Intra-industry trade, involuntary unemployment and indeterminacy

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Abstract

We study the impact of the opening to international trade of differentiated products (and of capital services) on steady state welfare and the local stability properties of two trading countries, who differ with respect to labor market characteristics and possibly with respect to taste for variety. We consider an overlapping generations model, where technologies of each differentiated product are identical across the world and exhibit increasing returns to scale. There is taste for variety and monopolistic competition in the output market, while the capital services market is perfectly competitive. In one country there is efficiency wages, whereas in the other there is full employment. Under autarky the full employment country is locally stable, but may suffer from indeterminacy when trade and capital movements are liberalized.

Keywords: Indeterminacy, Trade, Taste for variety, involuntary unemployment

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1 Introduction

The present work investigates whether free-trade in differentiated goods and the liberalization of capital movements between two countries (that differ in their labour market characteristics and may have different degrees of taste for variety), may stabilize/destabilize economies with respect to endogenous fluctuations, i.e., fluctuations that are driven by autonomous volatile changes in expectations. As shown by Woodford (1986), the existence of stochastic endogenous fluctuations (sunspot equilibria) is related to the indeterminacy of the equilibrium under perfect foresight, i.e. the existence of a continuum of equilibrium paths converging towards one steady state from the same initial value of the state variable. Although several studies have analyzed the link of free trade and capital mobility with indeterminacy and macroeconomic volatility, the macroeconomic literature has not yet addressed the implications of (intra-industry) trade between differentiated goods of a same industry. Our work aims at filling this gap.

The literature demonstrates that opening to inter-industry trade may have different impacts on the stability properties of trading countries. They may be classified into two subsets. In a first one, we find the contributions based on a two-country version of Benhabib and Nishimura (1985) who study the existence of local indeterminacy in a closed two-sector (consumption and investment) infinitely lived agent model with sector-specific externalities and social constant returns with international immobility of factors.[1] Nishimura and Shimomura (2002) consider a model where countries only differ with respect to their initial factor endowments. They show that international trade has no effect on the stability properties of the two countries.[2] By contrast, Sim and Ho (2007) consider that the technology are different across countries.[3] They prove that the world economy is characterized by saddle-point stability even if before trade one country exhibits sunspot fluctuations. Finally, Hu and Mino (2013) consider different trade structure with lending and borrowing, and show that international trade produces endogenous cycles in both countries, even if before trade sunspot

[1] They show that sunspot fluctuations arise provided that the investment good sector is more capital intensive than the consumption good section from the social perspective and less capital intensive from the private perspective, and that the elasticity of intertemporal substitution in consumption is large enough.

[2] Iwasa and Nishimura (2014) extend Nishimura and Shimomura (2002) by introducing a consumption capital good. They show that international trade can create sunspot fluctuations in the world economy.

[3] One country is characterized by sector-specific externality and the other country is not.

cycles do not emerge in the two countries. The second subset of models contains the contributions which deal with international capital mobility and international labor immobility. In a first paper, Nishimura *et al.* (2010) consider an infinitely-lived agent model with asymmetric technologies across countries and sector-specific externalities. They show that trade creates a contagion of sunspot cycles from one country to another. In a second paper, Nishimura *et al.* (2014) consider an infinitely-lived agent model with asymmetric technologies across countries and Cobb-Douglas decreasing returns to scale technologies. They analyze the existence of flip bifurcation and deterministic cycles and prove that the destabilizing effect of international trade and international capital mobility arises under certain parameter configurations. In other words, the opening of trade may create persistent endogenous fluctuations at the world level while the closed-economy equilibrium in each country is saddle-point stable. All of these papers assumed inter-industry trade, but nowadays a big percentage of trade between developed countries is trade of goods that belong to same industry.

In this paper, we focus on another possible feature of trade, i.e. intra-industry, by allowing differences in taste for variety across countries. We consider a two-country two-period overlapping generations model with imperfect competition a la Dixit and Stiglitz (1977) and increasing returns to scale due to fixed cost. This model is characterized by a constant markup and the number of varieties is endogenous and procyclical. Our model extends the closed economy framework with taset for variety developed in Seegmuller (2008) and the two country model considered in Aloi and Lloyd-Braga (2010) where one country has efficiency wages and involuntary unemployment. The novelty of our work is the consideration of intra-industry trade across two countries exhibiting different labour market characteristics and that may differ in their taste for variety. We first analyse autarkic equilibria in each of the two countries and then consider equilibria with trade and capital mobility between the two countries two levels of integration. We study the corresponding changes in steady state welfare and in the local stability properties when economies change from an autarkic environment to free trade and capital mobility.

This paper is organized as follows. Section 2 describes an economy in the autarky regime while Section 3 introduces the analysis of the local dynamics of the closed economy. Section 4 provides the analysis of the two-country model and the pattern of trade while Section 5 introduces the analysis of the local dynamics of the two-country model. Section 7 contains the concluding remarks and the proofs are gathered in the Appendix.

2 Autarky

The world consists of two countries, A and B, that share the same production structure. In both countries, the output market is subjected to monopolistic competition while the capital market is perfectly competitive. Households live for two periods and consume at each period a composite good. This composite good represents an aggregate consumption of all the differentiated goods. Agents supply one unit of effort when young. We consider that one economy is characterized by involuntray unemployment with efficiency wages and the other by full employment and perfectly competitive wages.

2.1 The model

We consider an infinite horizon discrete time economy populated populated by overlapping generations of agents living for two periods and firms. In the sequel, we will describe the household behavior and the technology available to the firms each firm produces a diferent variety of the good (i.e., a differentiated product).

2.1.1 Preferences

The two countries A and B have different labour markets characteristics and may differ with respect to their taste for variety. In country A the labour market is chacaterized by efficiency wages and involuntray unemployment and in country B there is full employmnet with perfectly competitive wages.

We assume an infinite horizon discrete time economy populated by overlapping generations of agents living for two periods: in the first one they are young, in the second one they are old. Population is constant over time, and in each period H individuals are born. In the first period, a young employed agent (that does not shirk and offers a unit of effort) receives a wage income, invest this income in capital K_{t+1} and consume the composite good C_t. In the second period, old agents are retired and purchase the composite good D_{t+1}. Agents have preferences defined over consumption in the first period of life C_t, consumption in the second period of life D_{t+1} and young age effort. A young agent born at period t solves the following dynamic program

$$\begin{array}{cl} \max\limits_{C_t, D_{t+1}, \tilde{K}_{t+1}} & C_t^{\alpha} D_{t+1}^{1-\alpha} - \nu e_t \\ s.t. & P_t C_t + P_t \tilde{K}_{t+1} = w_t \tilde{l}_t \\ & P_{t+1} D_{t+1} = r_{t+1} \tilde{K}_{t+1} \\ & C_t, D_{t+1}, \tilde{K}_{t+1} \geq 0, \end{array} \tag{1}$$

with $\nu > 0$, $\alpha \in (0, 1)$ and $e_t \in \{0, 1\}$. α is the propensity to consume of young agent, $e_t \in \{0, 1\}$ represents the effort supplied by agent, P_t is the price of the aggregate consumption good, r_{t+1} the nominal rental rate of capital and w_t the nominal wage.

The composite good C_t and of the composite good D_{t+1}.are represented by the aggregate of all varieties i

$$\begin{array}{l} C_t = (N_t)^{1+\beta} \left[\frac{1}{N_t} \sum_0^{N_t} c_{it}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \\ D_{t+1} = (N_{t+1})^{1+\beta} \left[\frac{1}{N_{t+1}} \sum_0^{N_t} d_{it+1}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \end{array} \tag{2}$$

where $\beta \geq 0$ denotes the share of varieties and $\varepsilon > 0$ is the intratemporal elasticity of substitution between varieties. Note that, when $\varepsilon > 1$, the differentiated goods are substitutes while when $\varepsilon < 1$ the differentiated goods are complementaries. Then, we will assume the following

Assumption 1. *The differentiated goods are substitutes, i.e.* $\varepsilon > 1$.

As in Benassy (1996), we define the function $t(N)$, which represents the gain from consuming one unit of N differentiated goods instead of consuming N units of a single variety. It follows from (2), $t(N)$ is defined by $C/N = N^{\beta}$ and taste for variety is defined by the elasticity of $t(N)$:

$$\tau(N) \equiv \frac{N t'(N)}{t(N)} = \beta \tag{3}$$

If β is set to zero, households have no taste for variety while if β is higher than zero, households have taste for variety. In this framework, the taste for variety, β, is independent of the intratemporal elasticity of substitution between varieties, ε. Each agent chooses consumption of the good i

$$\begin{array}{l} c_{it} = (N_t)^{\beta(\varepsilon-1)-1} \left(\frac{p_{it}}{P_t} \right)^{-\varepsilon} C_t, \\ d_{it+1} = (N_{t+1})^{\beta(\varepsilon-1)-1} \left(\frac{p_{it+1}}{P_{t+1}} \right)^{-\varepsilon} D_{t+1} \end{array} \tag{4}$$

$$P_t = \frac{1}{N_t^{\beta}} \left[\frac{1}{N_t} \sum_0^{N_t} p_{it}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, \forall t \tag{5}$$

where p_{it} is the price of a variety i.

Using the budget constraint, and defining the real wage $\omega_t \equiv \frac{w_t}{P_t}$, and the real interest rate $\rho_t \equiv \frac{r_t}{P_t}$ the first-order conditions are

$$C_t = \alpha \omega_t, D_{t+1} = (1-\alpha)\omega_t \rho_{t+1}, \tag{6}$$

Plugging the demand for the composite good C_t and D_{t+1} into the utility function defined by (1) yields to the indirect utility

$$V(\omega_t, \rho_{t+1}) = \alpha^{\alpha}(1-\alpha)^{1-\alpha} \rho_{t+1}^{1-\alpha} \left(\frac{w_t}{P_t} - \nu e_t \right) \tag{7}$$

In the case of a non shirking employed worker the indirect utility function becomes

$$V(\omega_t, \rho_{t+1}) = \alpha^{\alpha}(1-\alpha)^{1-\alpha} \rho_{t+1}^{1-\alpha} \left(\frac{w_t}{P_t} - \bar{\omega}_t \right) \tag{8}$$

with

$$\bar{\omega}_{A,t} \equiv \frac{\nu}{\alpha^{\alpha}(1-\alpha)^{1-\alpha} \rho_{A,t+1}^{1-\alpha}}$$

representing the real reservation wage.

2.1.2 Technologies

We consider that the good market is subject to monopolistic competition as in Dixit and Stiglitz (1977). In each period $t = 1, ..., \infty$, entry and exist are free and the zero profit conditions determine the number of firms. Furthermore, each firm produces one variety i of output using the following technology

$$y_{it} = \Theta \left[f\left(a_{it}\right) l_{it} - \phi \right] \tag{9}$$

where $\Theta > 0$ is a scaling parameter, $a_{it} = k_{it}/l_{it}$ the capital-labor ratio used by firm i and $\phi > 0$ a fixed cost. The technologies satisfy the following

$$f(a_{it}) = a_{it}^{s}$$

where s is the share of capital income.

The marginal cost is constant and the increasing returns are only due to the existence of the fixed cost ϕ. Aggregate production $P_t Y_t$ at period t is shared between consumption of young agent C_t, consumption of old agent D_t and investment I_t

$$P_t Y_t = \sum_0^{N_t} p_{it} y_{it} \ = \left(C_t + D_t + I_t\right) P_t. \tag{10}$$

We suppose full depreciation of capital and thus capital equals investment: $I_t = K_{t+1}$, and assume that I_t is defined by the same index of varieties than consumption

$$I_t = N_t^{1+\beta} \left[\frac{1}{N_t} \sum_0^{N_t} i_{it}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{11}$$

where i_{it} is the investment demand in variety of good i. i_{it} is optimally determined by

$$i_{it} = N_t^{\beta(\varepsilon-1)-1} \left(\frac{p_{it}}{P_t}\right)^{-\varepsilon} I_t. \tag{12}$$

It follows that the demand for a variety i is defined by $\upsilon_{it} = c_{it} + d_{it} + i_{it}$, where (c_{it}, d_{it}) and i_{it} are respectively given by (4) and (12). Then υ_{it} is given by

$$\upsilon_{it} = N_t^{\beta(\varepsilon-1)-1} \left(\frac{p_{it}}{P_t}\right)^{-\varepsilon} \left(C_t + I_t + D_t\right). \tag{13}$$

At each period t, each producer maximizes profits, i.e. $p_{it}y_{it} - r_t k_{it} - w_t l_{it}$, facing the demand function υ_t. The first order conditions are then

In country A we consider that labour market is characterized by involuntary unemployment due to the existence of effciency wages. Workers may shirk and in this case $e_{A,t} = 0$. Firms may caught workers who are shirking with the (ex-ante) probability $0 < \lambda < 1$. In case workers are caught shirking, they are fired without receiving any wage and they get zero utility. A young agent faces three possibilities: being unemployed, being employed and not shirking or being employed and shirking ($e_{A,t} = 0$, $w_{A,t} > 0$). Using () and (), the indirect utility of an employed worker that shirks is $V = (1-\lambda)\alpha^{\alpha}(1-\alpha)^{1-\alpha}\rho_{A,t+1}^{1-\alpha}\omega_{A,t}$. Using () we obtain that employed workers do not shirk under the Non Shirking Condition $\omega_{A,t} \geq \frac{\bar{\omega}_{A,t}}{\lambda}$. Since output of workers that shirk is zero, firms, when choosing w_A, l_A and k_A, y_{it} and p_{it} in order to maximize profits, take into account the Non Shirking Condition. Hence, the problem solved by firms is

$\underset{w_{At}, l_{At}, k_{At} \in \Re^3_{++}}{Max} \left(p_{it}y_{it} - w_{A,t}l_{A,t} - r_{A,t}k_{A,t}\right)$, $\quad s.t. w_{A,t} \geq \frac{\bar{\omega}_{A,t}}{\lambda} P_t$, facing the production function (9), the demand function (13) and considering $y_{it} = \upsilon_{it}$

The first order conditions are then given by (9), (13) and

$$\begin{array}{l} r_{A,t} = \Theta p_{it}\left(\frac{\varepsilon-1}{\varepsilon}\right) s a_{A,it}^{s-1}, \\ w_{A,t} = \Theta p_{it}\left(\frac{\varepsilon-1}{\varepsilon}\right)(1-s) a_{A,it}^{s}. \\ w_{A,t} = \frac{\bar{\omega}_{A,t}}{\lambda} P_t \end{array} \tag{14}$$

At the symmetric equilibrium, it holds that $l_{it} = l_t$, $k_{it} = k_t$, $a_{it} = a_t$ and $p_{it} = p_t$ for all firms i. Then, the real interest rate ρ_t, the real wage ω_t and the aggregate price P_t are given by

$$\begin{array}{l} \rho_{A,t} = \Theta N_{A,t}^{\beta}\left(\frac{\varepsilon-1}{\varepsilon}\right) s a_t^{s-1}, \\ \omega_{A,t} = \Theta N_{A,t}^{\beta}\left(\frac{\varepsilon-1}{\varepsilon}\right)(1-s) a_{A,t}^{s}, \\ P_{A,t} = N_{A,t}^{-\beta} p_{A,t}. \end{array} \tag{15}$$

Note that from (15), the markup factor is constant and given by $\varepsilon/(\varepsilon-1)$. This is the case because the elasticity of the demand for a variety i with respect to price is equal to ε. Note also that, when $\beta = 0$ the aggregate price P_t is equal to p_t at the symmetric equilibrium, meanwhile if $\beta > 0$ the aggregate price P_t decreases with the number of variety. Then, the real interest rate ρ_t and the real wage ω_t increase with respect to the number of variety N_t. The free-entry condition is determined by the zero profit condition, i.e. $p_t y_t - k_t r_t - l_t w_t = 0$, and given by

$$\frac{a_t^s l_t}{\varepsilon} = \phi. \tag{16}$$

From (9) and (16), we derive that the production level of each firm is constant at equilibrium: $y_t = \Theta[a_t^s l_t - \phi] = \Theta\phi(\varepsilon - 1)$.

2.2 Equilibrium

In this Section, we describe the competitive equilibrium of each country in autarky. We will assume that agents in country A are subjected to involuntary unemployment meanwhile agents in country B are all employed.

2.2.1 Country A: Involuntary unemployment

At intertemporal equilibrium all markets clear in each period t. Aggregate labor is given by $L_t^A = H^A \tilde{l}_t^A = N_t^A l_t^A$. Aggregate capital is determined by saving $K_t^A = H^A \tilde{K}_t^A = N_t^A k_t^A$. Using these market equilibrium conditions with the free-entry condition (16) and the fact that at the symmetric equilibrium $L_t^A = K_t^A / a_t^A$, we derive the number of variety (firm) N_t^A

$$N_t^A = \frac{\left(a_t^A\right)^{s-1} K_t^A}{\varepsilon\phi} \equiv N^A(a_t^A, K_t^A). \tag{17}$$

From the previous equation and equation (15), the real interest rate and the real wage can be written as function of K_t^A and a_t^A

$$\begin{array}{l}\rho_t = \Theta N^A(a_t^A, K_t^A)^{\beta^A}\left(\frac{\varepsilon-1}{\varepsilon}\right) s \left(a_t^A\right)^{s-1} \equiv \rho(a_t^A, K_t^A),\\ \omega_t = \Theta N^A(a_t^A, K_t^A)^{\beta^A}\left(\frac{\varepsilon-1}{\varepsilon}\right)(1-s)\left(a_t^A\right)^{s} \equiv \omega(a_t^A, K_t^A).\end{array} \tag{18}$$

Then an intertemporal equilibrium with perfect foresight is as sequence $\{a_t^A, K_t^A\}_{t=0}^{\infty}$, where the initial capital stock $K_{t=0}^A > 0$ is given, that satisfy the capital accumulation equation and the consumption-labor choice

$$K_{t+1}^A = (1-\alpha)\,\omega(a_t^A, K_t^A)\frac{K_t^A}{a_t^A}, \tag{19}$$

$$(1-\alpha)^{1-\alpha}\alpha^{\alpha}\omega(a_t^A, K_t^A)\rho(a_{t+1}^A, K_{t+1}^A)^{1-\alpha} = \frac{\nu}{\lambda}. \tag{20}$$

The dynamical system defined by (19)-(20) is a two-dimensional dynamical system with one predetermined variable, the aggregate capital.

2.2.2 Country B: Full employment

In country B, we consider that full employemnt exists and that the labour market is perfectly competitive ($L^B = H$). It follows that the number of variety is only function of the capital stock K^B

$$N_t^B = \frac{f(\frac{K_t^B}{L^B})L^B}{\varepsilon\phi} \equiv N^B(K_t^B). \tag{21}$$

We get also the real interest rate and the real wage can be written as function of K_t^B

$$\begin{array}{l}\rho_t = \Theta N^B(K_t^B)^{\beta^B}\left(\frac{\varepsilon-1}{\varepsilon}\right) s \left(\frac{K_t^B}{L^B}\right)^{s-1} \equiv \rho(K_t^B),\\ \omega_t = \Theta N^B(K_t^B)^{\beta^B}\left(\frac{\varepsilon-1}{\varepsilon}\right)(1-s)\left(\frac{K_t^B}{L^B}\right)^{s} \equiv \omega(K_t^B).\end{array} \tag{22}$$

Then an intertemporal equilibrium with perfect foresight is as sequence $\{K_t^B\}_{t=0}^{\infty}$, given the initial capital stock $K_{t=0}^B > 0$, that satisfy the capital accumulation equation, i.e. a first-order difference equation

$$K_{t+1}^B = (1-\alpha)\,\omega(K_t^B)L^B. \tag{23}$$

2.3 Steady State

2.3.1 Country A: involuntary unemployment

A steady state of the dynamic system (19)-(20) is a solution $(a^A, K^A) = (a_t^A, K_t^A)$ for all t, such that

$$a^A = (1-\alpha)\,\omega(a^A, K^A), \tag{24}$$

$$(1-\alpha)^{1-\alpha}\alpha^\alpha\omega(a^A, K^A)\rho(a^A, K^A)^{1-\alpha} = \tfrac{\nu}{\lambda}. \tag{25}$$

Proposition 1. *Under Assumption 1, there exists a unique stationary solution (a, K^A) of the dynamic system (19)-(20). The value of (a, K^A) is given by*

$$a = \left(\tfrac{\nu}{\lambda}\right)\tfrac{(1-\alpha)(1-s)^{1-\alpha}}{\alpha^\alpha s^{1-\alpha}} \tag{26}$$

$$K^A = \left[\frac{a^{(1-s)(1+\beta^A)}}{(1-\alpha)\Theta\left(\frac{\varepsilon-1}{\varepsilon}\right)(1-s)\left(\frac{1}{\varepsilon\phi}\right)^{\beta^A}}\right]^{\frac{1}{\beta^A}} \tag{27}$$

2.3.2 Country B: Full Employment

A steady state of the dynamic system (23) is a solution $K^B = K_t^B = K_{t+1}^B$ for all t, such that

$$K^B = (1-\alpha)\,\omega(K^B)L^B \tag{28}$$

3 Local analysis in autarky

For the purpose of our study, it is useful to introduce at this stage of the paper the following elasticities, the elasticity of the intensive production function $s(a)$ and the elasticity of capital-labor substitution $\sigma(a)$

$$s = \frac{f'(a)a}{f(a)}, \sigma = -\frac{f'(a)\left(1-\frac{f'(a)a}{f(a)}\right)}{f''(a)a}. \tag{29}$$

From these two elasticities we deduce the following relationship

$$\frac{f''(a)a}{f'(a)} = -\frac{1-s}{\sigma}. \tag{30}$$

Let us denote $s(a) = s$ and $\sigma(a) = \sigma$. We will assume, through the paper, that the elasticity of the intensive production function, evaluated at the steady state is less than one half.

Assumption 2. $1/4 < s < 1/2$.

Taking into account the first-order condition of the producer (14) and the free-entry condition (16), we get that s represents the share of capital in the economy, which takes a value limited by those of Cecchi and Garcia-Peñalosa (2010).

3.1 Country A: Involuntary Unemployment

In the system describing intertemporal equilibrium, there is one pre-determinate variable, the initial stock of capital. In such a configuration, the existence of local indeterminacy requires that the two characteristic roots associated with the linearization of the dynamic system defined by (19)-(20) around the normalized steady state have modulus less than one. In the opposite case the steady state is locally determinate.

Loglinearizing (19)-(20) and denoting percentage deviations from the steady state of K^A and a^A by $\widehat{K}_t^A \equiv \left(K_t^A - K^A\right)/K^A$ and $\widehat{a}_t^A \equiv \left(a_t^A - a^A\right)/a^A$, we derive the following

$$\begin{pmatrix} \widehat{K}_{t+1}^A \\ \widehat{a}_{t+1}^A \end{pmatrix} = \begin{pmatrix} (1+\beta^A) & -(1+\beta^A) + s\left(1+\beta^A\right) \\ \frac{\left[1+(1+\beta^A)(1-\alpha)\right]\beta^A}{(1-\alpha)(1-s)(1+\beta^A)} & \frac{1+\left[1+(1-\alpha)\beta^A\right]\left[s\left(1+\beta^A\right)-(1+\beta^A)\right]}{(1-\alpha)(1-s)(1+\beta^A)} \end{pmatrix} \begin{pmatrix} \widehat{K}_t \\ \widehat{a}_t \end{pmatrix} \tag{31}$$

The purpose of our analysis is to study local indeterminacy near a (normalized) steady state of (19)-(20). Then, we study the eigenvalues of the previous log-linearized system and determine the characteristic polynomial $P(\lambda) = \lambda^2 - \lambda T + D$ where T is the trace and is D the determinant of the associated Jacobian matrix. Then the following Proposition holds

Proposition 2. *Under Assumption 1, the characteristic polynomial is defined by $P(\lambda) = \lambda^2 - \lambda T + D$ where the trace T and the determinant D are given by*

$$T = \frac{1-(1-s)(1+\beta^A)\alpha}{(1-\alpha)(1-s)(1+\beta^A)}, D = \frac{s}{(1-\alpha)(1-s)}. \tag{32}$$

Proof: See Appendix 8.1.

In order to determine whether if the steady state is locally determinate or locally indeterminate, we need to determine under which conditions $D = 1$ and to analyze the sign of $1 - T + D$ and $1 + T + D$.

Using the determinant defined in (32), we get that $D \leq 1$ if and only if $\alpha \leq (1-2s)/(1-s) \equiv \tilde{\alpha}$. Furthermore, from (32), we get that:

$$\begin{array}{l} 1 - T + D = \frac{\beta^A}{(1-\alpha)(1-s)(1+\beta^A)} > 0, \\ 1 + T + D = \frac{[1-2\alpha(1-s)]\beta^A + 2[1-\alpha(1-s)]}{(1-\alpha)(1-s)(1+\beta^A)} \gtreqless 0. \end{array} \tag{33}$$

Let us define $\hat{\beta}^A$ one critical value of the taste for variety and $\hat{\alpha}$ one critical value of the propensity to consume that jointly determine the sign of $1 + T + D = 0$:

$$\tilde{\beta}^A = \frac{2[1-\alpha(1-s)]}{2\alpha(1-s)-1}, \;\; \hat{\alpha} = \frac{1}{2(1-s)}. \tag{34}$$

It follows, then, that if $\alpha < \hat{\alpha}$, $1 + T + D = 0 > 0$. On the contrary, when $\alpha > \hat{\alpha}$, $1 + T + D = 0 > 0$ for $\beta^A < \tilde{\beta}^A$ and $1 + T + D = 0 < 0$ for $\beta^A > \tilde{\beta}^A$. (Note that $\tilde{\beta}^A > 1$). Moreover under Assumption 2, it holds that $\hat{\alpha} > \tilde{\alpha}$. We then get the follopwing Proposition.

Proposition 3 *Under Assumptions 1 and 2, the following results hold*

i⌋ Let $\alpha < (1-2s)/(1-s)$ then the steady state is a sink;

ii⌋ Let $1/2(1-s) > \alpha > (1-2s)/(1-s)$ then the steady state is a source;

iii⌋ Let $\alpha > 1/2(1-s)$, then the steady state is a source for $\beta^A < \tilde{\beta}^A$ and then becomes a saddle for $\beta^A > \tilde{\beta}^A$.

3.2 Country B: Full Employment

In this case, the equilibrium is only determined by (23). By totally differentiate this equation with respect to K_t^B and K_{t+1}^B, we derive

$$\frac{dK_{t+1}^B}{dK_t^B} = s\left(1 + \beta^B\right). \tag{35}$$

Then the following proposition is immediately proved

Proposition 4. *The steady state of country B is satble when $\beta^B < \frac{1-s}{s}$.*

4 The Two-Country Model

In the Section, we describe the trade structure of the two-country model, we define the intertemporal equilibrium, the pattern of trade and analyze the steady state.

4.1 Trade Structure

We consider a economy composed of two countries, i.e. A and B, identical in every aspect except in their taste for variety and their labor market. Indeed, consumer tastes depend on customs, culture etc. and therefore taste for variety may vary across countries.[4] We suppose that capital is mobile across countries, i.e. $r_t^A = r_t^B$, while labor is internationally immobile across countries , i.e. $w_t^A \neq w_t^B$. Furthermore, goods are freely trade such that $p_{it}^A = p_{it}^B$. Let us define N_t^W the number of varieties in the world and K_t^W the capital stock in the world, which are given respectively by $N_t^W = N_t^A + N_t^B$ and $K_t^W = K_t^A + K_t^B$. In the open economy, the composite good C_t^j and D_{t+1}^j and the investment I_t^j in country $j \in \{A, B\}$ are defined over all varieties in the world N_t^W

$$\begin{aligned} C_t^j &= (N_t^W)^{1+\beta^j} \left[\frac{1}{N_t^W} \sum_0^{N_t^W} (c^j)_{it}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \\ D_{t+1}^j &= (N_{t+1}^W)^{1+\beta^j} \left[\frac{1}{N_{t+1}^W} \sum_0^{N_{t+1}^W} (d^j)_{it+1}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \end{aligned} \tag{36}$$

$$I_t^j = (N_t^W)^{1+\beta^j} \left[\frac{1}{N_t^W} \sum_0^{N_t^W} (i^j)_{it}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{37}$$

It follows that the demand of variety at the world level is now given by

$$\upsilon_{it}^W = (N_t^W)^{\beta^A(\varepsilon-1)-1} \left(\frac{p_{it}}{P_t}\right)^{-\varepsilon} \left(C_t^A + D_t^A + I_t^A\right) + (N_t^W)^{\beta^B(\varepsilon-1)-1} \left(\frac{p_{it}}{P_t}\right)^{-\varepsilon} \left(C_t^B + D_t^B + I_t^B\right). \tag{38}$$

The first-order conditions of the producer given by (14) with the fact that there exist free-trade $p_{it}^A = p_{it}^B$ and international capital mobility yields to $a_{it}^A = a_{it}^B = a_{it}$. It follows that the nominal wage are the same across. The real price at the world level are in country $j \in \{A, B\}$ defined over all varieties, then from (14) we get

$$\begin{aligned} \rho_t^{j,W} &= \Theta (N_t^W)^{\beta^j} \left(\frac{\varepsilon-1}{\varepsilon}\right) s \left(a_{it}^j\right)^{s-1}, \\ \omega_t^{j,W} &= \Theta (N_t^W)^{\beta^j} \left(\frac{\varepsilon-1}{\varepsilon}\right) (1-s) \left(a_{it}^j\right)^{s}. \end{aligned} \tag{39}$$

At the symmetric equilibrium it holds that $a_{it} = a_t$. We, then obtain the following relationship

[4]Taste for variety is an important determinant of trade flows in the European car industry. See for instance Auer (2013).

$$a_t = \frac{K_t^A}{L_t^A} = \frac{K_t^B}{L_t^B}. \tag{40}$$

The free-entry condition in each country is given by (16). Taking into account that $N_t^W = N_t^A + N_t^B$ and that the capital-labor ratio a is the same across country, we obtain that N_t^W is a function of K_t^W and a_t

$$N_t^W = \frac{a_t^{s-1} K_t^W}{\varepsilon \phi} \equiv N\left(a_t, K_t^W\right). \tag{41}$$

Finally, with the help of (40) and (41) the real wage and real interest are function K_t^W and a_t which can be written as $\rho_t^{j,W} = \rho^{j,W}(a_t, K_t^W)$ and $\omega_t^{j,W} = \omega^{j,W}(a_t, K_t^W)$.

4.2 Equilibrium

The world equilibrium is given by the capital accumulation that prevail under capital mobility and the intertemporal choice between consumption and labor in country A. The capital accumulation under capital mobility is

$$K_{t+1}^W = (1-\alpha)\left[\omega^{A,W}\left(a_t, K_t^W\right) L_t^A + \omega^{B,W}\left(a_t, K_t^W\right) L^B\right] \tag{42}$$

and the intertemporal choice between consumption and labor in country A is

$$(1-\alpha)^{1-\alpha}\alpha^\alpha \omega^{A,W}(a_t, K_t^W)\rho^{A,W}(a_{t+1}, K_{t+1}^W)^{1-\alpha} = \frac{\nu}{\lambda}. \tag{43}$$

Let us now express equations (42) and (43) in terms of a and K^W. Combining the world capital resource constraint $K_t^W = K_t^A + K_t^B$ and the capital-labor ratio $a_t = K_t^A/L_t^A = K_t^B/L^B$, we obtain that $K_t^A = K_t^W - a_t L^B$. It follows that

$$K_{t+1}^W = (1-\alpha)\left[\omega^{A,W}\left(a_t, K_t^W\right)\left(\frac{K_t^W - a_t L^B}{a_t}\right) + \omega^{B,W}\left(a_t, K_t^W\right) L^B\right], \tag{44}$$

and

$$(1-\alpha)^{1-\alpha}\alpha^\alpha \omega^{A,W}(a_t, K_t^W)\rho^{A,W}(a_{t+1}, K_{t+1}^W)^{1-\alpha} = \frac{\nu}{\lambda}. \tag{45}$$

An intertemporal equilibrium with perfect foresight is, then, a sequence $\left\{a_t, K_t^W\right\}_{t=0}^{\infty}$, given the initial capital stock $K_{t=0}^W > 0$, satisfying (44)-(45). System (44)-(45) defines a two dimensional dynamical system with one predetermined variable, the world aggregate capital.

4.3 Steady state

A steady state is defined as a sequence $\{a_t, K_t^W\}_{t=0}^{\infty} = (a, K^W)$ for all t satisfying

$$K^W = (1-\alpha)\left[\omega^{A,W}\left(a, K^W\right)\left(\tfrac{K^W - aL^B}{a}\right) + \omega^{B,W}\left(a, K^W\right)L^B\right], \tag{46}$$

and

$$(1-\alpha)^{1-\alpha}\alpha^{\alpha}\omega^{A,W}(a, K^W)\rho^{A,W}(a, K^W)^{1-\alpha} = \tfrac{\nu}{\lambda}. \tag{47}$$

Proposition 5. *Under Assumption 1*

When $\beta^A = \beta^B = \beta$, the steady state value of K^W and a are the same as the one of country A under autarky.

When $\beta^A \neq \beta^B$, the steady state value of a is different from that of autarky. Let χ^A and z^A be defined by (48) and (49). Then, the steady state values of a and K^W are given as follows:

$$a = \frac{(1-\alpha)(1-s)^{1-\alpha}}{\alpha^{\alpha}s^{1-\alpha}}\left(\frac{\chi^A}{z^A}\right)^{2-\alpha}$$

$$K^W = \left[\frac{\frac{z^A}{\chi^A}a^{(1-s)(1+\beta^A)}}{(1-\alpha)\Theta\left(\frac{\varepsilon-1}{\varepsilon}\right)(1-s)\left(\frac{1}{\varepsilon\phi}\right)^{\beta^A}}\right]^{\frac{1}{\beta^A}}$$

4.4 Pattern of Trade at the steady state

Let us denote z^j the share of country $j \in \{A, B\}$ in aggregate saving

$$z^A = \tfrac{\omega^A L^A}{\omega^{A,W}L^A + \omega^{B,W}L^B}, \;\; z^A + z^B = 1, \tag{48}$$

and χ^j the share of capital in country $j \in \{A, B\}$

$$\chi^A = \tfrac{K^A}{K^W}, \;\; \chi^A + \chi^B = 1. \tag{49}$$

When $z^A > z^B$, agents in country A save more than agents in country B meanwhile if $\chi^A < \chi^B$ capital uses in productiuon by country A holds is lower than the one uses in country B.

Taking the ratio of z^A/z^B gives:

$$\frac{z^A}{\chi^A} = \left(N^W\right)^{\beta^A - \beta^B} \frac{z^B}{\chi^B} \tag{50}$$

If $\beta^A > \beta^B$ we get that $z^A > \chi^A$. In that case, country A is a net importer of goods while country B is net importer of capital. On the contrary, when $\beta^A < \beta^B$ we get that $z^A < \chi^A$ implying that country A is a net importer of capital while country B is net importer of goods.

When $\beta^A = \beta^B$, each country produces in the long-run the amount of capital required to produce the consumption goods ($z^j = \chi^j$). In this case, international trade may occur during the dynamics transition but the long-run equilibrium is characterized with no net trade.

5 Local Dynamics in the Two-Country model

Let us denote $\beta_z^W = \beta^A z^A + \beta^B z^B$. Loglinearizing (44)-(45) and denoting percentage deviations from the steady state of K^W and a by $\widehat{K}_t^W \equiv \left(K_t^W - K^W\right)/K^W$ and $\widehat{a}_t \equiv (a_t - a)/a$.

$$\begin{pmatrix} \widehat{K}_{t+1}^W \\ \widehat{a}_{t+1} \end{pmatrix} = \begin{pmatrix} \frac{z^A}{\chi^A} + \beta_z^W & s\left(1+\beta_z^W\right) - \left(\frac{z^A}{\chi^A} + \beta_z^W\right) \\ \frac{\left[1+\left(\beta_z^W + \frac{z^A}{\chi^A}\right)(1-\alpha)\right]\beta^A}{(1-\alpha)(1-s)(1+\beta^A)} & \frac{s\left(1+\beta_z^W\right)(1-\alpha)\beta^A + s\left(1+\beta^A\right) - \beta^A\left[1+(1-\alpha)\left(\frac{z^A}{\chi^A}+\beta_z^W\right)\right]}{(1-\alpha)(1-s)(1+\beta^A)} \end{pmatrix} \begin{pmatrix} \widehat{K}_t^W \\ \widehat{a}_t \end{pmatrix} \tag{51}$$

The purpose of our analysis is to study local indeterminacy near a (normalized) steady state of (44)-(45). Then, we study the eigenvalues of the previous log-linearized system and determine the characteristic polynomial $P^W(\lambda^W) = \left(\lambda^W\right)^2 - \lambda^W T^W + D^W$ where T^W is the trace and is D^W the determinant of the associated Jacobian matrix. Then the following Proposition holds

Proposition 6. *Under Assumption 1, the characteristic polynomial is defined by* $P^W(\lambda^W) = \left(\lambda^W\right)^2 - \lambda^W T^W + D^W$ *with the trace* T^W *and the determinant* D^W*:*

$$T^W = \frac{s(1+\beta^A) + (1-\alpha)(1-s)\left(\frac{z^A}{\chi^A} + \beta_z^W\right) - \beta^A\left[1 + (1-\alpha)\left(\frac{z^A}{\chi^A} - 1\right)s\right]}{(1-\alpha)(1-s)\left(1+\beta^A\right)} \tag{52}$$

$$D^W = \frac{s\left[1 + \beta_z^W + (1+\beta^A)\left(\frac{z^A}{\chi^A} - 1\right)\right]}{(1-\alpha)(1-s)\left(1+\beta^A\right)} \tag{53}$$

Proof: See Appendix 8.2.

In the following, we will analyze first the case where there is no difference in taste for variety between countries and then consider the case where taste differs across country.

5.1 Case $\beta^A = \beta^B = \beta$

In the case in which the taste for variety is thr same across the world, i.e. $\beta^A = \beta^B = \beta$, it holds that $z^A/\chi^A = 1$. It follows that in this configuration, the local stability properties of the two-country model are the same from those of country A in autarky since the trace and the determinant correspond to (32). It follows that country B will import the local stability properties of country A. Then, country B might become locally indeterminate after opening to trade with country A. According to Proposition 3 and Proposition 4, Country B will exhibit local indeterminacy when $\alpha < \tilde{\alpha}$.

5.2 Case $\beta^A \neq \beta^B$

As in autarky in order to determine whether if the steady state is locally determinate or locally indeterminate, we need to determine under which conditions $D^W = 1$ and to analyze the sign of $1-T^W+D^W$ and $1+T^W+D^W$.

$$1 - T^W + D^W = \frac{[1-2s-\alpha(1-s)]\left(1 - \frac{z^A}{\chi^A} - \beta_z^W\right) + (2-\alpha)\left(1 - 2s + \frac{z^A}{\chi^A}s\right)\beta^A}{(1-\alpha)(1-s)\left(1+\beta^A\right)} \gtreqless 0 \tag{54}$$

$1 - T^W + D^W \gtreqless 0$ if $\frac{z^A}{\chi^A} \lesseqgtr \tilde{g}$
with $\tilde{g}$ given by:

$$\tilde{g} = \frac{\alpha[(1-s)(1-\beta_z^W) + \beta^A] - 1 + 2s - 2\beta^A}{\alpha(1-s) + 2s - 1 + \beta^A(2-\alpha)s}$$

$$1 + T^W + D^W = \frac{[1-\alpha(1-s)]\left(1 + \frac{z^A}{\chi^A} + \beta_z^W\right) - \left(1 - \frac{z^A}{\chi^A}s\right)\alpha\beta^A}{(1-\alpha)(1-s)\left(1+\beta^A\right)} > 0 \tag{55}$$

$D^W \gtreqless 1$ if and only if $\alpha \gtreqless \tilde{\alpha}^W$, where $\tilde{\alpha}^W$ is given by:

$$\tilde{\alpha}^W = 1 - \frac{s}{1-s}\left[1 + \beta_z^W + (1+\beta^A)\left(\frac{z^A}{\chi^A} - 1\right)\right]$$

Proposition 7. *Under Assumption 1 and 2, then the following results hold:*

i⌋ Let $\alpha < \tilde{\alpha}^W$, the steady state is a sink for $\frac{z^A}{\chi^A} < \tilde{g}$, and then becomes a saddle for $\frac{z^A}{\chi^A} > \tilde{g}$;

ii⌋ Let $\alpha > \tilde{\alpha}^W$, the steady state is a source for $\frac{z^A}{\chi^A} < \tilde{g}$, and then becomes a saddle for $\frac{z^A}{\chi^A} > \tilde{g}$.

In order to determine the effect of intra-industry trade, when agents have different taste of variety across countries, one need to compare results of Proposition 3 and of Proposition 7.

Proposition 8. *Under Assumption 1 and 2, then the following results hold:*

i⌋ Let $\frac{z^A}{\chi^A} < \tilde{g}$ country B will import the local stability properties of country A;

ii⌋ Let $\frac{z^A}{\chi^A} > \tilde{g}$ country A will import the local stability properties of country B.

Note that $\frac{z^A}{\chi^A} < \tilde{g}$ is equivalent to $\beta^B > \beta^A$. When countries differ with respect to their taste for variety, the channel of transmission of local indeterminacy across countries is intra-industry trade in goods. In other words, when one country has a higher taste for variety than the other country, this country will import goods from the other country as well as the local stability of the other country.

To be completed...

6 Welfare

In the following, we examine how opening to international trade impacts the welfare of agents in each countries.

To be completed...

7 Concluding remarks

In a two-country two-factor overlapping generations model with imperfect competition and increasing returns to scale, we have examined the impact of international trade on the existence of sunspot cycles.

To be completed.....

8 Appendix

8.1 Proof of Proposition 2

We substitute equations (17) and (18) into the dynamical system defined by (19)-(20). Letting $s(1) = s$ and $\sigma(1) = \sigma$, with the help of (29) and (30), the partial derivative of the real wage ω and the real interest rate ρ with respect to K and a are given by:

$$\frac{d\omega}{dK} = \frac{\beta^A}{K}\omega, \frac{d\omega}{da} = \frac{\left(1-\beta^A(1-s)\right)}{a}\omega, \tag{56}$$

$$\frac{d\rho}{dK} = \frac{\beta^A}{K}\rho, \frac{d\rho}{da} = -\frac{\left(\left(1+\beta^A\right)(1-s)\right)}{a}\rho. \tag{57}$$

Loglinearizing (19)-(20) we derive

$$\frac{dK_{t+1}^A}{K^A} = (1+\beta^A)\frac{dK_t^A}{K^A} + \left[s\left(1+\beta^A\right) - (1+\beta^A)\right]\frac{da_t}{a},$$

and

$$-(1-\alpha)\beta^A\frac{dK_{t+1}^A}{K^A} + (1-\alpha)(1-s)\left(1+\beta^A\right)\frac{da_{t+1}}{a} = \beta^A\frac{dK_t^A}{K^A} + \left[-\beta^A + s\left(1+\beta^A\right)\right]\frac{da_t}{a}.$$

Results follows.

8.2 Proof of Proposition 6

We substitute equations (17) and (18) into the dynamical system defined by (44)-(45) and Loglinearizing (44)-(45) we derive

$$\frac{dK_{t+1}^W}{K^W} = \left(\frac{z^A}{\chi^A} + \beta_z^W\right)\frac{dK_t^W}{K^W} + \left[s\left(1+\beta_z^W\right) - \left(\frac{z^A}{\chi^A} + \beta_z^W\right)\right]\frac{da_t}{a}$$

and

$$-(1-\alpha)\beta^A\frac{dK_{t+1}^W}{K^W} + (1-\alpha)(1-s)\left(1+\beta^A\right)\frac{da_{t+1}}{a} = \beta^A\frac{dK_t^W}{K^W} + \left[-\beta^A + s\left(1+\beta^A\right)\right]\frac{da_t}{a}$$

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