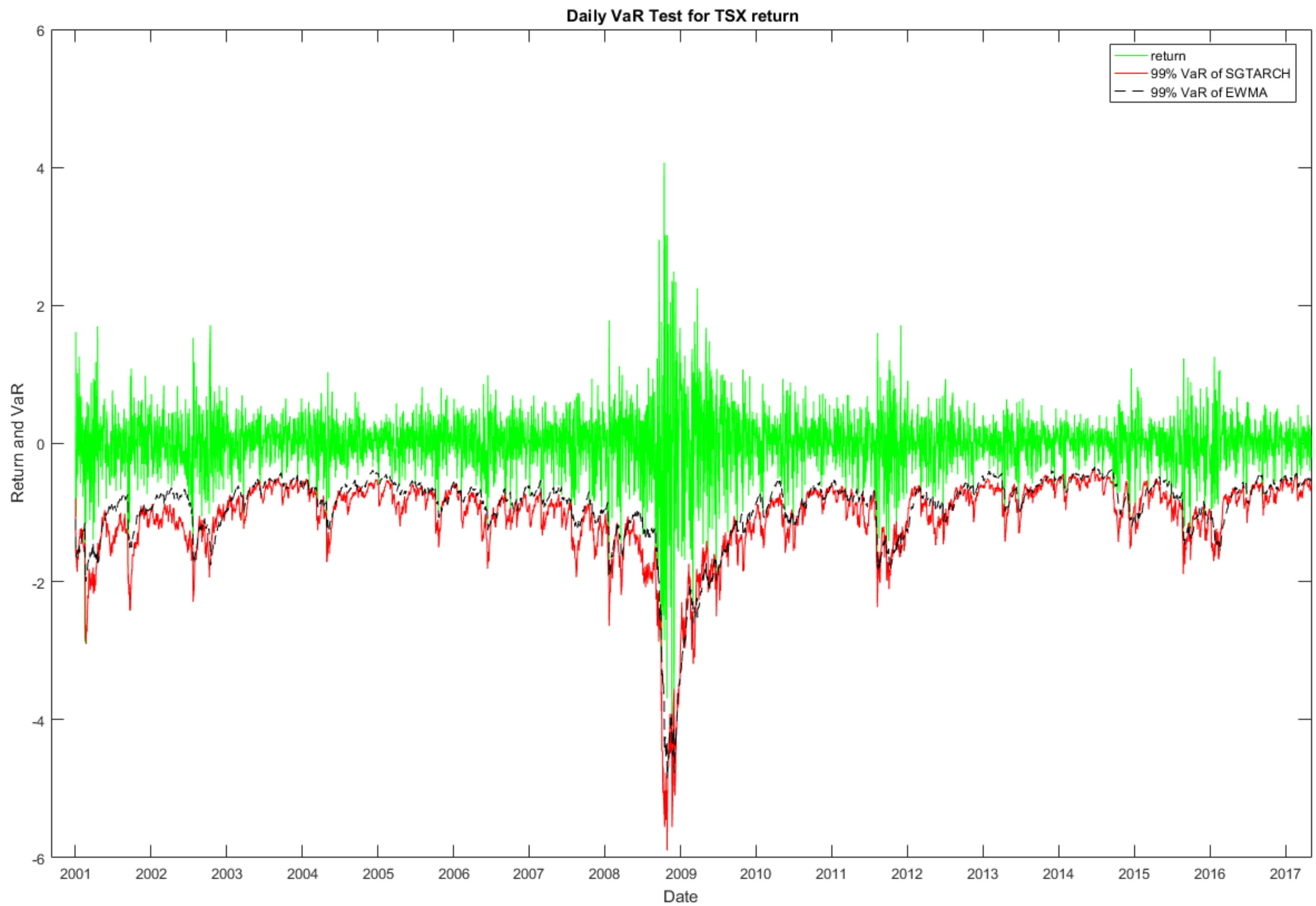

Figure 6: **Backtest the VaR for estimates of TSX**

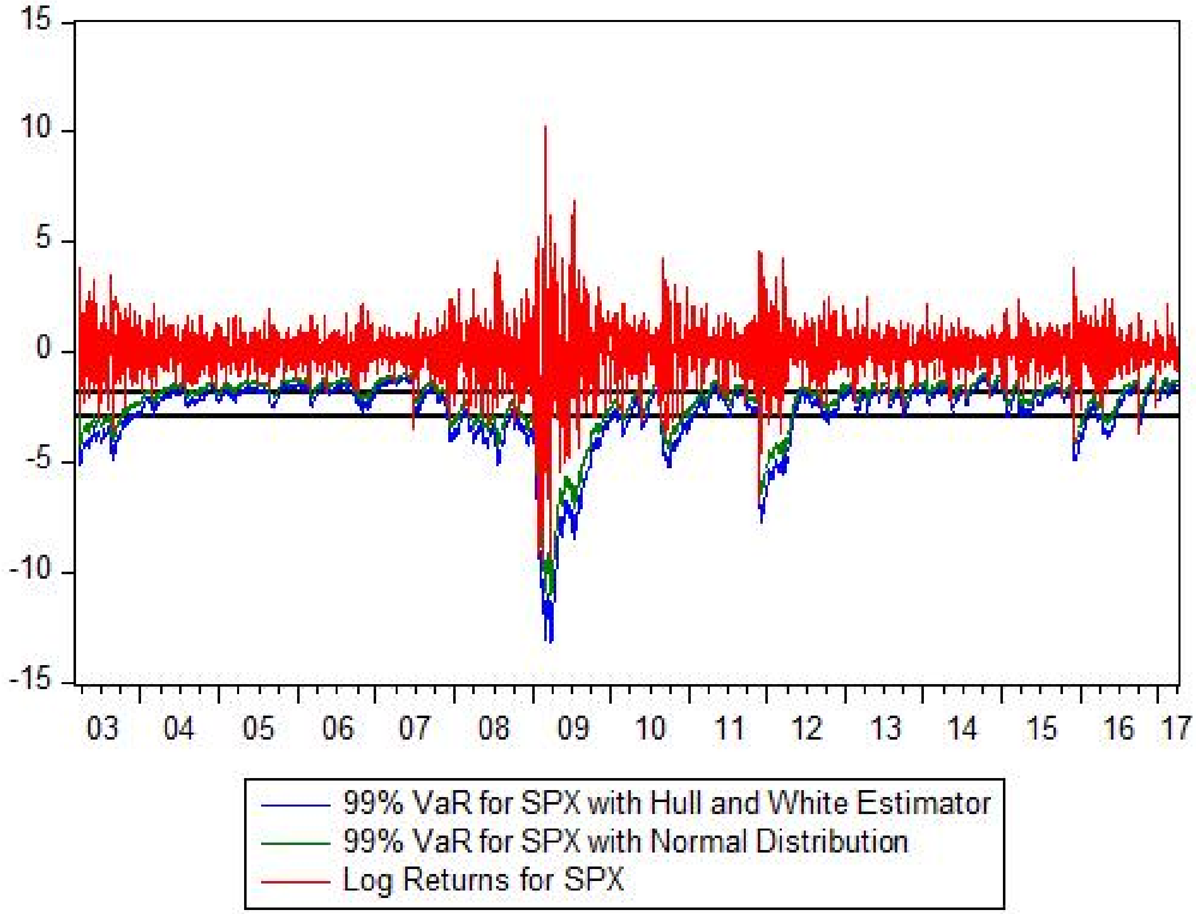

Figure 7: **Returns, VaR and Bands for SPX using EWMA Model**

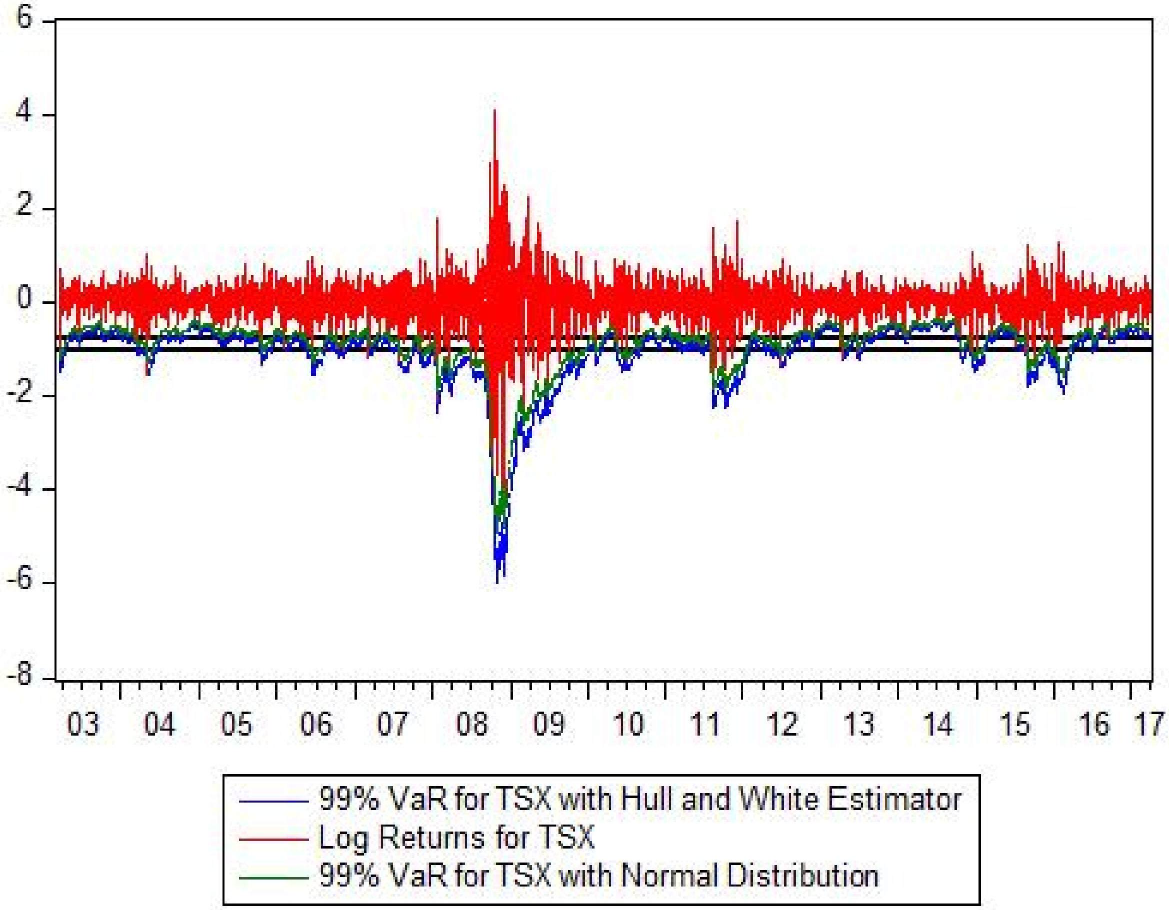

Figure 8: **Returns, VaR and Bands for TSX using EWMA Model**

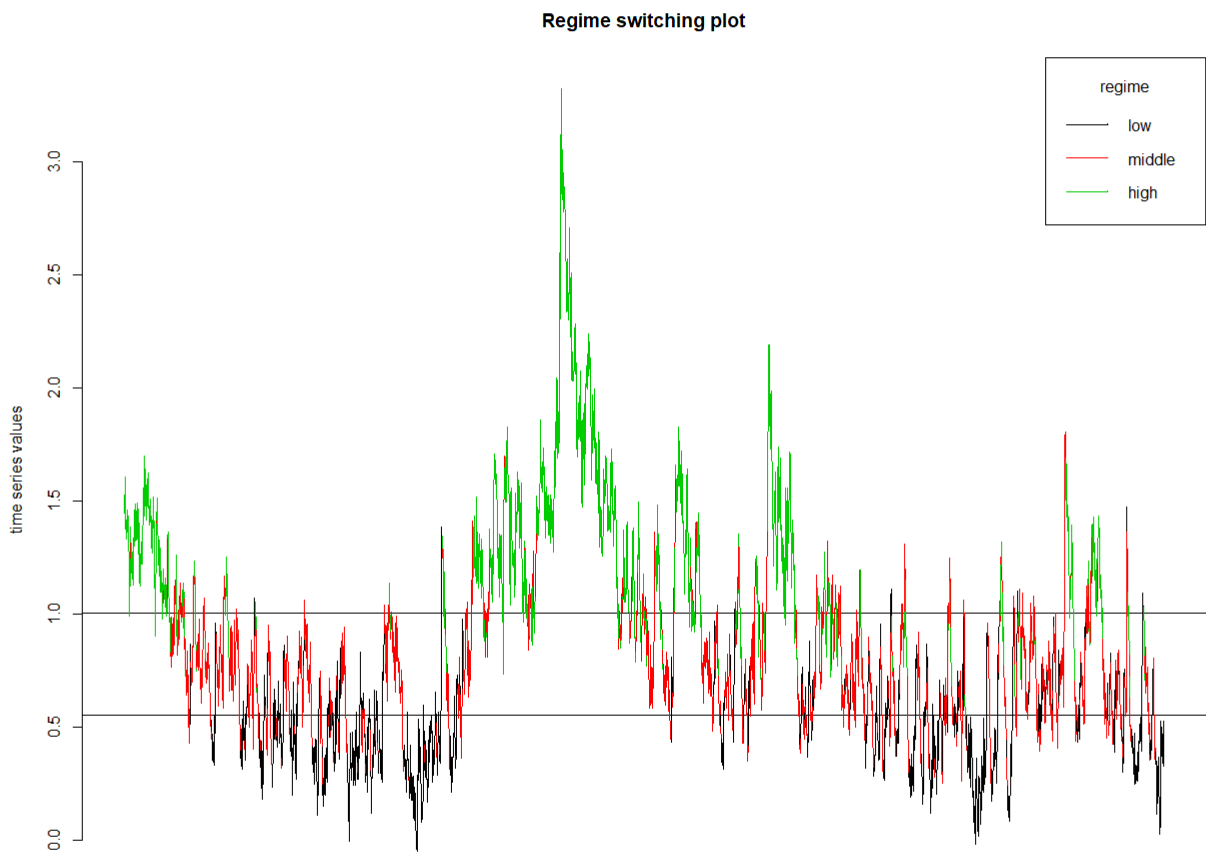

Figure 9: **Three regimes for log of VaR for SPX using Spline-Macro-GTARCH**

References

[1] Andersen, T. G., T. Bollerslev, F. X. Diebold, and P. Labys. (2003): “Modeling and forecasting realized Volatility,” *Econometrica, 71,579625.*

[2] Balduzzi, P., E. Elton, and T. Green. (2001): “Economic news and bond prices: evidence from the US treasury Market,” *The Journal of Financial and Quantitative Analysis, 36, 523-543*

[3] Bollerslev, T. (1986): “Generalized autoregressive conditional heteroskedasticity,” *Journal of Econometrics,31, 307-327.*

[4] Brownlees, C. and Engle, R. (2012): “Volatility, correlation and tails for systemic risk measurement,” *Working paper, Department of Finance, New York University.*

[5] Brunnermeire, M. and Pedersen, L. (2009): “Market liquidity adn funding liquidity,” *Review of Financial Studies, 22, 2201-2238.*

[6] Cai, J. (1994): “A markov model of switching-regime ARCH,” *Journal of Business and Economic Statistics, 13(3), 309-316.*

[7] Carpantier, J.F. and Dufays (2012): “Commodities volatility and the theory of storage,” *CORE discussion paper, University Libre de Bruxelles.*

[8] Candian derivatives clearing corporation, (2017): “Canadian derivatives clearing corporation manual,“ *CDCC website.*

[9] Engle, R. (1982): “ Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation,” *Econometrica 50 , 987-1007.*

[10] Engle, R. and Mezrich, J. (1995): "Grapping with GARCH," *Journal of Risk, September, 112-117.*

[11] Engle, R. and Rangel, J. (2008): "The spline-GARCH model for low-frequency volatility and its global macroeconomic causes," *Review of Financial Studies 21, 1187-1222.*

[12] Glosten, L., Jagannathan, R., and Runkle, D. (1993): "On the relation between the expected value and the volatility of the nominal excess return on stocks," *Journal of Finance 48, 1779-1801.*

[13] Goldman, E. (2012): "Bayesian analysis of the systemic risk rating using generalized threshold GARCH model ," *Working paper, Pace university.*

[14] Goldman, E. and Wang, T. (2012): "The Spline-Threshold-GARCH volatility model and tail risk ," *Working paper, Pace university.*

[15] Hamilton,J.D. and Susmel, R. (1994): "Autoregressive conditional heteroskedasticity and changes in regime," *Journal of Econometrics, 64,307-333.*

[16] Heller, D. and Vause, N. (2012): "Collateral requirements for mandatory central clearing of over-the-counter derivatives ," *BIS working paper, No.373.*

[17] Hull, J. and White, A. (1998): "Incorporating volatility updating into the historical simulation method for value-at-risk," *Journal of Risk, 1(1), 5-19.*

[18] Kashyap, A. and Stein. J (2014): "Cyclical implications of the Basel II capital standards ," *Federal Reserve Bank of Chicago Economic Perspectives.*

[19] Knott, R. and Polenghi, M. (2006): "Assessing central counterparty margin coverage on future contracts using GARCH models ," *Bank of England Working Paper, No.287.*

[20] Kupiec, p. (1995): “Techniques for verifying the accuracy of risk management models ,” *Journal of Derivatives, 3, 73-84.*

[21] Murphy, D., Vasio, M. and Vause, N. (2014): “An investigation into the procyclicality of risk-based initial margin models ,” *Bank of England working paper, No.29.*

[22] Murphy, D., Vasio, M. and Vause, N. (2016): “A comparative analysis of tools to limit the procyclicality of initial margin requirements ,” *Bank of England working paper, No.597.*

[23] Nelson, D.B. (1991): “Conditional heteroskedasticity in asset returns: a new approach,” *Econometrica, 59(2), 347-370.*

[24] Officer, R. F. (2012): “The factor-spline-GARCH model for high and low frequency correlations,” *Journal of Business and Economics Statistics, 30(1), 109-124.*

[25] Glasserman, P. and WU, Q. (2017): “Persistence and procyclicality in margin requirements,” *OFR Working paper series, 17-01*

[26] Rangle, J.G. and Engle, R.F. (1973): “The Variability of the Market Factor of the New York Stock Exchange,” *Journal of Business, 46,43453*

[27] Schwert, G. (1989): “Why Does Stock Market Volatility Change over Time?” *Journal of Finance, 44, 111553.*

[28] Roll, R. (1988): “R^2, ” *FJournal of Finance, 63,54166.*