Monetary-Fiscal Policy Mix and Risks of Nominal Bonds *

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Abstract

We propose a New Keynesian model with monetary-fiscal policy regime switch to explain the time-varying correlation between returns on stock and nominal Treasury bond found in the data. In the active monetary and passive fiscal policy (AMPF) regime, neutral technology (NT) and marginal efficiency of investment (MEI) shocks are the most important drivers of economic fluctuations. In the passive monetary and active fiscal policy (PMAF) regime, the effect of the NT shock is depressed due to the weak reaction of interest rate to inflation, while the effect of the MEI shock remains strong. Because the NT shock leads to positive, while the MEI shock leads to negative correlation between returns on stock and nominal bond, our model thus provides a coherent explanation for the negative correlation between stock and bond returns during 1950s and 2000s when the fiscal policies are active, and for the positive correlation during 1980-2000 when monetary policies are active.

Keywords: bond-stock return correlation, monetary-fiscal policy regime, inflation risk premium

JEL classification codes: E52, E62, G12.

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1 Introduction

Stocks and nominal Treasury bonds are the two largest classes of assets in the financial markets. Understanding the correlation between those two securities is tremendously important for the purpose of portfolio management and for the design of monetary policies. Empirically, an increasing amount of studies (Campbell et al., 2015, 2016; Christiansen and Ranaldo, 2007; Guidolin and Timmermann, 2007; Baele et al., 2010; David and Veronesi, 2013; Gourio and Ngo, 2016; Mumtaz and Theodoridis, 2017) have documented the time-varying feature of the correlation between returns on stock and nominal bonds. The overall stock-bond return correlation is slightly positive in the period 1953-2014. However, this correlation was negative in the 1950s, it becomes positive between the late 1960s to 2000, and after 2000, it turned negative again. The literature has yet provide a satisfactory explanation about the dynamics of this covariance during the post-War period. In this paper, we build a new Keynesian model with monetary-fiscal policy regime switch to explain the time-variation of the correlation between returns on stock and nominal Treasury bonds.

Since the seminal paper Leeper (1991), a large literature has been devoted to understand the effects of the monetary-fiscal policy regimes. Our model combines the monetary-fiscal policy regimes with the framework of Christiano, Motto and Rostagno (2014), which brings financial intermediaries and risk shocks into an otherwise standard New Keynesian model. Unlike Christiano et al. (2014), we incorporate the recursive preference to generate realistic asset prices and the mix of different monetary-fiscal regimes to match the structural change in the policies found in the literature. We include five structural shocks in the model to match the data: the neutral technology (NT) shock, marginal efficiency of investment (MEI) shock, investment price shock, risk shock, and monetary policy (MP) shock. Within the five shocks, the NT shock and MEI shock are the key driving forces of macro fluctuations.

The NT shock leads to positive correlation between stock and nominal bond, while the MEI shock leads to negative correlation. A positive NT shocks leads to higher output but lower inflation. The nominal interest rate reacts positively to inflation and also goes down.

Consequently, returns on stock and bond both go up, resulting a positive correlation. A positive MEI shock makes the transformation of investments into capital more efficiently, which leads to higher output, but as a demand shock, it also leads to higher inflation and higher nominal interest rate. As a result, return on stock goes up but return on nominal bond goes down, leading to negative correlation. Because the effect of the NT shock dominates that of the MEI shock, the correlation between returns on stock and bond is positive.

In the AMPF regime, nominal interest rate reacts strongly to inflation and real rate goes down after a positive NT shock, reinforcing its stimulus effect. However, in the PMAF regime, nominal interest rates only react weakly to inflation and real rate goes up after a positive NT shock. Consequently, the stimulus effect of the NT shock is largely muted and its impact on the economy becomes minimal. At the same time, the effect of the MEI shock remains strong in the PMAF regime, the correlation between returns on stock and bond is dominated by the MEI shock and becomes negative. Our model thus provides a coherent explanation for the negative correlation between stock and bond returns during 1950s and 2000s when the fiscal policies are active, and for the positive correlation during 1980-2000 when monetary policies are active.

We show that our results do not depend on the choice of the recursive preference. The beta of the nominal Treasury bonds show the same change of sign when the policy regime transits from the AMPF to the PMAF regimes under the constant relative risk aversion (CRRA) preference. We also show that the effective lower bound (ELB) is an extreme scenario of the PMAF regime, which resembles the Great Recession post-2007. Empirically, the beta of the nominal Treasury bonds drop to an extremely low level during that period.

There are three papers that are closely related to our work. Campbell et al. (2015) propose that the change in the bond-stock return correlation is driven by the changes in the sensitivity of monetary policies to inflation and output gap, and the persistence of the monetary policy shocks. However, their model cannot explain the negative beta of nominal Treasury bonds in the 1950s.

Similar to our model, Mumtaz and Theodoridis (2017) also uses the switch of monetary-fiscal policy interaction to explain the sign change in the bond-stock return correlation. However, our mechanisms are different. Their model replies on the fiscal policy shock, which leads to different sign of correlation during different regimes. Our model replies on the NT and MEI shocks, which have the largest contribution to business cycle fluctuations.

The closest one is Gourio and Ngo (2016), who focus on the zero lower bound (ZLB) period post 2008. Their New Keynesian model generates positive term premia and inflation risk premia during normal times, but these premia fall when the economy are at or close to the ZLB. The similarity of this paper and ours lies in the fact that the ZLB regime, where monetary policy is completely inactive, is an extreme case of the passive monetary policy in our setup. However, our results do not reply on the extreme inactiveness of the monetary policy. In fact, negative correlation between bond and stock returns happen not only during the ZLB period, but also in 1950's and the period between 2000 and 2008. Therefore, our framework is capable of explaining the dynamics of the correlation between stock and nominal bonds in the post-War period.

The rest of the paper proceeds as follows. Section 2 provides empirical evidence on the shifts of bond-stock return correlation and monetary-fiscal policy regime. Section 3 proposes a New Keynesian model with bond, equity, and different monetary-fiscal policy regimes, and discusses the asset pricing implications of the model. Section 4 discusses the calibration of the model, the quantitative results, and the determinants of the correlation of stock and bonds. Section 5 concludes.

2 Empirical Evidence

In this section, we discuss the regime switches of monetary-fiscal policies and the betas of the nominal and real Treasury bonds during the post-war period.

2.1 Policy regimes

It is the conventional to use Taylor rule representing central banks' monetary policy in the New Keynesian model:

$$i_t - i = \phi_i(i_{t-1} - i) + (1 - \phi_i)[\phi_\pi(\pi_t - \pi^*) + \phi_y(\Delta y_t - \mu_{z^+})] + \sigma_i e_{i,t}. \tag{2.1}$$

The policy rule has an interest-rate smoothing component captured by the sensitivity ϕ_i to the deviation of lagged interest rate, i_{t-1}, from the steady state value, i, and responds to the difference between inflation π_t and inflation target π^*, the deviation of output growth Δy_t from its steady state growth rate μ^{z^+}, and a policy shock $e_{i,t} \sim \text{IID}\mathcal{N}(0,1)$. The coefficients ϕ_π and ϕ_y capture the response of the monetary authority to the deviations of inflation and the output growth from their targets, respectively.

In standard New Keynesian models focusing on investigating monetary policy, tax policy is generally unspecified and the quantity of government bond is undetermined. When needed, tax policy is often specified as a response rule to economic fundamentals, similar to the form of the Taylor rule:

$$\tau_t - \tau = \varsigma_\tau(\tau_{t-1} - \tau) + \varsigma_b(b^\infty_{t-1} - b^\infty) + \varsigma_g(g_{yt} - g_y) + \varsigma_y(y_t - y), \tag{2.2}$$

where b^∞_{t-1} is the lagged government-debt-to-output ratio, g_t is the government-expenditure-to-output ratio, y_t is the detrended output, and y is the steady state of the detrended output. The coefficients ς_τ, ς_b, ς_g, and ς_y represent the persistency of the tax policy and the sensitivity of tax policy to government debt, government spending, and output, respectively. This specification of tax policy rule follows Davig and Leeper (2011) and Bianchi and Ilut (2016).

In most cases, we intend to investigate the impacts and effectiveness of monetary policy and fiscal policy separately, since they are implemented by separate and independent authorities, and monetary policy is more related to nominal variables, such as inflation and asset prices, while fiscal policy directly affects real variables, such as aggregate demand. When we mention policy cooperation, we usually only consider whether monetary and fiscal policies are expansionary or contractionary, but seldom discuss which policy is dominant and which policy accommodates the other one.

Leeper (1991) first categorized the policies as "active" or "passive" behavior, classified different combinations of active and passive monetary and fiscal policies, and analyzed their impacts on the economy. Generally speaking, two types of policy combinations or policy regimes yield determinacy and unique solution in models and are most relevant regimes in the reality: the active monetary policy and passive fiscal policy (AMPF) regime, and the passive monetary policy and active fiscal policy (PMAF) regime. The AMPF regime requires the Taylor principle is satisfied and the fiscal policy moves taxes to keep debt stable, that is, the nominal rate moves more than one-for-one with inflation with $\phi_\pi > 1$, and taxes responds strongly to the movements of government debt with $\varsigma_b > \beta^{-1} - 1$. The AMPF regime is the one we are familiar with. Under AMPF regime, monetary policy responds strongly to inflation to stabilize price level, while fiscal policy passively accommodates the monetary policy to stabilize debt. The PMAF regime is less familiar and requires opposite conditions: fiscal policy is no longer sensitive to debt level but take the role of stabilizing price, and monetary policy accommodates fiscal policy and allows inflation to move around to ensure debt stability. More specifically, in the PMAF regime, $\phi_\pi < 1$ and $\varsigma_b = 0$.[1]

Monetary and fiscal policy interactions are not only important for the existence and uniqueness of equilibria of models, but also have significant impacts on the economic dynamics. The rich implications on economic characteristics as well as policies of papers in this literature, such as Davig and Leeper (2011) and Bianchi and Ilut (2016) among many others, provide us a new angle to understand a number of puzzling time-varying economic dynamics in the data.

[1] See Leeper (1991) for more details.

It is easy to link the time-varying correlation between stock and bond returns to monetary policy, since government bond return is closely related to inflation, and inflation is directly impacted and targeted by monetary policy in conventional wisdom. However, the role played by fiscal policy is easily to be overlooked, since fiscal policy seems not directly related to inflation. This is true in the AMPF regime, which we are familiar with and is commonly considered as the sole regime in the real world. However, as we mentioned previously, in the PMAF regime, it is indeed fiscal policy that determines inflation. So given that both regimes exist, we have to take both monetary and fiscal policies into consideration to explain the correlation switch between stock and bond returns. Based on the estimation of Davig and Leeper (2011) among many others, both AMPF and PMAF regimes happened from time to time historically, for example, monetary policy switched from passive to active around 1980, and fiscal policy became active around 1956 and 2000. This estimated regime switches fit the sign switches of bond-stock return correlation almost perfectly. Based on coincidence of switches on policy regimes and bond-stock return correlation, it is natural to link the bond-stock return correlation literature and monetary-fiscal policy regime literature together, to provide one explanation for why bond-stock return correlation changed around 2000.

2.2 Risks of nominal Treasury bonds and TIPS

To explore the correlation between stock and bond returns, we follow Campbell et al. (2016) to estimate the realized betas of 5-year zero-coupon nominal U.S. Treasury bonds and Treasury inflation-indexed securities (TIPS) using rolling window regressions of daily data. We use the Capital Market Asset Pricing Model (CAPM) and the return on CRSP value-weighted stock index as the market return.

Yields of 5-year nominal Treasury bonds and TIPS in daily frequency are available from April 5th, 1962 to September 29th, 2017 and from January 2nd, 2003 to September 29th, 2017, respectively, both of which are obtained from the Wharton Research Data Services (WRDS). Yields of 5-year nominal Treasury bond yield between January, 1947 and April, 1962 are in monthly frequency and obtained from McCulloch and Kwon (1993). Daily return on the market portfolio is from Kenneth French's website.[2] The classification for policy regimes between the first quarter of 1949 to the third quarter of 2008 is based on the estimation in Davig and Leeper (2011).[3]

Figure A.2 plots the beta of 5-year Treasury bond for the period of 1947-2017, which is obtained by regressing daily return of 5-year Treasury bond on the return on market portfolio using 3-month rolling window. The same figure also shows that the two longest PMAF regimes during 1947 to 2017 are the periods of 1956-1965 and 2002-2017. Even though the estimation in Davig and Leeper (2011) ends in 2008, the PMAF regime is likely to go well beyond 2008. After the financial crisis, the U.S. government implemented the $787 billion American Recovery and Reinvestment Act, approved in February 2009, and provided large fiscal stimulus to the economy. Meanwhile, the interest rate has stayed at zero between 2008 to 2015. All of these facts are strong signals of the PMAF regime for the post-2008 period. For these aforementioned reasons, we will treat the period of 2002-2017 as a PMAF regime. During those two periods, 1956-1965 and 2002-2017, the beta of 5-year nominal Treasury bonds is largely negative. On the contrast, the bond beta is consistently positive during the AMPF regimes, i.e., the periods of 1984-1990 and 1995-2001.

Figure A.2 plots the beta of 5-year TIPS for the period of 2003-2017 while date is available. Since TIPS are not as liquid as nominal Treasury bonds, we regress weekly return of 5-year TIPS on the return on market portfolio using 6 month rolling window. Opposite of the beta of nominal bonds, the beta of TIPS is largely positive during this PMAF period.

In sum, the data seems to suggest that the correlation between stock and nominal bonds is negative in the PMAF regime while positive in the other regimes, especially in the AMPF regimes. On the contrary, the correlation between stock and real bond stays largely positive during the PMAF regime. In the rest of the paper, we explain these observed dynamics of the correlation between stock and nominal and real bonds in a DSGE model with different

[2]We thank Kenneth French for providing the data.
[3]We thank Eric Leeper for providing us the data.

policy regimes.

3 Model

In this section, we generate stock and bond returns in a DSGE model with microfoundations. The main structure of our model follows Christiano et al. (2014) in modeling the households, financial intermediaries, final good sector, and intermediate good sector, and the setup for monetary and fiscal policies is consistent with the convention of Leeper (1991) and Bianchi and Ilut (2016).

3.1 Household

Household maximizes life-time utility

$$V_t \equiv \max_{\{C_t, L_t, B_t/P_t, B_t^{\infty}/P_t, I_t\}} \quad (1-\beta_t)U(C_{h,t}, L_t) + \beta_t \mathbb{E}_t \left[V_{t+1}^{\frac{1-\gamma}{1-\psi}} \right]^{\frac{1-\psi}{1-\gamma}} \tag{3.1}$$

and

$$U_t \equiv U(C_{h,t}, L_t) = \frac{C_{h,t}^{1-\psi}}{1-\psi} - A_t^L \int_0^1 \frac{L_{j,t}^{1+\phi}}{1+\phi} dj \ ,$$

where $C_{h,t}$ is the habit adjusted consumption, defined as $C_{h,t} = C_t - b\bar{C}_t$ with $\bar{C}_t$ representing the aggregate consumption.[4] A_t^L is the disutility parameter of labor, growing at rate $(z_t^+)^{1-\psi}$, where (z_t^+) is the growth rate of the economy and is defined later in equation (3.17). $L_{j,t}$ is the number of household members with labor type j who are employed. The parameters are defined as follows: b is the habit parameter, ψ is the reciprocal of the degree of intertemporal elasticity of substitution, γ is the risk aversion parameter, and ϕ is the Frisch elasticity of labor supply parameter.

[4] In equilibrium, $C_t = \bar{C}_t$. However, when making decisions, households at time t take $\bar{C}_{t-1}$ as given.

Households' utility maximization is subject to the budget constraint

$$
\begin{aligned}
& P_t C_t + Q_t^\infty B_t^\infty + B_t + Q_t^k (1-\delta)\bar{K}_{t-1} + \frac{P_t}{\Psi_t} I_t \\
\leq \quad & B_{t-1}^\infty (Q_t^\infty \rho + 1) + R_{t-1} B_{t-1} + Q_t^k \bar{K}_t + P_t LI_t + P_t D_t - P_t T_t + P_t T_t^e \, ,
\end{aligned}
$$

and the law of capital accumulation

$$
\bar{K}_t = (1-\delta)\bar{K}_{t-1} + \left[1 - S\left(\frac{I_t}{\zeta_t^I I_{t-1}}\right)\right] I_t \, . \tag{3.2}
$$

where P_t is the price of consumption goods, Q_t^k is the price of raw capital at t, I_t is investment made at t, and Ψ_t is the relative price of consumption to investment goods defined later. $S(\cdot)$ is the investment adjustment cost, defined as

$$
S(x_t) = \frac{1}{2}\left\{\exp\left[\sigma_s \left(x_t - \exp(\mu_{z^+} + \mu_\psi)\right)\right] + \exp\left[-\sigma_s \left(x_t - \exp(\mu_{z^+} + \mu_\psi)\right)\right] - 2\right\} ,
$$

where $x_t = \frac{I_t}{\zeta_t^I I_{t-1}}$, and $\exp(\mu_{z^+} + \mu_\psi)$ is the steady state growth rate of investment. The parameter σ_s is chosen such that $S(\exp(\mu_{z^+} + \mu_\psi)) = 0$ and $S'(\exp(\mu_{z^+} + \mu_\psi)) = 0$. ζ_t^I measures the marginal efficiency of investment, and evolves as follows:

$$
\log\left(\frac{\zeta_t^I}{\zeta^I}\right) = \rho_{\zeta^I} \log\left(\frac{\zeta_{t-1}^I}{\zeta^I}\right) + \sigma_{\zeta^I} e_t^{\zeta^I} \, , \quad \text{and } e_t^{\zeta^I} \sim \text{IID}\mathcal{N}(0,1), \tag{3.3}
$$

where $e_t^{\zeta^I}$ denotes the MEI shock. Note that investment I_t is measured in terms of investment goods instead of consumption goods. LI_t is the real wage income defined as

$$
LI_t = \int \frac{W_{j,t}}{P_t} L_{j,t} \, dj \, ,
$$

D_t is the real dividend paid by firms, T_t is tax, T_t^e is the net transfer from entrepreneurs, and B_t is the face value of one-period debt lent to entrepreneurs at $t-1$ with gross nominal return R_t. To avoid numerical complication, we follow Woodford (2001) and define B_t^∞ as the amount of long-term government bonds issued at t, each of which has a stream of infinite coupon payments that starts in period $t+1$ with \$1 and decay every period at the rate of ρ. The price of one such long-term bond, Q_t^∞, is given by

$$Q_t^\infty = \mathbb{E}_t \left[\sum_{s=1}^{\infty} M_{t,t+s}^{\$} \rho^{s-1} \right] = \mathbb{E}_t \left[M_{t,t+1}^{\$} \left(1 + \rho Q_{t+1}^\infty \right) \right] ,$$

where $M_{t,t+s}^{\$}$ is the nominal stochastic discount rates (or pricing kernels) from period $t+s$ to t. The gross nominal return on long bond, R_t^B, is thus given by

$$R_t^B = \frac{1 + \rho Q_t^\infty}{Q_{t-1}^\infty} . \tag{3.4}$$

It can be easily shown that the yield y_d on this bond is given by $1/Q_t^\infty - (1-\rho)$ and the effective duration is $1/(1-(1+y_d)\rho)$.

3.2 Financial Intermediation

The entrepreneurs have the ability to turn raw capital into productive capital, which is used in production. How much productive capital can be produced by entrepreneur e depends on his net worth $N_{e,t}$, leverage ratio $\chi_{e,t}$, the optimal capital utilization rate $u_{e,t+1}$, and a random productive realized at the end of t after raw capital $\bar{K}_{e,t}$ is purchased:

$$K_{t+1} = \int_0^\infty dF(\omega) \int_0^1 de \ \left[u_{e,t+1} \omega_{e,t} \bar{K}_{e,t} \right] = \int_0^\infty f(\omega) d\omega \int_0^1 de \ \left[u_{e,t+1} \omega_{e,t} \frac{N_{e,t} \chi_{e,t}}{Q_t^k} \right] .$$

Entrepreneurs' productivity $\omega_{e,t}$ follows a lognormal distribution with time-varying standard deviation of $\sigma_{\omega,t}$, where

$$\log \left(\frac{\sigma_{\omega,t}}{\sigma_\omega} \right) = \rho_\omega \log \left(\frac{\sigma_{\omega,t-1}}{\sigma_\omega} \right) + \sigma_\omega e^{\omega,t}, \quad \text{and} \quad e_t^\omega \sim \text{IID}\mathcal{N}(0,1), \tag{3.5}$$

under the assumption that $\mathbb{E}_{t-1}[\omega_{e,t}] = 1$, $\log(\omega_{e,t}) \sim N(-\sigma^2_{\omega,t}/2, \sigma_{\omega,t})$.[5]

The leverage ratio that an entrepreneur can take is

$$\chi_{e,t} = \frac{N_{e,t} + B_{e,t}}{N_{e,t}}\,,$$

where $B_{e,t}$ is the one-period loan from the banking industry to entrepreneur e that matures at $t+1$. In aggregate, we have

$$N_t = \int_0^1 N_{e,t} d\, e \quad \text{and} \quad B_t = \int_0^1 B_{e,t} d\, e\,.$$

It can be shown that the leverage ratio χ is the same to all entrepreneurs

$$\chi_{e,t} = \chi_t = \frac{N_t + B_t}{N_t}, \tag{3.6}$$

$$Q^k_t \bar{K}_t = N_t + B_t. \tag{3.7}$$

Assume that the banking industry is competitive and banks earn risk-free interest rate on loans in every state of $t+1$, i.e.,

$$[1 - F(\bar{\omega}_{t+1})]\mathcal{Z}_{t+1} B_{e,t} + (1 - \mu_b) \int_0^{\bar{\omega}_{t+1}} \omega dF(\omega)\; R^k_{t+1} Q_{k,t} \bar{K}_{e,t} = R_t B_{e,t}\,,$$

where $\mathcal{Z}_{t+1}$ is the $t+1$ state-contingent nominal return on bank loan, and μ_b is the bankruptcy cost, $\bar{\omega}_{t+1}$ is the threshold above which entrepreneur is productive enough to pay back the loan, and R^k_t is the nominal return on raw capital at t from the perspective of entrepreneur. Entrepreneurs with the threshold productivity $\bar{\omega}_{t+1}$ can just pay back the interest and principal from what they produce:

$$R^k_{t+1} \bar{\omega}_{t+1} Q^k_t \bar{K}_{e,t} = B_{e,t} \mathcal{Z}_{t+1}, \Rightarrow$$

[5]If X follows a log-normal distribution with mean μ_X and standard deviation σ_X, that is, $\log(X) \sim \mathcal{N}(\mu_X, \sigma_X)$, then $\mathbb{E}[X] = \exp(\mu_X + \frac{1}{2}\sigma^2_X)$. According to the assumption, $\exp(\mu_X + \frac{1}{2}\sigma^2_X) = 1$, so $\mu_X = -\frac{1}{2}\sigma^2$.

$$\bar{\omega}_{t+1} = \frac{\mathcal{Z}_{t+1}(\chi_t - 1)}{\chi_t} \frac{1}{R^k_{t+1}}.$$

It can be shown that

$$R^k_{t+1}\left[\Gamma(\bar{\omega}_{t+1}) - \mu_b G(\bar{\omega}_{t+1})\right] = \frac{\chi_t - 1}{\chi_t} R_t \tag{3.8}$$

needs to hold in every state at period $t+1$. The definitions of $G(\bar{\omega}_{t+1})$ and $\Gamma(\bar{\omega}_{t+1})$ are given as follows:

$$G(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega_t dF(\omega_t), \tag{3.9}$$

$$\Gamma(\bar{\omega}_{t+1}) = [1 - F(\bar{\omega}_{t+1})]\bar{\omega}_{t+1} + G(\bar{\omega}_{t+1}). \tag{3.10}$$

The return on raw capital, R^k_t, is defined as

$$R^k_t = \frac{(1-\tau^k)\left[u_t r^k_t - a(u_t)/\Psi_t\right] P_t + (1-\delta)Q^k_t + \tau^k \delta Q^k_{t-1}}{Q^k_{t-1}}, \tag{3.11}$$

where r^k_t is the real rental rate of productive capital paid by producers, and τ^k is the tax rate on capital income. The nominal cost of utilization per unit of raw capital is $\frac{P_t}{\Psi_t} a(u_t)$, where

$$a(u) = r^k[\exp(\sigma_a(u_t - 1)) - 1]/\sigma_a\,,$$

with $\sigma_a > 0$. Note that the maintenance cost $a(u)$ is measured in terms of capital goods, whose relative price to consumption goods is $1/\Psi_t$. [6] The optimal value of u_t that maximizes the nominal return on raw capital, R^k_t, is

$$r^k_t = a'(u_t)/\Psi_t. \tag{3.12}$$

[6] Note that our definition of utilization cost is different from Christiano et al, which is $\frac{P_t}{\exp((t)\mu_\Psi)} a(u_t)\omega_t \bar{K}_t$.

For the entrepreneur, his total worth at the end of t is given by

$$\begin{aligned} N_t &= \gamma_e R_t^k Q_{k,t-1} \bar{K}_{t-1} \left[\int_{\bar{\omega}_t}^{\infty} (\omega_t - \bar{\omega}_t) dF(\omega_t) \right] + W_t^e \\ &= \gamma_e [1 - \Gamma(\bar{\omega}_t)] R_t^k Q_{k,t-1} \bar{K}_{t-1} + W_t^e \,, \end{aligned} \tag{3.13}$$

where $1 - \gamma_e$ is fraction of raw capital return transferred from entrepreneur to households, and W_t^e is the transfer from household to entrepreneur. The latter serves as an insurance to entrepreneurs so that they can consume even if they bankrupt. Therefore, the net transfer from entrepreneurs to household is

$$T_t^e = (1 - \gamma_e)[1 - \Gamma(\bar{\omega}_t)] R_t^k Q_{k,t-1} \bar{K}_{t-1} - W_t^e \,.$$

Entrepreneurs choose the default level $\bar{\omega}_{t+1}$ to maximize their profits, and the optimal default value satisfies

$$\mathbb{E}_t \left\{ [1 - \Gamma(\bar{\omega}_{t+1})] \frac{R_{t+1}^k}{R_t} + \frac{\Gamma'(\bar{\omega}_{t+1})}{\Gamma'(\bar{\omega}_{t+1}) - \mu G'(\bar{\omega}_{t+1})} \left[\frac{R_{t+1}^k}{R_t} \left(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1}) \right) - 1 \right] \right\} = 0. \tag{3.14}$$

Since all entrepreneurs choose the same utilization rate and leverage ratio, we have the following aggregation:

$$K_t = u_t \bar{K}_{t-1}. \tag{3.15}$$

3.3 Final-Good Production Sector

There are two industries in the production sector: the final goods industry and the intermediate goods industry. The production of the final consumption goods uses a continuum of intermediate goods, indexed by $i \in [0, 1]$, via the Dixit-Stiglitz aggregator:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{1}{\lambda_p}} \, di \right]^{\lambda_p} \,, \quad \lambda_p > 1 \,,$$

where Y_t is the output of the final goods, $Y_{i,t}$ is the amount of intermediate goods i used in the final goods production, which in equilibrium equals the output of intermediate goods i, and λ_p measures the substitutability among different intermediate goods. When λ_p is larger, the intermediate goods are more substitutable and the demand to intermediate goods is more price elastic. The final goods industry is perfectly competitive. Profit maximization of the final goods producers leads to the demand function for intermediate goods i:

$$Y_{i,t} = Y_t \left(\frac{P_{i,t}}{P_t} \right)^{-\frac{\lambda_p}{\lambda_p - 1}},$$

where P_t is the nominal price of the final consumption goods and $P_{i,t}$ is the nominal price of intermediate goods i. It can be shown that goods prices satisfy the following relation:

$$P_t = \left(\int_0^1 P_{i,t}^{-\frac{1}{\lambda_p - 1}} di \right)^{-(\lambda_p - 1)}.$$

3.4 Intermediate-Good Production Sector

The production of intermediate goods i uses both capital and labor via the following homogenous production technology:

$$Y_{i,t} = (z_t L_{i,t})^{1-\alpha} K_{i,t-1}^{\alpha} - z_t^{+} \varphi, \tag{3.16}$$

where z_t is the level of the neutral technology, $L_{i,t}$ and $K_{i,t}$ are the labor and capital services, respectively, employed by firm i. α is the capital share of the output, and φ is the fixed production cost. Finally, z_t^{+} is defined as:

$$z_t^{+} = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t, \tag{3.17}$$

where Ψ_t is the level of the investment-specific technology, measured as the relative price of consumption goods to investment goods. We assume that z_t and Ψ_t evolve as follows:

$$\mu_t^z = \mu_z(1-\rho_z) + \rho_z \, \mu_{t-1}^z + \sigma_z e_t^z \,, \quad \text{and } \; e_t^z \sim \text{IID}\mathcal{N}(0,1), \tag{3.18}$$

$$\mu_t^\psi = \mu_\psi(1-\rho_\Psi) + \rho_\psi \, \mu_{t-1}^\psi + \sigma_\psi e_t^\psi \,, \quad \text{and } \; e_t^\psi \sim \text{IID}\mathcal{N}(0,1), \tag{3.19}$$

where

$$\mu_t^z = \Delta \log z_t, \tag{3.20}$$

$$\mu_t^{z^+} = \Delta \log z_t^+, \tag{3.21}$$

$$\mu_t^\psi = \Delta \log \Psi_t. \tag{3.22}$$

e_t^z and e_t^ψ represent NT and investment price shocks, respectively. The intermediate goods industry is assumed to have no entry and exit, which is ensured by choosing a fixed cost ψ that brings zero profits to the intermediate goods producers in the steady state.

Cost minimization problem gives the relationship between capital rental rate and wage:

$$\frac{K_t}{L_t} = \frac{\alpha}{1-\alpha} \frac{W_t}{P_t r_t^k}. \tag{3.23}$$

Intermediate goods producer i rents capital service $K_{i,t}$ from entrepreneurs and its net profit at period t is given by $P_{i,t}Y_{i,t} - P_t r_t^k K_{i,t} - W_t L_{i,t}$, where $L_{i,t}$ is the labor service demanded by firms. $L_{i,t}$ is a combination of all labor types and will be defined later. The producer takes the nominal rent of capital service $P_t r_t^k$ and nominal wage rate W_t as given but has market power to set the price of its product in a Calvo (1983) staggered price setting to maximize profits. With probability ξ_p, producer i cannot reoptimize its price at period t, and has to set it according to the following rule,

$$P_{i,t} = \tilde{\pi}_{p,t} \, P_{i,t-1},$$

where

$$\tilde{\pi}_{p,t} = \left(\pi_t^*\right)^{\ell}\left(\pi_{t-1}\right)^{1-\ell} \tag{3.24}$$

is the inflation indexation, π_t^* is the target inflation rate or steady state inflation rate, and $\pi_t \equiv P_t/P_{t-1}$ is the inflation rate. Producer i sets price $P_{i,t}$ with probability $1-\xi_p$ to maximize its profits, i.e.,

$$\max_{\{P_{i,t}\}} \mathbb{E}_t \sum_{\tau=0}^{\infty} \xi_p^{\tau} M_{t,t+\tau}^{\$} \left[\tilde{\theta}_{p,t\oplus\tau} P_{i,t} Y_{i,t+\tau\,|\,t} - s_{t+\tau} P_{t+\tau} Y_{i,t+\tau\,|\,t}\right]$$

subject to the demand function

$$Y_{i,t+\tau} = Y_{t+\tau}\left(\frac{\tilde{\theta}_{p,t\oplus\tau} P_{i,t}}{P_{t+\tau}}\right)^{-\frac{\lambda_p}{\lambda_p-1}}$$

where $\tilde{\theta}_{p,t\oplus\tau} = \left(\prod_{s=1}^{\tau} \tilde{\pi}_{p,t+s}\right)$ for $\tau \geq 1$ and equals 1 for $\tau = 0$. Here, $Y_{i,t+\tau\,|\,t}$ is the output by producer i at time $t+\tau$ if the last time P_i is reoptimized is period t, and $s_{t+\tau}$ is the real marginal cost, given by

$$s_{t+\tau} \equiv MC_{t+\tau} = \frac{1}{z_{t+\tau}^{1-\alpha} P_{t+\tau}}\left(\frac{W_{t+\tau}}{1-\alpha}\right)^{1-\alpha}\left(\frac{r_{t+\tau}^{k}}{\alpha}\right)^{\alpha}. \tag{3.25}$$

The value of $s_{t+\tau}$ depends on the economic condition at $t+\tau$,and does not depend on firm i's actions.[7] The first order condition of this problem with respect to $P_{i,t}$ is

$$\sum_{\tau=0}^{\infty} \xi_p^{\tau} M_{t,t+\tau}^{\$}\left[\tilde{\theta}_{p,t\oplus\tau}^{1+\epsilon_p}(1+\epsilon_p) P_{i,t}^{\epsilon_p} P_{t+\tau}^{-\epsilon_p} Y_{t+\tau} - \epsilon_p s_{t+\tau} \tilde{\theta}_{p,t\oplus\tau}^{\epsilon_p} P_{i,t}^{\epsilon_p-1} P_{t+\tau}^{1-\epsilon_p} Y_{t+\tau}\right] = 0$$

[7]Equation (3.25) can be derived by minimizing input costs $W_{t+\tau} L_{j,t+\tau\,|\,t} + r_{t+\tau}^{k} K_{j,t+\tau\,|\,t}$ given $Y_{i,t+\tau\,|\,t}$.

where $\epsilon_p = \lambda_p/(1-\lambda_p)$. Define the following auxiliary variables

$$\begin{aligned} H_t &= \sum_{\tau=0}^{\infty} \xi_p^{\tau} M_{t,t+\tau}^{\$} \tilde{\theta}_{p,t\oplus\tau}^{1+\epsilon_p} \left(\frac{Y_{t+\tau}}{Y_t}\right) \left(\frac{P_{t+\tau}}{P_t}\right)^{-\epsilon_p}, \\ J_t &= \sum_{\tau=0}^{\infty} \xi_p^{\tau} M_{t,t+\tau}^{\$} \tilde{\theta}_{p,t\oplus\tau}^{\epsilon_p} \left(\frac{s_{t+\tau}}{s_t}\right) \left(\frac{Y_{t+\tau}}{Y_t}\right) \left(\frac{P_{t+\tau}}{P_t}\right)^{1-\epsilon_p}. \end{aligned}$$

Then the law of motion for inflation can be expressed as:

$$1 = (1-\xi_p) \left[\frac{\epsilon_p}{1+\epsilon_p} \frac{J_t}{H_t} s_t\right]^{\frac{1}{1-\lambda_p}} + \xi_p \left[\frac{\tilde{\pi}_{p,t}}{\pi_t}\right]^{\frac{1}{1-\lambda_p}}. \tag{3.26}$$

3.5 Labor Unions

There are labor contractors who hire workers of different labor types through labor unions and produce homogenous labor service L_t, according to the following production function:

$$L_t = \left[\int_0^1 L_{j,t}^{\frac{1}{\lambda_w}} \, dj\right]^{\lambda_w}, \quad \lambda_w > 1,$$

where λ_w measures the elasticity of substitution among different labor types. The intermediate goods producers employ the homogenous labor service for production. Labor contractors are perfectly competitive, and their profit maximization leads to the demand function for labor type j:

$$L_{j,t} = L_t \left(\frac{W_{j,t}}{W_t}\right)^{\frac{\lambda_w}{1-\lambda_w}}.$$

It is easy to show that wages satisfy the following relation:

$$W_t = \left(\int_0^1 W_{j,t}^{\frac{1}{1-\lambda_w}} \, dj\right)^{1-\lambda_w},$$

where $W_{j,t}$ is the wage of labor type j and W_t is the wage of the homogenous labor service.

Assume that labor unions face the same Calvo (1983) type of wage rigidities. In each period, with probability ξ_w, labor union j cannot reoptimize the wage rate of labor type j

and has to set the wage rate according to the following rule:

$$W_{j,t} = \tilde{\pi}_{w,t} e^{\tilde{\mu}_{w,t}} W_{jt-1} \,,$$

where

$$\tilde{\pi}_{w,t} = \left(\pi_t^*\right)^{\ell_w} \left(\pi_{t-1}\right)^{1-\ell_w} \tag{3.27}$$

is the inflation indexation and $\tilde{\mu}_{w,t} = \ell_\mu \mu_{z^+,t} + (1-\ell_\mu)\mu_{z^+}$ is the growth indexation. With probability $1-\xi_w$, labor union j chooses $W_{j,t}^*$ to maximize households' utility.

The optimal wage and the optimal wage markup $\mu_{w,t}$ are then given by

$$\left(W_t^*\right)^{1-\phi\epsilon_w} = \mu_w P_t C_{h,t}^{\varphi} A_t^L L_t^{\phi} W_t^{-\epsilon_w \phi} \left(\frac{J_{w,t}}{H_{w,t}}\right) \,, \quad \text{and} \quad \mu_w = \frac{\epsilon_w}{1+\epsilon_w} \,, \tag{3.28}$$

where

$$J_{w,t} = 1 + \xi_w \mathbb{E}_t \left[M_{t,t+1}^{\$} \frac{L_{t+1}}{L_t} \left(\frac{W_{t+1}}{W_t}\right)^{-\epsilon_w} \left(\tilde{\pi}_{w,t+1} e^{\tilde{\mu}_{w,t+1}}\right)^{\epsilon_w} J_{w,t+1} \right] , \tag{3.29}$$

$$H_{w,t} = 1 + \xi_w \mathbb{E}_t \Bigg[M_{t,t+1}^{\$} \frac{P_{t+1}}{P_t} \frac{A_{t+1}^L}{A_t^L} \left(\frac{C_{h,t+1}}{C_{h,t}}\right)^{\psi} \left(\frac{L_{t+1}}{L_t}\right)^{1+\phi} \left(\frac{W_{t+1}}{W_t}\right)^{-\epsilon_w(1+\phi)} \times \left(\tilde{\pi}_{w,t+i} e^{\tilde{\mu}_{w,t+i}}\right)^{\epsilon_w(1+\phi)} H_{w,t+1} \Bigg] . \tag{3.30}$$

The dynamics for aggregate wage level is:

$$W_t^{1/(1-\lambda_w)} = (1-\xi_w)(W_t^*)^{1/(1-\lambda_w)} + \xi_w \left(\tilde{\pi}_{w,t} \exp(\tilde{\mu}_t^w) W_{t-1}\right)^{1/(1-\lambda_w)} . \tag{3.31}$$

3.6 Policies

The central bank implements a Taylor (1993)-type monetary policy rule, specified as Equation 2.1, and the fiscal authority adjusts taxes according to the tax policy Equation 2.2.

Government's flow budget identity follows:

$$\frac{Q_t^\infty B_t^\infty}{P_t} = R_t^B \frac{Q_{t-1}^\infty B_{t-1}^\infty}{P_t} + G_t - T_t$$

holds at any time t. Equivalently, government budget constraint can be written in the following form:

$$b_t^\infty = \frac{R_t^B b_{t-1}^\infty Y_{t-1}}{\Pi_t Y_t} + g_{yt} - \tau_t \tag{3.32}$$

The government spending G_t is exogenously given to be a fixed proportion of output.

There are two relevant monetary/fiscal policy regimes that yield determinacy and unique solution of the model according to the policy regime literature, as discussed in Subsection 2.1: the AMPF regime and PMAF regime. The AMPF regime is the one we usually use in standard New Keynesian models: $\phi_\pi > 1$ and $\varsigma_b > \beta^{-1} - 1$, and the PMAF regime requires opposite conditions: $\phi_\pi < 1$ and $\varsigma_b = 0$.

3.7 Equilibrium

In equilibrium, all intermediate good producers take the same actions and all markets clear:

$$P_{i,t} = P_t, \quad Y_{i,t} = Y_t, \quad L_{i,t} = L_t.$$

— Resource constraint:

$$Y_t = C_t + I_t/\Psi_t + G_t + a(u_t)\bar{K}_{t-1} + \mathcal{D}_t, \tag{3.33}$$

where $\mathcal{D}_t$ is the bankruptcy cost, equal to $\mu G(\bar{\omega}_t) R_t^k \frac{Q_{k,t-1}\bar{K}_{t-1}}{P_t}$.

3.8 Asset Pricing Implications

In this section, we discuss the asset pricing implications of the model.

3.8.1 Returns on Stock

We define a stock in two ways. The first definition follows Abel (1999), in which a stock is the claim on consumption raised to a power of λ, C_t^λ, and $\lambda > 1$ reflects leverage. Since dividend in the data is four to five times more volatile than consumption, the leverage ratio λ is needed to create the wedge between dividend and consumption. The stock price and stock return are thus given by

$$\begin{aligned} S_t^c &= P_t C_t^\lambda + \mathbb{E}_t \left[M_{t,t+1}^{\$} S_{t+1}^c \right] && (3.34) \\ R_{s,t+1}^c &= \frac{S_{t+1}^c}{S_t^c - P_t C_t^\lambda} \, . && (3.35) \end{aligned}$$

The second definition follows Christiano et al. (2014), in which a stock is the claim on entrepreneur wealth. Assuming zero return at bankruptcy, the return on entrepreneur's wealth is given by

$$\begin{aligned} R_{s,t}^e(\omega) &= \max\{0, \omega R_t^k \chi_{t-1} - \mathcal{Z}_t(\chi_{t-1} - 1)\} \\ &= \max\{0, [\omega - \bar{\omega}_t]\} \times R_t^k \chi_{t-1} \, . \end{aligned}$$

It can be easily shown that the average return on entrepreneur's wealth is given by

$$R_{s,t}^e = [1 - G(\bar{\omega}_t) - \bar{\omega}_t] \, R_t^k \chi_{t-1} \, . \quad (3.36)$$

There are two main differences between the Christiano et al. (2014) and Abel (1999) definitions of a stock. First, the Christiano et al. (2014) definition allows for time-varying leverage ratio χ_t, financing cost $\mathcal{Z}_t$, and bankruptcy probability captured by $\bar{\omega}_t$, while the definition in Abel (1999) assumes a constant leverage ratio. Second, the value of a stock in the Christiano et al. (2014) definition crucially depends on the output of capital, captured by the rental rate of capital, and the resale value of capital Q_t^k, while the value of stock in the Abel (1999) definition only depends on consumption. Even though consumption is positively related to the output of capital, its correlation with the resale value of capital can be positive or negative, depending on both the shock and regime. As a result, the two stock returns $R^e_{s,t}$ and $R^c_{s,t}$ do not always react to shocks in the same manner.

3.8.2 Returns on Long-Term Real and Nominal Bonds

In our model, the long-term bond has a maturity of infinity and pays coupon every period. The duration of the bond is finite though because the coupon exponentially decays. To illustrate the intuition behind the return on the bond, we analyze the risk premium in a default-free zero-coupon bond with maturity of n periods. Real and nominal default-free zero-coupon bonds with maturity at $t+n$ pay a unit of real and nominal consumption, respectively, at maturity. Their prices are

$$B_t^{c,(n)} = \mathbb{E}_t[M_{t,t+n}], \quad \text{and} \quad B_t^{\$,(n)} = \mathbb{E}_t[M^{\$}_{t,t+n}], \tag{3.37}$$

for real and nominal bonds, respectively, where $M_{t,t+n}$ and $M^{\$}_{t,t+n}$ are the real and nominal discount factors for payoffs at $t+n$.[8] The associated real and nominal yields are defined, respectively, as

$$r_t^{(n)} = -\frac{1}{n}\log B_t^{c,(n)}, \quad \text{and} \quad i_t^{(n)} = -\frac{1}{n}\log B_t^{\$,(n)}.$$

The returns on real and nominal bonds are given by

$$R_{b,t+1}^{c,(n)} = \frac{B_{t+1}^{c,(n-1)}}{B_t^{c,(n)}}, \quad \text{and} \quad R_{b,t+1}^{\$,(n)} = \frac{B_{t+1}^{\$,(n-1)}}{B_t^{\$,(n)}}, \tag{3.38}$$

respectively.

It is useful to decompose expected excess returns on real and nominal bonds into real term and inflation risk premia, which are compensations for real and nominal risks, respectively.

[8]Notice that $B_t^{c,(n)}$ is the real price of the real bond, while $B_t^{\$,(n)}$ is the nominal price of the nominal bond.

The one-period real term premium of a bond with maturity n-period is defined as

$$rTP_t^{(n)} \equiv \log \mathbb{E}_t \left[R_{b,t+1}^{c,(n)} \right] - r_t \, , \tag{3.39}$$

and the one-period inflation risk premium $\pi TP_t^{(n)}$ is the log difference between the real returns for investing in an n-period nominal bond and an n-period real bond for one-period:

$$\pi TP_t^{(n)} \equiv \log \mathbb{E}_t \left[R_{b,t+1}^{\$,(n)} P_t / P_{t+1} \right] - \log \mathbb{E}_t \left[R_{b,t+1}^{c,(n)} \right] \, , \tag{3.40}$$

where r_t, $R_{b,t+1}^{\$,(n)}$, and $R_{b,t+1}^{c,(n)}$ are the net real interest rate, returns on nominal and real bonds, respectively.

Next, in order to illustrate the mechanism that drives the return on long-term bond, we derive the bond risk premium analytically under the simplifying assumption that all the variables follow log-normal distribution and are homoskedastic.

3.8.3 Real Term Premium and Inflation Risk Premium

The one-period real term premium of a bond with maturity n-period can be written as

$$rTP_t^{(n)} = -\text{cov}_t \left[m_{t,t+1}, \sum_{s=2}^{n} m_{t+s-1,t+s} \right] . \tag{3.41}$$

The above equation indicates that the real term premium of a long-term bond is positive if the stochastic discount factor (SDF) of the first period is negatively correlated with the SDFs of the future periods until maturity on average, and vice versa.

Inflation risk premium can be written as

$$\pi TP_t^{(n)} = \text{cov}_t \left(m_{t,t+1}, \sum_{s=1}^{n} \pi_{t+s} \right) . \tag{3.42}$$

Therefore, inflation risk premium of a bond with maturity n depends on the coveriance between the $t+1$-period pricing kernel and the inflation between $t+1$ to maturity.

The details of deriving the real term premium and inflation risk premium can be found in Section Appendix B.

4 Results and Analysis

4.1 Calibration

We calibrate the model to match key macro moments. The steady state growth rate of the technology μ_z is set to be 0.0041, and the steady state growth rate of the investment-specific technological change μ_ψ is set to 0.0042, implying the average annual growth rate of the economy is 2.4%. Steady state or targeted inflation rate, π^*, is 0.6075%, indicating the targeted annual inflation rate is 2.43%. Government spending is assumed to be 20% of total output. Following the convention of the macro literature, the power on capital in production function α is 0.33; deprecation rate on capital δ is 0.025; price and wage markup, λ_p and λ_w, are 1.2 and 1.05, respectively. The preference parameters are taken from the long-run risk literature: the reciprocal of elasticity of intertemporal substitution, ψ, is 1/1.5, and the risk aversion parameter γ is 60. The habit parameter, b, is 0.8, and the Frisch elasticity of labor supply ϕ is 1, both of which are taken from Christiano et al. (2014). The objective discount factor β is chosen to yield a 4.96% annual risk free rate. Price markup λ_p and wage markup λ_w are 1.2 and 1.05, respectively, from Christiano et al. (2014). The tax rate on capital income, τ^k, is set to 0 for simplicity. The persistence and volatility of shock processes are chosen to match the macroeconomic moments.

Policy parameters in different regimes are set according to the estimation in Bianchi and Ilut (2016). In the AMPF regime, monetary policy responds strongly to inflation to stabilize price and fiscal policy adjusts according to government debt position to satisfy government budget constraint: the sensitivity of interest rate to inflation ϕ_{pi} is 2.7372, the sensitivity of interest rate to output gap ϕ_y is 0.7037, the interest rate persistent parameter ϕ_r is 0.91, the sensitivity of tax to bond ς_b is 0.0609, the sensitivity of tax to output ς_y is 0.3504, the sensitivity of tax to government spending ς_g is 0.3677, and the tax persistent parameter ς_τ is 0.9844. In the PMAF regime, fiscal policy is active and responsible for stabilizing price. Tax-to-GDP ratio no longer adjusts according to debt position, meaning $\varsigma_b = 0$, and because less persistent, with $\varsigma_\tau = 0.8202$. At the same time, monetary policy is passive and has a lower sensitivity to macro fluctuation, $\phi_y = 0.1520$, and a lower persistency, where $\phi_r = 0.6565$. More important, the sensitivity of policy rate to inflation drops below one.

The magnitude of ϕ_π turns out to be critical to the dynamics of the economics in the PMAF, although not important to the stock-bond return correlation. When ϕ_π is too low, the model generates a counterfactual implication that consumption and output respond negatively to a positive NT shock. In fact, this is the most criticized feature of the new Keynesian model with the zero lower bound (ZLB), which is an extreme case of the PMAF regime. Wieland (2015) and Garín et al. (2017) demonstrate empirically that the sign of output response is the same as the sign of the shock both during normal time and at the ZLB, and Wu and Zhang (2017) proves that the economy has similar behavior when central banks implement unconventional monetary policy at the ZLB in a New Keynesian model. To avoid the aforementioned counterfactual implication, we choose $\phi_\pi = 0.5305$ according to Davig and Leeper (2011), instead of $\phi_\pi = 0.4991$ in Bianchi and Ilut (2016). Note that the negative correlation between stock and nominal bond is robust to the value of ϕ_π. In fact, the lower the value of ϕ_π is, the more negative the correlation is. We discuss the details in Subsection 4.6.

We compare the moments of key macro and finance variables in the data and generated in our model in Table A.3. Our model performs very well in matching the data.

4.2 Variance Decomposition

Table A.4 reports the variance decomposition for the AMPF and PMAF regimes, respectively. Results in both regimes show that the MEI shock contributes most to the dynamics of macroeconomic variables, followed by the NT shock. For example, the MEI shock contributes

69.03% and 97.48% of variations in consumption and investment growth rates, respectively, in the AMPF regime, and 84.28% and 98.10% in the PMAF regime. This is consistent with the finding in Justiniano et al. (2011) that the MEI shock contributes most to business cycle fluctuations and on the contrary, the investment price shock is of little importance. The risk shock does not play as important a role as it does in the model of Christiano et al. (2014) for the following reasons. First, our parameter values are calibrated mainly based on the estimation of a regime-switching model in Bianchi and Ilut (2016), while the parameter values in Christiano et al. (2014) are estimates from the model with a single AMPF regime. Second and quantitatively most important reason is that Christiano et al. (2014) includes both anticipated and unanticipated risk shocks. These two types of shocks are correlated and reinforce the effects of each other. The variance decomposition in Christiano et al. (2014) shows that anticipated risk shocks are much more important for business cycle fluctuations than the unanticipated shock. However, in order to keep our model simple and straightforward, we do not include the anticipated shocks.[9]

For a shock to have substantial effect on the correlation between stock and bond returns, it has to contribute significant amount of variations to both stock and bond returns. Table A.4 shows that in the AMPF regime, the correlation between the return on consumption claim and return on nominal bonds mainly depends on the NT shock, which contributes 61.39% and 25.84% of the variations in these two returns, while the correlation between the return on consumption claim and return on real bond returns mainly depends on the MP shock, which contributes 29.65% and 52.82% of the variations in these two returns. In contrast, the correlation between the return on entrepreneur wealth and returns on nominal and real bond returns mainly depends on the MEI shock, which contributes 25.74%, 49.81%, and 42.59% of the variations in these three returns, respectively.

In the PMAF regime, the effect of the NT shock on the return on consumption claim

[9]Another difference between our model and Christiano et al. (2014) is the preference function. We use recursive preference while they use log utility. However, our experiment shows that the preference function makes little difference in the contribution of risk shock to macroeconomic dynamics.

become significantly weaker while the effect of the MEI shock becomes the strongest. As a result, the correlation between stock and bond returns largely depends on the MEI shock, regardless of the definition of the stock and the nominal or real feature of the bonds. Specifically, the MEI shock contributes 78.08%, 27.07%, 40.96%, and 43.47% of the variations in returns on consumption claim, entrepreneur wealth, and nominal and real bonds, respectively.

Note that even though the risk shock contributes over 30% of the variations in the return on entrepreneur wealth R_s^e in both the AMPF and PMAF regimes, it never contributes more than 1% of the variations in the return on real and nominal bonds. Therefore, the risk shock is not crucial for understanding the correlation between stock and bond returns.

Given that the NT, MEI, and MP shocks are the most important drivers of the correlation of stock and bond returns, they are the focus of our analysis. The case of the MP shock is quite straight forward. A positive MP shock leads to higher nominal and real interest rates, and thus contracts the economy. Consequently, the values of stocks and bonds go down, resulting in a positive correlation between stock and bond returns, regardless of the definition of stock, the nominal and real features of the bonds, and the regimes. However, the effects of the NT and MEI shocks on the correlation between stock and bond returns are much more complex and we provide detailed analysis below.

4.3 The NT Shock

The AMPF regime — Under the AMPF regime, the values of the consumption claim and capital, the long-term nominal bond, and the long-term real bond all go up after a positive NT shock, resulting in a positive correlation between stock and bond returns, regardless of the definition of stock and the nominal or real feature of the bond.

Higher productivity leads to increase in consumption and investment, but decrease in inflation. Inflation goes down because nominal rigidity prevents real wage from rising as quickly as the productivity, resulting in a reduction in real marginal cost.In the AMPF regime, the nominal interest rate reacts strongly and positively to inflation and positively to output growth. Even though real interest rate rises at the beginning due to the reaction of nominal interest rate to output, it quickly drops due to the strong reaction of nominal interest rate to inflation. The lower real interest rate boosts the economy further. Taken all together, a positive NT shock under the AMPF regime leads to a strong and long-lasting boom in the economy. Consequently, the claim on consumption goes up and stock return gets higher.

The price of the real long-term bond crucially depends on the changes in average real interest rates from the current period till maturity. As argued before, real interest rate goes up at the beginning but goes down in the long-run. And the reduction in interest rate in the long-run dominates its increase in the short-run and the price of the real long-term bond goes up. Thus, return on the real bond goes up.

The price of the nominal long-term bond depends on both the inflation and the real interest rate. Since inflation drops, and the real interest rate from the current period till maturity also drops on average, the price of the nominal bond goes up and the return on this bond also goes up.

In sum, the NT shock leads to a positive correlation between stock and (real and nominal) bond returns under the AMPF regime. Moreover, the covariance of inflation and stochastic pricing kernel is positive.

The PMAF regime — After a positive NT shock, inflation again goes down. Under the Taylor rule in the PMAF regime, nominal interest rate weakly reacts to inflation. Consequently, nominal interest rate does not drop enough and the real interest rate rises consistently in the short and long run, leading to a lower price of the real long-term bond. The price of nominal bond still rises because of lower inflation, however at a much smaller magnitude.

More importantly, the contractionary effect of a higher real interest rate largely cancels out the stimulus effect of higher NT on the the economy. The demand for labor and capital decreases substantially so that the price of capital drops and the rise of consumption is significantly weakened after a positive NT shock. Consequently, th return on entrepreneur wealth goes down. The return on consumption claim still rises, however, the magnitude is less than 4% of that in the AMPF regime.

In sum, under the PMAF regime, the return on entrepreneur wealth is negatively correlated with the return on nominal bond, but positively correlated with the return on real bond. On the opposite, the return on consumption claim positively (negatively) correlates with return on nominal (real) bond. However, the effect of the NT shock on return on consumption claim in the PMAF regime is significantly weaker than that in the AMPF regime. Note that the covariance of inflation and stochastic pricing kernel stays positive.

4.4 The MEI Shock

The AMPF regime — After a positive MEI shock, the transformation of investment goods into raw capital becomes more efficient, leading to higher investments, lower price of capital, and larger amount of end-of-period capital. The substitution effect of a positive MEI shock leads to higher investment and lower consumption, while the wealth effect leads to higher consumption because the households anticipate a higher level of capital and consumption in the future. The substitution effect dominates the wealth effect and consumption drops initially, but quickly goes up afterwards. As a result, the return on consumption claims go up although the magnitude of the rise is small. The price of capital drops due to the lower cost of capital production and higher supply of capital, leading to a lower return on entrepreneur wealth. The negative effect of the MEI shock on the value of existing capital is discussed in Greenwood and Jovanovic (1999), which documents a deep drop of S&P 500 stocks in the 1990s when a new technology came out.

The wealth effect of a positive MEI shock leads to a higher capital utilization rate and labor supply, resulting in higher output. Consequently, the rental rate of capital and the marginal cost of output go up, resulting in higher inflation. As the MEI shock reverts back to its mean and capital level gets higher, the rental rate of capital goes down eventually and so does inflation. Under the Taylor rule, the nominal interest rate goes up first and then goes down in reaction to inflation. In the AMPF regime, the real interest rate also goes up, dampening the expansionary effect of the positive MEI shock.

The price of real long-term bond goes down due to the higher real interest rate.[10] Even though the nominal interest rate eventually goes up, the effect of the higher rate at the short run dominates and the price of the nominal long-term bond goes down. Therefore, the return on entrepreneur wealth positively correlates with, while the return on consumption claim negatively correlates with, the returns on nominal and real long-term bonds in the AMPF regime.

The PMAF regime — In this regime, the tax policy is active and responsible for price adjustments. After a positive MEI shock, output and inflation go up initially for the same reason as in the AMPF regime. However, the government increases taxes in reaction to higher output. Consequently, the price goes down to satisfy the government budget constraint.[11] Therefore, inflation goes up first after a positive MEI shock but quick goes down due to the active tax policy. The nominal interest rate moves with inflation according to the monetary policy rule. Since the policy rate reacts weakly to inflation, the real interest rate goes down first but goes up quickly. Due to the contractionary effect of higher taxes, consumption goes down sharply and consequently, the return on consumption claim goes down. The return on entrepreneur wealth goes down due to the drop in the price of capital.

Note that the stimulus effect of a positive MEI shock is also somewhat muted in the PMAF regime, similar to the case in the AMPF regime. The difference is that the strong reaction of the policy rate to inflation is the driving force in the AMPF regime, while the increase in taxes plays the role in the PMAF regime.

Since nominal interest rate quickly goes down after the initial rise, the price of nominal long-term bond goes up and so does the return on this bond. The real interest rate, on the

[10] The real interest rate goes down at the very beginning but stay higher afterwards, because the policy rate is persistent and only adjusts gradually.

[11] Another way to interpret the reduction in price is that after an increase in tax, government lowers money supply given that it has more fiscal income and needs not to rely on inflation to balance the budget.

opposite, goes down for the first couple of periods then goes up, resulting in a fall in the price of the real long-term bond and the return on this bond.

In sum, under the PMAF regime, the stock return negatively correlates with the return on nominal long-term bonds, but positively correlates with the return on real long-term bonds.

4.5 Correlation of Stock and Bond Returns

The above variance decomposition and impulse response analyses lead to the following proposition.

Hypothesis *The correlation of return on stock, in either definition, and return on nominal bond is positive in the PMAF regime, and negative in the AMPF regime. The correlation of return on stock and return on real bond is always positive.*

Table A.5 reports the correlation matrix of return on consumption claim(R_s^c), return on entrepreneur wealth(R_s^e), return on nominal bonds($R_b^\$$), and return on real bonds(R_b^c), in both the AMPF and PMAF regimes. Specifically, the correlation of the return on consumption claim and return on nominal bonds are positive in the AMPF regime, mainly due to the effect of the NT and MP shocks, while the correlation is negative in the PMAF regime, due to the effect of the MEI shock. The correlation of the return on entrepreneur wealth and return on nominal bonds are positive in the AMPF regime, and negative in the PMAF regime, both due to the effect of the MEI shock. The positive correlation between return on stock on consumption claim and return on real bond is due to the effect of the MP shock in the AMPF regime, and is due to the effect of the MEI shock in the PMAF regime. The positive correlation between return on consumption claim and entrepreneur wealth and return on real bond is due to the effect of the MP shock in both regimes.

4.6 The PMAF Regime at the Effective Lower Bound (ELB)

The zero lower bound is an extreme case of the PMAF regime where policy rate does not react to economic fluctuations at all, i.e., ϕ_π and ϕ_y are equal to zero. To keep the model simple and avoid the computational difficulty, we neither include additional preference or inflation shocks to create the ZLB environment nor discuss the case with completely inactive monetary policy, in which $\phi_\pi = 0$ and $\phi_y = 0$. Instead, we assume an ELB scenario, in which the policy rate is almost constant at its steady state level, and discuss the case with ϕ_π and ϕ_y close to zero. When ϕ_π is lower than certain threshold, the model implies that consumption and output respond negatively to a positive NT shock. Because the nominal interest rate is kept constant, the lower inflation caused by a positive NT shock induces higher real interest rate, which has a significant contractionary impact on the economy. In that case, stock prices decreases due to the pessimistic future economic outlook, hence bond and stock returns move in the opposite directions in response to NT shocks, which reinforces our result that in the PMAF regime — the bond-stock return correlation is even more negative since both NT shocks and MEI shocks generate negatively correlated bond and stock returns.

Therefore, our result does not rely on the extreme inactiveness of the monetary policy, as in Gourio and Ngo (2016). The negative correlation between stock and nominal Treasury bond holds as long as the monetary policy is passive (and fiscal policy is active to ensure tractability).

4.7 Inflation Risk Premium

Inflation risk premium (IRP) is positive in both the AMPF and PMAF regimes. The fact that IRP is not negative in the PMAF regime shows that the CAPM does not hold in our model. The reason is straightforward. The stochastic discount factor in our model depends on both the current period consumption growth and the return on wealth portfolio. That is, a one-factor asset pricing model such as the CAPM does not hold. As a result, the sign of the differences in the CAPM betas does not necessarily equal the sign of the differences in excess returns.

Our model also implies that the correlation between inflation and return on consumption claims is always positive, although the magnitude is smaller in the PMAF regime than it is in the AMPF regime. We plan to test this prediction in our future research.

4.8 Alternative Preference

In our benchmark model, we use recursive preference in order to generate a risk premium with reasonable magnitude. However, the relation between returns on stock and nominal bond is robust to the choice of preference. In this section, we change the preference to constant relative risk aversion (CRRA) preference. Figure A.7 to Figure A.10 plot the impulse responses to NT and MEI shocks in both regimes, respectively. These impulse responses are qualitatively similar to their counterparts under the recursive preference in the benchmark model. Specifically, a positive NT shock again leads to increases in the returns on consumption claims and long-term nominal bonds in both regimes, while a positive MEI shock leads to opposite movements in these two returns.

What does change under the CRRA preference is the sign of the IRP. The CAPM holds under the CRRA preference. If return on nominal (real) bond is negatively (positively) correlated with the return on consumption claim, the difference between nominal returns on nominal and real bonds, i.e., the IRP, is negative, and vice versa. Consequently, in the AMPF regime, the IRP is positive while it is negative in the PMAF regime. Table A.6 presents the correlation matrix under the CRRA preference, which shows that the bond-stock return correlation changes from positive to negative when the policy regime shifts from the AMPF to the PMAF.

5 Conclusion

We have built a New Keynesian model with the recursive preference, financial intermediaries, and monetary-fiscal policy interaction, which coherently explains the positive bond-stock return correlation during 1980-2000 when the monetary policy is active, and the negative correlation during 1950s and 2000s when the fiscal policy is active. When the monetary policy is active, the NT and MEI shocks together drive the economy, and both shocks induce a positive correlation between bond and stock return. Interestingly, when the fiscal policy is active, the MEI shock dominates in driving the economic dynamics, and induces a negative bond-stock return correlation. The negative correlation in the PMAF regime comes from the movement of bond return to the opposite direction compared to that in the AMPF regime, and negative inflation risk premium due to the strong reaction of fiscal policy to output fluctuations is the main driver of the opposite response of bond return.

In the next step, we plan to estimate our model using data from 1960's to the present with a deterministic switch in monetary and fiscal policy parameters in 2000. Through estimation, we can filter out the historical time series of the five structural shocks, calculate the historical decomposition of key macro and finance variables to support our simulation results that the MEI shocks indeed dominate in determining the correlation of bond and stock returns.

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Appendix A Tables and Figures

Table A.1: **Calibrated parameters in the quantitative model**

Parameter	Description	Value
Preference		
β	discount factor	0.9989
ψ	reciprocal of elasticity of intertemporal substitution	1/1.5
γ	risk aversion	60
ϕ	labor supply aversion	1
b	habit parameter	0.8
Production		
α	capital share	0.33
δ	capital depreciation rate	0.025
σ_s	investment adjustment cost parameter	10.78
σ_a	utilization rate cost parameter	2.6263
ξ_p	probability that cannot re-optimize price	0.74
ℓ	price indexation parameter	0.91
λ_p	degree of elasticity of substitution for goods aggregation	1.2
ξ_w	probatility that cannot re-optimize wage	0.81
ℓ_w	wage indexation parameter	0.94
λ_w	degree of elasticity of substitution for labor aggregation	1.05
μ_{z^+}	growth rate of permanent TFP	0.0041
μ_ψ	growth rate of investment specific technology	0.0042
π^*	target inflation rate	1.006
Policies		
ϕ_π^1	sensitivity of interest rate to inflation (AMPF)	2.7372
ϕ_π^2	sensitivity of interest rate to inflation (PMAF)	0.5305
ϕ_y^1	sensitivity of interest rate to output (AMPF)	0.7037
ϕ_y^2	sensitivity of interest rate to output (PMAF)	0.1520
ϕ_i^1	interest rate persistence (AMPF)	0.91
ϕ_i^2	interest rate persistence (PMAF)	0.6565
ς_b^1	sensitivity of tax to bond (AMPF)	0.0609
ς_b^2	sensitivity of tax to bond (PMAF)	0
ς_y^1	sensitivity of tax to output (AMPF)	0.3504
ς_y^2	sensitivity of tax to output (PMAF)	0.3504
ς_g^1	sensitivity of tax to government spending (AMPF)	0.3677
ς_g^2	sensitivity of tax to government spending (PMAF)	0.3677
ς_τ^1	tax persistence (AMPF)	0.9844
ς_τ^2	tax persistence (PMAF)	0.8202
g_y	steady-state government-spending-to-output ratio	0.20
τ^k	tax on capital income	0

Notes: The superscripts "1" and "2" for the policy coefficients represent the AMPF and PMAF regimes, respectively.

Table A.2: **Parameters for Shock Processes**

Parameter	Description	Value
Shocks		
$\rho_{\mu^{z+}}$	persistence of the NT shock	0.15
$\rho_{\mu^{\psi}}$	persistence of the investment price shock	0.16
$\rho_{\zeta^{I}}$	persistence of the MEI shock	0.77
ρ_{ω}	persistence of the risk shock	0.97
$\sigma_{\mu^{z+}}$	std of persistence of the NT shock	0.355
$\sigma_{\mu^{\psi}}$	std of persistence of the investment price shock	0.4
$\sigma_{\zeta^{I}}$	std of persistence of the MEI shock	2.75
σ_{ω}	std of persistence of the risk shock	7
σ_{i}	std of persistence of the MP shock	0.06

Table A.3: **Simulated Moments**

Column 1 are variable names. Column 2 and 3 give the mean and standard deviation in the data. Column 4 and 5 give the simulated mean and standard deviation from the model.

Variables	Data		Model	
	Mean	Std.Dev	Mean	Std.Dev.
consumption growth (ΔC)	1.88	2.72	2.48	2.8
investment growth (ΔI)	3.04	13.68	4.36	13.2
inflation (π)	1.40	1.00	2.12	0.92
stock(claim on consumption) return (R_s^c)	5.95	17.85	2.04	9.72
long-term nominal bond return ($R_b^{\$}$)	0.31	2.63	0.20	2.12

Table A.4: **Variance Decomposition (on impact, in %)**

The first column of this table lists key variables in the model. The second to sixth columns are contributions of the NT shocks, MEI shocks, risk shocks, shocks to investment price, and MP shocks, respectively. The numbers before and after the slash(/) represent the contribution of the shocks to fluctuations in variables in the AMPF regime and PMAF regime, respectively.

Variables	Shocks				
	e_z	e_{ζ^I}	e_{σ_ω}	e_ψ	e_r
stock(claim on consumption) return (R_s^c)	61.39/0.13	0.44/78.12	2.55/2.27	5.97/1.84	29.65/17.64
average return on entrepreneur wealth (R_s^e)	0.20/0.80	25.74/27.08	72.07/70.76	0.56/0.99	1.44/0.38
long-term nominal bond return ($R_b^\$$)	21.03/39.23	58.26/39.36	3.29/3.67	1.05/2.93	16.37/14.80
long-term real bond return (R_b^c)	0.90/36.53	67.27/47.32	0.63/3.13	2.35/3.95	28.85/9.06
inflation (π)	26.78/30.59	64.23/63.47	5.09/4.53	0.37/1.34	3.53/0.07
policy rate (i)	2.35/15.48	92.25/68.15	1.31/1.59	0.58/0.76	3.52/14.02
real interest rate (r)	3.08/22.26	88.62/60.70	1.51/3.82	0.57/0.96	6.22/12.26
nominal pricing kernel ($M^\$$)	78.55/76.68	0.04/1.47	0.09/0.08	21.32/21.76	0.00/0.01
consumption growth (ΔC)	21.09/9.24	69.03/84.29	2.92/1.37	1.43/0.97	5.52/4.13
investment growth (ΔI)	0.31/0.06	97.48/98.10	1.90/1.68	0.26/0.16	0.04/0.00
capital price (Q^k)	1.02/0.19	50.78/58.60	47.14/40.92	0.24/0.14	0.83/0.15
tax-to-GDP ratio (τ)	4.89/1.64	90.89/93.15	1.96/3.11	1.62/2.06	0.65/0.04

Table A.5: **Bond-Stock Return Correlation**

This table reports the correlation matrix of stock and bond returns. The numbers before and after the slash(/) represent the correlations in the AMPF regime and PMAF regime, respectively.

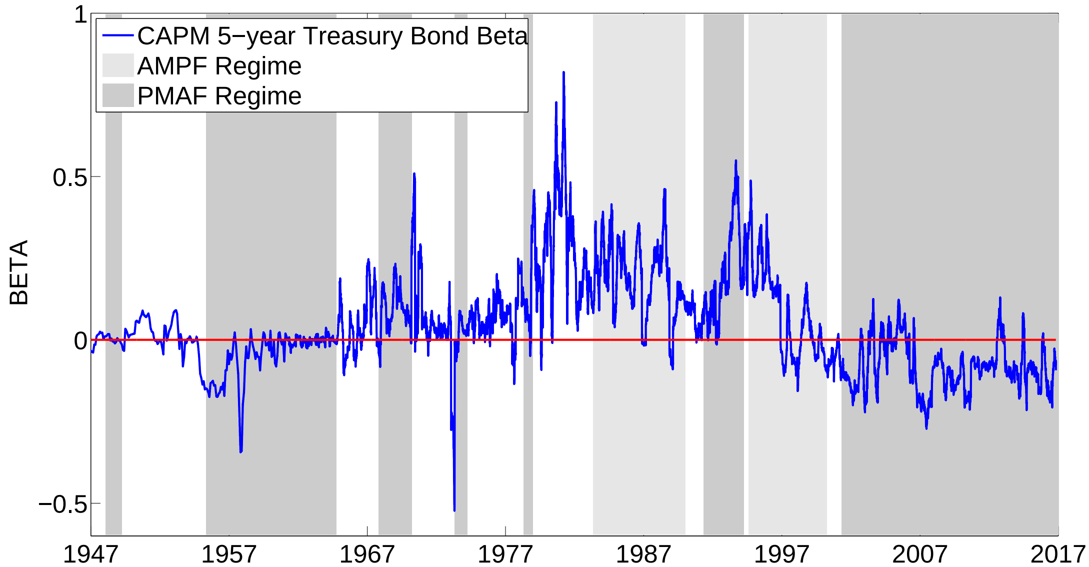
Variables	R_s^c	R_s^e	$R_b^\$$	R_b^c	π	ΔC	IRP
stock(claim on consumption) return (R_s^c)	1.00	0.18/0.34	0.53/-0.38	0.44/0.65	-0.22/-0.10	0.32/0.44	0.46/-0.60
average return on entrepreneur wealth (R_s^e)	–	1.00	0.61/-0.21	0.41/0.28	-0.28/-0.14	0.10/0.20	0.21/-0.27
long-term nominal bond return ($R_b^\$$)	–	–	1.00	0.66/-0.73	-0.41/-0.13	0.20/-0.21	0.28/0.92
long-term real bond return (R_b^c)	–	–	–	1.00	-0.17/0.35	0.44/0.09	-0.22/-0.91
inflation (π)	–	–	–	–	1.00	-0.61/-0.75	-0.37/-0.14
consumption growth (ΔC)	–	–	–	–	–	1.00	0.14/-0.31
inflation risk premium (IRP)	–	–	–	–	–	–	1.00

Table A.6: **Bond-Stock Return Correlation — CRRA Preference**

This table reports the correlation matrix of stock and bond returns in the model with a CRRA preference. The numbers before and after the slash(/) represent the correlations in the AMPF regime and PMAF regime, respectively.

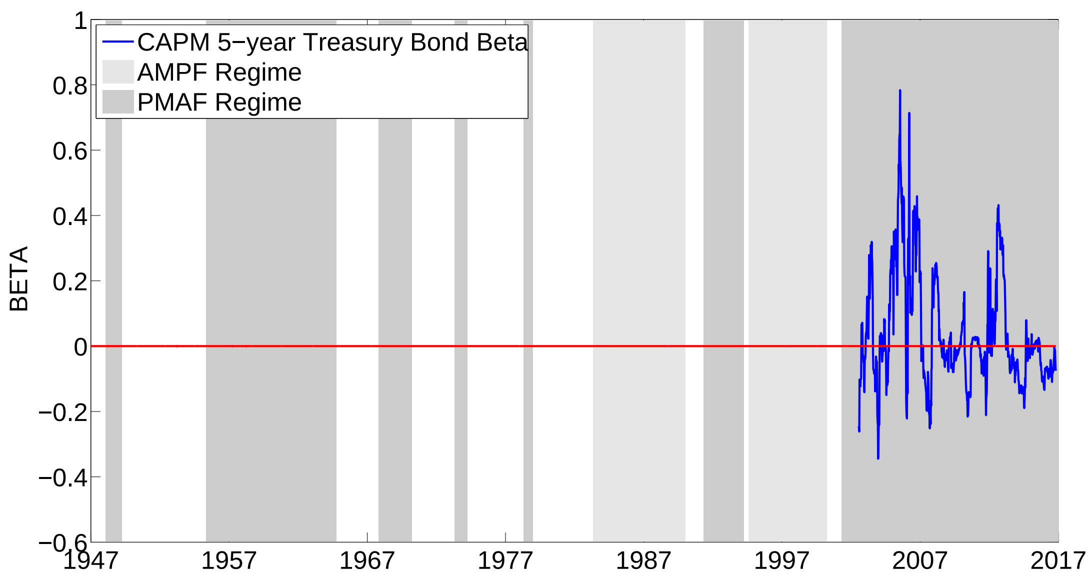
Variables	R_s^c	R_s^e	$R_b^\$$	R_b^c	π	ΔC	IRP
stock(claim on consumption) return (R_s^c)	1.00	0.25/0.36	0.31/-0.40	0.16/0.67	-0.15/-0.05	0.76/0.87	0.27/-0.62
average return on entrepreneur wealth (R_s^e)	–	1.00	0.58/-0.22	0.407/0.28	-0.27/-0.12	0.30/0.43	0.29/-0.28
long-term nominal bond return ($R_b^\$$)	–	–	1.00	0.73/-0.63	-0.35/-0.18	0.35/-0.27	0.29/0.85
long-term real bond return (R_b^c)	–	–	–	1.00	-0.17/0.37	0.57/0.63	-0.11/-0.92
inflation (π)	–	–	–	–	1.00	-0.36/-0.40	-0.43/-0.17
consumption growth (ΔC)	–	–	–	–	–	1.00	0.21/-0.62
inflation risk premium (IRP)	–	–	–	–	–	–	1.00

Figure A.1: **CAPM Beta of the 5-year Treasury Bond**



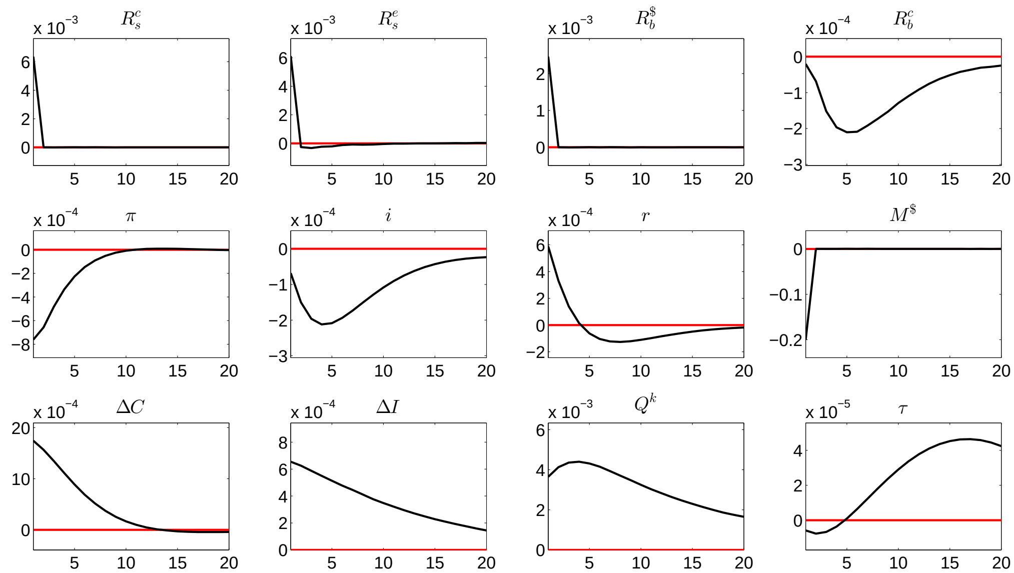
Notes: The beta in blue is estimated from a rolling window of 3 months of daily return from 1947 to 2017. The shaded areas with light and dark grey represent the AMPF and PMAF regimes, respectively. The x-axis is time, and y-axis is the size of beta.

Figure A.2: **CAPM Beta of the 5-year TIPS**



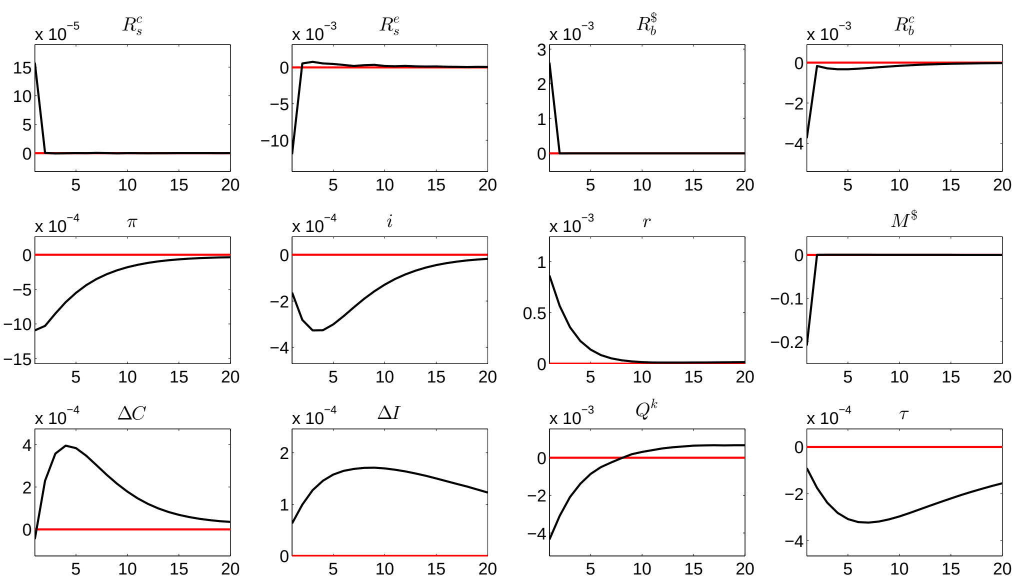
Notes: The beta in blue is estimated from a rolling window of 6 months of weekly return from 2003 to 2017. The shaded areas with light and dark grey represent the AMPF and PMAF regimes, respectively. The x-axis is time, and y-axis is the size of beta.

Figure A.3: **A Neutral Technology Shock in the AMPF Regime**



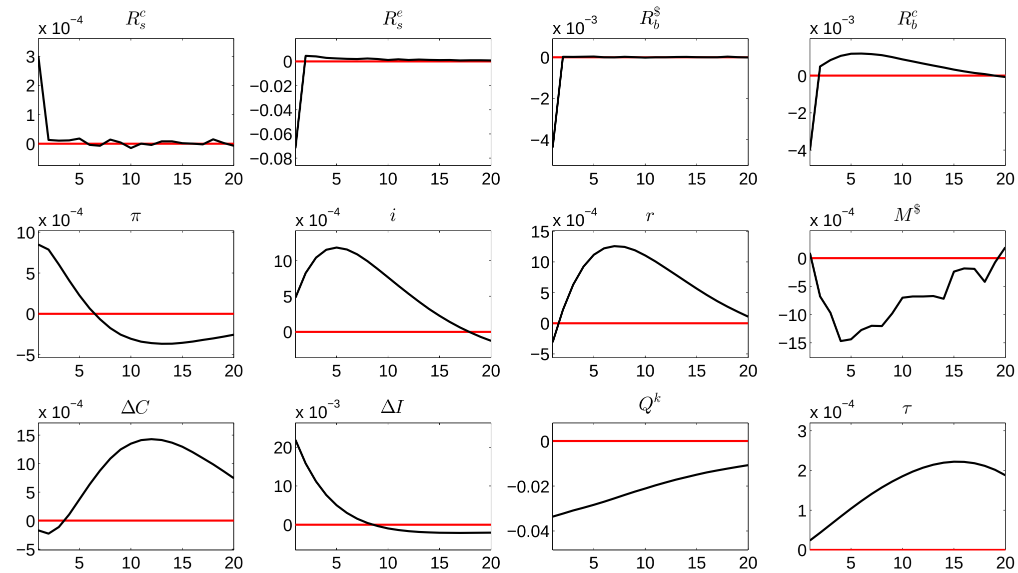
Notes: This figure plots the impulse responses of key macro and finance variables in the model under a positive NT shock when the monetary policy is active and fiscal policy is passive. The x-axis is the time in quarters, and y-axis represents percentage change from the steady state.

Figure A.4: **A Neutral Technology Shock in the PMAF Regime**



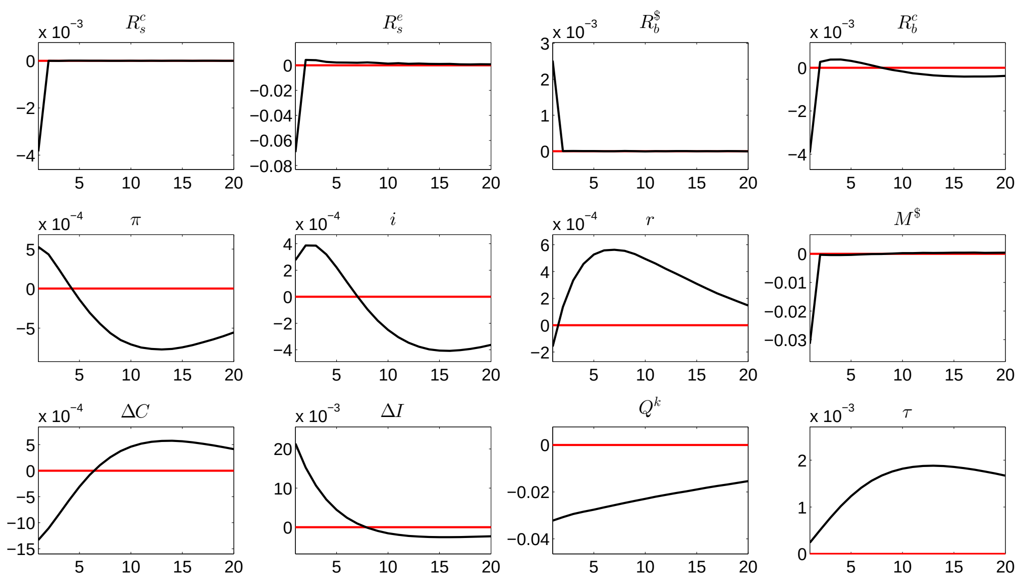
Notes: This figure plots the impulse responses of key macro and finance variables in the model under a positive NT shock when the monetary policy is passive and fiscal policy is active. The x-axis is the time in quarters, and y-axis represents percentage change from the steady state.

Figure A.5: **A Marginal Efficiency of Investment Shock in the AMPF Regime**



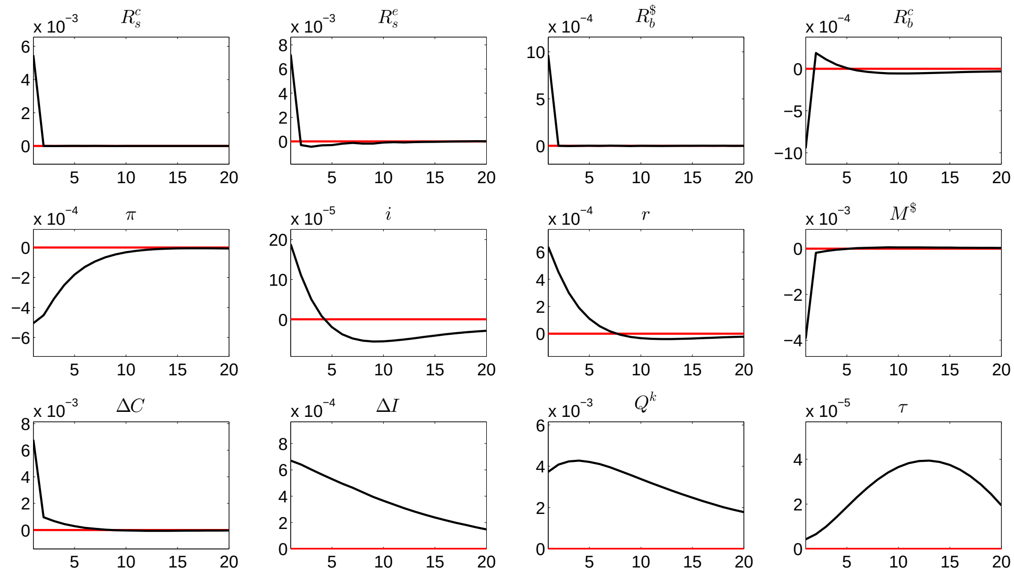
Notes: This figure plots the impulse responses of key macro and finance variables in the model under a positive MEI shock when the monetary policy is active and fiscal policy is passive. The x-axis is the time in quarters, and y-axis represents percentage change from the steady state.

Figure A.6: **A Marginal Efficiency of Investment Shock in the PMAF Regime**



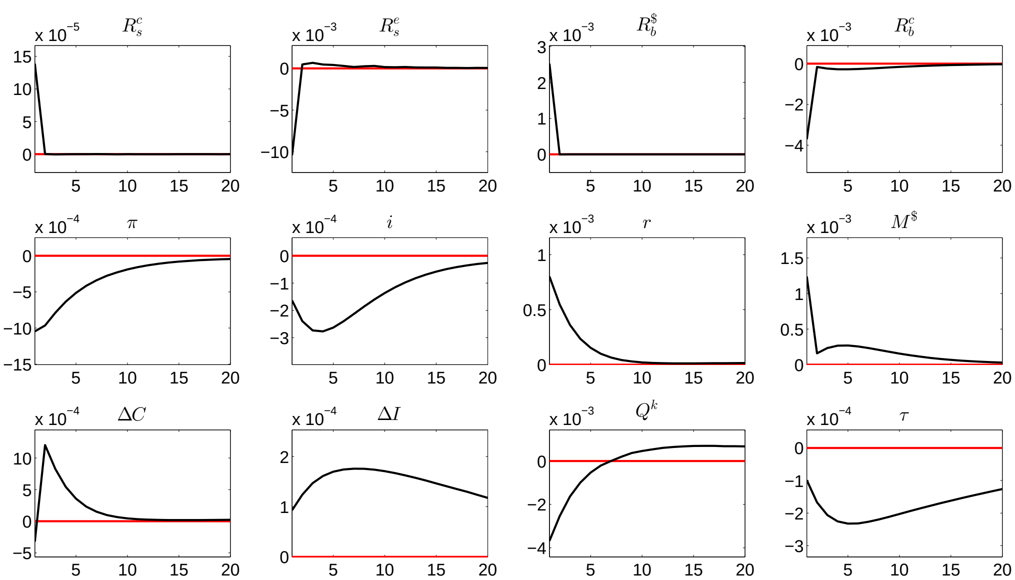
Notes: This figure plots the impulse responses of key macro and finance variables in the model under a positive MEI shock when the monetary policy is passive and fiscal policy is active. The x-axis is the time in quarters, and y-axis represents percentage change from the steady state.

Figure A.7: **A NT Shock in the AMPF Regime — CRRA Preference**



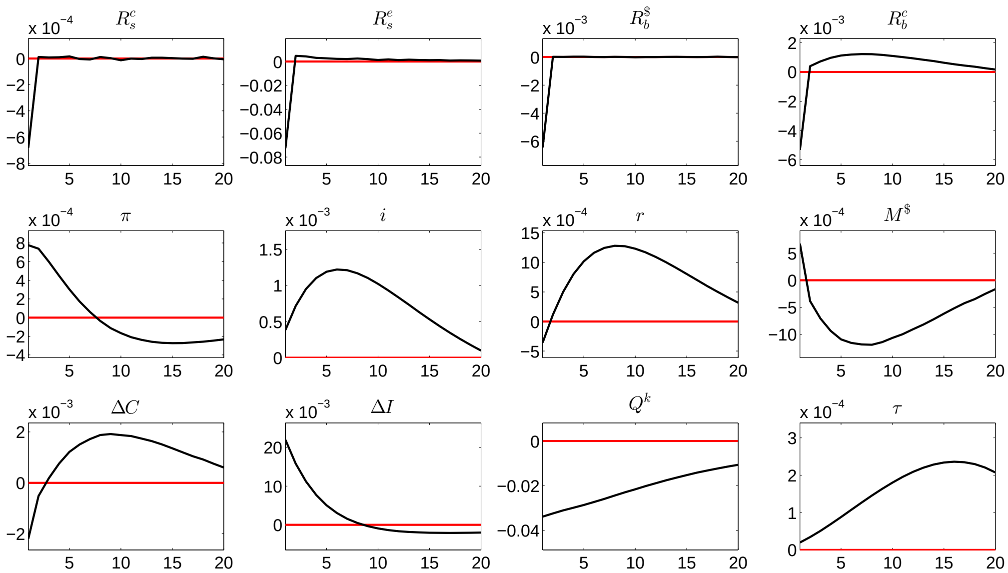
Notes: This figure plots the impulse responses of key macro and finance variables in the model with a CRRA Preference under a positive NT shock when the monetary policy is active and fiscal policy is passive. The x-axis is the time in quarters, and y-axis represents percentage change from the steady state.

Figure A.8: **A NT Shock in the PMAF Regime — CRRA Preference**



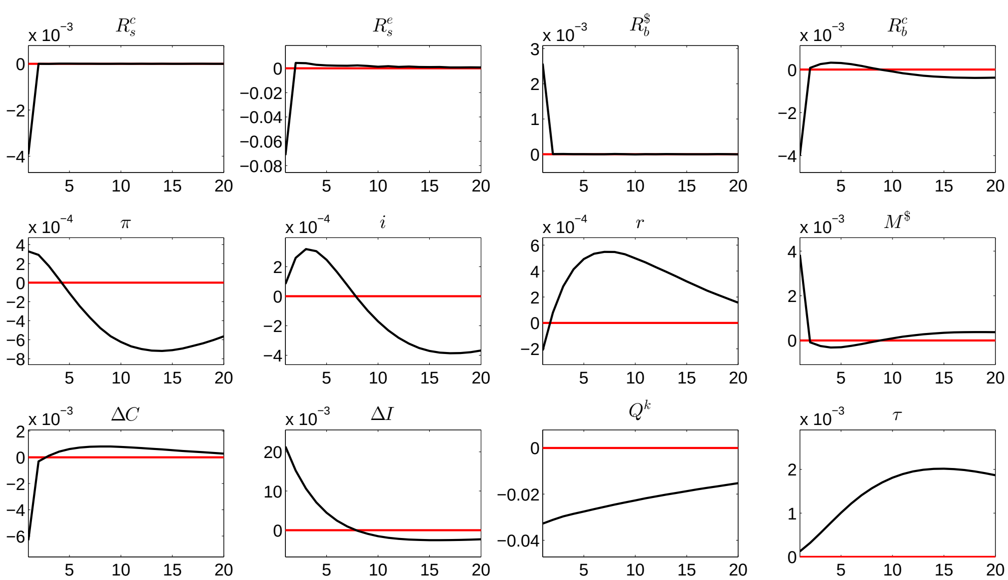
Notes: This figure plots the impulse responses of key macro and finance variables in the model with a CRRA Preference under a positive NT shock when the monetary policy is passive and fiscal policy is active. The x-axis is the time in quarters, and y-axis represents percentage change from the steady state.

Figure A.9: **A MEI Shock in the AMPF Regime — CRRA Preference**



Notes: This figure plots the impulse responses of key macro and finance variables in the model with a CRRA Preference under a positive MEI shock when the monetary policy is active and fiscal policy is passive. The x-axis is the time in quarters, and y-axis represents percentage change from the steady state.

Figure A.10: **A MEI Shock in the PMAF Regime — CRRA Preference**



Notes: This figure plots the impulse responses of key macro and finance variables in the model with a CRRA Preference under a positive MEI shock when the monetary policy is passive and fiscal policy is active. The x-axis is the time in quarters, and y-axis represents percentage change from the steady state.

Appendix B Bond Risk Premium

Appendix B.1 Real Term Premium

In equilibrium, $R^{c,(n)}_{b,t+1} = \exp\left(-(n-1)r^{(n-1)}_{t+1} + nr^{(n)}_t\right)$ satisfies

$$\mathbb{E}_t\left[M_{t,t+1}R^{c,(n)}_{b,t+1}\right] = 1\,.$$

Under the assumption of log-normality, the above equation leads to

$$-nr^{(n)}_t = -r_t - \mathbb{E}_t\left[(n-1)r^{(n-1)}_{t+1}\right] + \frac{1}{2}\text{var}_t\left[(n-1)r^{(n-1)}_{t+1}\right] - \text{cov}_t\left[m_{t,t+1},(n-1)r^{(n-1)}_{t+1}\right]\,. \tag{B.1}$$

The one-period real term premium of a bond with maturity n-period is thus given by

$$\begin{aligned} rTP^{(n)}_t &\equiv \log\mathbb{E}_t\left[\exp\left(-(n-1)r^{(n-1)}_{t+1} + nr^{(n)}_t\right)\right] - r_t \\ &= n\,r^{(n)}_t - \mathbb{E}_t\left[(n-1)r^{(n-1)}_{t+1}\right] + \frac{1}{2}\text{var}_t\left[(n-1)r^{(n-1)}_{t+1}\right] - r_t\,. \end{aligned}$$

Substituting equation (B.1) into the above equation leads to:

$$rTP^{(n)}_t = \text{cov}_t\left(m_{t,t+1},(n-1)r^{(n-1)}_{t+1}\right)\,. \tag{B.2}$$

Based on the definition of $r^{(n-1)}_{t+1}$, we have

$$\begin{aligned} -(n-1)r^{(n-1)}_{t+1} &= \log B^{c,(n-1)}_{t+1} = \log\mathbb{E}_{t+1}\left[e^{\sum_{s=2}^n m_{t+s-1,t+s}}\right] \\ &= \mathbb{E}_{t+1}\left[\sum_{s=2}^n m_{t+s-1,t+s}\right] + \text{var}_{t+1}\left[\sum_{s=2}^n m_{t+s-1,t+s}\right]\,. \end{aligned} \tag{B.3}$$

Under the assumption of log-mornality and homoskedasticity, variance and covariance are constant. Therefore, the combination of equations (B.2) and (B.3) leads to

$$rTP^{(n)}_t = -\text{cov}_t\left[m_{t,t+1},\sum_{s=2}^n m_{t+s-1,t+s}\right]\,. \tag{B.4}$$

The above equation indicates that the real term premium of a long-term bond is positive if the stochastic discount factor (SDF) of the first period is negatively correlated with the SDFs of the future periods until maturity on average, and vice versa. For example, the real term premium of a 2-period bond is given by

$$rTP^{(2)}_t = -\text{cov}_t\left(m_{t,t+1},m_{t+1,t+2}\right)\,.$$

If the SDF is negatively (positively) autocorrelated, the real term premium of the 2-period bond is positive (negative). The real term premium of a 3-period bond is given by

$$rTP^{(3)}_t = -\text{cov}_t\left(m_{t,t+1},m_{t+1,t+2}+m_{t+2,t+3}\right)\,.$$

Therefore, if $m_{t,t+1}$ is negatively (positively) correlated with $m_{t+1,t+2}$ and $m_{t+2,t+3}$ on average, the real term premium of a 3-period bond is positive (negative).

Appendix B.2 Inflation Risk Premium

Given that $R^{\$,(n)}_{b,t+1} = \exp\left(-(n-1)i^{(n-1)}_{t+1} + ni^{(n)}_t\right)$, inflation risk premium can be written as

$$\begin{aligned}\pi TP^{(n)}_t &= ni^{(n)}_t - nr^{(n)}_t + \log \mathbb{E}_t\left[\exp\left(-(n-1)i^{(n-1)}_{t+1} - \pi_{t+1}\right)\right] - \log \mathbb{E}_t\left[\exp\left(-(n-1)i^{(n-1)}_{t+1}\right)\right] \\ &= ni^{(n)}_t - nr^{(n)}_t - \mathbb{E}_t\left[(n-1)i^{(n-1)}_{t+1}\right] + \frac{1}{2}\mathrm{var}_t\left[(n-1)i^{(n-1)}_{t+1}\right] - \mathbb{E}_t\left[\pi_{t+1}\right] \\ &\quad + \frac{1}{2}\mathrm{var}_t\left[\pi_{t+1}\right] + \mathbb{E}_t\left[(n-1)r^{(n-1)}_{t+1}\right] - \frac{1}{2}\mathrm{var}_t\left[(n-1)r^{(n-1)}_{t+1}\right] \end{aligned} \quad \text{(B.5)}$$

From the equilibrium relation $\mathbb{E}_t\left[M^{\$}_{t,t+1}R^{\$,(n)}_{b,t+1}\right] = 1$, we get

$$-ni^{(n)}_t = -i_t - \mathbb{E}_t\left[(n-1)i^{(n-1)}_{t+1}\right] + \frac{1}{2}\mathrm{var}_t\left[(n-1)i^{(n-1)}_{t+1}\right] - \mathrm{cov}_t\left[m^{\$}_{t,t+1}, (n-1)i^{(n-1)}_{t+1}\right]. \quad \text{(B.6)}$$

In addition,

$$\begin{aligned} i_t - r_t &= -\log \mathbb{E}_t\left[e^{m^{\$}_{t,t+1}}\right] + \log \mathbb{E}_t\left[e^{m^{\$}_{t,t+1}+\pi_{t+1}}\right] \\ &= \mathbb{E}_t\left[\pi_{t+1}\right] + \frac{1}{2}\mathrm{var}_t\left[\pi_{t+1}\right] + \mathrm{cov}_t\left[m^{\$}_{t,t+1}, \pi_{t+1}\right]. \end{aligned} \quad \text{(B.7)}$$

Substituting equations (B.1), (B.6) and (B.7) into equation (B.5) leads to

$$\begin{aligned}\pi TP^{(n)}_t &= \mathrm{var}_t\left[\pi_{t+1}\right] + \mathrm{cov}_t\left[m^{\$}_{t,t+1}, \pi_{t+1}\right] + \mathrm{cov}_t\left[m^{\$}_{t,t+1}, (n-1)i^{(n-1)}_{t+1}\right] \\ &\quad - \mathrm{cov}_t\left[m_{t,t+1}, (n-1)r^{(n-1)}_{t+1}\right] + \mathrm{cov}_t\left[\pi_{t,t+1}, (n-1)i^{(n-1)}_{t+1}\right] \\ &= \mathrm{cov}_t\left[m_{t,t+1}, \pi_{t+1}\right] + \mathrm{cov}_t\left[m_{t,t+1}, (n-1)\left(i^{(n-1)}_{t+1} - r^{(n-1)}_{t+1}\right)\right], \end{aligned} \quad \text{(B.8)}$$

where the second equality follows from $m_{t,t+1} = m^{\$}_{t,t+1} + \pi_{t+1}$. Realizing that under the log-normality and homoskedasticity assumptions the nominal-real bond spread is

$$\begin{aligned}&(n-1)\left(i^{(n-1)}_{t+1} - r^{(n-1)}_{t+1}\right) = \log\left[B^{c,(n-1)}_{t+1}/B^{\$,(n-1)}_{t+1}\right] = \log \mathbb{E}_t[M_{t+1,t+n}] - \log \mathbb{E}_t[M^{\$}_{t+1,t+n}] \\ &= \log \mathbb{E}_{t+1}\left[\exp\left(\sum_{s=1}^{n-1} m_{t+s,t+s+1}\right)\right] - \log \mathbb{E}_{t+1}\left[\exp\left(\sum_{s=1}^{n-1} m_{t+s,t+s+1} - \pi_{t+s,t+s+1}\right)\right] \\ &= \sum_{s=1}^{n-1} \mathbb{E}_{t+1}[\pi_{t+1+s}] - \frac{1}{2}\mathrm{var}_t\left(\sum_{s=1}^{n-1}\pi_{t+1+s}\right) + \mathrm{cov}_{t+1}\left(\sum_{s=1}^{n-1} m_{t+s,t+s+1}, \sum_{s=1}^{n-1}\pi_{t+s+1}\right).\end{aligned}$$

Since the variance and covariance terms are constant under the log-normality and homoskedasticity assumptions, it follows that

$$\pi TP^{(n)}_t = \mathrm{cov}_t\left(m_{t,t+1}, \sum_{s=1}^{n}\pi_{t+s}\right). \quad \text{(B.9)}$$

Therefore, inflation risk premium of a bond with maturity n depends on the coveriance between the $t+1$-period pricing kernel and the inflation between $t+1$ to maturity.