The optimal rate of inflation: discounting matters

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Abstract

In the workhorse models used for monetary policy analysis, the presence of price stickiness implies a long-run relationship between the average markup and inflation. Thus, when output is inefficiently low, striking a balance between the cost and benefits of inflation suggests adopting a non-zero inflation rate. However, in most models, the optimal long-run inflation rate is exactly zero. This comes from a discounting effect; inflation in a given period has consequences in other periods which are weighted differently by the monetary authority. When social and private discount rates are equal, this discounting effect exactly offsets the effect of inflation on the average markup and the optimal inflation rate is equal to zero. This is no longer true when social and private discount rates differ, as is the case in numerous macroeconomic models. Moreover, this difference in discount rates implies that the optimal inflation rate is different from zero even in economies with a vertical long-run Phillips Curve.

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1 Introduction

In standard New Keynesian models, the monetary authority does not face a trade-off between stabilizing inflation and real activity. Stabilizing inflation also implies stabilizing the welfare-relevant output-gap, a result referred to as the "divine coincidence" (Blanchard and Galí 2007) in the literature. However these models do feature a long-run relation between the levels of inflation and output (see for example Ascari and Rossi 2012). This suggests that the central bank should take advantage of this relation and adopt a non-zero inflation target when output is inefficiently low. Yet, in most studies, the optimal long run inflation rate is found to be exactly zero. This paper shows that this comes from a discounting effect. The problem of the monetary authority is intrinsically dynamic; inflation in a given period t has an effect on both output at time t and output before time t through anticipated inflation. Under commitment, the central bank internalizes the effects of its choices on the behavior of the private sector. Thus, when fixing the time t inflation rate, it takes into account that this choice will have consequences in other periods and weights these consequences according to their distance in the future. In the standard New Keynesian model, this discounting effect offsets perfectly the effect of inflation on average output. This coincidence disappears as soon as social and private discount rates diverge. In this case, one of the two effects dominates and the optimal long-run inflation rate is different from zero.

This paper considers a standard New Keynesian model along the lines of Galí (2008). In this environment, inflation has an effect on economic activity both because it influences firms' markups (or inversely their marginal costs) and because it creates costs, through price dispersion or because resources are expended adjusting prices, that drive a "price wedge" between consumption and output. Given general assumptions about price-setting, the Ramsey problem of the monetary authority can be shown to simplify to a simple formula. The optimal inflation rate is chosen so that a weighted average of the discounted effects of inflation on marginal costs and on price wedges in all periods is equalized to zero, where the weights depend on the level of steady-state distortions. The paper then considers two different price setting models widely used in the literature, one in which firms face price adjustments costs à la Rotemberg (1982), and another in which prices are set on the basis of staggered Calvo (1983) contracts. In both cases, analytical expressions for the objects of interest, the discounted effects of inflation on marginal costs and on price wedges evaluated in steady-state, can be obtained. Their behavior can be analyzed to understand the determinants of the optimal long-run inflation rate.

Results are as follows. First, the price wedge is minimized for a zero inflation rate under the two price-setting mechanisms, irrespectively of the values of the private and social discount rates. Second, higher inflation in a given period has two opposite effects on marginal costs. Because some firms cannot reset their prices, or because they have to pay price adjustment costs, it erodes current markups. Moreover, firms choosing their optimal price in previous periods anticipate this higher inflation and set a higher markup. Thus, inflation has an effect on average marginal cost, which is non-zero in both models. However, when choosing the inflation rate, the Ramsey planner gives a higher weight to events that are closer in the future. These two effects perfectly offset each other when the private discount rate, which partly determines the slope of long-run Phillips curve, and the social discount rate, which determines the strength of the weighting effect, are equal. In that case, the discounted effects of inflation on marginal costs is equal to zero for an inflation rate of zero; long-run price stability is optimal. This result is no longer valid when private and social discount rates differ. When the private discount rate is higher, the monetary authority has more leverage over markups and the optimal inflation rate is positive. When the social discount rate is higher, the central bank puts a higher relative weight on earlier periods when anticipated inflation leads to higher markups and lower output, and the optimal inflation rate is negative.

The precise value of the optimal inflation rate depends on the gap between the social and private discount rates, and on the values of certain key parameters. In a baseline calibration, the optimal inflation rate is equal to 1.1% (-1.2%) under Rotemberg pricing and 0.7% (-1%) under Calvo pricing when the private discount factor is 10% lower (higher) than the social discount factor. Moreover, the absolute value of the optimal inflation rate is an increasing function of both the degree of steady-state distortions and the Frisch elasticity of labor supply. Intuitively, the benefits of using inflation to influence output decrease as output gets closer to its first-best level. Additionally, the number of hours worked is more sensitive to marginal cost when labor supply is more elastic. The output effects of a given increase in inflation operating through marginal costs are therefore larger in that case.

This paper is related to a wide literature expertly surveyed by Schmitt-Grohé and Uribe (2010) that has sought to understand the determinants of the optimal rate of inflation. Other more recent contributions by Bilbiie et al. (2014), Coibion and Gorodnichenko (2012), and Carlsson and Westermark (2016) show respectively that endogenous producer entry and exit, the zero lower bound on nominal interest rates, and the combination of labor market frictions and nominal wage staggering may provide justifications for moderatly positive inflation targets. Several papers, among which Goodfriend and King (1997) and King and Wolman (1999), have looked specifically at the consequences of the presence of sticky prices. Goodfriend and King (1997) showed that the rate of inflation that maximizes the average level of welfare is slightly positive in a model of staggered price adjustment à la Calvo (1983). King and Wolman (1999) make the distinction between such a "golden rule" rate of inflation that maximizes average utility and a "modified golden rule" rate of inflation, equal to zero, that maximizes the expected discounted sum of utilities. They make clear that the difference between these two inflation rates comes from discounting. The contribution of this paper is to show that the optimality of zero inflation rests on the equality of social and private discount factors. In numerous macroeconomic models, which are surveyed in section 5, this property is not verified and the optimal inflation rate implied by sticky prices is non-zero.

The paper is organized as follows. Section 2 presents a baseline New-Keynesian model. Section 3 derives the Ramsey problem faced by the central bank and shows that it amounts to choosing the inflation rate so as to equalize a weighted average of the discounted effects of inflation on marginal costs and on price wedges to zero. Section 4 provides expressions for the objects in this formula in two different price-setting models. It shows both analytically and quantitatively how the optimal inflation rate depends on the gap between the social and private discount factors. Section 5 surveys the macroeconomic literature and provides examples of studies in which social and private discount factors differ. Section 6 concludes.

2 A standard New Keynesian model

Consider a standard New Keynesian model without capital, as outlined for example in Galí (2008). The dynamics of the economy can be described by a Euler equation (1), an equation equating labor supply and labor demand (2), a resource constraint (3), as well as equations describing the price setting behavior of firms, the behavior of monetary policy, and the dynamics of a technological factor Z_t.

$$\beta_h E_t \frac{1+I_t}{\Pi_{t+1}} \frac{U_c(C_{t+1})}{U_c(C_t)} = 1 \tag{1}$$

$$\frac{v_L(L_t)}{U_c(C_t)} = mc_t F_L(L_t) \tag{2}$$

$$\frac{F(L_t)}{s_t} = C_t \tag{3}$$

whereβ_h is the discount factor of households, I_t the nominal interest rate set by the central bank, Π_t gross inflation, C_t consumption, Y_t output, L_t the number of hours worked, mc_t the marginal cost of firms, and s_t a wedge between consumption and output arising from the presence of costly price adjustment. Household utility depends positively on consumption and negatively on the number of hours supplied in a separable way, $U(C_t, L_t) = U(C_t) - v(L_t)$. Firms produce with labor according to the production technology $F(L_t)$. In what follows, I assume the following functional forms for analytical convenience; $F(L_t) = Z_t L_t$, $U(C_t) = log(C_t)$, and $v(L_t) = \chi \frac{L_t^{1+\varphi}}{1+\varphi}$, where φ is the inverse of the Frisch elasticity of labor supply and χ is a scaling factor. Depending on the model of price-setting behavior under consideration, the price wedge s_t and marginal cost mc_t are potentially functions of all past, current, and future inflation rates

$$mc_t = \mathit{\Omega}(\Pi_{t-i}, ..., \Pi_t, ..., \Pi_{t+i}) \tag{4}$$

$$s_t = \Upsilon(\Pi_{t-i}, ..., \Pi_t, ..., \Pi_{t+i}) \tag{5}$$

where Ω and Υ depend on the price-setting model, and $i \in \mathbb{N}^+$.

3 A simple formula for the optimal inflation rate

This section derives a simple formula that relates the optimal inflation rate to the discounted effects of inflation on marginal costs and on price wedges. I consider the problem of a monetary authority acting under commitment and look at the steady-state to which it would like to converge in the long-run. In this scenario, the central bank chooses a path $\{I_t\}_{t=0}^{\infty}$ in order to maximize intertemporal utility $E_t \sum_{i=0}^{\infty} (\beta_s)^i U(C_{t+i}, L_{t+i})$ subject to the constraints of the competitive economy, where β_s is the social discount factor. These constraints are equations (1), (2), (3), (4), and (5). The problem can be simplified in several ways. First, note that it is possible to use equations (2) and (3) to solve for consumption C_t and hours L_t as a function of mc_t and s_t

$$L_t = \left[\frac{mc_t s_t}{\chi}\right]^{\frac{1}{1+\varphi}} \tag{6}$$

$$C_t = Z_t \left[\frac{mc_t}{\chi}\right]^{\frac{1}{1+\varphi}} s_t^{-\frac{\varphi}{1+\varphi}} \tag{7}$$

Second, as emphasized in section 2, mc_t and s_t are functions of past, present, and future inflation rates. This implies, through equation (1), that choosing a sequence $\{I_t\}_{t=0}^{\infty}$ is equivalent to choosing a sequence for inflation $\{\Pi_t\}_{t=0}^{\infty}$. Thus, the Ramsey problem amounts to choosing $\{\Pi_t\}_{t=0}^{\infty}$ to maximize the following objective function

$$U = \sum_{t=0}^{\infty} \beta_s^t \left\{ log \left[Z_t \left[\frac{mc_t}{\chi}\right]^{\frac{1}{1+\varphi}} s_t^{-\frac{\varphi}{1+\varphi}} \right] - \frac{mc_t s_t}{1+\varphi} \right\} \tag{8}$$

where the link between s_t, mc_t and inflation rates in different periods is given by equations (4) and (5). In steady-state, the first-order condition of this problem can be expressed as

$$\left[mc(\Pi)^{-1} - s(\Pi)\right] \left(\sum_{j=-\infty}^{\infty} (\beta_s)^j \frac{\partial mc_{-j}}{\partial \Pi} \right) - \left[\varphi s(\Pi)^{-1} + mc(\Pi)\right] \left(\sum_{j=-\infty}^{\infty} (\beta_s)^j \frac{\partial s_{-j}}{\partial \Pi} \right) = 0 \tag{9}$$

where the absence of time subscript denotes steady-state variables. This equation makes clear that the optimal inflation rate depends on: 1) the discounted effect of current inflation on marginal costs in all periods; 2) the discounted effect of current inflation on price wedges in all periods; 3) weights that are related to the degree of steady-state inefficiency. In particular, note that mc is equal to 1 in an efficient steady-state with zero inflation, and lower than 1 when the steady-state is inefficient. Furthermore, s is exactly equal to 1 when inflation is zero, and larger than 1 when inflation is different from zero. Thus, the first term in equation (9) is equal to zero in an efficient steady-state with zero inflation. Indeed, in such a situation, output is at its optimal level and the monetary authority has no incentives to influence it through marginal costs. In the rest of the paper, I leave aside this particular case and consider that the steady-state is inefficient.

4 Two specificic cases: Rotemberg and Calvo pricing

This section takes a closer look at the elements in equation (9) in two models of price-setting. In the first one, firms face deadweight costs of adjusting prices à la Rotemberg (1982). In the second one, based on Calvo (1983), firms have an exogenous probability of being able to reset their price.

4.1 Rotemberg pricing

The deadweight costs of adjusting prices, expressed in units of consumption, are given by $\Gamma_t = \frac{\phi^p}{2}(\Pi_t - 1)^2 C_t$. In such a setting, the optimal pricing condition of firms leads to the following relationship between marginal cost and inflation

$$1 - \theta + \theta mc_t - \phi^p \Pi_t (\Pi_t - 1) + \beta_f \phi^p E_t \Pi_{t+1} (\Pi_{t+1} - 1) = 0 \tag{10}$$

where θ is the elasticity of substitution between goods, and β_f is the discount factor used by firms when discounting future profit flows. Moreover s_t is given by

$$s_t = 1 + \frac{\phi^p}{2}(\Pi_t - 1)^2 \tag{11}$$

Thus with Rotemberg pricing, s_t is a function of Π_t and mc_t is a function of Π_t and $E_t\Pi_{t+1}$. In this setting, (9) simplifies to

$$\Lambda - \Gamma = 0$$

where $\Lambda = \left[mc(\Pi)^{-1} - s(\Pi)\right]\left(\beta_s^{-1}\frac{\partial mc_{-1}}{\partial \Pi} + \frac{\partial mc}{\partial \Pi}\right)$ is the effect of inflation on utility operating through marginal cost and $\Gamma = \left[\varphi s(\Pi)^{-1} + mc(\Pi)\right]\frac{\partial s}{\partial \Pi}$ is the effect of inflation on utility operating through the resource costs of price adjustment. They are equal to

$$\Lambda = \left[mc(\Pi)^{-1} - s(\Pi)\right]\left(\frac{\beta_f}{\beta_s} - 1\right)\frac{\phi^p}{\theta}(1 - 2\Pi)$$

$$\Gamma = \left[\varphi s(\Pi)^{-1} + mc(\Pi)\right]\phi^p(\Pi - 1)$$

Not surprisingly, since resource costs depend only on current inflation, they are minimized when inflation is equal to zero ($\Pi = 1$). More interestingly, the effect of inflation on the discounted sum of marginal costs depends on the ratio of the social and private discount factors, β_s and β_f. When the two are equal, $\Lambda = 0$ for all values of Π. This result can be understood by considering the effect of current inflation on the sum of past and present marginal costs

$$\frac{\partial mc_{-1}}{\partial \Pi} + \frac{\partial mc}{\partial \Pi} = (1 - \beta_f)\frac{\phi^p}{\theta}(2\Pi_t - 1)$$

Since adjusting prices is costly, firms do not pass on the entirety of movements in marginal costs to prices and current inflation is associated with a reduction in markups. According to the same logic, expected future inflation leads firms to set higher markups in order to minimize future price adjustments costs. However, these effects are asymmetric. Since firms discount the future, higher inflation in t has a larger positive impact on marginal cost at time t than a negative impact at time $t-1$. In other words, the long-run Phillips curve of this model, which relates inflation to the level of activity (through the average marginal cost), is positively sloped. These outcomes, which happen in successive periods, are weighted differently by the Ramsey planner. Events occuring at time $t-1$, which are closer in the future, receive a higher weight than events occuring at time t. When social and private discount rates are equal, this weighting effect fully offsets the effect of inflation on average markups and the effect of inflation on the *discounted* sum of markups is equal to zero. The inflation rate is then set equal to zero in order to minimize resource costs. However, when social and private discount rates differ, one effect dominates the other and the optimal inflation rate is different from zero. This can be seen in figure 1, which plots Λ and Γ for different values of the ratio $\frac{\beta_f}{\beta_s}$.

When $\beta_f = \beta_s$, the effect of inflation on Λ is equal to zero for all values of Π (first panel). For an unchanged β_s, a decrease in β_f leads to a flattening of the long-run Phillips curve. The effect of steady-state inflation on the average markup is now larger and the average markup effect dominates the weighting effect; the optimal inflation rate is positive (panel 2). In the third case, the long-run Phillips Curve is identical as in the first case. However, the monetary authority now puts a larger relative weight on the adverse effects of inflation happening before time t than on its beneficial effects happening at time t; the optimal inflation rate is negative (third panel).

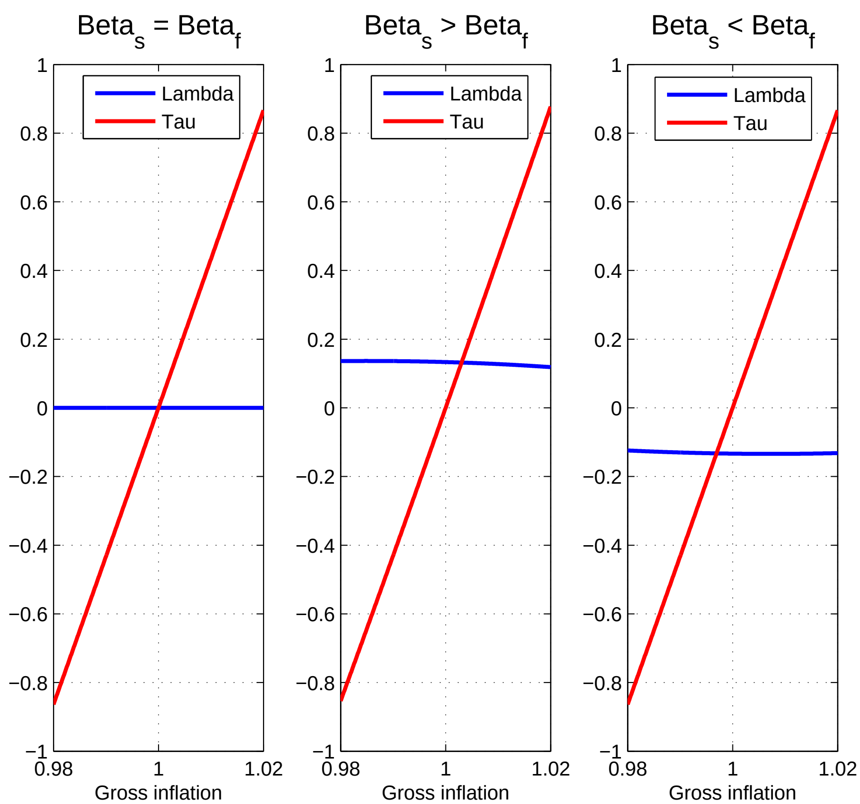

Figure 1: Λ and Γ according to the value of $\frac{\beta_f}{\beta_s}$. In the first case, $\beta_f = \beta_s = 0.99$. In the second case, $\beta_s = 0.99$ and $\beta_f = 0.9\beta_1$. In the third case, $\beta_f = 0.99$ and $\beta_s = 0.9\beta_f$. The other parameters are calibrated as follows: $\phi^p = 59$, $\theta = 6$, $\varphi = 0.25$.

4.2 Calvo pricing

In the second model of price-setting based on Calvo (1983), each firms may reset its price only with probability $1-\alpha$ in any given period, independent of the time elapsed since the last adjustment. The optimal pricing condition of firms is

$$E_t \sum_{s=t}^{\infty} (\alpha\beta_f)^{s-t} \left(\prod_{i=1}^{s} \Pi_{t+i}^{-1}\right)^{-\theta} \left\{ p_t^o \left(\prod_{i=1}^{s} \Pi_{t+i}^{-1}\right) - \frac{\theta}{\theta-1} mc_s \right\} = 0 \tag{12}$$

where $p_t^o = \frac{P_t^o}{P_t}$ is the relative price chosen by firms which can reset prices, and P_t is the aggregate price in the economy at time t. The dynamics of inflation are described by the following equation

$$1 = (1-\alpha)(p_t^o)^{1-\theta} + \alpha \Pi_t^{\theta-1} \tag{13}$$

Moreover, price dispersion leads to a dispersion in the quantity of the varieties produced by firms. Since aggregate output is a concave function of the quantity of each variety, this dispersion leads to a costly decrease in output which is captured by

$$s_t = (1-\alpha)\left(p_t^o\right)^{-\theta} + \alpha \Pi_t^{\theta} s_{t-1} \tag{14}$$

As shown in Appendix 7.1, in this environment s_t not only depends on the time t inflation rate but also on all past inflation rates Π_{t-i} $\forall i = \mathbb{N}^+$. Symmetrically, mc_t depends on the time t inflation rate and on all future inflation rates Π_{t+i} $\forall i \in \mathbb{N}^+$. As in the case of Rotemberg pricing, we can rewrite (9) as a function of the effect of inflation on utility operating through marginal costs Λ, and as the effect of inflation on utility operating through price dispersion Γ

$$\Lambda - \Gamma = 0$$

where

$$\Lambda = \left[mc(\Pi)^{-1} - s(\Pi)\right]\left(... + \beta_s^{-n}\frac{\partial mc_{-n}}{\partial \Pi} + ... + \beta_s^{-1}\frac{\partial mc_{-1}}{\partial \Pi} + \frac{\partial mc}{\partial \Pi}\right)$$

$$\Gamma = \left[\varphi s(\Pi)^{-1} + mc(\Pi)\right]\left(\frac{\partial s}{\partial \Pi} + \beta_s \frac{\partial s_{+1}}{\partial \Pi} + ... + \beta_s^n \frac{\partial s_{+n}}{\partial \Pi} + ...\right)$$

After using equations (12) to (14), and under the assumption that $\alpha\Pi^{\theta} < 1$[1], we can find an analytical solution for both expressions (see Appendix 7.1)

$$\Lambda = \left[mc(\Pi)^{-1} - s(\Pi)\right]\frac{(\theta-1)}{\theta}p^o(\Pi)\frac{\alpha\Pi^{\theta-2}}{1\text{-}\alpha\beta_f\Pi^{\theta-1}}\frac{\left[(1-\frac{\beta_f}{\beta_s}\Pi)(1-\alpha\frac{\beta_f}{\beta_s}\Pi^{\theta-1}) + (\theta-1)(1-\Pi)\frac{\beta_f}{\beta_s}(1-\alpha\Pi^{\theta-1})\right]}{(1-\alpha\Pi^{\theta-1})(1-\alpha\frac{\beta_f}{\beta_s}\Pi^{\theta-1})}$$

[1] Even for a high value of price stickiness $\alpha = 0.8$ and a high elasticity of substitution between goods $\theta = 11$, this is valid as long as quarterly steady-state inflation is lower than 2.05%.

$$\Gamma = \left[\varphi s(\Pi)^{-1} + mc(\Pi)\right] \frac{\theta(1-\alpha)(p^o(\Pi))^{-\theta}\alpha\Pi^{\theta-2}}{(1-\alpha\beta_s\Pi^\theta)(1-\alpha\Pi^\theta)(1-\alpha\Pi^{\theta-1})}(\Pi - 1)$$

While they are not as transparent as with Rotemberg pricing, several things can be learned from these expressions. First, the discounted effect of inflation on present and future price dispersion terms is equal to zero when $\Pi = 1$. When $\Pi < 1$, an increase in inflation reduces price dispersion today and in the future (Γ is negative). Symmetrically, when $\Pi > 1$, a decrease in inflation reduces price dispersion today and in the future (Γ is positive). Second, the discounted effect of current inflation on past and present marginal costs Λ is equal to zero when the social and private discount rates β_s and β_f are equal, and when steady-state inflation is equal to zero. The rationale for this result is similar to the one outlined above with Rotemberg pricing. It can be understood by writing the average markup μ_t, the inverse of marginal cost, as the product of the marginal markup μ_t^o of adjusting firms and a measure of the relative price of the firms that are not adjusting $\frac{1}{p_t^o}$

$$\mu_t = \frac{1}{mc_t} = \mu_t^o \frac{1}{p_t^o}$$

where $\mu_t^o = \frac{p_t^o}{mc_t}$. Higher inflation at time t lowers the average markup at time t both because it erodes the relative price of non-resetting firms (this is captured by the $1/p_t^o$ term) and because it leads firms that are able to reset their price to choose a lower marginal markup μ_t^o. Since any inflation rate is anticipated in steady-state, it also leads firms to set higher prices, and thus higher marginal markups, before time t. As emphasized in Goodfriend and King (1997), when unweighted, these two effects through current and anticipated inflation are equal for rates of inflation that are slightly positive [2]. However, events happening before time t loom larger for the Ramsey planner than events happening at time t. That is, the detrimental effects of current inflation on past markups is given a higher weight than its beneficial effects on current markups. When the social and private discount rates coincide, these average markup and weighting effects offset each other for an inflation rate of zero. In panel 1, the blue curve, which plots the discounted effects of inflation on marginal costs, crosses the horizontal zero line when $\Pi = 1$.

I now consider two other cases. First, for an unchanged β_s, assume a lower β_f. In that situation, firms put less weight on the future consequences of their current choices. As a result, the impact of higher anticipated inflation on markups is weaker, and the overall effect of inflation on the average markup is larger. As seen in panel 2, the blue curve no longer crosses the zero line when $\Pi = 1$, that is the weighting effect and the average markup effect no longer offset each other

[2] This can be shown by considering $\frac{\Lambda}{[mc^{-1}-s]}$ in the equation above. When $\beta_s = 1$, this expression is equal to zero for a rate Π^* that is larger than 1

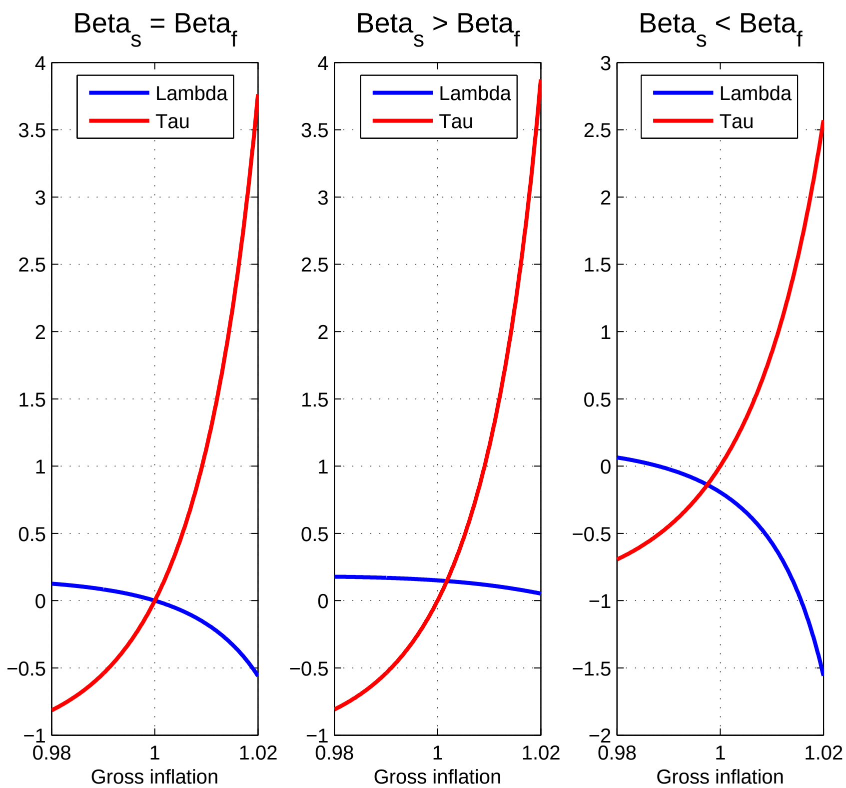

Figure 2: Λ and Γ according to the value of $\frac{\beta_f}{\beta_s}$. In the first case, $\beta_f = \beta_s = 0.99$. In the second case, $\beta_s = 0.99$ and $\beta_f = 0.9\beta_1$. In the third case, $\beta_f = 0.99$ and $\beta_s = 0.9\beta_f$. The other parameters are calibrated as follows: $\alpha = 0.75$, $\theta = 6$, $\varphi = 0.25$.

for an inflation rate of zero. The optimal inflation rate is positive. Second, the third panel presents a situation identical to that presented in the first panel except the planner now discounts the future to a greater extent. The weighting effect dominates the average markup effect, and the optimal inflation rate is negative.

4.3 Numerical analysis

In this section, I show numerically how the optimal inflation rate depends on the values of the social and private discount factors β_s and β_f. I also consider how variations in the degree of steady-state distortions and in the Frisch elasticity of labor supply affect the results. In all cases, I compare the results obtained with Rotemberg pricing to those obtained with Calvo pricing.

4.3.1 Baseline results

I consider the following baseline calibration. The Frisch elasticity of labor supply $\frac{1}{\varphi}$ is set equal to 4, a value commonly used in business cycle models (Chetty et al. 2011). Section 4.3.2 reports results when labor supply is less elastic. I assume that prices are reset on average every four quarters, which implies $\alpha = 0.75$. The price adjustment cost parameter ϕ^p is chosen according to the following logic. The linearized Phillips Curve of the model is identical in the two models of price-setting. With $\alpha = 0.75$ and a private discount factor of 0.99, we obtain a slope $\omega = \frac{(1-\beta_2\alpha)(1-\alpha)}{\alpha} = 0.0848$ with staggered Calvo contracts. With price adjustment costs à la Rotemberg, we have $\omega = \frac{\theta-1}{\phi^p}$. I thus set $\phi^p = \frac{\theta-1}{0.0848}$ so that the slope of the linearized Phillips Curve is identical in the two cases. The elasticity of substitution between goods θ is set equal to 6, which implies a steady-state markup of 20%. Using this calibration, Figure 3 plots the optimal steady-state inflation rate as a function of β_s and β_f.

These graphs provide a visual confirmation of the results of the previous section. When the social discount factor is lower than the private discount factor, the optimal inflation rate is negative. In the opposite case, the optimal inflation rate is positive. It is also possible to see that for a given combination (β_s, β_f), the absolute value of the optimal inflation rate is generally larger under Rotemberg pricing than under Calvo pricing. For example, when $\beta_s = 0.99$ and $\beta_f = 0.9$, the optimal net inflation rate π^* is equal to 1.1% under Rotemberg pricing and 0.67% under Calvo pricing. This is due to a combination of two factors. As shown in figures 1 and 2, Γ is convex with Calvo contracts whereas it is linear with price adjustments costs à la Rotemberg. Thus, the discounted effect of inflation on price wedges increase more rapidly with inflation in the Calvo case when $\Pi > 1$, and less rapidly when $\Pi < 1$. This explains why the optimal inflation rate is larger in the Rotemberg case when $\beta_s > \beta_f$. Symmetrically, we would expect the optimal inflation rate to be more negative in the Calvo case when $\beta_f > \beta_s$. However, since the discounted effect of inflation on marginal costs Λ is much lower with Rotemberg pricing than with Calvo pricing when $\beta_f > \beta_s$, we get the opposite result.

4.3.2 Sensitivity analysis

As shown in section 3, the expressions for Λ and Γ depend on weights related to the degree of steady-state distortions. I now explore how results change with the extent of steady-state distortions. I consider an economy with both price and wage markups, in which nominal wages are flexible. Equation (9) becomes

$$\left[mc^{-1} - \frac{s}{\mu^w}\right]\left(... + \beta_s^{-n}\frac{\partial mc_{-n}}{\partial \Pi} + ... + \beta_s^{-1}\frac{\partial mc_{-1}}{\partial \Pi} + \frac{\partial mc}{\partial \Pi}\right)$$

$$-\left[\varphi s^{-1} + \frac{mc}{\mu^w}\right]\left(\frac{\partial s}{\partial \Pi} + \beta_s\frac{\partial s_{+1}}{\partial \Pi} + ... + \beta_s^{n}\frac{\partial s_{+n}}{\partial \Pi} + ...\right) = 0$$

where μ^w is the wage markup. Since wages are assumed to be flexible, μ^w is independent of the inflation rate. I use the same calibration as before and set $\mu^w = 1.2$. The results are reported in figure 4. The absolute value of the optimal inflation rate π^* is now larger for a given combination (β_s, β_f). For example, for $\beta_s = 0.99$ and $\beta_f = 0.9$, the optimal net inflation rate π^* is equal to 2.3% under Rotemberg pricing and 1.28% under Calvo pricing[3]. Intuitively, the further away from the first best is the output level, the more beneficial it becomes to use the inflation rate to influence the level of activity through the level of average markups.

I also examine a situation in which labor supply is less elastic. In that case, I set φ equal to 1. As shown in equation (9), the relative weight on the resource costs of inflation in the utility function increases with φ. Thus, the benefits of deviating from price stability should be lower in that case. This is confirmed in figure 5, which plots the optimal inflation rate as a function of β_s and β_f. Finally, note that all the results reported in this section do not vary very much with the price stickiness parameters, α and ϕ^p.

5 When are private and social discount factors different?

If firms are owned by households, then they should use the household discount factor when evaluating future profit flows. This is the assumption that is made in most New Keynesian models. However in certain environments the "effective" discount factor used by firms may be slightly lower than the household discount factor. This happens when there is a positive probability that the firm or the

[3]Similar results obtain when the degree of steady-state inefficiency is varied by increasing the size of price markups instead of introducing wage markups.

product for which a pricing decision is made will disappear in the future. In that case, the discount factor is weighted by a factor $(1-\rho)$ where ρ is a mortality rate. Such a property arises in environments which explicitly take into account the flows of creation and destruction in the labor and goods markets. They include models of firm entry and exit (see for example Bilbiie et al 2014), and models with search and matching frictions in the labor and goods market. In one-worker firm models such as Kuester (2010), labor turnover implies firm turnover and the effective discount rate of firms includes the job separation rate. In goods market frictions models, relationships between producers and consumers may end abruptly because of changes in tastes or because consumers loose their purchasing power and are no longer able to buy the good. In all these cases, if the planner uses the household discount factor to evaluate future utility flows, the private discount factor is lower than the social discount factor.

A popular argument dating back to the work of Pigou (1920) and Ramsey (1928) is that the social discount rate should be equal to zero so as to give the same weight to current and future generations. Ramsey (1928) argued that discounting is not merely mistaken, but ethically indefensible. This argument has been contested on several grounds. First, a high social discount rate may be necessary to prevent current generations from being unfairly treated. In the presence of technological growth, future generations will be richer and consume more than present generations. In that case, discounting the future seems justified on the basis that it enables to rebalance consumption levels between generations. Marini and Scaramozzino (2000) consider an optimal growth model in which different generations coexist (see Yaari 1965 or Blanchard 1985). They show that positive discounting is necessary to maintain constant consumption over time when technological growth is exogenous. In the case of endogenous growth, they find that an equitable intertemporal allocation of resources can be reached only if the social discount rate exceeds the private discount rate. Second, many have argued that normative preferences should be based upon the revealed preferences of individuals, which indicate that the discount rate is positive. In that case, the social discount rate would be given by the household discount rate. However, Caplin and Leahy (2004) argue that revealed preferences may not adequatly represent tastes when individuals derive utility from past consumption. They show that in this setting the social planner should in general be more patient than private citizens.

6 Conclusion

To be completed

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7 Appendix

7.1 Λ with Calvo pricing

We start with the pricing condition

$$E_t \sum_{s=t}^{\infty} (\alpha\beta_f)^{s-t} \frac{U_c(C_s)}{U_c(C_t)} \left(\frac{P_t}{P_s}\right)^{-\theta} Y_s \left\{ p_t^o \frac{P_t}{P_s} - \frac{\theta}{\theta-1} mc_s \right\} = 0$$

This equation states that firms set their optimal price so that on average their relative price $\frac{P_t^o}{P_s}$ is equal to their marginal cost mc_s over the lifetime of the price. The weights used when averaging are $(\alpha\beta_f)^{s-t} \frac{U_c(C_s)}{U_c(C_t)} \left(\frac{P_t}{P_s}\right)^{-\theta} Y_s$. They depend on the stochastic discount factor of firms $\beta_f^{s-t} \frac{U_c(C_s)}{U_c(C_t)}$, on the probability that the price will still be effective α^{s-t}, and on the demand for its good that the firm will face in a given period $\left(\frac{P_t^o}{P_s}\right)^{-\theta} Y_s$. Imposing market clearing $C_s = Y_s$, and assuming log utility, we obtain

$$E_t \sum_{s=t}^{\infty} (\alpha\beta_f)^{s-t} \left(\frac{P_t}{P_s}\right)^{-\theta} p_t^o \frac{P_t}{P_s} = E_t \sum_{s=t}^{\infty} (\alpha\beta_f)^{s-t} \left(\frac{P_t}{P_s}\right)^{-\theta} \frac{\theta}{\theta-1} mc_s$$

We can express marginal cost as a function of the current period optimal price p_t^o, future inflation rates $\Pi_{t+1}, \Pi_{t+2}, ...$, and future marginal costs $mc_{t+1}, mc_{t+2}, ...$

$$mc_t = \frac{\theta-1}{\theta} p_t^o E_t \left[1 + \alpha\beta_f \Pi_{t+1}^{\theta-1} + (\alpha\beta_f)^2 (\Pi_{t+1}\Pi_{t+2})^{\theta-1} + ...\right]$$

$$-\alpha\beta_f E_t \Pi_{t+1}^{\theta} mc_{t+1} - (\alpha\beta_f)^2 E_t \left(\Pi_{t+1}\Pi_{t+2}\right)^{\theta} mc_{t+2} + ...$$

This equation is also valid one period ahead

$$mc_{t+1} = \frac{\theta-1}{\theta} p_{t+1}^o E_{t+1} \left[1 + \alpha\beta_f \Pi_{t+2}^{\theta-1} + (\alpha\beta_f)^2 (\Pi_{t+2}\Pi_{t+3})^{\theta-1} + ...\right]$$

$$-\alpha\beta_f E_{t+1} \Pi_{t+2}^{\theta} mc_{t+2} - (\alpha\beta_f)^2 E_{t+1} \left(\Pi_{t+2}\Pi_{t+3}\right)^{\theta} mc_{t+3} + ...$$

Substituting $\alpha\beta_f E_t \Pi_{t+1}^{\theta} mc_{t+1}$ in the equation for mc_t, we obtain

$$mc_t = \frac{\theta-1}{\theta} p_t^o E_t \left[1 + \alpha\beta_f \Pi_{t+1}^{\theta-1} + (\alpha\beta_f)^2 (\Pi_{t+1}\Pi_{t+2})^{\theta-1} + ...\right]$$

$$-\alpha\beta_f E_t \Pi_{t+1}^{\theta} \frac{\theta-1}{\theta} p_{t+1}^o \left[1 + \alpha\beta_f \Pi_{t+2}^{\theta-1} + (\alpha\beta_f)^2 (\Pi_{t+2}\Pi_{t+3})^{\theta-1} + ...\right]$$

Thus marginal cost is equal to the expected discounted sum of real revenues when prices are set optimally today minus the appropriately discounted expected discounted sum of real revenues when prices are set optimally tomorrow. This last expression can be rewritten as

$$mc_t = \frac{\theta-1}{\theta}p_t^o + \frac{\theta-1}{\theta}E_t\left(p_t^o - p_{t+1}^o\Pi_{t+1}\right)\left[\alpha\beta_f\Pi_{t+1}^{\theta-1} + (\alpha\beta_f)^2\left(\Pi_{t+1}\Pi_{t+2}\right)^{\theta-1} + ...\right]$$

We can now derive the effect of future expected inflation on marginal cost

- $\frac{\partial mc_t}{\partial \Pi_t} = \frac{\theta-1}{\theta}\frac{\partial p_t^o}{\partial \Pi_t}\left[1 + \alpha\beta_f\Pi_{t+1}^{\theta-1} + (\alpha\beta_f)^2(\Pi_{t+1}\Pi_{t+2})^{\theta-1} + ...\right]$
- $\frac{\partial mc_t}{\partial E_t\Pi_{t+1}} = \frac{(\theta-1)^2}{\theta}E_t\left(p_t^o - p_{t+1}^o\Pi_{t+1}\right)\Pi_{t+1}^{\theta-2}\left[\alpha\beta_f + (\alpha\beta_f)^2(\Pi_{t+2})^{\theta-1} + ...\right]$
- $-\frac{\theta-1}{\theta}\left(\frac{\partial p_{t+1}^o}{\partial \Pi_{t+1}}\Pi_{t+1} + p_{t+1}^o\right)\left[\alpha\beta_f\Pi_{t+1}^{\theta-1} + (\alpha\beta_f)^2\left(\Pi_{t+1}\Pi_{t+2}\right)^{\theta-1} ...\right]$
- $\frac{\partial mc_t}{\partial E_t\Pi_{t+2}} = \frac{(\theta-1)^2}{\theta}E_t\left(p_t^o - p_{t+1}^o\Pi_{t+1}\right)\Pi_{t+2}^{\theta-2}\left[(\alpha\beta_f)^2\Pi_{t+1}^{\theta-1} + (\alpha\beta_f)^3(\Pi_{t+1}\Pi_{t+3})^{\theta-1} + ...\right]$

Under the assumption that $\alpha\beta_f\Pi^\theta$is lower than 1, and using formulas for infinite series, we obtain in steady-state

- $\frac{\partial mc}{\partial \Pi} = \frac{\theta-1}{\theta}\frac{\partial p^o}{\partial \Pi}\frac{1}{1-\alpha\beta_f\Pi^{\theta-1}}$
- $\frac{\partial mc}{\partial \Pi_{+1}} = \frac{(\theta-1)^2}{\theta}p^o(1-\Pi)\frac{\Pi^{-1}\alpha\beta_f\Pi^{\theta-1}}{1-\alpha\beta_f\Pi^{\theta-1}} - \frac{\theta-1}{\theta}\alpha\beta_f\Pi^{\theta-1}\left(\frac{\partial p^o}{\partial \Pi}\Pi + p^o\right)\frac{1}{1-\alpha\beta_f\Pi^{\theta-1}}$
- $\frac{\partial mc}{\partial \Pi_{+2}} = \frac{(\theta-1)^2}{\theta}p^o(1-\Pi)\frac{\Pi^{-1}\left(\alpha\beta_f\Pi^{\theta-1}\right)^2}{1-\alpha\beta_f\Pi^{\theta-1}}$

For $n \geq 2$, all derivatives share a common form

- $\frac{\partial mc}{\partial \Pi_{+n}} = \frac{(\theta-1)^2}{\theta}p^o(1-\Pi)\frac{\Pi^{-1}\left(\alpha\beta_f\Pi^{\theta-1}\right)^n}{1-\alpha\beta_f\Pi^{\theta-1}}$

We can now compute what the planner cares about, the discounted effect of inflation on marginal cost

$$\Lambda = \sum_{i=0}^{\infty}(\beta_s^{-1})^i\frac{\partial mc}{\partial \Pi_{+i}}$$

After some algebra, we obtain

$$\Lambda = \frac{(\theta-1)}{\theta}p^o\frac{\alpha\Pi^{\theta-2}}{\text{1-}\alpha\beta_s\Pi^{\theta-1}}\frac{\left[(1-\frac{\beta_s}{\beta_f}\Pi)(1-\alpha\frac{\beta_s}{\beta_f}\Pi^{\theta-1}) + (\theta-1)(1-\Pi)\frac{\beta_s}{\beta_f}(1-\alpha\Pi^{\theta-1})\right]}{(1-\alpha\Pi^{\theta-1})(1-\alpha\frac{\beta_s}{\beta_f}\Pi^{\theta-1})}$$

where $p^o = \left[\frac{1-\alpha\Pi^{\theta-1}}{1-\alpha}\right]^{\frac{1}{1-\theta}}$.

7.2 Γ with Calvo pricing

We have

$$s_t = (1-\alpha)\left(p_t^o\right)^{-\theta} + \alpha \Pi_t^\theta s_{t-1}$$

Iterating backward, we obtain

$$s_t = (1-\alpha)\left[\left(p_t^o\right)^{-\theta} + \alpha \Pi_t^\theta \left(p_{t-1}^o\right)^{-\theta} + \alpha^2 \left(\Pi_t \Pi_{t-1}\right)^\theta \left(p_{t-2}^o\right)^{-\theta} + ...\right]$$

We can derive the effect of current and past inflation on markups

- $\frac{\partial s_t}{\partial \Pi_t} = -\theta(1-\alpha)\left(p_t^o\right)^{-\theta-1} \frac{\partial p_t^o}{\partial \Pi_t} + (1-\alpha)\theta \Pi_t^{\theta-1}\left[\alpha \left(p_{t-1}^o\right)^{-\theta} + \alpha^2 \Pi_{t-1}^\theta \left(p_{t-2}^o\right)^{-\theta} + ...\right]$
- $\frac{\partial s_t}{\partial \Pi_{t-1}} = -\theta(1-\alpha)\alpha \Pi_t^\theta \left(p_{t-1}^o\right)^{-\theta-1} \frac{\partial p_{t-1}^o}{\partial \Pi_{t-1}} + (1-\alpha)\theta \Pi_{t-1}^{\theta-1}\left[\alpha^2 \Pi_t^\theta \left(p_{t-2}^o\right)^{-\theta} + \alpha^3 \left(\Pi_t \Pi_{t-2}\right)^\theta \left(p_{t-3}^o\right)^{-\theta} + ...\right]$
- $\frac{\partial s_t}{\partial \Pi_{t-2}} = -\theta(1-\alpha)\alpha^2 \left(\Pi_t \Pi_{t-1}\right)^\theta \left(p_{t-2}^o\right)^{-\theta-1} \frac{\partial p_{t-2}^o}{\partial \Pi_{t-2}} + (1-\alpha)\theta \Pi_{t-2}^{\theta-1}\left[\alpha^3 \left(\Pi_t \Pi_{t-1}\right)^\theta \left(p_{t-3}^o\right)^{-\theta} + ...\right]$

Under the assumption that $\alpha \Pi^\theta$ is lower than 1, and using formulas for infinite series, we obtain in steady-state

- $\frac{\partial s}{\partial \Pi} = \theta(1-\alpha)(p^o)^{-\theta} \frac{\alpha \Pi^{\theta-2}}{(1-\alpha \Pi^{\theta-1})(1-\alpha \Pi^\theta)}(\Pi - 1)$
- $\frac{\partial s}{\partial \Pi_{-1}} = \alpha \Pi^\theta \frac{\partial s}{\partial \Pi}$
- $\frac{\partial s}{\partial \Pi_{-2}} = (\alpha \Pi^\theta)^2 \frac{\partial s}{\partial \Pi}$

For $n \geq 2$, all derivatives share a common form

- $\frac{\partial s}{\partial \Pi_{-n}} = (\alpha \Pi^\theta)^n \frac{\partial s}{\partial \Pi}$

We can now compute what the planner cares about, the discounted effect of inflation on price dispersion

$$\Gamma = \sum_{i=0}^{\infty} (\beta_s)^i \frac{\partial s}{\partial \Pi_{-i}}$$

After some algebra, we find

$$\Gamma = \frac{\theta(1-\alpha)(p^o)^{-\theta} \alpha \Pi^{\theta-2}}{(1-\alpha\beta_s \Pi^\theta)(1-\alpha \Pi^\theta)(1-\alpha \Pi^{\theta-1})}(\Pi - 1)$$

7.3 Deriving equation (12) with general functional forms for $U(C)$, $v(L)$, and $F(L)$

To be completed

7.4 Figures

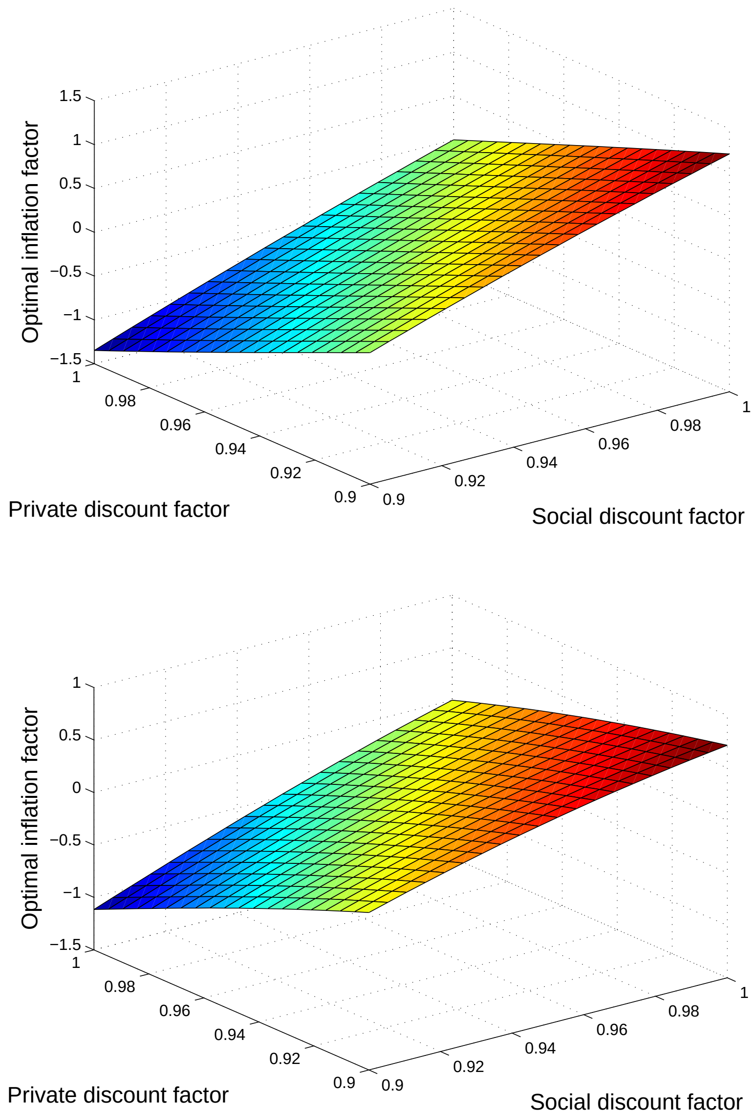


Figure 3: The optimal inflation rate (annualized and in percentage terms) as a function of the social and private discount rates. Top panel: Rotemberg pricing. Bottom panel: Calvo pricing. Baseline model.

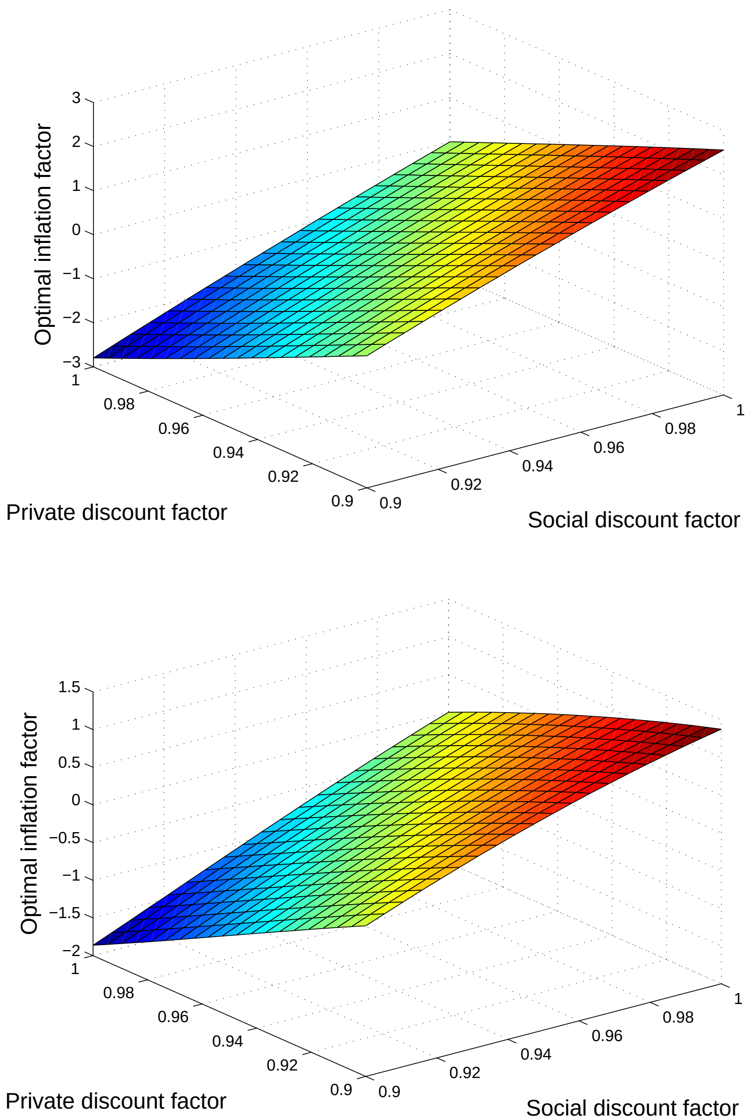
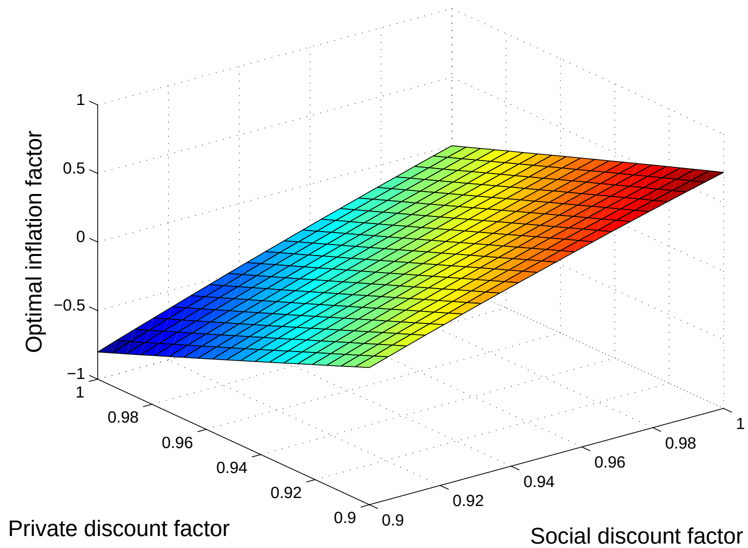


Figure 4: The optimal inflation rate (annualized and in percentage terms) as a function of the social and private discount rates. Top panel: Rotemberg pricing. Bottom panel: Calvo pricing. Model with both price and wage markups.



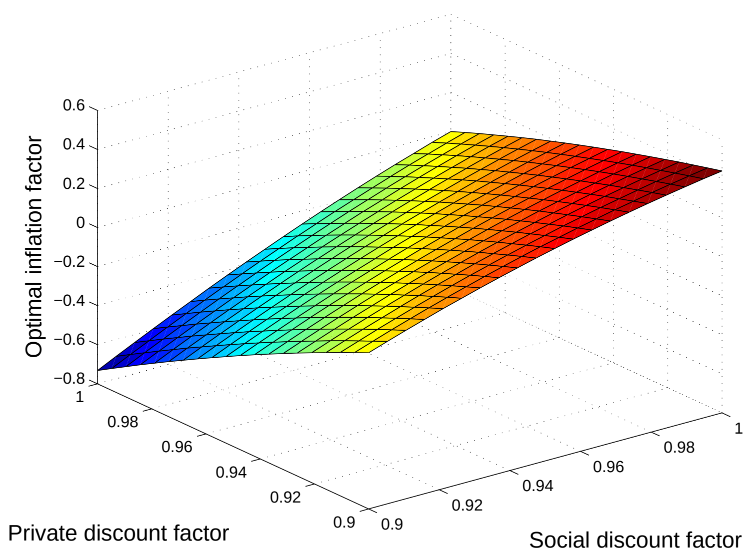


Figure 5: The optimal inflation rate (annualized and in percentage terms) as a function of the social and private discount rates. Top panel: Rotemberg pricing. Bottom panel: Calvo pricing. Model with low elasticity of labor supply.