

# Dynamic fiscal limits and monetary-fiscal policy interactions

Niccolò Battistini\*    Giovanni Callegari†    Luca Zavalloni‡ §

November 2017

## Abstract

We analyze fiscal sustainability and monetary-fiscal policy interactions through the lens of a DSGE model with risky sovereign debt à la [Bi \(2012\)](#). We find that a looser monetary policy stance implies larger inflation fluctuations, thus negatively affecting fiscal sustainability. As monetary policy is constrained, a binding ZLB further hampers fiscal sustainability, since fiscal policy acts only as an imperfect substitute for monetary policy in steering inflation. Moreover, in the analysis of the transmission of fiscal shocks, we find that positive spending shocks in normal periods tend to have adverse effects on the overall economy and fiscal sustainability, while the conclusion is reversed during periods of binding ZLB. Finally, the timing of the policy intervention is crucial: if shocks are implemented when exiting the ZLB, they risk trapping the economy in a persistent low-growth period and strongly increasing sustainability risks.

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\*European Central Bank. E-mail: [niccolo.battistini@ecb.europa.eu](mailto:niccolo.battistini@ecb.europa.eu).

†European Central Bank. E-mail: [giovanni.callegari@ecb.europa.eu](mailto:giovanni.callegari@ecb.europa.eu).

‡European Central Bank and University of Warwick. E-mail: [luca.zavalloni@ecb.europa.eu](mailto:luca.zavalloni@ecb.europa.eu).

§We would like to thank Roel Beetsma, Huixin Bi, Giancarlo Corsetti, Wouter Den Haan, Eric Leeper, and Stephanie Schmitt-Grohé for their helpful comments and suggestions. This paper has also benefited from several discussions with [...]. All errors are the sole responsibility of the authors. The views expressed in this paper are those of the authors only and do not represent those of the European Central Bank.

# 1 Introduction

Changes in the way economic agents perceive the sustainability of public debt can severely affect the way an economy responds to shocks. Monetary policy plays a key role in this respect. Indeed, throughout the recent years, the capacity of several advanced countries to service debt has been helped by the measures of monetary easing adopted by their central banks. Nowadays, on the contrary, the perspective of increases in interest rates is having the opposite effect, renewing the concerns on debt sustainability. In this context, there is little doubt that these changes can be more or less important depending on the initial levels of debt, or on the way agents expect the fiscal deficit to evolve over time.

In the wake of these considerations, this paper studies how different monetary policy stances affect the sustainability of public debt and the transmission of fiscal policy shocks, depending on the initial fiscal conditions and on the expectations on the future conduct of fiscal policy. In particular, we focus on the following set of questions:

1. In normal times, do different degrees of monetary policy activeness affect the maximum amount of public debt that a country is able to service? How is this debt limit affected by persistent periods of binding Zero-Lower Bound (ZLB)?
2. How does the transmission mechanism of fiscal policy shocks changes once we make public debt limits endogenous to monetary policy considerations?
3. How does this assessment change depending on different initial debt levels or expectations on spending policy?

To address these questions, our analysis relies on the debt limit framework pioneered first by [Bi \(2012\)](#). In this framework the maximum amount of debt that a country can tolerate as the present-discounted value of all maximum future primary surpluses. As the economy is subject to stochastic shock, the debt limit comes then out as a distribution rather than a single point estimate. This model suits us our needs because it allows us

to make the analysis contingent on the initial level of debt of a country, the assumptions on the path of both public transfers and government consumption and the nature and time-series characteristics of the shocks hitting the economy.

The main contribution of our paper is twofold. On the one hand, as an extension to [Bi, Leeper and Leith \(2013\)](#), we allow inflation to vary around its steady state level thus making fiscal limit distributions endogenous to monetary policy. This constitutes a significant progress compared to the existing literature, in which the fiscal limit was invariant to the different monetary policy regimes.

On the other hand, and on the basis of the results above, we propose a dynamic index for the evaluation of fiscal space that is contingent on the monetary policy stance and on the state of the economy. The objective of the index is to indicate whether fiscal policy is sustainable (i.e. public debt remains below the debt limit) not only at the time a change fiscal policy is implemented but also in the successive periods, given the size and nature of the fiscal shock. As an application, we apply our index to study the role of fiscal policy when monetary policy is exiting temporary periods of binding ZLB (edge-zones), taking into account different fiscal instruments and initial conditions.

Our results indicate that monetary policy can considerably affect fiscal sustainability. First, a looser monetary policy stance – i.e., a lower coefficient in the Taylor rule – implies larger fluctuations of inflation away from its target in response to exogenous shocks, higher price adjustment costs, lower firms' profits and, thus, a smaller tax base, which negatively affects fiscal sustainability. Similarly, a binding ZLB constraint implies that monetary policy cannot offset the adjustment costs of (downward) inflation fluctuations, which bring about significant revenue shortfalls.<sup>1</sup>

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<sup>1</sup>In fact, we show that, *ceteris paribus*, the presence of a ZLB constraint may even reverse our first finding about the beneficial effects of a more reactive monetary policy stance, since the latter increases the probability of the economy hitting the ZLB. In this respect, we find that higher steady state levels of inflation or of the risk-free nominal interest rate effectively reduce the frequency of a ZLB, thus helping fiscal sustainability.

Making the debt limit endogenous to monetary policy considerations also changes the transmission of fiscal shocks. We find that positive spending shocks in normal periods tend to have adverse effects on the economy, given the adverse impact on both output fluctuations and the debt limit. The conclusion is reversed during periods of binding ZLB, where spending shocks strongly reduces the fluctuation of output and inflation, and improves the capacity of the economy to support higher debt limits. The timing of the policy intervention, however, is crucial. If the shocks is implemented when exiting the ZLB, this risks to trap the economy in a persistent low-growth period and to strongly increase sustainability risks.

Our analysis is linked to the literature studying the implications of the fiscal theory of the price level on inflation determination, and the different combination of active/passive fiscal and monetary policy (see [Davig, Leeper and Walker \(2011\)](#) and [Leeper and Leith \(2016\)](#) among others). Differently from them, however, we study how *active* monetary policy regimes or simple suspension of monetary policy rules (i.e. periods of binding ZLB) affect debt limits. In this sense, we are also linked to the literature looking at the determination of sovereign risks in different monetary environments, like [Corsetti and Dedola \(2016\)](#) and [Aguiar et al. \(2013\)](#). Compared to these papers our analysis is mostly positive and abstract from the analysis of the conditions in which it is optimal for a government to inflate away debt or default.

The remainder of the paper is structured as follows. Section 2 presents some stylised facts on sustainability risks and monetary policy stance during the crisis, looking at the role of different initial conditions. Section 3 outlines the general equilibrium model. Section 4 presents the methodology for its numerical solution alongside the calibration parameters. Section 5 assesses the role of monetary policy and the ZLB in the determination of the debt limit. Section 6 introduces our forward-looking fiscal indicator. In section 7, finally, we apply our fiscal indicator to evaluate the role of spending shocks in a set of policy

scenarios, including the analysis of periods of binding ZLB. Section 8 concludes.

## 2 Sustainability risks and monetary policy: some facts

This section aims at providing some stylised facts on how changes in monetary policy affects the sustainability of public debt in advanced countries. This exercise is not straightforward, because debt sustainability is not an objective, measurable and observable measure, but rather a subjective judgement that crucially depends on the fiscal and macro-financial assumptions on which the underlying projections are based. Even our measures of fiscal sustainability, the maximum amount of public debt a country can service, is in itself unobservable and dependent not only on the long-term assumptions made, but also on the model that underlies the analysis.

For all these reasons, most of the literature (see [Greenlaw et al. \(2013\)](#), [Aizenman, Hutchison and Jinjarak \(2013\)](#) and [Santis \(2014\)](#), among others) on fiscal and monetary policy interactions focus on the dynamics of sovereign yields or risk premia. While a formal econometric analysis goes well beyond the scope of our work, in this section we want to present some descriptive facts on how countries with different fiscal stance reacted to common changes in monetary policy.

To this end, we focus on the euro area only and on the impact of monetary policy reaching the zero-lower bound. We focus on the euro area because it provides us with the setting for a natural experiment: how monetary policy changes affect different countries depending on their initial conditions. In particular, we check how fiscal, and sovereign yield dynamics evolved in the so called "core" and "periphery" countries, with the dividing line being the different perceptions on debt sustainability across the two blocks.

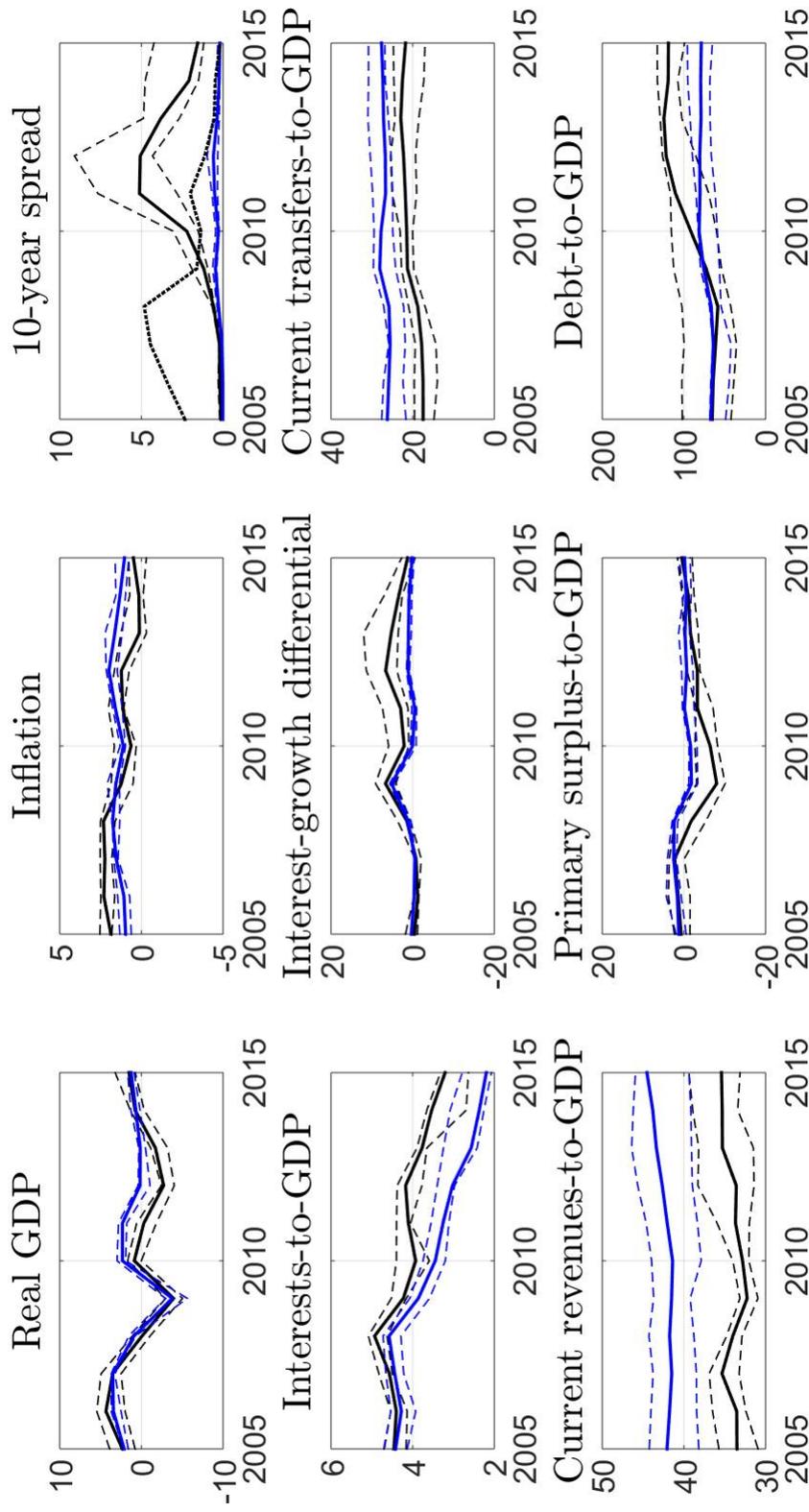


Figure 1: Source: Eurostat. Blue line: Core countries (Germany, France, Netherlands, Austria and Belgium). Black line: periphery countries (Italy, Spain, Ireland, Portugal, Cyprus and Ireland). The dashed lines around each solid line indicate the max and min values touched by the series in each year across the countries part of each group. In the 1-year chart, the dotted line indicates the evolution of the 1-year Euribor rate.

### 3 The model

The model builds on the works by Bi (2012) and Bi, Leeper and Leith (2013) by introducing agent preferences *à la* Greenwood, Hercowitz and Huffman (1988) and allowing for the presence of a ZLB on the nominal risk-free interest rate. Time is discrete and denoted as  $t = 0, 1, 2, \dots, \infty$ . The closed economy is populated by a representative household, who consumes, works, owns monopolistically competitive firms producing differentiated intermediate goods and perfectly competitive firms producing a homogeneous final good, and invests in two types of state-noncontingent assets, namely risk-free bonds and risky (i.e., defaultable) government bonds.

#### 3.1 The representative household

The representative household maximizes the following initial utility function:

$$\max_{c_t, n_t, b_t, b_t^F} E_0 \sum_{t=0}^{\infty} \beta^t \left( \prod_{s=-1}^{t-1} \xi_s \right) \frac{1}{1-\gamma} \left( c_t - \frac{n_t^{1+\frac{1}{\chi}}}{1+\frac{1}{\chi}} \right)^{1-\gamma}, \quad (1)$$

subject to the flow budget constraint:

$$c_t + \frac{b_t}{R_t} + \frac{b_t^F}{R_t^F} = (1 - \tau_t)(w_t n_t + \Upsilon_t) + z_t - m_t + \frac{b_{t-1}^d}{\pi_t} + \frac{b_{t-1}^F}{\pi_t}, \quad (2)$$

where  $E_0$  denotes the expectations operator,  $\beta$  the household's discount factor,  $\gamma$  its relative risk aversion and  $\chi$  its Frisch elasticity. Moreover,  $c_t$  denotes private consumption,  $n_t$  hours of labor,  $\tau_t$  the tax rate on wage income and profits,  $w_t$  the (real) wage rate,  $\Upsilon_t$  the representative firm's profits,  $\pi_t$  (gross) inflation,  $z_t$  transfers from the government to the households,  $m_t$  lump-sum taxes,  $b_t$  risky (i.e. defaultable) government bonds, with associated (gross) interest rate  $R_t$ , and  $b_t^F$  risk-free bonds, with associated (gross) interest

rate  $R_t^F$ , at time  $t$ . Notice that  $b_{t-1}^d \equiv (1 - \Delta_t)b_{t-1}$  denotes the part of real outstanding debt actually repaid and  $\Delta_t$  is the haircut on outstanding debt. Finally,  $\xi_t$  represent an exogenous discount factor shock given by

$$\log \xi_t = (1 - \rho^\xi) + \rho^\xi \log \xi_{t-1} + \varepsilon_t^\xi, \quad (3)$$

where  $\varepsilon_t^\xi \sim N(0, \sigma_\xi^2)$ . Trasfer

Labor supply:

$$n_t = [w_t(1 - \tau_t)]^\chi. \quad (4)$$

Euler equation (risk-free bonds):

$$\frac{1}{R_t^F} = \beta \xi_t E_t \left[ \frac{u_c(t+1)}{u_c(t)} \frac{1}{\pi_{t+1}} \right]. \quad (5)$$

Euler equation (risky government bonds):

$$\frac{1}{R_t} = \beta \xi_t E_t \left[ (1 - \Delta_{t+1}) \frac{u_c(t+1)}{u_c(t)} \frac{1}{\pi_{t+1}} \right]. \quad (6)$$

where  $\Delta_t = \begin{cases} 0 & \text{if } b_{t-1} < b^* \\ \delta_t & \text{if } b_{t-1} \geq b^*, ta \end{cases}$

Fisher identity:

$$R_t^F = r_t^F + E_t \pi_{t+1} - 1. \quad (7)$$

### 3.2 Final goods producers

Production function:

$$y_t = a_t n_t. \quad (8)$$

where  $y_t$  and  $a_t$  are the output and productivity levels. Final goods producers' minimization problem in symmetric equilibrium:

$$\min_{n_t} \{w_t n_t - \lambda_t (a_t n_t - y_t)\}. \quad (9)$$

Final goods producers' aggregate real marginal cost (equal to Lagrange multiplier  $-\lambda_t$ ):

$$mc_t = \frac{w_t}{a_t}. \quad (10)$$

### 3.3 Intermediate goods producers

Intermediate goods producer  $i$ 's maximisation problem with beginning-of-time (monopolistic) nominal profits (in symmetric equilibrium):

$$\max_{p_t(i)} E_0 \sum_{t=0}^{\infty} R_{0,t}^F \left[ p_t(i) y_t(i) - mc_t P_t y_t(i) - \frac{\phi}{2} \left( \frac{p_t(i)}{p_{t-1}(i)\pi} - 1 \right)^2 P_t y_t \right] \quad (11)$$

where  $R_{0,t}^F \equiv \beta^t \left( \prod_{s=-1}^{t-1} \xi_s \right) \frac{u_c(t)}{u_c(0)}$  and  $p_t(i)$  and  $P_t$  are the price chosen by firm  $i$  and the nominal aggregated price level, respectively.

The demand for each intermediate good is:

$$y_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\theta} y_t. \quad (12)$$

Phillips curve (in symmetric equilibrium):

$$(1 - \theta) + \theta mc_t - \phi \frac{\pi_t}{\pi} \left( \frac{\pi_t}{\pi} - 1 \right) + \phi \beta \xi_t E_t \left[ \frac{u_c(t+1)}{u_c(t)} \frac{y_{t+1}}{y_t} \frac{\pi_{t+1}}{\pi} \left( \frac{\pi_{t+1}}{\pi} - 1 \right) \right] = 0 \quad (13)$$

where  $\phi$  parametrizes Rotemberg (quadratic) price adjustment costs and  $\theta$  is the elasticity of substitution between goods.

Intermediate goods producers' (monopolistic) real profits (in symmetric equilibrium):

$$\Upsilon_t = y_t - mc_t y_t - \frac{\phi}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 y_t \quad (14)$$

### 3.4 The government

Public consumption:

$$\log g_t = (1 - \rho^g) \log g + \rho^g \log g_{t-1} + \varepsilon_t^g, \quad (15)$$

where  $\varepsilon_t^g \sim N(0, \sigma_g^2)$ .

Transfers:

$$\log z_t = \begin{cases} (1 - \rho^z) \log z + \rho^z \log z_{t-1} + \varepsilon_t^z & \text{if } x_t^z = 1 \\ \zeta \log z_{t-1} & \text{if } x_t^z = 2, \end{cases} \quad (16)$$

where  $\varepsilon_t^z \sim N(0, \sigma_z^2)$ ,  $|\rho^z| < 1$  and  $\zeta > 1$ .

Transfer regime transition matrix:

$$\begin{bmatrix} Pr(x_t^z = 1 | x_{t-1}^z = 1) & Pr(x_t^z = 1 | x_{t-1}^z = 2) \\ Pr(x_t^z = 2 | x_{t-1}^z = 1) & Pr(x_t^z = 2 | x_{t-1}^z = 2) \end{bmatrix} = \begin{bmatrix} p_1^z & 1 - p_1^z \\ 1 - p_2^z & p_2^z \end{bmatrix}. \quad (17)$$

Tax rate:

$$\tau_t = \tau + \mu^\tau (b_t - b) + \eta_t^\tau, \quad (18)$$

where  $b$  denotes the steady-state real debt level and

$$\eta_t^\tau = \rho^\tau \eta_{t-1}^\tau + \varepsilon_t^\tau, \quad (19)$$

with  $\varepsilon_t^\tau \sim N(0, \sigma_\tau^2)$ .

Lump-sum taxes:

$$m_t = m + \mu^m (b_t - b), \quad (20)$$

Tax revenues (from distortionary taxation):

$$T_t = \tau_t (w_t n_t + \Upsilon_t) \quad (21)$$

Government's budget constraint:

$$\frac{b_t}{R_t} + T_t + m_t = \frac{b_{t-1}^d}{\pi_t} + g_t + z_t \quad (22)$$

The central bank's (truncated) Taylor rule:

$$R_t^F = \max \left\{ R^F \left( \frac{\pi_t}{\pi} \right)^\alpha, 1 \right\} \quad (23)$$

Closing the model economy, the aggregate resource constraint is given by

$$c_t + g_t = y_t \left[ 1 - \frac{\phi}{2} \left( \frac{\pi_t}{\pi} - 1 \right)^2 \right], \quad (24)$$

whereby transfers and lump-sum taxes,  $z_t$  and  $m_t$ , cancel out as they simply redistribute resources between the household and the government.

Productivity:

$$\log a_t = (1 - \rho^a) \log a + \rho^a \log a_{t-1} + \varepsilon_t^a, \quad (25)$$

where  $\varepsilon_t^a \sim N(0, \sigma_a^2)$ .

## 4 Numerical solution and calibration

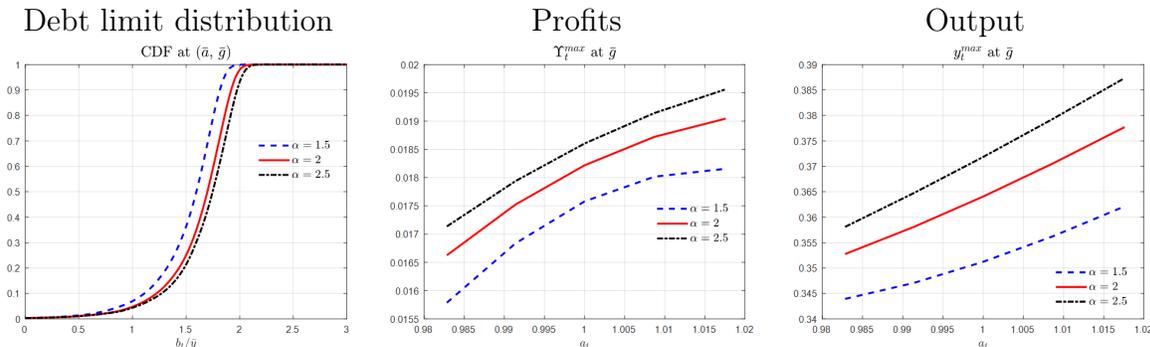
The use of a nominal model presents several problems. The most prominent one is that both the introduction of nominal rigidities and the relaxation of the log-utility assumption increase the computational burden of simulations. The specific problem lies in the fact that the revenue-maximising tax rate (i.e., the peak of the Laffer curve, which is crucial in the determination of the debt limit distribution) now depends non-linearly on the real wage and the inflation rate. No functional form is available to determine the equilibrium

level of these three endogenous variables. Hence, the equilibrium relationships of the three variables need to be solved numerically, given specific values for the state variables and the parameters.

Two assumptions allow us to simplify the computation of the maximum tax rate, the crucial element in the derivation of the debt limit, while preserving the unique equilibrium relation among the endogenous variables, given the state variables and the parameters of the model. First, the assumption of GHH preferences determines an explicit form for labour supply as a function of real wages and taxes. From an economic perspective, this assumption removes the wealth effect on labour supply. Hence, it is useful because otherwise the wealth effect pushes labour to display a counter-factual rise when TFP falls or when consumption drops sharply, as is the case in default episodes. Second, the introduction of a risk-free bond alongside the government bond allows us to pin down the path of consumption independently of the probability of default. Hence, when we solve the model for the maximum tax rate, the real wage and the inflation rate, we need not consider government debt, with considerable benefits in terms of computational time.

Given these two assumptions, we limit the number of state variables to two (TFP and government consumption) and the number of numerically determined control (or jump) variables to three, making the solution of the model feasible in a matter of seconds. Notice that a “solution of the model” includes a set (i.e., matrix) of one-to-one relationships between a specific value for the vector of state variables and a specific value for the vector of control variables. So, the model needs to be solved only once before the simulation of the debt limit distribution. When we calculate the debt limit distribution via simulations in our second step, all the control variables are readily available as a function of the state variables either through functional forms or through the one-to-one relationships between state and control variables established in our first step. The solution strategy is presented in [Appendix A](#).

Figure 2: Impact of the monetary policy stance on the debt limit distribution



Note: The left panel presents how the debt limit distributions change for three different values of the inflation coefficient in the Taylor rule (1.5, 2 and 2.5). The central and right panels show how profits and output change with productivity levels (in the x-axis) for the same three different Taylor coefficients. In all these simulations all other variables and shocks are set initially at their steady state value.

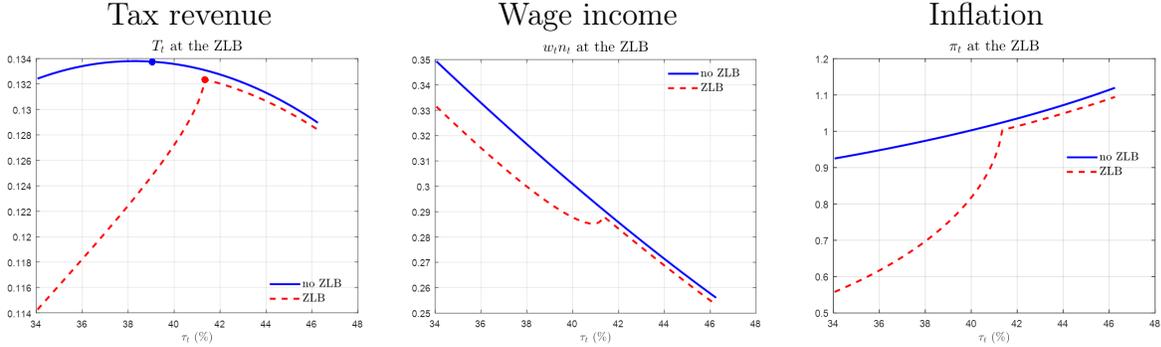
## 5 Monetary policy stance and debt limit determination

This section studies the role played by the monetary policy stance in the determination of the debt limit distribution. To this end, we analyze the impact of different monetary policy regimes, i.e. different degrees in the reactivity of the central bank to deviations of inflation from its target, on the debt limit distribution both. Further, we investigate how this impact changes under a binding ZLB constraint.

This analysis sheds light into an important aspect of fiscal-monetary policy interactions, often overlooked in the DSGE literature. Even in most recent studies (see, e.g., Bi, Leeper and Leith, 2013), inflation is fixed at its steady-state level when debt limit distributions are computed, so that no role is played by inflation and, thus, monetary policy in determining fiscal sustainability. In contrast, we allow inflation to drift away from its target in response to shocks in fundamentals and policy, so that we can evaluate the impact of these shocks on the sustainability of public debt under different macroeconomic conditions and policy regimes.

Figure 2 shows how different degrees of responsiveness of the risk-free interest rate

Figure 3: Binding ZLB in an economy at the debt limit



Note: The three charts show how tax revenues, wage income and inflation behave in equilibrium in an economy with binding ZLB. These three variables are plotted against different values of the tax rate in correspondence to a value of the discount factor shock that brings the economy at the ZLB. The solid blue line shows the behaviour of an economy in which the interest rates are left free to adjust, while the red-dotted line refer to an economy where interest rates cannot go below zero.

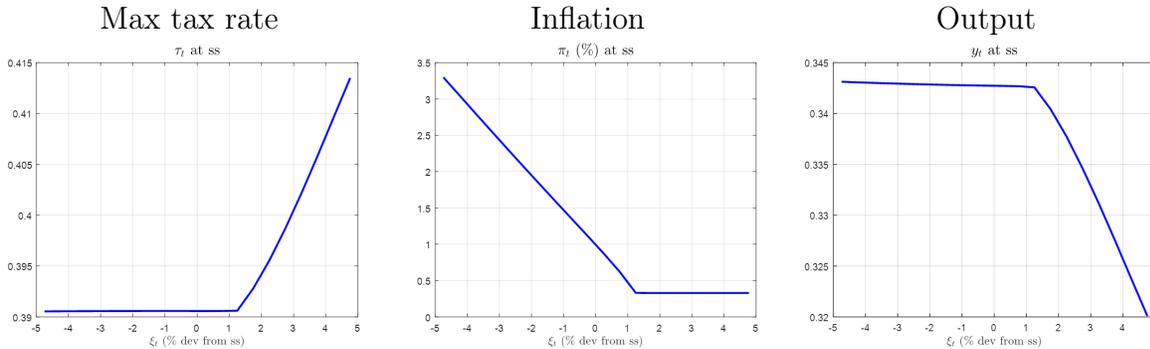
$R_t^F$  to inflation  $\pi_t$  affect the debt limit distribution and the determination of the tax base, essentially composed of output and profits. We do this by taking into account three different values of  $\alpha$ : 1.5 (consensus), 2 (strong) and 2.5 (very strong). The left panel presents how the debt limit distributions change for the three different values of the inflation coefficient in the Taylor rule (1.5, 2 and 2.5). The central and right panels show how profits and output change with productivity levels (in the x-axis) for the same three different Taylor coefficients.

The importance of inflation fluctuations is confirmed when looking at how binding ZLB affects the main determinants of the debt limit (maximum tax rate and tax base, i.e., profits and wages).

In the analysis that follows, we introduce a discount factor shocks  $\xi_t$  to bring the economy to a situation of binding ZLB. Higher values of  $\xi_t$  implies lower marginal propensity to consume, which in equilibrium is cleared through lower output and lower inflation. Values of  $\xi_t$  above 1.2 lead to a binding ZLB.

Figure 3 shows the impact of a binding ZLB on the maximum tax rate, inflation and

Figure 4: Binding ZLB in an economy at the debt limit



Note: The three panels describe the evolution of the revenue-maximising tax rate, inflation and output for different values of the discount factor shocks  $\xi_t$ . A discount factor shock above 1.2 leads to a binding ZLB. All other shocks and endogenous variables are initially set at their steady-state values.

profits. A binding ZLB tends to increase the maximum tax rate while reducing overall tax revenue. This is due to reduction of the tax base, especially of wage income (with the dotted line continuously below the solid line in the central panel of Figure 3). This, in turn, is accompanied by lower level of inflation (as expected in an economy with binding ZLB compared to an economy with interest rates allowed to fall below zero). This gap, however, falls considerably when the tax rate (depicted on the x-axis) reaches the revenue-maximising value and beyond.

Figure 4 shows why this is the case. The figure presents the evolution of the revenue-maximising tax rate, inflation and output for different values of the discount factor shocks  $\xi_t$ . Values of  $\xi_t$  above 1.2 brings the economy to the ZLB and makes monetary policy ineffective. The central panel of the figure shows that inflation stops falling once the discount fact reaches ZLB levels, despite monetary policy ineffectiveness. As shown in the left panel, this is due to the correspondent increase in the maximum tax rate  $\tau_t$ , which increases marginal costs and, with them, inflation.

In practice, in an economy at the ZLB, fiscal policy substitutes for monetary policy: with an active uses of the tax rate, the government supports inflation and avoids reductions

in profits that would otherwise reduce revenue collection. The increase in the tax rate, however, is not harmless as it reduces output through a reduction in labor supply, in turn due to the reduced disposable income received for every unit of labor supplied in the market. Despite the reduction in output, tax revenues are still substantially higher than what could be collected with tax rates below the maximizing value, because of the elevated tax cost of inflation fluctuations.

In synthesis, this section presented two important results: first, a more aggressive monetary policy stance can improve debt tolerance through a reduction of the price distortions in the economy; second, fiscal policy tends to replicate the work of monetary policy in an economy where the government wants to maximize tax revenues and the ZLB binds. These two considerations will be important in the simulations presented in section 6.

## 6 Forward-looking fiscal space index

The objective of this section is to derive a forward-looking measure of fiscal space. Any measure of fiscal space needs to be defined in terms of a specific fiscal instrument (i.e., tax rates, government consumption, government investment, etc.). In fact, in general equilibrium, each instrument has a different impact on the economy and, thus, on the debt-to-GDP ratio.

Let  $F$  be the fiscal instrument in relation to which we want to measure fiscal space. Let then  $s_t^{-F}$  be the state of the economy (the value taken by all exogenous shocks) at time  $t$ , with the exclusion of the value of the fiscal instrument  $F$ . For all variables other than  $F$ , the current state  $s_t^{-F}$  determines all their future values, in accordance with the respective assumed data generation process.

Let  $b_t$  be the debt-to-GDP ratio at time  $t$  and  $\hat{b}_t^\phi$  the debt limit ratio at time  $t$  corresponding to a given probability threshold  $\phi$ .

Prior to defining the index, we first need to define an intermediate but important fiscal variable: the fiscal policy signal  $FS$ .  $FS$  indicates whether the level of debt  $b_t$  is expected to reach or go beyond the debt limit  $\hat{b}_t^\phi$  between time  $t$  and time  $t + N$ . Formally, it is defined as follows:

$$FS_t = \begin{cases} 0 & \text{if } b_{t+i} < \hat{b}_{t+i}^\phi \quad \forall i \in [0, N] \\ 1 & \text{otherwise} \end{cases} \quad (26)$$

Let then be  $F_t^{FS}$  the level (maximum for tax rates, minimum for spending instruments) of the fiscal instrument  $F$  at which the fiscal signal  $FS$  takes the value of one given the state of the economy  $s_t^{-F}$ .<sup>2</sup> The fiscal space available at time  $t$  and relative to the fiscal instrument  $F$  can then be defined as

$$SP_t^F = (F_t - F_t^{FS} | s_t^{-F}) \quad (27)$$

Because of its evident non-linear nature (given the piecewise definition of the fiscal policy signal  $FS$  and of the threshold  $F_t^{FS}$ ) there is no analytical solution for  $SP_t^F$ . It needs to be simulated, based on a specific set of initial shocks.

In synthesis, this fiscal index is completely forward looking and takes into account the general equilibrium implications of changes to the fiscal instrument  $F$ . Moreover, it has the interesting property of being state-contingent, as it depends on the state  $s_t^{-F}$ .

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<sup>2</sup>Of course, in this section we assume that such level actually exists. This needs to be verified.

## 7 The impact and sustainability of government spending shocks

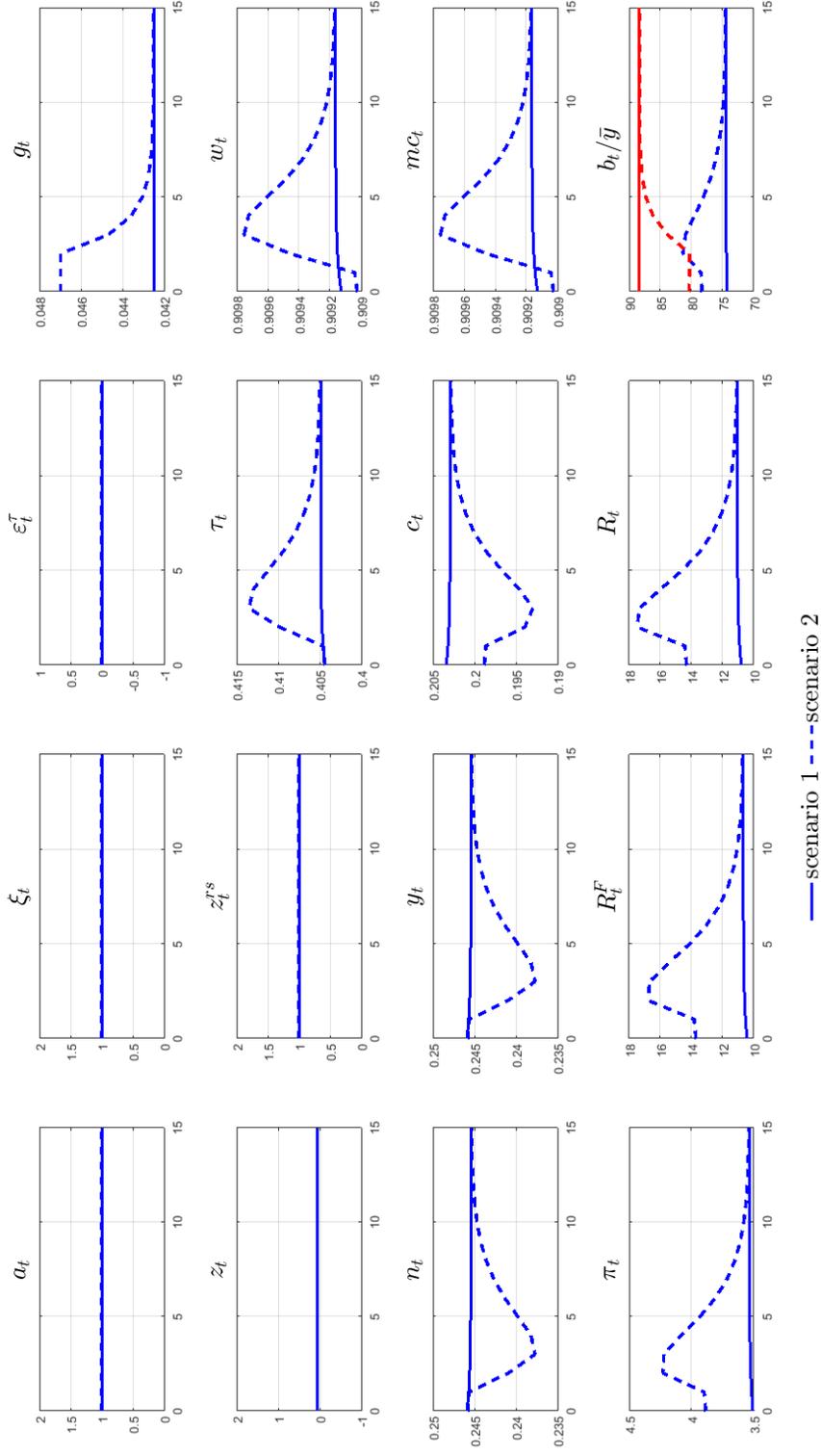
This section presents a concrete application of the tools presented above, focusing on government consumption shocks in our model economy. We perform two exercises: in the first we evaluate  $SP^G$ , that is, the policy space available for increases in spending; in the second we investigate how the sustainability of spending shocks changes when the ZLB is binding. In this context, we analyze also the importance of the timing of the shocks, also in relation with the trade-off between sustainability and stabilization.

In line with the spirit of the  $SP^G$  indicator, figure 5 shows the maximum extent to which government spending can increase before hitting the 30% default probability threshold during a time horizon of 15 years. In order to emphasize the forward-looking nature of the indicator, the shock has been assumed to last 3 years, after which it follows an AR(1) process with a 0.5 decay parameter, thus going back to zero after 7 years from the initial increase.

The novelty of our analytical framework is that it evaluates the sustainability of the shock not only by looking at the relative increase in public debt but at the reduction in the debt limit. This reduction occurs because of two main reasons.

First, the increase in spending reduces the primary surplus thus containing the amount of resources available to finance debt. The increase in expenditure is coupled with a reduction in revenues due to the output contraction that follows the shock. The contraction, in turn, is directly linked to the increase in interest rates and the expected increase in the income tax rate, which amplifies the economy's distortions. The primary surplus response would still be negative even with a (positive) output response more in line with the existing literature.

Figure 5: Maximum sustainable spending shock with endogenous debt limits



Note: The figures represent a scenario in which spending is increased for 3 years for the extent allowed by the indicator  $SP^G$ . This corresponds to a shock of approximately 1.8% of (steady-state) GDP. After 3 years, the shock follows an AR(1) process with a 0.5 parameter. The solid line refers to the economy's steady state, while the dotted line describe the dynamics following the spending shock. In the bottom right panel, the red line indicates the evolution of the 30% debt limit threshold, i.e., the value on the debt limit distribution that corresponds to a 30% default probability.

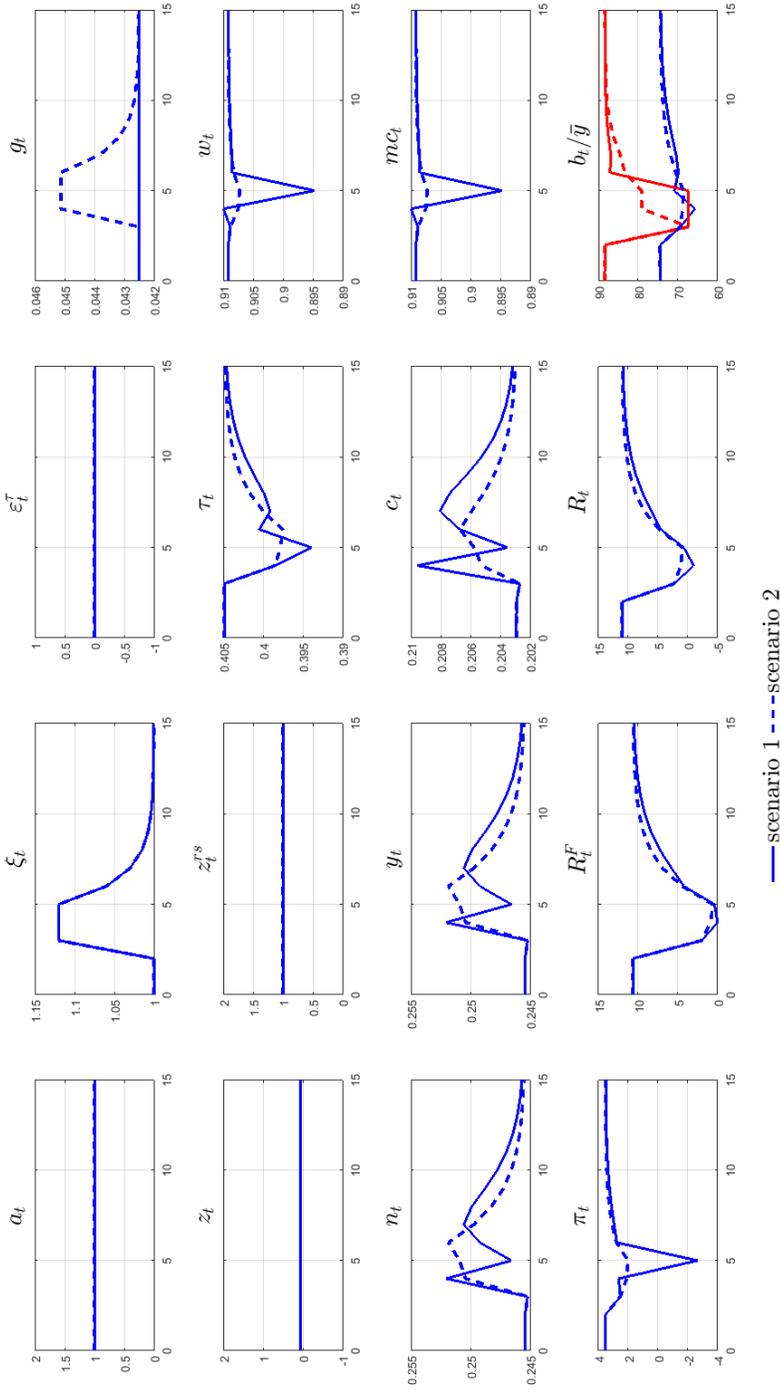
Second, the increase in demand triggers an increase in inflation to which the central bank responds with an increase in the interest rate. The increase in the risk-free rate is coupled with an increase in the risk-premium, due to the increase in the public debt. *Ceteris paribus*, higher rates lead to a lower present-discounted value of future primary surpluses, thus further shifting to the left the debt limit distribution (and reducing the 30% probability threshold).

In such a context, the space available for increases in spending is much smaller than what would be implied by a simple difference between the initial level of debt and a specific probability threshold in the debt limit distribution. As the bottom right panel of figure 5 shows, the largest government spending shock available to stimulate the economy while keeping debt below the 30% threshold, i.e. the fiscal margin associated with a 30% default probability, amounts to about 1.8% of (steady-state) GDP for 3 years. The importance of a forward-looking indicator is highlighted by the fact that the debt limit is hit after 3 years, and not as the shock initially occurs.

After the analysis of the sustainability of the spending shock, we now turn to analyse the trade-off between sustainability and stabilization and its interaction with a binding ZLB. We look at the case in which a government spending shock together with an increase in the discount factor has compressed the risk-free interest rate.

From a stabilization point of view, the spending shock has a positive impact. Compared to a scenario in which only the discount factor occurs, and in line with the existing literature (see Leeper and Leith, 2016, Christiano et al., 2011, and Woodford, 2011), the volatility of both output and inflation is decisively dampened. The shock offsets the drop in inflation, thus reducing real interest rates (given the lack of response from the central bank) and lifts output.

Figure 6: Impact of a 3-year government spending shock in presence of binding ZLB



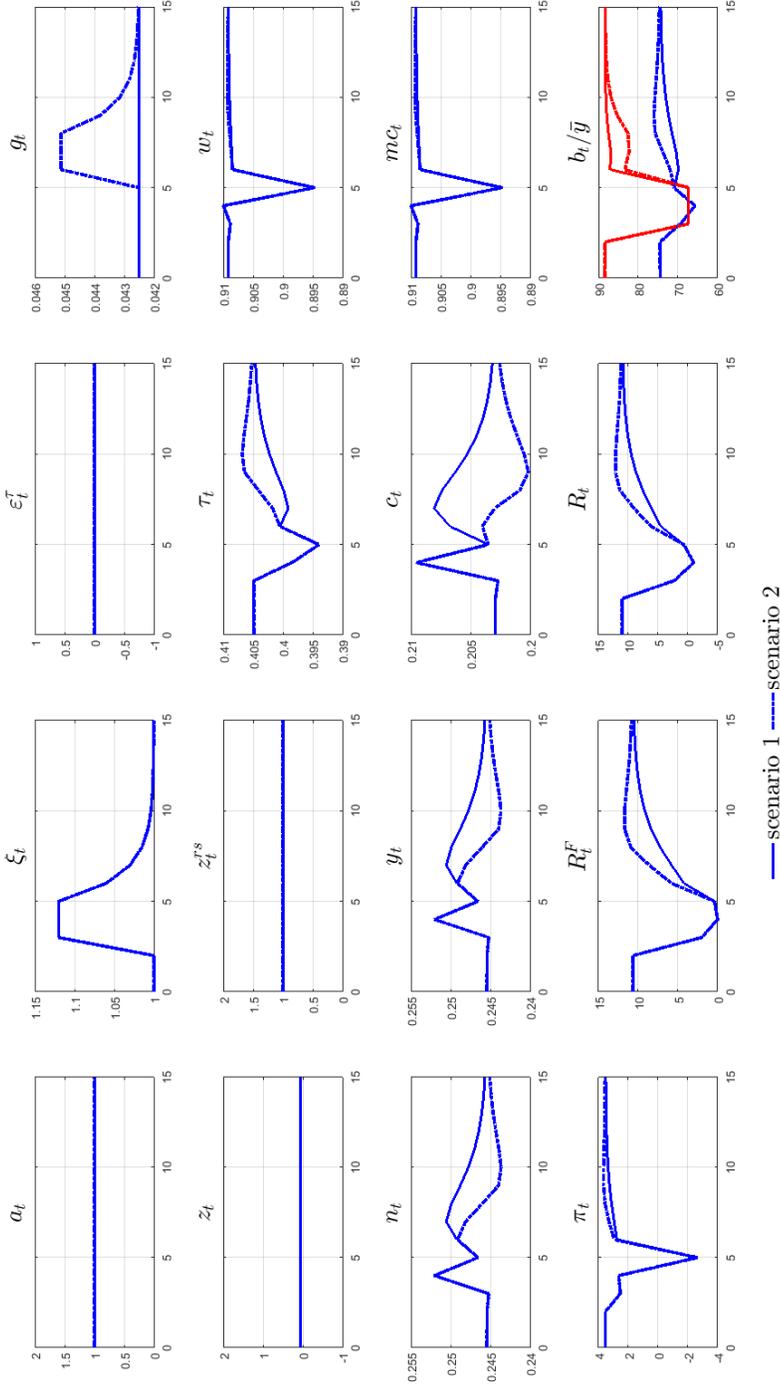
Note: The figures represent a scenario in which spending is increased by approximately 1.1% of (steady-state) GDP for 3 years, after which the shock follows an AR(1) process with a 0.5 parameter. The spending shock occurs in a scenario in which the ZLB is binding, due to a persistent increase in the households' time discount factor, which reduces consumption and thus output. The solid line describes the evolution of the economy after the discount factor shock and with government spending at its steady-state level. The dotted line describes the dynamics following the spending shock. In the bottom right panel, the red line indicates the evolution of the 30% debt limit threshold, i.e., the value on the debt limit distribution that corresponds to a 30% default probability.

From a sustainability point of view, the spending shock leads to a minimal increase in debt compared to the starting scenario with only the discount factor shock. The impact on the debt limit, on the other hand, is substantial, as the increase in output and the smaller inflation volatility increases the tax base and thus the capacity of the economy to service debt.

This result, however, strongly depends on the *timing* of the spending shock in relation with the period in which the ZLB is binding. Figure 7 shows the impact of a spending shock occurring as the economy is exiting a period of binding ZLB. In this case, monetary policy is free to offset the demand increase through increases in interest rates. This has a lasting impact on government debt and on the debt limit, significantly shrinking the space available (and thus increasing the sustainability risks and the risk premia).

In practice, while fiscal policy has a clear scope for stabilization purposes during periods of binding ZLB, our results suggest prudence in their use. The implementation lags of fiscal policy interventions and the uncertainty that surrounds the timing of monetary policy decisions (and the persistence of the ZLB) can greatly increase the risk of policy mistakes that could lead to a prolonged period of low growth, as shown in Figure 7.

Figure 7: Impact of a 3-year government spending shock when exiting periods of binding ZLB



Note: The figures represent a scenario in which spending is increased by approximately 1.1% of (steady-state) GDP for 3 years, after which the shock follows an AR(1) process with a 0.5 parameter. The spending shock occurs in a scenario in which the ZLB is binding, due to a persistent increase in the households' time discount factor, which reduces consumption and thus output. The solid line describes the evolution of the economy after the discount factor shock and with government spending at its steady-state level. The dotted line describes the dynamics following the spending shock. In the bottom right panel, the red line indicates the evolution of the 30% debt limit threshold, i.e., the value on the debt limit distribution that corresponds to a 30% default probability.

## 8 Conclusions

We analyzed fiscal sustainability and monetary-fiscal policy interactions through the lens of a DSGE model with risky sovereign debt à la [Bi \(2012\)](#). We found that a looser monetary policy stance implies larger inflation fluctuations, thus negatively affecting fiscal sustainability. As monetary policy is constrained, a binding ZLB further hampers fiscal sustainability, since fiscal policy acts only as an imperfect substitute for monetary policy in steering inflation. Moreover, in the analysis of the transmission of fiscal shocks, we found that positive spending shocks in normal periods tend to have adverse effects on the overall economy and fiscal sustainability, while the conclusion is reversed during periods of binding ZLB. Finally, the timing of the policy intervention is crucial: if shocks are implemented when exiting the ZLB, they risk trapping the economy in a persistent low-growth period and strongly increasing sustainability risks.

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## Appendix A Solution algorithms

The absence of functional forms for all the endogenous variables requires the solution of part of the non-linear model (i.e., for the real wage, the inflation rate and the maximum tax rate, without considering debt and risk-free assets) with numerical methods before calculating the debt limit distribution via Monte Carlo simulations. The solution is based on the monotone mapping method, developed by Coleman (1991) and Davig (2004), which discretizes the state space and conjectures candidate decision rules that reduce the system to a set of first-order expectational difference equations. The decision rules for the real wage  $w_t^* = f^w(\psi_t^1)$ , the inflation rate  $\pi_t^* = f^\pi(\psi_t^1)$  and the corresponding maximum tax rate  $\tau_t^* = f^\tau(\psi_t^1)$  are solved in the following steps.

1. Discretize the state space  $\psi_t^1 = \{a_t, \xi_t, g_t\}$  with grid points  $n_a = 5, n_\xi = 5, n_g = 5$ .
2. For  $i = 1, 2, \dots$ , make a guess for the decision rules  $(f_g^w, f_g^\pi, f_g^\tau)$  over the state space. If  $i = 1$ , set  $(f_g^w, f_g^\pi, f_g^\tau)$  to their steady state values; if  $i > 1$ , set  $(f_g^w, f_g^\pi, f_g^\tau)$  to the solutions in the previous iteration  $(f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau)$ .
3. At each grid point, solve the model and obtain the updated rule  $(f_i^w, f_i^\pi, f_i^\tau)$  using the given rule  $(f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau)$  as a guess. Given equations (3), (7), (11), (18) and (19) to pin down  $(n_t, mc_t, \Upsilon_t, R_t^F, c_t)$ , respectively, use the model equations (4), (10) and (16) to solve the non-linear model and determine the decision rules  $(f_i^w, f_i^\pi, f_i^\tau)$ . In particular, maximise (16) subject to the non-linear constraints (4) and (10) and non-negativity constraints on endogenous variables as appropriate. The integrals implied by the expectation terms are evaluated using numerical quadrature. The exogenous AR(1) processes are approximated as first-order Markov processes according to the quadrature approach by Tauchen and Hussey (1991).
4. Notice that  $(f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau)$  are assumed to be decision rules at  $t+1$  when evaluating expectations, as they provide a set of intra-temporally consistent solutions for the op-

timising agents. To ensure that the solution is also inter-temporally consistent, establish a rule to check convergence of the decision rules  $(f_i^w, f_i^\pi, f_i^\tau)$  and  $(f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau)$  as follows:

- (a) if  $\max\{|(f_i^w, f_i^\pi, f_i^\tau) - (f_{i-1}^w, f_{i-1}^\pi, f_{i-1}^\tau)| > 1e - 6\}$ , go back to step 2;
- (b) otherwise,  $(f_i^w, f_i^\pi, f_i^\tau)$  are the decision rules.