A Robust LM Test for Long Memory

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Abstract

We propose a nonparametric time-domain based Lagrange Multiplier test for long memory with robustness against short-run dynamics and heteroskedasticity of quite general form. We show that its limiting null distribution is standard normal and demonstrate its consistency via simulation study. Moreover, we compare its performance to other existing time-domain based testing procedures in the literature in a large scale Monte Carlo simulation setup. Regardless of the several robustness features, our test shows good power results. In an empirical application on several types of ex-post Variance Risk Premium series, we find that long memory remains in the fractional cointegration relationship between implied and realized variance, additionaly to conditional and unconditional heteroskedasticity.

Key words: Long Memory · Lagrange Multiplier Test · Heteroskedasticity · Robust Statistic.

JEL classification: C12; C32

Outline

We propose a nonparametric and robust time-domain based Lagrange Multiplier (LM) test for long memory. Our procedure is constructed by recasting the test by Harris et al. (2008) in a regression framework in the spirit of Breitung and Hassler (2002) and subsequently standardizing the resulting t-statistic with a HAC estimator of Andrews (1991).

By leaving out an increasing number of low-order autocovariances when computing the LM statistic, the test is based on the standardized weighted sum of higher-order autocovariances, whose behavior is controlable under the null, whereas it diverges under the alternative. Firstly, this enables us to achieve robustness against any kind of short-run dynamics in the data generating process (DGP) via a nonparametric correction and secondly by employing a HAC estimator, we achieve robustness against a wide class of heteroskedasticity in the error terms, by simultaneously retaining a standard normal limiting null distribution. Both issues are very useful in applied work, since most prior proposed LM tests in this setup (e.g. Tanaka (1999), Breitung and Hassler (2002), Nielsen (2005) etc.) on one hand follow a parametric approach which poses the threat of a potential misspecification of the model and on the other hand rely in general on conditional homoskedastic error terms, which clearly rules out any kind of conditional and/or unconditional heteroskedasticity.

In particular the latter case has been studied in recent applied work and has been found in several macroeconomic and financial data (e.g. Loretan and Phillips (1994), McConnell and Perez-Quiros (2000), Sensier and Van Dijk (2004), Stărică and Granger (2005) and Cavaliere et al. (2015b) among others). Complementary research emerged, analyzing the severe influece that such time-varying conditional and unconditional volatility has on standard inference procedures like unit root, cointegration and stationarity tests, as well as tests on the fractional integration parameter in a $ARFIMA$ model (e.g. Cavaliere and Taylor (2007, 2008, 2009), Cavaliere et al. (2015a), Demetrescu and Hanck (2012a,b), Demetrescu and Sibbertsen (2016) etc.). We show that our test displays standard normal asymptotics under the null under a set of very mild assumptions. Contrary to the few existing tests displaying similar robustness features, our test additionally considers locally stationary processes (e.g. Beran (2009), Dahlhaus (2000) or Palma and Olea (2010)). Therefore, aside from allowing for breaks in the unconditional variance and a wide array of conditionally heteroskedastic models, we additionally allow for indirectly induced heteroskedasticity via one or multiple switching in the short-run dynamics of the DGP. To the best of our knowledge no existing testing procedure in the literature considers such a wide modelling class.

We perform an extensive Monte Carlo simulation study demonstrating the good performance of our test compared to other existing procedures in the literature. Our results are fairly robust against different user-specific choices like kernel implementation in the HAC estimator or to the truncation bandwidth, chosen to balance robustness against power. Regardless of the several robustness features, our procedure shows good power results. Additionally our test is easy to implement and has a fast computation speed.

We briefly illustrate our findings in table 1 and 2 to display the performance of various LM testing procedures in the literature under a stationary $ARFIMA$ model with various short-run dynamics and a break in the unconditional variance at the beginning ($\tau = \lfloor t/T \rfloor = 0.25$) of the sample, based on a nominal significance level $\alpha = 5\%$ and 10,000 Monte Carlo replications. Reported are results for the tests by Tanaka (1999) (Ta), Breitung and Hassler (2002) (BH) and Harris et al. (2008) (HCL) which all require conditional homoskedasticity, as well as Demetrescu et al. (2008) (ALM), Cavaliere et al. (2015b) (WB^{LM}) and different versions of our new test DSW that are robust against heteroskedasticity of quite general form. As one can observe the former procedures display weaknesses due to potential misspecification and due to the unconditional heteroskedasticity, whereas the latter procedures clearly control their size with our test showing a very good performance. Complementary table 2 shows that our test displays good power results.

As an empirical application, we analyze different types of ex-post Variance Risk Premium series of the Standard and Poors 500 index, constructed via a fractional cointegration relationship of implied volatility, represesented by the volatility index (VIX) issued by the Chicago Board of Options Exchange (CBOE), and realized volatility. In addition to long memory, we clearly find conditional and unconditional heteroskedasticity in the residuals of the series, showing the need for tests displaying robustness features to such forms of non-stationary volatility.

Our current research concentrates on extending the test to a multivariate framework and analyzing its applicability to an even wider model class.

																$s=8$				$s=4$			
		Ta		Ta_{AR}		BH		BH_{AR}		HCL		ALM		WB^{LM}		DSW_{QS}		DSW_B		DSW_{QS}		DSW_B	
ϕ/θ	T	ar	ma	ar	ma	ar	ma	ar	ma	ar	ma	ar	ma	ar	ma	ar	ma	ar	ma	ar	ma	ar	ma
-0.5	250	0	0	7.13	0	99.82	99.99	17.30	84.51	12.29	13.72	2.67	2.32	4.79	4.24	4.71	4.33	4.69	4.20	6.81	4.18	6.46	4.25
-0.5	500	0	0	9.25	0	100	100	18.52	96.74	14.30	13.92	2.84	1.92	4.01	4.91	5.08	4.43	5.33	4.46	8.17	4.70	8.12	5.06
-0.5	1000	0	0	10.03	0	100	100	19.07	99.86	14.93	14.65	2.88	2.68	4.38	4.83	5.47	4.98	5.23	4.41	6.68	4.62	6.27	4.70
-0.5	2000	0	0	11.40	0	100	100	20.02	100	15.28	14.75	3.47	3.20	3.88	5.19	5.37	4.70	5.49	4.65	5.47	4.79	5.13	4.62
0	250	9.47		5.48		18.17		16.61		8.72		2.78		2.06		4.05		3.69		3.96		3.90	
0	500	10.41		7.31		19.16		17.98		9.62		2.87		2.43		4.28		4.18		4.42		4.33	
0	1000	11.74		8.70		19.29		18.66		10.70		3.07		2.72		4.67		4.47		4.71		4.74	
0	2000	12.31		10.32		19.84		19.06		11.75		3.18		2.79		4.59		4.88		4.75		4.77	
0.5	250	99.94	96.21	1.40	0.52	99.94	95.21	13.01	22.47	8.70	9.97	3.07	2.73	4.07	2.21	4.49	4.02	4.55	4.18	4.84	4.40	5.24	4.68
0.5	500	100	99.94	3.08	1.00	100	99.86	15.29	29.95	10.79	11.88	2.97	2.84	5.11	3.07	4.64	4.28	5.10	4.71	4.96	4.50	4.59	4.30
0.5	1000	100	100	5.37	0.86	100	100	16.35	38.58	12.01	13.63	2.69	2.89	4.84	3.38	4.62	4.70	5.34	5.00	4.50	5.08	5.15	4.80
0.5	2000	100	100	7.57	0.51	100	100	17.93	51.04	13.09	13.61	3.11	3.32	5.00	3.69	4.67	4.73	4.51	4.75	4.95	4.97	4.59	4.57

Table 1: Size for testing $d = 0$ for various LM tests under unconditional heteroskedasticity.

		$d=0.2$								$d=0.4$							
		$s=8$				$s=4$				$s=8$				$s=4$			
		DSW_{QS}		DSW_B		DSW_{QS}		DSW_B		DSW_{QS}		DSW_B		DSW_{QS}		DSW_B	
ϕ/θ	T	ar	ma	ar	ma	ar	ma	ar	ma	ar	ma	ar	ma	ar	ma	ar	ma
-0.5	250	22.99	15.18	22.18	14.94	31.09	18.34	29.63	17.62	72.72	66.67	72.44	65.22	82.59	73.90	82.06	74.11
-0.5	500	31.28	20.67	29.58	20.32	43.90	26.46	43.62	25.17	86.19	80.84	85.70	80.09	95.86	90.11	95.24	89.11
-0.5	1000	38.17	26.96	37.27	26.10	52.40	33.85	50.74	34.17	95.04	91.53	94.78	92.20	99.12	97.11	98.93	97.25
-0.5	2000	51.79	37.98	49.47	37.51	65.87	49.41	64.74	48.41	99.24	98.70	99.17	98.65	99.97	99.80	99.93	99.87
0	250	29.99		30.19		36.80		36.87		71.15		71.61		80.76		81.85	
0	500	38.02		37.75		51.03		50.84		83.83		83.72		94.03		94.20	
0	1000	47.83		48.05		62.50		62.95		92.84		93.29		98.80		98.74	
0	2000	62.64		61.55		79.52		79.34		98.64		98.61		99.99		99.93	
0.5	250	28.73	28.64	31.24	30.08	35.42	34.41	38.18	34.48	65.02	65.65	66.96	69.33	76.80	76.77	79.67	78.39
0.5	500	35.77	35.55	36.57	36.76	50.01	46.95	52.60	47.39	77.26	79.06	79.61	81.76	91.02	91.59	93.25	92.60
0.5	1000	44	43.98	45.63	44.64	60.58	58.97	63.20	60.09	88.72	90.40	90.27	91	96.78	97.28	97.77	97.85
0.5	2000	59.31	59.03	58.63	58.76	80.27	77.98	80.99	77.65	97.39	97.95	97.56	98.23	99.74	99.86	99.77	99.83

Table 2: Power for testing $d = 0$ for the DSW test under unconditional heteroskedasticity.

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