

Income Inequality and Asset Prices*

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-preliminary and incomplete-

Abstract

We study the relationship between income inequality and stock market returns. We develop a quantitative general equilibrium model that includes two groups of agents: top 10% income group (capital owners) and workers who consume their after-tax labor income. Inequality in the model arises from two sources: capital and labor, and its increase is modeled as growth in the capital owners' capital and labor shares of income. The model and the data predict positive relationship between capital income inequality and real equity returns and negative relationship between labor income inequality and real equity returns. We find that real cumulative return on ex-dividend S&P 500 index would have been lower by 40% if there was no rise in capital income inequality. If the labor share of top decile would not grow, the same S&P 500 index would have experienced cumulative growth higher by 32%. The effects of capital and labor incomes' changes partially offset each other, and S&P 500 index displays a cumulative increase of 290% between 1970 and 2014 and 275% if there was no increase in income inequality.

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1 Introduction

The increase in income inequality in the United States over the past four decades triggered a vivid debate on its causes. A large body of research has initially suggested that skill-biased technical change and resulting skill-premium is the main driver of the observed increase in income inequality. More recently, an alternative explanation to the skill premium has been suggested by Piketty (2014), among others. Using Survey of Consumer Finances (SCF), Kacperczyk et al. (2016) show that the main driver of the recent increase in income inequality is the growing dispersion in capital income inequality reflecting the widening wealth distribution. In fact, Saez and Zucman (2016) emphasize the strong correlation between top labor and capital incomes. Specifically, the top wealth holders of today are also the top earners. For instance, they show that in 2012 the top 0.1% of wealth holders earned 31 times the average labor income and their pre-tax income share almost tripled between 1960 and 2012. Given that equities are concentrated in hands of households at the top of the U.S. income distribution, changes in their labor and capital income would be expected to affect the movements in asset prices.¹

In this paper, we study the relationship between income inequality and stock market returns. We develop a quantitative general equilibrium model that reflects the importance of limited stock market participation.^{2,3} Our framework includes two groups of agents. The top income group (capital owners) owns 100% of the economy’s financial wealth—a setup that roughly approximates the highly-skewed distribution of U.S. financial wealth.⁴ The rest of economy is populated by workers who consume their after-tax labor income.

As in the data, inequality in the model arises from two sources: capital and labor, and its increase is modeled as growth in the capital owners’ capital and labor income shares.⁵ Income shares enter the model via stochastic exponents in a Cobb-Douglas aggregate production function, as in Young (2004), Ríos-Rull and Santaella-Llopis (2010), Lansing (2015), and Lansing and Markiewicz (2016). Their movements, re-

¹Chen and Stafford (2016) argue that even fewer than 20 percent of households own stock directly.

²Vissing-Jorgensen (2002) and Lettau and Ludvigson (2009), among others emphasize the importance of limited stock market participation for modeling asset price dynamics.

³Models with these features include Danthine, and Donaldson (2002) and Guvenen (2009), among others.

⁴The share of total financial wealth owned by the top 10% of households is around 80% in the sample period (Wolf, 2010).

⁵Figure 5 shows how top income shares, real income and capital income share evolved since 1970 in the U.S.

flecting changes in relative factor productivities affect capital owner's decision how much to invest and hence equity values.⁶

The model predicts positive relationship between capital income inequality and real equity returns and negative relationship between labor income inequality and real equity returns. The impact of total income inequality on the stock market return is thus unclear. The data confirms this prediction. In a set of contemporaneous regressions, we find a significant, positive relationship between capital share of income and the excess real S&P 500 return and a negative one between labor share of income and the excess real equity return. We do not find any significant relationship between total income inequality and stock market returns.

The quantitative model is designed to match exactly (1) real per capita output, (2) real per capita aggregate consumption, (3) real per capita private nonresidential investment, (4) real per capita government consumption and investment, and (5) real per capita government transfer payments to individuals and (6) real, per capita equity value of S&P 500 between 1970 and 2014. In addition, it delivers satisfying quantitative performance in both macroeconomic and financial dimensions tracking well the cyclical movements of after-tax and transfers income and replicating the main unconditional asset pricing moments.

The general equilibrium framework that exactly replicates the U.S. data paths allows us also to evaluate the quantitative impact of the observed changes in income inequality in the U.S. on asset price dynamics. For this purpose, we compare the baseline simulation replicating U.S. data dynamics to three counterfactuals. In the first one, the capital share of income is held constant at year 1970 value. In the second, labor share of income of capital owners is kept constant. In the third counterfactual both capital and labor shares of income of capital owners are kept unchanged since 1970.

We find that labor income inequality is the key driver of the increase in the U.S. income inequality between 1985 and 2000 and capital income inequality mainly accounts for inequality growth after 2000. In line with theoretical and empirical predictions, capital share of income is associated with higher stock market index. Specifically, real cumulative return on ex-dividends S&P 500 index would have been lower by 40% if there was no rise in capital income inequality. If the labor share of top decile would not grow, the same S&P 500 index would have experienced cumulative

⁶Greenwald, Lettau, and Ludvigson (2014) and Lansing (2015), find that the shocks to the factor shares account for a substantial amount of the dynamics of the U.S. stock prices.

growth higher by 32%. Since the effects of capital and labor incomes' changes partially offset each other, the impact of total increase in income inequality on stock market returns is quantitatively small. S&P 500 displays a cumulative increase of 290% during the sample period and 275% under the counterfactual scenario.

The paper is organized as follows. In Section 2, we provide intuition for the link between income inequality and stock returns using a simple deterministic model. Specifically, in this section we analyze how different sources of income inequality: labor and capital affect asset prices. Section 3 tests the theoretical predictions via a set of contemporaneous regressions between excess real equity returns and income inequality measures. In Section 4, we build a stochastic model which exactly replicates the U.S. data paths and generates realistic asset price dynamics. Section 5 evaluates the quantitative performance of the model in both macroeconomic and financial dimensions. In Section 6, we carry out a set of counterfactual exercises which allow us to assess the quantitative impact of the recent increase in income inequality on the stock market returns. Section 7 concludes.

2 Simple model

We start our analysis with a simple model that features the main components necessary to understand the relationship between income inequality and stock returns. The framework we use builds on the empirical evidence highlighting the high concentration of capital in the U.S. economy at the top of the income distribution (Chen and Stafford, 2016). Income inequality in the model comes from two sources: labor income and capital income. Capital stock is held by capital owners who also supply labor to their own firms. Workers simply supply labor and consume their wage income. Changes in income inequality are driven by shifts in relative factor productivities reflected by movements in Cobb-Douglas factor shares parameters.

The pricing kernel in this class of models is uniquely determined by the capital owners consumption and investment choices.

Recent empirical study by Greenwald, Lettau, and Ludvigson (2014) finds that highly persistent factor share shocks, which redistribute income between stockholders and non-stockholders are an important driver of U.S. stock prices over the period 1952 to 2012. Similarly Lansing (2015) shows that distribution shocks can account for a considerable part of equity premium. Given that the shares of income proxy for income inequality, these studies imply a relationship between dynamics of inequal-

ity and asset prices. We first study this relationship in a framework of a simple, deterministic model.

2.1 Workers

The individual worker's decision problem is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t^w), \quad (1)$$

subject to the budget constraint

$$c_t^w = w_t^w \ell_t^w, \quad (2)$$

where E_t represents the mathematical expectation operator, β is the subjective time discount factor, c_t^w is the individual worker's consumption, w_t^w is the worker's competitive market wage, and $\ell_t^w = \ell^w$ is the constant supply of labor hours per worker. Workers simply consume their labor income $w_t^w \ell_t^w$.

2.2 Capital Owners

Capital owners maximize their consumption stream

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t^c), \quad (3)$$

subject to the budget constraint

$$c_t^c + i_t = w_t^c \ell_t^c + r_t k_t, \quad (4)$$

where c_t^c is the individual capital owner's consumption and $\ell_t^c = \ell^c$ is the constant supply of labor hours. The symbol i_t represents investment in physical capital k_t . For simplicity, we assume that the functional form of the utility function and the discount factor β are the same for both capital owners and workers. Capital owners earn income by supplying labor and capital services to firms. They earn a wage w_t^c for each unit of labor employed by the firm and receive the rental rate r_t for each unit of physical capital used in production.

2.3 Firms

Identical competitive firms are owned by the capital owners and produce output according to the technology

$$y_t = A k_t^{\theta_t} \left[(\ell_t^c)^{\alpha_t} (n \ell_t^w)^{1-\alpha_t} \right]^{1-\theta_t}, \quad A > 0, \quad (5)$$

The shifts in the production function exponents θ_t and α_t represent changes in shares of income and hence proxy for income inequality shifts. Given the Cobb-Douglas form of the production function, θ_t is capital's share of income, $\theta_t + \alpha_t(1 - \theta_t)$ is the top decile income share, $\alpha_t(1 - \theta_t)$ is the labor income share of the capital owners, and $(1 - \alpha_t)(1 - \theta_t)$ is the income share of the workers. Given that capital stock is held by capital owners, an increase in capital share of income θ_t directly implies an increase in both capital income and total income inequalities, everything else unchanged. An increase in α_t implies growing labor income inequality.

Profit maximization by firms yields the following factor prices

$$r_t = \theta_t y_t / k_t, \quad (6)$$

$$w_t^c = \alpha_t(1 - \theta_t) y_t / \ell^c, \quad (7)$$

$$w_t^w = (1 - \alpha_t)(1 - \theta_t) y_t / (n\ell^w). \quad (8)$$

Capital dynamics are described by a following law of motion:

$$k_{t+1} = (1 - \delta)k_t + i_t, \quad (9)$$

where δ is the capital depreciation rate. In this simple setup, we abstract from the capital adjustment costs. They will be introduced in the full version of the model and will appear important for mapping from investment to the asset pricing variables. In this version of the model the equity price p_t simply equals investment i_t and the closed-form investment decision rule for capital owners is:

$$i_t = p_t = \beta\theta_t y_t - (1 - \beta)(1 - \delta)k_t. \quad (10)$$

In equilibrium, the dividends equal $d_t = \theta_t y_t - i_t$. Using (10) we can write the dividends as

$$d_t = \theta_t y_t (1 - \beta) - (1 - \beta)(1 - \delta)k_t. \quad (11)$$

The excess return is defined as the difference between return on equity and risk free rate:

$$R_{t+1}^e = R_{t+1} - R_{t+1}^f \quad (12)$$

where $R_{t+1} = \frac{p_{t+1} + d_{t+1}}{p_t}$, $R_{t+1}^f = \frac{1}{E_t M_{t+1}^c}$ and $M_{t+1}^c = \beta (c_{t+1}^c / c_t^c)^{-1}$.

2.4 Income Inequality and Asset Pricing Variables

An increase in θ_t measures an increase in capital income inequality. Taking partial derivatives with respect to θ_t we find the following:

$$\frac{\partial p_t}{\partial \theta_t} = \beta y_t [1 + \theta_t \ln(k_t) - \alpha \ln(l^c) - (1 - \alpha_t) \ln(l^w)], \quad (13)$$

and

$$\frac{\partial d_t}{\partial \theta_t} = (1 - \beta) y_t [1 + \theta_t \ln(k_t) - \alpha_t \ln(l^c) - (1 - \alpha_t) \ln(l^w)] \quad (14)$$

so the response of real equity prices and dividends depends positively on the economy's output y_t and how this output is distributed between the factors of production. The response is positive in the share of income of accumulated capital stock $\theta_t \ln(k_t)$ and negative in the labour shares of income of capital owners $\alpha_t \ln(l^c)$ and workers $(1 - \alpha_t) \ln(l^w)$. The responses of real equity prices and dividends to increase in capital share of income translate into the response of the contemporaneous excess return as defined in (12). Specifically

$$\frac{\partial R_t^e}{\partial \theta_t} = y_t [1 + \theta_t \ln(k_t) - \alpha \ln(l^c) - (1 - \alpha_t) \ln(l^w)]. \quad (15)$$

Assuming that the capital share of income θ_t does not change, an increase in α_t implies an increase in the labor share of income of capital owners $\alpha_t(1 - \theta)$. Taking the partial derivatives with respect to α_t we find the following:

$$\frac{\partial p_t}{\partial \alpha_t} = \beta \theta_t (1 - \theta_t) y_t [\ln(l^c) - \ln(l^w)], \quad (16)$$

$$\frac{\partial d_t}{\partial \alpha_t} = (1 - \beta) \theta_t (1 - \theta_t) y_t [\ln(l^c) - \ln(l^w)], \quad (17)$$

and

$$\frac{\partial R_t^e}{\partial \alpha_t} = \theta_t (1 - \theta_t) y_t [\ln(l^c) - \ln(l^w)]. \quad (18)$$

The change in the expected equity return in response to the change in labor income inequality depends on the relative number of capital owners l^c and workers l^w in the economy. If they are equal, $l^c = l^w$, there will be no impact of increase in labor income inequality on the real equity return. If there are fewer workers in the economy than there are capital owners, the impact will be positive. In the data however capital stock is highly concentrated among the richest and hence the number of capital owners l^c is lower than the number of workers l^w thus implying the negative impact of increase in the labor income inequality on expected equity return.

3 Income Inequality and Asset Pricing Variables in the Data

We test the relationships between income inequality and equity return implied by the model via a set of simple regressions. It is important to emphasize that these relationships are contemporaneous. Thus, in contrast to the large body of empirical asset pricing literature, we do not seek to identify a new asset pricing factor and therefore we do not carry out predictive regressions.⁷

As in the stylized model presented in the previous section, we use the shares of income to proxy for inequality.

We first regress the capital share of income and labor share of income on the excess real equity return. The slope coefficient should indicate if the equity returns are positively or negatively related to the capital and labor income inequalities. The annual real excess return series has been constructed using data from Welch and Goyal (2008, updated) where we used the S&P 500 value weighted index adjusted for inflation and the risk-free rate is the Treasury-bill rate. The shares of income data come from the Bureau of Economic Analysis (BEA). Capital's share of income is computed as 1 minus the ratio of employee compensation to gross value added of the corporate business sector. As there is a clear time trend in the computed series, we use the HP filter to remove it.⁸ We expect that there is a relationship between real equity and cyclical components of shares of income.

Labour income inequality is measured as the labor share of income of the top decile of the U.S. income distribution and comes from the U.S. Census Bureau. Again, we remove the time-trend using HP filter. In addition to the inequality proxies, we include in the regressions a set of controls: GDP growth, price-earnings ratio, price-dividend ratio and the Lettau-Ludvigson consumption-wealth ratio, commonly referred to as CAY. The sample covers post-war period between 1945 and 2014, for which the CAY series are available and 1967-2014 for which the U.S. Census Bureau provides the labor and total income shares of the top decile. Table 1 below shows the results of the regressions including only inequality proxies and Table A.1 in Appendix A reports all the additional regressions.

The first column of Table 1 describes the regressor and the second column indi-

⁷Toda and Walsh (2016) for instance show that the top income share predicts subsequent excess stock market returns.

⁸Karabarbounis and Neiman (2014) show that capital's share increased in 42 out of 59 countries over at least 15 last years.

Table 1: **Slope Coefficients in Regressions of Returns on Inequality Measures**

Inequality Measure		
Capital income inequality	θ_t	7.10***
U.S. capital share of income		(1.76)
Labor income inequality	$\alpha_t(1 - \theta_t)$	-6.59**
Labor share of income of top 10		(1.94)
Total Income inequality		
Top income share	$\theta_t + \alpha_t(1 - \theta_t)$	
Top 10		-1.07
		(4.23)
Top 20		4.45
		(6.05)
Top 5		-2.07
		(3.66)

cates the corresponding variable in the stylized model. The third column reports the slope coefficient obtained from the regression and the corresponding standard errors in brackets. The results suggest that capital income inequality is positively related to the real excess returns and that labor income inequality is negatively related to the real excess return. However when we regress the excess real equity return on the total share of income of top decile, the slope coefficient becomes insignificant. This is not surprising. If almost entire capital stock is held by the top decile of the U.S. income distribution and increase in capital income inequality has significant positive impact on equities, and an increase in the labor share of the same top decile has a significant negative effect, they both cancel each other out.

We check if the income shares do not proxy for some other variable driving real equity return. In the contemporaneous equity regressions, we also include real GDP growth, price dividend ratio, price earnings ratio and the Lettau-Ludvigson consumption-wealth ratio, commonly referred to as CAY. The table with the results of additional regressions is contained in Appendix A. Table A.1 shows that our results are robust to the inclusion of additional variables.

4 Complete Model

The complete version of the model includes extensions of the simple framework in three dimensions that we find crucial to study the relationship between income inequality and asset price dynamics. First, in order to generate extra volatility in the asset returns, we employ a non-linear capital adjustment costs specification, in the spirit of Cassou and Lansing (2006) and Lansing (2012).

Second, the fact that the pre-tax income inequality in the U.S. increased more than after-tax inequality highlights the importance of progressive taxation and transfers for the model. We therefore introduce a government which collects tax revenue to finance expenditures on public consumption and redistributive transfers.

Third, in order to bring the model closer to the data both in macro and financial dimensions, we introduce three shocks: a labor-augmenting productivity shock, a capital accumulation shock and a distribution shocks along the lines of Young (2004), Ríos-Rull and Santaaulàlia-Llopis (2010), and Lansing and Markiewicz (2016). In what follows, we describe all the model's elements in detail.

4.1 Workers

The individual worker's decision problem is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t^w), \quad (19)$$

subject to the budget constraint

$$c_t^w = (1 - \tau_t^w) w_t^w \ell_t^w + T_t/n, \quad (20)$$

where τ_t^w is the worker's personal income tax rate. Workers are assumed to incur a transaction cost for saving or borrowing small amounts which prohibits their participation in financial markets. As a result, they simply consume their resources each period, consisting of after-tax labor income $(1 - \tau_t^w) w_t^w \ell_t^w$ and a per-worker transfer payment T_t/n received from the government.

4.2 Capital Owners

Capital owners represent the top decile of income distribution. Their decision problem is to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t^c), \quad (21)$$

subject to the budget constraint

$$c_t^c + i_t = (1 - \tau_t^c)(w_t^c \ell_t^c + r_t k_t) + \tau_t \phi_t i_t, \quad (22)$$

where capital and labour incomes are taxed and the capital owner's personal income tax rate is τ_t^c . The term $\tau_t \phi_t i_t$ captures the degree to which business investment can be deducted from business taxable income, where τ_t is the effective business tax rate (which may differ from τ_t^c), and ϕ_t is an index number that captures elements of the tax code that encourage saving or investment. In the quantitative analysis, we calibrate the average value of the term $\tau_t \phi_t i_t$ to reflect a standard depreciation allowance for physical capital.

Because standard frictionless production economies cannot generate sufficient return volatility, we introduce a non-linear adjustment costs specification for capital, which can create fluctuations in the price of capital and increase returns' volatility:

$$k_{t+1} = B k_t^{1-\lambda_t} i_t^{\lambda_t}, \quad (23)$$

$$\lambda_t = \bar{\lambda} + \exp(v_t) \quad (24)$$

$$v_t = \rho_v v_{t-1} + \epsilon_t, \epsilon_t \sim NID(0, \sigma_\epsilon^2), \quad (25)$$

with k_0 given. The parameter $\lambda \in (0, 1]$ is the elasticity of new capital with respect to new investment. When $\lambda_t < 1$, equation (23) reflects the presence of capital adjustment costs.⁹

⁹Similar formulation is employed by Cassou and Lansing (2006) in a welfare analysis of tax reform. Since equation (23) can be written as $k_{t+1}/k_t = B (i_t/k_t)^\lambda$, our adjustment cost specification can be viewed as a log-linearized version of the following law of motion employed by Jermann (1998): $k_{t+1}/k_t = 1 - \delta + \psi_0 (i_t/k_t)^{\psi_1}$.

4.3 Firms

As in the simple model firms are owned by the capital owners and produce output according to the Cobb-Douglas technology

$$y_t = A k_t^{\theta_t} \left[\exp(z_t) (\ell_t^c)^{\alpha_t} (n \ell_t^w)^{1-\alpha_t} \right]^{1-\theta_t}, \quad A > 0, \quad (26)$$

$$z_t = z_{t-1} + \mu + \varepsilon_t, \quad \varepsilon_t \sim NID(0, \sigma_\varepsilon^2), \quad (27)$$

$$s_t \equiv \frac{\theta_t}{\theta_t + \alpha_t(1 - \theta_t)}, \quad (28)$$

$$s_t = (s_{t-1})^\rho (\tilde{s})^{1-\rho} \exp(u_t), \quad \begin{aligned} \tilde{s} &\equiv \exp\{E[\log(s_t)]\}, \\ |\rho| &< 1, \\ u_t &\sim NID(0, \sigma_u^2), \end{aligned} \quad (29)$$

with z_0 and s_0 given. In equation (26), z_t represents a labor-augmenting productivity shock that evolves as a random walk with drift. The drift parameter μ determines the trend growth rate of the economy. The shock innovation ε_t is normally and independently distributed (*NID*) with mean zero and variance σ_ε^2 . Stochastic shifts in the the production function exponents θ_t and α_t represent distribution shocks along the lines of Young (2004), Ríos-Rull and Santaeulàlia-Llopis (2010), and Lansing (2015). Given the Cobb-Douglas form of the production function, θ_t is capital's share of income, $\theta_t + \alpha_t(1 - \theta_t)$ is the top decile income share, $\alpha_t(1 - \theta_t)$ is the labor income share of the capital owners, and $(1 - \alpha_t)(1 - \theta_t)$ is the income share of the workers, representing the bottom four deciles.

Profit maximization by firms yields the factor prices decribed by (6), (7) and (8).

4.4 Government

The government collects tax revenue to finance expenditures on public consumption and redistributive transfers. We assume that the government's budget constraint is balanced each period, as given by

$$g_t + T_t = n \tau_t^w \underbrace{w_t^w \ell^w}_{= y_t^w} + \tau_t^c \underbrace{(w_t^c \ell^c + r_t k_t)}_{= y_t^c} - \tau_t \phi_t i_t, \quad (30)$$

where g_t is public consumption, T_t is aggregate redistributive transfers, and y_t^i for $i = w, c$, is the pre-tax income for workers and capital owners, respectively. The balanced-budget constraint can be viewed as an approximation to the consolidated

budgets of federal, state, and local governments. Public consumption does not provide direct utility to either capital owners or workers. Nevertheless, we include g_t in our analysis to obtain quantitatively realistic tax rates during the transition period from 1970 to 2014.

Following Guo and Lansing (1998) and Cassou and Lansing (2004), we introduce progressive income taxation via the formulation

$$\tau_t^i = 1 - (1 - \tau_t) \left(\frac{y_t^i}{\bar{y}_t} \right)^{-\kappa}, \quad (31)$$

where τ_t^i is the personal income tax rate of agent type i , y_t^i is the individual agent's pre-tax income, and \bar{y}_t is the average per capita income level in the economy which the agent takes as given. The parameter $\kappa \geq 0$ governs the slope of the tax schedule while τ_t governs the level of the tax schedule. When $\kappa > 0$, the agent's personal tax rate is increasing in the agent's income, reflecting a progressive tax schedule. When $\kappa = 0$, the tax schedule is flat such that all agents face the same tax rate τ_t regardless of their income. For simplicity, we assume that τ_t also pins down the effective business tax rate which exhibits no progressivity.

The agent's marginal personal tax rate MTR_t^i is defined as the change in taxes paid divided by the change in income, that is, the rate applied to the last dollar earned. The expression for the agent's marginal personal tax rate is

$$MTR_t^i = \frac{\partial (\tau_t^i y_t^i)}{\partial y_t^i} = 1 - (1 - \kappa) (1 - \tau_t^i), \quad (32)$$

which implies $MTR_t^i > \tau_t^i$ when $\kappa > 0$.

The average per capita income level in the economy is given by $\bar{y}_t = y_t / (n + 1)$, where $n + 1$ is the total number of agents. Making use of the Cobb-Douglas production function (26) and the factor prices (6), (7) and (8), the equilibrium personal income tax rates for each type of agent are given by:

$$\tau_t^w = 1 - (1 - \tau_t) \left[(1 - \alpha_t) (1 - \theta_t) (n + 1) \frac{1}{n} \right]^{-\kappa} \quad (33)$$

$$\tau_t^c = 1 - (1 - \tau_t) \{ [\theta_t + \alpha_t (1 - \theta_t)] (n + 1) \}^{-\kappa}. \quad (34)$$

All else equal, higher values of θ_t or α_t will serve to increase the capital owner's tax rate, but decrease the worker's tax rate.

4.5 Decision Rules

Given that labor supply is inelastic, workers simply consume their after-tax income plus transfers each period according to their budget constraint (20). In equilibrium, the individual worker's ratio of consumption to total output is given by

$$\frac{c_t^w}{y_t} = \frac{1}{n} \left[(1 - \tau_t^w) (1 - \alpha_t) (1 - \theta_t) + \frac{T_t}{y_t} \right], \quad (35)$$

where τ_t^w is given by equation (33) and we have substituted in the worker's equilibrium real wage (8).

For capital owners, we first use the capital law of motion (23) to eliminate i_t from the budget constraint (22). The capital owner's first-order condition with respect to k_{t+1} is given by

$$\frac{(1 - \tau_t \phi_t) i_t}{\lambda} = E_t M_{t+1}^c [(1 - \kappa) (1 - \tau_{t+1}^c) r_{t+1} k_{t+1} - (1 - \tau_{t+1} \phi_{t+1}) i_{t+1} + \frac{(1 - \tau_{t+1} \phi_{t+1}) i_{t+1}}{\lambda}], \quad (36)$$

where $M_{t+1}^c \equiv \beta (c_{t+1}^c / c_t^c)^{-1}$ is the capital owner's stochastic discount factor and τ_{t+1}^c is given by equation (34) evaluated at time $t + 1$.¹⁰ In deciding how much to invest, the capital owner takes into account the slope of the personal tax schedule, as reflected by the term $(1 - \kappa)$.

Capital owners must only decide the fraction of their after-tax income to be devoted to investment, with the remaining fraction devoted to consumption. Given the decision rule $x_t = x(s_t)$, the equilibrium version of the capital owner's budget constraint (22) can be used to derive the following expressions for the capital owner's allocations:

$$\frac{c_t^c}{y_t} = \frac{1}{1 + x(s_t)} (1 - \tau_t^c) [\theta_t + \alpha_t (1 - \theta_t)], \quad (37)$$

$$\frac{i_t}{y_t} = \frac{x(s_t)}{1 + x(s_t)} \left(\frac{1 - \tau_t^c}{1 - \tau_t \phi_t} \right) [\theta_t + \alpha_t (1 - \theta_t)]. \quad (38)$$

A convenient property of our setup is that we do not need to specify the laws of motion for the tax wedges in order to solve for the capital owner's allocations. This is because the income and substitution effects of changes in either τ_t or ϕ_t are offsetting. In equilibrium, τ_t^c depends only on τ_t and the income share variables θ_t and α_t .

¹⁰After taking the derivative of the capital owner's Lagrangian with respect to k_{t+1} , we have multiplied both sides of the resulting expression by the ratio k_{t+1}/c_t^c which is known at time t .

4.6 Asset Pricing Variables

The first order condition of capital owners (36) takes the form of a standard asset pricing equation where $p_t = (1 - \tau_t \phi_t) i_t / \lambda$ is the market value of the capital owner's equity shares in the firm:

$$\underbrace{\frac{(1 - \tau_t \phi_t) i_t}{\lambda}}_{p_t} = E_t M_{t+1}^c \left[\underbrace{(1 - \kappa) (1 - \tau_{t+1}^c) r_{t+1} k_{t+1} - (1 - \tau_{t+1} \phi_{t+1}) i_{t+1}}_{d_{t+1}} + \underbrace{\frac{(1 - \tau_{t+1} \phi_{t+1}) i_{t+1}}{\lambda}}_{p_{t+1}} \right], \quad (39)$$

In deciding how much to invest, the capital owner takes into account the slope of the personal tax schedule, as reflected by the term $(1 - \kappa)$. The equity shares entitle the capital owner to a perpetual stream of dividends d_{t+1} starting in period $t + 1$. The model's adjustment cost specification (23) implies a direct link between equity values and investment. This feature is consistent with the observed low-frequency comovement between the real S&P 500 stock market index and real business investment in recent decades, as documented by Lansing (2012).

The excess return is defined by equation (12). Using (39) and (38) we can write price dividend ratio in terms of stationary variables:

$$\frac{p_t}{d_t} = \frac{1}{\lambda} x_t [\theta_t + \alpha_t (1 - \theta_t)]. \quad (40)$$

4.7 Computation

We employ methodology similar to Chari, McGrattan, and Kehoe (2007) who develop a quantitative model with four "wedges" that relate to labor, investment, productivity, and government consumption. First, given the observed paths for the income shares in the data, we solve for the time series of τ_t and ϕ_t that allow the model to exactly replicate the observed time paths of the four U.S. macroeconomic ratios plotted in Figure 6 included in the appendix.

Second, we compute the paths of productivity shock z_t and capital accumulation shock v_t . We solve for time series of productivity shocks z_t to exactly replicate the path of U.S. real per capita output over the period 1970 to 2014, where the level of real output in 1970 is normalized to 1.0. The time series of capital accumulation shock v_t is computed so that the model matches the empirical S&P 500 index path between 1970 and 2014. The time series for the state variable s_t is taken directly from U.S. data.

Next, we use the laws of motion for the shocks(25), (27), and (29) to recover the time series of innovations ε_t , u_t and ϵ_t . For periods beyond 2014, we assume that all shock innovations are zero, while income shares, tax wedges, and the various macroeconomic ratios remain constant at year 2014 values. Details regarding the simulation procedure are contained in Appendix B.

5 Quantitative Performance of the Model

We first consider a baseline simulation that exactly replicates the observed trajectories of the following U.S. macroeconomic variables over the period 1970 to 2014: (1) real per capita output, (2) real per capita aggregate consumption, (3) real per capita private nonresidential investment, (4) real per capita government consumption and investment, and (5) real per capita government transfer payments to individuals. On the asset pricing side, the model exactly replicates real, per capita equity value of S&P 500 between 1970 and 2014. In this section, we compare the model’s predictions for additional macroeconomic variables and a set of asset pricing empirical facts.

5.1 Macroeconomic Variables

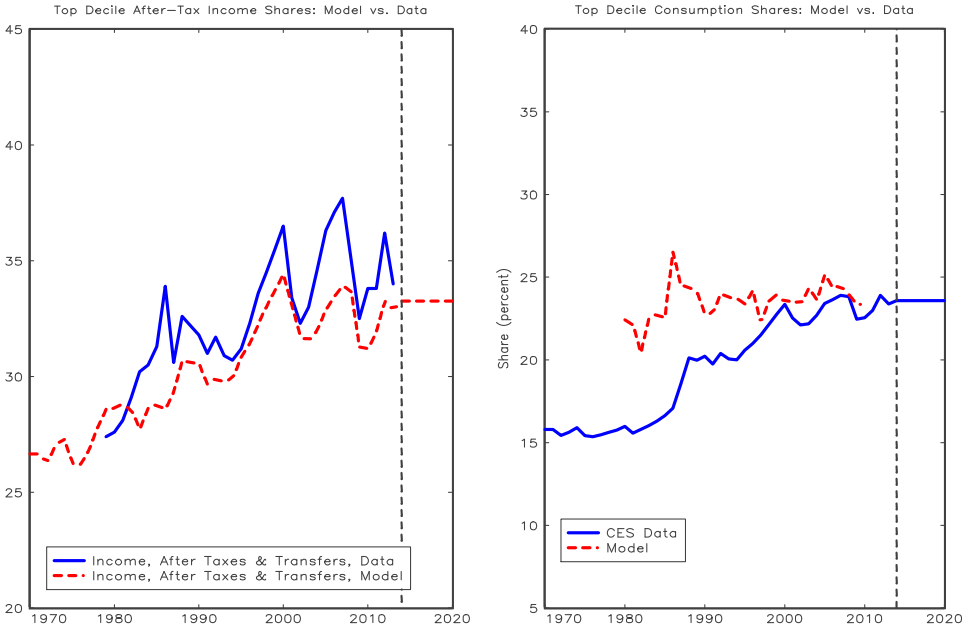
We know that after-tax and transfers income inequality and consumption inequality increased less than pre-tax income inequality reflecting U.S. progressive tax system. We check to what extent the model is able to capture this feature of the U.S. data and, in Figure 1, we compare the paths of after-tax and transfer income and consumption shares of the U. S. top decile.

The left panel of the figure shows the path of after-tax and transfers income share of the capital owners in the baseline model simulation (dotted line) and the corresponding time-series from CBO between 1979 and 2013 (solid line). The model tracks well the cyclical movements of after-tax and transfers income in the data reflecting an appropriate degree of progressivity in the modeled tax system. The fluctuations implied by the model are however much smoother than their empirical counterparts.

The right panel of Figure 1 plots consumption share of the U.S. top decile for the period 1980 and 2010 and is computed using the reported expenditure data from the Consumer Expenditure Survey (CES).¹¹ The model predicts a much larger increase

¹¹The CES data and associated Stata codes are the same as those used by Aguiar and Bils (2015) and are available from Mark Aguiar’s website. The data excludes the top and bottom five percent of households sorted by before tax income. For comparison with the model, we treat households in the

Figure 1: *Shares of Income and Consumption in the Model and the Data*



in the capital owners' consumption share than in the data for the same 1980 to 2010 time period (6% vs 1%). The initial level of consumption inequality in the model is largely underestimated and matches the one in the data only at the end of the sample. The model could potentially deliver a higher level in consumption inequality if a fraction of government transfer payments were distributed to capital owners rather than being wholly distributed to workers.

5.2 Asset Pricing Variables

In order to gauge the ability of the model to match the dynamics of financial variables, we compare model's unconditional moments of real returns and price-dividend ratio to their empirical counterparts. Table 2 reports mean, volatility, first-order and second order autocorrelations, skewness and kurtosis for the model generated series and in the data for the period between 1970 and 2014. The first column reports the variable in question, the second the moment, the third the value in the data and the last column shows the corresponding value in the model.

Table 2: **Unconditional asset pricing moments between 1970 and 2014**

Variable	Moment	Data	Model
$R_t - 1$	Mean real return: R_t	7%	3%
	Volatility of real return: σ_{R_t}	17%	16%
	1st ord. autocorr: $\rho_{R_t, R_{t-1}}$	0.03	-0.02
	2nd ord. autocorr: $\rho_{R_t, R_{t-2}}$	-0.17	-0.17
	Skew	-0.66	-0.56
	Kurt	2.79	2.73
$\frac{p_t}{d_t}$	Mean: $\frac{p_t}{d_t}$	38.47	50.52
	Volatility: $\sigma_{\frac{p_t}{d_t}}$	17.71	36.44
	1st ord. autocorr: $\rho_{\frac{p_t}{d_t}, \frac{p_{t-1}}{d_{t-1}}}$	0.93	0.95
	2nd ord. autocorr: $\rho_{\frac{p_t}{d_t}, \frac{p_{t-2}}{d_{t-2}}}$	0.83	0.88
	Skew	0.97	0.60
	Kurt	3.4	1.9

The top panel of Table 2 reports the values for the real returns and the bottom panel for the price-dividend ratio. The baseline simulation of the model matches the volatility of stock returns (σ_{R_t}), and somewhat exceeds the volatility of the price-dividend ratio ($\sigma_{\frac{p_t}{d_t}}$). The model is also able to generate close to zero first-order

90th through 95th percentiles as the top quintile and households in the 5th through 90th percentiles as the remainder.

autocorrelation of stock returns $\left(\rho_{R_t, R_{t-1}}\right)$ and high persistence of price-dividend ratio $\left(\rho_{\frac{p_t}{d_t}, \frac{p_{t-1}}{d_{t-1}}}\right)$. The model has some difficulty in replicating the mean stock return and mean price-dividend ratio.

6 Quantitative Impact of Income Inequality on Asset Prices

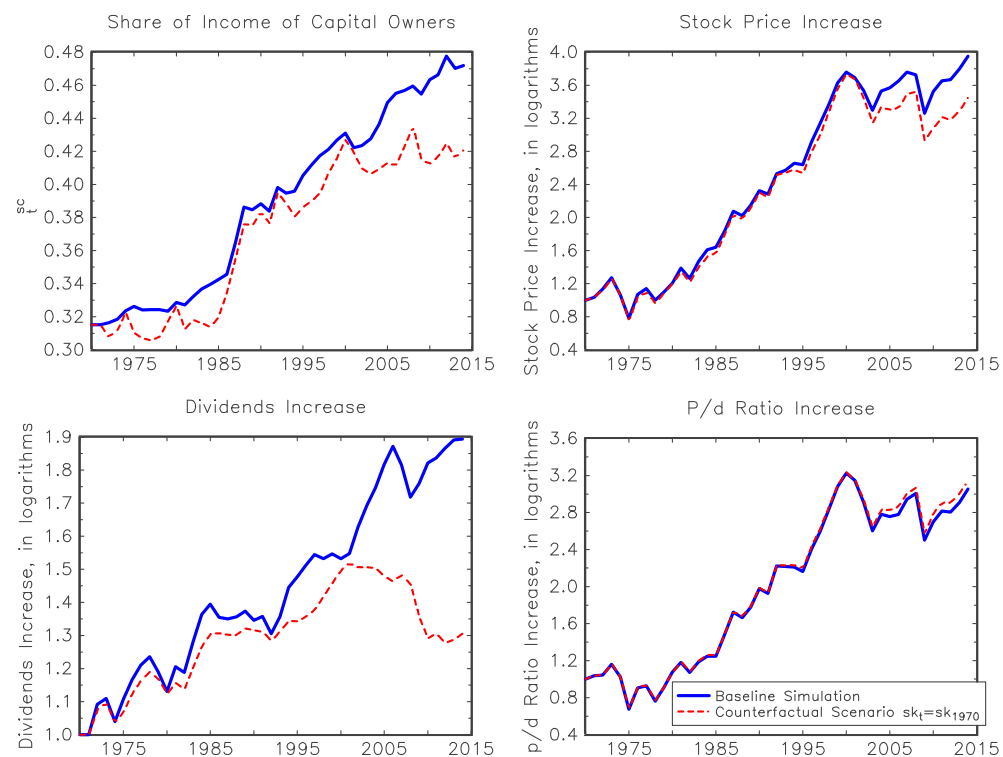
The general equilibrium framework that exactly replicates the U.S. data paths allows us also to evaluate the quantitative impact of the observed changes in income inequality in the U.S. on asset price dynamics. For this purpose, we simulate the baseline model which matches the dynamics of U.S. macroeconomic and financial quantities and compare it to three counterfactual simulations. The first counterfactual keeps capital income inequality (capital share of income) constant at the 1970 level while preserves all the remaining parametrization of the baseline simulation. The second counterfactual assumes no increase in the labour share of income and the third keeps total income inequality constant at the 1970 level.

6.1 Impact of Capital Income Inequality on Asset Pricing Variables

In the first counterfactual exercise, we keep capital share of income θ_t constant at the level of 1970, $\theta_t = \theta_0 = 0.28$. The labour share of income of capital owners is allowed to move in line with the U.S. data as defined in the baseline simulation. Figure 2 shows the outcomes of this exercise. The solid lines plot the variables under the baseline scenario while the dashed lines show the paths of variables under the counterfactual. The top left panel plots changes in the shares of income in the U.S. data (baseline) and counterfactual. While before 2000, both paths moved relatively close together, starting from 2000, the divergence between the baseline total share of income of capital owners and the counterfactual one widens indicating a major role of capital share of income in driving income inequality since this date.

The remaining panels of Figure 2 plot the main asset pricing variables generated by the model in the baseline scenario and in the counterfactual. Recall that the baseline scenario replicates yearly S&P 500 index and the counterfactual keeps the capital share of income at 1970 value: $\theta_t = \theta_0 = 0.28$. The top right panel plots the observed increase in the S&P 500 index (solid line) and the dashed line plots the increase that the index would have experienced if capital income inequality would have remained unchanged since 1970. Because increase in capital income inequality

Figure 2: *Asset pricing variables in the baseline scenario and when capital share of income is kept at 1970 value*



occurred mainly after 2000, the two stock market paths start to diverge around this date. In 2014 under the counterfactual, the nominal stock market index would have experienced a cumulative increase of 250% in contrast to 290% that occurred in the S&P 500 series. Thus, our model suggests that an increase in capital income inequality between 1970 and 2014 accounts for roughly one sixth of the cumulative increase in S&P 500 index.

The bottom left panel of Figure 2 plots the dividends' cumulative growth between 1970 and 2014. Here the divergence between the two paths is even larger. Recall the impact of an increase in capital share of income θ_t on dividends is summarized by equation (14). A lower capital share of income relative to the baseline simulation implies lower dividend payments to capital owners, given that the labour shares of income of both groups of agents are assumed to replicate the baseline simulation.

Finally, the bottom right panel of Figure 2 shows how lower θ_t affects the price dividend ratio. The price dividend ratio would have been higher under the counterfactual than under the baseline, the result due to the larger drop of dividends than prices in the counterfactual relative to the baseline scenario.

6.2 Impact of Labour Income Inequality on Asset Pricing Variables

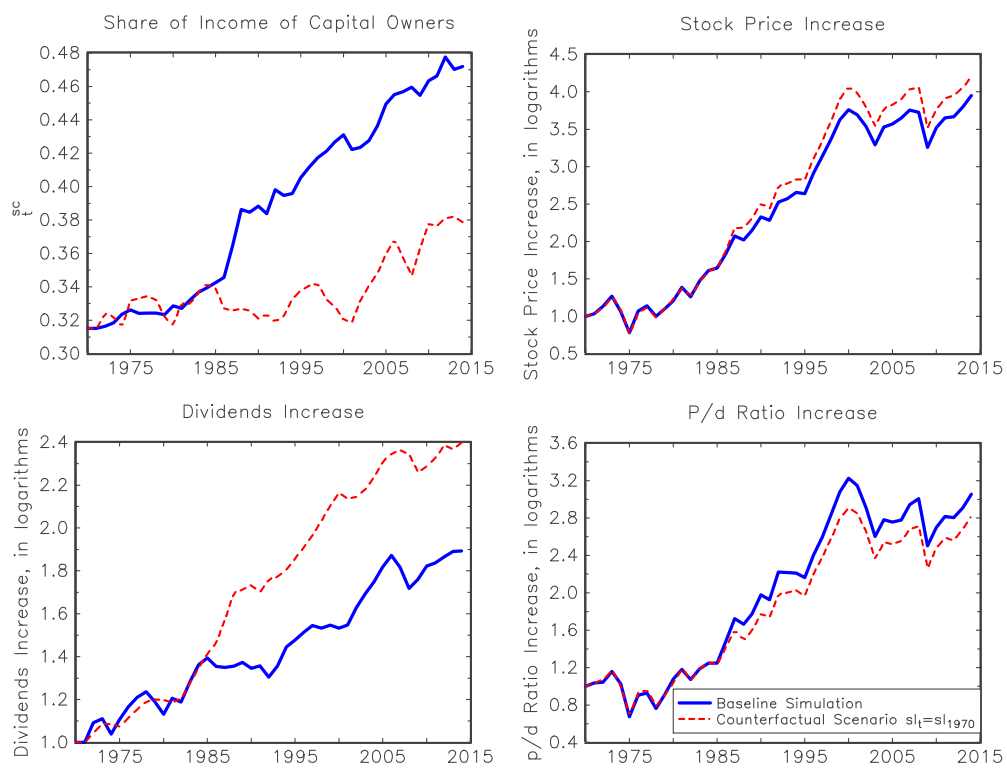
The second counterfactual exercise keeps the labour share of income of capital owners constant at the 1970 level. Figure 3 shows the results of this exercise.

The top left panel of the figure plots the path of the total share of income of capital owners in the baseline scenario (solid line) and when the labour share of income is kept unchanged at 1970 value (dotted line). The solid line starts to diverge from the dotted line around 1985 showing that the labour income inequality in the U.S. started to increase before capital income inequality. Recall from Figure 2 that the main surge in capital share of income occurred after 2000.

The remaining three plots of Figure 3 show cumulative stock index increase, dividends increase and price-dividends ratio increase. The three panels show that labour share of income has the opposite impact on asset pricing variables in comparison to the effect of the capital share of income. This finding is naturally in line with the small model intuition summarized in section 2.4 and the empirical evidence described in section 3.

The top right panel of Figure 3 shows that the stock price index would have been higher under unchanged labour income share of capital owners. Specifically, a cumulative increase in S&P 500 index would have been equal to 322% in contrast

Figure 3: Asset pricing variables in the baseline scenario and when labour share of income of capital owners is kept at 1970 value



to 290% observed in the data. Note that the negative impact of labour income inequality increase on the stock returns is quantitatively smaller than the impact of capital income inequality on the stock returns.

6.2.1 Impact of Total Income Inequality on Asset Pricing Variables

The final exercise keeps both sources of inequality unchanged at 1970 value so that the share of income of capital owners is constant during the entire sample period: $s_t^c = \bar{s}^c = 0.315$.

Figure 4: *Asset pricing variables in the baseline scenario and when total share of income of capital owners is kept at 1970 value*

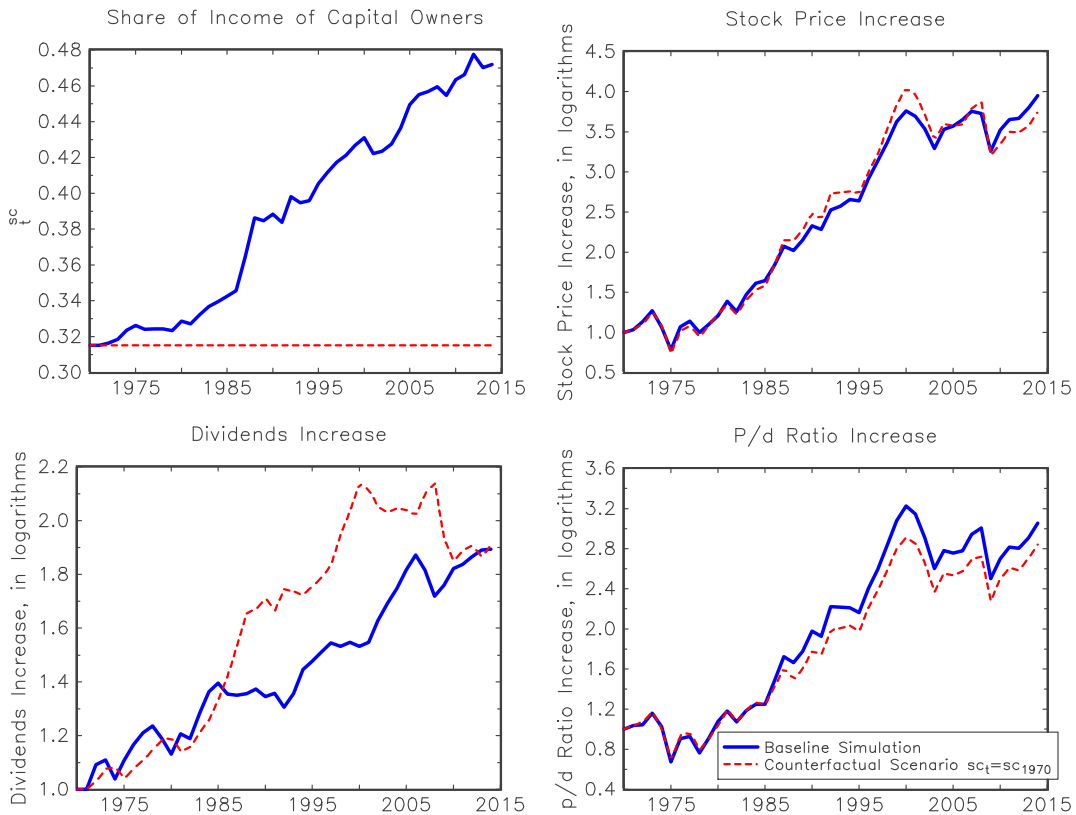


Figure 4 contains a top left panel with top decile income shares under the baseline scenario (solid line) and under the counterfactual (dotted line) and three panels

displaying asset pricing variables. The top right panel shows interesting dynamics of cumulative growth in the real equity value under counterfactual scenario. Specifically, between 1985 and 2000, the counterfactual stock market index (dotted line) grows at the faster rate than S&P 500 (solid line). This result is due to the fact that during this period income inequality was mainly driven by labour income inequality which has negative impact on real equity value. Since 2005, however, the counterfactual path of the stock market cumulative growth is below the one observed in the U.S. data. In this period, top decile income share increase is mainly driven by the rise in capital share of income.

The total impact of the increase in income inequality on real equity value is relatively small. S&P 500 displays a cumulative increase of 290% during the sample period and 275% under counterfactual. This result is due to the fact that, between 1970 and 2014, both capital and labour incomes contributed to the increase in the share of income of top decile and their impact on the real equity values partially offsets each other out. The baseline cumulative stock market value is higher at the end of the sample than that under the counterfactual because capital income inequality had a larger quantitative impact on the stock market value than labour income inequality.¹²

7 Conclusion

The increase in U.S. income inequality over the past 45 years has been driven by increase in capital and labor shares of income of those at the top of income distribution. In this paper, we study how the observed increase in inequality is related to stock market returns.

We design the model with two types of agents and concentrated-ownership of physical capital which exactly replicate the observed time paths of numerous U.S. macroeconomic variables from 1970 to 2014. In addition, the model delivers satisfying quantitative performance in both macroeconomic and financial dimensions tracking well the cyclical movements of after-tax and transfers income and replicating the main unconditional asset pricing moments.

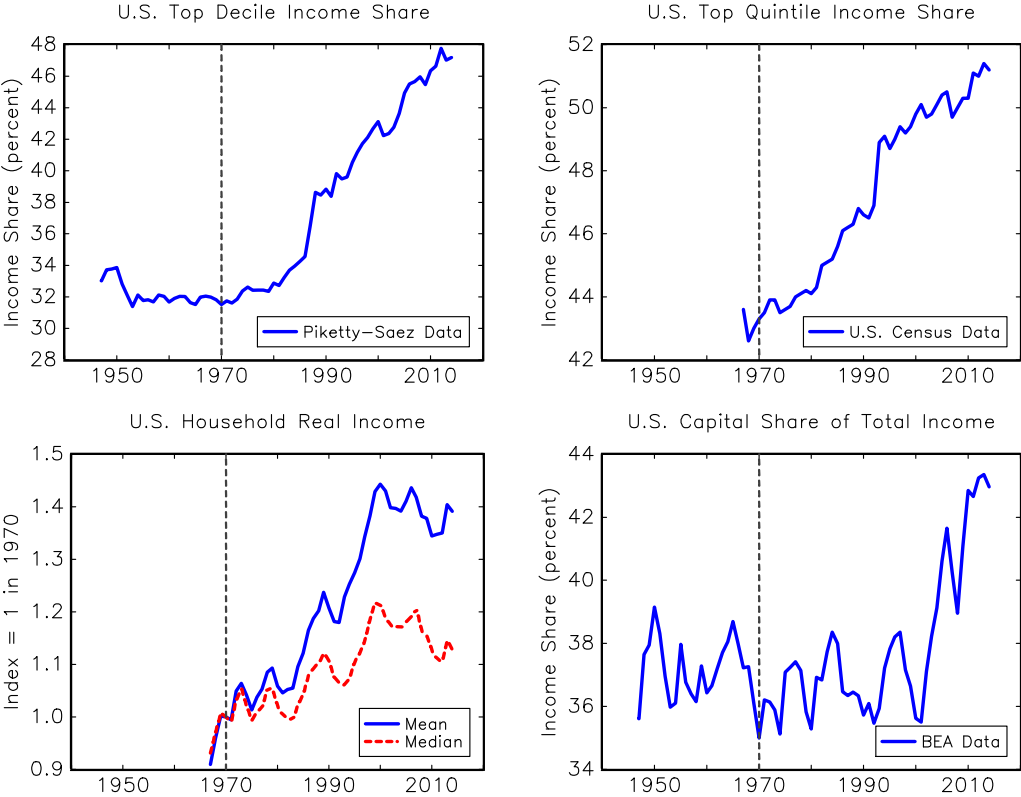
Both in the model and the data, we find that real equity returns are positively associated with capital share of income and negatively with labor share of income of capital owners.

According to our analysis, labor income inequality is the key driver of the increase

¹²This result is in line with micro-data based findings of Kacperczyk et al (2016).

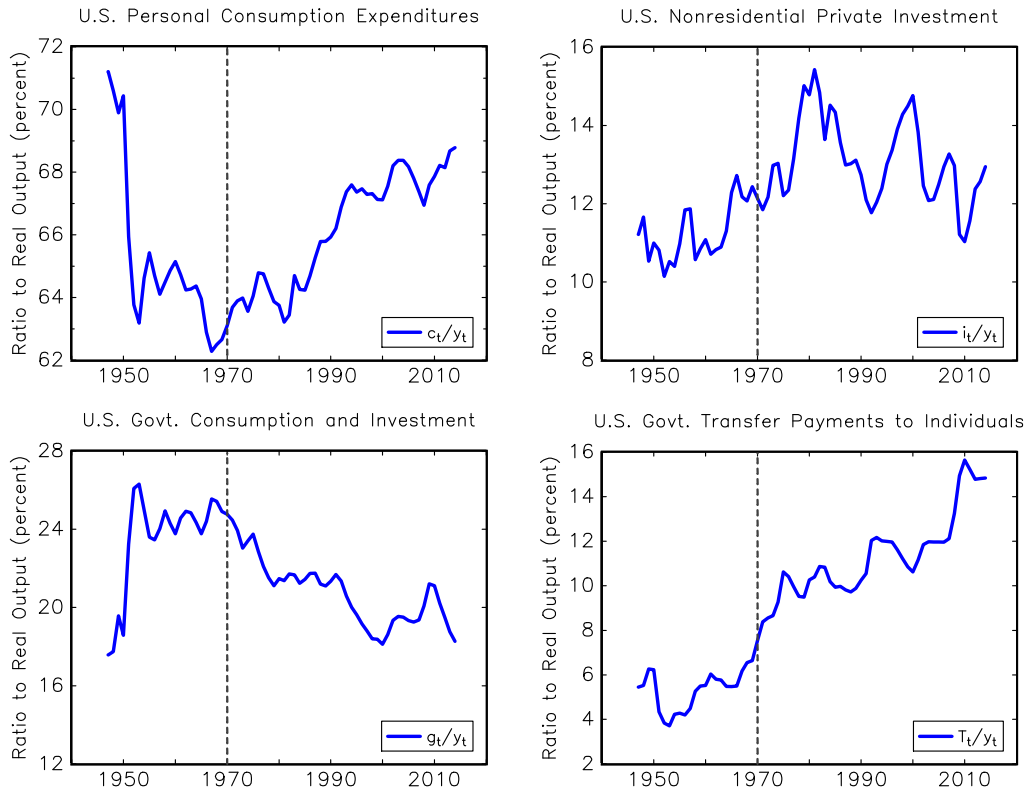
in the U.S. income inequality between 1985 and 2000 and capital income inequality mainly accounts for inequality growth after 2000. In line with theoretical and empirical predictions, capital share of income is associated with higher stock market index. We find that real cumulative return on ex-dividends S&P 500 index would have been lower by 40% if there was no rise in capital income inequality. If the labor share of top decile would not grow, the same S&P 500 index would have experienced cumulative growth higher by 32%. Since the effects of capital and labor incomes' changes partially offset each other, the impact of total increase in income inequality on stock market returns is quantitatively small. S&P 500 displays a cumulative increase of 290% during the sample period and 275% under the counterfactual scenario.

Figure 5: *Tracking the Rise in U.S. Income Inequality.*



The increase in U.S. income inequality over the past 45 years can be traced to gains made by those near the top of the income distribution where financial wealth and corporate stock ownership is highly concentrated.

Figure 6: *U.S. Macroeconomic Ratios.*



The baseline simulation exactly replicates the observed U.S. time paths for the ratios c_t/y_t , i_t/y_t , g_t/y_t , and T_t/y_t from 1970 to 2014. The vertical dashed line marks $t_0 = 1970$. Data series are constructed as described in footnote 9.

A Appendix: Additionnal Regressions

Table A.1. Additionnal Regressions of Returns on Inequality Measures						
Slope Coefficients in Equity Premium Regressions with Capital Income Inequality Proxy						
Regressors	(1)	(2)	(3)	(4)	(5)	(6)
Capital Share	7.10***	10.66***	7.08***	7.14***	7.17***	10.10***
of Income	(1.76)	(1.89)	(1.91)	(1.93)	(1.81)	(2.43)
Real GDP		-1.85**				-1.80**
growth		(0.64)				(0.67)
p/d			0.15			0.26
			(0.11)			(0.17)
p/e				-2.02		-0.36
				(2.71)		(0.41)
CAY					0.31	0.41
					(0.72)	(0.73)
R^2	0.14	0.19	0.16	0.15	0.14	0.23
Sample	1945-2014	1945-2014	1945-2014	1945-2014	1945-2014	1945-2014
Slope Coefficients in Equity Premium Regressions with Labor Income Inequality Proxy						
Top 10 Labor Share	-6.59**	-7.21***	-6.97**	-7.55**	-6.62**	-8.18***
of Income	(1.94)	(1.67)	(2.21)	(2.14)	(1.97)	(2.03)
Real GDP		-1.17				-1.70*
growth		(0.91)				(0.96)
p/d			0.32**			0.36**
			(0.11)			(0.13)
p/e				0.31		-0.14
				(0.22)		(0.38)
CAY					0.72	1.50**
					(0.89)	(0.62)
R^2	0.19	0.21	0.29	0.25	0.20	0.38
Sample	1967-2014	1967-2014	1967-2014	1967-2014	1967-2014	1967-2014
Slope Coefficients in Equity Premium Regressions with Total Income Inequality Proxy						
Top 10 Share	4.45	4.59	2.60	3.03	4.28	0.40
of Income	(6.05)	(6.17)	(5.94)	(5.91)	(6.15)	(5.75)
Real GDP		-0.11				-0.39**
growth		(0.98)				(0.84)
p/d			0.28*			0.39**
			(0.17)			(0.20)
p/e				-2.02		-0.37
				(2.71)		(0.48)
CAY					0.68	0.97
					(1.02)	(0.77)
R^2	0.007	0.007	0.008	0.15	0.10	0.11
Sample	1967-2014	1967-2014	1967-2014	1967-2014	1967-2014	1967-2014

B Appendix: Capital Owner Decision Rule and λ Calibration

By combining equations (22), (6), and (7), and then dividing both sides of the expression by c_t^c , we obtain the following transformed version of the capital owner's budget constraint:

$$1 + x_t = (1 - \tau_t^c) [\theta_t + \alpha_t (1 - \theta_t)] y_t / c_t^c, \quad (\text{A.1})$$

where $x_t \equiv (1 - \tau_t \phi_t) i_t / c_t^c$. Solving the above equation for c_t^c / y_t yields equation (37) in the text. Equation (38) in the text follows directly from the definition of x_t .

The capital owner's first-order condition (36) can be re-written as follows

$$\begin{aligned} x_t &= E_t \beta \left[\frac{\lambda (1 - \kappa) (1 - \tau_{t+1}^c) \theta_{t+1} y_{t+1}}{c_{t+1}^c} + (1 - \lambda) x_{t+1} \right], \\ &= E_t \beta [\lambda (1 - \kappa) s_{t+1} (1 + x_{t+1}) + (1 - \lambda) x_{t+1}] \end{aligned} \quad (\text{A.2})$$

where $s_{t+1} \equiv \theta_{t+1} / [\theta_{t+1} + \alpha_{t+1} (1 - \theta_{t+1})]$ and we have eliminated $(1 - \tau_{t+1}^c) y_{t+1} / c_{t+1}^c$ using equation (A.1). Notice that the rational expectation solution for x_t will depend on the state variable s_t but not on the tax wedges. The tax wedges are subsumed within the definition of x_t .

To solve for the approximate decision rule $x_t = x(s_t)$, we first log linearize the right-side of equation (A.2) to obtain

$$x_t = E_t a_0 \left[\frac{x_{t+1}}{\tilde{x}} \right]^{a_1} \left[\frac{s_{t+1}}{\tilde{s}} \right]^{a_2}, \quad (\text{A.3})$$

where a_0 , a_1 , and a_2 are Taylor-series coefficients. The expressions for the Taylor-series coefficients are

$$a_0 = \beta [\lambda (1 - \kappa) \tilde{s} (1 + \tilde{x}) + (1 - \lambda) \tilde{x}], \quad (\text{A.4})$$

$$a_1 = \frac{[\lambda (1 - \kappa) \tilde{s} + (1 - \lambda)] \tilde{x}}{\lambda (1 - \kappa) \tilde{s} (1 + \tilde{x}) + (1 - \lambda) \tilde{x}}, \quad (\text{A.5})$$

$$a_2 = \frac{\lambda (1 - \kappa) \tilde{s} (1 + \tilde{x})}{\lambda (1 - \kappa) \tilde{s} (1 + \tilde{x}) + (1 - \lambda) \tilde{x}}, \quad (\text{A.6})$$

where the approximation is taken around the ergodic mean such that $\tilde{x} \equiv \exp \{E [\log (x_t)]\}$ and $\tilde{s} \equiv \exp \{E [\log (s_t)]\}$.

We conjecture that the decision rule for x_t takes the form $x_t = \tilde{x} [s_t / \tilde{s}]^\gamma$. The conjectured solution is iterated ahead one period and then substituted into the right-side of equation (A.3) together with the law of motion for s_{t+1} from equation (29).

After evaluating the conditional expectation and then collecting terms, we have

$$x_t = \underbrace{a_0 \exp \left[\frac{1}{2} (a_2 + \gamma a_1)^2 \sigma_u^2 \right]}_{=\tilde{x}} \times \left[\frac{s_t}{\bar{s}} \right]^{\underbrace{\rho (a_2 + \gamma a_1)}_{=\gamma}} \quad (\text{A.7})$$

which yields two equations in the two unknown solution coefficients \tilde{x} and γ .

Using the decision rule for x_t and the definition of s_t from equation (28), we have

$$\frac{\partial x_t}{\partial \theta_t} = \frac{\partial x_t}{\partial s_t} \frac{\partial s_t}{\partial \theta_t} = \frac{\gamma x_t}{\theta_t + \alpha_t (1 - \theta_t)} > 0, \quad (\text{A.8})$$

which shows that an increase in θ_t causes the capital owner to devote more resources to investment instead of consumption.

From equations (A.4) through (A.7), we see that the value of λ will influence the value of the Taylor-series coefficients and hence the value of \tilde{x} . To calibrate the value of λ , we first use the investment decision rule (38) to eliminate the term $\tau_t \phi_t i_t$ from the government budget constraint (30). Next, we substitute in the equilibrium expressions for τ_t^w and τ_t^c from equations (33) and (34) and then solve the resulting expression for $1 - \tau_t$ to obtain

$$1 - \tau_t = \frac{1 - (g_t/y_t + T_t/y_t + i_t/y_t)}{q_t^{1-\kappa} (n+1)^{-\kappa} (1+x_t)^{-1} + (1-q_t)^{1-\kappa} [(n+1)/n]^{-\kappa}}, \quad (\text{A.9})$$

where $q_t \equiv \theta_t + \alpha_t (1 - \theta_t)$ is the top decile income share. Using equation (A.9), we construct an expression for the term $1 - \tau_t \phi_t$ which appears in the investment decision rule (38). Substituting for $1 - \tau_t^c$ and $1 - \tau_t \phi_t$ in the investment decision rule and then solving for x_t yields a complicated expression for the target value of x_t in terms of the top decile income share q_t , the macroeconomic ratios g_t/y_t , T_t/y_t , and i_t/y_t , and the investment tax wedge ϕ_t .

We choose a target value for ϕ_t that is based on a standard depreciation allowance with a depreciation rate of $\delta = 0.06$ and an investment-capital ratio of $i_t/k_t = 0.0803$. Specifically, we choose $\phi_t i_t = \delta k_t$ such that $\phi_t = \delta (k_t/i_t) = 0.06/(0.0803) = 0.747$. Given this target value for ϕ_t together with the 1970 to 2014 average top decile income share of $q_t = 0.4737$ and the 1970 to 2014 average values for the U.S. macroeconomic ratios g_t/y_t , T_t/y_t , and i_t/y_t , we solve for the corresponding target value $x_t = 0.5326$. Using equation (A.7), we then solve for the value of $\lambda = 0.0394$ to achieve the ergodic mean target value $\tilde{x} = 0.5326$. Also using equation (A.7), we obtain the decision rule coefficient $\gamma = 0.3602$ for the baseline calibration.

C Appendix: Numerical Simulation Procedure and Calibration

C.1 Baseline Simulation

Given the agents' decision rules (35), (37), and (38), together with the government budget constraint (30) and the equilibrium personal income tax rates (33) and (34), we solve for the time series of tax wedges τ_t and ϕ_t so that the model exactly replicates the observed time paths of the four U.S. macroeconomic ratios plotted in Figure 2. The resulting changes in τ_t and ϕ_t feed through to bring about changes in the personal income tax rates τ_t^c and τ_t^w levied on capital owners and workers.

Given the observed U.S. time series for s_t from Figure 3, we use the decision rule (A.8) to compute $x_t = x(s_t)$ for each period from 1970 to 2014. The time series for τ_t is computed using equation (A.10), where q_t , g_t/y_t , T_t/y_t , and i_t/y_t are the observed U.S. values. Given x_t , q_t , i_t/y_t , and τ_t , we use the investment decision rule (38) to compute the time series for the investment tax wedge ϕ_t .

The aggregate resource constraint for the model economy implies $c_t/y_t = 1 - g_t/y_t - i_t/y_t$. The computed time series for τ_t and ϕ_t ensure that we exactly replicate the observed U.S. time paths for g_t/y_t and i_t/y_t . Since we define y_t in the data as $c_t + i_t + g_t$ (footnote 9), our procedure ensures that we also replicate the observed U.S. time path for c_t/y_t , as plotted in Figure 2.

The final step is to compute a time series of productivity shocks z_t that cause the model to exactly replicate the path of U.S. real per capita output from 1970 to 2014. The level of real output in the data is normalized to 1.0 in the year 1970. We calibrate the value of A in the production function (26) to yield $y_t = 1$ at $t_0 = 1970$. The calibration assumes $k_t/y_t = 1.51$ in 1970 which is obtained by combining the observed U.S. value of $i_t/y_t = 0.1214$ in 1970 with the calibration target of $i_t/k_t = 0.0803$. Given the computed time series for τ_t and ϕ_t described above, we conjecture a time series for z_t from 1970 to 2014 with $z_0 = 0$. Using the agents' decision rules, we then simulate the model. After each simulation, we compute a new time series for z_t as follows

$$z_t = \frac{\log(y_t) - \log \left\{ A k_t^{\theta_t} \left[(\ell^c)^{\alpha_t} (n \ell^w)^{1-\alpha_t} \right]^{1-\theta_t} \right\}}{1 - \theta_t}, \quad (\text{B.1})$$

where y_t is given by the normalized real output series from the U.S. data, θ_t and α_t are pinned down by the income share data, and k_t is the model capital stock series implied by the law of motion (23) with i_t determined by the capital owner's decision rule (38). We repeat this procedure until the computed time series for z_t does not change from one simulation to the next. In practice, convergence is achieved after about 12 simulations. For $t > 2014$, we assume that the shock innovations ε_t and u_t are zero each period while θ_t , α_t , τ_t , and ϕ_t are held constant at year 2014 values.

As a result, the macroeconomic ratios c_t^c/y_t , c_t^w/y_t , i_t/y_t , g_t/y_t , and T_t/y_t all remain constant at year 2014 values.

C.2 Model Calibration

Table 2 summarizes the parameter values for the baseline simulation. Values are set to achieve targets based on observed U.S. variables within the sample period 1970 to 2014.

Table 3: Baseline Parameter Values

Parameter	Value	Description/Target
n	9	Capital owners = top income decile.
θ_0	0.2800	Capital's share of income = $0.350 \times 0.8 = 0.28$ in 1970.
α_0	0.1277	Top decile income share = 0.433 in 1970.
ℓ^c/ℓ^w	0.2928	Mean relative wage $w^c/w^w = 2$ in 1970.
μ	0.0201	Mean per capita consumption growth = 2.01%, 1970 to 2014.
β	0.9634	Mean log equity return = 5.74%, 1970 to 2014.
A	0.4274	$y_t = 1$ with $k_t/y_t = 1.51$ in 1970.
B	1.1375	Estimated from U.S. data on k_t and i_t , 1970 to 2014.
λ	0.0441	Estimated from U.S. data on k_t and i_t , 1970 to 2014.
\tilde{s}	0.7991	$\tilde{s} \equiv \exp\{E[\log(s_t)]\} = 0.7991$, 1970 to 2014.
κ	0.1204	Estimated tax schedule slope = 1.214, Cassou and Lansing (2004).
τ_0	0.3354	$g_t/y_t + T_t/y_t = 0.323$ in 1970.
ϕ_0	0.5421	$i_t/y_t = 0.121$ in 1970.

The time period in the model is one year. The number of workers per capital owner is $n = 9$ so that capital owners represent the top decile of earners. In the model, capital owners possess 100% of the physical capital wealth.¹³

The initial capital income share θ_0 is set to match the 1970 observed value of 0.35, as shown in Figure 1. We apply a scale factor of 0.8 to the 1980 capital income share of 0.35, resulting in an initial steady-state capital share in the model of 0.28. The initial production elasticity of the capital owner's labor supply α_0 is set to achieve an initial top decile income share of $\theta_0 + (1 - \theta_0)\alpha_0 = 0.433$, corresponding to 1970 observed value as shown in Figure 1. Given these values, the labor supply ratio ℓ^c/ℓ^w is set so that the initial wage ratio in 1970 is $w^c/w^w = 2$ with ℓ^w normalized to 1. For comparison, Heathcote, Storesletten, and Violante (2010), p. 686 report a male college wage premium of about 1.5 in 1970, whereas Gottschalk and Danziger (2005), p. 238 report a male wage ratio of 4.1 in 1979 when comparing the top decile to the bottom decile. The wage ratio w^c/w^w in our model compares the top decile to the remainder of households, so one would expect it to fall somewhere in between the values reported by the two studies, but likely closer to the value reported by

¹³See Wolff (2010, Table 2, p. 44).

Heathcote, Storesletten, and Violante (2010). The quantitative results exhibit little sensitivity to the value of the initial wage ratio.

The value $\mu = 0.0201$ matches the average growth rate of real per capita aggregate consumption over the period 1970 to 2014, where the consumption series is constructed as described in footnote 9. Given μ , we choose β to achieve a mean log equity return of 5.74%, coinciding with the corresponding real return delivered by the S&P 500 stock index over the period 1970 to 2014.¹⁴

The level of real per capita output in the U.S. data is normalized to 1.0 in the year 1970. We calibrate the value of A in the production function (26) to yield $y_t = 1$ at $t_0 = 1970$. The calibration assumes $k_t/y_t = 1.51$ in 1970 which is obtained by combining the observed U.S. value of $i_t/y_t = 0.121$ in 1970 with a reasonable value for the investment-capital ratio i_t/k_t . For example, in a model with no capital adjustment costs, we have $i_t/k_t = k_{t+1}/k_t - 1 + \delta$, where δ is the annual depreciation rate of physical capital. For the calibration, we employ the value $\delta = 0.06$ which in turn yields the target value $i_t/k_t = \exp(0.0201) - 1 + 0.06 = 0.0803$. Given the calibrated value for λ (described below), we set the parameter B in the capital law of motion (23) such that $B = (k_{t+1}/k_t) (i_t/k_t)^{-\lambda}$, where $k_{t+1}/k_t = \exp(0.0201)$ and $i_t/k_t = 0.0803$.

The parameter λ governs the strength of capital adjustment costs and depreciation. The value of λ influences the coefficients in the capital owner's decision rule $x_t = x(s_t)$, where $x_t \equiv (1 - \phi_t \tau_t) i_t / c_t^c$. We choose the value of λ to achieve the target value $\tilde{x} \equiv \exp\{E[\log(x_t)]\} = 0.5326$. This target value is computed using the 1970 to 2014 average values for the U.S. income shares and the U.S. macroeconomic ratios plotted in Figure 2. Details of the calibration procedure for λ are contained in the appendix.

Recall that s_t represents the ratio of capital's share of income to the top decile income share (Figure 3). We choose the parameters \tilde{s} , ρ , and σ_u in the law of motion (29) to match the mean, persistence, and volatility of $\log(s_t)$ in U.S. data from 1970 to 2014.

After computing the time series of productivity shocks z_t that cause the model to exactly replicate the path of U.S. real per capita output, we use the law of motion (27) to recover the implied sequence of innovations ε_t , with $z_t = 0$ at $t_0 = 1970$. The standard deviation of the implied shock innovations turns out to be $\sigma_\varepsilon = 0.0440$. The corresponding standard deviation of output growth in both the model and the data is 1.73% from 1970 to 2014.

The slope parameter for the progressive tax schedule is set to $\kappa = 0.125$ so that the hypothetical average-income agent in the model with $y_t^i/\bar{y}_t = 1$ faces a tax schedule slope of $MTR_t^i/\tau_t^i = 1.124$ when the top decile income share and the macroeconomic

¹⁴The mean log equity return in the model is given by $E[\log(R_{t+1}^s)] = -\log(\beta) + \mu$. Data on real log equity returns for the U.S. are from Welch and Goyal (2008), updated through 2014 using data available from www.hec.unil.ch/agoyal/.

ratios i_t/y_t , g_t/y_t , and T_t/y_t take on their average values from 1970 to 2014. The target slope corresponds to the value estimated by Cassou and Lansing (2004) using the 1994 U.S. tax schedule for married taxpayers with no children, filing IRS form 1040 jointly.¹⁵ Given the many significant changes to the U.S. tax code that have taken place since 1970, we examine the sensitivity of our results to different values for κ .¹⁶ Table 2 shows the personal income tax rates faced by each type of agent for the baseline calibration with $\kappa = 0.125$ and an alternative calibration with $\kappa = 0.224$. The alternative calibration implies a more progressive tax schedule such that the average-income agent now faces a steeper slope of $MTR_t^i/\tau_t^i = 1.4$. The income ratios y_t^i/\bar{y}_t that determine the personal income tax rates from equations (31) and (32) are based on the average top decile income share from 1970 to 2014. For both calibrations, the tax rates faced by capital owners are higher than those for workers.

Given values for κ , λ , \tilde{x} , and \tilde{s} , and the capital owner's decision rules for x_t and i_t , we solve for ϕ_0 and τ_0 such that the model delivers the observed U.S. values $i_t/y_t = 0.121$ and $g_t/y_t + T_t/y_t = 0.323$ at $t_0 = 1970$. A similar procedure is used to solve for ϕ_t and τ_t for each $t > t_0$, as described in Appendix B.

¹⁵The tax schedule, taken from Mulligan (1997, Table 5-2), displays twelve different tax brackets that derive from the combined effects of the federal individual income tax, the earned income tax credit, and employee and employer contributions to Social Security and Medicare.

¹⁶Significant tax code changes were enacted by the Economic Recovery Tax Act of 1981 (ERTA81) and the Tax Reform Act of 1986 (TRA86). ERTA81 imposed a 23 percent across-the-board cut in all marginal tax rates and reduced the top marginal rate for individual income from 70 to 50 percent. TRA86 further lowered marginal rates for individuals and corporations, dramatically reduced the number of tax brackets, and eliminated or reduced many tax breaks. For additional details, see Guo and Lansing (1997).

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