The Evolution of Regional Economic Interlinkages in Europe*

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Abstract

This paper studies the dynamics of the propagation of regional business cycle shocks in Europe and uncovers new features of its underlying mechanisms. Motivated by the lack of high frequency data at the regional level, we propose a new method to measure time-varying synchronization in small samples that combines regime-switching models and dynamic model averaging. The results indicate three main findings: (i) in only two years, the Great Recession synchronized Europe twice as much as the European Union integration process did in several decades, (ii) Ile de France is the region acting as the main channel in the transmission of business cycle shocks in Europe, followed by Inner London. (iii) We uncover a nonlinear effect of sectoral composition in explaining regional synchronization, which significantly increased after the Great Recession. Moreover, we identify similarities in services-related sectors as what is primarily responsible for this significant and nonlinear effect.

Keywords: Business Cycles, Sectoral Composition, Regime-switching, Model Averaging.

JEL Classification: C31, C32, E32, R11.

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1 Introduction

The regional dimension has been a relevant concern for European institutions since the establishment of the European community. Since the Treaty of Rome in 1957, there has been an increasing interest in using regional policies to reduce differences among European regions. The regional policy of the European Union (EU), known as the Cohesion Policy, makes up more than one third of the total EU budget and, for decades, has been focused on increasing regional competitiveness and fostering economic growth and employment at a regional level.\footnote{In the 80s, structural funds were established to adapt the EU to the arrival of some of the more backward regions. In the 90s, the resources for the structural and cohesion funds increased (reaching a third of the total EU budget). In the 2000, the ‘Lisbon Strategy’ shifted the cohesion policies’ priorities towards growth, employment and innovation. Since 2010, in the context of the ‘Europe 2020 Strategy’, the bulk of Cohesion Policy funding has focused on less developed regions, trying to reduce the economic, social and territorial disparities that still exist within the EU. This policy also addresses issues such as climate change, energy supply and globalization.}

In spite of the relevance of the analysis of regional business cycles and their comovement, the attention given to this issue has been scarce. In contrast, numerous studies have examined the business cycles and the degree of synchronization among countries within the European Monetary Union and among European countries in general (see Camacho et al. (2008), Giannone et al. (2010) and De Haan et al. (2008) for a survey). It has to be borne in mind that, due to a richer set of information based on a higher disaggregation, regional level analysis may unmask important aspects of the comovement of business cycles that can not be observed at the country level, as shown in Gadea et al. (2011) for the Spanish case.

Most of the studies that have focused on describing overall regional cyclical patterns can be divided into two strands. The first focuses on analyzing regional convergence (see Ramajo et al. (2008), Quah (1996) and Sala-i-Martin (1996)) and on identifying the determinants of long-term economic performance at the regional level (Ozyurt and Dees, 2015). The second, which is directly related to our work, focuses on the synchronization of short-term fluctuations in regional real activity. These studies usually compute pairwise correlations of different measures of the regional cycles with respect to country-specific cycles or a reference cycle, such as the European one, to identify ‘border effects’. The measure of regional economic activity has also varied across studies. Fatas (1997), Barrios and De Lucio (2003) and Belke and Heine (2006) use employment data, while Montoya
and de Haan (2008) and Barrios et al. (2003) use gross value added and Clark and van Wincoop (2001) employ on both measures of real activity to compare synchronization patterns among European countries and US Census regions. Marino (2013) uses a different methodological approach that is based on on dynamic factor models to analyze regional fluctuations of GDP and employment. For a partial summary review of this literature, see Montoya and de Haan (2008).

It is worth noting that Europe has not only experienced significant policy changes to encourage its unification process, but has also been exposed to large contractionary business cycle shocks, coming from the foreign side, during the ‘Great Recession’, and from the domestic side, during the recent ‘Debt Crisis’. These, and other factors have produced significant changes in the overall patterns of regional business cycle synchronization. Although the studies mentioned above provide a better understanding of business cycle comovements at the regional level, they are not able to endogenously identify changes in the patterns of regional synchronization. This is an important limitation in evaluating the effect of policy and business cycle shocks over time.

This paper has some advantages over the previous literature and provides a threefold contribution. First, we use data with a broader coverage in the time and space dimensions than the previous studies. In particular, our study is the first to capture the effect of the Great Recession (GR, henceforth) on regional synchronization, which allows us to compare its effects to those of important milestones in Europe, including the Maastricht Treaty and the introduction of the euro.2 Furthermore, we analyze a wider and more disaggregated geographical coverage, considering a range of 213 European regions based on the NUTS2 classification (Nomenclature of territorial units for statistics, at the second level) and corresponding to 16 European countries for a period of 32 years (1980-2011) with a yearly frequency.3 Most of the previous studies look at a smaller number of European regions (NUTS1 level) and a shorter period of time.4 Finally, we employ the most comprehensive measure of real economic activity, that is, real GDP data as the literature on national business cycle synchronization usually does.

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2Our study covers the last three global recessions, which started in 1982, 1991 and 2009, respectively. See IMF (2009), for a definition of global recessions

3The NUTS 2013 classification lists 98 regions at NUTS1 level, 276 regions at NUTS2 level and 1,342 regions at NUTS 3 level. Although socioeconomic analysis of regions is made up to NUTS3 level, the regions eligible for the support of cohesion policy are defined at NUTS 2 level

Second, we propose a new methodology to measure time-varying synchronization in small samples that allows us to overcome the short length of the series produced by the lack of high frequency data. We combine the regime-switching synchronization approach, proposed in Leiva-Leon (2016), with the dynamic model averaging framework for Markov-switching models, proposed in Guerin and Leiva-Leon (2015), to produce a flexible method to estimate business cycles time-varying synchronization. The alternative rolling approach is not suitable in this case because of the short number of observations (31 data points). Our approach not only deals with this limitation, but also provides measures of uncertainty for the estimated dynamic synchronizations. Most of the papers that analyze synchronization among European regions use standard measures of pairwise correlation, only differing in some cases in the measure of the cycle (percentage changes or different filters, such as the Hodrick-Prescott filter).

Third, our results provide endogenous identification of changes in regional synchronization patterns over time and assessments about the main sources of these changes. We analyze the bilateral linkages information across time -by computing dynamic cohesion measures- and space -by assessing the degree of country-specific cohesion. This bilateral analysis allows us to identify, for the first time, the regions that play a more important role in the propagation of business cycle shocks. Moreover, since the sectoral composition has a significant role in explaining synchronization, we analyze it in greater depth. We focus on assessing the changes in the relationship between sectoral composition similarities and business cycles similarities across European regions. To provide robust assessments, first, we control for additional factors and employ on parametric methods, and second, we make very general assumptions using nonparametric methods. The latter technique allows us to identify an asymmetric effect of sectoral composition on synchronization which increases after the GR. An additional disaggregated analysis shows that financial and business services are main responsible for this phenomenon.

The main findings of the paper are the following. First, the impact of the GR on increasing synchronization among European regional cycles was twice as big as that of the European Union integration process until then. A gradual increase in synchronization is also observed during the five years prior to the introduction of the euro. Most of the core countries have experienced an increase in cohesion over time, while the Nordic countries, Greece, Ireland and the Netherlands show a different pattern, with a general decrease of internal synchronization from the mid 90s until 2007. Second, Ile de France is the region acting as the main channel in the propagation of business cycle shocks in Europe. The regional linkages have increased over time. At the beginning of the sample, Ile de
France was connected to Inner London and some regions of East Germany and Italy; the introduction of the euro led to some Spanish regions also being connected; during the GR, the topological structure of the European network become even more connected (some Spanish regions becoming more connected to some French, Italian and English regions). Third, increases in regional sectoral composition similarity have a positive effect on business cycle synchronization only for regions that already experience high levels of similarity in their productive structure. This effect was significantly amplified during 2010-2011, implying that the propagation of business cycle shocks among European regions became more dependent on the regional production structure after the GR.

The remainder of the paper is organized as follows. Section 2 presents the methodology for measuring time-varying synchronization. Section 3 describes the dataset and shows the results of computing bilateral similarities of regional business cycles, taking into account the time and space dimensions as well as the economic size of regions. Section 4 investigates the relationship between the degree of synchronization and regional similarity and controls for additional factors underlying the different degrees of synchronization. Finally, Section 5 concludes.

2 Measuring regional business cycle synchronization

Regime-switching models have been widely used to infer endogenous changes in the synchronization of business cycle phases, at the country level (Ductor and Leiva-Leon (2016)), and at the state (Leiva-Leon (2016)) and sectoral level (Camacho and Leiva-Leon (2016)), for the US. However, the framework used in these studies requires a relatively large number of observations to provide inferences on regimes of synchronization (high or low). This is an important limitation to analyze endogenous changes in the synchronization of European regional business cycles, since regional real GDP data is only available annually from 1980 to 2011, leaving only 31 time observations for our analysis. Alternative rolling approaches are not suitable for analyzing endogenous changes in synchronization in the present case because the window would cut most of the available sample. In this section, we propose a framework that allows us to deal with the short sample limitation, and that, moreover, provides statistical assessments about the changes in synchronization.

2.1 The model

To measure business cycle synchronization, we follow the line proposed in Harding and Pagan (2006) who focus on testing the hypothesis that cycles are either unsynchronized
or perfectly synchronized during a given sample period. In this section, we propose an econometric framework to assess the degree of business cycle synchronization over time using Dynamic Model Averaging to account for both polar cases of synchronization. Let \( Y_{a,t} \) and \( Y_{b,t} \) be the real GDP of European regions \( "a" \) and \( "b" \), respectively. To focus on cyclical fluctuations, we define \( y_{k,t} = 100 \times \Delta \log(Y_{k,t}) \) as the real GDP growth of region \( "k" \), and model their joint dynamics following Leiva-Leon (2016), that is

\[
\begin{bmatrix}
  y_{a,t} \\
  y_{b,t}
\end{bmatrix} =
\begin{bmatrix}
  \mu_a(S_{a,t}) \\
  \mu_b(S_{b,t})
\end{bmatrix} +
\begin{bmatrix}
  e_{a,t} \\
  e_{b,t}
\end{bmatrix},
\begin{bmatrix}
  e_{a,t} \\
  e_{b,t}
\end{bmatrix} \sim N(0, \Sigma)
\] (1)

where

\[
\mu_k(S_{k,t}) = \mu_{k,0} + \mu_{k,1}S_{k,t}, \quad \text{for } k = a, b,
\] (2)

and \( S_{k,t} \) denotes a latent state variable that indicates the state of the economy of region \( k \). It takes the value \( S_{k,t} = 0 \) if region \( k \) is in recession, or the value of \( S_{k,t} = 1 \) if region \( k \) is in expansion. Each state variable is assumed to follow a first-order Markov process with transition probabilities given by

\[
p(S_{k,t} = j_k | S_{k,t-1} = i_k, S_{k,t-2} = h_k, ...) = p(S_{k,t} = j_k | S_{k,t-1} = i_k),
\] (3)

and the variance-covariance matrix \( \Sigma \) is assumed to be non-diagonal.

The dependency relationship between the state variables provides information about the synchronization of the economic cycles of the two regions. Despite the complexity involved in modelling the exact dependency relationship between \( S_{a,t} \) and \( S_{b,t} \), we are able to model their joint dynamics, summarized in the state variable \( S_{ab,t} \), under the two extreme cases. The first corresponds to the independent case, where the joint probability is the product of the marginal probabilities,

\[
p(S^I_{ab,t} = i_{ab}) = p(S_{a,t} = i_a)p(S_{b,t} = i_b).
\]

The second corresponds to the fully dependent case, where \( S_{a,t} = S_{b,t} = S_t \), and the joint probability is modelled as

\[
p(S^D_{ab,t} = i_{ab}) = p(S_t = i),
\]

where \( S_t \) is a state variable that governs the whole dynamics of the system in Equation (1) and has its own transition probability

\[
p(S_t = j | S_{t-1} = i, S_{t-2} = h, ...) = p(S_t = j | S_{t-1} = i).
\] (4)
Our goal is to provide assessments about the degree of business cycle synchronization between regions “a” and “b”, which can be interpreted as a linear combination between the two extreme cases. Moreover, we are interested in providing information about the degree of synchronicity for each period of time. Therefore, we model the joint probability of state variables as follows

\[ p(S_{ab,t} = i_{ab}) = p(S^{D}_{ab,t} = i_{ab})\delta_{t} + p(S^{I}_{ab,t} = i_{ab})(1 - \delta_{t}), \quad (5) \]

where the weight \( \delta_{t} \) can be interpreted as the degree of synchronization between the two regions at time \( t \). To model \( \delta_{t} \), Leiva-Leon (2016) introduces another state variable that indicates either regimes where the independent case, \( p(S^{I}_{ab,t} = i_{ab}) \), provides a better characterization of the model’s dynamics or regimes where the dependent case, \( p(S^{D}_{ab,t} = i_{ab}) \), is the most appropriate characterization. However, inferences about the time-varying synchronization using this framework are less accurate when the number of observations contained in \( y_{t} \) is very limited. This is because synchronicity regimes (independent or fully dependent) considered to be a sequence of several time periods where the latent variables follow similar dynamics. Due to the data limitations at regional level, our available information consists of only 32 data points (years).

To overcome this drawback, in this paper, we propose a flexible way to compute \( \delta_{t} \), which is not based on the assumption of regimes of dependency. Instead, we use Dynamic Model Averaging to infer time periods where one polar case, the independent or fully dependent, provides the best characterization of the data in \( y_{t} \). This procedure allows us to capture changes in European regional synchronization with the 32 observations available at a yearly frequency.

### 2.2 Dynamic model averaging

Dynamic model averaging (DMA) was initially proposed by Raftery et al. (2010) and motivated by taking into account time variation in model uncertainty. DMA has been applied to a variety of contexts including forecasting inflation in the context of time-varying parameter (TVP) regression models (see Koop and Korobilis (2012), linear vector autoregressive (VAR) models (Koop (2014)) and large TVP-VAR models (Koop and Korobilis (2013)) to forecast inflation, real output and interest rates. Recently, Guerin and Leiva-Leon (2015) proposed a framework to use DMA in the context of Markov-switching (MS) models to predict recessions under model uncertainty. Accordingly, we can think of equation (5) as the regime probabilities obtained from two different models, one with
independent state variables, \( p(S^I_{ab,t} = i_{ab}) \), and another with fully dependent state variables, \( p(S^D_{ab,t} = i_{ab}) \), averaged over time with a time-varying weight, \( \delta^ab_t \). Therefore, we can interpret our synchronization framework as a dynamic average of two MS models and follow the line of Guerin and Leiva-Leon (2015) to compute \( p(S_{ab,t} = i_{ab}) \).

Taking the model’s parameters as known and suppressing the indexes ‘a’ and ‘b’ for ease of notation, the algorithm used to obtain the elements in equation (5) consists of iteratively computing the following steps.

**Step 1: predicting regime probability.** Using the corresponding transition probabilities for the independent model, \( p^I = p(S_{a,t}|S_{a,t-1}) \times p(S_{b,t}|S_{b,t-1}) \) and for the fully dependent model, \( p^D = p(S_t|S_{t-1}) \), as defined in equations (3) and (4), respectively, compute the predicted regime probabilities, \( p(S^m_{t}|\psi_{t-1}) \), for \( m = I, D \), given past information \( \psi_{t-1} \),

\[
\begin{align*}
 p(S^m_t, S^m_{t-1}|\psi_{t-1}) &= p^m p(S^m_{t-1}|\psi_{t-1}) \\
 p(S^m_t|\psi_{t-1}) &= \sum_{S^m_{t-1}} p(S^m_t, S^m_{t-1}|\psi_{t-1})
\end{align*}
\]

Then, the predictive likelihood is calculated from the predicted regime probabilities:

\[
f^m(y_t|\psi_{t-1}) = \sum_{S^m_t} \sum_{S^m_{t-1}} f^m(y_t|S^m_t, S^m_{t-1}, \psi_{t-1}) p(S^m_t, S^m_{t-1}|\psi_{t-1}).
\]

**Step 2: updating model probability.** Let \( M_t \in \{I, D\} \) denote which model applies at each period of time. To simplify notation, let \( \pi_{t-1,m} = p(M_t = m|\psi_{t-1}) \) be the predictive probability associated with the \( m \)-th MS model at time \( t \), given the information up to \( t-1 \). Starting with an equal-weight initial-model probability \( p(M_0) \), we follow the updating criterion of Raftery et al. (2010), which is based on a measure of model fit for \( y_t \), that is, the predictive likelihood:

\[
\pi_{t|m} = \frac{\pi_{t-1,m} f^m(y_t|\psi_{t-1})}{\sum_{r=1}^{2} \pi_{t-1,r} f^r(y_t|\psi_{t-1})}.
\]

**Step 3: updating regime probability.** Use the predictive likelihood, \( f^m(y_t|\psi_{t-1}) \),

\footnote{The Hamilton filter is initialized with the ergodic probabilities \( P(S_0) \).}
to compute the updated regime probabilities, $p(S^m_t | \psi_t)$, for both models, as follows:

$$p(S^m_t, S^m_{t-1} | \psi_t) = \frac{f_m(y_t, S^m_t, S^m_{t-1} | \psi_{t-1})}{f_m(y_t | \psi_{t-1})} = \frac{f_m(y_t | S^m_t, S^m_{t-1}, \psi_{t-1}) p(S^m_t, S^m_{t-1} | \psi_{t-1})}{f_m(y_t | \psi_{t-1})}$$

$$p(S^m_t | \psi_t) = \sum_{S^m_{t-1}} p(S^m_t, S^m_{t-1} | \psi_t),$$

which are used in Step 1 of the next iteration.

**Step 4: predicting model probability.** Compute the predicted probability associated with the $m$-th model, $\pi_{t+1|m,t}$, following Raftery et al. (2010) and using the forgetting factor $\alpha$, as follows:

$$\pi_{t+1|m,t} = \frac{\pi^\alpha_{t|m,t}}{\sum_2 \pi^\alpha_{t|t,r}},$$

which are used in Step 2 of the next iteration. The forgetting factor $\alpha$ is the coefficient that governs the amount of persistence in the models’ weights, the higher the $\alpha$, the greater the weight attached to past predictive performance. It is commonly set to a fixed value slightly less than one. However, in our context, due to the small sample size, small variations in $\alpha$ may influence the dynamics of $\pi_{t+1|m,t}$. Therefore, instead of simply imposing a given forgetting factor, we estimate it along with the other parameters of the model using Bayesian methods. Our estimation strategy consists of linearizing equation (12) and applying regression analysis to estimate $\alpha$. The Appendix provides further details about the estimation methods employed.

We repeat the steps above for $t = 1, ..., T$. The output of the algorithm consists of the regime probabilities for each model, $p(S^m_t | \psi_t)$, for $m = I, D$, and the model probabilities for each time period, $\pi_{t|m,t}$. Therefore, we compute the expected joint regime probabilities by averaging across models:

$$p(S_{ab,t} = i_{ab} | \psi_t) = p(S^D_t | \psi_t) \pi_{t|t,D} + p(S^I_t | \psi_t) \pi_{t|t,I},$$

where $\pi_{t|t,D} = \delta_{t|t}$, and $\pi_{t|t,I} = (1 - \delta_{t|t})$, following the notation in equation (5).

### 2.3 Simulations

In this section, we conduct Monte Carlo simulations to compare the finite sample performance of the method proposed in Leiva-Leon (2016) to measure time-varying synchroniza-
tion and the method proposed in this paper for the same purpose. Our goal is to examine the accuracy of the two methods in inferring synchronization changes under different scenarios, regarding (i) the gap between the state-dependent parameters, (ii) the volatility of the series, and (iii) the size of the sample.

2.3.1 Design

The experiment consists of generating two observed series, \( y_{a,t} \) and \( y_{b,t} \), governed by two unobserved state variables, \( S_{a,t} \) and \( S_{b,t} \), respectively, according to the following parsimonious system,

\[
\begin{bmatrix}
  y_{a,t} \\
  y_{b,t}
\end{bmatrix} = \begin{bmatrix}
  \mu_{S_{a,t}} \\
  \mu_{S_{b,t}}
\end{bmatrix} + \begin{bmatrix}
  e_{a,t} \\
  e_{b,t}
\end{bmatrix},
\]

where \( \mu_{S_{i,t}} = \mu_{i,0} + \mu_{i,1}S_{i,t} \), for \( i = a, b \), and the innovations \( e_t = [e_{a,t}, e_{b,t}]' \), are normally distributed, that is, \( e_t \sim N(0, \Omega) \). The latent state variables are assumed to undergo changes in their synchronization over time, and they are generated as follows.

First, let \( \tilde{S}_{a,t} \) be a state vector of sequence \( a \) at time \( t \). If the sequence \( a \) is in state 1 at time \( t \) then we write \( \tilde{S}_{a,t} = (1, 0)' \), and if it is in state 2 at time \( t \) then we write \( \tilde{S}_{a,t} = (0, 1)' \). The vector \( \tilde{S}_{a,t} \) is assumed to follow a first-order Markov chain. That is, for time \( t \), compute \( (q_a, 1-q_a)' = P_a\tilde{S}_{a,t} \), where

\[
P_a = \begin{pmatrix}
  p_{a,11} & 1-p_{a,22} \\
  1-p_{a,11} & p_{a,22}
\end{pmatrix},
\]

is the transition probability matrix, and the realization of the sequence at time \( t+1 \) is defined as

\[
\tilde{S}_{a,t+1} = \begin{cases}
  (1, 0)', & \text{If } q_a \geq \theta \\
  (0, 1)', & \text{Otherwise}
\end{cases}
\]

where \( \theta \) is drawn from a \( U[0, 1] \). Second, in an analogous way, generate an independent Markovian sequence, \( \tilde{S}_{b,t} \), with its corresponding transition probability matrix, \( P_b \). Third, generate another Markovian sequence, \( \tilde{V}_t \), with its corresponding transition probability matrix, \( P_V \), that governs the changes of synchronization between \( \tilde{S}_{a,t} \) and \( \tilde{S}_{b,t} \) according to the following rule:

\[
S_{a,t} = \tilde{S}_{a,t+1},
\]

\[
S_{b,t} = \begin{cases}
  \tilde{S}_{a,t+1}, & \text{If } V_t = (1, 0)' \\
  \tilde{S}_{b,t+1}, & \text{If } V_t = (0, 1)'
\end{cases}
\]

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where $\tilde{S}_{i,t[1]}$ denotes the first element of the sequence vector for $i = a, b$.

For simplicity, we assume that $p_{a,11} = p_{b,11} = 0.9$, and $p_{a,22} = p_{b,22} = 0.8$. Since our application involves dealing with a small sample, we are interested in generating a small number of random changes of synchronization during the time span, $T$. In doing so, we relate the transition probabilities of the state variable that indicates changes in synchronization, $p_{v,11} = p_{v,22} = p_v$, with the sample size, $T$, based on an expected duration of one change over the entire sample.\(^6\)

We control for three dimensions in our simulations. First, we focus on the dispersion between the state-dependent means, defined as $\bar{\mu} = |\mu_{S_{i,t}=1} - \mu_{S_{i,t}=0}|$, keeping a mean between $\mu_{S_{i,t}=1}$ and $\mu_{S_{i,t}=0}$ equal to zero. In particular, we consider a set given by $\bar{\mu} = \{0.5, 1, 2\}$. Second, we assess the effect of the volatility associated with the innovations of the data. For simplicity, we assume that $\sigma_{11}^2 = \sigma_{22}^2 = 0.5\sigma_{12}^2 = \sigma^2$, and study different scenarios of volatility given by the set $\sigma^2 = \{0.5, 1, 2\}$. Our main interest is in the performance of the models with different sizes of the available sample. Therefore, we evaluate the finite sample properties by repeating the simulations associated with each configuration of parameters for the set of sample sizes, $T = \{30, 50, 70, 100, 200, 400\}$. We perform $M = 1000$ simulations for each configuration of parameters $\bar{\mu}$, and $\sigma^2$, and for each sample size, $T$, under consideration.

2.3.2 Results

At each $m$-th replication, we compute the time-varying synchronization estimated with the forgetting factor (FF) method proposed in this paper and define it as $\delta^m_t$. We also compute the time-varying synchronization estimated with the Markov-chain (MC) method, proposed in Leiva-Leon (2016), and define it as $d^m_t$. For each replication we compute the Quadratic Probability Score (QPS) associated with each method, taking as reference the variable indicating the true synchronization changes, $V_t = \tilde{V}_{t[1]}$. Our object of interest is

\[^6\]We relate the expected duration of a synchronization regime (high or low), $p_v^E$, defined as $p_v^E = \frac{1}{1-p_v}$, with the number of time periods that the series would experience such regime, $\tau$. Since we are interested in generating (on average) one change of synchronization, $\tau = \frac{T}{4}$.\[^6\]
the average QPS over the $M$ replications associated with each method, calculated as,

\[
QPS_\delta = \frac{1}{M} \sum_{m=1}^{M} \left[ \frac{1}{T} \sum_{t=1}^{T} (V^m_t - \hat{\delta}^m_t)^2 \right],
\]

\[
QPS_d = \frac{1}{M} \sum_{m=1}^{M} \left[ \frac{1}{T} \sum_{t=1}^{T} (V^m_t - d^m_t)^2 \right].
\]

Table 1 reports the simulation results based on $M = 1000$ replications. The results indicate that, for small samples ($T = 30$), the FF method outperforms the MC method in 78% of the cases. However, when the sample increases, the relative performance of the FF method with respect to the MC method tends to decrease. In particular, for $T = 50$, $T = 100$, and $T = 200$, the FF method outperforms the MC method in 67%, 56%, and 44%, of the cases, respectively. Therefore, the method to assess changes in synchronization proposed in this paper tends to perform significantly better than the MC method in small samples but it is also competitive in large samples.

3 Assessing changes in regional interdependence

3.1 Data

The sample consists of 213 NUTS-2 regions corresponding to 16 European countries: the EU-12 (Austria (AT), Belgium (BE), Finland (FI), France (FR), Germany (DE), Ireland (IE), Italy (IT), Luxembourg (LU), Netherlands (NL), Portugal (PT), Spain (ES) and Greece (GR)) and Denmark (DK), Norway (NO), Sweden (SE) and the UK (UK). Regarding Germany, data of eastern Landers and Berlin (DE3, DE4, DE8, DED, DEE and DEG) are not available prior to 1991. Therefore, they are not included in our analysis.

The availability of regional data on a high frequency basis and for a long span is scarce. To analyze the synchronization of the regional business cycles, we employ annual real GDP data, as quarterly data is not available. The series cover a period of 32 years, from 1980 to 2011. Thus, for the first time, we analyze the possible effect of the GR on the regional business cycle of the European countries.

The source of the data is the Cambridge Econometrics database, which has carried out some minor adjustments to data originally collected from Eurostat’s REGIO database using data from AMECO, a dataset provided by the European Commission’s Directorate General Economic and Financial Affairs (DG EcFin).\footnote{In particular, the GDP series is deflated to 2005 constant price euros using price deflators obtained}
Summing up, we have data of 213 regions, representing a total of 22,578 pairwise combinations.

3.2 Bilateral economic linkages

Much of the literature about business cycle synchronization rely on dynamic factor models to infer changes in the comovement between the real activity of different economies and some common (or global) factors. Some examples are Kose et al. (2012) and Del Negro and Otrok (2008), among others, at a country level and Marino (2013) at a regional level. However, these studies do not provide information about changes in the bilateral similarities of the business cycles of these economies, which would provide a more detailed picture to assess the propagation patterns of contractionary and expansionary shocks.

To investigate the overall spectrum of the aggregate economic interlinkages between the European regions, we estimate the model (1)-(5) for each pair of regions. Figure 1 displays the time-varying business cycle synchronization for some selected pairs of regions, showing some similarities and some dissimilarities among them. Chart A displays the pattern of synchronization between two capital regions of neighboring countries, Ile de France (FR) and Inner London (UK). The chart shows a decrease in the degree of synchronization between the two regions at the end of the 80s and an upward trend after the Maastricht Treaty was signed in 1992, followed by a significant jump during the GR (2008-2009).\(^8\) Chart B displays the synchronization between two capital regions of non-neighbor countries, Madrid (ES) and Lazio (IT), it increased in the mid-80s, remained relatively stable after the mid 90s, and substantially increased during the GR period. Chart C displays a scenario similar to the one described in Chart B for two non-capital regions of neighboring countries, Cataluña (ES) and Languedoc Rousillon (FR). Finally, Chart D displays the synchronization between a capital and a non-capital region of the same country, Sør-Ostlandet (NO) and Oslo og Akershus (NO), respectively. Unlike the previous examples, Chart D shows a marked decline in synchronization between the two regions during the late 90s, at the beginning of the monetary integration.\(^9\)

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\(^8\)A look at the GDP growth rates of the two series shows that Inner London (UK) began to grow sharply until the mid 80s, decreasing rapidly afterwards until 1991 when it reached its lowest level. Ile de France (FR) experienced a less volatile behavior during the same period: its upward trend during the 80s lasted almost till the beginning of the 90s recession and the minimum was far below that reached during the GR.

\(^9\)The GDP growth rates of both series were almost identical until the mid-90s. However, during the 2000s, Sør-Ostlandet (NO) shows a steady downward trend, while Oslo og Akershus (NO) experienced a deceleration at the beginning of the 2000s, expanding afterwards till the beginning of the GR.
To have a glimpse about the regions experiencing the highest levels of synchronization with the rest of Europe, we compute the following statistics:

\[ d_i^t = \frac{1}{n} \sum_{j=1}^{n} \delta_{ij}^t, \quad \text{for } i = 1, 2, ..., n, \]

where \( n = 213 \) regions and \( d_i^t \) provides information about the overall degree of synchronization of region \( i \) at time \( t \). Figures 2, 3 and 4 plot maps of Europe, identifying the regions with high levels of aggregate synchronization for the years 1981 (beginning of our sample), 1999 (introduction of the euro) and 2009 (last global recession), respectively. The figures show that, in 1981, relatively few regions were highly synchronized with each other. However, during the implementation of the euro, many regions of Spain, France, Italy, and even some regions of the United Kingdom, became more synchronized. Finally, in the middle of the GR, in 2009, almost all the regions under consideration in this study experienced high levels of synchronization, showing evidence of the propagation of a contractionary shock during this time. Although this information provides only a visual assessment of the effect of the euro and the GR in Europe, the next section focuses on quantifying more precisely the overall effect that the two events had across European regions.

3.3 Synchronization across time and space

Having estimated all the economic bilateral linkages between the European regions, we will now analyze this information from two different perspectives. First, we focus on the time dimension and “collapse” the space dimension. Croux et al. (2001) investigate synchronization between European countries at the frequency domain using cohesion measures. Cohesion measures are defined as indexes that summarize information about bilateral relations and that are useful to provide an overall assessment of the interrelations between a set of elements (markets, countries, etc.).\(^\text{10}\) We follow Croux et al. (2001) in computing indexes of cohesion to measure the overall degree of European synchronization. Nevertheless, our analysis differs from Croux et al. (2001) in two important features. First, we focus on the time rather than the frequency domain and, second, we are interested in measuring cohesion based on regional rather than national disaggregation. Accordingly,\(^\text{10}\)

\(^{10}\)Croux et al. (2001) find that countries experience stronger comovements with neighbors than with other countries, especially in the long run (cycles longer than four years).
the time-varying European cohesion is defined as follows

\[ c_{t, \text{europe}} = \frac{\sum_{a \neq b} \omega_{a,t} \omega_{b,t} \delta_{t}^{ab}}{\sum_{a \neq b} \omega_{a,t} \omega_{b,t}} \text{, for } a, b = 1, \ldots, n \]  

(15)

where, \( \omega_{a,t} \) and \( \omega_{b,t} \) denote the weights that regions “a” and “b” receive, respectively, defined as the GDP share of each region with respect to the total GDP in Europe.

Chart A of Figure 5 displays the time-varying European cohesion, showing a relatively stable pattern during the 80s at levels around 0.50. After the early 90s, the cohesion starts to increase slowly, in particular, after the Maastricht treaty was signed in 1992. This pattern continues after the introduction of the euro in 1999, reaching a level of 0.56 by the end of 2007. The increase in cohesion of 12\% between 1981 and 2007 can be, at least, partially attributed to the process of European unification in institutional, economic and many other terms. However, between 2008 and 2009, the cohesion increased by 31\%, reaching a maximum level of 0.74. This leap in the cohesion was obviously influenced by the GR, since contractionary business cycle shocks were propagated through most of the European regions.\textsuperscript{11} These results imply that neither the Maastricht treaty nor the introduction of the euro managed to synchronize European regions in decades as much as the GR did in only two years.\textsuperscript{12}

To get a better understanding about the space dimension of European regional synchronization, we measure the degree of cohesion associated with each country. This will allow us to identify the countries containing more and less synchronized regions. The time-varying cohesion of a given country, \( \kappa \), at time, \( t \), is defined as

\[ c_{t}^{\kappa} = \frac{\sum_{a \neq b \in \kappa} \omega_{a,t} \omega_{b,t} \delta_{t}^{ab}}{\sum_{a \neq b \in \kappa} \omega_{a,t} \omega_{b,t}} \]

(16)

Chart B of Figure 5 shows the case of countries that have experienced an increasing cohesion over time, progressively growing during the 90s and experiencing a hike at the beginning of the GR. They are Austria, France, Italy, Belgium, Germany, Portugal, Spain, and the United Kingdom. Most of them are countries included in the Eurozone, which would explain this upward trend. Amongst them, Portugal shows the lowest degree of

\textsuperscript{11}Previous results are not comparable to ours due to their different datasets and methodologies. The closest regarding datasets is that of Montoya and de Haan (2008). They compute the degree of synchronization between 53 NUTS1 regions and the Euro Area and observe that the correlation decreased during the 80s and at the very beginning of the 90s, increasing afterwards till the end of their sample, 2005.

\textsuperscript{12}This result agrees with Canova et al. (2012) who find that, for the European case, the institutional events have no effects on the real fluctuations.
synchronization during the whole sample while Spain, on the contrary, stands out for its high cohesion during most of the period considered. Chart C of Figure 5 shows a different cohesion pattern for Greece, Ireland, Denmark, Norway, Sweden, Finland, and the Netherlands. These countries experienced a decreasing internal synchronization from the mid 90s until 2007 -being especially sharp for Norway since 1997 and for Greece since 2004. Finland is an exception as it has a very high degree of synchronization during the sample time. When the GR arrived in 2008, all the European countries in our sample experienced significant increases in their respective cohesions. These results imply that, in general, core countries have become more internally synchronized than Nordic countries.

3.4 The role of the economic size of regions

In the previous sections, we have analyzed the synchronization of regional GDP growth assigning the same importance to all the regions in Europe and, consequently, treating any bilateral relationship in the same way. However, some regions may play a more important role in the propagation of business cycle shocks due to their size in economic terms, i.e. their GDP share with respect to the total in Europe. For example, two highly synchronized regions that also have a large economic size would be channels in the propagation of business cycle shocks in a more prominent way than two also highly synchronized regions of small economic size. Therefore, in this section we focus on identifying the bilateral cyclical relationships between regions that simultaneously comply with two features, (i) high synchronization and (ii) high economic size.

To identify the linkages of regions accounting for their economic size, we construct a measure of weighted synchronization based on the same notion of cohesions described in equation (16). Therefore, our modified synchronization measure is given by

$$\tilde{\delta}_{ab} = \frac{(\omega_{a,t} \omega_{b,t} \delta_{ab}^{t}) - \delta_{\text{min}}}{\delta_{\text{max}} - \delta_{\text{min}}}$$

(17)

where $\delta_{\text{min}}$ and $\delta_{\text{max}}$ are coefficients used to normalize $\tilde{\delta}_{ab}^{t}$ between 0 and 1 for an easier interpretation, and denote the minimum and maximum value of the term $(\omega_{a,t} \omega_{b,t} \delta_{ab}^{t})$ for $\forall a \neq b$ and for $t = 1, 2, ..., T$, respectively. Accordingly, pairs of regions experiencing high values $\tilde{\delta}_{ab}^{t}$ can be interpreted as the most prominent channels in the transmission of business cycle shocks.

Figure 6 plots the main linkages between regions in 1981\textsuperscript{13}. These relationships can

\textsuperscript{13}The threshold for this graphical analysis is 0.3. See Bailey (2015) et al. for a threshold selection with econometric techniques.
be described as European network with a star topological structure, in which the central region is Ile de France and the arrows around it are Inner London, some regions located in West Germany and other regions in Northern and Central Italy. Figure 7 plots the main linkages for 1999, when the euro was introduced. The figure shows a topological structure similar to that of 1981, the difference being that, during this period, some Spanish regions became connected to the center (Ile de France). These results imply that the implementation of the euro converted Spain, which at that time was following a prolonged expansionary path, into a significant contributor to the European business cycle dynamics.\footnote{Notice that during this period the relative weight of the Spanish regions was very similar. Hence, synchronization increased.}

Figure 8 plots the main regional linkages for 2009, in the middle of the GR, showing a significantly more connected topological structure of the European network. In particular, some Spanish regions also became connected to French, Italian and English regions.

4 Synchronization and sectoral composition

In previous sections we have studied the synchronization of the European regions across time and space, and have assessed the role played by the economic size of regions in characterizing the propagation of business cycle shocks. In this section, we examine the role played by other factors in explaining the evolution of the cyclical synchronization between regions, paying particular attention to the similarities of their productive structure. Sectoral composition could play a significant role in explaining synchronization since, in the absence of data on intra-industry trade at the regional level, sectoral composition may be viewed as a proxy for the transmission channel of shocks between regions.

The literature on synchronization has mainly focused on analyzing the role of sectoral composition on determining business cycle synchronization at the country level.\footnote{Clark and Wincoop (2001) examine several measures of sectoral dissimilarity. They find that these measures explain, to some extent, the low cross-country correlation of employment between the US and the EU, but they find no correlation with GDP. Imbs (2004) computes a specialization index of industries and identifies a low business cycle correlation between highly specialized regions.} However, studies that investigate up to which extent sectoral patterns can explain regional synchronization are quite scarce and their findings are not conclusive. For example, Barrios et al. (2003) compute industrial similarity using a sample of eleven UK regions and six euro area countries for the period 1966-1997 and find that industrial similarity does not explain a decline in the UK-EU business cycles correlations. Instead, Belke and Heine
(2006) test the impact of industrial structure on the regional employment cycles of thirty European regions for the period 1975-1996, finding that differences in regional industry structure account for the decline in synchronicity among regions.\textsuperscript{16} Since these studies relied simple linear regression approaches, a plausible reason explaining the differences in their results could be the existence of a nonlinear relationship between sectoral composition similarities and business cycle synchronization, or a potential instability in such relationship over time, or both of them. In this section, we tackle these features in a parsimonious way to provide a robust assessment about the relationship between sectoral composition and business cycle synchronization at the regional level.

4.1 Changes in sectoral composition

First, we proceed to quantify the degree of similarity in bilateral sectoral composition experienced by the European regions. In particular, we compute the variable sectoral, which captures the similitude between the productive structures of the regions. We adapt the expression used by Imbs (2004) and measure sectoral composition similarity as follows,

\begin{equation}
C_{ab}^{t} = 1 - \frac{\sum_{i=1}^{k} |c_{i;a}^{t} - c_{i;b}^{t}|}{2},
\end{equation}

where $c_{i,j}^{t}$ is the employment share of sector $i$ in region $j$ at time $t$. Given the available data, we consider 6 sectors: Agriculture; Industry; Construction; Wholesale, retail, transport and distribution, communications, hotels and catering (WRTDCHC); Financial and business services (FBS); and Non-market services. $C_{a,b}^{t}$ ranges from 0, when the sectoral structures of $a$ and $b$ are radically different, to 1, when the sectoral structures are identical.

In Figure 9, we plot the estimated density of $C_{t}^{a} = \{C_{a,b}^{t} : \forall a \neq b\}$ for selected years. The figure indicates an increase in the sectoral similarity over time, which can be observed in the displacement of the kernel density mass towards the right tail. Moreover, the data shows that this has been a gradual but persistent pattern over the entire sample period (1980-2011), rising from 0.78 on average in 1980 to 0.85 in 2011, as can be see in the heat maps of Figure 13, in Appendix B. The heat maps offer richer information if one is interested in analyzing a particular set of regions. For instance, in the map of 1980, it is possible to identify cold areas that correspond to Greek regions that report degrees of sectoral similarity around 0.3.\textsuperscript{17} This result has important implications for the

\textsuperscript{16}Also, Barrios and De Lucio (2003) show that the more similar the sectoral structures of Iberian regions (1975-1998) are, the more correlated employment cycles are.

\textsuperscript{17}Figures 13 (a) and (b) plot the sectoral similarity matrix in the form of heat maps for beginning and
propagation of shocks throughout the entire European economy. The more similar is the economic structure of regions, the more similar would be their responsiveness to shocks, potentially amplifying their effects at the aggregate level.

4.2 Parametric Regression Analysis

We start analyzing the relationship between synchronization and sectoral composition by controlling for additional potential determinants of regional synchronization. To do so, we work with the original synchronization measures, $\delta_{t}^{ab}$, and model it as a function of sectoral composition, $C_{t}^{ab}$, and a set of regressors. The regressors can be sorted into two groups. Institutional and geographical factors:

- **EMU**: defined as a dummy that takes value 1 if both regions belong to the EMU and 0 otherwise.
- **Groups**: defined as a dummy that takes value 1 if both regions belong to the same group of countries and 0 otherwise; the groups are defined as “Central countries” (BE, DE, FR, NL, LU, AT), “Nordic countries” (DK, SE, NO, FI), “Mediterranean countries” (EL, IT, PE, ES), “Island countries” (IE, UK).
- **Country**: defined as a dummy that takes value 1 if both regions belong to the same country and 0 otherwise.

Factors of economic importance of regions, proxied by their economic size:

- **Size-reg**: defined as the economic weight of each pair of regions: $\frac{Y_{a,t} + Y_{b,t}}{\sum_{i=1}^{n} Y_{i,t}}$, where $Y_{i}$ is the real GDP of region $i$ and $n$ is the total number of regions.
- **Size-country**: defined as the economic weight of the countries to which each pair of regions belong: $\frac{Z_{a,t} + Z_{b,t}}{\sum_{i=1}^{\eta} Z_{i,t}}$, where $Z_{i}$ is the real GDP of the country to which of region $i$ belong and $\eta$ is the number of countries.

Notice that the first three variables are dummies that represent geographical and institutional characteristics so they have no time dimension. However, the remaining three are time-varying.\(^{18}\)

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end of the sample, 1980 and 2011, respectively. Intermediate maps, which are not presented to save space, and the detail by region are available upon request.

\(^{18}\)We have calculated the covariance matrix in order to discard multicolinearity problems. We find that correlations are low, except in the cases of groups and country (0.50), groups and emu (0.41) and size-country and size-reg (0.29).
In order to account for the heterogeneity in the cross-sectional dimension and over time, we estimate a panel regression with fixed time effects.\footnote{We have tested the presence of unit roots in time-varying variables, such as synchronization and the sectoral index. The Augmented Dickey-Fuller test (Dickey and Fuller, 1979) and the Mz test of Ng and Perron (2001) do not reject the null hypothesis in almost all cases. However, we can accept the existence of cointegration between time-varying variables using the test of Phillips and Moon (1999). Moreover, Phillips and Moon (1999) show that the fact that \( n \gg t \), as in our case, guarantees the consistency of our estimates.}

\[
\delta_{it}^{ab} = \beta X_{it}^{ab} + \tau_t + \epsilon_{it}^{ab},
\]

(19)

where \( X_{it}^{ab} = [\text{Constant}_{it}^{ab}, EMU_{it}^{ab}, Groups_{it}^{ab}, Country_{it}^{ab}, Size-reg_{it}^{ab}, Size-country_{it}^{ab}, Sectoral_{it}^{ab}] \). The estimation results are reported in Table 2, and indicate that all the explanatory factors exhibit a positive and statistically significant relationship with the synchronization between regions. Although, notice that the magnitudes of the coefficients are in general small, with the exceptions of the variables size-reg and sectoral, implying that after controlling for other factors, sectoral composition significantly explains changes in synchronization.

To investigate potential instabilities over time in the relationship between regional synchronization and its driving factors, we estimate a cross-sectional regression for each time period (year) in the sample.

\[
\delta_{jt}^{ab} = \beta_X X_{jt}^{ab} + \epsilon_{jt}^{ab}.
\]

(20)

Accordingly, we run \( T \) regressions by OLS and save the associated time-varying coefficients, \( \beta_t \), for \( t = 1, 2, ..., T \). The dynamics of the estimated regression coefficients for all the years are displayed in Figure 10. The results indicate that the estimated dynamic coefficients, \( \hat{\beta}_t \), in general, fluctuate closely around the constant estimates from the panel regression, \( \hat{\beta} \), implying that the relationship between synchronization and its driving factors has remained relatively stable over time. However, there are some important exceptions, especially since the GR.

Regarding the institutional factors, the EMU membership (\( emu \)) has a positive and significant effect that begins in the 90s and increases after the Maastricht Treaty, remaining high during the creation of the euro. Nevertheless, the effect becomes negative when the GR affected with different intensities to eurozone countries. Belonging to the same group of countries (\( group \)) in general does not explain regional synchronization since its coefficient is hardly ever significant, with the exception of the subsequent years after the GR, when
the effect is significantly positive. The variable *country* is positive and significant, pointing to a strong intra-country synchronization. These results imply that the arrival of the GR triggered a stronger synchronization between regions of the same group of countries.

Regarding the economic factors, Figure 10 shows that their corresponding relationships with regional synchronization are relatively stable over time, with the exception in the last part of the sample. Since the GR the correlation between synchronization and the economic factors under consideration, *size-reg*, *size-country* and *sectoral*, significantly increased. To sum up, these results show that, accounting for additional factors, the importance of sectoral composition in explaining synchronization seems to have significantly increased since the Great Recession.

### 4.3 Nonparametric Regression Analysis

Previous studies have used econometric approaches similar to the ones described in Section 4.2 to provide evidence about that greater similarities in sectoral composition across countries are positively related to business cycle synchronization at the country level (Ductor and Leiva-Leon (2016) and Imbs (2004)). However, when dealing with data at a higher level of disaggregation, e.g. at the regional level, the associated larger heterogeneity inherent in the data may be characterized by nonlinear relationships between the variables under study. Therefore, to provide robust assessments about the relationship between similarities in business cycles, \( \delta_t^{ab} \), and similarities in sectoral composition, \( C_t^{ab} \), we take advantage of the rich set of information composed the 22,578 pairwise linkages between the 213 NUTS-2 European regions, and rely on minimal assumptions to employ nonparametric methods.

Our focus is on assessing the role that similarities in regional sectoral composition play in explaining regional business cycle synchronization patterns, and how this role has evolved over time. In doing so, we first compute the average synchronization over each possible level of sectoral composition similarity, \( E_t(\delta_t^{ab}|C_t^{ab} = c) \). Notice that the conditional mean can be calculated for every period of time, \( t = 1, 2, ..., T \). To compute the conditional expectation, we use the Nadaraya-Watson estimator (Local Constant) with a Gaussian kernel. The selection of the optimal bandwidth is based on cross-validation using the Quartic kernel for each year. To guarantee robustness of our results we add-

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\(^{20}\)Clark and van Wincoop (2001) confirm the existence of a border effect on within-country correlations (of some French and German regions) larger than on cross-country correlations. Montoya and de Haan (2008) find that within-country cycles are always more correlated than with respect to the Euro area cycle, being their sample 53 regions (12 countries). The previous findings contradict those of Fatas (1997), who suggests than the effect of national borders has been reduced over time for 38 regions (4 countries).
tionally use the Local Linear estimator proposed by Stone (1977)\textsuperscript{21}. Chart (a) of Figure 11 plots the Local Constant estimator of the conditional average synchronization for selected years, showing two main features. First, since the early 80s until the mid 2000s the expected synchronization remained at moderated levels, around 0.5. However, since 2009 the levels of synchronization significantly increased. Second, before the GR the expected synchronization remained relatively constant across the different levels of sectoral composition, while since the GR, there has been a clear positive relationship between sectoral similarities and business cycle similarities.\textsuperscript{22}

Notice that if $\delta^{ab}$ is independent of $C^{ab}$, then $E(\delta^{ab}|C^{ab}) = E(\delta^{ab})$. Therefore, we can proxy the role of sectoral similarities in explaining regional synchronization with the difference between the conditional and the unconditional expectations,

\begin{equation}
\Delta^{ab}_t(c) = E_t(\delta^{ab}_t|C^{ab}_t = c) - E_t(\delta^{ab}_t),
\end{equation}

where the unconditional expectation is calculated as the simple cross-sectional average. Since $\Delta^{ab}_t(c)$ can be also computed for $t = 1, 2, ..., T$, we are able to investigate how the role of sectoral similarities has evolved over time. When $\Delta^{ab}_t(c)$ is close to zero, it implies that the explanatory power of sectoral similarities is negligible. Conversely, the larger is $\Delta^{ab}_t(c)$ the more informative is sectoral similarities to explain synchronization patterns. Chart (b) of Figure 11 plots $\Delta^{ab}_t(c)$ from selected years, reinforcing the evidence about a significantly increase in the explanatory power of sectoral composition similarities on regional business cycle interlinkages since the GR. These results are consistent with the findings in Section 4.2.

Moreover, Chart (b) of Figure 11 unveils that the GR triggered a nonlinear relationship between synchronization and sectoral similarities, as can be seen in $\Delta^{ab}_t(c)$ for $t = 2010, 2011$. In particular, such relationship is convex for values of sectoral similarity lower than around 0.8, while it becomes concave for values of sectoral similarity higher than 0.8. It implies that increases in sectoral similarity between regions that already exhibit a very similar productive structures are expected to yield small increases in business cycle synchronization. While, increases in the sectoral similarity between regions with different productive structures are expected to yield larger increases in regional business cycle synchronization. Notice that this asymmetric effect is also present in the years for

\textsuperscript{21}For a discussion of both methods see Liu (2011).

\textsuperscript{22}Figure 11 shows the expected synchronization conditional on the domain of sectoral similarities corresponding to the interval $[0.7, 0.9]$ since most of the mass of the distribution is concentrated in that interval, as shown in Figure 9.
the years prior the GR but at a much smaller magnitude. For robustness, we repeat the same exercises but using the Local Linear method instead of the Local Constant method to estimate the conditional expectation. The results are plotted in Chart (a) of Figure 15 in Appendix B, pointing to the same conclusions. These results have important implications for the Cohesion Policy of the European Union, that has as main objectives avoiding regional disparities, such as dissimilarities in regional business cycle fluctuations, and restructuring declining industrial areas and diversify rural areas.

From these results we can conclude that the propagation of business cycle shocks among European regions have become more dependent on the regional production structure after the GR. The non-parametric analysis has provided information about the type of relationship between $\delta_{ab}$ and $C_{ab}$, making minimal assumptions, and has allowed us to quantify the sectoral composition effect more accurately than using parametric regression analysis. This method has enabled us to confirm the asymmetric effect of the sectoral composition that was hidden in panel data analysis or time-dependent regressions. Next, we analyze more in depth where such asymmetric effect is coming from. In particular, we aim to identify the sector or sectors contributing the most to such asymmetric effect of sectoral composition on business cycle synchronization. In doing so, we disaggregate the production structure of the regions into the corresponding sectors, as follows,

$$C_{t}^{i,ab} = 1 - \frac{|c_{t}^{i,a} - c_{t}^{i,b}|}{c_{t}^{i,a} + c_{t}^{i,b}},$$

where $C_{t}^{i,ab}$ is the contribution of sector $i$ to the sectoral composition similarity between regions $a$ and $b$. As in the aggregate case, this disaggregated index ranges from 0 to 1, taking the value of 0 when the weights associated to sector $i$ of regions $a$ and $b$ are radically different, and taking the value of 1 when the weights of sector $i$ are identical for both regions. Accordingly, we measure the contribution of similarities in each sector in explaining regional synchronization by computing,

$$\Delta_{t}^{i,ab}(c) = E_{t}(\delta_{t}^{ab}|C_{t}^{i,ab} = c^{i}) - E_{t}(\delta_{t}^{ab}),$$

for $i = \text{Agriculture, Industry, Construction, WRTDCHC, FBS, NM services}$. Figure 12 shows the contributions of each sector estimated with the Local Constant method for every period of time in 3-D plots. The figure indicates that the sectors of Agriculture, Industry and Construction have an almost flat pattern around zero. While, the sectors associated to wholesale, retail, transport, distribution, communications, hotels and catering
(WRTDCHC), Financial and business services (FBS), and Non-market services exhibit an asymmetric effect of business cycles synchronization, especially after 2009. This implies that the increase in the ability of sectoral composition in explaining European regional synchronization since the Great Recession is mainly associated to the similarity between services-related industries. We also perform the same exercise but relying on the Local Linear estimator, but the results remained unchanged (see Figure 15 in Appendix B).

5 Conclusions

This paper analyzes changes, across time and space, in the synchronization of European regional business cycles and investigates the role that the sectoral composition, and other explanatory factors, have played in those changes. It also identifies the regions that are the main channels in the transmission of business cycle shocks. Our sample has a wider and more disaggregated geographical and temporal coverage than previous literature. To deal with regional data characteristics, we propose a new method to measure time-varying synchronization in small samples that combines regime-switching models and dynamic model averaging.

The results show that, in only two years, the GR synchronized Europe twice as much as the European Union process did in decades. A gradual increase in synchronization is also observed during the five years previous to the introduction of the euro. We find that most of the core countries experienced an increase in cohesion over time while the Nordic countries, Greece, Ireland and the Netherlands show a different pattern, with a general decrease of internal synchronization from the mid 90s until 2007. Considering the role of the economic size of the regions in our analysis, we observe that Ile de France is the region that is the main channel in the propagation of business cycles shocks in Europe. The regional linkages have grown over time. Increases in sectoral composition similarity have a positive effect on business cycle synchronization only for regions that already have high levels of similarity in their productive structure. Moreover, we find that European regions became more dependent on the production structure after the GR. At a disaggregated level, services, and especially services related to industry, are primarily responsible for this significant and asymmetric effect.

Our findings provide crucial information for policymakers in the implementation of Cohesion Policy as we not only offer a comprehensive framework of regional dynamics in the last three decades but also determine which economic regions are the most sensitive to policy changes or shocks.
References


Tables

Table 1: Simulation results

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Note. The table reports the average QPS for both methods based on 1,000 replications.

Table 2: Estimation of panel

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Note. HAC estimation; time fix-effects included.
Figures

Figure 1: Time-varying Regional Business Cycle Synchronization

(a) Ile de France vs. Inner London

(b) Madrid vs. Lazio

(c) Cataluña vs. Languedoc Roussillon

(d) Oslo og Akershus vs. Sør-Østlandet

Note. The figure plots the time-varying synchronization between pairs of European regions. Solid lines represent the 0.5 quantile and dashed lines represent the 0.75 and 0.25 quantiles of the distribution of $\delta_{t}^{ab}$. 
Figure 2: Region-specific aggregate synchronization in Europe

Note: In red, regions with an aggregate synchronization higher than 0.5.
Figure 3: Region-specific aggregate synchronization in Europe (cont.)

Note. In red, regions with an aggregate synchronization higher than 0.5.
Figure 4: Region-specific aggregate synchronization in Europe (cont.)

Note. In red, regions with an aggregate synchronization higher than 0.5.
Figure 5: European and Regional Intra-synchronization across Countries

(a) Time-varying European cohesion

(b) Time-varying country-specific cohesion

(c) Time-varying country-specific cohesion

Note. Chart (a) plots the European cohesion measure over time. Charts (b) and (c) plots the country-specific cohesion measures over time. The right axis in Chart (c) corresponds to Finland.
Note. A red line between two regions indicates a weighted synchronization higher than or equal to 0.3 between those regions.
Figure 7: Main European regional economic linkages (cont.)

1999

Note. A red line between two regions indicates a weighted synchronization higher than or equal to 0.3 between those regions.
Figure 8: Main European regional economic linkages (cont.)

2009

Note. A red line between two regions indicates a weighted synchronization higher than or equal to 0.3 between those regions.
Figure 9: Density of sectoral similarity across regions

Note. The figure plots the kernel estimates of the distribution of sectoral similarities across the European regions for selected periods. Sectoral similarity is measured following Imbs (2004).
Figure 10: Coefficients and confidence interval of time regressions

Note. Solid lines in each chart show the OLS coefficient estimates and dashed lines show the 95% confidence interval.
Figure 11: Expected synchronization conditional to sectoral similarity

(a) Conditional mean

(b) Conditional minus unconditional mean

Note. Chart (a) shows the conditional expectation of synchronization for selected periods of time, that is, $$E_t(\hat{\gamma}^{ab}_t | C^{ab}_t = c)$$. Chart (b) shows the conditional minus the unconditional expectation of synchronization for selected periods of time, that is, $$\Delta_t^{ab} = E_t(\hat{\gamma}^{ab}_t | C^{ab}_t = c) - E_t(\hat{\gamma}^{ab}_t)$$. 

39
Figure 12: Disaggregated local constant estimator

Note. The charts show the conditional minus the unconditional expectation of synchronization over time for different sectors.
Appendix A

Bayesian estimation

Since the likelihood function of this model is conditional on several states, the estimation of parameters obtained with the maximum likelihood approach could become cumbersome. Therefore, we use a Bayesian approach to estimate this model. This approach allows us to provide a measure of uncertainty about the parameter estimates and about the degree of synchronization $\delta_t$.

The approach to estimate the vector of parameters $\theta$ relies on a bivariate extended version of the multi-move Gibbs-sampling procedure implemented by Kim and Nelson (1999) for Bayesian estimation of univariate Markov-switching models. In this setting, (i) the parameters of the model $\theta$, (ii) the Markov-switching variables $\tilde{S}_{a,T} = \{S_{a,t}\}_T^1$, $\tilde{S}_{b,T} = \{S_{b,t}\}_T^1$, $\tilde{S}_T = \{S_t\}_T^1$, and (iii) the synchronization degree $\tilde{\delta}_T = \{\delta_t\}_T^1$, are treated as random variables given the data in $\tilde{y}_T = \{y_t\}_T^1$. The purpose of this Markov chain Monte Carlo simulation method is to approximate the joint and marginal distributions of these random variables by sampling from conditional distributions.

The Gibbs sampler used in the estimation procedure can be briefly described in the following steps:

**Step 1**: Generate the latent variables $\tilde{S}_{a,T}$, $\tilde{S}_{b,T}$, $\tilde{S}_T$, and the weights $\tilde{\delta}_T$, conditional on the data $\tilde{y}_T$ and the vector of parameters, $\theta$.

**Step 2**: Generate the transition probabilities associated with each latent variable, $p_{00,a}$, $p_{11,a}$, $p_{00,b}$, $p_{11,b}$, $p_{00}$, $p_{11}$, conditional on $\tilde{S}_{a,T}$, $\tilde{S}_{b,T}$, $\tilde{S}_T$, and $\tilde{\delta}_T$.

**Step 3**: Generate the means associated with the factors, $\tilde{\mu}_{a,0}$, $\tilde{\mu}_{a,1}$, $\tilde{\mu}_{b,0}$, $\tilde{\mu}_{b,1}$, conditional on $\tilde{S}_{a,T}$, $\tilde{S}_{b,T}$, $\tilde{S}_T$, $\tilde{\delta}_T$ and $\tilde{y}_T$.

**Step 4**: Generate the variance-covariance matrix, with elements $\sigma_{a}^2$, $\sigma_{b}^2$, $\sigma_{ab}$, conditional on $\tilde{\mu}_{a,0}$, $\tilde{\mu}_{a,1}$, $\tilde{\mu}_{b,0}$, $\tilde{\mu}_{b,1}$, $\tilde{S}_{a,T}$, $\tilde{S}_{b,T}$, $\tilde{S}_T$, $\tilde{\delta}_T$ and $\tilde{y}_T$.

**Step 5**: Generate the forgetting factor, $\alpha$, conditional on $\tilde{\delta}_T$ and the variance of its linearized innovations $\sigma_\delta$.

**Step 6**: Generate the variance $\sigma_\delta$, conditional on $\tilde{\delta}_T$ and $\alpha$.

Steps 1 through 6 can be iterated $L + M$ times, where $L$ is large enough to ensure that the Gibbs sampler has converged. Thus, the marginal distributions of the state variables, synchronization variable and the parameters of the model can be approximated by the empirical distribution of the $M$ simulated values. For our empirical application, we set $M = 6000$ and $L = 1000$. 

41
Priors

For the mean and variance parameters in vector $\theta$, the independent Normal-Wishart prior distribution is used:

$$p(\mu, \Sigma^{-1}) = p(\mu)p(\Sigma^{-1}),$$

(24)

where

$$\mu \sim N(\mu, V_{\mu})$$

$$\Sigma^{-1} \sim W(\mathcal{S}^{-1}, \upsilon),$$

and the associated hyperparameters are given by $\mu = (-1, 2 - 1, 2)'$, $V_{\mu} = I$, $\mathcal{S}^{-1} = I$, $\upsilon = 0.23$.

For the transition probabilities $p_{a,00}, p_{a,11},$ of $S_{a,t}$, $p_{b,00}, p_{b,11},$ of $S_{b,t}$, and $p_{00}, p_{11},$ of $S_t$, Beta distributions are used as conjugate priors:

$$p_{k,00} \sim Be(u_{k,11}, u_{k,10}), p_{k,11} \sim Be(u_{k,00}, u_{k,01}),$$

(25)

$$p_{00} \sim Be(u_{11}, u_{10}), p_{11} \sim Be(u_{00}, u_{01}),$$

(26)

where the hyperparameters are given by $u_{a,01} = u_{b,01} = u_{01} = 2$, $u_{a,00} = u_{b,00} = u_{00} = 8$, $u_{a,10} = u_{b,10} = u_{10} = 1$ and $u_{a,11} = u_{b,11} = u_{11} = 9$.

To be able to generate draws of the forgetting factor, we linearize the equation associated to the dynamics of $\tilde{\delta}_T$ and treat it as a linear regression, for which we use a Normal-Gamma prior distribution given by:

$$\alpha \sim N(\alpha, \sigma_\alpha)$$

$$\sigma_{\tilde{\delta}}^{-1} \sim G(\mathcal{S}^{-1}, \upsilon),$$

with hyperparameters associated to the Normal distribution, $\alpha = 0.9$, $\sigma_\alpha = 0.1$, and to the Gamma distribution, $\mathcal{S}^{-1} = 0$, $\upsilon = 0$.

23 For the empirical application, due to the substantial heterogeneity in the magnitude of growth rates at the regional level, we use $\mu = (\mu_{a-}, \mu_{a+}, \mu_{b-}, \mu_{b+})$, as hyperparameter priors for the means, where $\mu_{\cdot -}$ and $\mu_{\cdot +}$ are the means of positive and negative values of $y_{i,t}$, respectively, for $i = a, b$. 

42
**Gibbs sampler**

**Draw** \( \tilde{S}_a, \tilde{S}_b, \tilde{S}_T, \tilde{\delta}_T \).

In order to make inferences on the variables \( \tilde{S}_a, \tilde{S}_b, \tilde{S}_T \) and \( \tilde{\delta}_T \), we follow the line of Kim and Nelson (1999) and compute draws from the conditional distributions:

\[
g(\tilde{S}_k|\theta, \tilde{y}_T) = g(S_k|\tilde{y}_T) \prod_{t=1}^{T} g(S_{k,t}|S_{k,t+1}, \tilde{y}_t), \quad \text{for } k = a, b \tag{27}
\]

\[
g(\tilde{S}_T|\theta, \tilde{y}_T) = g(S_T|\tilde{y}_T) \prod_{t=1}^{T} g(S_t|S_{t+1}, \tilde{y}_t). \tag{28}
\]

In order to obtain the two terms in the right-hand side of Equations (27) and (28), the following two steps can be employed:

**Step 1**: Conditional on the parameters \( \theta \), we run the filtering algorithm proposed in Section 2.2, get a draw of \( \tilde{\delta}_T \), and compute the terms \( g(\tilde{S}_k|\tilde{y}_t) \) for \( k = a, b \), and \( g(\tilde{S}_t|\tilde{y}_t) \) for \( t = 1, 2, \ldots, T \), save them and take the elements for which \( t = T \).

**Step 2**: The product in the second term of the right-hand side of Equations (27) and (28) can be obtained for \( t = T - 1, T - 2, \ldots, 1 \), by following the result:

\[
g(S_t|\tilde{y}_t, S_{t+1}) = \frac{g(S_t, S_{t+1}|\tilde{y}_t)}{g(S_{t+1}|\tilde{y}_t)} \propto g(S_{t+1}|S_t)g(S_t|\tilde{y}_t), \tag{29}
\]

where \( g(S_{t+1}|S_t) \) corresponds to the transition probabilities of \( S_t \) and \( g(S_t|\tilde{y}_t) \) was saved in Step 1.

Then, we compute

\[
\Pr[S_t = 1|S_{t+1}, \tilde{y}_t] = \frac{g(S_{t+1}|S_t = 1)g(S_t = 1|\tilde{y}_t)}{\sum_{j=0}^{1} g(S_{t+1}|S_t = j)g(S_t = j|\tilde{y}_t)}, \tag{30}
\]

and generate a random number from a \( U[0,1] \). If that number is less than or equal to \( \Pr[S_t = 1|S_{t+1}, \tilde{y}_t] \), then \( S_t = 1 \), otherwise \( S_t = 0 \). The same procedure applies for \( S_{a,t} \) and \( S_{b,t} \).

**Draw** \( p_{a,00}, p_{a,11}, p_{b,00}, p_{b,11}, p_{00}, p_{11} \).

Conditional on \( \tilde{S}_k \) for \( k = a, b \), and \( \tilde{S}_T \), the transition probabilities are independent from the data set and the model’s parameters. Hence, focusing on the case of \( \tilde{S}_T \), the likelihood
function of \( p_{00}, p_{11} \) is given by

\[
L(p_{00}, p_{11} | \tilde{S}_T) = p_{00}^{n_{00}} (1 - p_{00}) p_{11}^{n_{11}} (1 - p_{11}),
\]

(31)

where \( n_{ij} \) refers to the transitions from state \( i \) to \( j \), accounted for in \( \tilde{S}_T \).

Combining the prior distribution with the likelihood, the posterior distribution is given by

\[
p(p_{00}, p_{11} | \tilde{S}_T) \propto p_{00}^{n_{00} + n_{00} - 1} (1 - p_{00})^{n_{01} + n_{01} - 1} p_{11}^{n_{11} + n_{11} - 1} (1 - p_{11})^{n_{10} + n_{10} - 1},
\]

(32)

which indicates that draws of the transition probabilities will be taken from

\[
p_{00} | \tilde{S}_T \sim \text{Be}(u_{00} + n_{00}, u_{01} + n_{01}), \quad p_{11} | \tilde{S}_T \sim \text{Be}(u_{11} + n_{11}, u_{10} + n_{10}).
\]

(33)

The same procedure applies for the cases of \( \tilde{S}_{k,T} \) for \( k = a, b \).

**Draw** \( \mu_{0,a}, \mu_{1,a}, \mu_{0,b}, \mu_{1,b} \).

The bivariate Markov-switching model can be compactly expressed as

\[
\begin{bmatrix}
y_{a,t} \\ y_{b,t}
\end{bmatrix} =
\begin{bmatrix}
1 & S_{a,t} & 0 & 0 \\
0 & 0 & 1 & S_{b,t}
\end{bmatrix}
\begin{bmatrix}
\mu_{a,0} \\ \mu_{a,1} \\ \mu_{b,0} \\ \mu_{b,1}
\end{bmatrix}
+ \begin{bmatrix}
e_{a,t} \\ e_{b,t}
\end{bmatrix},
\begin{bmatrix}
e_{a,t} \\ e_{b,t}
\end{bmatrix} \sim N\left(\begin{bmatrix}0 \\ 0\end{bmatrix}, \begin{bmatrix}
\sigma_a^2 & \sigma_{ab} \\ \sigma_{ab} & \sigma_b^2
\end{bmatrix}\right)
\]

(34)

stacking as

\[
y =
\begin{bmatrix}
y_1 \\ y_2 \\ \vdots \\ y_T
\end{bmatrix},
\tilde{S} =
\begin{bmatrix}
\tilde{S}_1 \\ \tilde{S}_2 \\ \vdots \\ \tilde{S}_T
\end{bmatrix},
\text{and } \xi =
\begin{bmatrix}
\xi_1 \\ \xi_2 \\ \vdots \\ \xi_T
\end{bmatrix}.
\]

The model in Equation (34) remains written as a normal linear regression model with an error covariance matrix of a particular form:

\[
y = S \mu + \xi, \quad \xi \sim N(0, I \otimes \Sigma)
\]

(35)

Conditional on the covariance matrix parameters, state variables and the data, by using
the corresponding likelihood function, the posterior distribution $p(\mu | \tilde{S}_{a,T}, \tilde{S}_{b,T}, \tilde{S}_T, \tilde{\delta}_T, \Sigma^{-1}, \tilde{y}_T)$ takes the form

$$
\mu | \tilde{S}_{a,T}, \tilde{S}_{b,T}, \tilde{S}_T, \tilde{\delta}_T, \Sigma^{-1}, \tilde{y}_T \sim N(\bar{\mu}, \nabla_\mu),
$$

(36)

where

$$
\begin{align*}
\nabla_\mu &= \left( V_\mu^{-1} + \sum_{t=1}^{T} \bar{S}_t \Sigma^{-1} \bar{S}_t \right)^{-1} \\
\bar{\mu} &= V_\mu^{-1} \mu + \sum_{t=1}^{T} \bar{S}_t \Sigma^{-1} y_t.
\end{align*}
$$

After drawing $\mu = (\mu_{a,0}, \mu_{a,1}, \mu_{b,0}, \mu_{b,1})'$ from the above multivariate distribution, if the generated value of $\mu_{a,1}$ or $\mu_{b,1}$ is less than or equal to 0, that draw is discarded; otherwise, it is saved, in order to ensure that $\mu_{a,1} > 0$ and $\mu_{b,1} > 0$.

**Draw $\sigma_a^2, \sigma_b^2, \sigma_{ab}$.**

Conditional on the mean parameters, state variables and the data, by using the corresponding likelihood function, the posterior distribution $p(\Sigma^{-1} | \tilde{S}_{a,T}, \tilde{S}_{b,T}, \tilde{S}_T, \tilde{\delta}_T, \mu, \tilde{y}_T)$ takes the form

$$
\Sigma^{-1} | \tilde{S}_{a,T}, \tilde{S}_{b,T}, \tilde{S}_T, \tilde{\delta}_T, \mu, \tilde{y}_T \sim W(\overline{\Sigma}^{-1}, \overline{\sigma}),
$$

(37)

where

$$
\begin{align*}
\overline{\sigma} &= T + \nu \\
\overline{\Sigma} &= \overline{S} + \sum_{t=1}^{T} \left( y_t - \tilde{S}_t \mu \right) \left( y_t - \tilde{S}_t \mu \right)',
\end{align*}
$$

After $\Sigma^{-1}$ is generated, the elements in $\Sigma$ are recovered.

**Draw $\alpha$**

In our context of finite samples, small variations of in $\alpha$ may end up affecting significantly the dynamics of $\delta_t$. Therefore, to avoid this drawback, instead of simply imposing a given forgetting factor $\alpha$, we take advantage of the Bayesian procedure to estimate the other elements of the model and generate draws to simulate the posterior distribution of $\alpha$. In particular, we linearize the predicted model probability $\pi_{t+1|t,D} = \delta_{t+1|t}$, in equation (13), to relate it with the updated model probability $\pi_{t|t,D} = \delta_{t|t}$, in equation (9), with the
following linear regression:
\[
\ln(\delta_{t+1|t}) = \alpha \ln(\delta_{t|t}) + \epsilon_t, \tag{38}
\]
where \(\epsilon_t\) is assumed to be normally distributed with variance \(\sigma_\delta\). Conditional on the variance \(\sigma_\delta\), by using the corresponding likelihood function, the posterior distribution \(p(\alpha|\sigma_\delta, \tilde{\delta}_T)\) takes the form
\[
\alpha|\sigma_\delta, \tilde{\delta}_T \sim N(\overline{\alpha}, \overline{\sigma}_\alpha), \tag{39}
\]
where
\[
\overline{\sigma}_\alpha = \left( \sigma_\alpha^{-1} + \sigma_\delta^{-1} \sum_{t=1}^T \ln(\delta_{t|t})^2 \right)^{-1},
\]
\[
\overline{\alpha} = \sigma_\alpha \left( \sigma_\alpha^{-1} \alpha + \sigma_\delta^{-1} \sum_{t=1}^T \ln(\delta_{t|t}) \ln(\delta_{t+1|t}) \right).
\]

After obtaining a draw of \(\alpha\) from the above posterior distribution, if \(0 < \alpha < 1\), it is saved; otherwise the draw is discarded to ensure the interpretation of a forgetting factors.

**Draw \(\sigma_\delta\)**

Conditional on the parameter \(\alpha\), and by using the corresponding likelihood function, the posterior distribution \(p(\sigma_\delta^{-1}|\alpha, \tilde{\delta}_T)\) takes the form

\[
\sigma_\delta^{-1}|\alpha, \tilde{\delta}_T \sim G(\overline{\sigma}^{-1}, \overline{\tau}), \tag{40}
\]
where
\[
\overline{\tau} = T + \overline{v},
\]
\[
\overline{\sigma} = \overline{s} + \sum_{t=1}^T \left( \ln(\delta_{t+1|t}) - \alpha \ln(\delta_{t|t}) \right)^2.
\]

After \(\sigma_\delta^{-1}\) is generated, the variance \(\sigma_\delta\) is recovered.