Idiosyncratic vs. Systematic Risk: A Network Risk Model via Portfolio Risk Decomposition

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Abstract: Departing from the earlier literature which relies heavily on the Capital Asset Pricing Model (CAPM) framework, this paper attempts to utilize the network approach to decompose the portfolio risk into its systematic and idiosyncratic components. First, we construct a weighting procedure that makes use of both the variance-decompositions-based Diebold-Yilmaz connectedness framework as well as the information on the structure of the variance-covariance matrix. Second, to differentiate systematic and idiosyncratic risk we take advantage of a covariance-decomposition-based approach. Finally, we propose a procedure to calculate the individual systematic risk contribution for each asset in the portfolio. The implementation of proposed methodology on a portfolio of major U.S. Financial Institution (FI) stocks during the 2006-2009 and a portfolio of US and EU FI stocks during the 2009-2016 period generate intuitive results in line with the actual developments in the banking industries.

Key Words: CAPM, Vector Autoregression, Variance and Covariance Decomposition, Connectedness, portfolio risk, idiosyncratic and systematic risk.

JEL codes: C32, G21

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1 Introduction

Financial institutions played a key role in the evolution of the U.S. financial crisis and its propagation through the global financial system. Disproportionate risks taken by big financial institutions have caused serious trouble for the whole financial system. As a consequence, one of the most important priorities of the literature on financial crises is to identify and measure systemic risk in the financial and real economic systems.

An important first step in the analysis of systemic risk is to come up with a workable definition of systemic risk. Smaga (2014), who surveyed the literature on the definitions of systemic risk, concluded that before the global financial crisis the main emphasis for the systemic risk was on contagion within the financial system. According to Bandt and Hartmann (2000), systemic risk was regarded in a narrow sense as contagion working from one institution, market or system to the others through linkages. However, in light of the experience of global financial crisis, attempts to define systemic risk started to put more emphasis on negative impact of the financial shocks on the entire economy. Consistent with this approach, the literature on systemic risk considered system-wide shocks which by themselves adversely affect many institutions or markets simultaneously. However, there is still no consensus about the definition of systemic risk.

This study provides a methodology to undertake network-based evaluation of riskiness of assets (or institutions). We expand connectedness framework of Diebold and Yilmaz (2014) incorporating with risk measures of individual agents/assets. Proposed methodology combines both the network structure and the variance-covariance matrix of a portfolio. It decomposes total portfolio risk to its systematic and idiosyncratic components. In this case, systematic component is made up of risk stem from the interaction of linkages and riskiness of agents in the system. To interpret risk, we follow portfolio risk approach which can be summarized as volatility of the portfolio return. In that regard, we assume the system to exist as a portfolio of assets. Network information is deciphered through the connectedness matrix which are obtained from variance decompositions.

The definitions of systematic and idiosyncratic risk have been developed under traditional Capital Asset Pricing Model (CAPM) construction. According to this approach, total risk of a portfolio can be decomposed into systematic risk -the risk that is captured by mapping the portfolio to risk factors- and idiosyncratic (or specific) risk that is not captured by the portfolio mapping but unique factors imputed to each individual assets. Methodologically, CAPM based models rely on the regression equation and obtain measures of common market factors which implies systematic risk, while the residual term for each individual stock reflects
idiosyncratic variation.

Apart from this approach, this paper is an attempt to utilize the ideas developed in the network literature, in particular the Diebold-Yilmaz connectedness approach, to decompose the portfolio risk into its systematic and idiosyncratic components. Diebold and Yilmaz (2014) developed a methodology to measure connectedness of financial assets through variance decompositions from approximating VAR models. Using financial market information on individual assets or asset classes, their methodology reveals the underlying network structure across assets or asset classes. The resulting information about the structure of the underlying network is used to understand the transmission mechanism of shocks throughout the corresponding financial and/or real economic system. This paper adds risk dimension to this approach. We decompose risk with the help of familiar variance decomposition technique applied in Diebold-Yilmaz framework and covariance decomposition technique which is proposed by Goto and Volkanov (2002). In this case, systematic risk implies the portfolio risk captured by the interrelations (or simply linkages) of agents in the system. On the other hand, idiosyncratic risk is regarded as the self-effectuating portion of portfolio risk instead of residual risk. We further identify the individual asset contributions to the systematic risk and the diversification among the systematic and idiosyncratic risk components.

The proposed methodology and the resulting measures enable us to assess systemic risk, yet it ignores the factors like liquidity and leverage which are accepted as crucial for systemic risk in literature\(^1\). Moreover, it does not impose any condition on the state of the system in order to calculate the risk measure in case of systemic event unlike Marginal Expected Shortfall (MES) and Systemic Expected Shortfall (SES) of Acharya et al. (2010) or Delta Conditional Value-at-Risk (ΔCoVaR) of Adrian and Brunnermeier (2014). Hence, this methodology diverges from the systemic risk literature in that sense. Incorporation of factors such as liquidity and leverage or stress-test like applications conditional on systemic events are left for future work.

In Section 2, we discuss the related literature and explain the foundation of our methodology. After that, in Section 3, we build up our model by using Diebold and Yilmaz (2014) and forecast error covariance decomposition which is borrowed from Goto and Volkanov (2002) is an extension of vector autoregression toolbox. In Section 4, we demonstrate the detail of estimation procedure and report decomposition results. In addition to these we proposed two important implication in Section 5 and we also provide estimation results of these implications. At last, in Section 6 we concluded our study in light of the results.

\(^1\)See Billio et al. (2011).
2 Literature and Methodology

Decomposition of risk to its systematic and idiosyncratic components has been intensively studied in the literature with different objectives and methodologies. We can categorise the contributions to this literature into two main blocks, depending on whether they focus on systematic or idiosyncratic risk per se.

Studies that focus specifically on idiosyncratic risk broadly aim to decompose risk and question the effect of idiosyncratic risk on different set of key variables, such as stock returns and volatility of returns in different markets. The most commonly observed analyses in this strand of literature focus on how the idiosyncratic risk affects stock returns. This research question have prevailed especially after early 2000s. Campbell et al. (2001) attempted to characterize the behavior of stock market volatility not only at the market level, but also at the industry and firm levels. They used CAPM as the baseline model in their analysis and tried to quantify the idiosyncratic components of risk. Furthermore, they showed that the number of stocks needed to obtain any given amount of portfolio diversification has increased over time.

In a paper that appeared shortly after Campbell et al. (2001), Goyal and Santa-Clara (2003) studies the relationship between the equity risk and returns in the stock market. In particular, they found that the lagged variance of the average market return has no forecasting power for market returns, whereas lagged average stock variance has significant positive explanatory power for the market return. Based on this finding, Goyal and Santa-Clara (2003) concluded that idiosyncratic risk matters for market returns. Bali et al. (2005) computed the idiosyncratic risk and construct the firm-level volatility measures a la Campbell et al. (2001) to determine its contribution to the prediction of excess market returns. They found that value-weighted average idiosyncratic volatility measure does not have explanatory power for the time-series variation in the value-weighted market returns.

Moreover, other than investigating the effect of idiosyncratic volatility on stock returns, Fuertes et al. (2015) examined the role of momentum, term structure and idiosyncratic volatility by using CAPM based techniques whether they signals in commodity futures markets.

In addition to studies cited above, there is a strand of the literature that emphasized the role of systematic risk rather than the idiosyncratic one and analyzed its role by employing similar CAPM based techniques such as Gencay et al. (2005), Marshall (2015).

Different from the popular CAPM-based decomposition methodology Gagliardini and Gourireux (2013) used distributional Value–at–Risk, expected shortfall and the other related
risk measurement approaches for static and dynamic factor models to decompose risk.

In this paper, analogous to asset pricing literature, the volatility (or standard deviation) of a portfolio is taken as the measure of risk. We use the estimated risk parameter, based on the variance-covariance structure of portfolio.

To be more precise, we define variance of a portfolio as

$$\sigma^2_p = \text{var}(x'R) = x'\Sigma x$$

where \(x\) is the portfolio weight vector, \(R\) is the net return vector of assets included in the portfolio and \(\Sigma\) is the resulting variance-covariance matrix. By taking the square root of variance we obtain the volatility of the portfolio return. The fact that variance-covariance matrix is the key input of the risk calculation procedure, it is used as the basis for decomposition of total risk into its idiosyncratic and systematic components.

Our main approach aims to break total variance-covariance matrix into idiosyncratic and systematic variance-covariance matrices such as

$$\Sigma = \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \ldots & \sigma_{1N} \\
\sigma_{21} & \sigma_{22} & \ldots & \sigma_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_{N1} & \sigma_{N2} & \ldots & \sigma_{NN}
\end{bmatrix}$$

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$$\Sigma_{Total} = \Sigma_{idiosyncratic} + \Sigma_{Systematic}$$

Here we assume that the variances and covariances, conditional on idiosyncratic and systematic parts, are linearly additive but volatilities are sub-additive. Linear additivity assumption can be justified by variance decomposition. In variance decompositions, the resulting variance-covariance matrix is just the sum of squared of impulse responses. Besides, for a portfolio of imperfectly colinear assets, portfolio risk must be smaller than the sum of the risks of individual assets, which implies in turn sub-additivity.

### 3 Portfolio Risk Decomposition

#### 3.1 Decomposition via Diebold-Yilmaz Framework

Diebold and Yilmaz (2014) developed a methodology to obtain connectedness measure
(Diebold-Yilmaz Connectedness Index – DYCI) via variance decompositions from approximating VAR models. In line with the variance decomposition analysis, connectedness measures are defined as the shares of forecast error variation of a factor due to shocks arising from any other factor. Formally, for an \( N \)-dimensional vector autoregression, connectedness measure is defined as \( d^H_{ij} \) as the \( ij \)-th element of \( H \)-step forecast error variance decomposition as the fraction of variable \( i \)'s \( H \)-step forecast error variance due to shocks in variable \( j \), where \( i \neq j \).

\[
\begin{array}{cccc}
  x_1 & x_2 & \ldots & x_N & \text{From Others} \\
  x_1 & d^H_{11} & d^H_{12} & \ldots & d^H_{1N} & \sum_{j=1}^{N} d^H_{1j}, j \neq 1 \\
  x_2 & d^H_{21} & d^H_{22} & \ldots & d^H_{2N} & \sum_{j=1}^{N} d^H_{2j}, j \neq 2 \\
  \vdots & \vdots & \ddots & \vdots & \vdots \\
  x_N & d^H_{N1} & d^H_{N2} & \ldots & d^H_{NN} & \sum_{j=1}^{N} d^H_{Nj}, j \neq N \\
  \text{To Others} & \sum_{i=1}^{N} d^H_{i1}, i \neq 1 & \sum_{i=1}^{N} d^H_{i2}, i \neq 2 & \sum_{i=1}^{N} d^H_{iN}, i \neq N & \frac{1}{N}\sum_{i,j=1}^{N} d^H_{ij}, i \neq j \\
\end{array}
\]

Table 1: The Connectedness Table

Table 1 is called connectedness table, which is not different than any forecast error variance decomposition matrix except the last row and the column, and each entry shows connectedness measures of each variable in the system. The rightmost column which is called "From Others" is the row sums and bottom row which is called "To Others" is just the column sums of off-diagonal elements of the forecast-error variance matrix. The off-diagonal elements of the connectedness table are the measures of pairwise directional connectedness. The diagonal elements of the connectedness table \( d^H_{ii} \) measure the contribution of shocks to each variable to its own forecast error variance is correctly kept out of the Total To and From directional measures of connectedness. The bottom-right element is the system-wide connectedness measure.

DYCI framework produces weighted and directed networks defined by variance decompositions. Connectedness measures can be used in understanding the nature of connectedness among asset returns included in a particular portfolio. As Diebold and Yilmaz (2014) argued networks defined by variance decompositions provide more information about the sophisticated network relationships compared to undirected, un-weighted network structures that uses only binary variables.
Another important feature of DYCI is that links are not only defined between variables or factors, but also defined for the variable itself. More formally, DYCI framework allows for the measurement of own-connectedness. Thus, own-connectedness relations in our framework can be defined as the projection of variance into the idiosyncratic variance. In order to follow-up on that argument, suppose that $D_{H}^{N}$ is an adjacency matrix constructed through variance decomposition as

$$D_{H}^{N} = \begin{bmatrix} d_{11}^{H} & \cdots & d_{1N}^{H} \\ \vdots & \ddots & \vdots \\ d_{N1}^{H} & \cdots & d_{NN}^{H} \end{bmatrix} \quad n = 1, 2, \ldots, N$$

Now, in order to acquire idiosyncratic variance, we pick up the diagonal elements of $D_{H}^{N}$, which measures own-connectedness. $\Delta_{H}^{N}$ is the diagonal matrix which includes the diagonal elements of $D_{H}^{N}$, the variance contribution of particular variables to themselves, separates idiosyncratic variance contribution from the total variance contribution.

$$\text{diag}(D_{N}^{H})I = \Delta_{N}^{H} = \begin{bmatrix} \delta_{11}^{H} & 0 & \cdots & 0 \\ 0 & \delta_{22}^{H} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \delta_{NN}^{H} \end{bmatrix}$$

where $I$ is an $N$-dimensional identity matrix.

Now, think any variance-covariance matrix for weighting by $\Delta_{N}^{H}$, which can be computed by any other methodology or directly forecast error covariance matrix obtained by VAR estimation, $\Sigma$. It is coherently preferable to use forecast error covariance matrix, which is the main input of variance decomposition procedure, but it is also possible to use $D_{N}^{H}$ independent from forecast error covariance matrix by assuming it as exogenous network structure. We prefer to use forecast error covariance matrix (or is commonly called as MSE matrix) in order to hold the ties between variance decomposition and variance-covariance matrix consistent.

Weighting procedure relies on two basic properties of variances and covariances, namely, $Var(aX + b) = a^2 Var(X)$ and $Cov(aX + b, cY + d) = ac Cov(X, Y)$. In standard applications using actual data of $X$ and $Y$ and known parameters $a, b, c,$ and $d$, one can compute the resulting variance in a straightforward fashion. However, in our case, we are dealing with the so-called second moments and our inputs are comprised of just the right-hand side of variance and covariance properties. Thus, we apply sort of reverse logic at this stage to
weigh the variance-covariance matrix.

To see procedure more clearly suppose that $\Gamma_H^H$ is the square-root of $\Delta_H^H$. Notice that $\Delta_H^H$ is a diagonal matrix and it is appropriate to take square root of elements individually to get $\Gamma_H^H$.

$$\Gamma_H^H = \sqrt{\Delta_H^H} = \begin{bmatrix}
\gamma_{11}^H & 0 & \cdots & 0 \\
0 & \gamma_{22}^H & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \gamma_{NN}^H
\end{bmatrix}$$

where $(\gamma_{ii}^H)^2 = \delta_{ii}^H$.

At that moment the weighting procedure ends up with pre- and post-multiplication of the variance-covariance matrix $\Sigma$ by $\Gamma_H^H$. Lets define idiosyncratic variance-covariance matrix, $\Sigma_{id}$ as

$$\Sigma_{id} = \Gamma_H^H \Sigma^H \Gamma_H^H$$

which can be explicitly rewritten as

$$\Sigma_{id} = \begin{bmatrix}
\gamma_{11}^H & 0 & \cdots & 0 \\
0 & \gamma_{22}^H & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \gamma_{NN}^H
\end{bmatrix} \begin{bmatrix}
\sigma_{11} & \cdots & \sigma_{1N} \\
\vdots & \ddots & \vdots \\
\sigma_{N1} & \cdots & \sigma_{NN}
\end{bmatrix} \begin{bmatrix}
\gamma_{11}^H & 0 & \cdots & 0 \\
0 & \gamma_{22}^H & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \gamma_{NN}^H
\end{bmatrix}$$

Consequently, we get the idiosyncratic variance-covariance matrix as

$$\Sigma_{id} = \begin{bmatrix}
\sigma_{11}(\gamma_{11}^H)^2 & \sigma_{12}(\gamma_{11}^H)(\gamma_{22}^H) & \cdots & \sigma_{1N}(\gamma_{11}^H)(\gamma_{NN}^H) \\
\sigma_{21}(\gamma_{22}^H)(\gamma_{11}^H) & \sigma_{22}(\gamma_{22}^H)^2 & \cdots & \sigma_{2N}(\gamma_{22}^H)(\gamma_{NN}^H) \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_{N1}(\gamma_{NN}^H)(\gamma_{11}^H) & \sigma_{N2}(\gamma_{NN}^H)(\gamma_{22}^H) & \cdots & \sigma_{NN}(\gamma_{NN}^H)^2
\end{bmatrix}$$

which is the part of total variance-covariance matrix extracted using the network structure. Now, the non-idiosyncratic component of variance-covariance matrix is just the systematic variance-covariance matrix such as

$$\Sigma_{sys} = \Sigma_{Total} - \Sigma_{id}$$
Because variance decompositions normalized into [0, 1] interval, weighting procedure do not violate semi-positive definiteness property of variance covariance matrix. Additionally, symmetry property of variance-covariance matrix is also preserved based on $\sigma_{ij}(1 - (\gamma_{ii}^H) \gamma_{jj}^H) = \sigma_{ji}(1 - (\gamma_{jj}^H) \gamma_{ii}^H)$.

However, it is implicitly assumed that variances and covariances can be weighted straightforwardly, and by using only the own-connectedness measures for each variable one can decompose covariances as well. In line with the covariance property, $Cov(aX + b, cY + d) = acCov(X, Y)$, we multiply the covariance by the square root of diagonal elements of variance decomposition to obtain the appropriate decomposition of covariance.

Yet, one can raise an objection to this implicit assumption and argue that by reversing the procedure of weighting covariance, one can ignore the relationships among variables. However, we need to emphasize that our methodology does not suffer from this problem. Instead of decomposing covariances by the variance weights, we use general form of variance decomposition; covariance decomposition.

### 3.2 Decomposition via Forecast Error Covariance Decomposition

In this section, we attempt to decompose variance-covariance matrix into idiosyncratic and systematic components directly by using the covariance decomposition instead of using the connectedness matrix. To do that, we borrowed forecast error covariance decomposition methodology from Goto and Volkanov (2002). In their study, Goto and Volkanov (2002) applied covariance decomposition in order to investigate the dynamic response of the covariance between excess returns and inflation by using orthogonalized monetary policy shocks. They stated that for a serially and contemporaneously uncorrelated $w_t$, $h$-step forecast error covariance matrix is simply $\Sigma_y(h) = \sum_{i=0}^{h-1} (\Theta_s \Theta_s')$. Alternatively, the elements of that matrix can be calculated as $\sum_{k=1}^{K} \sum_{s=0}^{h-1} \theta_s(i, k)^2$. Then, they inferentially concluded that $h$-step forecast error covariance between $i$-th and $j$-th variables, $\Sigma_{ij}^y(h)$ is calculated by using the formula of $\Sigma_{ij}^y(h) = \sum_{k=1}^{K} \sum_{s=0}^{h-1} \theta_s(i, k)\theta_s(j, k)$. In that formulation, $K$ represents...
the number of endogenous variables in the system and \( \theta_s(j,k) \) is the \((j,k)\)-th element of VMA coefficients for each lag obtained by VAR estimation, \( \Theta_s \). Different than the DYCI framework, for covariance decomposition, we do not need to have normalized decomposition weights. Instead, to measure risk of a portfolio we only need the decomposition itself. Owing to that we just use the numerator of normalization formula for the covariance decomposition, which is suggested by Goto and Volkanov (2002) such as

\[
\frac{\sum_{s=0}^{h-1} \theta_s(i,k)\theta_s(j,k)}{\sqrt{\sum_{k=1}^{K} \sum_{s=0}^{h-1} \theta_s(i,k)^2(\sum_{k=1}^{K} \sum_{s=0}^{h-1} \theta_s(j,k)^2)}}
\]

Numerator of the expression is just the \( h \)-step forecast error covariance between \( i \)-th variable and \( j \)-th variable accounted for by shocks in \( k \)-th variable. By looking carefully to formula, it is easy to see that covariance decomposition is just generalized form of variance decomposition. By setting \( i = j \), we get variance decomposition. Additionally, one of the main differences between the covariance decomposition and the variance decomposition is the number of parameters to calculate. While variance decompositions end up with \( N \times N \) matrices, covariance matrix decompositions result in \( N \times N \times N \) arrays.

Goto and Volkanov (2002) also showed the relationship between the impulse responses of variance decomposition and the covariance decomposition framework. They claimed that they can infer the net effect of shocks on covariances by studying the conditional moment profile. By using standard VAR estimation, conditional \( h \)-step forecast, given an impulse in the \( k \)-th shock is \( y_t^{(k)}(h) = y_t(h) + \Theta_{h-1}\epsilon_{t+1}^{(k)} \) where \( \epsilon_{t+1}^{(k)} \) is the impulse vector with one in the \( k \)-th element and zeros in the others. Then, conditional moment profile is

\[
E\left[(y_{t+h} - y_t^{(k)}(h))(y_{t+h} - y_t^{(k)}(h))'\right] - \Sigma_y(h) = \theta_{h-1}(\cdot, k)\theta_{h-1}(\cdot, k)'
\]

due to a unit impulse in the \( k \)-th innovation where \( \theta_{h-1}(\cdot, k) \) is the \( k \)-th column of \( \theta_{h-1} \). Hence, it is clear that covariance decomposition is just cumulative sum of the multiplication.

Unlike the standard DYCI framework, which extracts information about the underlying network structure from variance decompositions, covariance decomposition methodology can also be used to extract additional information about the network structure. However, the main motivation of this study is just to quantify the systematic vs. idiosyncratic risks of a portfolio of assets. Therefore, deriving a network structure is beyond the objective of our study. Instead, in the rest of the paper, we will use covariance decomposition to extract idiosyncratic variance-covariance contribution of assets.

In principle any variance-covariance matrix can be used with connectedness matrix to
decompose portfolio risk as mentioned above. Yet, there are some practical differences due to nature of covariance decomposition. As the denominator of the covariance decomposition equation is not linear, it is hardly possible to obtain normalized weights. Thus, it is possible to proceed with covariance decomposition using only the forecast error covariance matrix without normalization.

By using directly the level of covariances instead of weights, we have covariance decomposition without normalization such as

$$\omega_{ij,k,h} = \sum_{s=0}^{h-1} \theta_s(i, k)\theta_s(j, k)$$

where \( \omega_{ij,k,h} \) is the \( h \)-step ahead forecast error covariance between the \( i \)-th and \( j \)-th variables accounted for by shocks in the \( k \)-th variable.

After estimating the VAR system and applying the covariance decomposition, we get \( \omega_{ij,k,h} \) which is an \( N \times N \times N \) array that will be used for the portfolio risk decomposition. To decompose portfolio risk, we use important property of covariance decomposition that is being the cumulative sum of the multiplication. This property enables us to obtain conditional covariances. In line with the previous section we commence our procedure by obtaining self-effectuating relations. In order to get that relation, we pick up covariances conditional on the covariate variables. Explicitly, we are interested in the covariate variables contribution on their own covariances. In a formal way, in order to get idiosyncratic component of \( h \)-step forecast error covariance of \( i \)-th and \( j \)-th, by using the summation property, we sum up each \( \omega_{ij,k,h} \) for \( k = i, j \).

$$\omega_{ij,ij,h} = \omega_{ij,i,h} + \omega_{ij,j,h}$$

The result, \( \omega_{ij,ij,h} \) is just the idiosyncratic covariance contribution of variables \( i \) and \( j \). If we do this for all variables in the system we get

$$\Sigma_{id,h} = \begin{bmatrix} \omega_{11,11,h} & \omega_{12,12,h} & \cdots & \omega_{1N,1N,h} \\ \omega_{21,21,h} & \omega_{22,22,h} & \cdots & \omega_{2N,2N,h} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{N1,N1,h} & \omega_{N2,N2,h} & \cdots & \omega_{NN,NN,h} \end{bmatrix}$$

Once the contributions of \( i \) and \( j \) to the idiosyncratic covariance are obtained, what remains is to subtract their covariance contributions from the forecast error covariance matrix, to obtain the remaining contributions to the covariance. This gives us the non-idiosyncratic
component of forecast error covariance matrix, by definition, systematic variance-covariance matrix is

\[ \Sigma_{sys,h} = \Sigma_{Total,h} - \Sigma_{id,h} = \begin{bmatrix} \sigma_{11} - \omega_{11,11,h} & \cdots & \sigma_{1N} - \omega_{1N,1N,h} \\ \vdots & \ddots & \vdots \\ \sigma_{N1} - \omega_{N1,N1,h} & \cdots & \sigma_{NN} - \omega_{NN,NN,h} \end{bmatrix}. \]

Notice that symmetry property of variance–covariance matrix continues to hold because \( \theta_s(i,k)\theta_s(j,k) = \theta_s(j,k)\theta_s(i,k) \). However, semi-positive definiteness property is not guaranteed. In applications, each systematic and idiosyncratic variance–covariance matrices must be checked before proceed to calculate each type of risk.

4 Estimation and Data

4.1 Shrinkage and Selection

Both the DYCI framework and covariance decomposition framework rely on the estimation of an approximating VAR model. However, given the fact that high number of endogenous variables in the vector autoregression model, it is problematic to go ahead with the classical least squares estimation of the VAR model. In order to overcome the degrees of freedom problem, one needs to use one of the available shrinkage and selection procedures. While there are many ways to apply shrinkage and selection such as pure shrinkage (which employs traditional informative-prior Bayesian analyses or ridge regression) or pure selection (using more traditional information criteria), in our study we use the least absolute shrinkage and selection operator (LASSO) methodology, which has been widely used by time series econometricians recently. (Demirer et al., 2015)

Tibshirani (1996) defines lasso methodology such as a linear model that minimizes the residual sum of squares subject to the sum of the absolute value of the coefficients being less than a constant. Classical linear regression model is defined as

\[ \hat{\beta} = \arg \min_\beta \sum_{i=1}^{N} \left( y_t - \sum_i \beta_i x_{it} \right)^2 \]

The difference of the lasso estimation is the constraint imposed for coefficients such as
By including constraint on coefficients into the optimization function, problem turns out to be a constrained optimization such as

\[ \hat{\beta} = \arg \min_{\beta} \left( \sum_{i=1}^{N} \left( y_t - \sum_{i} \beta_i x_{it} \right)^2 + \lambda \sum_{i=1}^{K} |\beta|^q \right) \]

The degree of absolute value of \( \beta \)'s, \( q \), determines the shape of penalty such as concave \((q \to 0)\) or smooth convex penalties \((q = 2)\). As Demirer et al. (2015) did, we chose the case \( q = 1 \) which blends both selection and shrinkage, too.

4.2 Generalized Forecast Error Covariance Decomposition

One of the main problem in impulse response analysis is the identification of impulse responses. The Choleski factorization approach to identification impulses responses imposes the orthogonalization of shocks to each variable. In the Choleski approach the identification of impulse responses (or simply, vector moving average coefficients) is achieved by assuming a specific order of variables, either guided by a structural economic model or by the additional knowledge of the exogeneity of the variables in the system. However, the fact that Choleski factorization assumes a specific order renders its use in a directional connectedness framework. As here we consider the contribution of each variable to the systematic risk, we cannot rely on Choleski factorization for the identification of impulse responses.

In order to deal with this problem Diebold and Yilmaz (2014) used Generalized Forecast Error Variance Decomposition methodology, developed by Pesaran and Shin (1998). They defined \( m \times 1 \) vector of the generalized impulse response function of a unit shock to the \( j \)-th equation as

\[ \psi^\beta_j(n) = \sigma_{jj}^{-1/2} A_p \Sigma e_j \]

instead of using orthogonalized impulse response function, \( \psi^o_j(n) = A_p Pe_j \) where \( A \) is \( N \times N \) vector autoregression coefficient matrix for lag \( p \), \( P \) is Choleski decomposition of variance-covariance matrix \( \Sigma \), an \( m \times m \) lower triangular matrix and \( e_j \) is the selection vector which contains \textit{one} in the \( j \)-th entry and \textit{zeros} in the remaining entries and \( n = 1, 2, \ldots, N \).

By using generalized impulse responses, we modified forecast error covariance decompo-
sition methodology and defined $h$-step ahead forecast error covariance between $i$-th and $j$-th variable which is accounted for by the innovations in variable $k$ such as

$$\omega_{ij,k,h} = \sigma_{kk}^{-1} \sum_{l=0}^{h} (e_i' A_l \Sigma e_k)(e_j' A_l \Sigma e_k)$$

### 4.3 Data and Risk Decomposition

We apply our approach to decompose portfolio risk into its idiosyncratic and systematic components on two separate sample of stock returns. First sample consists on 76 U.S. financial institutions until Bear Stearns is delisted, then we dropped Bear Stearns and Wachovia from our sample. Yet, we used 74 FIs in remaining. 72 of these are the investment and commercial banks, bank holding companies and other financial service companies in the U.S. measured by their average market capitalization for the 2006-2010 period. We intentionally added two financial institutions into this sample without regarding the market capitalization. One is AIG, an insurance company that was affected substantially from Lehman Brothers collapse in mid-September 2008. The second one is the American Express, the card company, which is also one of the largest financial services firms in the United States. Using this sample, we emphasized the 2008 financial crisis. Second sample includes 62 banks (commercial and investment banks) and financial institutions such as bank holding companies, investment companies and different kind of financial institutions from European Union and United States. So that we hesitated to use market capitalizations due to the differences across countries in this sample.

For the first sample, we used the time interval from 11 August 2006 to 28 May 2010. This sample period captures the 2008 financial crises, and helps us show dramatic changes in risk and hence allows us to see the results of our methodology crystal clear. For the second sample, we used the time interval from 5 January 2009 to 18 July 2016. This period captures break points such as European debt crisis, Greek referendum and Brexit and other important dates.

First, we constructed $100$ equally weighted portfolios for each sample. We got the price data from Thompson Reuters. In order to produce return series for assets, we used arithmetic returns instead of the logarithmic returns because the dramatic changes in the price of some particular assets such as Bear Stearns is exaggerated by logarithmic returns due to improper approximation for big price changes.

In the remainder of this section we present risk decompositions using our covariance
decomposition approach. First, in Figure 1 we present the decomposition of risk into idiosyncratic and systematic components for an $100 equally-weighted portfolio of major U.S. financial institution stocks by using first sample period. Similarly, Figure 2 represents the decomposition result for the second sample period and portfolio is constructed in the same way yet includes both US and EU financial institutions.

Figure 1 depicts that total risk as well as its systematic and idiosyncratic components for global financial crisis and eurozone crisis samples. For global financial crisis, three types of risks started to increase gradually with the first signs of the U.S. sub-prime mortgage crisis in early 2007. The first tremors of the subprime crisis were followed by the so-called liquidity crisis of August 2007, spreading the virus across the U.S. financial system as well as the international investors who invested in the U.S. market. After steadily increasing in early 2008, the total portfolio risk and its systematic component increased significant both in mid-summer of 2008, and in mid-September, due to troubles of Wachovia Bank and the collapse of Lehman Brothers, respectively. With collapse of Lehman Brothers, AIG, Citigroup, Wachovia and many other banks and indeed the whole U.S. banking system were under big systemic threat.

The total risk increased further from late March to early May 2009, with investors anxiously waiting for the outcome of U.S. banks’ stress test results conducted by the Federal Reserve. Once the stress test results for U.S. banks in the first week of May, both the total portfolio risk as well as its systematic and idiosyncratic components turned sharply down. Then, we see significant decline in all three risk types.

Not surprisingly, using 76 stocks in the first portfolio enables high level of risk diversification. This is evident in Figure 1 where the systematic component dominates the total

Figure 1: Decomposition of Portfolio Risk
risk and seems is the main driver of total portfolio risk. Besides, one can see that the sum of systematic and idiosyncratic risk is exceeding total risk, which is expected due to the sub-additivity property of risk.

Figure 1 shows risk results for US and EU banks from 2009 to mid-2016, as well. Initially, we witnessed a decline in risk by the end of 2009. This fall overlaps with the latest motion in previous graph. Following to that period, financial stress has risen due to the concerns about Greece in April 2010 and immediately spread to Portugal, Spain, Ireland and Italy. Nevertheless, bailout plan and new austerity measures introduced for Greece and announcement of forming EFSF/ESM to stabilize financial turmoil throughout the Europe have restrained the fear of the solvency and stopped the sudden increase in risk after mid-2010. The big drop in the beginning of 2011 stem from ruling out of Greek shock from the rolling window. After a relatively calm period following mid-2011, we see a steadily upward movement in risk until first quarter of 2012 and due to lack of significant recovery deepened concerns about Euro area. Besides, ECB’s rate hike action also contributed to elevate risk. This affected mainly Europe banks and U.S. banks holding Eurozone debt. Then, through successive bailout programs for particular countries, haircut agreement with holders of Greek sovereign debt and ECB’s step back from interest rate action gradually choked off the risk level and rolled back lower levels. Moreover, the upheaval against the dictated bailout plans in Greece increased the tension again at the end of 2014 and got the fear to the top by the Greek bailout referendum. On the other hand concerns about especially Italian and German banks fueled risk level by the end of 2015. The very last increase in the risk in financial system stemmed from the Brexit, Britain’s withdrawal from the European Union. Notice that Brexit burst overall risk to Greek crisis exposure level but it is still lower than the level exposed in 2012. Similar to global financial crisis, we found evidence of the dominance of systematic risk in total risk, and sub-additivity is also valid for that case.
The dominance of systematic risk in the total risk and the difference between the level of systematic and idiosyncratic risk in figure may have potential to misdirect the intuition especially for idiosyncratic risk. In order to deal with this possibility, we differenced both systematic and idiosyncratic risk and calculated the moving average for 150 days.

Figure 2 reveals pattern of moving averages for both sample. In both figure it is obvious systematic risk step up on idiosyncratic one in each systemic events pointed out above. When risk arises contagiously, changes in the systematic risk climb over idiosyncratic risk. On contrary, idiosyncratic risk exceed systematic one in relatively peaceful times. In these periods, concerns about the system do not prevail, the source of risk seems to be asset specific developments.

Next we focus on the ratio of the systematic component to the total portfolio risk reported
in Figure 1. This ratio shows us the daily relationship between systematic and total risk. For a high dimensional portfolio, the share of the systematic risk component is sensibly high for both sample, but by focusing on the margins, we see an increase in the systematic risk also drives total risk to be high, as we observe in both figures. In global financial crisis, in Figure 3, share of the systematic component is relatively low in the third and fourth quarter of 2006 however in March 2007, systematic component of the portfolio risk increases dramatically and stays high until the third quarter of 2009. Even though, total banking portfolio risk started declining in May 2009, the ratio stayed very high until the end of September 2009. This is a result of the fact that, as the total risks declined both the systematic and idiosyncratic components declined as well, keeping the ratio more or less unchanged. However, towards the end of September 2009, both total risk and its systematic component declined sharply, where as the drop in the idiosyncratic component was much slower which can also be verified by Figure 3.

On the other hand, in eurozone crisis, systematic risk over total risk follows similar pattern. But one may notice the level of systematic risk over total risk is slightly lower than previous one. This is a direct result of using 10 less stock which decreases the diversification. Additionally, in eurozone crisis sample we selected both EU and US financial institutions rather than US financial institutions only. In that case correlation among financial institutions from different countries may be less positively than correlation of financial institutions in one single country even though this country is a significant actor of global financial system.

5 Implications

There can be found several implications of this methodology. First, model can decompose systematic and idiosyncratic risk and this findings can be used to explore the effect of systematic and idiosyncratic risk on different kind of objectives as it is done in the literature. Second, methodology allows to monitor systematic risk. This is important especially in market downturns for portfolio management strategies of financial institutions, central banks and other regulatory institutions. Third, prevailing regulation such as Basel Accord distinguishes equity and interest rate risk as general market risk and specific risk. This definitions are substitute to systematic and idiosyncratic risk. To satify reporting demand of regulatory institutions, this methodology provides model based estimation of systematic and idiosyncratic risk rather than standardized methods.

Furthermore, we can extent the implications with help of certain assumptions. First,
we can obtain individual systematic risk contribution. Second, we can detect the diversification among systematic and idiosyncratic risk which implies the correlation among these components. Details are explained in the following subsections.

5.1 Individual Systematic Risk Contribution

One of the objectives of our risk decomposition exercise is to identify the contribution of each individual stock to the systematic risk using the information embedded in the systematic variance-covariance matrix. In order to do that we follow an ad hoc procedure where we isolate an asset from the so-called portfolio and treat it autarchical. By definition of the systematic-variance covariance matrix, this procedure helps us calculate the systematic risk before and after the isolation of the stock from the portfolio. The difference between the systematic risk measures before and after the isolation of the stock is regarded as the systematic risk contribution of that individual asset. The stock is isolated from the portfolio by setting the stocks variance and covariances to zero in the systematic variance-covariance matrix. If anyone interested in the total risk, it is possible to add the deleted variance and covariances to the idiosyncratic variance-covariance matrix. Linear additivity assumption justifies the deletion and addition. After setting the rows and columns of the variance-covariance matrix that correspond to the isolated asset equal to zero, post-isolation risk estimation is straightforward. The difference between the full portfolio systematic risk and the revised systematic risk gives the individual systematic risk contribution.

This procedure which relies on isolation of an asset at a time has its side effect. It effectively wipes out the diversification effect of the so-called autarchic variable. Broadly, in line with the sub-additivity of risk, excluding an asset from the portfolio can potentially increase the variance of portfolio and hence renders the pre- and post-isolation systematic risk levels incomparable. But in a well-diversified portfolio with a large number of assets, marginal diversification effect of an asset is relatively low. Therefore, using high dimensional system, in principle, allows us to ignore the effect of diversification for a particular asset. Conversely, for a low dimensional system, one must pay attention to the marginal diversification effect of a variable, which can be high. As a consequence, the systematic risk contribution of each asset can be exaggerated by the drop in variance due to diversification.

As we have already discussed above, we developed our framework to be able to measure the systematic risk contributions of particular financial institutions. For equally weighted portfolios, we show the percentage systematic risk contributions for twelve selected U.S. companies from the global financial crisis sample in Figure 4.
In the first sub-figure (upper left), we see Bear Stearns has contributed systematic risk above average value for whole period like Lehman Brothers. The significant change coincides with the subprime mortgage crisis which Bear Stearns’ funds invested heavily on these kind of assets. Then in March 2008, Bear Stearns failed to survive and contribution to the systematic risk dramatically increased and eventually taken over by JP Morgan.

Then, we reported the systematic risk contribution of Lehman Brothers. Evidently, Lehman Brothers had historically higher level of systematic risk contribution compared to other institutions. Particularly, at the end of the first quarter of 2008, Lehmans systematic risk contribution rises and stays almost unchanged until its collapse on September 15, 2008. The giant peak in the figure shows that the collapse of Lehman Brothers contributed a big proportion of the systematic risk with stimulating the fear about the collapse of the whole financial system.

In following, we report the systematic risk contributions of Goldman Sachs and J.P. Morgan. Both banks were regarded as rock solid during the crisis. Hereby, systematic risk contribution of these investment banks could not be much higher than the average expected systematic risk contribution, namely, 1/76. Goldman indicates consistent decline until the end of third quarter of 2008. However, Goldman is converted to bank holding company to reach FED facilities in September 2008. Besides, US Government’s TARP (Troubled Asset Repurchasing Program) bailout action to increase capitalization of US banks has included Goldman as participant with other banks such as J.P. Morgan, Citibank, Bank of America, Morgan Stanley and Wells Fargo. This action has signalled that supported banks were may be in worse condition than it is perceived and boosted concerns. In addition to this, as Black and Hazelwood (2013) pointed out large banks supported by government became more susceptible to the moral hazard incentive for risk taking associated with being perceived as "too-big-to-fail" phenomenon. This effect is clear for all big banks in the system.

Next, we see Bank of America and Citibank, which are two of the largest banks in the U.S. banking system. They both had a rough ride during the stages of crises in 2007 and especially in 2008. Both banks systematic contribution started with around 1% after mid-2006 and at the third quarter of 2009, they are both above 2%.

Seventh and eighth sub-figures (third row of Figure 4) shows two important actors of US mortgage market, Fannie Mae (The Federal National Mortgage Association) and Freddie Mac (Federal Home Loan Mortgage Corporation). At the beginning of the period Fannie Mae contributes systematic risk relatively higher than Freddie Mac. However Freddie Mac caught up Fannie Mae due to concerns about the housing and mortgage market in the last quarter
of 2007. Their systematic risk contributions continued to stay high throughout 2008 until early September when both of them were taken to U.S. government custody. Government’s action has decreased their systematic risk contribution dramatically at the third quarter of 2008 and restored confidence to these companies.

Then, we reported the systematic risk contribution of one of the biggest issuers of CDS on corporate debt, namely AIG. At first, AIG has systematic risk contribution around 1%, which is lower than $1/76$, the average expected systematic risk contribution of any stock in the equally-weighted portfolio. However, its systematic risk contribution started to increase as the financial markets lived through each step of the U.S. crisis in 2007 and 2008. At the third quarter of 2008, AIGs systematic risk explodes relatively to prior dates with the concerns about the likely default on its CDS. After the extension of bailout credit to AIG, its contribution to systematic risk started to decline.
In Figure 4, estimation results are reported for Morgan Stanley, Wells Fargo and American Express, as well. Morgan Stanley also has significant contribution to the systematic risk yet we do not observe significant change throughout the period. Similar to the Goldman case, Morgan Stanley has also converted to bank holding company in September 2008 and participated in TARP. This made similar impact on systematic risk contribution of Morgan Stanley.

Wells Fargo is one of the largest bank in US performed well in financial crisis yet we observe gradual increase in the systematic risk contribution in 2009. This movement is compatible with other large banks in the US financial system. On the other hand, American Express do not change too much in the system and consistent with the expected average systematic risk contribution.
Further, we have done similar application for Eurozone crisis sample. In this case, U.S. financial institutions do not show dramatic changes with respect to European counterparts. Due to this fact we report results for only European FIs leave results for selected US FIs in appendix section. We have 62 financial institution in this sample and expected average systematic risk contribution is consequently much closer to 2%.

Figure 5 shows estimation results for 18 selected banks from Europe. First two sub-graph reveals Greek banks' systematic risk contribution. From late 2009, Greece has faced a depression due to the combination of its long lasting structural problems and spillover from the turbulence of 2008. Revelation of accounting and statistical misreporting scandal in 2010 was also a breakpoint. Several factors compounded and eventually in April 2010, Greece became the main source of risk in global financial markets. To see Greek banks' systematic risk contributions, Alpha Bank and National Bank of Greece (NBG) is chosen to be reported. Both banks shared a common destiny with Greece economy in the beginning of period. As Greek economy fell down, so they have become more exposed and systematic risk contributions increased. After second quarter of 2010, rise in systematic risk is bounded owing to the agreement (so-called **Economic Adjustment Programme**) with European Commission, ECB and IMF together. Meanwhile, market panic gradually cooled off and overall systematic risk fell. Nevertheless, these days were not like the days just before the turmoil. They have faced prolonged under-capitalization problem. Further, higher risk premium for their bonds and securities had made international markets vigilant to the developments in Greece and Greek FIs. Moreover, bailouts directly affected liability side of their balance sheets and thus they have become more connected to international financing so far. Until end of 2012, we observe volatile but horizontal paths near expected average share. Notice that even if their systematic risk contribution decreased in this period, its share have not kept this track at all.

Banking crisis in Cyprus and recapitalization of major banks by Hellenistic Stability Fund have affected Greek banks in 2013. Volatility in systematic risk contribution for each bank was far more higher than in the past. Further in September 2012, NBG offered to merge with the third biggest bank in Greece, Eurobank. This initiative was precluded due to deficiency of required capital in April 2013. Nevertheless, this initiative had two important impact on NBG. First, the possibility of merger boosted **too big to bail-out** fear in markets. Second, the failure of initiative have emphasized the further recapitalization requirement for NBG. At that moment, NBG's systematic risk has taken off with respect to Alpha Bank based on especially increase in its systematic role. Soon after mid-2013, systematic risk contribu-
Figure 5: EU FIs: Systematic Risk Contributions for Eurozone Crisis Period
tions of Greek banks have calmed down. But then, social and economic effects of austerity policies which was dictated by Troika in exchange for bail-out funding gradually prompt disturbance in Greece. In December 2014, election of a radical left-wing party, Syriza, came up for criticism against these policies and questioned the benefit of economic and political integration to Europe. Fear of the Grexit exploded the systematic risk contributions dramatically. The possibility of withdrawal from European monetary system boosted systematic risk contributions due to current and future cost of disintegration to Greece and other countries in the system. After rejection of bailout program in July 2015 referendum, systematic risk contribution peaked. Yet, following renegotiations allowed to diminish the possibility of Grexit and systematic risk contributions have faded out gently. The last upward movements in Greek banks coincide with the Brexit. Not surprisingly this recalls Greece to mind. In that regard, Greece suffer from even financial and economic problems but the main systematic risk contribution originated by the disintegration risk. This reminds us the validity of diversification argument in EU especially for smaller countries.

In Figure 5, we also reported systematic risk contribution of two main Portuguese banks: Banco Comercial Portugues (BCP) and Banco BPI (BPI). First, we want to note that Portuguese banks, like Greek, Spanish and Italian ones show more alliance to the macroeconomic fundamentals than other US and core EU countries. Domestic government bond holding behavior, lending heavily on domestic market and low connectedness through assets relatively to other FIs in the system are some of the reasons behind this. Owing to this, considerable amount of developments in these banks’ systematic risk contributions can be explained by macroeconomic factors and policy actions taken by governments and international institutions. For instance, liquidity and capital funding of ECB and bail-out agreements enabled a funding relationship between peripherals and core countries by 2012. Besides, government bond yield spread has already widened between them. By reaccess of these countries to international bond market after first chapter of Eurozone crisis, yield spread attracted foreign investors and increased the connectedness with foreign counterparts and so systematic relations. We see similar positive trend for both Portuguese banks in the last quarter of 2009. This period coincides with the increase in the budget deficit expectations of Portuguese government and successive rating reduction by top rating institutions S&P, Moodys and Fitch. In response, government declared its commitment to reduce budget deficit and public sector imbalances in the second quarter of 2010. However, the increase in the risk perception about Portugal remained high during this period. Hence, rating institutions continued to degrade its rating in the first half of 2010. At the end of second quarter, government actions so
far so good prevented upward movements and systematic risk contributions remained to be stable at this level. After austerity package announced in November 2010 systematic risk contributions of both banks tend to decrease. Overall recovery of financial markets following to the Greek crisis also helped to relieve Portuguese banks’ positions.

When political tension climbed up, it was resulted as rejection of new austerity package in March 2011. Portuguese government resigned and systematic risk contributions went up again. EU and IMF agreed upon bailout package in May and successful parliamentary elections in June cooled down the tension. Systematic risk contributions turned back to relatively lower levels. Notice that Portuguese banks share of systematic risk contribution remains mostly below expected average\(^2\) until the end of 2012. The main breakpoint began with ECB’s aggressive stance to deal with the sovereign debt crisis and reaccess of Portuguese banks to international bond market. Based on abundant funding opportunity, easy access to international bond market and wider yield spread between core and peripheral countries created incentive to make carry trade for Portuguese banks. By the end of 2012, systematic relations scaled up, connectedness with foreign counterparts increased. In both graphs we see similar rise in systematic risk contributions. Therefore, their shares dramatically exceed the expected average share. BCP which is the largest bondholder of Portuguese sovereign bonds have performed more volatile and high systematic risk contribution in this period. This evidence indicates how sovereign debt crisis transmitted to banking industry in case of bailouts and ECB’s liquidity policy. When Portuguese government bond yield began to go up again due to political conflict between ruling coalition parties on new austerity packages in May 2013, heavy losses boosted risk for both banks. At that moment, increase in systematic linkages was compounded with uprising risk perception. This exploded systematic risk contribution of Portuguese banks. Another impacts have taken place in mid-2014 such as exiting international bailout program, bailing out Banco Espirito Santo and corruption inquiry linkage of Interior Minister Miguel Macedo. For both BPC and BPI we see the highest systematic contributions in that period. Then, the effect of Greece controversy and inconclusive elections in the second half of 2015 have prompted concerns for Portugal. We observe last highest record for the both banks’ systematic risk contributions in this period. Furthermore, fundamental problems of Portugal are still deadlock and financial system suffers high non–performing loan portfolios, low quality assets and low profitability. Nevertheless, we observe divergent movements for both banks’ systematic risk contributions. While contribution of BCP remains to be high and volatile, contribution of BPI tends to

\(^2\)In this case expected average share of systematic contribution is 1/62.
decrease. The fall in the systematic risk contribution of BPI probably stems from the
initiative for BPI’s takeover by a Spanish bank, CaxiaBank. This initiative has positively
affected BPI’s stock returns with respect to BCP and risk level stayed relatively low.

In order to see the Spanish banks’ role in this system, we reported systematic risk con-
tributions of Banco Santander (BS) and Banco Bilbao Vizcaya Argentaria (BBVA). Similar
to Portuguese ones, Spanish banks’ stories is much more linked to the macroeconomic facts
and government actions rather than banking operations. In the third quarter of 2008, Spain
entered recession and it is followed by S&P’s rating downgrade. In return, Spanish govern-
ment has taken some stimulus action against recession which was not successful at all. In
this period, Bank of Spain bailed out a respectively small regional bank in Spanish banking
system. Then in June 2009, Spanish government established a bail-out fund so-called Fondo
De Reestructuración Ordenada Bancaria (FROB) to reconstruct banks which faced difficulties
or in the worst case, solvent ones. Even Spain exited recession in the beginning of 2010, high
public deficit and increasing unemployment remained to be problematic. In May 2010, just
after Greece’s fall down, unemployment rate exceed 20% and government announced some
new austerity measures. These developments provoked concerns about Spanish economy and
government bond yield went up immediately. Losses on Spanish banks which held sovereign
bonds in their accounts cut back optimism about the future. Systematic risk contributions
of both Spanish banks surpassed expected average share of systematic risk contribution at
that moment and remained to be above until mid-2011. Expansion of Long Term Refinanc-
ing Operations credit to Spanish banks by Eurosystem in March and Draghi’s encouraging
speech in July 2012, systematic relations with EU financing mechanisms increases. For this
reason, systematic risk contributions of both banks went up like in Portugal case. Following
to that period, share of systematic risk contributions of both banks have oscillated around
2% which is close to expected average share.

It is obvious that 2013-2014 period was relatively calm for Spanish banks. Both bank
experienced slightly decreasing trend in their systematic risk contributions. In the beginning
of 2014, they succeeded to report high profits and this positively affected their stock returns
with respect to other peripherals. However, notice that in 2009, Spanish banks had 1%
share for each in portfolio systematic risk. Overall increase in systematic risk contributions
of Spanish banks similarly happened mostly due to increasing connectedness international
markets. One another and related reason is based on the slow recovery of Spanish economy.
Even Spanish economy managed to produce more and reduce unemployment in some ex-
tent after Eurozone crisis yet it is still far below the pre-crisis levels. This created incentive
for Spanish banks to convert their business lines from traditional banking activities such as taking deposits and lending towards trading more in equity and bond markets. In addition, due to relatively higher profits, their stocks and bonds could favourably be traded in international markets and they succeeded to find new investors for their CoCo bonds. Owing to that Spanish banks have became more connected to international financial system and increased their systematic importance. This fact can be verified by checking both graphs. At last, CoCo bond turmoil leaded by German Deutsche Bank affected Spanish banks in a remarkable way in the beginning of 2016.

Next, we report the systematic risk contribution of two important bank from Ireland, Allied Irish Bank and Bank of Ireland. Heavy losses absorbed capital accounts of Irish banking sector in 2008-2009 due to high exposure on bursting Irish property market, instability in global financial markets and macroeconomic instability such as increasing government bond yield, fall in employment, low demand and growth rate. First, to stabilize financial sector Irish government headed to take action and provided blanket guarantee to cover debt of domestic banks in September 2008. Furthermore, they established National Asset Management Agency to purchase and manage especially non-performing loan assets of Irish banks in April 2009. After NAMA business plan announced in mid-September 2009 market reacted favorably and Irish banks stocks gained value in global markets. This prompt confidence and Irish banks restarted to lever their systematic role in the system again. Nevertheless, Irish banks utilized by government action, questions about the financial sector stability remained to be exist. Owing to this systematic risk contribution of Irish banks has begun to increase. These actions supported by takeover of loan payments in exchange for indirect stake of Bank of Ireland in February 2010. First, NAMAs purchases of loans from banks have taken place in condition of raising capital. Moreover, Irish government nationalized Anglo Irish Bank at the end of March 2010, purchase of 18% stake in Allied Irish Bank in May 2010 and additional purchase of Bank of Irelands stake in exchange for capital injection to satisfy capital requirement ratio were some of the examples. Also, Greek crisis hit Irish banks in April 2010. It is obvious in both graphs, systematic risk contributions went up. This escalation can similarly be explained by increasing connectedness due to increase in bank borrowings from international bond market, ECB actions and overall increase in risk due to Greece. Then, increasing financial transactions also strengthened financial positions and risk perception about Irish banks went down. This caused a decline in systematic risk contributions even it remained high during 2010.

At the end of 2010, Allied Irish Bank which needed far more capital injection than other
banks in Irish banking system according to stress test results held at March 2011, ultimately nationalized by government. After that we see the systematic risk contribution of AIB has a downward trend until 2011, more stable and lower than Bank of Ireland contrarily to the previous periods. On the other hand, Bank of Ireland had already been regarded as risky prior to the stress test results. Beyond, it has continued to climb and peaked after announcement and remained to be high in second quarter of 2011. By liquidity assist of ECB and Central Bank of Ireland, further support provided by Troika-leded bailout to Irish banks in August 2011, BoI gained strength and risk perception decreased till the end of 2011. Story of the Bank of Ireland do not diverge from other peripherals after 2012. Reaccessing international bond market again in 2012 and high government yield spread between peripheral and core EU countries enabled Ireland and Irish banks to generate more capital for their businesses. Relatively fair economic condition for Ireland made it preferable for investment opportunities. The only difference in here is that Irish banks were already more connected to the international financial system prior to the Eurozone crisis with respect to other peripherals. The very last hike in systematic risk contribution for Bank of Ireland coincides with the Brexit. This result is coherent to the expectations.

Then, we report the systematic risk contributions of Intesa Saopaolo and Unicredit from Italy. Until 2010, Italian banks emerged volatile but below and close to expected average systematic risk contribution. They remained relatively sound such that they refused to take the government aid and could issue their stocks and bonds to international market to boost their capital level. When Greek crises deepened in April 2010, risk escalated for Italy due to common deficiencies with Greece like high sovereign debt, low growth rate and political instability. At that moment, Italian banks’ contribution to the systematic risk exceed 2% due to their holding of large sum of Italian sovereign debt.

Furthermore, during this period Italy diverged from other troubled countries with low current account deficit, lack of property speculation earlier on crisis and Italian banks also used to hold nearly half of their sovereign debt in their own hands. Moreover, they have slightly better stress test results in the first chapter of Eurozone crisis. Due to that Intesa and Unicredit did not exploded in their systematic risk contribution further and showed more persistence in the following period.

In 2011, even both Italian banks continued to boost their capital to satisfy regulatory needs, high volatility in the Eurosystem in mid-2011 also hit Italian banks. At that moment, Intesa and Unicredit exceed 3% of systematic risk contribution for each.

Especially following to the expansion of LTRO (Long Term Refinancing Operation) by
ECB in December 2011, February 2012 and so on, ECB’s aggressive stance over Eurozone crisis enabled Italian corporates and banks to enjoy convenient conditions for issuing corporate bonds to international markets. In the meantime, government bond market has been profitable and Italian banks continued to buy and hold high amount of government debt in their accounts using similar carry trade structure. This behaviour amplified their roles in financial system. Besides, in late 2012, tension on Eurozone banks also hit Intesa and Unicredit. These banks experienced their second peaks in systematic risk contributions at the end of 2012.

Notice that following to the Greek crisis, Italian banks have endured their contribution to systematic risk above expected average without any significant exception. We have three possible and potentially complementary explanation for this. First, as stated above, Italian banks have not restricted by any barrier in international financial markets. This prevented disintegration from the international financial system. Second, in the wake of Eurozone crisis, sovereign debt of Italy was far more greater than Greece, Portugal and Ireland and large amount of debt has consisted till today. This kept concerns alive throughout Eurozone crisis period. Third, increasing amount of non-performing loans sustained to grow in the Italian banking system especially they concentrated on Intesa and Unicredit during this period. These factors can explain sustained high systematic risk contribution between 2013-2015 period for Italian banks.

Lastly, CoCo bond turmoil also hit Italian banks in the beginning of 2016. The last uprising movements in systematic risk contributions stem from increase in their risks. In that period we see that their systematic risk contribution headed towards 3% level. In mid-2016, Brexit also boosted risk for both bank but this change is lower than others. So their shares of systematic risk contribution did not change too much.

In following, we report the systematic risk contributions of BNP Paribas (BNP) and Societe Generale (SocGen) from France. In the wake of Greek crisis, one of the largest non-residential sovereign bondholders of Greek debt was France. Particularly, both bank have revealed that they had large amount of Greek sovereign debt (5bn and 3bn respectively) in May 2010 and SocGen has already owned 54% share of Geniki Bank in Greece in its assets. After April 2010, we see similar upwards trend in systematic contribution of BNP and SocGen. Additional to Greek sovereign debt, both banks have large amount of peripheral countries’ debt especially Italian debt in their accounts. This was the reason behind both banks have stuck in the middle of eurozone crisis. As a consequence, SocGen’s high exposure to Greek bonds and bank ownership in Greece caused a significant reaction to Greek
crisis much more than BNP Paribas and carried its systematic risk contribution to 3% of total systematic risk. In the last quarter of 2012, flat patterns in shares of systematic risk contribution were disturbed by downgrade of BNP’s credit rating and change in the outlook of SocGen to negative by S&P. In addition to that Societe Generale was under investigation due to Libor Scandal in second half of 2012. This triggered a volatile period which can be seen in both graph. Following to 2013, both banks’ systematic risk contributions have crawled around 2% level which is higher than expected average systematic contribution level. Moreover, notice that SocGen had consistently higher systematic risk contribution than BNP throughout this period. Last, we see noisy pattern for both banks in 2016 due to CoCo bond tension and eventually systematic risk contribution hike after Brexit.

We also report the systematic risk contributions of Deutsche Bank (DB) and Commerzbank from Germany. Story of German banks goes further back to previous periods before Eurozone crisis. Heavy exposure in property market and sovereign bonds of Eastern Europe countries hit severely prior to 2010. However, they managed to survive. At the beginning of Greek crisis, German banks also shocked but in a short time after that both bank have revealed that they had minor exposure to Greek debt with respect to their total assets. In both graph we see decline in the systematic risk contributions. As Greek crisis pull other peripheral countries over especially by 2011, German banks especially Commerzbank faced trouble with sovereign debt held in their accounts particularly Italian and additionally Portuguese and Spanish debt. By 2012, ECB’s action on easing monetary and liquidity policy slowed down the contribution of both bank on systematic risk however it was not permanent. The last important case for German Banks initiated in the beginning of 2016 which is incorporated with a fancy type of security, CoCo bonds (contingent claim bonds). As concerns about the Italy and other peripheral countries, due to exposures of DB has experienced decline in share price. Besides, increased operational risk stemmed from mortgage market transactions has contributed this decline. The fear about the conversion of CoCo bonds to shares with low prices forced investors to sell them in the market. This rocketed the DB CoCo bond yield. Even though DB asserted that they remain rock solid, market fear had already grown up. One can notice that German banks contribution to systematic risk considerably high at that moment. Especially for DB, we observe maximum level of systematic risk contribution. On a side note, Brexit seems to have little impact on German banks.

We then give systematic risk contribution results for UK banks such as Barclays, Lloyds, HSBC and Royal Bank of Scotland (RBS). By loosely looking to four graphs, one can see
that HSBC diverges from all other UK and other countries’ banks mentioned above with its low level of contribution to system. This is mostly due to their diversified operations across countries not only in Europe but also in East Asia, Latin America, Middle East and US. HSBC’s contribution to systematic risk is about 1% throughout the period. In global financial crisis UK banks were harshly wounded however they succeeded to survive anyway. At the end of 2009, we observe decline in systematic risk contributions for each bank. When Greek crisis hit, especially Barclays and RBS exposed heavily on peripheral countries and systematic risk contributions gained momentum. In that case, Lloyds had negligible amount of exposure with respect to both banks. Following to this period, their systematic risk contribution crawled and tended to decrease eventually. For Barclays, this movement ceased with fall in profits in 2011. Additionally, depreciation of reputation based on mis-sold of payment protection insurance in UK banking sector deteriorated customer satisfaction for all Barclays, Lloyds and RBS. Thus, we see volatile and higher level of systematic risk contribution. On the other hand, RBS’s credit rating was downgraded by Moody’s in October 2011. Therefore, especially foreign investors has dropped off their holdings of RBS stock and price fell down. We observe sharp movements in RBS’ systematic risk contribution with respect to other UK banks. One important development in mid-2012 changed Barclays’ involvement of Libor scandal. This also pump up its systematic risk contribution to higher levels but it is not permanent. In the beginning of 2013, when all three reported heavy losses in their financial statements for 2012, the capital structure of these banks became a concern again. Graphs indicate that they faced high volatility in their systematic risk contribution in early 2013. After relatively calm period, by late 2015, increasing turmoil in financial markets and growing controversy especially about the migration policy of EU have shaken UK banks once again. Their contribution to systematic risk have began to accelerate especially after the announcement of Brexit referendum in February 2016. Referendum took place in June and slight majority of votes opt for leaving EU membership. This decision hiked up systematic contribution of UK banks.

5.2 Diversification of Systematic and Idiosyncratic Risk

Diversification is one of the main argument for portfolio risk especially since Markowitz (1952). There are bunch of literature on diversification which studies different aspects and implication about diversification on portfolio selection. Campbell et al. (2001) is one of the important studies discusses the properties of diversification. They argued the adequacy of the conventional wisdom about diversification depends on the level of idiosyncratic volatility
in the stocks making up the portfolio. They concluded that even the number of stocks included in portfolio is important for diversification, the composition of stocks making up the portfolio has also significant effect on diversification. Our framework has direct implication on identifying the systematic and idiosyncratic risk which are the basics of diversification argument.

However, we can produce an additional diversification implication by virtue of decomposition findings. In traditional CAPM based techniques, due to regression construction one must assume the independence between systematic and idiosyncratic components of risk. However in our methodology we do not have such requirement. By virtue of distinctive systematic and idiosyncratic variance-covariance structure we can calculate systematic and idiosyncratic risk separately. Without independence assumption, we can easily generate the correlation between systematic and idiosyncratic risk. This measure is helpful to understand the relationship between systematic and idiosyncratic risk. For instance, in systemic events individual firms may influenced by systematic effects and idiosyncratic risks may arise. On contrary, individual shocks may accumulate and generate systematic risk in some cases.

To measure this kind of relationship we employed diversification argument with a set of assumptions. First we assumed that separate variance-covariance structure as hypothetical individual assets such as fully idiosyncratic and fully systematic assets. In that regard, increasing positive correlation between idiosyncratic and systematic risk implies a direct decrease in the diversification among them. On contrary, decreasing positive correlation produces diversification effect. This argument is justified by the coefficient of diversification by Perignon and Smith (2010). In their paper coefficient of diversification is defined by

$$\delta = \frac{\sum_{i=1}^{N} VaR_i - DVaR}{\sum_{i=1}^{N} VaR_i}$$

where $VaR$ is Value-at-Risk amount and $DVaR$ is the diversified VaR. In that regard we used total risk as diversified risk and systematic and idiosyncratic risk series as two separate individual risk result. Estimation results of coefficient of diversification for both sample is reported in Figure 6.

In both figure we see similar but reverse pattern in coefficient of diversification with respect to systemic events. Its straightforward to make inference about the correlation between systematic and idiosyncratic risk. In systemic event, we see decrease in diversification which implies increase in the correlation among them. This result shows that systematic risk and idiosyncratic risk affects each other, bilaterally. This result can be extended by causality.
analysis but we stop our application here and left the remaining one for future studies.

6 Conclusions

In this paper we proposed a network-based methodology to decompose portfolio risk with its theoretical foundation and applied on two main financial crisis period experienced in last decade. This methodology provides an alternative technical approach to decompose portfolio risk in addition to other models in literature. One of the main significant difference of this methodology is inclusion of linkages rather than common market factors to obtain systematic component of risk. We think this kind of analysis is more consistent to the definition of systemic risk. To decompose portfolio risk we explained two method. First one incorporates both the variance–covariance matrix as a measure of risk and connectedness matrix as a measure of linkages. This method is based on weighting the variance–covariance matrix with connectedness table and extracting the idiosyncratic variance–covariance matrix. Furthermore, the systematic variance–covariance matrix is regarded as the total variance–covariance matrix minus idiosyncratic one. Yet the weighting procedure has a potential shortcoming such as ignoring the correlation among variables. To overcome, we employed forecast error covariance decomposition. In this methodology, the portfolio variance–covariance matrix is decomposed directly without any need of weighting.

In application, we estimated two separate sample period which focused on two important crisis period in last decade. In first sample we estimated data of 2006-2010 period to cover financial crisis period and US financial institutions only. In second sample, we used 2009-2016 period to include especially Eurozone crisis and other important developments until recent
dates. In second sample, we used both US and EU financial institutions. Both estimation results show that in a well-diversified portfolio of assets systematic risk prevails. However, changes in systematic and idiosyncratic risk depends on the structure of the financial system. If there is a concern about contagion of risk in the system, then systematic risk step up on idiosyncratic risk. However, in ordinary times idiosyncratic changes overtake systematic one. In such periods shocks in the system do not convert into systemic events.

Our methodology produces direct and indirect implications. Direct implications can be explained by three reasons. First, as estimation results indicate that it provides a viable decomposition framework which can be employed to acquire systematic and idiosyncratic risk separately. This enables us to use both risk results in order to investigate effects of each on several objectives as it is done in literature. Second, methodology give an opportunity to monitor systematic risk in system. Especially in market downturns financial institutions can use this data in order to compose their market strategies and portfolios. Moreover, central banks and regulatory institutions can track systematic risk measure to monitor overall systematic risk level in the financial systems. Third, prevailing regulation such as Basel Accord regards equity and interest risk as a combination of general market risk and specific risk. These definitions are substitutes of systematic and idiosyncratic risk. This methodology provides a model based estimation technique to decompose equity and interest rate risk into their systematic and idiosyncratic components to satisfy reporting demand of financial market regulators.

There are also two main implications which can be obtained by using some additional procedures as individual systematic risk contribution and diversification of systematic and idiosyncratic risk. Individual systematic risk contribution methodology relies on the ad hoc procedure by isolating an asset from the so-called portfolio and treating it autarchical. Systematic risk contribution of a specific asset is just the difference between the actual systematic risk result and post-treatment risk result. Estimation results for individual systematic risk contributions are reasonable and compatible with actual developments. On the other hand, we can obtain the coefficient of diversification for hypothetical fully-systematic and fully-idiosyncratic risk. This enables us to compute a measure of relation between systematic and idiosyncratic risk factors.
7 Appendix

In this section, we represent US FIs systematic risk contribution separately from EU FIs. In Figure 7-8, we put Goldman Sachs, JP Morgan, Bank of America, Citibank, Fannie Mae, Freddie Mac, AIG, Morgan Stanley, Wells Fargo and American Express. In these figures, Bear Stearns and Lehman Brothers unfortunately do not exist due to their collapse in global financial crisis. During the eurozone crisis many of major US financial institutions seemed to be stationary in their systematic risk contributions at level of expected average or below. However, some of them demonstrate abnormal deviations above expected average. First, we see that systematic risk contributions fell with respect to European countries. However, some of them tended to increase their contribution due to their transatlantic exposure to Europe countries such as Citibank, JP Morgan, Bank of America and Morgan Stanley. Nevertheless, their exposure to European countries are low proportionally to their balance sheets and Greek effect did not hurt them too much. Major effect has taken place at mid-2011 with world-wide plunge of stock markets. Besides, in August 2011, AIG sued Bank of America because of the troubled mortgages caused billions of dollars loss for AIG. This exceptional impact is apparent in Figure 7 and Figure 8. On the other hand, the most turbulent times are experienced by Fannie Mae and Freddie Mac which are located in Figure 7. There are two significant jumps which exceed expected average level which can be seen in both graphs.
Figure 7: Individual Systematic Risk Contributions for First Sample
References


