

Level and Volatility Shocks to Fiscal Policy: Term Structure Implications

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Abstract

We build and estimate a New-Keynesian model with heterogeneous agents to study the impact of level and volatility shocks to fiscal policy on the term structure of interest rates and bond risk premia. We derive three key insights from the estimated theoretical model. First, government spending level shocks generate positive covariance between marginal utility to consume and inflation, making nominal bonds poor hedges against consumption risk. Therefore, investors demand positive risk premium for holding inflation risk. Second, government spending volatility shocks generate negative inflation risk premium as inflation declines when the marginal utility to consume is high. Increased uncertainty in government spending causes savings to go down while investment to go up, which in turn lowers the marginal cost of production and inflation. Third, variability in the nominal term premium is driven by variation in the real term premium while inflation risk premium is remarkably stable over time. We find that fluctuation of the real term premium is entirely driven by government spending volatility shocks.

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1 Introduction

Fiscal policy shocks and fiscal volatility shocks have first order effects on economic activity. Government spending and taxation can impact corporate borrowing-investment choices, household consumption-saving behavior, and economic aggregates such as inflation. Similarly, uncertainty about government spending and tax rates can alter the decision-making process faced by economic agents. The study of fiscal policy commands a large area of literature in economics.¹ The majority of papers focuses on optimal taxation or government spending and its impact on the output multiplier or consumption. More recently, Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) show that unexpected increase in the return on capital tax rate uncertainty has strong negative impact on output.² Bachmann, Bai, Lee, and Zhang (2015) quantify the welfare costs of fiscal uncertainty in a neo-classical stochastic growth model.

The link between fiscal policy and the term structure of interest rates, on the other hand, is less well established. Dai and Philippon (2005) provide empirical evidence of fiscal deficits driving nominal yield curve dynamics in a no-arbitrage affine macro-finance model, but the model does not accommodate endogenous inflation, which Piazzesi and Schneider (2007) document to be the main risk factor in generating bond risk premium. Given the high political uncertainty and low interest rate environment in the U.S. towards the end of 2016, the impact of fiscal volatility shocks on bond risk premium remains an important but open question. In this paper, we estimate a dynamic stochastic general equilibrium (DSGE) model to investigate the effects of fiscal policy and policy uncertainty on the term structure of interest rates and bond risk premia. We focus on three specific aspects of fiscal policy: government spending, taxation on the return of capital, and the systematic response of the fiscal authority to debt and spending.

¹Papers in this field is too numerous to list. See Barro (1974), Aschauer (1985), Aiyagari, Christiano, and Eichenbaum (1992), Baxter and King (1993), Ramey and Shapiro (1998), Gali, Valles, and Lopez-Salido (2007), Christiano, Eichenbaum, and Rebelo (2011) for equilibrium examples.

²Following the literature (see e.g., Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015)) we interpret the unexpected changes in the time-varying volatility of the fiscal instrument (e.g. government expenditure) innovations as a representation of unexpected variations in uncertainty about fiscal policy. We also use the term “uncertainty” as shorthand for what would more precisely be referred to as “risk”.

Through the lens of the model, we document three main findings in this paper. First, shocks to government spending generate positive inflation risk premium as inflation is high precisely when consumption declines. Second, shocks to the conditional volatility of government spending generate negative inflation risk premium: inflation is low when consumption is low. However, the overall nominal term premium resulting from the spending volatility shock is still positive, caused by the positive real term premium. Third, time-series variation of nominal term premium is largely driven by time varying real term premium, while inflation risk premium is much more stable. Government spending volatility shock is the primary factor in generating real term premium fluctuations.

The theoretical analysis is conducted in a general equilibrium model with production. Ricardian equivalence in the model is disrupted by introducing a class of agents with limited market participation. The heterogeneity of the consumers follows Mankiw (2000), such that they consist of optimizing households who have the ability to save by purchasing nominal bonds and invest in capital, as well as rule-of-thumb households who are constrained to consume their entire after-tax labor income. The households are called savers and spenders, respectively, and the presence of the spenders leads to fiscal policy non-neutrality such that debt versus taxes financing by the government is nontrivial. Each period, both types of households decide the quantity of goods to consume and the hours of labor to supply in order to maximize their lifetime utilities.

The production sector is in line with the standard New-Keynesian stochastic growth model. The productive function is Cobb-Douglas employing transitory productivity shocks and permanent labor productivity shocks. The intermediate-good firms adjust prices according to the Calvo (1983) process, under which only a fraction of the firms are allowed to maximize present value of their expected profits by choosing the optimal price each period. This mechanism induces monetary policy non-neutrality with respect to the real economy such that comparison between fiscal policy and monetary policy impacts is meaningful. The monetary authority sets the nominal short-term interest rate using a simple Taylor rule with contemporaneous feedbacks from inflation and the output gap plus a monetary policy shock

which represents any unexpected deviations of the nominal short rate. The fiscal authority chooses the amount of current period lump-sum taxes to collect. Government revenue is a combination of the lump-sum transfer and tax on the return of capital such that the government budget constraint is satisfied. The fiscal instruments, namely government spending and capital tax rates, are exogenous and their shocks exhibit stochastic volatility following an autoregressive process.

We solve the model using the standard macroeconomic technique of approximation around the nonstochastic steady state, or perturbation methods (see Schmitt-Grohe and Uribe (2004)). A first-order approximation of the model and bond price (i.e., a log-linearization) eliminates the term premium entirely and a second-order approximation to the solution of the model and bond price produces a term premium that is nonzero but constant. Since in this paper we are not only interested in the level of the term premium but also in its volatility and variation over time, we compute a third-order approximate solution of the model and bond prices around the non-stochastic steady state using the pruning algorithm suggested by Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2016). Importantly, provided the linearized solution is stable, Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2016) provide closed-form solutions for first and second unconditional moments of the pruned DSGE model. This allows us to estimate our model, approximated up to third-order, using means, variances and contemporaneous covariances of macro and financial series within a generalized method of moments (GMM) setting.

There are seven economic shocks driving the dynamics of the theoretical model: transitory and permanent productivity shocks, monetary policy shocks, as well as level and volatility shocks to government spending (as a share of output) and the tax rate of return on capital. Since the impact of both productivity shocks and monetary policy shocks have been examined in the equilibrium term structure literature, our analysis is centered on the four fiscal shocks.³ A positive level shock to government spending drives up demand of output, and it also crowds out consumption of the savers as demand of debt increases. The wealth effect of

³For example, see Rudebusch and Swanson (2012), Kung (2015), and Hsu, Li, and Palomino (2015).

lower consumption increases the labor supply and depresses real wage. Investment declines initially as labor supply increases but quickly rebounds to go higher. Increase in return on capital generates a spike in inflation immediately after the positive shock is realized, producing positive average inflation risk premium. On the other hand, a positive shock to government spending volatility lowers government debt and inflation in our benchmark model. Increase in spending volatility makes capital investment more attractive over debt for consumption smoothing because government spending is expected to be high implying higher future taxes. The oversupply of capital causes the real return on capital to decline, while increase in labor supply puts downward pressure on real wage. This leads to lower inflation as marginal cost of production decreases, generating negative average inflation risk premium. Although the average risk premium is orders of magnitude larger stemming from the level shock, the volatility shock to government spending generates almost all the time variation in bond risk premium according to our variance decomposition.

Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) examine the impact of fiscal volatility shocks on economic activity in a model without the saver-spender dichotomy but with distortionary taxation on consumption, labor income, and return on capital. They document that shocks to the return on capital tax rate volatility are the dominant fiscal volatility shocks. Without complicating the model for asset pricing purposes, we add distortionary tax on capital return only in the model. However, in our model, level and volatility shocks to the tax rate on return of capital contribute very minimally to the magnitude and volatility of the nominal term premium.

The rest of the paper is structured as follows. The next section reviews the related literature. Section 3 introduces the model. Section 4 discusses data and presents our solution method and estimation approach. Section 5 presents our results. Section 6 concludes. Detailed derivations are deferred to the Appendix.

2 Related Literature

This paper belongs to a growing literature examining the relation between government policies, economic activity, and asset prices. The joint modeling of the yield curve and macroeconomic variables has received much attention since Ang and Piazzesi (2003), where the authors connect latent term structure factors to inflation and the output gap. More recently, many term structure studies incorporate monetary policy elements in their models using the fact that the nominal short rate is the monetary policy instrument. However, these models are generally silent on the effects of fiscal policy on the term structure despite evidence suggesting that it has nontrivial effects on interest rates. The primary contribution of this paper is establishing the link between fiscal policy and risk premia on nominal bonds, namely the term premium and the inflation risk premium. The model shows loose fiscal policy and high government spending cause investors to demand higher returns in exchange for holding Treasury securities.

This paper is most closely related to the literature on term structure and bond risk premia in equilibrium. Campbell (1986) specifies an endowment economy in which utility maximizing agents trade bonds of different maturities. When the exogenous consumption growth process is negatively autocorrelated, term premia on long-term bonds are positive, generating upward sloping yield curves because they are bad hedges against consumption risk compared to short-term bonds. More recently, Piazzesi and Schneider (2007), using Epstein and Zin (1989) preferences, show that inflation is the driver that generates a positive term premium on nominal long-term bonds. Negative covariance between consumption growth and inflation translates into high inflation when consumption growth is low and marginal utility to consume is high. Wachter (2006) generates upward sloping nominal and real yield curves employing habit formation. In her model, bonds are bad hedges for consumption as agents wish to preserve previous level of consumption as current consumption declines.

Rudebusch and Swanson (2008) and Rudebusch and Swanson (2012) examine bond risk premia in general equilibrium where utility-maximizing agents supply labor to profit-maximizing

firms to produce consumption goods. The best-fit model in the latter paper is successful in matching the basic empirical properties of the term structure using only transitory productivity shocks. Palomino (2010) studies optimal monetary policy and bond risk premia in general equilibrium. More specifically, he shows that the welfare-maximizing monetary policy affects inflation risk premia depending on the credibility of the monetary authority in the economy as well as the representative agent's preference. Kung (2015) builds a equilibrium model with stochastic endogenous growth to explain the impact of monetary policy shocks on bond risk premium. Hsu, Li, and Palomino (2015) examine risk premia on real bonds in general equilibrium. Calibrated to TIPS data, they find that productivity growth shocks alone generate negative term premium on real bonds, but the presence of wage rigidities makes term premium positive.

This paper is also related to the literature on the interaction between fiscal policy and asset pricing. Croce, Kung, Nguyen, and Schmid (2012) study the effects of fiscal policies in a production-based general equilibrium model in which taxation affects corporate decisions. They find that tax distortions have negative effects on the cost of equity and investment. Dissanayake (2016) empirically documents that firms with higher exposure to government spending shocks earn higher risk premia. Whereas the empirical implications of Dissanayake (2016) are derived within a two-sector model (similar in spirit to Papanikolaou, 2011) our modeling approach is different, relying on heterogeneous agents with EZ preferences.

Our interest is different. We analyze the impact of government spending level and uncertainty shocks on the term structure of interest rates. Our interest in fiscal volatility shocks is motivated by Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) who uncover evidence of time-varying volatility in tax and government spending processes for the United States and, using both a VAR and a New Keynesian model, document that these fiscal volatility shocks can have a sizable adverse effect on economic activity. To the best of our knowledge, our paper is the first attempt to evaluate the dynamic consequences on the term premium of unexpected changes to fiscal volatility shocks.

Our paper also fits within a large literature that analyzes the importance of fiscal and

monetary policy shocks for explaining US macroeconomic fluctuations. Whereas most of the existing empirical literature separately considers either monetary policy or fiscal policy (see Rossi and Zubairy (2011) for a discussion), our framework allows for a joint analysis of these two shocks.

Our quantitative exercise relies on a full estimation of the model. In terms of estimation, we follow the lead of Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2016) and employ a GMM method that matches data-driven moments to model-implied moments obtained upon a third-order perturbation approximation of the policy functions. Closer to our work is that of Rudebusch and Swanson (2012), who also use perturbation methods to solve a DSGE model with EZ preferences and endogenous inflation to account for the dynamics of the yield curve. Importantly Rudebusch and Swanson (2012) approximate the yields on bonds through a consol. Since Andreasen and Zabczyk (2015) show that a consol approximation of yields may introduce approximation biases, this paper solves for the nominal bond yield at each maturity.

3 The Benchmark Model

The New-Keynesian model is the workhorse of modern monetary economics. The micro-foundations underlying the New-Keynesian framework generate frictions in the firm's first order condition relating its real marginal cost to its marginal product of labor. The resulting output gap between actual output and output under flexible prices summarizes aggregate economic activity. Furthermore, the output gap is forward-looking and is driven by the expectation of future output gap as well as the real short-term interest rate.⁴ The monetary authority implements the Taylor rule and sets the nominal short rate as a function of inflation and output gap. The nominal short rate in turn affects the real short rate thus making monetary policy non-neutral with respect to output gap and the real economy. Finally, we close the model with a fiscal authority which implements a fiscal policy rule that determines

⁴For a detailed exposition on the New-Keynesian framework, see Clarida, Gali, and Gertler (1999).

the fraction of government obligations to be financed through taxes rather than new debt issuance.

3.1 The Saver's Problem

There are two types of households in the economy. Following Mankiw (2000), we will call them savers and spenders according to their budget constraints.⁵ The savers have the ability to save current income in order to smooth future consumption by purchasing government bonds. In contrast, the spenders are required to consume their entire after tax income. With Epstein and Zin (1989), the representative agent of the savers maximizes lifetime utility by solving the following:

$$\begin{aligned} \max \quad & V(C_t^o, N_t^o) = \left[\frac{C_t^{o1-\psi}}{1-\psi} - \lambda_t \frac{N_t^{o1+\omega}}{1+\omega} \right] + \beta E_t \left[\{V_{t+1}^o\}^{\frac{1-\gamma}{1-\psi}} \right]^{\frac{1-\psi}{1-\gamma}}, \\ \text{s.t.} \quad & P_t C_t^o + P_t I_t^o + Q_t^{(1)} B_t(t+1) + P_t \tau_t \\ & = P_t W_t N_t^o + (1 - \tau_t^k) P_t R_t^k K_{t-1}^o + B_{t-1}(t) + P_t \Psi_t, \end{aligned}$$

where the “*o*” superscript denotes the optimizing household. β is the time discount factor, ψ is the inverse of the elasticity of intertemporal substitution (EIS), the Epstein-Zin parameter γ is related to the coefficient of relative risk aversion (we discuss this parameter in detail in Section 5), and ω is the inverse of the Frisch elasticity of labor supply. λ_t is the time varying parameter as a function of the permanent technology shock ($A_t^{1-\psi}$) in order to achieve balanced path in the wage demand equation.

C_t^o and N_t^o are real consumption and labor, respectively. I_t^o denotes investment in real terms. P_t is the price level in the economy. $B_t(t+1)$ is the amount of nominal bonds outstanding at the end of period t and due in period $t+1$. W_t refers to real labor income, which is the same across households in the economy. τ_t is real lump-sum tax collected by the fiscal authority to keep the real debt process from exploding, and Ψ_t is dividend income

⁵For the rest of the paper, we will interchangeably refer to savers and spenders as optimizing households and rule-of-thumb households, respectively.

coming from the firms. K_t^o is capital, R_t^k is the return on capital, δ is the depreciation on capital.

V_t^o is the value function of the dynamic programming problem for the representative saver, and V_{t+1}^o is the “continuation utility” of the value function. The budget constraint states that the agent has periodic after-tax income from labor, capital, and dividends as well as bonds maturing at time t . The agent then decides how much to consume after taxes, how much to invest, and how much to pay for newly issued bonds at time t at price $Q_t^{(1)}$.

The nominal pricing kernel written in terms of return on consumption and return on labor income of the savers-spenders economy with distortionary taxes is

$$M_{t,t+1}^{\$} = \left[\beta \left(\frac{C_{t+1}^o}{C_t^o} \right)^{-\psi} \right]^{\frac{1-\gamma}{1-\psi}} \left(\frac{P_t}{P_{t+1}} \right) [(1 - share_t)R_{t+1}^c + share_t R_{t+1}^l]^{\frac{\psi-\gamma}{1-\psi}},$$

where

$$R_{t+1}^c = \frac{(1 + P_{t+1}^c)C_{t+1}^o}{P_t^c C_t^o} \text{ and } R_{t+1}^l = \frac{(1 + P_{t+1}^l)LI_{t+1}}{P_t^l LI_t}.$$

P^c and P^l are prices of the consumption and labor claims, and LI is labor income.

3.2 Wage Rigidities and Saver’s Optimal Wage Setting

Optimal price setting in the presence of wage stickiness is done through the following optimization problem. There is a continuum of optimizing households in the economy, indexed by k . Each period, only a fraction, $1 - \theta$, of the optimizing households has the ability to adjust wage demand optimally. The objective function is:

$$\begin{aligned} \max_{W_t^{\$,*}(k)} \quad & E_t \left[\sum_{s=0}^{\infty} \theta^s M_{t,t+s}^{\$} \left\{ I_{t,t+s}^w W_t^{\$,*}(k) N_{t+s}^o(k) - P_{t+s} MRS_{t+s}(k) N_{t+s}^o(k) \right\} \right] \\ \text{s.t.} \quad & N_{t+s}^o(k) = \left(\frac{W_t^{\$,*}(k)}{W_{t+s}^{\$}} \right)^{-\eta_w} N_{t+s}^d \\ & W_t^{\$} = \left[\int_0^1 W_t(k)^{1-\eta_w} dk \right]^{\frac{1}{1-\eta_w}} = \left[(1 - \theta) W_t^{\$,*1-\eta_w} + \theta (I_{t-1,t}^w W_{t-1}^{\$})^{1-\eta_w} \right]^{\frac{1}{1-\eta_w}}, \end{aligned}$$

where $W_t^{\mathbb{S},*}$ is the optimal *nominal* wage chosen at time t and I_{t+s}^w is the wage index in the case when $W_t^{\mathbb{S},*}$ is not adjusted optimally in following periods. $MRS_{t+s}(k)$ is the marginal rate of substitution between consumption and labor dis-utility. $W_t^{\mathbb{S}}$ is the prevailing nominal market-clearing wage at time t , and N_{t+s}^d is the aggregate labor demand of the savers. The Calvo (1983) style staggered wage setting is standard in the macroeconomic literature.

The optimal wage demand equation for the savers is:

$$\left[\frac{1}{1-\theta} \left\{ W_t^{1-\eta_w} - \theta \left(I_{t-1,t}^w \frac{W_{t-1}}{\Pi_t} \right)^{1-\eta_w} \right\} \right]^{\frac{1}{1-\eta_w}} H_t^o = \nu_w A_t^{1-\psi} C_t^{o\psi} N_t^{o\omega} G_t^o,$$

where

$$\begin{aligned} H_t^o &= 1 + \theta \mathbb{E}_t \left[M_{t,t+1}^{nom} I_{t,t+1}^w^{-\eta_w} \left(\frac{N_{t+1}^d}{N_t^d} \right) \left(\Pi_{t+1} \frac{W_{t+1}}{W_t} \right)^{\eta_w} H_{t+1}^o \right] \\ G_t^o &= 1 + \theta \mathbb{E}_t \left[M_{t,t+1}^{nom} I_{t,t+1}^w^{-\eta_w} \left(\frac{A_{t+1}}{A_t} \right)^{1-\psi} \left(\frac{C_{t+1}^o}{C_t^o} \right)^\psi \left(\frac{N_{t+1}^o}{N_t^o} \right)^\omega \left(\frac{N_{t+1}^d}{N_t^d} \right) \Pi_{t+1}^{1+\eta_w} \left(\frac{W_{t+1}}{W_t} \right)^{\eta_w} G_{t+1}^o \right]. \end{aligned}$$

In the above formulation, W_t is real wage, Π_t is inflation, and ν_w is the wage markup. The equilibrium condition states that the optimal real wage is equal to the marginal cost of providing an extra unit of labor ($A_t^{1-\psi} C_t^{o\psi} N_t^{o\omega}$) multiplied by a time-varying markup ($\nu_w \frac{G_t^o}{H_t^o}$) stemming from the monopolistic behavior of the savers in the labor market.

3.3 The Saver's Investment Decision

In the saver-spender economy, the savers are also the owners of capital. They rent out capital to the firms in exchange for earning the return on capital, R_t^k . The capital accumulation equation is standard with convex quadratic adjustment cost, Φ :

$$K_t^o = (1 - \delta) K_{t-1}^o + \Phi \left(\frac{I_t^o}{K_{t-1}^o} \right) K_{t-1}^o,$$

where δ is the rate of capital depreciation.

The saver's optimal investment strategy has to satisfy the following equation :

$$Q_t^{inv} = \mathbb{E}_t \left[M_{t,t+1} \left[(1 - \tau_t^k) R_{t+1}^k + Q_{t+1}^{inv} \left\{ (1 - \delta) + \Phi \left(\frac{I_{t+1}^o}{K_t^o} \right) - \Phi' \left(\frac{I_{t+1}^o}{K_t^o} \right) \frac{I_{t+1}^o}{K_t^o} \right\} \right] \right],$$

where Q_t^{inv} is the shadow price of investment, and Φ' is the first derivative of the quadratic adjustment cost function.

Similar to the standard investment first order condition from Q -theory, we derive here the intertemporal relationship of investment's Q as a function of the return on capital, the rate of depreciation, and the marginal rate of investment adjustment cost.

3.4 The Spender's Problem

Without the ability to save through purchasing government bonds or investing in capital, the representative agent of the spenders cannot smooth consumption intertemporally and maximizes the following within-period utility over his/her lifetime:

$$\max E_t \left[\sum_{j=0}^{\infty} \beta^j \left(\frac{C_{t+j}^r}{1 - \psi} - \lambda_t \frac{N_{t+j}^r}{1 + \omega} \right) \right],$$

while following a budget constraint such that

$$P_t C_t^r + P_t \tau_t = P_t W_t N_t^r,$$

where the “ r ” superscript now refers to the rule-of-thumb household. After dividing through both sides of the equality by the price level P_t , this budget constraint says that, every period, the saver is forced to consume his/her entire after-tax labor income sans the lump-sum transfer. Notice that the parameters β , γ , and ω are the same across both types of households. The first order condition relating wage and labor supply is:

$$W_t = \frac{1 + \tau_t^c}{1 - \tau_t^l} A_t^{1-\psi} C_t^{r\psi} N_t^{r\omega}.$$

This “intra-temporal” substitution between real consumption and labor supply is the same across the two households. Therefore, there exists an unique real market wage in the economy.

3.5 Aggregation

The aggregate real consumption, labor supply and taxes in the economy can be expressed by the following weighted average of the corresponding variables of each type of households:

$$\begin{aligned}\log\left(\frac{C_t}{A_t}\right) &= \mu \log\left(\frac{C_t^r}{A_t}\right) + (1 - \mu) \log\left(\frac{C_t^o}{A_t}\right) \\ \log(N_t) &= \mu \log(N_t^r) + (1 - \mu) \log(N_t^o)\end{aligned}$$

where μ denotes the fraction of rule-of-thumb consumers in the economy. Furthermore, since investment and capital are unique only to the optimizing households, we differentiate the aggregate quantities of investment and capital by the weighted averages below:

$$\begin{aligned}Inv_t &= (1 - \mu)Inv_t^o \\ K_t &= (1 - \mu)K_t^o.\end{aligned}$$

Lastly, tax collection is the weighted average of tax revenue from the savers and the spenders plus a lump sum transfer, τ_t ,

$$Tax_t = \mu Tax_t^r + (1 - \mu)Tax_t^o,$$

where

$$\begin{aligned}Tax_t^o &= \tau_t + \tau_t^k R_t^k K_{t-1}^o \\ Tax_t^r &= \tau_t.\end{aligned}$$

3.6 The Firm's Problem

There is a dispersion of firms, denoted by j , with identical production technology in the economy. With nominal price stickiness and monopolistic competition, each firm is faced with the following optimization problem:

$$\max_{P_t^*(j)} E_t \left[\sum_{s=0}^{\infty} \alpha^s M_{t,t+s}^{\$} \{ P_t^*(j) Y_{t+s}(j) - P_{t+s} [W_{t+s} N_{t+s}(j) + R_{t+s}^k K_{t+s}(j)] \} \right] \quad (1)$$

$$\text{s.t. } Y_{t+s}(j) = Z_{t+s} K_{t+s-1}(j)^\kappa (A_t N_{t+s}(j))^{1-\kappa} \quad (2)$$

$$Y_{t+s}(j) = \left(\frac{P_t^*(j)}{P_{t+s}} \right)^{-\eta} Y_{t+s} \quad (3)$$

$$P_t = \left[\int_0^1 P_t(j)^{1-\eta} dj \right]^{\frac{1}{1-\eta}} = [(1-\alpha)P_t^{*1-\eta} + \alpha P_{t-1}^{1-\eta}]^{\frac{1}{1-\eta}}. \quad (4)$$

Using Calvo (1983) pricing, a firm can choose to optimally adjust price to $P_t^*(j)$ with probability $(1-\alpha)$ each period independent of the time elapsed between adjustments. The objective function of the firm is simply profit maximization: revenue minus labor cost and rent on capital. The within-period profits are discounted by the nominal pricing kernel and the probability that the firm has not been allowed to adjust its price optimally up to that period. Each period, with probability α , the firm is stuck with the price from the previous period. The cash-flow stream is discounted by the nominal stochastic discount factor between times t and $t+s$, $M_{t,t+s}^{\$}$. $P_t^*(j)Y_{t+s}(j)$ is total sales for firm j at time $t+s$. $W_{t+s}N_{t+s}(j)$ and $R_{t+s}^k K_{t+s}(j)$ are the real labor cost of and the real rental cost of capital, respectively. Notice real wage and real return on capital are determined in equilibrium with the households and are common across all firms.

There are three constraints faced by the firm in optimizing its profit. Equation (2) is the production function of firm j , where Z_t is the transitory productivity shock, the parameter κ is the capital share of input in the Cobb-Douglas production function, and A_t is the permanent productivity shock driving growth in the economy. Equation (3) is the demand equation for firm j 's output as a function of the optimal price it sets at time t . Lastly, equation (4) is the price aggregator as a weighted average of the optimal price at time t and the sticky price

from time $t - 1$.

$P_t^*(j)$ is the optimal price the firm j charges for one unit of the consumption good set at time t . α is the probability in each period $t + s$ that the firm is not allowed to adjust its price optimally so it has to keep charging $P_t^*(j)$. If a firm is not allowed to adjust its price optimally, then it charges $P_t^*(j)$ at time $t + s$, as the price is not indexed. All variables indexed by j is firm-specific. For example, $Y_{t+s}(j)$ means output of firm j at time $t + s$ given the last time firm j was able to set its optimal price was at time t . Without the index j , the variable is common across all firms, such as the price level P_{t+s} and the productivity shock Z_{t+s} . Finally, η determines the markup charged by the firm when it sets $P_t^*(j)$ due to monopolistic competition.

Z_t is the economy-wide productivity shock on output. Log productivity is an exogenous AR(1) process such that

$$z_t = \log(Z_t) = \phi_z z_{t-1} + \sigma_z \epsilon_{z,t},$$

with $\epsilon_t^z \sim \text{iid } \mathcal{N}(0, 1)$. The log growth rate of the permanent productivity shock is expressed is the following AR(1) process with mean growth rate g_a :

$$\Delta a_t = (1 - \phi_a)g_a + \phi_a \Delta a_{t-1} + \sigma_a \epsilon_{a,t}.$$

The firm's optimal price setting behavior has to satisfy the following equation in the presence of nominal price rigidities such that it can only adjust its price optimally each period with probability α .

$$\left[\frac{1}{1 - \alpha} \left(1 - \alpha \left(\frac{1}{\Pi_t} \right)^{(1-\eta)} \right) \right]^{\frac{1}{(1-\eta)}} F_t = \frac{\nu \kappa^{-\kappa} (1 - \kappa)^{-(1-\kappa)} R_t^{K\kappa} W_t^{(1-\kappa)} J_t}{Z_t A_t^{1-\kappa}}, \quad (5)$$

where $\nu = \frac{\eta}{\eta-1}$ is the frictionless markup and Π^* is the inflation target of the central bank.

F_t and J_t are recursively defined as

$$F_t = 1 + \alpha \mathbb{E}_t \left[M_{t,t+1}^{nom} \left(\frac{Y_{t+1}}{Y_t} \right) \Pi_{t+1}^\eta F_{t+1} \right]$$

$$J_t = 1 + \alpha \mathbb{E}_t \left[M_{t,t+1}^{nom} \left(\frac{Z_t}{Z_{t+1}} \right) \left(\frac{A_t}{A_{t+1}} \right)^{1-\kappa} \left(\frac{R_{t+1}^K}{R_t^K} \right)^\kappa \left(\frac{W_{t+1}}{W_t} \right)^{(1-\kappa)} \left(\frac{Y_{t+1}}{Y_t} \right) \Pi_{t+1}^{(1+\eta)} J_{t+1} \right].$$

3.7 The Market Clearing Condition

In this economy, total output has to equal to total private consumption and private investment plus total government spending:

$$Y_t = C_t + I_t + G_t. \quad (6)$$

3.8 The Monetary Authority

Disengaging monetary policy neutrality by augmenting the saver-spender model with the New-Keynesian framework, we assess the implications of fiscal policy on bond risk premia in the presence of an effective monetary authority. The Taylor rule used by the monetary authority to set the nominal short rate, $R_t^{(1)}$, in the model is:

$$\frac{R_t^{(1)}}{R} = \left(\frac{R_{t-1}^{(1)}}{R} \right)^{\rho_r} \left(\frac{\Pi_t}{\Pi^*} \right)^{(1-\rho_r)\rho_\pi} \left(\frac{Y_t/A_t}{Y/A} \right)^{(1-\rho_r)\rho_x} e^{u_t},$$

where R is the steady state nominal rate, Π_t is inflation or $\left(\frac{P_t}{P_{t-1}} \right)$, Π^* is the long-run inflation target, Y is the steady state output, and u_t is the monetary policy shock. The parameter ρ_r is the autoregressive coefficient used for interest rate smoothing. The parameter ρ_π has to be greater than 1 in order to satisfy the Taylor principle for the existence of a determinant equilibrium. Finally, the monetary policy shock follows an autoregressive process of order one

$$u_t = \phi_u u_{t-1} + \sigma_u \epsilon_t^u,$$

with $\epsilon_t^u \sim \text{iid } \mathcal{N}(0, 1)$.

3.9 The Government's Budget Constraint

The government's flow budget constraint balances resources with uses:

$$P_t Tax_t + Q_t^{(1)} B_t(t+1) = B_{t-1}(t) + P_t G_t,$$

where G_t is consumption by the government or government spending. G_t is not productive in the model economy. Government spending as a fraction of output, $g_t = \frac{G_t}{Y_t}$, follows the $AR(1)$ process such that

$$\begin{aligned} g_{t+1} &= (1 - \phi_g) \theta_g + \phi_g g_t + e^{\sigma_{g,t+1}} \epsilon_{g,t+1} \\ \sigma_{g,t+1} &= (1 - \phi_\sigma^g) \theta_\sigma^g + \phi_\sigma^g \sigma_{g,t} + \sigma_\sigma^g \epsilon_{\sigma,t+1}, \end{aligned}$$

where $\sigma_{g,t}$ is stochastic volatility specific to the government spending shock, $\epsilon_{g,t}$, with mean θ_σ^g . σ_σ^g is the time-invariant volatility-of-volatility.

The lump-sum tax is meant to be collected to keep the borrowing path of the government from exploding. The amount is trivial compared to the revenue from the distortionary taxes. Following standard procedure in the literature, we specify the lump-sum tax as a function of real debt.

$$\tau_t = \rho_b D_{t-1}(t) + \rho_g G_t.$$

The capital tax rate is an exogenous $AR(1)$ process with stochastic volatility:

$$\begin{aligned} \tau_{t+1}^k &= (1 - \phi_{\tau^k}) \theta_{\tau^k} + \phi_{\tau^k} \tau_t^k + e^{\sigma_{\tau^k,t+1}} \epsilon_{\tau^k,t+1} \\ \sigma_{\tau^k,t+1} &= (1 - \phi_\sigma^{\tau^k}) \theta_\sigma^{\tau^k} + \phi_\sigma^{\tau^k} \sigma_{\tau^k,t} + \sigma_\sigma^{\tau^k} \epsilon_{\sigma,t+1}, \end{aligned}$$

3.10 Equilibrium

The competitive equilibrium is characterized by the set of market clearing conditions: composite labor, capital stock, bonds, and final goods. Furthermore, given prices and wages of other households, each optimizing household chooses the optimal allocation to solve his/her utility maximization problem. Finally, given wages and prices of other firms, each firm chooses the optimal production input to solve its profit maximization problem. In equilibrium, $N_t^d = N_t^o$.

4 Inference and the observable variables

To estimate the deep structural parameters of our dynamic, stochastic general equilibrium model (DSGE) we rely on the generalized method of moments (GMM) using first and second unconditional moments of macroeconomic and financial data. This section provides a detailed description of the estimation method and discusses the data used to evaluate the unconditional moments.

4.1 Data and Moments for GMM

The time unit is defined to be one quarter. We estimate the model using the following quarterly time series: (i) log output growth, Δy_t (henceforth, Δ denotes the temporal difference operator); (ii) log investment growth, Δinv_t ; (iii) log consumption growth, Δc_t ; (iv) the 1-quarter nominal interest rate, r_t ; (v) inflation, π_t ; (vi) the slope of the term structure proxied by the difference between the 10-year nominal interest rate, $y_t^{(40)}$, and the 1-quarter nominal interest rate; (vii) the 10-year term premium from Adrian, Crump, and Moench (2013). Our sample goes from 1970.Q1 to 2007.Q4. The starting date follows Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015).⁶ The end date is set to

⁶We have also repeated our estimation exercise with moments computed from a longer sample period, from 1961.Q3 to 2007.Q4, and find that the results remain qualitatively the same.

avoid the complications created by the zero lower bound of the nominal interest rate. See Appendix A for a description of the data series.

To estimate model parameters we use the mean, the variance and the contemporaneous covariances in the data as moments.⁷ Hence, we let

$$q_t = \begin{bmatrix} \mathbf{data}_t \\ \mathit{diag}(\mathbf{data}_t \mathbf{data}_t') \\ \mathit{vech}(\mathbf{data}_t \mathbf{data}_t') \end{bmatrix} .$$

Letting θ contain the structural parameters, our GMM estimator is given by

$$\theta_{GMM} = \mathop{\text{argmin}}_{\theta \in \Theta} \left(\frac{1}{T} \sum_{t=1}^T q_t - E[q_t(\theta)] \right)' W \left(\frac{1}{T} \sum_{t=1}^T q_t - E[q_t(\theta)] \right)$$

Here, W is a positive definite weighting matrix and $E[q_t(\theta)]$ contains the model-implied moments computed as described in the following subsection. We use the conventional two-step implementation of GMM by letting $W_T = \mathit{diag}(\hat{S}^{-1})$ in a preliminary first step to obtain $\widehat{\theta^{\text{step } 1}}$ where \hat{S} denotes the long-run variance of $\frac{1}{T} \sum_{t=1}^T q_t$ when re-centered around its sample mean. Our final estimates $\widehat{\theta^{\text{step } 2}}$ are obtained using the optimal weighting matrix $W_T = \mathit{diag}(\hat{S}_{\widehat{\theta^{\text{step } 1}}}^{-1})$, where $\hat{S}_{\widehat{\theta^{\text{step } 1}}}$ denotes the long-run variance of our moments re-centered around $E[q_t(\widehat{\theta^{\text{step } 1}})]$. The long-run variances in both steps are estimated by the Newey-West estimator using 10 lags, but our results are robust to using more lags.

4.2 Inducing Stationarity and Solution Method

The exogenous forcing processes A_t displays a stochastic trend. This random trend is inherited by the endogenous variables of the model. We focus our attention on equilibrium fluctuations around these stochastic trends. To this end, we perform a stationarity-inducing transformation of the endogenous variables by dividing them by their trend component.

⁷We have also repeated our procedure adding to the first and second moments used in the baseline estimation the first and fifth autocovariances to capture the persistence in the data. Our point estimates do not significantly change and the conclusion from model-implied moments remain qualitatively the same. Results are available upon request.

Appendix B.3.1 describes this transformation and presents the complete set of equilibrium conditions in stationary form.

To analyze the role of fiscal shocks and the implications for time-varying risk premia, we solve our DSGE model using perturbation method (see Schmitt-Grohe and Uribe (2004)).⁸ Perturbation method approximates the solution of DSGE models by higher-order Taylor series expansions around the steady state. Given our interest in analyzing time-varying risk premia, we employ a third-order Taylor approximation of the policy functions that characterize the equilibrium dynamics of the model (see propositions 3 and 4 in Andreasen (2012) for how stochastic volatility affects any type of risk premia in a wide class of DSGE models). However, higher-order terms may generate explosive sample paths thus precluding any estimation method that, like GMM, relies on finite moments from stationary and ergodic probability distribution (see e.g. Sims, Kim, Kim, and Schaumburg (2008) for a discussion of this issue within the context of second-order approximations). To ensure stable sample paths (and existence of finite unconditional moments) we adopt the pruned state-space system for non-linear DSGE models suggested by Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2016). Intuitively, pruning means we are going to omit terms of higher-order effects than the considered approximation order (third-order, in our case) when the system is iterated forward in time.⁹ Provided the linearized solution is stable, Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2016) derive closed-form solutions for first and second unconditional moments of the pruned state-space of the DSGE. This is important since it allows us to compute in a reasonable amount of time the unconditional moments for our DSGE model solved up to third-order.

Since in our estimation we use bond prices for short-term and long-term maturities we need to compute the yield curve implied by our DSGE model. To do so we exploit the fact that bond prices beyond the policy rate, r_t , do not affect allocations and prices (i.e.,

⁸Our model has a relatively large number of state variables and five shocks. Because of this high dimensionality, discretization and projection methods are computationally infeasible, so we solve the model using the standard macroeconomic technique of approximation around the nonstochastic steady state – so-called perturbation methods.

⁹For details on the pruning method, see Sims, Kim, Kim, and Schaumburg (2008) for second-order and Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2016) for higher-order approximations to the solutions of DSGE models.

consumption, inflation, etc). Taking advantage of this property, we follow Andreasen and Zabczyk (2015) and we approximate our model by a two-step procedure, where we first solve the model without bond prices exceeding one period and second recursively compute all remaining bond prices based on

$$Q_t^{(k)} = E_t \left[M_{t,t+1}^{\$} Q_{t+1}^{(k-1)} \right] ,$$

where $M_{t,t+1}^{\$} = M_{t,t+1} \frac{1}{\pi_{t+1}}$ denotes the nominal stochastic discount factor, and $M_{t,t+1}$ denotes the real stochastic discount factor. We let $k = 2, \dots, 40$ quarters. The nominal yield curve with continuous compounding is then given by $y_t^{(k)} = -\frac{1}{k} \log Q_t^{(k)}$. We also compute the real term structure based on

$$Q_{t,real}^{(k)} = E_t \left[M_{t,t+1} Q_{t+1,real}^{(k-1)} \right] .$$

Finally, we define the 10-year nominal term premium to be the difference between the 10-year interest rate and the yield-to-maturity on the corresponding bond under risk-neutrality. The latter is computed by discounting payments by $R_t^{(1)}$ instead of the stochastic discount factor.

5 Estimation results

This section presents our main results in terms of parameter estimates, model's fit and impulse responses.

5.1 Parameter estimates

Given that we are dealing with a large model, we fix a small number of parameters to values commonly used in the literature, see Table 1-Panel A. In particular, we set the average fraction of spenders in the model to 50%, which is consistent with the value used by Mankiw (2000) and Gali, Valles, and Lopez-Salido (2007) and empirically verified by others. The rate of depreciation on capital is 0.021 as employed by Kaltenbrunner and Lochstoer (2010) in the LRR I model with transitory productivity shocks. This value implies a steady-state

investment-output ratio of 21 percent. The capital share of intermediate output, κ , is 0.33. The following parameter values are standard in New-Keynesian models. The price rigidity parameter, α , is 0.66. This means every period, two thirds of the firms in the economy are not able to adjust their prices to the optimal level. The higher the α , the stickier the nominal prices are. We also set the wage rigidity parameter, θ , to 0.66. The price markup parameter resulting from monopolistic competition, η , and the wage markup parameter in union wage setting, η_w , are both equal to 6. Hence, steady-state price and wage markup are both equal to 20%. Consistent with previous studies, our calibrated parameters imply a steady-state capital-output ratio, $\frac{Y}{4K^o}$, of about 2. Finally, according to the data, we set the government spending-output ratio, θ_g , to 20.2%, and the mean of the tax rate, θ_{τ^k} , to 40%.¹⁰

We estimate offline the processes for capital tax rate and for government spending. This procedure has the benefit of ensuring that the latent fiscal (tax and government spending) volatility factors maintain their intended economic interpretation.¹¹ Below we report again the dynamics of our two policy instruments for reader's convenience:

$$x_{t+1} = (1 - \phi_x)\theta_x + \phi_x x_t + e^{\sigma_{x,t+1}} \epsilon_{x,t+1} \quad (7)$$

$$\sigma_{x,t+1} = (1 - \phi_\sigma^x)\theta_\sigma^x + \phi_\sigma^x \sigma_{x,t} + \sigma_\sigma^x \epsilon_{\sigma,t+1} \quad (8)$$

for $x \in \{g, \tau^k\}$ where g is government spending as a share of output, and τ^k is the tax rate on capital income. These equations incorporate time-varying volatility in the form of stochastic volatility. Namely, the log of the standard deviation, $\sigma_{x,t}$, of the innovation to each policy instrument is random, and not a constant, as traditionally assumed. The parameter θ_σ^x determines the average standard deviation of a fiscal shock to the policy instrument x , $\frac{\sigma_\sigma^x}{\sqrt{1-(\phi_\sigma^x)^2}}$ is the unconditional standard deviation of the fiscal volatility shock to instrument x , and ϕ_σ^x controls the shock's persistence. Following Fernández-Villaverde, Guerrón-

¹⁰The unconditional mean of a model solved with third-order approximation generally differs from the the steady-state value. The unconditional mean of investment-output and capital-output ratios are 26% and 2.3, respectively.

¹¹Alternatively we could have used macro and financial variables (bond yields) to estimate the full fledged model and to smooth out the time-varying volatility in fiscal rules. This approach has two drawbacks. First, the size of the state space in that exercise makes this strategy too onerous. Second, bond yields may potentially compromising the interpretation of the filtered volatility in government spending and capital tax rate. On the other hand, our approach disciplines the stochastic volatility to fit the observed government spending and capital tax rate data only.

Quintana, Kuester, and Rubio-Ramírez (2015), we estimate Eqs. (7) and (8) for each fiscal instrument separately, and we set the means in equation (7) to each instrument’s average value (see Table 1-Panel A). We estimate the rest of the parameters following a Bayesian approach by combining the likelihood function with uninformative priors and sampling from the posterior with a Markov Chain Monte Carlo.¹² Table 1-Panel B reports the posterior median for the parameters along with 95 percent probability intervals. Both tax rates and government spending as a share of output are persistent. E.g., the half-life of government spending is around $-\log(2)/\log(0.97) = 22.75$ quarters. Deviations from average volatility last also for some time. The $\epsilon_{x,t}$ s have an average standard deviation of $100 \times \exp(-4.987) = 0.68$ and $100 \times \exp(-6.172) = 0.21$ percentage point for tax and government spending, respectively. A positive one-standard-deviation innovation $\epsilon_{\sigma,t}$ increases the standard deviation of the innovation to the fiscal shock to about $100 \times (\exp(-4.987 + 0.230)) = 0.86$ and $(100 \times \exp(-6.172 + 0.112)) = 0.23$ percentage point for tax and government spending, respectively. These results are in line with Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) (see in particular their Table 1).

Table 1–Panel C reports the estimates of the structural parameters in our model.

[Insert Table 1 about here.]

The estimation assign a relatively high value of 0.999 to β . This value is needed in order to obtain a sufficiently low mean value for the one-period nominal interest rate. The parameter γ is estimated to be 150. Since the representative agent in the model can earn labor income as a mean to smooth consumption, his/her attitude toward risk is different than those who do not supply labor. Following Swanson (2012), we adjust the risk aversion parameter by taking into account the labor margin using the closed-form formula $\frac{\psi}{1+\frac{\psi}{\omega\nu}} + \frac{\gamma-\psi}{1-\frac{1-\psi}{1+\omega}}$ with $\nu = \frac{\eta}{\eta-1}$. The representative saver’s true coefficient of relative risk aversion is therefore ≈ 52 . This may seem like a high value; however, other studies using Epstein-Zin preferences also typically

¹²Specifically, for government spending we adopt a beta distribution for ϕ_g^g and ϕ_g with mean 0.8 and 0.85 respectively, a uniform distribution between -11 and -3 for θ_g^g , and an inverse gamma for σ_g^g with mean 0.1. Correspondingly, for capital tax we use a beta distribution for ϕ_{σ^k} and ϕ_{τ^k} with mean 0.85 and 0.8 respectively, a uniform distribution between -8 and -3 for θ_{σ^k} , and an inverse gamma for σ_{σ^k} with mean 0.2.

estimate a high coefficient of relative risk aversion: Piazzesi and Schneider (2007) estimate a value of 57, van Binsbergen, Fernández-Villaverde, Koijen, and Rubio-Ramírez (2012) a value of about 66, and Rudebusch and Swanson (2010) a value near 110.

The estimation procedure picks a low value for the IES, ≈ 0.23 . The low value for the IES helps to make consumption less volatile and real interest rates more volatile, both of which improve the fit to the macro moments in the data. The higher interest rate volatility also increases bond price volatility and improves the model's fit to the finance moments. The monetary policy rule coefficients on inflation, ρ_π , is slightly higher than the typical value of 1.5 used in the literature but still consistent with the estimates in Smets and Wouters (2007). The shock persistence and variance for transitory productivity, ϕ_z and σ_z , are also in line with, e.g., the estimates in Smets and Wouters (2007).

5.2 Model's fit

Given our GMM estimates, how well does the model fit the data? We address this question by comparing a set of statistics implied by the model to those measured in the data. In particular, we study the mean and standard deviation of the seven observable variables included in the estimation, as well as of the long-term, 10-year yield. We also look at the autocorrelations of these set of variables.

Table 2 report various model-implied moments, along with the corresponding empirical moments for quarterly US data from 1970.Q1 to 2007.Q3. Panel A displays results for our baseline model with fiscal uncertainty. Panel B displays results for the model where we shut down the stochastic volatility channel. The table reports the median and the 90 percent probability intervals that account for both parameter and small-sample uncertainty.¹³

[Insert Table 2 about here.]

¹³We draw the structural parameters from a Normal distribution with a variance-covariance matrix obtained from our second step GMM estimation procedure. The parameters governing the processes for the fiscal instruments are obtained from the posterior distribution reported in Table 1-Panel B. For each parameter draw, we generate an artificial sample of the observable variables with same length as our data-set (152 observations) after discarding 50 initial observations.

Our benchmark model matches the mean and variability of the short-term rate and the slope for the nominal term structure (all values fall within the 90% confidence interval). In particular, the model is able to produce a sizable slope of 1.6%, a value almost identical to the empirical one. The model also fits the mean of the long-term 10-year rate, but it falls short when it comes to explain its variation. The good fit on the financial side is obtained while simultaneously matching key moments for standard real macro variables. In particular, the model slightly over-predicts the volatility of output but it matches fairly well the standard deviation of consumption and investment as well as the mean of all the macro variables (output, investment, consumption and inflation).

It is important at this point to recall that the unconditional mean of a model solved with higher-order terms generally differs from its steady-state value. Our results for unconditional mean in Table 2 correct for third-order effects and, thus, account for the non-linearity in the model. The contribution of second-order effects to the mean of the variables in the pruned approximation is particularly important for financial variables, and only to a lesser extent for macroeconomic quantities.¹⁴ As an example, the median short rate as implied by our 2nd-step GMM parameter estimates when we approximate the policy functions with first-order terms only, is about 15.5% (with 90% confidence bands at 10% and 18%); this is a much higher value than the one obtained in the pruned approximation that accounts for second- and third-order effects (see again Table 2 – Panel A).¹⁵

With respect to term premium, the model is overall quite successful in reproducing the mean 10-year term premium of about 2%. In terms of second moments, the model is able to account for up to $.210/.544 \approx 39\%$ of the term premium unconditional standard deviation.

It is important to observe that the good properties of the model with respect to the volatility of the short rate, slope, and term premium are generated by stochastic volatility in fiscal instruments and not by a higher variance in the shocks to the policy instruments.

¹⁴The unconditional mean of the state vector is not further corrected by the third-order effects when, as in our case, all innovations have zero third moments.

¹⁵The results for the moments of the baseline model approximated to the first- and second-order are available upon request.

To further substantiate this statement and to explore the effects of stochastic volatility, Table 2–Panel B considers the benchmark model without time-varying volatilities in fiscal policies. In particular we set the unconditional variance in shocks to government spending and capital tax rate $\sigma_{x,t+1} = \sigma_x = \theta_\sigma^x + \sigma_\sigma^x$, with $x \in \{g, \tau^k\}$. In this way the unconditional variance in fiscal instruments is comparable to the specification of our benchmark model with stochastic volatility. Comparing Panel B with Panel A, we see that many of the moments are not affected by the presence of stochastic volatility: e.g., the mean level of the term premium and the slope of the term structure are almost unchanged by stochastic volatility. A key difference, however, is a sizable decrease in the standard deviation of the short-rate, the slope for the nominal term structure, and the term premium. Quantitatively, a model without fiscal uncertainty is not able to match the variability in the short-rate and the slope for the nominal term structure (the model-implied 90% confidence intervals do not include the data values). With respect to bond risk premium, a model without time-varying uncertainty induces a decrease in term premium volatility from 9 basis points (see benchmark model, Panel A) to 2 basis points, and it is able to account for at most $.073/.544 \approx 13\%$ of the overall term premium volatility. Finally, not only a slightly higher variance in fiscal shocks does not generate good properties in terms of second moments for the financial variables but also it generates a too volatile investment series. Overall, time-varying volatility in fiscal shocks seem to be an important driver for variation in the U.S. yield curve and term premia.

Returning to our benchmark case with fiscal uncertainty, we observe that, in terms of autocovariances, the model strikes a balance between matching the high persistence of financial variables and the low persistence of macroeconomic quantities.¹⁶ Table 2 reports the numerical value of the first-order autocorrelation coefficient for all the quantities in the model while Figure 1 displays the entire autocovariance function of the data (black line) and the model (red line), along with the 90 percent intervals that account for parameter and small-sample uncertainty within the model. The Figure includes all the observable quantities used to estimate the model, as well as the long-term, 10-year yield. We see that the model captures the decaying autocorrelation structure of these eight variables reasonably well. The

¹⁶These moments are not used in the estimation and constitute an out-of-sample test of the model’s fit.

success is particularly impressive for the long-term rate and the term premium, for which the data and model-implied autocorrelations lie virtually on top of each other. The model does a fairly good job for output, consumption, and inflation, but does not capture the full extent of the persistence in investment growth, and to a lesser extent, in the nominal short-term interest rate.

[Insert Figure 1 about here.]

The model also generates a reasonable cross-correlation for consumption and term premia. Figure 2 displays the cross-correlation function between the term premium and Δc_{t-k} , a function of the lag k . For negative values of k , this is the correlation of the financial indicator with future economic growth. If these correlations are nonzero, we say the financial indicator leads the business cycle. The correlations on the left show that, in the data, the term premia is positively correlated with future economic growth up to five year in the future. This cross-correlation is positive and quite persistent in our model as well. The correlations on the right are generally smaller, i.e. the data also suggest essentially zero values for the correlations of the term premia with past economic growth. Consistently with this fact, our model assigns a higher degree of uncertainty (i.e. the confidence bands widen) to the correlations on the right (i.e. the relation between the term premia and past consumption growth) as compared to the ones on the left (i.e. the relation between term premia with future economic growth).¹⁷

[Insert Figure 2 about here.]

¹⁷We could have replaced consumption with output or the term-premium with slope and little changes. The consumption/term premium figure is a bit sharper and output/slope less so. Also, although the correlations in the data are modest individually (the largest is about 0.23) they exhibit a clear pattern. See also Backus, Routledge, and Zin (2010) for further evidence that a variety of equity and bond portfolios lead the business cycle. Finally, following Backus, Routledge, and Zin (2010), Figure 1 displays the cross correlation of quarterly year-on-year growth rates and term premia; this is a crude approximations to the Hodrick-Prescott filter often used in business cycle analysis. The correlations are larger and smoother if we use year-on-year growth rates. However, the raw data would not change the conclusion.

5.3 Impulse responses

A large literature in financial economics finds that bond risk premia are substantial and vary significantly over time (see Campbell and Shiller (1991) and Cochrane and Piazzesi (2005)); however, the economic forces that can justify such large and variable term premium are less clear. In this section, we shed some light on this issue by examining the model's impulse responses to shocks. Importantly for our analysis, the pruned method applied to higher-order approximations of DSGE models yields closed-form expressions not only for first and second unconditional moments but also for impulse response functions (see Andreasen et al. , 2016).

To understand the role of shocks for the term premium Figure 3 shows the impulse responses of consumption, inflation, long-term bond yield, and the term premium to a positive one-standard-deviation shock to government spending level (column 1) and volatility (column 2), and to capital tax rate level (column 3) and volatility (column 4); Figure 4 shows the impulse responses to monetary policy (column 1), transitory and permanent productivity shocks (columns 2 and 3, respectively). Each panel reports the impulse responses obtained by considering only the linear terms (solid black line) as well as the impulse responses including second- and third-order terms (red line). At the outset, it is interesting to observe that first-order terms are the dominant ones for the impulse responses of macroeconomic quantities (i.e. consumption and inflation) to transitory and permanent productivity shocks (see columns 3 and 4 in Figure 4): the solid black line shows that the first order terms provide a very good approximation, indeed. However this is not anymore the case for government spending level, capital tax rate, and, to a lesser extent, for monetary shocks: in this case the linear terms often touch or even lie outside the confidence bands for the third-order approximation. Also, for the term premium, the impulse response is zero to first and second order, so the third-order terms are dominant. Similarly, non-linear terms are necessary to study impulse responses to fiscal uncertainty shocks (see columns 2 and 4 in Figure 3).

[Insert Figures 3 and 4 about here.]

There are two main takeaways from Figures 3 and 4. First, government spending volatility shocks are the main driver of variability in the nominal term premium. By looking at the last row, we see that government spending volatility shocks induce the largest fluctuations in term premium; fluctuations induced by capital tax (level and volatility) as well as monetary shocks are puny. Both government spending level and volatility shocks demand a positive, and quite persistent risk premium. To understand the behavior of the overall term premium we look at the covariance between the stochastic discount factor and the long-term bond price (i.e. the first and third rows in Figure 3). Both level and volatility shocks to government spending imply a negative covariance between the stochastic discount factor and the long-term bond price, and hence a positive term premium.

It is important to recall that the overall term premium can be decomposed into inflation risk premium and real term premium. Next, we investigate the behavior of inflation risk premium in our model. To understand the sign of inflation risk premium, we analyze the behavior of consumption and inflation in response to a shock. The second key and novel result conveyed by Figure 3 is that the relationship between consumption and inflation depends critically on the nature of the underlying fiscal shocks: government spending level shocks imply a negative correlation between consumption and inflation; government spending uncertainty shocks imply exactly the opposite relation. Therefore, in our model, an increase in government level spending implies that inflation is high exactly when savers wish to consume more; but high inflation makes payoffs on nominal bonds low in real terms, and the *positive* covariance between marginal utility of consumption and inflation generates positive inflation risk premia. On the other hand, following a positive government uncertainty shock, consumption and inflation move in the same direction, which in turn delivers a negative inflation risk premia. The fact that fiscal volatility shocks command a positive nominal term premium despite the negative inflation risk premium suggests that the real term premia account for the bulk of variations of nominal term premia. This finding is consistent with Abrahams, Adrian, Crump, and Moench (2015).¹⁸ We will return to this fact in Section 5.5.

¹⁸Abrahams, Adrian, Crump, and Moench (2015) present an affine term structure model for the joint pricing of Treasury Inflation-Protected Securities (TIPS) and Treasury yield curves. Their baseline specification with six principal components extracted from Treasury and TIPS yields, in combination with a liquidity factor, generates

Figure 4 shows the impulse responses to monetary, transitory and permanent productivity shocks. In short, these responses are in line with the literature. Column 1 shows the impact of a one-standard deviation positive shock to monetary policy. The monetary policy shocks cause consumption and inflation to fall, generating negative inflation risk premium. However, the monetary policy shock generates minimal fluctuations in the nominal term premium, see the fourth subplot in column 1. Column 2 of Figure 3 shows that an increase in the transitory technology shock causes consumption to rise while causing inflation to fall, generating positive inflation risk premium. The fall in inflation pushes up nominal bond prices. Nominal bonds are risky because they lose value at exactly those times when the marginal utility is the highest. This mechanism has been highlighted by Piazzesi and Schneider (2007), and our results for transitory technology shocks are consistent with the finding of Rudebusch and Swanson (2008). Finally, the bottom panels in columns 2 and 3 show the time variation in term premium due to productivity shocks. A positive transitory productivity shock decreases the overall term premium while a positive permanent productivity shock elevates the term premium.

We conclude this section by quantifying the relative contribution of each shock to the moments of macroeconomic and financial variables. To this end, we Table 3 reports the (generalized) forecast error variance decomposition (see Lanne and Nyberg (2016)) obtained from our model.¹⁹ Similar to Table 2, we account for both parameter and small-sample uncertainty.

[Insert Table 3 about here.]

Table 3 shows that transitory productivity shocks are a key determinant of the volatilities for macroeconomic quantities; shocks to the *level* of government spending and capital tax rate represent an important important driver for output and investment standard deviation, respectively. Turning to the term premium it is shocks to the *volatility* of government spend-

negligibly small pricing errors for both real and nominal yields. For the contribution of real term and inflation risk premia to the overall term premium see Figure 7 in their paper.

¹⁹We report the GFEVDs for a forecast horizon $h = 20$ quarters. Our conclusions do not qualitatively change when we consider horizons $h = 4$, or $h = 12$ quarters.

ing that drive the variation in the model, however. This result differs from Rudebusch and Swanson (2008) who instead argue for a key role of transitory technology shocks. The Table also shows the importance of computing term premium as the difference between the observed yield to maturity on the bond and the risk-neutral yield to maturity. Had we approximated the term premium with the slope of the yield curve, i.e. with the difference between the yield to maturity on the long-term bond and the one-period risk-free rate, we would have attributed a more prominent role to productivity, monetary and fiscal policy level shocks at the expense of government spending volatility ones. The reason for this is that the slope is an imperfect measure of the riskiness of the long-term bond because it can vary in response to shocks even if all investors in the model are risk-neutral.

The results so far support the view that shocks to government spending volatility are the main source of *variation* in term premium. With respect to the *level* of the slope and term premium, in untabulated results, we observe that permanent productivity shocks counteract the effect of transitory productivity shocks, consistent with Hsu, Li, and Palomino (2015). Monetary and capital tax rate shocks are puny. Shocks to transitory productivity and government spending are the key drivers of the average term-premium in our model.

To summarize, we find that stochastic volatility in government spending shocks can generate sizable variation in the term premium without distorting the ability of the model to match key macroeconomic moments.

5.4 Inspecting the mechanism

We expand on the importance of fiscal shocks in our model by examining the impulse responses of other endogenous variables. They are presented in Figures 5 and 6 for government spending level and volatility shocks, and in Figures 7 and 8, for capital tax rate level and volatility, respectively.

[Insert Figures 5 about here.]

In Figure 5, following a positive government spending level shock, output rises but consumption falls as the optimizing households saves more via government debt while anticipating higher taxes in the future. The negative wealth effect causes labor supply to increase for both savers and spenders immediately after the positive spending shock is realized. The precautionary savings motive also pushes returns on capital higher as government bonds are preferred over capital investment for consumption smoothing. At the same time, the positive spending shock drives up demand for final goods, and firms increase labor input while decrease investment in the short run.

[Insert Figure 6 about here.]

Volatility shocks to government spending, on the other hand, have different effects on economic aggregates than the level shocks. In Figure 6, following a positive volatility shock, consumption declines, but debt level also drops. Increased uncertainty about future government spending makes debt less attractive than capital investment for consumption smoothing. Unlike a positive level shock, a positive volatility shock causes returns on capital to fall, making capital input relatively cheap for firms in producing final goods. This puts downward pressure on labor demand and real wages. As both marginal costs of capital and labor diminish, inflation declines.

The impulse response functions to a volatility shock of the capital income tax rate are broadly in line with the findings of Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015): output, consumption, investment, hours, the real wage, and the short rate (not reported) significantly fall after a positive standard-deviation innovation to tax rate volatility.

[Insert Figures 7 and 8 about here.]

Next, we investigate the role played by rigidities in our model. In particular, Table 4 reports the median of the moments implied by a model without price rigidities (column (3)),

a model without wage rigidities (column (4)), and a fully flexible model (column (5)). The value of the structural parameters is fixed at our GMM estimates in all cases.

[Insert Table 4 about here.]

We first focus on the trade-off implied by wage rigidities. Compared with our baseline model (see column (2)), a model without wage rigidities (see column (4)) would increase the mean of the term premium (see Panel A), and it would allow for a lower level of risk aversion. A model without wage rigidities would also achieve a better fit in terms of financial variables volatility. In particular, a lower degree of wage rigidities increases the volatility of short- and long-rates, as well as that of term premium (see Panel B). However, the presence of wage rigidities is necessary to achieve a good fit in terms of mean and standard deviation of the slope, as well as a good fit for the investment volatility. Wage rigidities are also key to generate a persistent series for inflation, short rate, and slope (see Panel C). In terms of first moment (see Panel A), the models with and without rigidities (columns 3 to 5) have all very similar implications for output, consumption and investment, as well as for long rates. Also, rigidities do not significantly influence the persistence of long-term rates and of the term premium. Comparing price and wage rigidities, we see that they mostly act in opposite direction: absence of wage rigidities increases the volatility of macroeconomic and financial variables, whereas absence of price rigidities dampens the standard deviations. Analogously, when we look at the first moment, the presence of wage rigidities decreases the average term premium and slope, whereas price rigidities (together with risk aversion) helps to increase the term premium and the slope.

5.5 The Importance of Fiscal Shocks for the Term Premium

Ex ante, productivity, fiscal and monetary policy shocks could all be very important drivers of the term premium. Table 3 has already highlighted that, ex post, government spending volatility shocks turn out to be the key driver of variation in term premium within our model. To further substantiate our claim that government spending shocks represent a

key determinant of the term premium, we feed our model with the filtered shocks from the estimated government spending dynamics, see Eqs. (7) and (8). Figure 9 presents the model's prediction for the term premium, together with the actual term premium. The left Panel presents the term premium implied by our model fed with only shocks to the government spending level, whereas the right Panel presents the premium implied by our model fed with level and volatility shocks to the government spending.²⁰

[Insert Figure 9 about here.]

The figure shows how the government spending level shocks make the model able to track the average term premium whereas the volatility shocks helps in capturing the variability of the term premium. Also, the interaction of shocks to level and volatility captures the trending down in the late 90s. To our eyes, (the fiscal shocks in) the model provides a tantalizing account of the cyclical and longer-term fluctuations in the term premium.

We also quantify the relative contribution of real and inflation risk premia to the overall nominal compensation. Figure 10 shows the result. The Figure superimposes the nominal and real term premium, as well as their difference, the inflation risk premium.

[Insert Figure 10 about here.]

The right chart shows that (at low and intermediate frequencies) real term premia account for the bulk of variations of nominal term premia. The estimated real term premium declined from about 1.5 percent to around one percent in 2007. Instead, inflation risk premia mostly capture a level effect in nominal term premia. Hence, the compensation investors require for bearing real interest rate risk – the risk that real short rates don't evolve as they expected – is the main driving force behind movements in nominal term premia according to our model.

²⁰We estimate the parameters in Eqs. (7) and (8) following a Bayesian approach. The particle filter delivers draws for the shocks. We feed each draw into the model; then, we compute the median and 95 percent probability intervals for the model-implied premium.

5.6 Policy Experiment

We perform policy analysis to examine the impact of fiscal and monetary policy rules on the first and second moments of the nominal term premium in this section. Specifically, we focus on the three parameters: the fiscal policy response to debt (ρ_b), the fiscal policy response to government spending (ρ_g), and the monetary policy response to inflation (ρ_π). We plot the comparative statics of these three parameters in Figure 11, where the top subplot is for ρ_b , the middle subplot is for ρ_g , and the bottom subplot is for ρ_π .

[Insert Figure 11 about here.]

We make the following observations in Figure 11. First, when the fiscal authority responds aggressively to repay real debt by increasing ρ_b , tax collection goes up and new debt issuance declines. As a result, the unconditional mean and volatility of term premium decrease as bond supply drops. Second, stronger fiscal policy response to government spending by increasing ρ_g leads to higher average term premium but negligibly lower variability of said term premium. Financing of government spending through taxes as opposed to debt affects the intertemporal consumption substitution of the savers. Because they are Ricardian, the savers would expect lower taxes and higher consumption in the future under rational expectations. Thus, they would want to consume more now and demand less Treasury debt, resulting in higher average term premium.

Third, consistent with the findings of Hsu, Li, and Palomino (2015), robust inflation response by the monetary authority, with higher ρ_π in the Taylor rule, generates greater level of average term premium while making inflation and inflation risk premium more stable. The stability of inflation risk premium is reflected in the decline of nominal term premium volatility with ρ_π in the bottom subplot in Figure 11. On the other hand, stronger inflation response also signals higher expected consumption growth for a given realization of shocks, *ceteris paribus*. Therefore, households have the incentive to consume now rather than later, causing real and nominal yield curves to be more upward-sloping and term premia to be

higher.

6 Conclusion

Recent literature has highlighted that fiscal policy shocks and fiscal volatility shocks have first order effects on economic activity. The link between fiscal policy and the term structure of interest rates, however, is less well established.

To fill this gap, we propose and estimate a New-Keynesian model with heterogeneous agents to study the effects of fiscal policy on the term structure of interest rates and associated risk premia. We focus on two specific aspects of fiscal policy: shocks to government spending level and shocks to government spending uncertainty. The agents' heterogeneity in the model is needed to break Ricardian equivalence, and, following Mankiw (2000), it is implemented by introducing a class of agents with limited market participation.

We document that our DSGE model is successful in matching both basic macroeconomic and financial moments in the data. Importantly for our purpose, the model is quite successful in reproducing the average 10-year term premium, as well as its dynamic properties as captured by the autocorrelation function. Stochastic volatility in government spending allows to capture up to 36% of the overall term premium variability, whereas a model with no stochastic volatility would account for at most 15% of the term premium volatility. Not only fiscal level and volatility shocks produce a large and variable term premium; fiscal shocks also emerge within the model as the largest source of term premium fluctuations, and drive away other potential drivers of the term premium such as productivity and monetary shocks.

Through the lens of the model we also show that the relationship between consumption and inflation depends critically on the nature of the underlying fiscal shocks: government spending level shocks imply a negative correlation between consumption and inflation; government spending uncertainty shocks imply exactly the opposite relation. Since the empirical relation between consumption and inflation was large and negative in the 1970s and early

1980s, but much smaller in the 1990s and 2000s (see Piazzesi and Schneider (2007) and Benigno (2007)), our finding suggests that the relative importance of transitory technology and government level shocks may have been larger in the 1970s and early 1980s than over the rest of the sample where monetary and government spending uncertainty shocks may have become dominant.

In all, we view our estimated DSGE model as an important step forward to understand what state variables drive variation over time in bond risk premia. Our findings speak to the key role played by shocks to the level and the uncertainty about fiscal policy.

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Figures

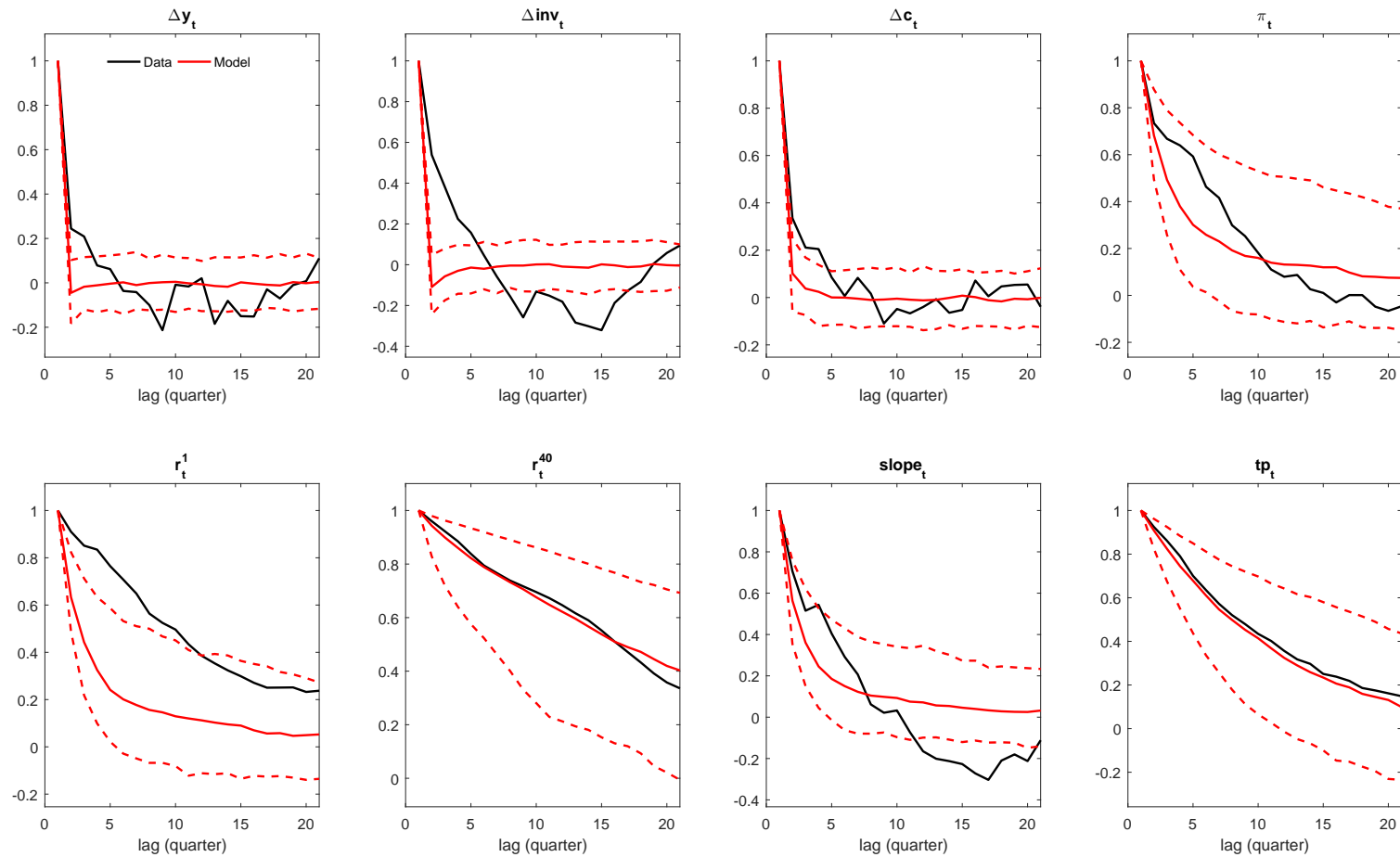


Figure 1: **Autocorrelation Functions**

Autocorrelation function of the observable variables in the baseline model and the data. The black line is the data. The solid red line is the model's median and the dashed lines are the model's 5th and 95th percentiles. The sample period for the data runs from 1970.Q1 to 2007.Q4.

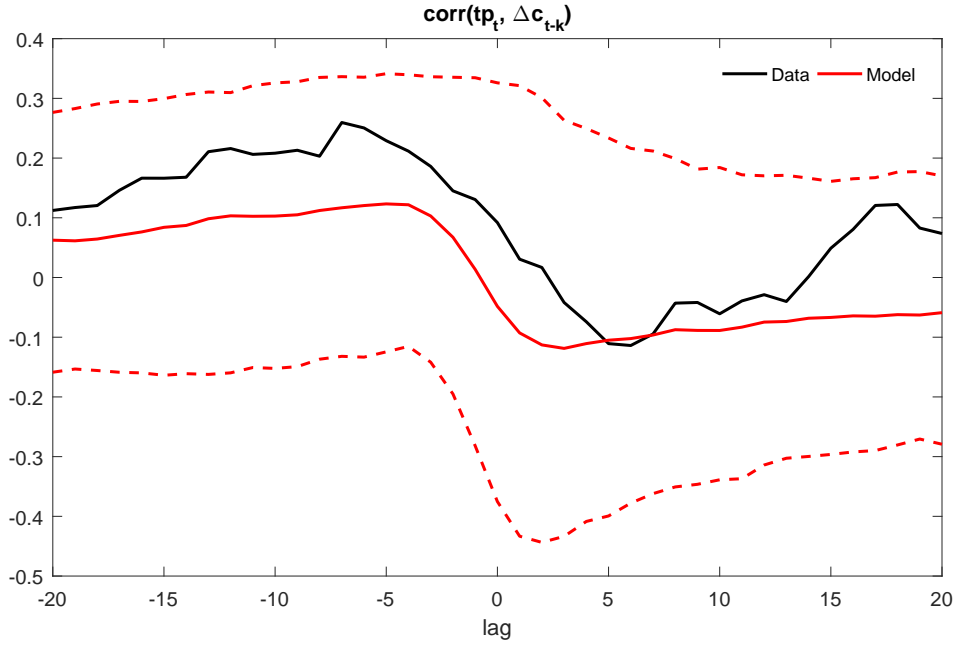


Figure 2: **Cross-Correlation Function - Consumption Growth and Term Premium**

This figure plots the cross-correlation function between the term premium and Δc_{t-k} , a function of the lag k , in the baseline model and the data. The black line is the data. The solid line is the model's median and the dashed lines are the model's 5th and 95th percentiles. The sample period for the data runs from 1970.Q1 to 2007.Q4.

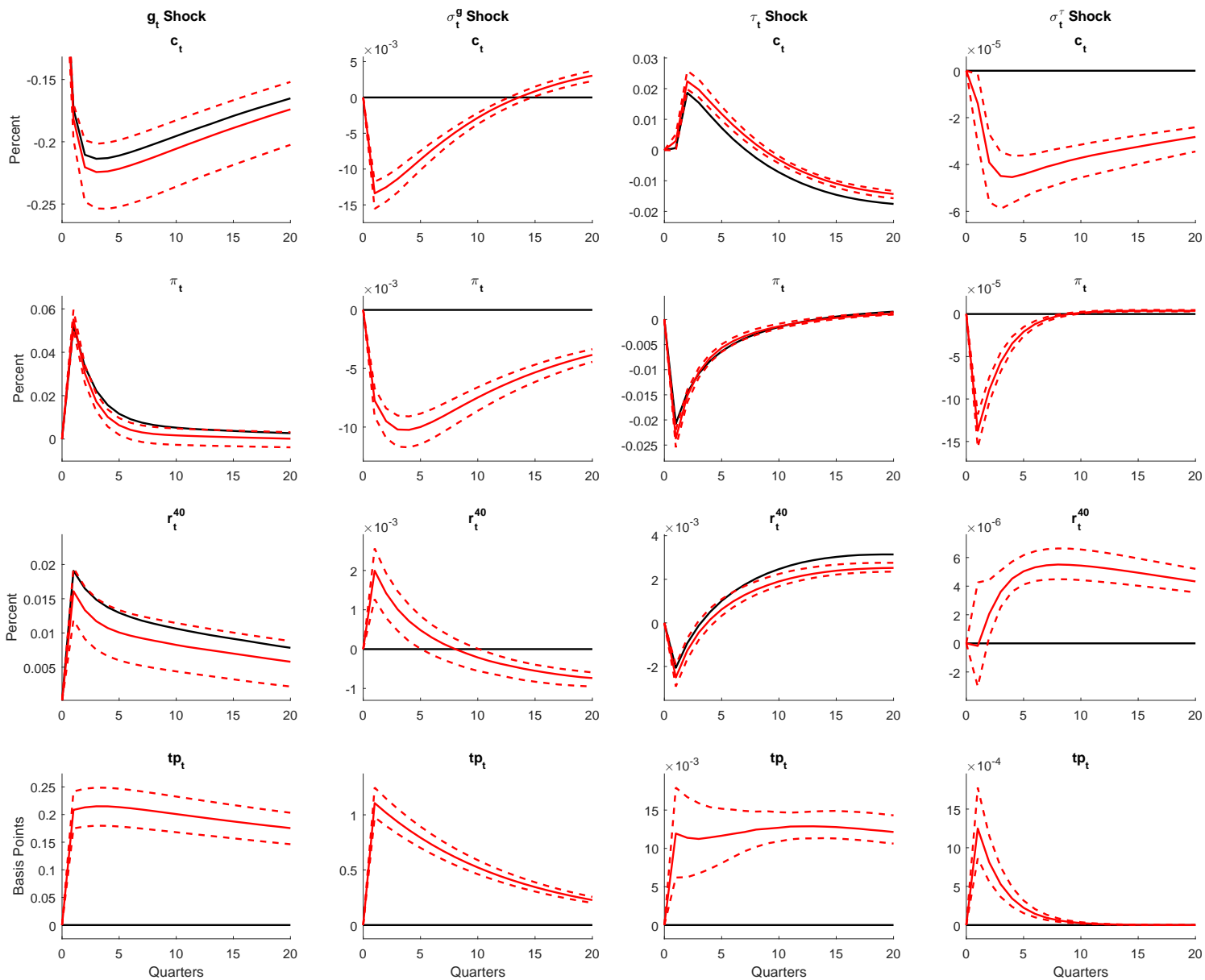


Figure 3: Unconditional Impulse Responses to Structural Shocks

This figure plots the impulse responses of consumption, inflation, long-term term bond yields and the term premium to positive one standard deviation shocks to government spending level (g_t), government spending volatility (σ_t^g), capital income tax level (τ) and capital income tax volatility (σ_t^τ).

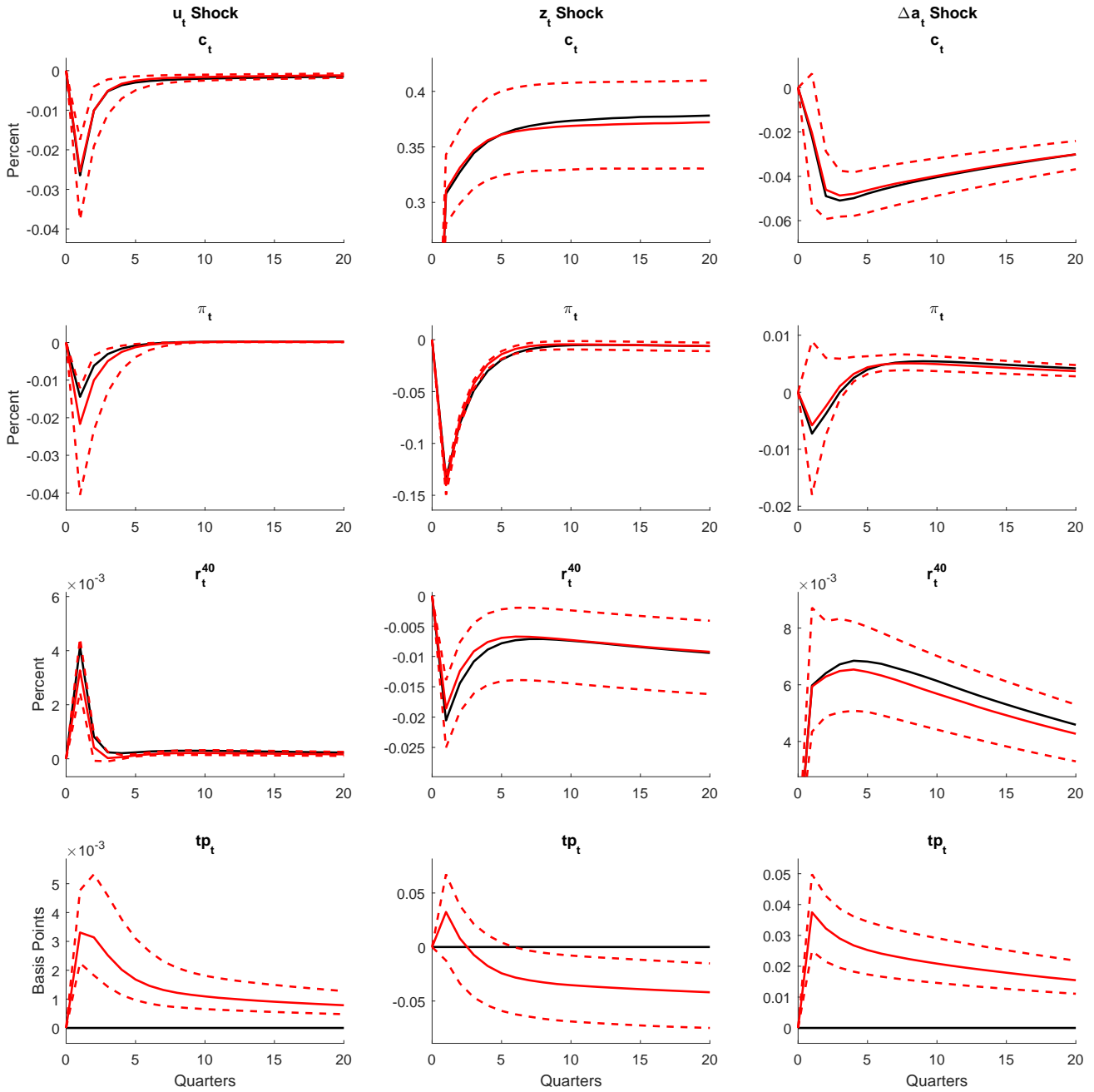


Figure 4: **Unconditional Impulse Responses to Structural Shocks**

This figure plots the impulse responses of consumption, inflation, long-term term bond yields and the term premium to positive one standard deviation shocks to monetary policy (u_t), transitory productivity to a one standard deviation transitory productivity (z_t) and permanent productivity (Δa_t).

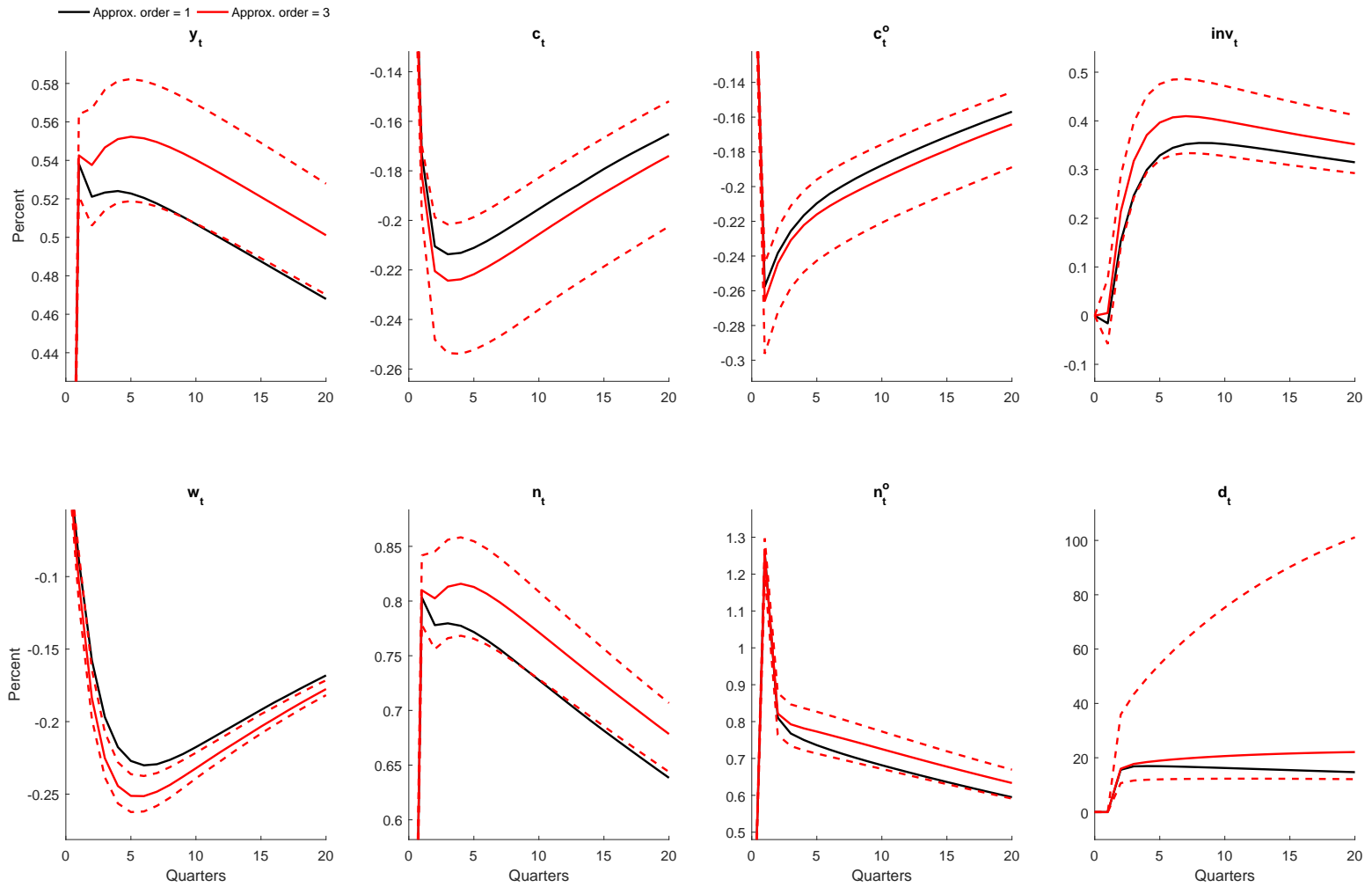


Figure 5: Unconditional Impulse Response Function - Government Spending Shock

This figure plots the impulse responses to a one standard deviation government spending level shock. The solid line is the median, while the dotted lines are the 5th and 95th percentiles.

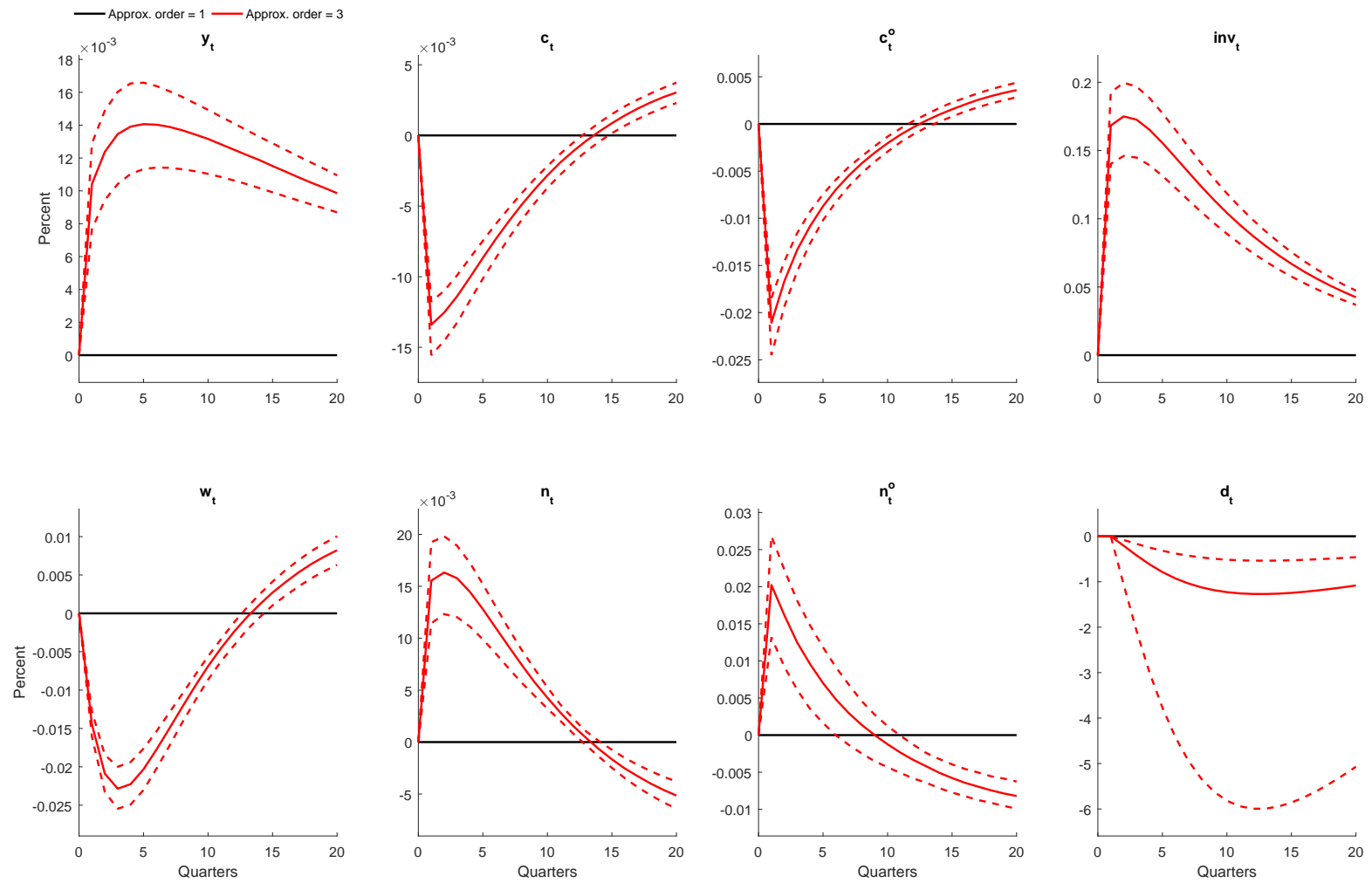


Figure 6: Unconditional Impulse Response Function - Government Spending Uncertainty Shock

This figure plots the impulse responses to a one standard deviation government spending uncertainty shock. The solid line is the median, while the dotted lines are the 5th and 95th percentiles.

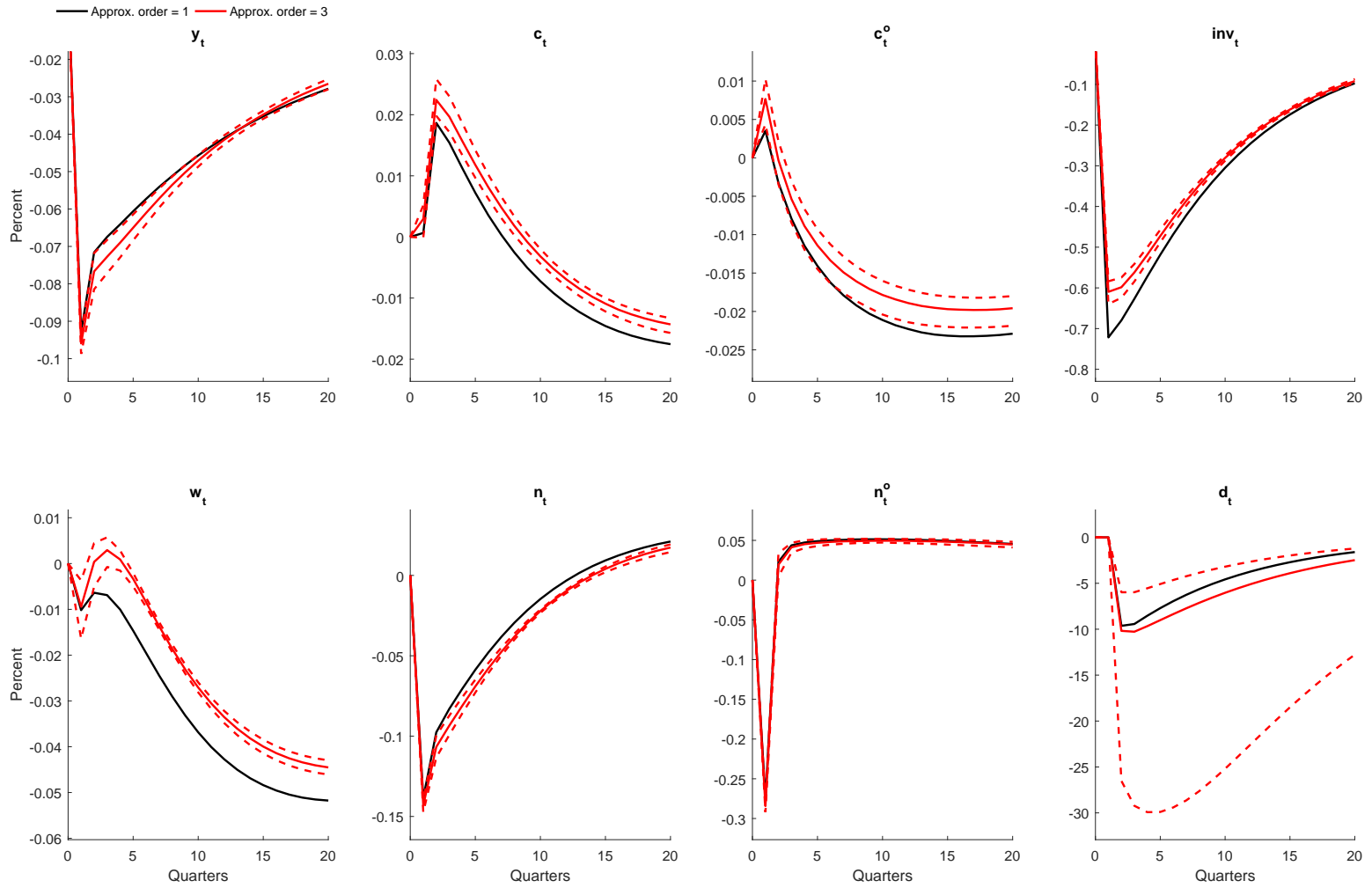


Figure 7: Unconditional Impulse Response Function - Capital Income Tax Shock

This figure plots the impulse responses to a one standard deviation Capital Income Tax level shock. The solid line is the median, while the dotted lines are the 5th and 95th percentiles.

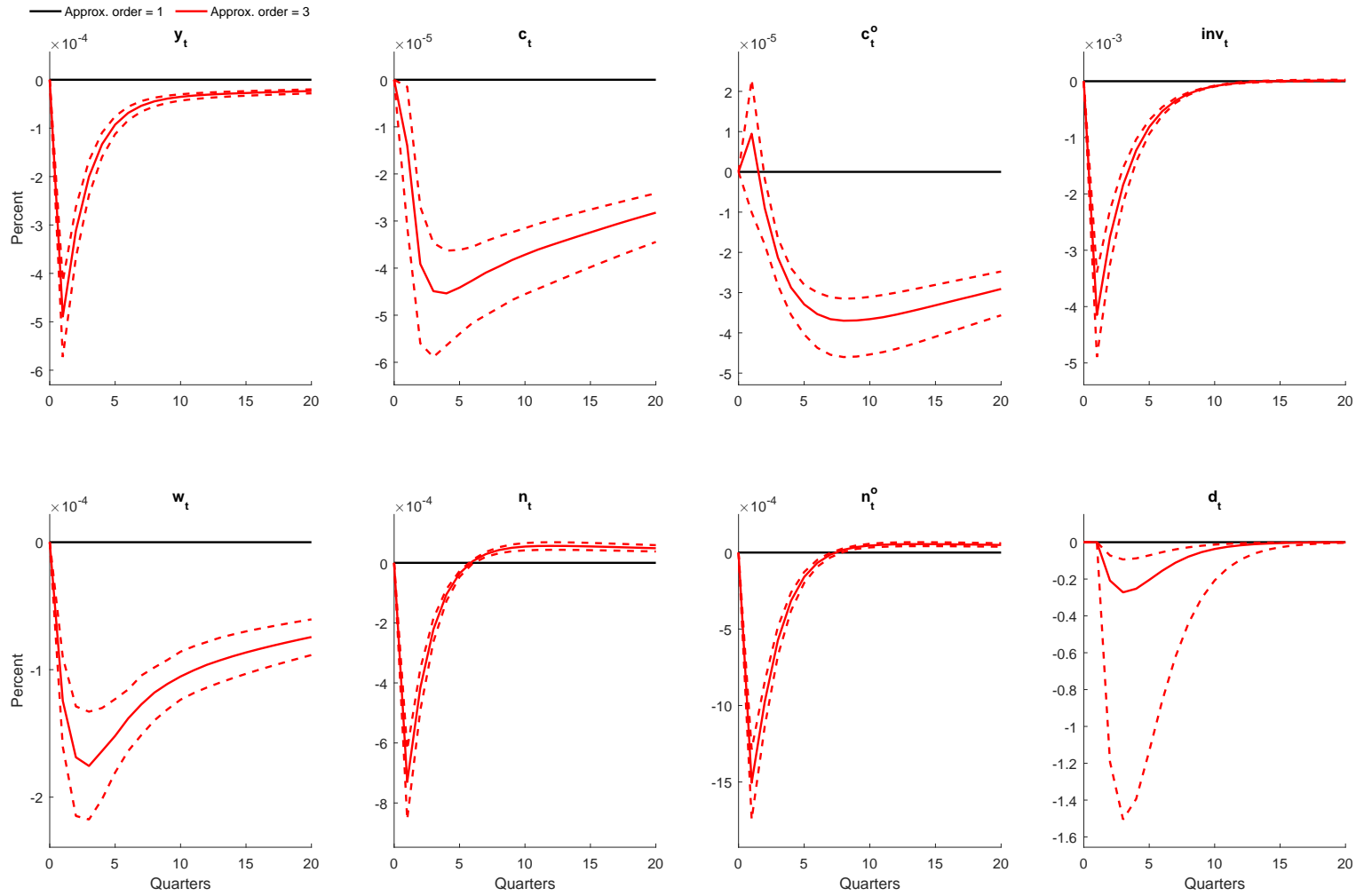


Figure 8: Unconditional Impulse Response Function - Capital Income Tax Uncertainty Shock

This figure plots the impulse responses to a one standard deviation Capital Income Tax uncertainty shock. The solid line is the median, while the dotted lines are the 5th and 95th percentiles.

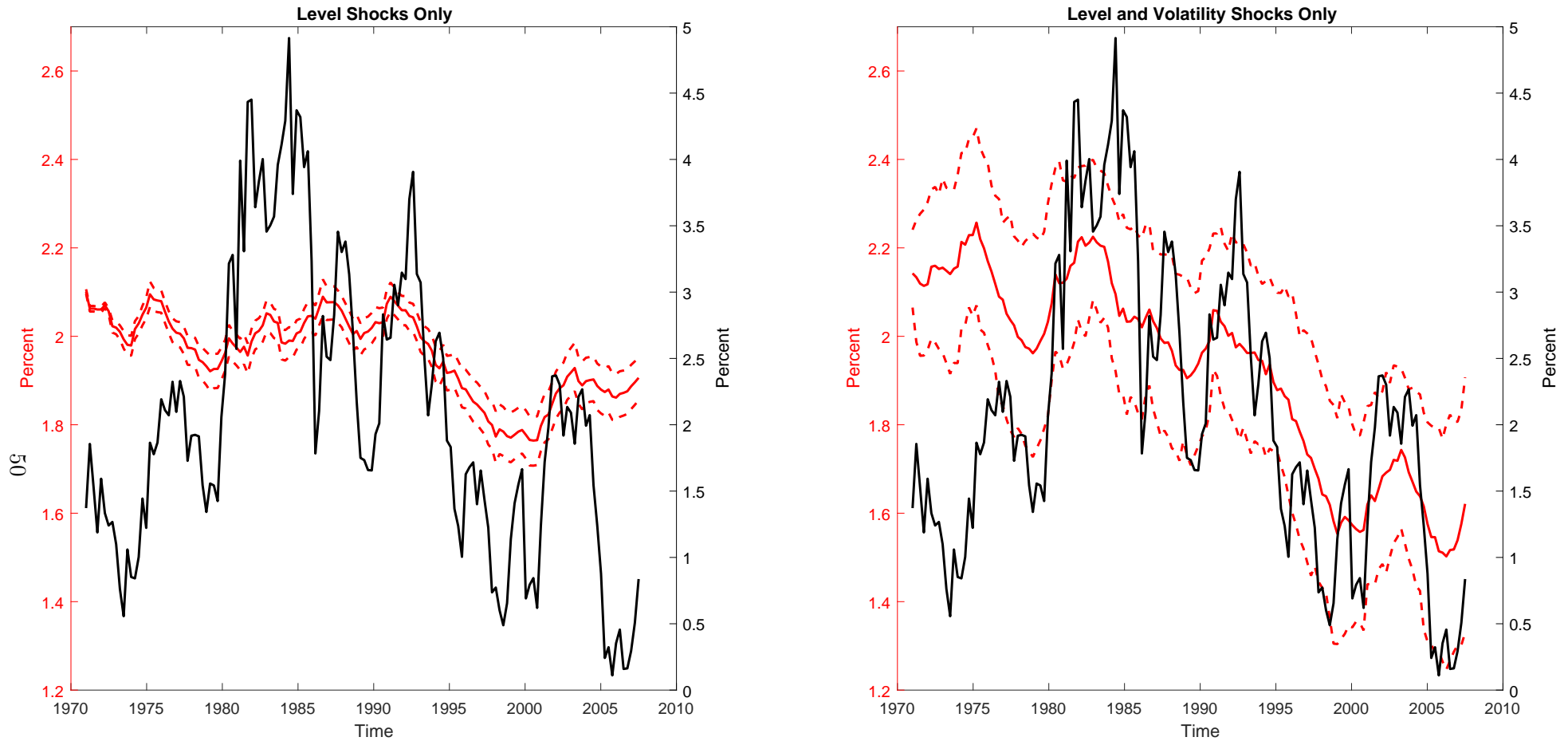


Figure 9: **Term Premium - Counterfactual Analysis**

This figure plots the model-implied term premium against the actual term premium for the period from 1970.Q1 to 2007.Q4. The solid red line is the median, while the dotted lines are the 5th and 95th percentiles. The correlation between the data and the model-implied term premium is 0.50 in the left panel and 0.54 in the right panel.

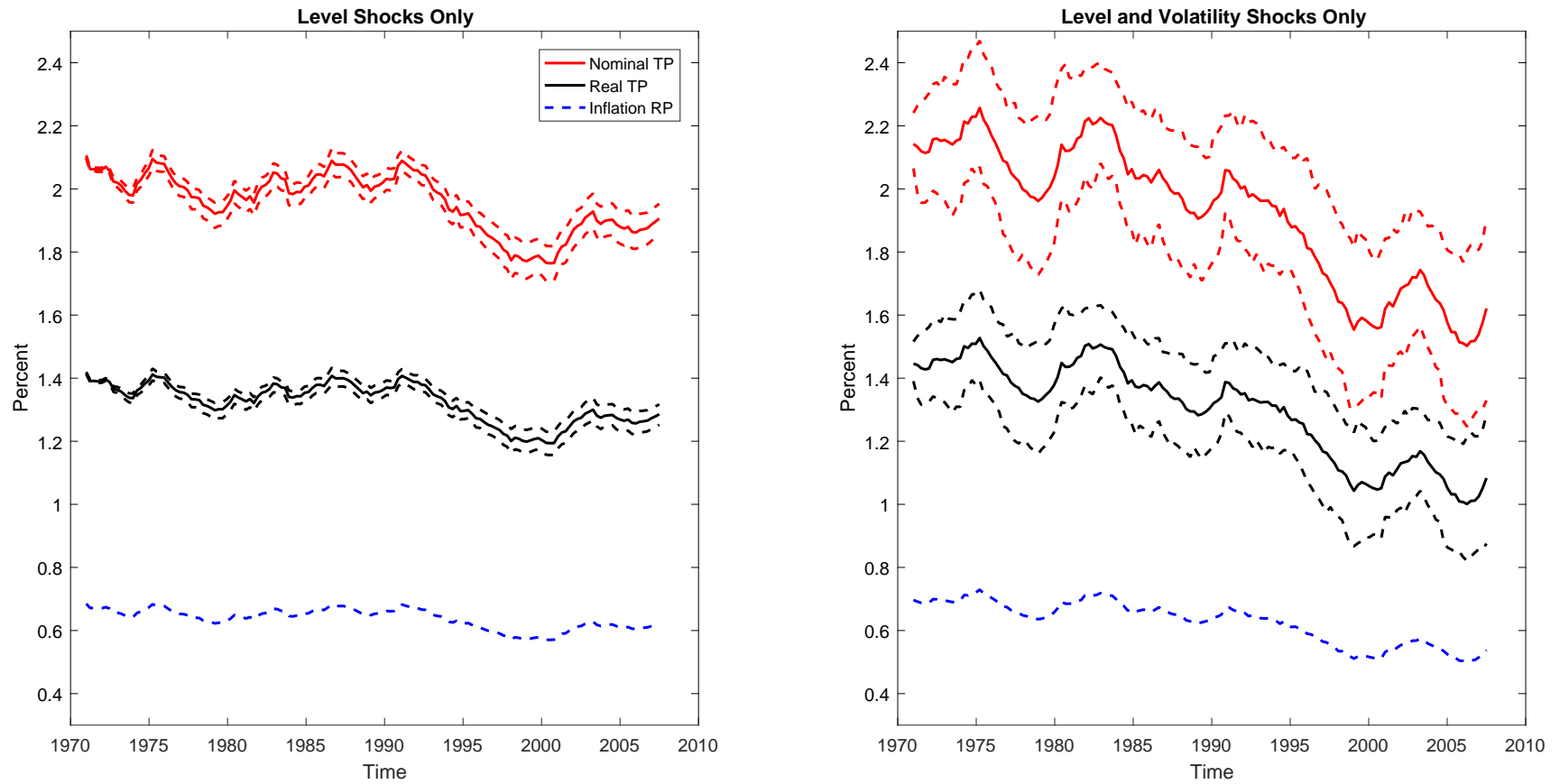


Figure 10: **Nominal vs Real Term Premium - Counterfactual Analysis**

This figure plots the model-implied nominal and real term premium as well as the inflation risk premium from 1970.Q1 to 2007.Q4. The solid lines correspond to the median, while the dotted lines are the 5th and 95th percentiles. The dotted blue line is the difference between the median nominal and median real term premium.

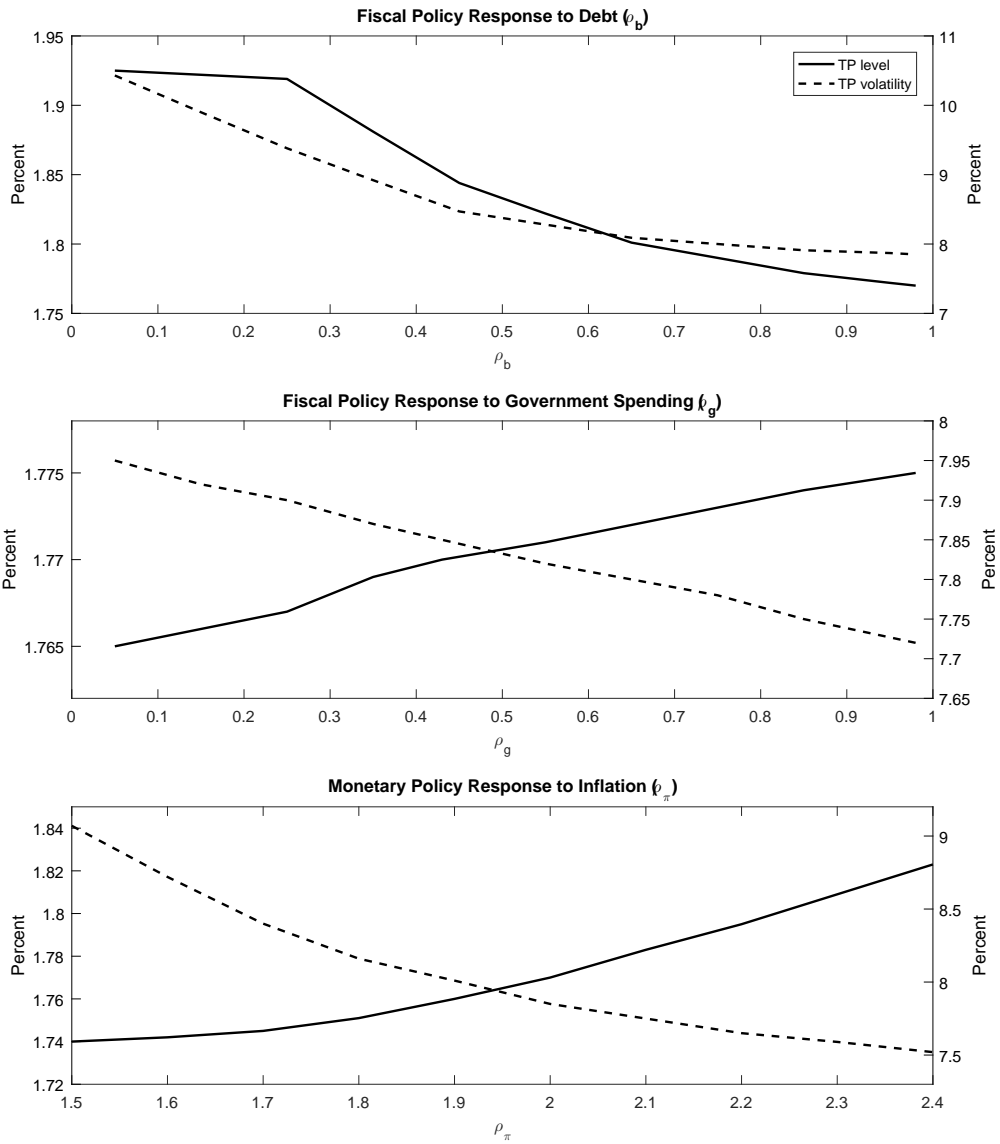


Figure 11: **Policy Parameter Analysis**

This figure plots comparative statics for term premium from the benchmark model. The responses of term premium are shown by varying the fiscal policy response to maturing real debt (ρ_b), the fiscal policy response to government spending shocks (ρ_g), and the monetary policy response to inflation (ρ_π).

Tables

Table 1: Calibrated and Estimated Parameters

This table reports the parameter values for the baseline model.

Panel A: Calibrated Parameters		
Coefficient	Description	Value
μ	share of spenders	0.5
δ	capital depreciation	0.021
κ	capital share of production	0.33
α	share of firms with rigid prices	0.66
θ	share of households that cannot set wage optimally	0.66
η	markup parameter	6
η_w	markup parameter	6
θ_g	steady-state of government spending level	0.2022
θ_{τ^k}	steady-state of capital tax rate	0.3737
Panel B: Separately Estimated Parameters		
Coefficient	Description	Value
ϕ_g	autocorrelation of government spending level	0.975 [0.955;0.995]
ϕ_σ^g	autocorrelation of government spending volatility	0.916 [0.829;0.973]
θ_σ^g	steady-state of government spending volatility	-6.172 [-6.435; -5.905]
σ_σ^g	volatility of government spending volatility shock	0.112 [0.049;0.236]
ϕ_{τ^k}	autocorrelation of capital income tax level	0.948 [0.902;0.993]
$\phi_\sigma^{\tau^k}$	autocorrelation of capital income tax volatility	0.867 [0.781;0.933]
$\theta_\sigma^{\tau^k}$	steady-state of capital income tax volatility	-4.987 [-5.281; -4.669]
$\sigma_\sigma^{\tau^k}$	volatility of capital income tax volatility	0.230 [0.124;0.376]

Table 1 (continued): Calibrated and Estimated Parameters

This table reports the parameter values for the baseline model.

Panel C: Estimated Parameters			
Coefficient	Description	1. Step GMM	2. Step GMM
β	time discount parameter	0.9997 (0.046)	0.9999 (0.006)
ψ	inverse of the elasticity of intertemporal substitution	4.5573 (8.664)	4.3581 (1.243)
γ	risk aversion	148.2430 (133.773)	149.9470 (19.212)
ω	inverse of Frisch labor supply elasticity	0.8162 (1.621)	0.8160 (0.207)
ζ	capital adjustment cost	3.3265 (8.875)	3.2255 (0.342)
π	inflation target	1.0080 (0.013)	1.0090 (0.001)
ρ_π	Taylor rule coefficient on inflation	1.9612 (0.669)	1.9933 (0.085)
ρ_x	Taylor rule coefficient on output gap	0.0138 (0.062)	0.0142 (0.006)
ρ_b	fiscal response to debt	0.9823 (0.801)	0.9801 (0.452)
ρ_g	fiscal response to government spending	0.4043 (0.925)	0.4326 (0.093)
ϕ_z	autocorrelation of transitory productivity shock	0.9943 (0.040)	0.9985 (0.002)
σ_z	volatility of transitory productivity shock	0.0079 (0.011)	0.0079 (0.001)
ϕ_a	autocorrelation of permanent productivity shock	0.1969 (1.812)	0.1702 (0.179)
g_a	steady-state of permanent productivity shock	0.0025 (0.026)	0.0025 (0.003)
σ_a	volatility of permanent productivity shock	0.0050 (0.001)	0.0050 (0.001)
ϕ_u	autocorrelation of monetary policy shock	0.2482 (0.472)	0.2617 (0.205)
σ_u	volatility of monetary policy shock	0.0015 (0.010)	0.0016 (0.001)

Table 2: Baseline Model Moments

This table reports the mean, standard deviations and correlations for observable variables in the baseline model. The sample period for the data is 1961.Q3 to 2007.Q3.

Series	Mean Baseline model				Standard deviation Baseline model				First autocorrelation Baseline model			
	Data	Median	5th	95th	Data	Median	5th	95th	Data	Median	5th	95th
Output growth	1.655	2.035	1.446	2.540	1.671	2.017	1.575	3.960	0.244	-0.045	-0.183	0.103
Investment growth	1.992	2.024	1.341	2.632	4.071	4.853	3.166	6.646	0.539	-0.109	-0.243	0.048
Consumption growth	2.016	2.046	1.749	2.344	0.972	0.978	0.838	1.559	0.335	0.102	-0.060	0.248
Short rate	5.908	5.451	2.817	7.707	1.446	0.916	0.696	1.605	0.910	0.633	0.488	0.824
Long rate	7.545	7.108	4.097	9.000	1.175	0.246	0.112	0.612	0.960	0.943	0.829	0.980
Inflation	2.007	2.089	0.620	3.716	0.429	0.521	0.361	1.189	0.734	0.682	0.494	0.878
Slope	1.637	1.633	0.010	2.616	0.927	0.791	0.619	1.171	0.706	0.583	0.431	0.740
Term Premium	2.064	1.685	0.011	2.375	0.544	0.086	0.022	0.210	0.924	0.908	0.824	0.962

Panel B: GMM 2.Step without SV

Series	Mean Baseline model				Standard deviation Baseline model				First autocorrelation Baseline model			
	Data	Median	5th	95th	Data	Median	5th	95th	Data	Median	5th	95th
Output growth	1.655	2.044	1.713	2.385	1.671	1.772	1.555	2.174	0.244	-0.047	-0.169	0.088
Investment growth	1.992	2.030	1.604	2.428	4.071	4.413	3.978	4.903	0.539	-0.176	-0.287	-0.053
Consumption growth	2.016	2.045	1.773	2.303	0.972	0.915	0.804	1.073	0.335	0.102	-0.046	0.237
Short rate	5.908	4.989	2.770	6.729	1.446	0.793	0.650	1.198	0.910	0.579	0.441	0.790
Long rate	7.545	6.789	4.362	8.204	1.175	0.177	0.095	0.452	0.960	0.931	0.816	0.975
Inflation	2.007	1.717	0.388	2.976	0.429	0.438	0.349	1.046	0.734	0.621	0.484	0.874
Slope	1.637	1.782	0.013	2.480	0.927	0.710	0.591	0.879	0.706	0.541	0.417	0.689
Term Premium	2.064	1.814	0.018	2.603	0.544	0.024	0.009	0.073	0.924	0.958	0.911	0.986

Table 3: Variance Decomposition - The Effect of Structural Shocks

This table reports the variance decomposition for the different structural shocks in the baseline model.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Only z	Only Δa	Only u	Only g	Only σ_g	Only τ	Only σ_τ
Output growth	75.68 [70.94,78.71]	0.90 [0.50, 1.40]	0.02 [0.01, 0.06]	23.30 [20.38, 27.53]	0.02 [0.02, 0.03]	0.08 [0.07, 0.09]	0.00 [0.00,0.01]
Investment growth	76.78 [71.09,81.48]	4.49 [1.74,8.60]	8.13 [4.36,14.03]	1.25 [1.10,1.43]	0.62 [0.43,0.81]	8.27 [7.62,9.04]	0.24 [0.00,0.54]
Consumption growth	49.96 [40.63,60.63]	31.07 [17.07,43.69]	0.47 [0.24,0.97]	17.77 [14.91,21.67]	0.10 [0.08,0.13]	0.25 [0.17,0.40]	0.00 [0.00,0.01]
Short rate	67.45 [60.49,72.00]	1.08 [0.53,2.00]	9.12 [5.03,16.61]	16.12 [13.80,18.18]	2.78 [2.11,3.67]	2.96 [2.47,3.53]	0.00 [0.00,0.00]
Long rate	41.16 [11.31,76.02]	14.39 [8.29,21.30]	0.27 [0.16,0.44]	40.88 [9.45,72.61]	0.28 [0.12,0.50]	1.83 [1.27,2.27]	0.00 [0.00,0.01]
Inflation	78.33 [73.85,82.48]	1.25 [0.62,2.26]	1.65 [0.48,6.00]	12.10 [9.42,15.41]	3.03 [2.28,3.95]	2.81 [2.35,3.42]	0.00 [0.00,0.02]
Slope	68.91 [61.05,74.31]	0.48 [0.09,1.96]	10.62 [5.63,19.30]	12.31 [11.06,13.69]	3.47 [2.66,4.51]	3.49 [2.97,4.06]	0.01 [0.00,0.07]
Term Premium	0.29 [0.09,1.06]	0.12 [0.06,0.22]	0.00 [0.00,0.00]	9.31 [7.84,10.34]	90.23 [89.40,91.16]	0.04 [0.03,0.05]	0.00 [0.00,0.01]

Table 4: Data and Baseline Model Implied Statistics - The Effect of Rigidities

This table reports the mean, standard deviations and first auto-correlations for observable variables in the baseline model.

Panel A: Mean					
	Rigidities				
	(1)	(2)	(3)	(4)	(5)
	Data	Baseline	No PR	No WR	NoR
Output growth	1.655	2.035	2.052	2.026	2.032
Investment growth	1.992	2.024	2.017	2.022	2.036
Consumption growth	2.016	2.046	2.048	2.025	2.043
Short rate	5.908	5.451	7.842	4.280	7.714
Long rate	7.545	7.108	7.572	7.528	7.543
Inflation	2.007	2.089	3.326	1.048	3.274
Slope	1.637	1.633	-0.140	3.248	0.382
Term Premium	2.064	1.685	-0.053	3.125	0.001

Panel B: Standard Deviation					
	Rigidities				
	(1)	(2)	(3)	(4)	(5)
	Data	Baseline	No PR	No WR	NoR
Output growth	1.671	2.017	1.864	2.328	1.757
Investment growth	4.071	4.853	3.117	7.857	3.113
Consumption growth	0.972	0.978	1.057	1.149	1.059
Short rate	1.446	0.916	0.749	1.781	0.781
Long rate	1.175	0.246	0.238	0.415	0.244
Inflation	0.429	0.521	0.484	0.932	0.512
Slope	0.927	0.791	0.620	1.634	0.655
Term Premium	0.544	0.086	0.066	0.118	0.076

Panel C: First Autocorrelation					
	Rigidities				
	(1)	(2)	(3)	(4)	(5)
	Data	Baseline	No PR	No WR	NoR
Output growth	0.244	-0.045	-0.004	-0.085	0.055
Investment growth	0.539	-0.109	0.085	-0.320	0.091
Consumption growth	0.335	0.102	0.053	-0.004	0.042
Short rate	0.910	0.633	0.667	0.313	0.622
Long rate	0.960	0.943	0.951	0.974	0.951
Inflation	0.734	0.682	0.653	0.343	0.609
Slope	0.706	0.583	0.587	0.250	0.548
Term Premium	0.924	0.908	0.900	0.938	0.904

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A Data for the Estimation

We follow Schmitt-Grohé and Uribe (2012) and construct the macroeconomic observable variables used in the estimation as:

1. Real Gross Domestic Product, BEA, NIPA table 1.1.6., line 1, billions of chained 2000 dollars seasonally adjusted at annual rate. Downloaded from www.bea.gov.
2. Gross Domestic Product, BEA NIPA table 1.1.5., line 1, billions of dollars, seasonally adjusted at annual rates.
3. Personal Consumption Expenditure on Nondurable Goods, BEA, NIPA table 1.1.5., line 4, billions of dollars, seasonally adjusted at annual rate. Downloaded from www.bea.gov.
4. Personal Consumption Expenditure on Services, BEA NIPA table 1.1.5., line 5, billions of dollars, seasonally adjusted at annual rate. Downloaded from www.bea.gov.
5. Gross Private Domestic Investment, Fixed Investment, Nonresidential, BEA NIPA table 1.1.5., line 8, billions of dollars, seasonally adjusted at annual rate. Downloaded from www.bea.gov.
6. Gross Private Domestic Investment, Fixed Investment, Residential, BEA NIPA table 1.1.5., line 11, billions of dollars, seasonally adjusted at annual rate. Downloaded from www.bea.gov.
7. Civilian Noninstitutional Population Over 16, BLS LNU0000000Q. Downloaded from www.bls.gov.
8. Nonfarm Business Hours Worked, BLS, PRS85006033, seasonally adjusted, index 1992=100. Downloaded from www.bls.gov.
9. GDP Deflator = (2) / (1).
10. Real Per Capita GDP = (1) / (7).
11. Real Per Capita Consumption = [(3) + (4)] / (9) / (7).
12. Real Per Capita Investment = [(5) + (6)] / (7) / (9).

Government spending data are from Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015).

With regard to the financial variables, the Treasury yield data are from Gurkaynak, Sack, and Wright (2007) (data are available for download on the website <http://www.federalreserve.gov/Pubs/feds/2006/200628/feds200628.xls>) and the series for the 10-year Term premia is from Adrian, Crump, and Moench (2013) (data available at https://www.newyorkfed.org/research/data_indicators/term_premia.html). We thank the authors for making these data available for download.

B Solving the Extended Model

B.1 Households with Epstein-Zin Preference

The savers' optimization problem is:

$$\begin{aligned} \max \quad & V(C_t, N_t) = \left\{ (1 - \beta)U(C_t, N_t)^{1-\psi} + \beta E_t [V_{t+1}^{1-\gamma}]^{\frac{1-\psi}{1-\gamma}} \right\}^{\frac{1}{1-\psi}} \\ \text{s.t.} \quad & E_t \left[\sum_{s=0}^{\infty} M_{t,t+s}^{\$} P_{t+s} C_{t+s} \right] \leq E_t \left[\sum_{s=0}^{\infty} M_{t,t+s}^{\$} (W_{t+s} P_{t+s} N_{t+s} - P_{t+s} T_{t+s} + P_{t+s} \Psi_{t+s}) \right], \end{aligned}$$

where

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$$

and

$$U(C_t, N_t) = \left[\frac{C_t^{1-\psi}}{1-\psi} - A_t^{1-\psi} \frac{N_t^{1+\omega}}{1+\omega} \right]^{\frac{1}{1-\psi}}.$$

The first order conditions are:

$$\frac{\partial V_t}{\partial C_t} : \frac{1}{1-\psi} \left[V_t^{1-\psi} \right]^{\frac{1}{1-\psi}-1} (1-\beta) C_t^{-\psi} - \lambda M_{t,t}^{\$} P_t = 0 \quad (9)$$

$$\frac{\partial V_t}{\partial N_t} : \frac{1}{1-\psi} \left[V_t^{1-\psi} \right]^{\frac{1}{1-\psi}-1} (1-\beta) (-A_t^{1-\psi} N_t^{\omega}) + \lambda M_{t,t}^{\$} W_t P_t = 0 \quad (10)$$

$$\frac{\partial V_t}{\partial C_{t+1}} : \frac{1}{1-\psi} \left[V_t^{1-\psi} \right]^{\frac{1}{1-\psi}-1} \beta \left(\frac{1-\psi}{1-\gamma} \right) E_t [V_{t+1}^{1-\gamma}]^{\frac{1-\psi}{1-\gamma}-1} (1-\gamma) V_{t+1}^{-\gamma} \frac{\partial V_{t+1}}{\partial C_{t+1}} - \lambda M_{t,t+1}^{\$} P_{t+1} = 0. \quad (11)$$

Furthermore,

$$\frac{\partial V_{t+1}}{\partial C_{t+1}} = \frac{1}{1-\psi} \left[V_{t+1}^{1-\psi} \right]^{\frac{1}{1-\psi}-1} (1-\beta) C_{t+1}^{-\psi}. \quad (12)$$

Combining (9) and (10), I have the household's intratemporal consumption and labor supply optimality condition:

$$\frac{\lambda(1-\psi)}{V_t^{\psi}(1-\beta)} = \frac{C_t^{-\psi}}{P_t} = \frac{A_t^{1-\psi} N_t^{\omega}}{W_t P_t} \Rightarrow W_t = A_t^{1-\psi} C_t^{\psi} N_t^{\omega}.$$

Finally, combining (9), (11) and (12), I obtain the intertemporal consumption optimality condition:

$$\frac{\lambda(1-\psi)}{V_t^{\psi}(1-\beta)} = \frac{C_t^{-\psi}}{P_t} = \beta \left(\frac{C_{t+1}^{-\psi}}{P_{t+1}} \right) \left(\frac{V_{t+1}^{\psi-\gamma}}{M_{t,t+1}^{\$}} \right) E_t \left[V_{t+1}^{\frac{1}{1-\gamma}} \right]^{\frac{\gamma-\psi}{1-\gamma}}.$$

To get the nominal pricing kernel, I solve for $M_{t,t+1}^{\$}$,

$$M_{t,t+1}^{\$} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\psi} \left(\frac{P_{t+1}}{P_t} \right)^{-1} \left[\frac{V_{t+1}}{E_t [V_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right]^{\psi-\gamma}. \quad (13)$$

B.2 Monopolistic Producers and Price Rigidities

There is a dispersion of firms, denoted by j , with identical production technology in the economy. With nominal price stickiness and monopolistic competition, each firm is faced with the following optimization problem:

$$\max_{P_t^*} E_t \left[\sum_{s=0}^{\infty} \alpha^s M_{t,t+s}^s (P_t^* (\Pi^*)^s Y_{t+s|t}(j) - W_{t+s|t}(j) P_{t+s} N_{t+s|t}(j)) \right] \quad (14)$$

$$\text{s.t. } Y_{t+s|t}(j) = Z_{t+s} N_{t+s|t}(j) \quad (15)$$

$$Y_{t+s|t}(j) = \left(\frac{P_t^* (\Pi^*)^s}{P_{t+s}} \right)^{-\theta} Y_{t+s} \quad (16)$$

$$P_t = \left[\int_0^1 P_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}} = \left[(1-\alpha) P_t^*{}^{1-\theta} + \alpha (P_{t-1} \Pi^*)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \quad (17)$$

Using Calvo (1983) pricing, a firm can choose to optimally adjust price to P_t^* with probability $(1-\alpha)$ each period independent of the time elapsed between adjustments. Furthermore, $t+s|t$ denotes the value in period $t+s$ given that the firm last adjusted price in period t . Π^* is the natural level of inflation that firms use to adjust their prices to from period to period if they cannot optimally set the price, and Z_t is the productivity shock on output. Log productivity is an exogenous AR(1) process such that

$$z_{t+1} = \ln(Z_{t+1}) = \phi_z z_t + \sigma_z \epsilon_{z,t+1}.$$

The first order condition for firm j is:

$$E_t \left[\sum_{s=0}^{\infty} \alpha^s M_{t,t+s}^s Y_{t+s|t}(j) \left(P_t^* (\Pi^*)^s - \nu P_{t+s} \frac{W_{t+s|t}(j)}{Z_{t+s}} \right) \right] = 0, \quad (18)$$

where $\nu = \frac{\theta}{\theta-1}$ is the frictionless markup in the absence of price adjustment constraint. Utilizing (16) and the fact that $W_{t+s|t}(j) = W_{t+s}$, (18) can be rewritten as:

$$\left(\frac{P_t^*}{P_t} \right) F_t = \nu \frac{W_t}{Z_t} J_t$$

or after manipulating (17):

$$\left[\frac{1}{1-\alpha} \left(1 - \alpha \left(\frac{\Pi^*}{\Pi_t} \right)^{1-\theta} \right) \right]^{\frac{1}{1-\theta}} F_t = \nu \frac{W_t}{Z_t} J_t. \quad (19)$$

F_t can be recursively expressed as:

$$\begin{aligned} F_t &= 1 + E_t \left[\sum_{s=1}^{\infty} (\alpha \Pi^*)^s M_{t,t+1}^s M_{t+1,t+s}^s \left(\frac{Y_{t+s}}{Y_{t+1}} \right) \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{P_t \Pi^*}{P_{t+1}} \right)^{-\theta} \left(\frac{P_{t+1} (\Pi^*)^{s-1}}{P_{t+s}} \right)^{-\theta} \right] \\ &= 1 + \alpha \Pi^* E_t \left[M_{t,t+1}^s \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{P_t \Pi^*}{P_{t+1}} \right)^{-\theta} E_{t+1} \left[\sum_{s=1}^{\infty} (\alpha \Pi^*)^{s-1} M_{t+1,t+s}^s \left(\frac{Y_{t+s}}{Y_{t+1}} \right) \left(\frac{P_{t+1} (\Pi^*)^{s-1}}{P_{t+s}} \right)^{-\theta} \right] \right] \\ &= 1 + \alpha \Pi^* E_t \left[M_{t,t+1}^s \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{\Pi^*}{\Pi_{t+1}} \right)^{-\theta} F_{t+1} \right], \end{aligned}$$

Similarly, J_t has the following recursive formulation:

$$J_t = 1 + \alpha \Pi^* E_t \left[M_{t,t+1}^{\$} \left(\frac{Z_t}{Z_{t+1}} \right) \left(\frac{W_{t+1}}{W_t} \right) \left(\frac{Y_{t+1}}{Y_t} \right) \left(\frac{\Pi^*}{\Pi_{t+1}} \right)^{-1-\theta} J_{t+1} \right].$$

B.3 The System of Equations for the Model with Growth

The full model presented in this section has thirty-three endogenous variables:

$\{M, R^{cl}, R^c, R^l, share, P^c, P^l, C^o, LI, N^o, W, \tau, D, C^r, N^r, C, N, K, Inv, K^o, Inv^o, Y, \Phi, \Phi', R^I, R^K, Q, P^{real}, \Pi, F, J, M^{nom}, y^{(1)}\}$.

I have a system of thirty-three equations resulting from equilibrium conditions, first order conditions and policy rules:

Pricing kernel,

$$M_{t-1,t} = \left[\beta \left(\frac{C_t^o}{C_{t-1}^o} \right)^{-\psi} \right]^{\frac{1-\gamma}{1-\psi}} \left[R_t^{cl} \right]^{\frac{\psi-\gamma}{1-\psi}} \quad (20)$$

$$R_t^{cl} = (1 - share_{t-1})R_t^c + share_{t-1}R_t^l \quad (21)$$

$$share_t = \frac{1}{\left(1 - \frac{(1+\omega)P_t^c C_t^o}{(1-\psi)P_t^l LI_t} \right)} \quad (22)$$

$$R_t^c = \frac{(1 + P_t^c)C_t^o}{P_{t-1}^c C_{t-1}^o} \quad (23)$$

$$1 = \mathbb{E}_t[M_{t,t+1}R_{t+1}^c] \quad (24)$$

$$R_t^l = \frac{(1 + P_t^l)LI_t}{P_{t-1}^l LI_{t-1}} \quad (25)$$

$$1 = \mathbb{E}_t[M_{t,t+1}R_{t+1}^l] \quad (26)$$

Labor income,

$$LI_t = W_t N_t^o \quad (27)$$

Fiscal rule,

$$Tax_t^o = \tau_t + \tau_t^k R_t^k K_{t-1}^o \quad (28)$$

$$Tax_t^r = \tau_t \quad (29)$$

$$\tau_t = \rho_b D_{t-1}(t) + \rho_g G_t \quad (30)$$

Wage setting of the saver,

$$\left[\frac{1}{1-\theta} \left\{ W_t^{1-\eta_w} - \theta \left(I_{t-1,t}^w \frac{W_{t-1}}{\Pi_t} \right)^{1-\eta_w} \right\} \right]^{\frac{1}{1-\eta_w}} H_t^o = \nu_w A_t^{1-\psi} C_t^{o\psi} N_t^{o\omega} G_t^o \quad (31)$$

$$H_t^o = 1 + \theta \mathbb{E}_t \left[M_{t,t+1}^w I_{t,t+1}^w - \eta_w \left(\frac{N_{t+1}^d}{N_t^d} \right) \left(\Pi_{t+1} \frac{W_{t+1}}{W_t} \right)^{\eta_w} H_{t+1}^o \right] \quad (32)$$

$$G_t^o = 1 + \theta \mathbb{E}_t \left[M_{t,t+1}^w I_{t,t+1}^w - \eta_w \left(\frac{A_{t+1}}{A_t} \right)^{1-\psi} \left(\frac{C_{t+1}^o}{C_t^o} \right)^\psi \left(\frac{N_{t+1}^o}{N_t^o} \right)^\omega \left(\frac{N_{t+1}^d}{N_t^d} \right) \Pi_{t+1}^{1+\eta_w} \left(\frac{W_{t+1}}{W_t} \right)^{\eta_w} G_{t+1}^o \right] \quad (33)$$

Wage indexing,

$$I_{t-1,t}^w = e^{g_a} \quad (34)$$

Marginal rate of intratemporal substitution of the spender,

$$W_t = A_t^{1-\psi} C_t^{r\psi} N_t^{r\omega} \quad (35)$$

Rule-of-thumb budget constraint,

$$C_t^r = W_t N_t^r - \tau_t \quad (36)$$

Aggregation,

$$\log\left(\frac{C_t}{A_t}\right) = \mu \log\left(\frac{C_t^r}{A_t}\right) + (1-\mu) \log\left(\frac{C_t^o}{A_t}\right) \quad (37)$$

$$\log(N_t) = \mu \log(N_t^r) + (1-\mu) \log(N_t^o) \quad (38)$$

$$Tax_t = \mu Tax_t^r + (1-\mu) Tax_t^o \quad (39)$$

$$K_t = (1-\mu) K_t^o \quad (40)$$

$$Inv_t = (1-\mu) Inv_t^o \quad (41)$$

Production function,

$$Y_t = Z_t K_{t-1}^\kappa (A_t N_t)^{1-\kappa} \quad (42)$$

Capital accumulation,

$$K_t^o = ((1-\delta) + \Phi_t) K_{t-1}^o \quad (43)$$

Capital adjustment cost,

$$\Phi_t = b1 + \frac{b2}{(1-1/\zeta)} \left(\frac{Inv_t^o}{K_{t-1}^o} \right)^{1-1/\zeta} \quad (44)$$

$$\Phi_t' = b2 \left(\frac{Inv_t^o}{K_{t-1}^o} \right)^{-1/\zeta} \quad (45)$$

Return on investment,

$$1 = \mathbb{E}_t[M_{t,t+1} R_{t+1}^I] \quad (46)$$

$$R_t^I Q_{t-1} = (1-\tau_t^k) R_t^K + Q_t \left(1-\delta + \Phi_t - \Phi_t' \frac{Inv_t^o}{K_{t-1}^o} \right) \quad (47)$$

$$1 = Q_t \Phi_t' \quad (48)$$

Market clearing condition,

$$Y_t = C_t + Inv_t + G_t \quad (49)$$

Government budget constraint,

$$D_{t-1}(t) = Tax_t - G_t + P_t^{real} D_t(t+1) \quad (50)$$

Capital labor ratio,

$$W_t = \frac{(1-\kappa) R_t^K K_{t-1}}{\kappa N_t} \quad (51)$$

Optimal price setting,

$$\left[\frac{1}{1-\alpha} \left(1 - \alpha \left(\frac{1}{\Pi_t} \right)^{(1-\eta)} \right) \right]^{\frac{1}{(1-\eta)}} F_t = \frac{\nu \kappa^{-\kappa} (1-\kappa)^{-(1-\kappa)} R_t^{K\kappa} W_t^{(1-\kappa)} J_t}{Z_t A_t^{1-\kappa}} \quad (52)$$

$$F_t = 1 + \alpha \mathbb{E}_t \left[M_{t,t+1}^{nom} \left(\frac{Y_{t+1}}{Y_t} \right) \Pi_{t+1}^\eta F_{t+1} \right] \quad (53)$$

$$J_t = 1 + \alpha \mathbb{E}_t \left[M_{t,t+1}^{nom} \left(\frac{Z_t}{Z_{t+1}} \right) \left(\frac{A_t}{A_{t+1}} \right)^{1-\kappa} \left(\frac{R_{t+1}^K}{R_t^K} \right)^\kappa \left(\frac{W_{t+1}}{W_t} \right)^{(1-\kappa)} \left(\frac{Y_{t+1}}{Y_t} \right) \Pi_{t+1}^{(1+\eta)} J_{t+1} \right] \quad (54)$$

Nominal pricing kernel,

$$M_{t-1,t}^{nom} = \frac{M_{t-1,t}}{\Pi_t} \quad (55)$$

Euler equation,

$$\frac{1}{R_t^{(1)}} = \mathbb{E}_t [M_{t,t+1}^{nom}] \quad (56)$$

Real bond price,

$$P_t^{real} = \mathbb{E}_t [M_{t,t+1}] \quad (57)$$

Taylor rule,

$$\frac{R_t^{(1)}}{R} = \left(\frac{R_{t-1}^{(1)}}{R} \right)^{\rho_r} \left(\frac{\Pi_t}{\Pi^*} \right)^{(1-\rho_r)\rho_\pi} \left(\frac{Y_t/A_t}{Y/A} \right)^{(1-\rho_r)\rho_x} e^{u_t}, \quad (58)$$

where g_t , u_t and z_t are exogenous shocks to government spending, monetary policy and productivity, respectively:

$$\begin{aligned}
g_{t+1} &= (1 - \phi_g)\theta_g + \phi_g g_t + \sigma_{g,t}\epsilon_{g,t+1} \\
\sigma_{g,t+1}^2 &= (1 - \phi_\sigma^g)\theta_\sigma^g + \phi_\sigma^g \sigma_{g,t}^2 + \sigma_{\sigma,t}^g \epsilon_{\sigma,t+1} \\
\tau_{t+1}^k &= (1 - \phi_{\tau^k})\theta_{\tau^k} + \phi_{\tau^k} \tau_t^k + \sigma_{\tau^k,t}\epsilon_{\tau^k,t+1} \\
\sigma_{\tau^k,t+1}^2 &= (1 - \phi_\sigma^{\tau^k})\theta_\sigma^{\tau^k} + \phi_\sigma^{\tau^k} \sigma_{\tau^k,t}^2 + \sigma_{\sigma,t}^{\tau^k} \epsilon_{\sigma,t+1} \\
u_{t+1} &= \phi_u u_t + \sigma_u \epsilon_{u,t+1} \\
z_{t+1} &= \phi_z z_t + \sigma_z \epsilon_{z,t+1} \\
\Delta a_{t+1} &= (1 - \phi_a)g_a + \phi_a \Delta a_t + \sigma_a \epsilon_{a,t+1};
\end{aligned}$$

B.3.1 The Stationary Model

To make the model stationary, output, consumption, investment, capital stock, real wage, real debt, government revenue, and government spending need to be detrended by the permanent component of productivity, A_t .

Pricing kernel,

$$M_{t-1,t} = \left[\beta \left(\frac{\frac{C_t^o}{A_t} A_t}{\frac{C_{t-1}^o}{A_{t-1}} A_{t-1}} \right)^{-\psi} \right]^{\frac{1-\gamma}{1-\psi}} \left[R_t^{cl} \right]^{\frac{\psi-\gamma}{1-\psi}} \quad (59)$$

$$\Rightarrow M_{t-1,t} = \left[\beta \left(\frac{\widetilde{C}_t^o e^{\Delta a_t}}{\widetilde{C}_{t-1}^o} \right)^{-\psi} \right]^{\frac{1-\gamma}{1-\psi}} \left[R_t^{cl} \right]^{\frac{\psi-\gamma}{1-\psi}} \quad (60)$$

$$R_t^{cl} = (1 - share_{t-1})R_t^c + share_{t-1}R_t^l \quad (61)$$

$$share_t = \frac{1}{\left(1 - \frac{(1+\omega)P_t^c \frac{C_t^o}{A_t}}{(1-\psi)P_t^l \frac{LI_t}{A_t}} \right)} \Rightarrow share_t = \frac{1}{\left(1 - \frac{(1+\omega)P_t^c \widetilde{C}_t^o}{(1-\psi)P_t^l \widetilde{LI}_t} \right)} \quad (62)$$

$$R_t^c = \frac{(1 + P_t^c) \frac{C_t^o}{A_t} A_t}{P_{t-1}^c \frac{C_{t-1}^o}{A_{t-1}} A_{t-1}} \Rightarrow R_t^c = \frac{(1 + P_t^c) \widetilde{C}_t^o}{P_{t-1}^c \widetilde{C}_{t-1}^o} e^{\Delta a_t} \quad (63)$$

$$1 = \mathbb{E}_t[M_{t,t+1} R_{t+1}^c] \quad (64)$$

$$R_t^l = \frac{(1 + P_t^l) \frac{LI_t}{A_t} A_t}{P_{t-1}^l \frac{LI_{t-1}}{A_{t-1}} A_{t-1}} \Rightarrow R_t^l = \frac{(1 + P_t^l) \widetilde{LI}_t}{P_{t-1}^l \widetilde{LI}_{t-1}} e^{\Delta a_t} \quad (65)$$

$$1 = \mathbb{E}_t[M_{t,t+1} R_{t+1}^l] \quad (66)$$

Labor income,

$$\frac{LI_t}{A_t} = \frac{W_t}{A_t} N_t^o \Rightarrow \widetilde{LI}_t = \widetilde{W}_t N_t^o \quad (67)$$

Fiscal rule,

$$\frac{Tax_t^o}{A_t} = \frac{\tau_t}{A_t} + \tau_t^k R_t^k \frac{K_{t-1}^o}{A_{t-1}} \frac{A_{t-1}}{A_t} \implies \widetilde{Tax}_t^o = \widetilde{\tau}_t + \tau_t^k R_t^k \widetilde{K}_{t-1}^o e^{-\Delta\alpha_t} \quad (68)$$

$$\frac{Tax_t^r}{A_t} = \frac{\tau_t}{A_t} \implies \widetilde{Tax}_t^r = \widetilde{\tau}_t \quad (69)$$

$$\frac{\tau_t}{A_t} = \rho_b \frac{D_{t-1}(t)}{A_{t-1}} \frac{A_{t-1}}{A_t} + \rho_g \frac{G_t}{A_t} \implies \widetilde{\tau}_t = \rho_b \widetilde{D}_{t-1}(t) e^{-\Delta\alpha_t} + \rho_g \widetilde{G}_t \quad (70)$$

Wage setting of the saver,

$$\left[\frac{1}{1-\theta} \left\{ W_t^{1-\eta_w} - \theta \left(I_{t-1,t}^w \frac{W_{t-1}}{\Pi_t} \right)^{1-\eta_w} \right\} \right]^{\frac{1}{1-\eta_w}} H_t^o = \nu_w A_t^{1-\psi} C_t^{o\psi} N_t^{o\omega} G_t^o \quad (71)$$

$$\implies \left[\frac{1}{1-\theta} \left\{ \widetilde{W}_t^{1-\eta_w} - \theta \left(I_{t-1,t}^w \frac{\widetilde{W}_{t-1}}{\Pi_t} \frac{A_{t-1}}{A_t} \right)^{1-\eta_w} \right\} \right]^{\frac{1}{1-\eta_w}} H_t^o = \nu_w \widetilde{C}_t^{o\psi} N_t^{o\omega} G_t^o \quad (72)$$

$$H_t^o = 1 + \theta \mathbb{E}_t \left[M_{t,t+1}^{nom} I_{t,t+1}^{w} \left(\frac{N_{t+1}^d}{N_t^d} \right) \left(\Pi_{t+1} \frac{W_{t+1}}{W_t} \frac{A_{t+1}}{A_t} \right)^{\eta_w} H_{t+1}^o \right] \quad (73)$$

$$\implies H_t^o = 1 + \theta \mathbb{E}_t \left[M_{t,t+1}^{nom} I_{t,t+1}^{w} \left(\frac{N_{t+1}^d}{N_t^d} \right) \left(\Pi_{t+1} \frac{\widetilde{W}_{t+1}}{\widetilde{W}_t} e^{\Delta\alpha_{t+1}} \right)^{\eta_w} H_{t+1}^o \right] \quad (74)$$

$$G_t^o = 1 + \theta \mathbb{E}_t \left[M_{t,t+1}^{nom} I_{t,t+1}^{w} \left(\frac{C_{t+1}^o}{A_{t+1}} \frac{A_{t+1}}{A_t} \right)^{\psi} \left(\frac{N_{t+1}^o}{N_t^o} \right)^{\omega} \left(\frac{N_{t+1}^d}{N_t^d} \right) \Pi_{t+1}^{1+\eta_w} \left(\frac{A_{t+1}}{A_t} \right)^{1-\psi} \left(\frac{W_{t+1}}{W_t} \frac{A_{t+1}}{A_t} \right)^{\eta_w} G_{t+1}^o \right] \quad (75)$$

$$\implies G_t^o = 1 + \theta \mathbb{E}_t \left[M_{t,t+1}^{nom} I_{t,t+1}^{w} \left(\frac{\widetilde{C}_{t+1}^o}{\widetilde{C}_t^o} \right)^{\psi} \left(\frac{N_{t+1}^o}{N_t^o} \right)^{\omega} \left(\frac{N_{t+1}^d}{N_t^d} \right) \Pi_{t+1}^{1+\eta_w} \left(\frac{A_{t+1}}{A_t} \right)^{1+\eta_w} \left(\frac{\widetilde{W}_{t+1}}{\widetilde{W}_t} \right)^{\eta_w} G_{t+1}^o \right] \quad (76)$$

Wage indexing,

$$I_{t-1,t}^w = e^{g_a} \quad (77)$$

Marginal rate of intratemporal substitution of the spender,

$$\frac{W_t}{A_t} = A_t^{1-\psi} \left(\frac{C_t^r}{A_t} \right)^{\psi} N_t^{r\omega} \frac{1}{A_t^{1-\psi}} \implies \widetilde{W}_t = \widetilde{C}_t^{r\psi} N_t^{r\omega} \quad (78)$$

Rule-of-thumb budget constraint,

$$\frac{C_t^r}{A_t} = \frac{W_t}{A_t} N_t^r - \frac{\tau_t}{A_t} \implies \widetilde{C}_t^r = \widetilde{W}_t N_t^r - \widetilde{\tau}_t \quad (79)$$

Aggregation,

$$\log(\widetilde{C}_t) = \mu \log(\widetilde{C}_t^r) + (1 - \mu) \log(\widetilde{C}_t^o) \quad (80)$$

$$\log(N_t) = \mu \log(N_t^r) + (1 - \mu) \log(N_t^o) \quad (81)$$

$$\frac{Tax_t}{A_t} = \mu \frac{Tax_t^r}{A_t} + (1 - \mu) \frac{Tax_t^o}{A_t} \implies \widetilde{Tax}_t = \mu \widetilde{Tax}_t^r + (1 - \mu) \widetilde{Tax}_t^o \quad (82)$$

$$\frac{K_t}{A_t} = (1 - \mu) \frac{K_t^o}{A_t} \implies \widetilde{K}_t = (1 - \mu) \widetilde{K}_t^o \quad (83)$$

$$\frac{Inv_t}{A_t} = (1 - \mu) \frac{Inv_t^o}{A_t} \implies \widetilde{Inv}_t = (1 - \mu) \widetilde{Inv}_t^o \quad (84)$$

Production function,

$$\frac{Y_t}{A_t} = Z_t \left(\frac{K_{t-1}}{A_{t-1}} \right)^\kappa N_t^{1-\kappa} \left(\frac{A_{t-1}}{A_t} \right)^\kappa \implies \widetilde{Y}_t = Z_t \left(\widetilde{K}_{t-1} e^{-\Delta a_t} \right)^\kappa N_t^{1-\kappa} \quad (85)$$

Capital accumulation,

$$\frac{K_t^o}{A_t} = ((1 - \delta) + \Phi_t) \frac{K_{t-1}^o}{A_{t-1}} \left(\frac{A_{t-1}}{A_t} \right) \implies \widetilde{K}_t^o = ((1 - \delta) + \Phi_t) \widetilde{K}_{t-1}^o e^{-\Delta a_t} \quad (86)$$

Capital adjustment cost,

$$\Phi_t = b1 + \frac{b2}{(1 - 1/\zeta)} \left(\frac{\frac{Inv_t^o}{A_t} \frac{A_t}{K_{t-1}^o} \frac{A_t}{A_{t-1}}}{\frac{K_{t-1}^o}{A_{t-1}}} \right)^{1-1/\zeta} \implies \Phi_t = b1 + \frac{b2}{(1 - 1/\zeta)} \left(\frac{\widetilde{Inv}_t^o e^{\Delta a_t}}{\widetilde{K}_{t-1}^o} \right)^{1-1/\zeta} \quad (87)$$

$$\Phi_t' = b2 \left(\frac{\frac{Inv_t^o}{A_t} \frac{A_t}{K_{t-1}^o} \frac{A_t}{A_{t-1}}}{\frac{K_{t-1}^o}{A_{t-1}}} \right)^{-1/\zeta} \implies \Phi_t' = b2 \left(\frac{\widetilde{Inv}_t^o e^{\Delta a_t}}{\widetilde{K}_{t-1}^o} \right)^{-1/\zeta} \quad (88)$$

Return on investment,

$$1 = \mathbb{E}_t[M_{t,t+1} R_{t+1}^I] \quad (89)$$

$$R_t^I Q_{t-1} = (1 - \tau_t^k) R_t^K + Q_t \left(1 - \delta + \Phi_t - \Phi_t' \frac{\frac{Inv_t^o}{A_t} \frac{A_t}{K_{t-1}^o} \frac{A_t}{A_{t-1}}}{\frac{K_{t-1}^o}{A_{t-1}}} \right) \quad (90)$$

$$\implies R_t^I Q_{t-1} = (1 - \tau_t^k) R_t^K + Q_t \left(1 - \delta + \Phi_t - \Phi_t' \frac{\widetilde{Inv}_t^o e^{\Delta a_t}}{\widetilde{K}_{t-1}^o} \right) \quad (91)$$

$$1 = Q_t \Phi_t' \quad (92)$$

Market clearing condition,

$$\frac{Y_t}{A_t} = \frac{C_t}{A_t} + \frac{Inv_t}{A_t} + \frac{G_t}{A_t} \implies \widetilde{Y}_t = \widetilde{C}_t + \widetilde{Inv}_t + \widetilde{G}_t \quad (93)$$

Government budget constraint,

$$\frac{D_{t-1}(t)}{A_{t-1}} \frac{A_{t-1}}{A_t} = \frac{Tax_t}{A_t} - \frac{G_t}{A_t} + P_t^{real} \frac{D_t(t+1)}{A_t} \implies \widetilde{D}_{t-1}(t) e^{-\Delta a_t} = \widetilde{Tax}_t - \widetilde{G}_t + P_t^{real} \widetilde{D}_t(t+1) \quad (94)$$

Capital labor ratio,

$$\frac{W_t}{A_t} \frac{A_t}{A_{t-1}} = \frac{(1-\kappa)}{\kappa} R_t^K \frac{K_{t-1}}{A_{t-1} N_t} \implies \widetilde{W}_t e^{\Delta a_t} = \frac{(1-\kappa)}{\kappa} R_t^K \frac{\widetilde{K}_{t-1}}{N_t} \quad (95)$$

Optimal price setting,

$$\left[\frac{1}{1-\alpha} \left(1 - \alpha \left(\frac{1}{\Pi_t} \right)^{(1-\eta)} \right) \right]^{\frac{1}{(1-\eta)}} F_t = \frac{\nu \kappa^{-\kappa} (1-\kappa)^{-(1-\kappa)} R_t^{K\kappa} \left(\frac{W_t}{A_t} \right)^{(1-\kappa)} J_t}{Z_t} \quad (96)$$

$$\implies \left[\frac{1}{1-\alpha} \left(1 - \alpha \left(\frac{1}{\Pi_t} \right)^{(1-\eta)} \right) \right]^{\frac{1}{(1-\eta)}} F_t = \frac{\nu \kappa^{-\kappa} (1-\kappa)^{-(1-\kappa)} R_t^{K\kappa} \left(\widetilde{W}_t \right)^{(1-\kappa)} J_t}{Z_t} \quad (97)$$

$$F_t = 1 + \alpha \mathbb{E}_t \left[M_{t,t+1}^{nom} \left(\frac{Y_{t+1}}{A_{t+1}} \frac{A_{t+1}}{A_t} \right) \Pi_{t+1}^\eta F_{t+1} \right] \quad (98)$$

$$\implies F_t = 1 + \alpha \mathbb{E}_t \left[M_{t,t+1}^{nom} \left(\frac{\widetilde{Y}_{t+1}}{\widetilde{Y}_t} e^{\Delta a_{t+1}} \right) \Pi_{t+1}^\eta F_{t+1} \right] \quad (99)$$

$$J_t = 1 + \alpha \mathbb{E}_t \left[M_{t,t+1}^{nom} \left(\frac{Z_t}{Z_{t+1}} \right) \left(\frac{A_t}{A_{t+1}} \right)^{1-\kappa} \left(\frac{R_{t+1}^K}{R_t^K} \right)^\kappa \left(\frac{W_{t+1}}{A_{t+1}} \frac{A_{t+1}}{A_t} \right)^{(1-\kappa)} \left(\frac{Y_{t+1}}{A_{t+1}} \frac{A_{t+1}}{A_t} \right) \Pi_{t+1}^{(1+\eta)} J_{t+1} \right] \quad (100)$$

$$\implies J_t = 1 + \alpha \mathbb{E}_t \left[M_{t,t+1}^{nom} \left(\frac{Z_t}{Z_{t+1}} \right) \left(\frac{R_{t+1}^K}{R_t^K} \right)^\kappa \left(\frac{\widetilde{W}_{t+1}}{\widetilde{W}_t} \right)^{(1-\kappa)} \left(\frac{\widetilde{Y}_{t+1}}{\widetilde{Y}_t} e^{\Delta a_{t+1}} \right) \Pi_{t+1}^{(1+\eta)} J_{t+1} \right] \quad (101)$$

Nominal pricing kernel,

$$M_{t-1,t}^{nom} = \frac{M_{t-1,t}}{\Pi_t} \quad (102)$$

Euler equation,

$$\frac{1}{R_t^{(1)}} = \mathbb{E}_t [M_{t,t+1}^{nom}] \quad (103)$$

Real bond price,

$$P_t^{real} = \mathbb{E}_t [M_{t,t+1}] \quad (104)$$

Taylor rule,

$$\frac{R_t^{(1)}}{R} = \left(\frac{R_{t-1}^{(1)}}{R} \right)^{\rho_r} \left(\frac{\Pi_t}{\Pi^*} \right)^{(1-\rho_r)\rho_\pi} \left(\frac{\widetilde{Y}_t}{\widetilde{Y}} \right)^{(1-\rho_r)\rho_x} e^{u_t}, \quad (105)$$

where g_t , u_t and z_t are exogenous shocks to government spending, monetary policy and productivity, respectively:

$$\begin{aligned}
g_{t+1} &= (1 - \phi_g)\theta_g + \phi_g g_t + \sigma_{g,t}\epsilon_{g,t+1} \\
\sigma_{g,t+1}^2 &= (1 - \phi_\sigma^g)\theta_\sigma^g + \phi_\sigma^g \sigma_{g,t}^2 + \sigma_{\sigma^g,t}\epsilon_{\sigma,t+1} \\
\tau_{t+1}^k &= (1 - \phi_{\tau^k})\theta_{\tau^k} + \phi_{\tau^k} \tau_t^k + \sigma_{\tau^k,t}\epsilon_{\tau^k,t+1} \\
\sigma_{\tau^k,t+1}^2 &= (1 - \phi_{\sigma^{\tau^k}})\theta_{\sigma^{\tau^k}} + \phi_{\sigma^{\tau^k}} \sigma_{\tau^k,t}^2 + \sigma_{\sigma^{\tau^k},t}\epsilon_{\sigma,t+1} \\
u_{t+1} &= \phi_u u_t + \sigma_u \epsilon_{u,t+1} \\
z_{t+1} &= \phi_z z_t + \sigma_z \epsilon_{z,t+1} \\
\Delta a_{t+1} &= (1 - \phi_a)g_a + \phi_a \Delta a_t + \sigma_a \epsilon_{a,t+1}; \cdot
\end{aligned}$$

B.3.2 The Steady State System of the Model with Growth

The steady state of the pricing kernel block, with the exception of *share*, can be determined right away by noting $M = \beta e^{-\psi g_a}$: Pricing kernel,

$$M = \beta e^{-\psi g_a}, \quad (106)$$

$$R^{cl} = \frac{1}{\beta e^{-\psi g_a}}, \quad (107)$$

$$R^c = \frac{1}{\beta e^{-\psi g_a}}, \quad (108)$$

$$P^c = \frac{\beta e^{(1-\psi)g_a}}{1 - \beta e^{(1-\psi)g_a}}, \quad (109)$$

$$R^l = \frac{1}{\beta e^{-\psi g_a}}, \quad (110)$$

$$P^l = \frac{\beta e^{(1-\psi)g_a}}{1 - \beta e^{(1-\psi)g_a}}. \quad (111)$$

In steady state, capital cancel out in the capital accumulation equation such that

$$\Phi = e^{g_a} + \delta - 1. \quad (112)$$

Given $\Phi = \delta$, the investment-capital ratio and Φ' can be found using the adjustment cost functions

$$\widetilde{Inv} = \left[(e^{g_a} + \delta - 1 - b1) \frac{(1 - 1/\zeta)}{b2} \right]^{\zeta/(\zeta-1)} e^{-g_a} \tilde{K} \quad (113)$$

$$\Phi' = b2 \left[(e^{g_a} + \delta - 1 - b1) \frac{(1 - 1/\zeta)}{b2} \right]^{-1/(\zeta-1)}. \quad (114)$$

Return on investment is also $\frac{1}{\beta e^{-\psi g_a}}$ which allows us to find the rental cost of capital,

$$R^I = \frac{1}{\beta e^{-\psi g_a}} \quad (115)$$

$$Q = \frac{1}{b2} \left[(e^{g_a} + \delta - 1 - b1) \frac{(1 - 1/\zeta)}{b2} \right]^{1/(\zeta-1)} \quad (116)$$

$$R^K = \frac{1}{1 - \tau^k} \left[\frac{1}{\beta e^{-\psi g_a}} - e^{g_a} + [(e^{g_a} + \delta - 1 - b1)(1 - 1/\zeta)] \right] Q. \quad (117)$$

To solve for the steady state inflation, we notice the following:

$$M^{nom} = \frac{\beta e^{-\psi g_a}}{\Pi} \quad (118)$$

$$R^{(1)} = \frac{\Pi}{\beta e^{-\psi g_a}} \quad (119)$$

$$P^{real} = \beta e^{-\psi g_a}. \quad (120)$$

From the Taylor rule,

$$\Pi = \left(R \frac{\beta e^{-\psi g_a}}{\Pi^{*\rho\pi}} \right)^{\frac{1}{1-\rho\pi}}. \quad (121)$$

With steady state inflation given, equilibrium wage offer is:

$$F = \frac{1}{1 - \alpha \beta e^{(1-\psi)g_a} \Pi^{\eta-1}} \quad (122)$$

$$J = \frac{1}{1 - \alpha \beta e^{(1-\psi)g_a} \Pi^\eta} \quad (123)$$

$$\widetilde{W} = \left\{ \left[\frac{1}{1 - \alpha} \left(1 - \alpha \left(\frac{1}{\Pi} \right)^{(1-\eta)} \right) \right]^{\frac{1}{(1-\eta)}} \frac{\kappa^\kappa (1 - \kappa)^{(1-\kappa)} F R^{K-\kappa}}{\nu J} \right\}^{\frac{1}{(1-\kappa)}}. \quad (124)$$

With steady state inflation given, equilibrium wage demand is:

$$H^o = \frac{1}{1 - \theta \beta e^{-\psi g_a} \Pi^{\eta_w-1}} \quad (125)$$

$$G^o = \frac{1}{1 - \theta \beta e^{(1-\psi)g_a} \Pi^{\eta_w}} \quad (126)$$

$$\widetilde{W} = \underbrace{\widetilde{C}^{\psi} N^{\omega} \nu_w \frac{G^o}{H^o} \left[\frac{1}{1 - \theta} \left\{ 1 - \theta \left(\frac{1}{\Pi} \right)^{1-\eta_w} \right\} \right]}_{\psi}^{\frac{-1}{1-\eta_w}}. \quad (127)$$

Capital labor ratio delivers capital in terms of labor input,

$$\widetilde{K} = \frac{\kappa}{1 - \kappa} \frac{\widetilde{W} e^{g_a}}{R^K} N. \quad (128)$$

Combining the production function and the market clearing condition, we can solve for steady state labor by

writing consumption, investment, and capital in terms of labor:

$$(\tilde{K}e^{-g_a})^\kappa N^{1-\kappa} = \frac{\tilde{C} + \tilde{I}nv}{1 - \theta_g} \quad (129)$$

$$\left[\frac{\kappa}{1 - \kappa} \frac{\tilde{W}}{R^K} \right]^\kappa N = \frac{1}{1 - \theta_g} \left\{ \left(\frac{\tilde{W}}{N^\omega \Psi^{1-\mu}} \right)^{1/\psi} + \left[(e^{g_a} + \delta - 1 - b1) \frac{(1 - 1/\zeta)}{b2} \right]^{\zeta/(\zeta-1)} \frac{\kappa}{1 - \kappa} \frac{\tilde{W}}{R^K} N \right\} \quad (130)$$

$$\left(\frac{\tilde{W}}{N^\omega \Psi^{1-\mu}} \right)^{1/\psi} = \left\{ (1 - \theta_g) \left[\frac{\kappa}{1 - \kappa} \frac{\tilde{W}}{R^K} \right]^\kappa - \left[(e^{g_a} + \delta - 1 - b1) \frac{(1 - 1/\zeta)}{b2} \right]^{\zeta/(\zeta-1)} \frac{\kappa}{1 - \kappa} \frac{\tilde{W}}{R^K} \right\} N \quad (131)$$

$$\frac{\tilde{W}}{N^\omega \Psi^{1-\mu}} = \left\{ (1 - \theta_g) \left[\frac{\kappa}{1 - \kappa} \frac{\tilde{W}}{R^K} \right]^\kappa - \left[(e^{g_a} + \delta - 1 - b1) \frac{(1 - 1/\zeta)}{b2} \right]^{\zeta/(\zeta-1)} \frac{\kappa}{1 - \kappa} \frac{\tilde{W}}{R^K} \right\}^\psi N^\psi \quad (132)$$

$$N^{\psi+\omega} = \frac{\tilde{W}}{\Psi^{1-\mu}} \left\{ (1 - \theta_g) \left[\frac{\kappa}{1 - \kappa} \frac{\tilde{W}}{R^K} \right]^\kappa - \left[(e^{g_a} + \delta - 1 - b1) \frac{(1 - 1/\zeta)}{b2} \right]^{\zeta/(\zeta-1)} \frac{\kappa}{1 - \kappa} \frac{\tilde{W}}{R^K} \right\}^{-\psi} \quad (133)$$

$$N = \left[\frac{\tilde{W}}{\Psi^{1-\mu}} \left\{ (1 - \theta_g) \left[\frac{\kappa}{1 - \kappa} \frac{\tilde{W}}{R^K} \right]^\kappa - \left[(e^{g_a} + \delta - 1 - b1) \frac{(1 - 1/\zeta)}{b2} \right]^{\zeta/(\zeta-1)} \frac{\kappa}{1 - \kappa} \frac{\tilde{W}}{R^K} \right\}^{-\psi} \right]^{\frac{1}{\psi+\omega}} \quad (134)$$

where the second equality uses the fact that $\tilde{W} = \tilde{C}^\psi N^\omega \Psi^{1-\mu}$ ²¹. Labor is now written in terms of parameters and known variables. Steady state capital can be calculated using the capital labor ratio. Steady state investment can be found using the adjustment cost function relating investment to capital.

Production function delivers the steady state output,

$$\tilde{Y} = (\tilde{K}e^{-g_a})^\kappa N^{1-\kappa}. \quad (135)$$

Market clearing condition pins down the steady state aggregate consumption,

$$\tilde{C} = (1 - \theta_g)\tilde{Y} - \tilde{I}nv. \quad (136)$$

²¹Combining $\tilde{W} = \tilde{C}^\psi N^\omega \Psi$ and $\tilde{W} = \tilde{C}^\psi N^\omega$ by taking logs and multiplying the former by $1 - \mu$ and the latter by μ . The sum of the two equations become:

$$\begin{aligned} (1 - \mu)\tilde{w} + \mu\tilde{w} &= (1 - \mu)(\psi\tilde{c}^\psi + \omega n^\omega + \log(\Psi)) + \mu(\psi\tilde{c}^\psi + \omega n^\omega) \\ \tilde{w} &= \psi[(1 - \mu)\tilde{c}^\psi + \mu\tilde{c}^\psi] + \omega[(1 - \mu)n^\omega + \mu n^\omega] + (1 - \mu)\log(\Psi) \\ \tilde{w} &= \psi\tilde{c} + \omega n + (1 - \mu)\log(\Psi) \\ \tilde{W} &= \tilde{C}^\psi N^\omega \Psi^{1-\mu}, \end{aligned}$$

where the third equality uses the fact that $\log(\tilde{C}) = \mu\log(\tilde{C}^\psi) + (1 - \mu)\log(\tilde{C}^\omega)$ and $\log(N) = \mu\log(N^\psi) + (1 - \mu)\log(N^\omega)$

Steady state real debt can be calculated from the fiscal rule and the government budget constraint:

$$\widetilde{D}e^{-g_a} = \widetilde{Tax} - \theta_g \widetilde{Y} + \beta e^{-\psi g_a} \widetilde{D} \quad (137)$$

$$\widetilde{D}e^{-g_a} = \rho_b \widetilde{D}e^{-g_a} + \rho_g \theta_g \widetilde{Y} + (1 - \mu) \tau^k R^k \widetilde{K}^o e^{-g_a} - \theta_g \widetilde{Y} + \beta e^{-\psi g_a} \widetilde{D} \quad (138)$$

$$(e^{-g_a} - \rho_b e^{-g_a} - \beta e^{-\psi g_a}) \widetilde{D} = (\rho_g - 1) \theta_g \widetilde{Y} + \theta_{\tau^k} R^k \widetilde{K} e^{-g_a} \quad (139)$$

$$\widetilde{D} = \frac{(1 - \rho_g) \theta_g \widetilde{Y} - \theta_{\tau^k} R^k \widetilde{K} e^{-g_a}}{(\beta e^{-\psi g_a} + \rho_b e^{-g_a} - e^{-g_a})}. \quad (140)$$

Steady state lump-sum transfer is:

$$\widetilde{\tau} = \left[\rho_b \frac{(1 - \rho_g) \theta_g \widetilde{Y} - \theta_{\tau^k} R^k \widetilde{K} e^{-g_a}}{(\beta e^{(1-\psi)g_a} + \rho_b - 1)} + \rho_g \theta_g \widetilde{Y} \right]. \quad (141)$$

Tax revenues are:

$$\widetilde{Tax}^o = \widetilde{\tau} + \theta_{\tau^k} R^k \widetilde{K}^o e^{-g_a} \quad (142)$$

$$\widetilde{Tax}^r = \widetilde{\tau} \quad (143)$$

$$\widetilde{Tax} = \mu \widetilde{Tax}^r + (1 - \mu) \widetilde{Tax}^o \quad (144)$$

Substituting the marginal rate of intratemporal substitution between consumption and labor for the constrained type into the rule-of-thumb budget constraint, steady state consumption of the hand-to-mouth type is:

$$\widetilde{C}^r = \widetilde{W} \left(\frac{\widetilde{W}}{\widetilde{C}^r \psi} \right)^{\frac{1}{\omega}} - \widetilde{\tau}, \quad (145)$$

which is a nonlinear function of C^r and can be solved numerically. Once W and C^r are known, steady state N^r is

$$N^r = \left(\frac{\widetilde{W}}{\widetilde{C}^r \psi} \right)^{\frac{1}{\omega}}, \quad (146)$$

and since C and N are known, the aggregation relationships provide steady state C^o and N^o :

$$\widetilde{C}^o = \exp \left[\frac{\log(\widetilde{C}) - \mu \log(\widetilde{C}^r)}{(1 - \mu)} \right] \quad (147)$$

$$N^o = \exp \left[\frac{\log(N) - \mu \log(N^r)}{(1 - \mu)} \right]. \quad (148)$$

Finally, the following steady states are trivial:

$$\widetilde{K}^o = \frac{\widetilde{K}}{(1 - \mu)} \quad (149)$$

$$\widetilde{Inv}^o = \frac{\widetilde{Inv}}{(1 - \mu)} \quad (150)$$

$$\widetilde{LI} = \widetilde{WN}^o \quad (151)$$

$$share = \frac{1}{1 - \frac{1+\omega}{1-\psi} \frac{P_c \widetilde{C}^o}{F_l \widetilde{LI}}}. \quad (152)$$