Pseudo-wealth and Consumption Fluctuations

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This paper provides an explanation for situations in which the state variables describing the economy do not change, but aggregate consumption experiences significant changes. We present a theory of pseudo-wealth—individuals’ perceived wealth that is derived from heterogeneous beliefs and expectations of gains in a bet. This wealth is divorced from real assets that may exist in society. The creation of a market for bets will imply positive pseudo-wealth. Changes in the differences of prior beliefs will lead to changes in expected wealth and hence to changes in consumption, implying ex-post intertemporal individual and aggregate consumption misallocations and instabilities. Thus, in the environment we describe, completing markets increases macroeconomic volatility, raising unsettling welfare questions.

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I. Introduction

This paper provides a new explanation for an important macro-economic phenomenon: There are many occasions in which the (physical) state variables describing the economy (the level of human and natural capital, the amount of plant and equipment) do not exhibit large changes but the state of the economy, including the levels of consumption of the society, changes dramatically.

This paper puts forward the hypothesis that at least some of this volatility arises from fluctuations in what we call pseudo-wealth—wealth that individuals perceive they have, but which is to some extent divorced from the physical assets that exist in society. We show that there can be sudden changes in the aggregate value of this pseudo-wealth, and that these fluctuations in turn can lead to high levels of volatility in aggregate consumption and to ex-post intertemporal consumption misallocations (in the sense of having paths of individual and aggregate consumption that are not as smooth as the individuals and the society wished ex-ante). We show, moreover, that the persistence of pseudo-wealth can lead to increasing levels of debt.

There is a challenge, however, in creating a persuasive theory of pseudo-wealth. If one assumes expectations are simply arbitrarily given, then a sudden change in expectations (the probability distribution of future states of nature) can obviously give rise to marked changes in the value of wealth. There is some evidence that especially dramatic changes in perceptions occur during times of crisis (Hoffman, Post, and Pennings, 2013; Kozlowski, Veldkamp, and Venkateswaran, 2015). The recent US financial crisis is just one more example of this. Prior to the crisis, many financial experts believed that the likelihood of a housing bubble was negligible (or at least that its effects would be contained), and presumably they persuaded many others that that was the case. Between 2007 and early 2009 there was a massive
change in beliefs. By early 2009, it was hard for anyone to maintain that there had not been a bubble. It is, of course, a challenge to reconcile these beliefs and their evolution over time with any theory of rational expectations.

The problem with this theory is that the task of explaining consumption volatility is too easy. This is a legitimate critique of “animal spirits.” More refined theories try to explain how distributions of beliefs change over time as a function of the new information the economy receives.

In this paper, we explore an explanation that is more tethered and less arbitrary. It is based on two key hypotheses:

1. There can exist large differences in prior beliefs that are sustained over time. Differences in views can exist even when individuals have rational expectations, so long as they have access to different information that leads to different priors, and as long as the structure of the economy is such that differences in priors persist over time.\(^1\) All that we require is that the assumptions which give rise to “common knowledge”—a state of affairs in which all individuals agree about the probabilities of different events—are not satisfied.

2. Differences in views, with betting markets, give rise to the creation of pseudo-wealth, with the aggregate expected wealth of market participants exceeding true wealth—i.e., a level of wealth consistent with societal beliefs that are feasible. Each side “expects” to win. Betting markets also lead to more uncertainty. If the positive effect of pseudo-wealth creation on demand is larger than the negative effect of uncertainty (due to the increase

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\(^1\) Differences in priors are a necessary condition for speculative trade. In models with heterogeneous information but common priors, no-trade theorems apply (Milgrom and Stokey, 1982). Differences in priors could also arise as the consequences of behavioral biases. For our analysis what matters is that the priors are different, independently of what creates those differences.
in precautionary savings) on demand, the result will be an increase in current levels of consumption. It is inevitable that (later) someone's expectations will be disappointed—indeed, in any betting market, someone is disappointed. The point here is that the disappearance of the bet will lead to destruction of pseudo-wealth. If the pseudo-wealth component was significant, then this moment will have macroeconomic significance.

We present a model of two agents who disagree in the probability that a sunspot event occurs. There is a market for short-term bets, and given the disagreement of beliefs, in equilibrium both agents will trade a bet. They both believe they have a relatively larger chance of winning, hence they both feel wealthier. However, this cannot be true for the aggregate, as the bet is not creating any real wealth. Once the sunspot occurs, it cannot occur again; hence the bet disappears, and even though no real wealth is destroyed, the expected wealth of both agents discontinuously decreases.

A financial innovation as the creation of a market for bets in a context of heterogeneous beliefs will lead to increases in expected wealth that will tend to increase consumption but also increases in volatility that will tend to increase savings for precautionary motives. We restrict the analysis to the cases where the wealth effects dominate over the precautionary savings effects. Our main result is that the interaction of disagreement of beliefs with a market for bets will create excessive ex-post aggregate consumption volatility, excessive either with respect

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2 We focus on those events because we want to isolate the effects of pseudo-wealth changes from those of other changes. In practice, many bets are about matters of economic substance—such as weather there is a housing or oil price bubble.

3 The sunspot can be taken as a metaphor for an event that rarely occurs, like a structural transformation, over which there is not a long history to have properly learned the true probability distribution that governs it.

4 One can extend the model to situations where new betting opportunities may open up. The central point is that spending will be related to the magnitude of betting opportunities (itself related to the magnitude of differences in judgments).
to a world of common beliefs or to a world with no market for bets. At the moment in which the bet disappears, the agent betting in favor of the sunspot will experience an increase in wealth, and the other agent will experience a decrease in wealth. However, the “pseudo-wealth” component of expected wealth will vanish, as the difference in views that was leading to a perception of higher wealth at the individual level will no longer be relevant. At this moment, aggregate consumption will decrease discontinuously.

Notably, the financial innovation that completes the market will create risk in an economy that would be otherwise stable—but from the viewpoint of individual beliefs, this will be Pareto efficient. But if the planner had prohibited the bet, the society would have experienced a smoother path of consumption, and each individual's consumption profile would have been smooth. If we allow the planner to take a stance on beliefs, and the planner uses beliefs that are consistent, then the betting equilibrium would be Pareto inferior.

The rest of the paper is organized as follows. Section I.A frames our paper in the existing literature. In section II, we present a simple framework that displays the presence of positive pseudo-wealth and analyze the implications for spending and savings of a financial innovation as the creation of a market for bets in an environment featured by heterogeneous beliefs. Section III solves the model for the certainty equivalence case. Our goal in that section is to illustrate the effects of the dynamics of pseudo-wealth on the dynamics of consumption in a tractable setting in which wealth effects dominate over precautionary savings effects. Section IV summarizes the main results of the model in terms of consumption volatility. Section V discusses the limitations of our assumptions and possible extensions. We show that in richer economic structures, fluctuations in pseudo-wealth can also have effects on actual wealth as well as distributional consequences. We also
analyze a case in which heterogeneous beliefs can give rise to negative pseudo-
wealth. Section VI presents the conclusions.

A. Related Literature

The issue of “excessive” consumption volatility has received much attention in the
macroeconomics literature. The term “excessive” indicates that the actual
consumption volatility cannot be explained by a benchmark model that would
imply a more stable path of consumption relative to output. Since Friedman, one of
the central hypotheses of modern macroeconomics is that agents smooth
consumption. The benchmark model typically invoked features a representative
agent model with rational expectations and transitory shocks to output. The existing
literature offers different types of deviations from that benchmark to explain the
higher levels of consumption volatility observed in times of high output volatility.

Aguiar and Gopinath (2007) introduce trend shocks in a real business cycle
framework. The volatility of trend shocks is larger in emerging economies than in
advanced economies, which implies the higher consumption volatility observed in
the former set of economies. But this approach does not really solve the quandary
noted above. Unlike our approach, this approach requires large changes in the state
variables (represented as permanent productivity shocks) to explain large changes
in consumption.

Another branch of related literature provides an explanation for changes in current
behavior as a response to today’s changes on expectations concerning the evolution

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5 This approach has been criticized by Garcia Cicco, Pancrazi, and Uribe (2010). They perform an econometric
estimation (using long time-series) of the parameters that govern the productivity processes, and obtain a lower
variance and persistence of the permanent component of growth shocks, making the model incapable of explaining
the high volatility of consumption relative to output in those economies.
of state variables in the future. For example, Beaudry and Portier (2004, 2006) and Jaimovich and Rebelo (2008) present a class of models where news about future total factor productivity drive changes in individuals’ decisions that could lead to a downturn in the present. Relatedly, Lorenzoni (2009) presents a theory of “news shocks”, in which business cycles are driven by changes in the expectations of the individuals about the present state of the economy—but expectations are formed on the basis of noisy public sources of information regarding long-run shifts in aggregate productivity. Although in this family of models it is possible to have situations in which the state of the economy changes with no changes in the current state variables, these frameworks—unlike our framework—still rely on changes in the future state variables that are anticipated in the present for explaining changes in the state of the economy observed also in the present.

Angeletos and La’O (2013) provide an explanation that can account for shifts in the expectations of economic activity with no changes in the fundamentals that describe the economy. Their theory accommodates the notion of animal spirits in a model of rational expectations with a unique equilibrium in an environment featured by incomplete information and imperfect communication. What drives fluctuations in the economy is an extrinsic shock⁶ that they dub “sentiment shock”, that can effectively alter beliefs in equilibrium. These shocks can be interpreted either as shocks to beliefs of endogenous economic outcomes — an interpretation corresponding to shocks to first-order beliefs — or as shocks to the beliefs that each agent (or island) forms about the beliefs of other agents (or islands) about her productivity — an interpretation corresponding to shocks to second-order beliefs.⁷ A positive correlation in the degree of agents or islands’ optimism or pessimism

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⁶ An extrinsic shock is a shock to residuals that does not affect any payoff.

⁷ Angeletos and La’O (2013)’s preferred interpretation is as shocks to first-order beliefs, a concept that is more amenable for empirical analysis, as surveys generally only contain information on first-order beliefs.
about the terms of trade with the other islands that each of them will face may emerge endogenously as agents learn from realized market outcomes. The pseudo-wealth theory that we develop in this paper does not require any shock to the expectations of endogenous market outcomes. The increase in individual and aggregate consumption that occurs when the market for bets is created is the pure consequence of the speculative trade, that is unrelated with market outcomes different than the expectation of a gain associated with the bet. In turn, the fall in aggregate consumption that occurs when pseudo-wealth is destroyed is also unrelated to expectations of market outcomes, but it is the consequence of the realization of a rare event that eliminates any difference in prior beliefs. In our framework, agents fully agree on any possible market outcome rather than the betting outcome probabilities at all moments. Besides, in the theory of pseudo-wealth the evolution of second-order beliefs is uninteresting: the premise is that before the realization of the rare event, agents “agree to disagree”.

The literature on learning as the basis of formation of expectations introduces deviations from the full information rational expectations hypothesis (cf. Evans and Honkapohja, 2001). Models with learning can account for a larger volatility of expected wealth due to the possibility of revisions of expectations. Accordingly, these models lead to larger macroeconomic volatility and endogenous business cycles (for example, Boz, Bora Durdu, and Daude (2011); Heymann and Sanguinetti (1998), Pintus and Suda (2015), Guzman and Howitt, 2016). Both this paper and the learning literature are predicated on imperfect knowledge. In both, changes in beliefs have real macroeconomic effects and can lead to volatility. While in the learning literature, macroeconomic fluctuations are related to changes in average beliefs that have macroeconomic consequences, here, fluctuations can arise even if there are no changes in average beliefs: it is changes in the dispersion
of beliefs which drives changes in aggregate consumption, and these changes can be triggered in a variety of ways.

There is a large literature that analyzes the consequences of heterogeneous beliefs. Geanakoplos (2010) offers an approach for explaining excessive volatility of asset prices based on the interaction between heterogeneous expectations, collateral constraints, and leverage. Bad news in the economic environment can be amplified through the interaction between leverage and collateral constraints, leading to large changes in the “marginal buyer” of an asset, and thus in asset market prices. In this approach, not everyone's expected wealth is reduced after the shock. (Only the expected wealth of the optimists who owned assets decreases.) In our approach, both optimists and pessimists will suffer a decrease in expected pseudo-wealth after the shock, as the betting market had previously allowed them to exploit differences in beliefs in a way that all of them were feeling too “optimistic” about their future wealth. Scheinkman and Xiong (2003) show how speculative behavior—defined as the agent’s willingness to pay a price for an asset above her valuation of it, due to the belief that he will be possible to sell it at an even higher price in the future—in a context of short-sale constraints and overconfidence—defined as the belief of an agent that her information is more accurate than it is—creates asset prices bubbles. In our framework, agents do not speculate on the behavior of others—they simply make betting and consumption decisions that they believe are optimal, independent of the potential behavior of others.

More generally, the paper is related to a sizable literature that investigates the consequences of financial innovations and financial constraints in an environment of heterogeneous beliefs, such as Fostel and Geanakoplos (2012), Simsek (2013a, 2013b), among others. The closest contribution to this paper is Iachan, Nenov, and

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8 The marginal buyer, who is the least optimist of the agents who buy the asset, is a more pessimistic agent after a bad shock.
Simsek (2015) (INS henceforth). While INS focus on the effects of a financial innovation (that could be equivalent to the creation of a market for bets in this paper) on savings, we focus both on the moment of the innovation and especially on the moment in which the innovation either disappears or is no longer relevant, and analyze the effects that those moments have on consumption, betting, and savings. Also, while INS focus on cases in which substitution effects dominate over income effects, we focus on the opposite case. Both papers could be seen as complementary.

This paper is the first step of a research agenda outlined in Stiglitz (2015) and Guzman and Stiglitz (2015a), and advanced in Guzman and Stiglitz (2015b) that intends to offer a general framework for understanding situations in which large changes in macroeconomic behavior are observed with no counterpart in the size of changes of the state variables describing the economy.

II. A baseline model of pseudo-wealth

A. Environment

The environment features a small open economy with two infinitely lived representative agents, indexed by \( i = A, B \).

In every period, each agent receives the same constant exogenous endowment of a single consumption good, denoted by \( y > 0 \).

\[ ^9 \text{Diamond and Stiglitz (1974) have demonstrated under what general conditions the optimal value of the control variable increases or decreases when there is a mean utility preserving increase in risk.} \]
There is a Poisson probability $\lambda$ for the arrival of a one-time exogenous event, a sunspot. There is disagreement on the value of $\lambda$: agent $A$ believes that the sunspot is more likely to occur than agent $B$ does, i.e., $\lambda^A > \lambda^B$.

Agents can borrow in the international credit market at the risk-free interest rate $r$.

We assume the instantaneous utility function $u\left(c^i_t\right)$ is continuous and strictly concave, $u'(c^i_t) > 0$ and $u''(c^i_t) < 0$, where $c^i_t \geq 0$ is the level of consumption of individual $i$ in period $t$.

**B. States**

The set of spaces in period $t, Z_t$, is dependent on the history of previous states. Until a sunspot occurs, there are two possible states: $Z_t = \{S, O\}$ if $z_j = O \forall j < t$, $z_t$ is the state realization in period $t$, $S$ refers to the sunspot state and $O$ to the no sunspot state. Once the sunspot occurs, it cannot occur again, and the state will be $z_t = O$ forever, i.e. $Z_t = \{O\}$ if $\exists z_j = S$, for any $j < t$. The sunspot can then be interpreted as a shock to prior beliefs—a shock that vanishes any difference in agents’ priors.

Figure 1 depicts the tree of possible states before the occurrence of the sunspot.
C. Bets and pseudo-wealth

We assume there is a market for short-term bets. Given the disagreement on $\lambda$, agents will trade a bet in equilibrium.

The existence of the market for bets completes the market, in the sense that for all agents and for all states there is an available asset such that the payoff in one state is positive and in the other state is zero.

Bets have two effects on the agents’ decisions:

1. Given the disagreement on the true value of $\lambda$, bets will create pseudo-wealth. Because of the bet, each party believes that he is wealthier. Thus, the perceived aggregate wealth exceeds the total “true” wealth. The larger
the size of the bet and the larger the discrepancy of beliefs on the probability of occurrence of the sunspot, the larger will be the pseudo-wealth. This effect will tend to increase consumption.

2. Bets will create uncertainty about expected wealth. In a general setup, this effect will tend to increase precautionary savings.

Therefore, every additional dollar of bet will be associated with a marginal benefit that comes from the perceived increase in wealth, and a marginal cost that comes from the increase in the variance of future expected wealth.

In every period the bet gets resolved. One side or the other wins the bet, and the pseudo-wealth that was created gets destroyed. If that were the whole process, pseudo-wealth would be ephemeral, with no real macroeconomic consequences.

If, however, differences in beliefs persist, individuals may once again engage in a bet, and so new pseudo-wealth is created. This destruction of pseudo-wealth but creation of new pseudo-wealth will occur until \( z_t = S \).

In the betting equilibrium, agent A will pay \( p_t \) to agent B in period \( t \). If \( z_t = S \), agent B pays 1 to agent A, while if \( z_t = O \), agent B pays nothing. Formally, the bet net returns \( \psi_t^A \) are described as follows:

\[
\psi_t^A = \begin{cases} 
1 - p_t, & z_t = S \\
-p_t, & z_t = O 
\end{cases}
\]

\[
\psi_t^B = \begin{cases} 
-(1 - p_t), & z_t = S \\
p_t, & z_t = O 
\end{cases}
\]

The pseudo-wealth of agents A and B in period \( t \) is described by

\[
PW_t^A(z_t) = (\lambda^A - p_t)b_t(z_t)
\]
and

\[ PW_t^B(z_t) = (p_t - \lambda_t^B)b_t(z_t) \]

where \( b_t \geq 0 \) is the equilibrium level of betting and \( PW_t^I \) is the level of pseudo-wealth of individual \( i \) in period \( t \). Correspondingly, the expected present discounted value of pseudo-wealth for both agents is described by

\[
E_t PW^A(z_t) = \begin{cases} 
E_t \sum_{j=t}^{\infty} [\beta(1 - \lambda^A)]^{j-t} (\lambda^A - p_j)b_j(z_j) > 0 & \text{if } z_j = O \text{ } \forall j \leq t \\
0 & \text{if } \exists j \leq t : z_j = S
\end{cases}
\]

and

\[
E_t PW^B(z_t) = \begin{cases} 
E_t \sum_{j=t}^{\infty} [\beta(1 - \lambda^B)]^{j-t} (p_j - \lambda^B)b_j(z_j) > 0 & \text{if } z_j = O \text{ } \forall j \leq t \\
0 & \text{if } \exists j \leq t : z_j = S
\end{cases}
\]

where \( \beta \in (0,1) \) is the discount factor (identical for all agents), and the expected aggregate pseudo-wealth is

\[
E_t PW(z_t) = E_t PW^A(z_t) + E_t PW^B(z_t)
\]

The aggregate pseudo-wealth is strictly positive before the sunspot, and as noted, depends on the magnitude of the disparity in the agents’ beliefs and the size of the bet.

\[ D. \text{ Optimization} \]

Consumers are forward-looking. In period \( t \), each agent chooses a sequence of consumption, borrowing, and bets in order to maximize the expected present discounted value of utility,
\[
\max_{\{c_j^i(z_j), d_j^i(z_j), v_j^i(z_j)\}_{j=t}} E_t \sum_{j=t}^{\infty} \beta^{j-t} u(c_j^i(z_j)), \quad i = A, B
\]

subject to the budget constraints
\[
c_t^i(z_t) + (1 + r)d_{t-1}^i(z_{t-1}) = y + d_t^i(z_t) + \psi_t^i(z_t)b_t(z_t) \quad \forall t = A, B
\]

where \( z_t \) denotes the history of states until period \( t \) and \( d_t^i \in \mathbb{R} \) denotes net borrowing by agent \( i \) in period \( t \); and also subject to the transversality condition
\[
\lim_{j \to \infty} \frac{d_j^i}{(1 + r)^j} = 0, \quad i = A, B
\]

Every time the no sunspot state occurs, there will be a winner and a loser of the bet but the tree of future states will be the same as one period before. That is, the realizations of states act as wealth shocks, which implies a need to re-optimize in each period.

We will also assume (for simplicity) that \( \beta (1 + r) = 1 \).

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**E. Consumption responses to increases in risk and expected wealth**

The response of the individuals’ consumption and savings to the creation of the market for bets will depend on the agents’ preferences. Suppose for the sake of simplicity and only for the analysis of this section that there are only two periods, \( t = 0, 1 \), and in \( t = 0 \) the creation of a market for bets that will be opened in the morning of \( t = 1 \) is announced. Suppose individuals’ beliefs are symmetric: \( \lambda^A = \frac{1}{2} = \frac{1}{2} - \lambda^B \). Agent A will maximize
\[
u(c_0^A) + \beta \left[ \lambda^A u(c_0^A(S)) + (1 - \lambda^A)u(c_0^A(O)) \right]
\]

subject to the budget constraints
\[ c^A_0 = y + d^A_0 \]
\[ c^A_1(S) = y + (1 - p)b^A - (1 + r)d^A_0 \]
\[ c^A_1(O) = y - pb^A - (1 + r)d^A_0 \]

Before the creation of the market for bets, \( c^A_t(z_t) = y \ \forall t, \forall z_t \). But betting will create a wealth effect and an increase in the variance of consumption in \( t = 1 \), that will turn affect consumption and savings decisions in \( t = 0 \). Lemma 1 shows that when utility is strictly increasing in consumption, the creation of the market for bets will lead to positive betting in equilibrium, thus creating a positive wealth effect.

**Lemma 1**: Suppose \( u \) is differentiable and \( u' > 0, \lambda^A > \lambda^B \). Then, \( b^i > 0 \ \forall i \) when the market for bets is created.

**Proof**: From the first order conditions of the utility maximization problem,

\[ \lambda^A(1 - p)u'(c^A_1(s)) - (1 - \lambda^A)pu'(c^A_1(O)) \leq 0, \ b^A_0 \geq 0, \text{ and} \]
\[ [\lambda^A(1 - p)u'(c^A_1(s)) - (1 - \lambda^A)pu'(c^A_1(O))]b^A_0 = 0 \]

Suppose that \( b^A_0 = 0 \). Then,

\[ \lambda^A(1 - p)u'(c^A_1(s)) - (1 - \lambda^A)pu'(c^A_1(O)) = (\lambda^A - p)u'[y - (1 + r)d^A_0] > 0 \]

a contradiction. Due to symmetry, the same holds for agent B. QED

The creation of the market for bets will increase (decrease) savings in \( t = 0 \) in the open economy if in the absence of savings or borrowing the marginal utility of consumption in the first period is smaller (greater) than the expected marginal utility of consumption in the second period. Formally, following the creation of the market for bets savings will increase (decrease) if
\[ u'(y) < (>)\lambda^A u'(y + (1-p)b) + (1 - \lambda^A)u'(y - pb) \]

The direction of the inequality will in general depend on the form of the utility function and the level of risk aversion. For the quadratic utility function, \( u(c_t(z_t)) = c_t(z_t) - \gamma c_t(z_t)^2 \), savings will decrease when the market for bets is created, as

\[ u'(y) = 1 - 2\gamma y > 1 - 2\gamma y - 2\gamma b(\lambda^A - p) = E u'(c_t^A) \]

Thus, under quadratic preferences the creation of the market for bets will increase consumption. This is a case in which there is no precautionary savings effect, hence the only relevant effect at the time of the creation of the market for bets is the wealth effect.

But this is not a general result. For example, for utility functions that feature precautionary savings or substitution effects, the response of savings to the creation of the market for bets may be positive. The next proposition describes sufficient conditions under which this is indeed the case.

**Proposition 1**: Suppose \( u \) is three times differentiable, \( u' > 0, u'' < 0, u''' > 0 \). Then, \( \exists \lambda^* \in \left(\frac{1}{2}, 1\right) \) such that savings will increase in \( t = 0 \) when the market for bets is created if \( \lambda^A < \lambda^* \).

**Proof**: Let \( g(\lambda^A) = \lambda^A u'(y + (1-p)b) + (1 - \lambda^A)u'(y - pb) \). As agents’ beliefs are symmetric, take \( p = \frac{1}{2} \). Due to the strict convexity of \( u' \), \( g\left(\frac{1}{2}\right) > u'(y) \) for \( b > 0 \). Also, as \( u'' < 0 \) and due to lemma 1, \( g(1) = u'(y + (1-p)b) < u'(y) \). Then, due to the continuity of \( u' \), \( \exists \lambda^* \in \left(\frac{1}{2}, 1\right) \) such that \( g(\lambda^*) = u'(y) \). Then, savings will increase in \( t = 0 \) if \( \lambda^A \in \left(\frac{1}{2}, \lambda^*\right) \). QED
Proposition 1 shows that given a precautionary savings motive, for a sufficiently low dispersion of beliefs the wealth effect will be dominated by the precautionary savings effect when the market for bets is created.

Note that in the analysis above we described the force that counteracts with the positive wealth effect as a precautionary savings effect. Under a utility function that allows for a certainty-equivalent representation for every risky consumption profile (as it is the case with Epstein-Zin preferences), it is then possible to isolate the risk effects and focus on the certainty-equivalent payoffs of the bet. The counteracting forces must then be described as a wealth effect and a substitution effect, as in INS. But with more general preferences, we must then focus on the actual consumption profile, and think of the counteracting forces as a wealth effect and a precautionary savings effect.

III. The certainty equivalence case

We are interested in analyzing a case in which the pseudo-wealth effect induced by the creation of the market for bets dominates over the substitution effect. For simplicity, we will assume that preferences take the form of a quadratic utility function.

\[ u(c_t^i(z_t)) = c_t^i(z_t) - \gamma c_t^i(z_t)^2, i = A, B \]

It is well known that the quadratic utility function cannot be globally correct, since it implies that the marginal utility of consumption becomes negative for \( c_t > 1/2\gamma \). However, it will be useful for our analysis as long as we restrict the value of
y sufficiently as to ensure that consumption lies in the area in which the marginal utility of consumption is positive.10

A. Expected wealth and the intertemporal budget constraint

The expected wealth of each agent in period \( t \), \( E_t W^i \), will be composed of three parts: the expected value of the endowment the agent receives, the expected value of pseudo-wealth, and the (negative of the) debt payments that must be paid in period \( t \). Then,

\[
(EW) \quad E_t W^i (z_t) = \frac{y}{1-\beta} + E_t PW^i (z_t) - (1 + r)d^i_{t-1} (z_{t-1}) \forall i, \forall t
\]

With a quadratic utility function, agent \( i \) faces the following intertemporal budget constraint:

\[
\sum_{j=t}^{\infty} \beta^j c^i_j (z_j) = \frac{y}{1-\beta} + E_t PW^i (z_t) - (1 + r)d^i_{t-1} (z_{t-1})
\]

B. Individual consumption and borrowing

In this case of quadratic utility function, the consumption optimal rule is

\[
c^i_t (z_t) = y + (1 - \beta)[E_t PW^i (z_t) - (1 + r)d^i_{t-1} (z_{t-1})]
\]

where the individuals’ debt dynamics before the sunspot occurs are described by the following expressions (assuming \( d^i_{t-1} = 0 \ \forall i \)):

10 Note that the expected wealth is bounded from above, as \( d_t \geq -\frac{y}{1-\beta} \ y < \infty \), and the level of debt must satisfy the transversality condition. Then, as consumption will be a linear function of expected wealth, \( \exists \ c < \infty \ such that c^i_t < \ c, \forall \ t \ \forall i \), which implies that we could restrict \( y \) to \( y < \frac{1}{2c} \) ensuring that that the marginal utility of consumption is always positive.
\[ d_t^A(z_t) = (1 - \beta) \sum_{j=0}^{t} E_j PW^A(O) + \sum_{j=0}^{t-1} p_j b_j - \psi_t^A(z_t) b_t \]

and

\[ d_t^B(z_t) = (1 - \beta) \sum_{j=0}^{t} E_j PW^B(O) - \sum_{j=0}^{t-1} p_j b_j - \psi_t^B(z_t) b_t \]

Note that at time 0, given that the outstanding debt is zero and expected pseudo-wealth is positive for both agents, \( c_0^i > y \forall i \).

As shown in Hall (1979), if the change in marginal utility from one period to the next is small, under a quadratic utility function each individual’s consumption path will be approximately described by a random walk.\(^\text{11}\) In every period, the bet winner (loser) will experience a positive (negative) wealth shock, and individual consumptions adjust accordingly.

C. Aggregate consumption and borrowing

Aggregate consumption is governed by the following expression:

\[ (AC) \quad c_t(z_t) = y + (1 - \beta) [E_t PW(z_t) - (1 + r)d_{t-1}(z_{t-1})] \]

Aggregate outstanding debt is given by

\[ (AD) \quad d_{t-1}(z_{t-1}) = (1 - \beta) \sum_{j=0}^{t-1} E_j PW(z_j) \]

Aggregate debt will be increasing over time until the sunspot occur, and will decrease since then in order to satisfy the transversality condition.

\(^{11}\) This requires that the level of consumption in period \( t \) is sufficiently far from the bliss point \( 1/2\gamma \).
IV. Pseudo-wealth and consumption volatility

Proposition 2 shows that the disappearance of betting opportunities leads to a discontinuous decrease in aggregate consumption. Therefore, the process of creation and destruction of pseudo-wealth will increase the ex-post consumption volatility with respect to the world in which betting is prohibited.

Proposition 2: At $z_t = S$, there is a discontinuous decrease in aggregate consumption.

Proof: It comes from (PW), (EW), and (AC).

At the moment the sunspot occurs there will be a discontinuous decrease in aggregate expected wealth due to the disappearance of pseudo-wealth.

Corollary 1: Aggregate consumption will be lower after the sunspot the longer it takes for the sunspot to occur.

The longer it takes for the sunspot to occur, the larger will be the level of aggregate debt when it occurs (cf. equation (AD)), hence the lower will be aggregate consumption in every period after the sunspot—the economy will need to generate a larger current account surplus to repay the larger external debt, hence given the constant endowment, consumption will have to fall by more.

Corollary 2: Aggregate consumption volatility is larger when there exists a market for bets.
With no market for bets, \( c_t = 2y \ \forall t \), hence \( Var(c_t) = 0 \). With the market for bets, \( c_t(z_t) > 2y \) as far as expected pseudo-wealth is larger than outstanding debt (which happens with certainty in \( t = 0 \)) or \( c_t(z_t) < 2y \) in the opposite case.

Our next question of interest is if the above results imply that prohibiting bets would be optimal. Addressing this welfare question requires a criterion for dealing with heterogeneous beliefs.

**Definition 1 (Stiglitz, 1982).** We say that beliefs satisfy group rationality if

\[
\frac{1}{2}\lambda^A + \frac{1}{2}\lambda^B = \lambda
\]

where \( \lambda \) is the true probability of occurrence of sunspot.

**Proposition 3:** Suppose the planner computes welfare using average beliefs and suppose beliefs satisfy group rationality. Then, under a utilitarian social welfare function, the creation of a market for bets leads to a decrease in the expected present value of welfare.

**Proof:** The planner’s intertemporal budget constraint at time 0 is \( \sum_{t=0}^{\infty} \beta^t c_t(z_t) = \frac{2y}{1-\beta} \). The planner’s optimal solution is \( c_t^A(z_t) = c_t^B(z_t) = y \), \( c_t^P(z_t) = c_t^A(z_t) + c_t^B(z_t) = 2y \ \forall t \). The creation of the market for bets leads to \( c_t(z_t) > 2y \) before \( z_t = S \), and \( c_t(z_t) < 2y \) afterwards. Group rationality and strict concavity of \( u(\cdot) \) imply the proposition.

Proposition 3 establishes that prohibiting the bet can increase welfare from the viewpoint of beliefs that are consistent in the aggregate—as it would be the case of
the beliefs of the planner that considers the true probability to be $\lambda$, given by
definition 1. But such a prohibition would not increase ex-ante expected utility for
agents $A$ and $B$ given their beliefs—indeed, both agents would be strictly worse-off
ex-ante with the bet prohibition given their beliefs.

This proposition is a particular case of the case of “reasonable beliefs”—i.e., beliefs
that are a convex combination of agents’ beliefs (see Brunnermeier, Simsek, and
Xiong (2014)). In fact, if the planner uses any convex combination of the agents’
beliefs to compute welfare, there will always be a transfer between agents such that
from the viewpoint of the planner’s beliefs, it is optimal to prohibit the bet and
implement such a transfer. If the planner’s beliefs are closer to the beliefs of agent
$A$ (or $B$), then the prohibition of the bet will be accompanied by a transfer from $B$
to $A$ (or $A$ to $B$) such that both agents are weakly better-off from the viewpoint of
the planner’s belief, with at least one agent being strictly better-off.

V. Extensions

Our results raise important questions regarding the effects of markets that allow
for speculation in a context of heterogeneous beliefs. We have assumed a very
simple environment of an endowment economy with two agents with the same
preferences that feature no precautionary savings effects. In spite of the simplicity
of the environment, we showed that the cycles of pseudo-wealth lead to
discontinuous changes in the control variables even when there are no changes in
the state variables of the economy. However, this environment does not permit to
analyze other macroeconomic consequences of those discontinuous changes or the
distributional consequences that fluctuations in pseudo-wealth could imply.
For example, different discount factors for agents A and B would affect the
dynamics of aggregate consumption. In particular, if agent A had a smaller (larger)
discount factor than agent B, the no occurrence of the sunspot would have a
contractionary (expansionary) effect on aggregate consumption, as it would change
the distribution of expected wealth in favor (against) of the agent with a larger
(smaller) marginal propensity to consume.

More generally, fluctuations in pseudo-wealth can give rise to a variety of richer
effects in more complex environments. They could have macroeconomic effects
that go beyond an intertemporal misallocation of resources. Besides, pseudo-wealth
could also be negative. The rest of the section provides a set of simple examples
that show how the range of applicability of the pseudo-wealth theory could be
extended.

**Example 1: Output fluctuations in a one sector production economy**

The first example considers a one sector economy where an internationally
tradable good is produced using only labor, and there are constant returns to scale:

\[ y_{T,t} = l_t \]

where \( y_{T,t} \) denotes the production of the tradable good that is sold at at the
international price, that is constant and that we assume is equal to 1, and \( l_t \) is the
amount of labor employed, all in period \( t \).

The agents’ preferences are now defined over consumption of the single good and
leisure, according to \( U_t^i = u(c_t^i) - v(1 - l_t^i) \), with \(-v' > 0\), \(-v'' < 0\). Workers
receive a wage \( w_t \) per unit of labor in period \( t \). We assume there is perfect
competition in the production sector. Thus, in equilibrium \( w_t = 1 \).

The consumers-workers budget constraints are now
\[ c^i_t(z_t) + (1 + r)d^i_{t-1}(z_{t-1}) = l^i_t + d^i_t(z_t) + \psi^i_t(z_t)b^i_t(z_t) \forall t = A, B \]

The optimal choice of consumption and labor of the consumers-workers must satisfy the conditions

\[ \frac{v'(1 - l^i_t)}{u'(c^i_t)} = 1 \]

\[ u'(c^i_t) = E_t^i u'(c^i_{t+1}) \]

and

\[ v'(1 - l^i_t) = E_t^i v'(1 - l^i_{t+1}) \]

Maintaining the assumption \( u\left(c^i_t\right) = c^i_t - \gamma c^i_t^2 \), the creation of the market for bets will still lead to an increase in the individuals’ and aggregate consumption, but will also decrease the labor supply at the fixed wage; hence it will decrease the level of employment and output in equilibrium. Following the realization of the state \( S \), the bet’s winner will decrease the labor supply and the loser will increase it. The absolute size of the negative wealth shock for the loser is larger than the absolute size of the positive wealth shock for the winner (the total negative wealth shock is the destruction of aggregate pseudo-wealth). Thus, destruction of pseudo-wealth will affect both the individuals’ and aggregate labor supply: the level of employment and output will increase.

As in RBC models, the labor supply will respond negatively to positive wealth shocks (and vice versa) and this in turn will affect employment and output in equilibrium. Under different assumptions, the sign of the changes could be different. But the general point of this example is that in a context of heterogeneous beliefs in a production economy the creation of the betting market will increase the volatility of output—that would be zero if betting were not allowed.
Example 2: Distributional effects in a two-sectors production economy

The example of this section describes an economic structure in which pseudo-wealth fluctuations lead to fluctuations in the real economy and also have distributional effects.

We now assume that the economy has two sectors, one that produces a tradable good and the other produces a non-tradable good. Production in both sectors requires labor, but production in the tradable sector also requires a factor of production that is in fixed supply, that can be interpreted as land. There is a continuum of workers of mass 1: Half of them believe the probability of occurrence of the sunspot in each period is $\lambda^A$ and the other half believes it is $\lambda^B$ (we refer to them as type A and type B workers, respectively). We assume perfect labor mobility across sectors.

The production function of the non-tradable good features decreasing returns to scale:

$$y_{N,t} = l_{N,t}^\alpha$$

where $l_{N,t}$ denotes labor in the non-tradable sector and $\alpha \in (0,1)$. A fraction $(1 - \alpha)$ of the production of the non-tradable good is kept (and consumed, not traded) by an agent that is not explicitly modeled, that can be thought of as a manager or owner of the firm that produces the non-tradable good.\textsuperscript{12} The rest is traded domestically at a price $p_{N,t}$.

The production function for the tradable good takes the Leontieff form,

$$y_{T,t} = \min \{l_{T,t}, X\}$$

\textsuperscript{12} We make this assumption for simplicity. We just want to describe a situation where labor exhibits decreasing returns.
where \( l_{T,t} \) denotes labor in the tradable sector and \( X \) is a factor in fixed supply owned by a capitalist that we assume it is a foreign agent that does not spend in the domestic economy. Then, taking the price of the tradable good as the numeraire,

\[
L_t = \begin{cases} 
X, & \text{if } w_t \leq 1 \\
0, & \text{if } w_t > 1 
\end{cases}
\]

The capitalist’s profit function is

\[
\Pi_t = \begin{cases} 
(1 - w_t)X, & \text{if } w_t \leq 1 \\
0, & \text{if } w_t > 1 
\end{cases}
\]

If \( w_t < 1 \) there will be rents in the tradable sector.

Suppose consumers-workers’ preferences are described by \( U_t^i = u^T(c^i_{T,t}) + u^N(c^i_{N,t}) + v(1 - l^i_t) \), with \( u^{k'} > 0, u^{k''} < 0, k = T, N \). Given that all workers are identical in all dimensions but on the beliefs about the probability of occurrence of the sunspot, they will all resolve an identical problem at the moment the market for bets is created. Each worker offers an amount of labor \( l^i_t \) in period \( t \) and is randomly assigned to one of the two production sectors of the economy, and every worker of the same beliefs-type works the same amount of time independently of the sector where she is placed.

The positive wealth shock implied by the creation of the market for bets will raise the relative price of the non-tradable good and wages in equilibrium.\(^{13}\) There is a distributional effect between classes: the distribution of the production of the tradable good changes in favor of the workers and against the capitalist (or

\(^{13}\) The vector of equilibrium prices in this dynamic small open economy must satisfy the Euler equations for the consumption of the tradable good, the non-tradable good, and the time worked; it must satisfy the equalization between marginal rates of substitution and relative prices, \( \frac{w}{p^T} = w_t, \frac{w}{p^N} = \frac{w}{p_{N,t}}, \) and \( \frac{w'}{p'N} = \frac{1}{p_{N,t}} \); it must satisfy the equilibrium condition in the labor market \( l_{T,t} + \left( \frac{q_{N,t}}{w_t} \right)^{\gamma'} = \int_{t_0}^{t} l^i_t \, dt \); and it must satisfy the equilibrium condition of equalization of demand for and supply of the non-tradable good. When the wealth effect dominates, the new equilibrium will feature higher \( p_{N,t} \) and \( w_t \); and as analyzed earlier the presence of pseudo-wealth will lead to external borrowing (in terms of tradable goods) for satisfying the Euler equation for the consumption of the tradable good.
equivalently, the capitalist’s rent decreases). When the occurrence of the sunspot destroys pseudo-wealth, the opposite effect will occur: as the aggregate labor supply increases, wages will fall and rents will increase. And these distributional effects could have richer macroeconomic effects in an environment that features aggregate demand externalities.¹⁴

Example 3: Negative pseudo-wealth

Differences in priors could also lead to situations where there is negative pseudo-wealth – situations where the sum of the perceived wealth of the members of the society is smaller than the actual wealth capacity of the society. In turn, negative pseudo-wealth could have negative consequences for the process of creation of actual wealth. We will show a simple example that illustrates this phenomenon. We previously showed that completing markets in a context of heterogeneous beliefs could lead to creation of positive pseudo-wealth, that in turn could increase consumption volatility, and that under a criterion for welfare analysis that uses any measure of reasonable beliefs (as defined in Brunnermeier, Simsek, and Xiong, 2014) it could even reduce welfare. Unlike that case, the example that follows will show an environment with heterogeneous beliefs in which moving in the direction of complete markets could eliminate the negative pseudo-wealth, enhancing efficiency and improving welfare.

Suppose that an individual owns an asset that can be used for productive purposes. The owner of the asset - agent X - will produce one unit of services if it holds the asset, while another agent - agent Y - by holding the same asset would produce \( A \) units, \( A > 1 \). Both agents are consumers of the services produced by the asset. It is

¹⁴ As workers had borrowed when the market for bets was created due to the presence of pseudo-wealth, the decrease in wages will increase the value of real debts. This Fisher effect can amplify the real effects of the negative pseudo-wealth shock, as discussed in Guzman and Stiglitz (2015b).
efficient that agent X rents the asset to agent Y and agent Y employs it for production. Suppose that at the time of signing a rental contract there is uncertainty about the market price of the services. Let’s assume that the market price, \( p \), may take two values: \( p = \{ p^L, p^H \} \), with \( p^L < p^H \). Let \( R \) denote the rental cost. The values of agents X and Y’s utilities if the rental contract is signed are denoted by \( V^X \) and \( V^Y \), respectively:

\[
V^X = \lambda^X u \left( \frac{R}{p^H} \right) + (1 - \lambda^X) u \left( \frac{R}{p^L} \right) - u(1)
\]

\[
V^Y = \lambda^Y u \left( \frac{p^H A - R}{p^H} \right) + (1 - \lambda^Y) u \left( \frac{p^L A - R}{p^L} \right) - u(0)
\]

where \( u'(x) > 0 \), \( x \in \mathbb{R} \), and \( u(1) \) and \( u(0) \) would be the utility levels of agents X and Y in the absence of a rental contract.

Let \( \{ R \} \) denote the set of fixed rental contracts that make both agents weakly better off ex-ante, i.e. \( \{ R \} = \{ R \in \mathbb{R}_{\geq 0} : V^X(R) \geq 0 \land V^Y(R) \geq 0 \} \). We can show that under a sufficiently large disagreement of beliefs there are conditions that imply \( \{ R \} = \emptyset \). To illustrate this case, suppose disagreement is maximum: \( \lambda^X = 1 \) and \( \lambda^Y = 0 \). Then, agent Y will only sign a rental contract if \( R \leq p^L A \). But if \( \frac{p^L}{p^H} A < 1 \), there will be no mutually beneficial contract. Agent’s X (expected and realized) wealth will be one unit of the service and agent’s B will be zero: the implication is that realized wealth will be below the society’s capacity of wealth creation.

Note that the likelihood that the set of feasible fixed rental contracts is empty depends on the nature of the beliefs disagreement. If agents were optimistic about their own upside states (\( p^L \) for agent X and \( p^H \) for agent Y), there would be more
feasible fixed rental contracts. This result is related to Simsek (2013) that shows that the nature of beliefs disagreements matters for economic outcomes.\textsuperscript{15}

However, under this scenario of maximum beliefs disagreement there are feasible contingent rent contracts: Any contingent contract $R(p)$ with $R(p^L) \in [p^L, p^L A]$ and $R(p^H) \in [p^L, p^L A]$ will be feasible. In this case, moving towards the direction of complete markets will align rather than divorce the society’s expectations of future wealth and the society’s potential wealth capacity.

This example illustrated a simple situation where dispersion of beliefs may have negative macroeconomic consequences, not through increases in speculative trade but on the contrary because of a decrease in the cardinality of feasible intertemporal contracts due to heterogeneous priors.

The pseudo-wealth theory can be used to analyze other situations where disagreement over events has negative macroeconomic effects. For example, in an already established borrower-lender relationship, a shock to the economy that creates dispersion of beliefs such that the lender becomes more pessimistic than the borrower about the capacity of repayment of borrowings will create negative pseudo-wealth: the lender will think he is receiving less (in expected terms) than the borrower thinks he is paying. In turn, this will decrease the lender’s consumption with no counteracting increase in the borrower’s consumption, implying a decrease in aggregate consumption. If later the borrower turns out to be correct, the lender will experience a positive wealth shock, that will lead to an increase in his consumption, again not counteracted by a change in the lender’s

\textsuperscript{15} Simsek (2013) proves that the tightness of collateral constraints depends on the nature of belief disagreements. Different than in this case, the relevant asset price effects occur when there is a bias in favor of the probability of upside states because this relaxes collateral constraints.
consumption. In a richer environment these effects could further affect the state of the macroeconomy.\textsuperscript{16}

VI. Conclusions

This paper has shown that, when there are differences in beliefs, the amounts that betting individuals expect to receive from other individuals may differ markedly from the amounts that these same individuals expect to pay. This disparity in (the present discounted value of) expected transfer payments we refer to as pseudo-wealth. We have noted that there can be large changes in the aggregate value of pseudo-wealth, and that these changes in aggregate pseudo-wealth can give rise to large fluctuations in consumption.

Our analysis does not need to assume that there is a well-defined distribution of probabilities that it is correct, or that such a distribution is known by the agents of the economy. Our approach is reasonable when agents form beliefs over a one-time event. As the “rare” event only happens one time there would be nothing to learn from its no occurrence.

Indeed, even after the event has occurred, we cannot be sure which of the two individuals was “right.” And it is sensible to think that all individuals would not share the same beliefs about the likelihood of these occurrences, that there is no common knowledge. If agents don't share the same beliefs, then there is room for a bet that increases pseudo-wealth.

\textsuperscript{16} The macroeconomic effects would be richer in the presence of collateral constraints, and in an environment where pseudo-wealth fluctuations led to fluctuations in the price of the asset that is used as collateral. Relatedly, Hirano, Inaba, and Yanagawa (2015) show that in the presence of collateral constraints, policy interventions that make the emergence of bubbles more likely and the bust of bubbles less likely may positively affect ex-ante production efficiency due to the crowding in effect on investments that a bubble may have when collateral constraints are binding.
The theory of pseudo-wealth is complementary of other theories of macroeconomic fluctuations, but generates a set of testable hypotheses that distinguishes it from those other theories. For example, representative agent models with learning can also feature aggregate wealth misperceptions that lead to consumption booms followed by busts. However, by construction, fluctuations in those models only occur at the aggregate level. Instead, the pseudo-wealth theory predicts both increases in aggregate consumption volatility as well as larger variance of consumption at the cross-section level.

Unlike other theories of macroeconomic fluctuations, for the pseudo-wealth theory of fluctuations what matters is the dispersion of beliefs rather than average beliefs: Any changes in average beliefs would be irrelevant in our model. Besides, unlike other complementary theories, in the theory of pseudo-wealth the distribution of the individuals’ consumption will change in every period before the rare event occurs even if there are no changes in the dispersion of beliefs or in average beliefs from one period to another. This occurs because in every period in which betting occurs there will be a winner and a loser, and betting will continue occurring as long as disagreement persists.

The relevant population of economic agents for empirical tests will in general be different for different situations in which pseudo-wealth can be created. For example, in the case of trade in speculative assets, it is likely that the relevant population will consist of the wealthiest households that have an easier access to those markets. But we have noted that the model can be extended in directions such that the waves of creation and destruction of pseudo-wealth can also have macroeconomic effects that could affect the population that in principle did not participate in either the pseudo-wealth creation or destruction processes.

This paper is the first step of a research agenda that intends to offer a general framework for understanding situations in which large changes in macroeconomic
behavior are observed together with no (or very small) changes in the state variables describing the economy. The dynamics of pseudo-wealth—its formation, dissolution, and its aggregate persistence both positively and negatively—help explain macroeconomic volatility and gives insight into the nature of persistent booms and busts—a fact still not fully explained by the existing literature.

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