

Risk, Uncertainty, and Financial Frictions*

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Abstract

This paper studies the role of financial shocks and uncertainty in causing business cycle fluctuations using a New Keynesian style model. The model features financial frictions, incorporated through a financial accelerator mechanism, and stochastic volatility, which allows the standard deviation of exogenous shocks to change over time. The nonlinear model is estimated using Bayesian techniques and a particle filter. The estimated model is then used to study the impact that financial and uncertainty shocks have on the economy. I find that uncertainty shocks can have a significant impact and are important for explaining business cycles. While most uncertainty shocks have smaller impacts than the traditional shocks commonly found in DSGE models, increases in uncertainty surrounding future intertemporal tradeoffs and wealth can lead to a sizable drop in consumption, investment, and output. I also find that wealth shocks reduce investment and output, showing the importance of financial frictions. The impacts of these shocks are larger than the “risk shocks,” which make borrowers more risky and are modeled after those found in Christiano, Motto, and Rostagno (2014). Using a variance decomposition study, I find that stochastic volatility, especially stochastic volatility associated with wealth, intratemporal, and intertemporal shocks, is important in explaining fluctuations in output, inflation, investment, consumption, and credit spreads. These results suggest that uncertainty shocks associated with entrepreneurial wealth and household behavior are important drivers of the business cycle, but uncertainty is less important than traditional shocks typically found in DSGE models.

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1 Introduction

The “Great Recession” has raised questions about the importance of traditional sources of business cycle fluctuations and has increased interest in alternative sources of fluctuations, including changing financial conditions and increased uncertainty. During the 2008-2009 financial crisis, financial markets tightened and uncertainty rose. Both changes in financial conditions and uncertainty have been promoted as key causes for the recession that followed the crisis, and are discussed as key sources of business cycle fluctuations. While both shocks may be important, it is difficult to disentangle one from the other as financial conditions tend to tighten at the same time uncertainty rises. Therefore, it is important to study the relative importance of the uncertainty and financial shocks within the same framework. Additionally, it is important know which types of uncertainty have the biggest impact on the economy because uncertainty can take many forms.

This paper seeks to further understand the relationship between financial shocks and uncertainty. This is done using an estimated New Keynesian DSGE model with a financial accelerator mechanism similar to BGG. The model is similar to the model used by Fuentes-Albero (2013), Christiano et al. (2014), and Higgins (2016). The model incorporates the risk shocks from Christiano et al. (2014) as well as stochastic volatility as in Fernández-Villaverde et al. (2010). This paper will view changes in the standard deviation of exogenous shocks, which are determined by the stochastic volatility terms to be a change in uncertainty.¹ The inclusion of stochastic volatility allows the model to incorporate uncertainty in the financial sector as well as other sectors, which makes the model well equipped to study the importance of uncertainty and financial shocks within the same model. This paper is most closely related to Christiano et al. (2014) and Gilchrist et al. (2014). This paper differs from Christiano et al. (2014) since the model allows for uncertainty in sectors outside of the financial sector, allowing for uncertainty for all economic shocks. It differs from Gilchrist et al. (2014) as the model in this study is a medium DSGE

¹This differs from true Knightian uncertainty since it can be measured by agents in the model. One could view changes in standard deviations as changes in risk, but this paper will refer to these as changes in uncertainty to avoid confusion with the “risk shocks” in the paper.

model that has been estimated and allows for a much broader range of uncertainty shocks.

The results indicate that uncertainty shocks can have an important impact on the economy. The uncertainty shocks that have the largest impact on output are shocks to the standard deviation of the wealth and intertemporal shocks. The magnitude of the impact of a one standard deviation intertemporal uncertainty shock is similar to a traditional productivity shock. The response of investment to a one standard deviation wealth uncertainty shock is about half the magnitude and more persistent than the response of investment to a traditional monetary policy shock. Additionally, the variance decomposition study shows that stochastic volatility plays a key role in explaining variation in output, investment, consumption, inflation and credit spreads. While I do find that “risk shocks,” as in Christiano et al. (2014), negatively impact output and investment, these shocks are not as important as uncertainty shocks in explaining the variation in the variance decomposition study. In fact, risk shocks play little to no role in explaining the variance of output, inflation, and consumption. In addition to the uncertainty shocks, a shock that reduces wealth impacts the economy by reducing investment due to the inclusion of financial frictions. The importance of shocks associated with the financial sector provide support to the concept that shocks, both traditional and uncertainty shocks, within the financial sector are important drivers of business cycles, as in Caldara et al. (2016). The results also point to the importance of uncertainty about intertemporal shocks, where increased uncertainty dampens consumption and investment, lowering output.

The rest of the paper is organized as follows. Section 2 provides a brief literature review and section 3 presents the model to be estimated. Sections 4 and 5 describe the data sources and present the estimation strategy. Section 6 presents the priors and estimation results, while section 7 studies the IRFs from the model and section 8 discusses the variance decomposition. Finally, section 9 provides some concluding remarks and describes some potential future related research.

2 Literature Review

The importance of uncertainty in driving economic fluctuations has been well studied. Several studies have focused on the impact of uncertainty under investment, including Lucas and Prescott (1971).² Bloom et al. (2007), Bloom (2009), and Bloom et al. (2014) show that uncertainty is an important driver of economic fluctuations. Bloom et al. (2014) show that uncertainty rises during recessions and increases in uncertainty can cause 2.5% reductions of GDP. Fernández-Villaverde et al. (2015a) and Davig and Foerster (2015) show that fiscal uncertainty can have a large, adverse effect on economic activity. Uncertainty is often closely associated with volatility, with both often being measured as standard deviations. Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramírez (2010) show that stochastic volatility, where the standard deviation of exogenous shocks are allowed to change over time, helps with model fit and plays an important role in explaining the Great Moderation in the United States.³

Many macroeconomic models have assumed frictionless financial markets using the Modigliani-Miller paradigm. In recent years, many studies have shown the importance of financial frictions in explaining business cycle fluctuations. Theoretical models often use a framework similar those found in Kiyotaki and Moore (1997) or Bernanke, Gertler and Gilchrist (1999), which I will refer to as BGG. Several studies have shown the importance of financial frictions and credit spreads both theoretically and empirically.⁴ In a closely related paper, Christiano, Motto, and Rostagno (2014) show that risk shocks, which is modeled as changes in the idiosyncratic uncertainty about the productivity of borrowers, is important in explaining business cycle fluctuations. In other studies, Fuentes-Albero (2013) and Higgins (2016) show that changing financial frictions were an important factor in reducing volatility during the Great Moderation.

While both uncertainty and financial shocks may be important drivers of business cycles,

²For an extended discussion of the topic see Dixit and Pindyck (1993).

³Stochastic volatility is used in this study to model changes in uncertainty.

⁴Among others, see Iacoviello (2005), Mueller (2009), Arellano, Bai and Kehoe (2012), Gilchrist and Zakrajšek (2012), Faust et al. (2013), Iacoviello and Pavan (2013), and Gilchrist and Mojon (2014).

it is difficult to empirically differentiate between the two. This is because uncertainty and financial tightness tend to comove. This can be seen in Figure 1, where the VIX, a measure of uncertainty, and the spread between Baa corporate bond and 10 year treasury yields, a measure of financial tightness, tend to follow a similar pattern. The difficulties involved with disentangling the impact of uncertainty and financial tightness is emphasized by Stock and Watson (2012) and Caldara et al. (2016). Caldara et al. (2016) uses a penalty function to differentiate between the two sources and find that financial shocks have a significant effect on the economy. They also find that uncertainty impacts the economy when it elicits a persistent tightening of financial conditions. This is supported theoretically by Gilchrist, Sim, and Zakrajšek (2014), who show that uncertainty can impact the economy through financial distortions.

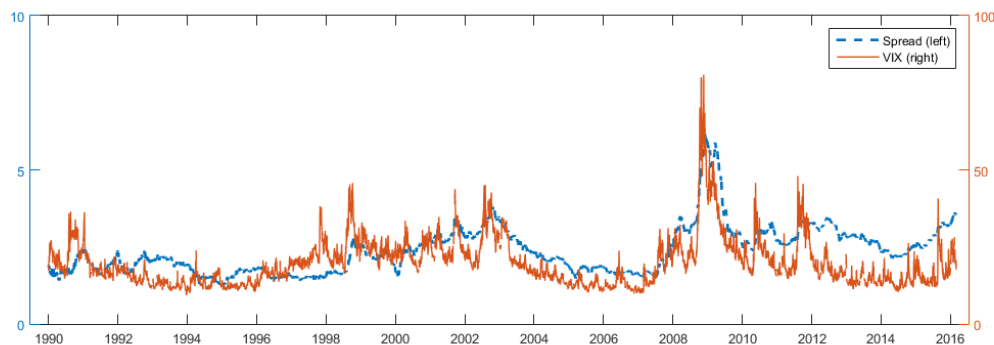


Figure 1: VIX and Baa-10 Year Treasury Spread

This paper builds on the literature by looking at uncertainty and financial shocks within an estimated DSGE model. A key feature of the model is that it allows for several types of uncertainty through the inclusion of stochastic volatility. The model is estimated using Bayesian techniques and data from the United States. Due to the inclusion of stochastic volatility, the model is nonlinear in nature, so the model has to be approximated using a second order perturbation before it can be estimated.⁵ The model is then estimated using Bayesian techniques and a particle filter as in Fernández-Villaverde, Guerrón-Quintana,

⁵Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006) and Caldara, Fernández-Villaverde, Rubio-Ramírez, and Yao (2012) show that perturbation methods are both fast and accurate in general and for models featuring stochastic volatility.

and Rubio-Ramírez (2015b) and Higgins (2016). While a second order approximation is sufficient for estimating the model, it cannot be used to study impulse response functions associated with uncertainty shocks.⁶ Therefore, a third-order perturbation is used to calculate the impulse response functions (IRFs). Using calibrations drawn from the estimation results, the IRFs are calculated using the pruning and generalized impulse response function (GIRF) procedures detailed in Andreasen, Fernández-Villaverde and Rubio-Ramírez (2014).⁷ The pruned third-order approximation is also used to simulate data for the variance decomposition study.

3 Model

The model economy is populated by a continuum of households, financial intermediaries, entrepreneurs, capital producers, final good producers, and a government. A New Keynesian model serves as the basis for the model, with aspects of the financial accelerator model from BGG. Stochastic volatility is incorporated into the model so the standard deviation of the underlying shocks are allowed to drift over time, which allows for the level of uncertainty in the economy to change over time. The monetary policy rule parameters are allowed to drift over time in order to pick up changes in monetary policy.⁸ The model is based on the models of Smets and Wouters (2007), Christiano, Motto and Rostagno (2010), Fernández-Villaverde et al. (2010), Fuentes-Albero (2013), and Higgins (2016).

The main departure from a standard New-Keynesian or RBC model is the inclusion of a financial accelerator à la BGG. The financial accelerator introduces entrepreneurs who use their own funds and borrow funds in order to purchase capital at the end of every period. After capital is purchased, entrepreneurs face a productivity shock that the entrepreneur costlessly observes, but the financial intermediary must pay a fee in order to observe the

⁶Due to computing and time limitations, a third-order approximation is not used for the estimation. Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramírez (2015b) show that the stochastic volatility terms can still be estimated with a second-order approximation even if impulse response functions cannot be calculated from the second-order approximation.

⁷Code for how to implement the procedure using Dynare is available on Andreasen's personal website and was used in this paper.

⁸This is important since data from before the Great Moderation is used for the estimation. Studies have indicated that monetary policy may have changed during the 1980s. For more information see Clarida, Galí, and Gertler (2000) and Fernández-Villaverde et al. (2010).

entrepreneur's productivity. This asymmetric information introduces new dynamics into the model and can amplify business cycles.

3.1 Household

There is a continuum of households indexed by j who maximize expected utility represented by

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s d_{t+s} \left[\log (c_{j,t+s} - hc_{j,t+s-1}) - \varphi_t \psi \frac{l_{j,t+s}^{1+\vartheta}}{1+\vartheta} \right] \quad (1)$$

subject to the budget constraint

$$c_{j,t+s} + \frac{D_{j,t+s+1}}{P_{t+s}} + \frac{NB_{j,t+s+1}}{P_{t+s}} \leq w_{j,t+s} l_{t+s} + R_{t+s-1} \frac{D_{j,t+s}}{P_{t+s}} + R_{t+s-1} \frac{NB_{j,t+s}}{P_{t+s}} + div_{t+s} - T_{t+s} - Trans_{t+s}, \quad \forall s.$$

The utility function is separable in consumption, $c_{j,t}$ and hours worked, $l_{j,t}$. β is the discount factor, R_t is the risk-free, gross nominal interest rate paid on deposits and government bonds, $NB_{j,t}$ is the nominal value of government bonds held, $D_{j,t}$ is the nominal value of deposits at the financial intermediary, div_t is the real value of dividends obtained from ownership of firms, T_t are taxes paid, and $Trans_t$ is the real value of wealth transfers to/from the entrepreneurial sector. The household faces an intertemporal preference shock, d_t , and an intratemporal preference shock, φ_t , that evolves as

$$\log d_t = \rho_d \log d_{t-1} + \sigma_{d,t} \varepsilon_{d,t} \quad (2)$$

$$\log \varphi_t = \rho_\varphi \log \varphi_{t-1} + \sigma_{\varphi,t} \varepsilon_{\varphi,t}. \quad (3)$$

With the inclusion of stochastic volatility, the standard deviation of the innovation is allowed to change over time and follows the process

$$\log \sigma_{d,t} = (1 - \rho_{\sigma_d}) \log \sigma_d + \rho_{\sigma_d} \log \sigma_{d,t-1} + \eta_d u_{d,t} \quad (4)$$

$$\log \sigma_{\varphi,t} = (1 - \rho_{\sigma_{\varphi}}) \log \sigma_{\varphi} + \rho_{\sigma_{\varphi}} \log \sigma_{\varphi,t-1} + \eta_{\varphi} u_{\varphi,t}. \quad (5)$$

The exogenous innovations, $\varepsilon_{d,t}$, $\varepsilon_{\varphi,t}$, $u_{d,t}$, and $u_{\varphi,t}$, and all other innovations described in the model are i.i.d. $N(0, 1)$. Allowing for $\sigma_{d,t}$ and $\sigma_{\varphi,t}$ to drift through time will allow the model to pick up changes in the nature of underlying shocks over time as in Fernández-Villaverde et al. (2010) and will be interpreted as changes in uncertainty. This specification of stochastic volatility will be present for all shocks, excluding parameter drift shocks and the risk shock, in the model.

Households provide differentiated labor to a “labor packer” in a monopolistically competitive market. The labor is aggregated using the following production function

$$l_t^d = \left(\int_0^1 l_{j,t}^{\frac{\eta-1}{\eta}} dj \right)^{\frac{\eta}{\eta-1}}. \quad (6)$$

The “packer” maximizes profits given the wage, w_t , and the differentiated wage, $w_{j,t}$ using the following equation

$$\max_{l_{j,t}} w_t l_t^d - \int_0^1 w_{j,t} l_{j,t} dj. \quad (7)$$

Households face a Calvo pricing mechanism for their wages. Each period, a fraction $1 - \theta_w$, of households are able to optimize their prices. Households that are not able to optimize their wages partially index wages based on previous inflation using the indexation parameter χ_w , setting prices as $w_{i,t} = \Pi_{t-1}^{\chi_w} w_{i,t-1}$, where $\Pi_t = P_t/P_{t-1}$ is the gross inflation rate.

3.2 Final Good Production

A perfectly competitive final good producer aggregates a continuum of intermediate goods using the production function

$$y_t^d = \left(\int_0^1 y_{i,t}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \quad (8)$$

where ϵ is the elasticity of substitution for the intermediate goods. The final goods producer minimizes costs given this production function, the price of intermediate goods, $p_{i,t}$, and the price of the final good, p_t . This minimization leads to the following demand function

$$y_{i,t} = \left(\frac{p_{i,t}}{p_t} \right)^{-\epsilon} y_t^d \quad \forall i$$

where the price of the final good is defined by

$$p_t = \left(\int_0^1 p_{i,t}^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}.$$

3.3 Intermediate Good Production

Intermediate goods are produced by a monopolistic competitor with a Cobb-Douglas production function defined as

$$y_{i,t} = A_t k_{i,t}^\alpha (l_{i,t}^d)^{1-\alpha} \quad (9)$$

where $k_{i,t}$ is the capital rented and $l_{i,t}^d$ is the “packed” labor rented by firm i . The log of the productivity term, A_t , follows a random walk and features stochastic volatility. The process is defined as

$$\log A_t = \log A_{t-1} + \Upsilon_A + \sigma_{A,t} \varepsilon_{A,t} \quad (10)$$

where the standard deviation of the innovation follows the process

$$\log \sigma_{A,t} = (1 - \rho_{\sigma_A}) \log \sigma_A + \rho_{\sigma_A} \log \sigma_{A,t-1} + \eta_d u_{A,t}. \quad (11)$$

Intermediate good producers face a Calvo pricing mechanism. Each period, a fraction $1 - \theta_p$, of firms are able to optimize their prices. Firms that are not able to optimize their prices partially index their prices based on previous inflation using the indexation parameter χ , setting prices as $p_{j,t} = \Pi_{t-1}^\chi p_{j,t-1}$.

3.4 Capital Producers

Capital producers are perfectly competitive, infinitely lived agents who produce new capital and purchase capital from entrepreneurs. Capital is produced using a linear production function, where one unit of investment in period t produces ζ_t units of time $t + 1$ physical capital. The log of the productivity of capital production, ζ_t , follows a random walk with drift defined by the process

$$\log \zeta_t = \log \zeta_{t-1} + \Upsilon_\zeta + \sigma_{\zeta,t} \varepsilon_{\zeta,t} \quad (12)$$

where the standard deviation of the innovation follows the process

$$\log \sigma_{\zeta,t} = (1 - \rho_{\sigma_\zeta}) \log \sigma_\zeta + \rho_{\sigma_\zeta} \log \sigma_{\zeta,t-1} + \eta_{\zeta} u_{\zeta,t}.^9 \quad (13)$$

Capital producers face a capital adjustment cost defined as $V \left[\frac{I_t}{I_{t-1}} \right] = \frac{\chi}{2} \left(\frac{I_t}{I_{t-1}} - \Upsilon_z \right)^2$. Capital producers chose a level of investment to maximize profits, which results in the following relationship

$$1 = Q_t \zeta_t \left(1 - V \left[\frac{I_t}{I_{t-1}} \right] - V' \left[\frac{I_t}{I_{t-1}} \right] \frac{I_t}{I_{t-1}} \right) + \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \zeta_{t+1} Q_{t+1} V' \left[\frac{I_{t+1}}{I_t} \right] \left(\frac{I_{t+1}}{I_t} \right)^2. \quad (14)$$

Where Q_t represents the relative price of capital.

The aggregate capital stock evolves as

$$K_{t+1} = (1 - \delta) K_t + \zeta_t \left(1 - V \left[\frac{I_t}{I_{t-1}} \right] \right) I_t. \quad (15)$$

Where δ is the depreciation rate.

3.5 Financial Intermediaries and Entrepreneurs

Entrepreneurs are risk-neutral, finitely lived agents who can borrow funds from financial intermediates. Entrepreneurs survive from one period to the next with probability ν . Fi-

⁹To ease notation later, define $z_t = A_t^{(1/1-\alpha)} \zeta_t^{(\alpha/1-\alpha)}$ and $\Upsilon_z = (\Upsilon_A + \Upsilon_\zeta)/(1 - \alpha)$.

financial intermediaries attain funds from households. At the end of period t , entrepreneurs purchase physical capital. At the beginning of period $t + 1$, entrepreneurs observe an idiosyncratic shock that affects the productivity of their capital. Entrepreneurs then determine how much of their capital to rent. At the end of the period entrepreneurs sell off undepreciated capital and pay of debts to the financial intermediary.

Each entrepreneur faces an idiosyncratic productivity shock, ω_t^j , at the beginning of each period which is only observed by the entrepreneur and affects the productivity of his capital holdings. This shock is assumed to have a lognormal distribution with c.d.f. $F(\omega)$ with parameters μ_ω and $\sigma_{\omega,t}$ satisfying $\mathbb{E}(\omega^j) = 1$ in the steady state. As in Christiano et al. (2014), the standard deviation of the distribution changes overtime through a “risk shock” and evolves according to the following AR(1) process

$$\log \sigma_{\omega,t} = (1 - \rho_{\sigma_\omega}) \log \sigma_\omega + \rho_{\sigma_\omega} \log \sigma_{\omega,t-1} + \eta_\omega u_{\omega,t} \quad (16)$$

where $u_{\omega,t} \sim_{iid} N(0, 1)$.

Entrepreneurs maximize profits by determining the fraction of capital to utilize, u_t^j , to solve

$$\max_{u_t^j} \left[u_t^j r_t^{j,k} - \zeta_t^{-1} a(u_t^j) \right] \omega_t^j K_t^j. \quad (17)$$

The rental rate of capital is denoted $r_t^{j,k}$. $a(\cdot)$ represents the cost of utilizing capital and is defined as $a(u_t) = \gamma_1 (u_t - 1) + \frac{1}{2} \gamma_2 (u_t - 1)^2$.

Capital demand for entrepreneur j is determined by the gross nominal returns on holding capital

$$R_{t+1}^{j,k} = \left[\frac{\left(u_{t+1}^j r_{t+1}^{j,k} - \zeta_t a(u_{t+1}^j) \right) + \omega_{t+1}^j (1 - \delta) Q_{t+1}}{Q_t} \right] \frac{P_{t+1}}{P_t}. \quad (18)$$

The gross return on capital is denoted $R_{t+1}^{j,k}$ and $\omega_{t+1}^j (1 - \delta) Q_{t+1}$ is the return from selling undepreciated capital.

An entrepreneur can use his own net worth, N_{t+1}^j , or external financing to purchase new physical capital. Lenders are unable to observe the returns of entrepreneurs unless they

pay an auditing cost. Due to cost minimization, lenders will only audit entrepreneurs when the loan is not fully repaid. The auditing cost is represented by a fraction, μ , being lost in the process of liquidation leaving $(1 - \mu)P_t\omega_{t+1}^j R_{t+1}^k Q_t K_{t+1}^j$.

The debt contract is set to maximize expected entrepreneurial profits subject to a participation constraint. The maximization problem is defined as

$$\max_{\bar{\omega}_{t+1}, k_{t+1}} \mathbb{E}_t \left\{ [1 - \Gamma_t(\bar{\omega}_{t+1})] \frac{R_{t+1}^k}{R_t} (B_{t+1} + N_{t+1}) \right\} \quad (19)$$

subject to the constraint

$$\frac{R_{t+1}^k}{R_t} [\Gamma_t(\bar{\omega}_{t+1}) - \mu_{t+1} G_t(\bar{\omega}_{t+1})] = \frac{Q_t K_{t+1} - N_{t+1}}{Q_t K_{t+1}} \quad (20)$$

where $\Gamma_t(\bar{\omega}_{t+1})$ is the expected share of gross entrepreneurial earnings going to the lender and $\mu G_t(\bar{\omega}_{t+1})$ is the expected monitoring cost.¹⁰ The size of the loan is defined as B_{t+1} , the net worth of the borrower is defined as N_{t+1} , Q_t is the shadow cost of capital, and $\bar{\omega}_{t+1}$ represents the productivity that must be drawn in order for the borrower to be able to repay the loan. From the first-order conditions of the debt contract, the average gross interest rate for entrepreneurs, Z_{t+1} , is calculated as

$$Z_{t+1} = R_{t+1} + \frac{\mu_{t+1} G_t(\bar{\omega}_{t+1}) K_{t+1} Q_{t+1}}{K_{t+1} Q_t - N_{t+1}}. \quad (21)$$

The net worth of entrepreneurs evolves as

$$P_t N_{t+1} = x_t \nu V_t + P_t W_t^e \quad (22)$$

where x_t represents a wealth shock, ν is the survival probability, V_t represents entrepreneurial equity, and W_t^e is the value of wealth transfers made by exiting firms.¹¹ The wealth shock changes over time according to $\log x_t = (1 - \rho_x) \log x + \rho_x \log x_{t-1} + \sigma_{x,t} \varepsilon_{x,t}$ and $\log \sigma_{x,t} = (1 - \rho_{\sigma_x}) \log \sigma_x + \rho_{\sigma_x} \log \sigma_{x,t} + \eta_x u_{x,t}$. The entrepreneurial sector provides trans-

¹⁰ $\Gamma_t(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega f_t(\omega) d\omega + \bar{\omega}_{t+1} \int_{\bar{\omega}_{t+1}}^{\infty} f_t(\omega) d\omega$ and $G_t(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega f_t(\omega) d\omega$.

¹¹ $V_t = P_{t-1} R_t^k Q_{t-1} K_t - R_{t-1} B_t - \mu G_t(\bar{\omega}_t) P_{t-1} R_t^k Q_{t-1} K_t$

fers to/from the private sector equal to $Trans_t = N_{t+1} - \nu V_t - W^e$.

3.6 Monetary policy

The central bank sets the interest rate using a Taylor rule with drifting parameters. The rule is defined as

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{1-\gamma_R} \left(\left(\frac{\Pi_t}{\Pi}\right)^{\psi_{1,t}} \left(\frac{Y_t}{Y_{t-1}}\right)^{\psi_2} \right)^{1-\gamma_R} m_t. \quad (23)$$

The Taylor coefficient on inflation drifts according to the processes

$$\log(\psi_{1,t}) = \rho_{\psi_1} \log(\psi_{1,t-1}) + (1 - \rho_{\psi_1}) \log(\psi_1) + \eta_{\psi_1} \varepsilon_{\psi_{1,t}}. \quad (24)$$

The monetary policy shock is defined as $\log m_t = \sigma_{m,t} \varepsilon_{m,t}$ and $\sigma_{m,t}$ follows the process

$$\log \sigma_{m,t} = (1 - \rho_{\sigma_m}) \log \sigma_m + \rho_{\sigma_m} \log \sigma_{m,t} + \eta_m u_{m,t}. \quad (25)$$

3.7 Market clearing

The market clearing condition for the goods market is defined by

$$Y_t = C_t + I_t + G_t + \frac{1}{\zeta_t} a(u_t) K_t + \mu G_t (\bar{\omega}_t) R_t^k Q_{t-1} K_t. \quad (26)$$

Government spending is allowed to change over time and is defined by $G_t = (1/(1 + \tilde{g}g_t)) Y_t$ where the government spending shock follows

$$\log(g_t) = \rho_g \log(g_{t-1}) + \sigma_{g,t} \varepsilon_{g,t} \quad (27)$$

where the standard deviation of the innovation follows the process

$$\log \sigma_{g,t} = (1 - \rho_{\sigma_g}) \log \sigma_g + \rho_{\sigma_g} \log \sigma_{g,t} + \eta_g u_{g,t}. \quad (28)$$

Credit market clearing is defined by

$$\frac{D_{t+1}}{P_t} = \frac{B_{t+1}}{P_t} = Q_t K_{t+1} - N_{t+1}. \quad (29)$$

3.8 Equilibrium

The equilibrium can be characterized by the first order conditions, the Taylor rule monetary policy and market clearing conditions. The equilibrium is not stationary due to the unit root in the processes for technology and capital producer productivity shocks, so some variables must be normalized. To do this let $\tilde{c}_t = c_t/z_t$, $\tilde{I}_t = I_t/z_t$, $\tilde{K}_t = K_t/(z_t\zeta_t)$, $\tilde{B}_{t+1} = B_{t+1}/z_t$, $\tilde{N}B_{t+1} = NB_{t+1}/z_t$, $\tilde{N}_{t+1} = N_{t+1}/z_t$, $\tilde{div}_t = div_t/z_t$, $\tilde{T}_t = T_t/z_t$, $\tilde{\lambda}_t = \lambda_t z_t$, $\tilde{r}_t = r_t\zeta_t$, $\tilde{Q}_t = Q_t\zeta_t$, $\tilde{w}_t = w_t/z_t$, $\tilde{w}^*_t = w^*_t/z_t$, and $\tilde{y}_t = y_t/z_t$.¹²

4 Data

The data used to estimate the model spans from 1954.Q4 to 2015.Q2.¹³ Eight data series are used: growth rate of real output, growth rate of real per capita investment, growth rate of real per capita consumption, the growth rate of real wages, inflation (measured as the logged ratio of the price level this period and last period), the gross federal funds rate, the spread between the Moody's Baa corporate bond interest rate and the 10 year treasury yield, and the growth rate of net worth.¹⁴

All observables, except the growth rate of net worth, are assumed to be measured without measurement error.¹⁵ While measurement errors are a potential feature of the data, not using measurement error sharpens the estimation strategy and makes for cleaner counterfactual studies.

¹²The first order conditions can be found in the Appendix.

¹³In order to include data from the financial crisis, data from a period where policy is at the zero lower bound has to be used. Since the estimation strategy is already computationally taxing and time consuming, no attempts are made to adjust for this.

¹⁴For more details on how the data is calculated, see the Appendix. In the Appendix, there is also discussion of the measure of the external finance premium.

¹⁵The measurement error is assumed to be normal, mean 0 and independent across observations. The standard deviation of the measurement error is estimated with the other parameters.

5 Model Solution and Estimation Strategy

Based on the complexity, size and nonlinearities of the model, estimation is difficult. The model does not have a closed-form solution and the likelihood function cannot be analytically solved, so numerical approximations must be used. To achieve this, I first do a second-order approximation of the model around the steady-state using perturbation methods and then approximate the likelihood using a particle filter. This is then used in a Metropolis-Hastings algorithm to estimate the posterior.

The estimation follows Higgins (2016) and Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramírez (2015b), who show that many dynamic equilibrium models can be written as

$$\mathbb{E}_t f(\mathcal{Y}_{t+1}, \mathcal{Y}_t, \mathcal{S}_{t+1}, \mathcal{S}_t, \mathcal{Z}_{t+1}, \mathcal{Z}_t; \gamma) = 0 \quad (30)$$

where \mathbb{E}_t represents the conditional expectation operator at time t , \mathcal{Y}_t represents the $k \times 1$ vector of observables at time t , \mathcal{S}_t represents the $n \times 1$ vector of endogenous states at time t , \mathcal{Z}_t represents the $m \times 1$ vector of structural shocks at time t , γ represents the vector of parameters in the model, and f maps $\mathbb{R}^{2(k+n+m)}$ into \mathbb{R}^{k+n+m} . In this model, the endogenous state is defined as

$$\mathcal{S}_t = (\log R_{t-1}, \log \tilde{c}_{t-1}, \log \tilde{I}_{t-1}, \log \tilde{y}_{t-1}, \log R_{t-1}^k, \log \tilde{N}_{t-1}, \log q_{t-1}, \log \Pi_{t-1})'$$

The observables are defined as

$$\mathcal{Y}_t = \left(\Delta \log \tilde{y}_t, \Delta \log \tilde{I}_t, \Delta \log \tilde{c}_t, \Delta \log \tilde{w}_t, \Pi_t, R_t, Z_t - R_t, \Delta \log \tilde{N}_t \right)'$$

The structural shocks are defined as

$$\mathcal{Z}_t = (\log x_t, \log d_t, \log g_t, \log \tilde{\zeta}_t, \log \tilde{A}_t, \log \sigma_{\omega,t}, \log \varphi_{1,t})'$$

The structural shocks are assumed to follow the process

$$\mathcal{Z}_{it+1} = \rho_i \mathcal{Z}_{it} + \Lambda \sigma_i \sigma_{it+1} \epsilon_{it+1} \quad (31)$$

for all $i \in 1, \dots, m$ where σ_{it+1} denotes the stochastic volatility shock and Λ represents the perturbation parameter. The stochastic volatility shock evolves as

$$\log \sigma_{it+1} = \rho_{\sigma_i} \log \sigma_{it} + \Lambda \eta_i u_{it+1}.^{16} \quad (32)$$

To approximate the model it is first put into a state space representation. The endogenous state vector evolves as

$$\mathcal{S}_{t+1} = h(\mathcal{S}_t, \mathcal{Z}_{t-1}, \Sigma_{t-1}, \mathcal{E}_t, \mathcal{U}_t, \Lambda; \gamma) \quad (33)$$

while two policy functions evolve according to the following processes

$$\mathcal{Y}_t = g(\mathcal{S}_t, \mathcal{Z}_{t-1}, \Sigma_{t-1}, \mathcal{E}_t, \mathcal{U}_t, \Lambda; \gamma) \quad (34)$$

$$\mathcal{Y}_{t+1} = g(\mathcal{S}_{t+1}, \mathcal{Z}_t, \Sigma_t, \Lambda \mathcal{E}_{t+1}, \Lambda \mathcal{U}_{t+1}, \Lambda; \gamma). \quad (35)$$

The functions h and g map \mathbb{R}^{n+4m+1} into \mathbb{R}^n and \mathbb{R}^k . The vector of volatility shocks is defined as

$$\Sigma_t = (\hat{\sigma}_{x,t}, \hat{\sigma}_{d,t}, \hat{\sigma}_{g,t}, \hat{\sigma}_{\zeta,t}, \hat{\sigma}_{m,t}, \hat{\sigma}_{A,t}, 0, 0)'$$

where $\hat{\sigma}_{x,t} = \sigma_{x,t}/\sigma_x$. The innovations are divided between an $m \times 1$ vector of innovation to structural shocks, \mathcal{E}_t and an $m \times 1$ vector of innovations to volatility shocks, \mathcal{U}_t . For this model, the vector of innovations are defined as

$$\mathcal{E}_t = (\varepsilon_{xt}, \varepsilon_{dt}, \varepsilon_{gt}, \varepsilon_{\zeta t}, \varepsilon_{mt}, \varepsilon_{At}, \varepsilon_{\mu t}, \varepsilon_{\psi_1 t})'$$

¹⁶This differs slightly from Fernández-Villaverde et al. (2015b) in that the innovation to the stochastic volatility shock is not normalized by the level of persistence, but this could be factored in when interpreting results.

and

$$\mathcal{U}_t = (u_{xt}, u_{dt}, u_{gt}, u_{\zeta t}, u_{mt}, u_{At}, 0, 0)'$$

It is important to note that the zeros in \mathcal{E}_t and \mathcal{U}_t are due to $\sigma_{\omega,t}$ and $\psi_{1,t}$ not being subject to stochastic volatility.

As in Fernández-Villaverde et al. (2015b), equations 31 to 33 are stacked into the following transition equation

$$\mathbb{S}_{t+1} = \tilde{h}(\mathbb{S}_t, \Lambda; \gamma) + \Xi \mathbb{W}_{t+1} \quad (36)$$

where $\mathbb{S}_t = (\mathcal{S}'_t, \mathcal{Z}'_{t-1}, \Sigma'_t, \mathcal{E}'_{t-1}, \mathcal{U}'_{t-1})'$ and \tilde{h} maps \mathbb{R}^{n+4m+1} into \mathbb{R}^{n+4m} . The vector $\mathbb{W}_{t+1} = (\mathcal{W}'_{1t+1}, \mathcal{W}'_{2t+1})'$ is a $2m \times 1$ vector of random variables, where \mathcal{W}_{1t+1} and \mathcal{W}_{2t+1} are $m \times 1$ vectors with $N(0, I)$ distributions and Ξ is a $(n + 4m) \times 2m$ matrix, where the top $n + 2m$ rows equal zero and the bottom of the matrix equal to a $2m \times 2m$ identity matrix. The policy function can also be rewritten as

$$\mathcal{Y}_t = g(\mathbb{S}_t, \Lambda; \gamma). \quad (37)$$

Equations 36 and 37 are approximated around the steady-state using the perturbation method to find a second-order approximation.¹⁷ The approximate solution is then used in a particle filter with 10,000 particles to approximate the likelihood.

The particle filter used is very similar to the one described in Fernández-Villaverde et al. (2015b), but there is one key difference: in this paper there is one more observable than stochastic volatility shocks so measurement error must be included in the model. In order to integrate measurement error into the estimation procedure, changes have to be made to the particle filter. The differences between this particle filter with measurement error can be seen when calculating $p(\mathcal{Y}_t = \mathbb{Y}_t^{data} | s_t^i, z_{t-1}^i, \sigma_{t-1}^i, \varepsilon_t^i; \gamma)$ using simulated draws $(s_t^i, z_{t-1}^i, \sigma_{t-1}^i, \varepsilon_t^i)$. Fernández-Villaverde et al. (2015b) show that this can be calculated

¹⁷For more information on second order perturbation techniques see Kim et al. (2005), Schmitt-Grohe and Uribe (2004) and Gomme and Klein (2011). Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006) and Caldara, Fernández-Villaverde, Rubio-Ramírez, and Yao (2012) show that perturbation methods are both fast and accurate in general and for models featuring stochastic volatility.

using terms from the approximated observation equation to define

$$\mathbb{A}_t (\mathbb{Y}_t, s_t^i, z_{t-1}^i, \sigma_{t-1}^i, \varepsilon_t^i) =$$

$$\mathbb{Y}_t - \mathcal{Y} - \begin{pmatrix} \tilde{\Psi}_{Y_1}^1 \tilde{\hat{S}}_t^i \\ \vdots \\ \tilde{\Psi}_{Y_k}^1 \tilde{\hat{S}}_t^i \end{pmatrix} + \frac{1}{2} \begin{pmatrix} \tilde{\hat{S}}_t^{i'} \tilde{\Psi}_{y,1}^{2,1} \tilde{\hat{S}}_t^i \\ \vdots \\ \tilde{\hat{S}}_t^{i'} \tilde{\Psi}_{y,k}^{2,1} \tilde{\hat{S}}_t^i \end{pmatrix} + \frac{1}{2} \begin{pmatrix} \Psi_{y,1}^\Lambda \\ \vdots \\ \Psi_{y,k}^\Lambda \end{pmatrix} + \begin{pmatrix} \mathcal{E}_t^{i'} \tilde{\Psi}_{y,1}^{2,2} \\ \vdots \\ \mathcal{E}_t^{i'} \tilde{\Psi}_{y,k}^{2,2} \end{pmatrix} \sigma_{t-1}^i$$

where $\tilde{\hat{S}}_t^i = (s_t^{i'}, z_{t-1}^{i'}, \sigma_{t-1}^{i'}, \varepsilon_t^{i'})'$ represents the simulated states without the stochastic volatility components and $\tilde{\hat{S}}_t^i$ represents these states in deviation from mean form. Also, $\tilde{\Psi}_{Y_j}^1$ represents the first-order components of the approximation to the observation equation that relates to $\tilde{\hat{S}}_t^i$, while $\tilde{\Psi}_{y,j}^{2,1}$ represents the second order components relating to $\tilde{\hat{S}}_t^i$, $\Psi_{y,j}^\Lambda$ represents the linear terms of the approximation, and $\tilde{\Psi}_{y,j}^{2,2}$ represent the second order components related to the stochastic volatility shocks for $j = 1, \dots, k$. In a departure from Fernández-Villaverde et al. (2015b), it is also important to define

$$\mathbb{B}(\mathcal{E}_t) = \begin{pmatrix} \mathcal{E}_t^{i'} \hat{\Psi}_{y,1}^{2,2}, \mathbf{0}_{m \times 1} \\ \vdots \\ \mathcal{E}_t^{i'} \hat{\Psi}_{y,k-m}^{2,2}, \mathbf{0}_{m \times 1} \\ \mathcal{E}_t^{i'} \hat{\Psi}_{y,k}^{2,2}, 1, \mathbf{0}_{m-1 \times 1} \\ \mathcal{E}_t^{i'} \hat{\Psi}_{y,k}^{2,2}, 0, 1, \mathbf{0}_{m-2 \times 1} \\ \vdots \\ \mathcal{E}_t^{i'} \hat{\Psi}_{y,k}^{2,2}, \mathbf{0}_{m-1 \times 1}, 1 \end{pmatrix} \quad (38)$$

where $\hat{\Psi}_{y,j}^{2,2}$ represents the second order components of the approximation the observation equation relating to \mathcal{E}_t and Σ_{t-1} . Equation 38 differs from Fernández-Villaverde et al. (2015b) due to the inclusion of measurement error for m observables. In this paper $m = 1$ and it is important that the observable that is assumed to have measurement error, $\Delta \log N_t$,

is listed last in \mathcal{Y}_t . These equations are used to calculate the measurement density as

$$p(\mathcal{Y}_t = \mathbb{Y}_t | s_t^i, z_{t-1}^i, \sigma_{t-1}^i; \gamma) = \left| \det \left(\mathbb{B}^{-1} \left(\mathcal{E}_t^{i'}; \gamma \right) \right) \right| p \left((\mathcal{U}_t, me_t) = \mathbb{B}^{-1} \left(\mathcal{E}_t^{i'}; \gamma \right) \mathbb{A}_t \left(s_t^i, z_{t-1}^i, \sigma_{t-1}^i; \gamma \right) \right) \quad (39)$$

where me_t is the $m \times 1$ vector of measurement error. This can be evaluated since the distribution of \mathcal{U}_t and me_t are both known. The measurement density can be applied to the particle filter as in Fernández-Villaverde et al. (2015b) in order to approximate the likelihood.

6 Priors and Estimation Results

The model is estimated using flat priors in order to minimize the impact of pre-sample information.¹⁸ Flat priors are also chosen since it is difficult to know what reasonable priors are for the stochastic volatility terms. Flat priors do not come without concerns, as they can make identification difficult. To avoid potential identification issues and ease the computational burden, some parameters are fixed to certain values. As is common in the literature, I fix $\delta = 0.025$, $\psi = 8$, $F(\bar{\omega}_{ss}) = 0.003$, $\sigma_\omega^2 = 0.24$, $\beta = 0.995$, $\alpha = 0.35$, $\eta = 6$, $\varepsilon = 6$, $\vartheta = 1.17$, and $\nu = 0.973$.¹⁹

The posteriors for the stochastic volatility and risk shock measures are presented in Table 1, while all results can be found in the Appendix. The results are presented with the means of the posterior distribution along with their standard deviation. Based on the posterior distributions, it is likely that there were changes in the standard deviation of the exogenous shocks; suggesting that uncertainty is changing over time. Additionally, the posteriors suggest that there are persistent risk shocks.

¹⁸The priors are described in the Appendix. In addition to the priors, the sample is restricted to only coefficients that result in a unique stationary equilibrium where c_{ss} and W^e are greater than 0.

¹⁹These numbers are similar to those used by Fernández-Villaverde et al. (2010) and Christiano et al. (2010) in similar studies. Since there is no standard measure for hours in the model, steady state labor, l , is normalized to 1, which determines the value for ψ .

Table 1: Posterior, Stochastic Volatility and Risk Shock

$\log(\sigma_m)$	$\log(\sigma_d)$	$\log(\sigma_x)$	$\log(\sigma_\zeta)$	$\log(\sigma_A)$	$\log(\sigma_g)$	$\log(\sigma_\varphi)$	σ_ω
-4.40 (0.12)	-1.34 (0.25)	-4.26 (0.15)	-4.55 (0.07)	-5.05 (0.14)	-0.95 (0.17)	-2.19 (0.13)	0.65 (0.04)
ρ_{σ_m}	ρ_{σ_d}	ρ_{σ_x}	ρ_{σ_ζ}	ρ_{σ_A}	ρ_{σ_g}	ρ_{σ_φ}	ρ_{σ_ω}
0.55 (0.19)	0.85 (0.07)	0.18 (0.12)	0.79 (0.16)	0.76 (0.16)	0.62 (0.12)	0.36 (0.05)	0.55 (0.25)
$\log(\eta_m)$	$\log(\eta_d)$	$\log(\eta_x)$	$\log(\eta_\zeta)$	$\log(\eta_z)$	$\log(\eta_g)$	$\log(\eta_\varphi)$	$\log(\eta_\omega)$
-1.27 (0.32)	-1.20 (0.35)	-0.58 (0.14)	-0.69 (0.38)	-1.28 (0.13)	-0.77 (0.16)	-0.58 (0.15)	-2.96 (0.34)

7 Impulse Response Functions

Using values from the posterior, IRFs are generated using the pruning and generalized impulse response function (GIRF) procedures detailed in Andreasen et al. (2014). The IRFs for output, consumption and investment for all of the traditional shocks can be seen in Figures 7 to 11 in the Appendix, except for the IRFs for the wealth shock, which can be seen below in Figure 2. A general theme from the IRFs is that traditional DSGE shocks typically produce larger responses than uncertainty shocks. However, a closer look reveals that some uncertainty shocks have effects that are similar in magnitude and sometimes larger effects than some of the traditional shocks, as can be seen in Figures 3 to 4. The impact of these uncertainty shocks are larger and more persistent than those associated with “risk shocks.”

To get a better understanding of the importance that financial frictions play in the model, it is important to study the response of output and investment to a wealth shock, which can be seen in Figure 2. An increase in wealth means that entrepreneurs have to borrow less in order to purchase the same amount of capital, so the increased wealth allows entrepreneurs to purchase more capital, which causes investment to increase. The increase in wealth also reduces the problems associated with asymmetric information, which causes a drop in the credit spread. The reduction in the credit spread, leads to a further increase in investment. The increased investment leads to more demand and an increase in capital

so output increases.

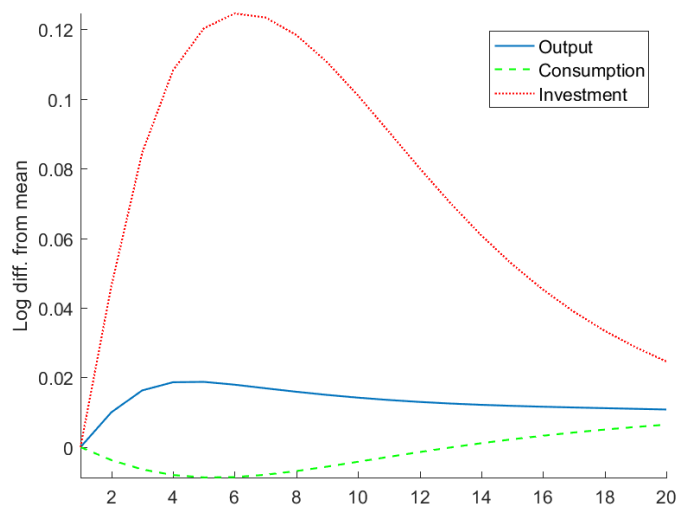


Figure 2: IRFs to wealth shock (ε_x)

An increase in intertemporal uncertainty has a large, negative impact on investment and also causes a drop in output and consumption, as can be seen in Figure 3. The magnitude of the impacts are similar to those of a productivity shock, as can be seen in the Appendix in Figure 7. An intertemporal shock causes households to increase consumption today and reduce saving. Uncertainty surrounding the shock can impact the economy through two main mechanisms. First, households have habit persistence, so individuals need to consider how consumption today impacts utility in the future. Therefore, households need to consider how much they plan to consume in the next period before consuming today. The increased uncertainty about the future causes households to use precaution and consume less today. The second mechanism is through the investment adjustment cost. Since it is expensive for the capital producer to adjust investment from period to period, uncertainty about savings and investment next period causes them to reduce investment this period.

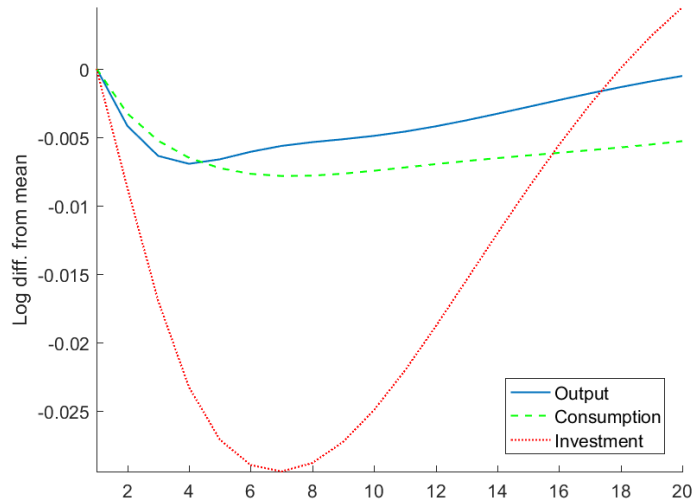


Figure 3: IRFs to intertemporal uncertainty shock (u_d)

As can be seen in Figure 4, an increase in uncertainty of wealth shocks reduces investment, output, and consumption. While the impact of this shock is smaller in magnitude than a traditional wealth shock, the size of the fall in investment is about half as large as the impact of a traditional monetary policy shock, while being more persistent than the monetary policy shock, as can be seen in the Appendix in Figure 11. The increase in wealth uncertainty causes there to be increased uncertainty about future investment because wealth shocks cause the effect of asymmetric information problems to change as discussed above. This means that there may be bigger swings in the credit spread in the future, which has impacts the ability of entrepreneurs to borrow and therefore impacts the level of investment. Due to the adjustment costs associated with investment, this causes investment to fall today in order to avoid future adjustment costs. The reduction in investment causes a slight fall in output.

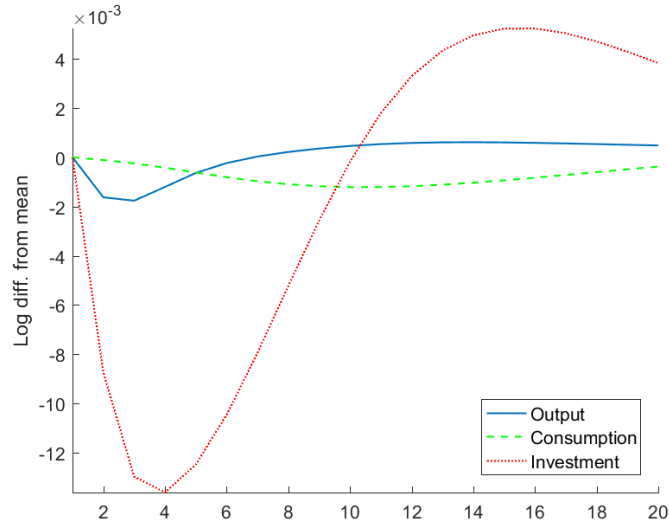


Figure 4: IRFs to wealth uncertainty shock (u_x)

Figure 5 shows that increases in risk cause output, consumption and investment to all fall. The magnitude of these effects is smaller and less persistent than the intertemporal and wealth uncertainty shock and slightly larger than the effect of a wealth uncertainty shock. This suggests that the distribution of entrepreneurial productivity may be important, but less important than other sources of uncertainty.

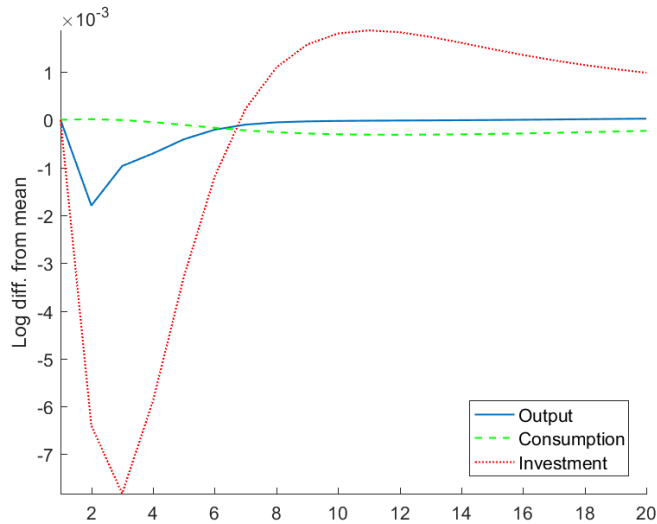


Figure 5: IRFs to risk shock ($u_{\omega,t}$)

To better understand the differing effects of the financial shocks described above, it is important to study the IRF of the credit spread to each of these shocks. As can be seen in Figure 6, a negative one standard deviation wealth shock leads to a much bigger increase in the credit spread than a positive one standard deviation wealth uncertainty or risk shock. The response to wealth shocks is also much more persistent than the other two shocks. Since wealth shocks also decrease the need for entrepreneurs to borrow, this can help explain why wealth shocks have a much bigger impact on investment. Comparing the wealth uncertainty shock and risk shock, it is clear that the risk shock has a much larger initial impact but it is much less persistent than the response to increased wealth uncertainty. This difference can partially explain the difference in persistence of the IRF for investment. It is important to note that in addition to increasing credit spread, the increase in wealth uncertainty also reduces current investment in order to avoid future capital adjustment costs, as described above. This helps explain why wealth uncertainty shocks lower investment by more than risk shocks despite having a smaller peak impact on credit spreads.

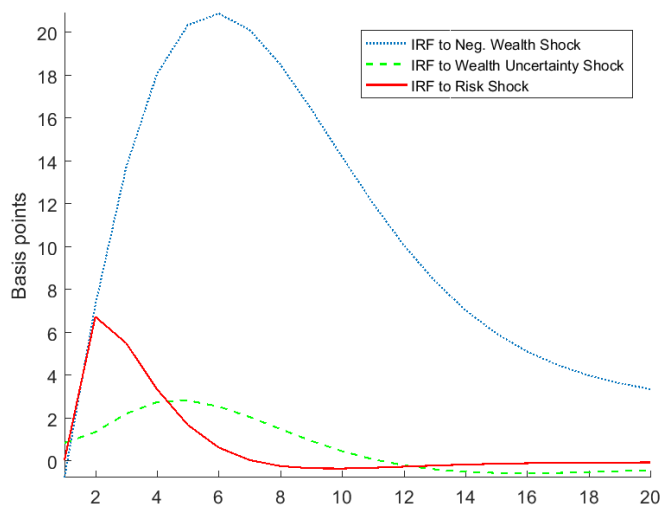


Figure 6: IRF of credit spread

8 Variance Decomposition

While it is important to understand the impact of shocks through IRFs, it is also important to understand the importance of each shock in explaining the variance of different variables. This is commonly done through variance decomposition. Since this model is nonlinear, the standard methods of variance decomposition are not applicable. Therefore, a method of simulation using the pruned third-order approximation is used. Data is simulated feeding all of the shocks through the model in order to get a baseline standard deviation for all variables of interest.²⁰ Then, individual innovations to shocks are set to 0 for all time periods in order to see how much the standard deviation of the relevant variables fall when the shocks are removed..²¹

Table 2: Variance Decomposition

	Standard Deviation of:				
	Output	Inflation	Investment	Consumption	Spread
Baseline	0.27	0.058	1.07	0.27	0.015
No "Risk Shock"	0.27	0.058	1.07	0.27	0.014
No s.v.	0.20	0.044	0.91	0.21	0.013
No s.v. x	0.26	0.057	1.03	0.27	0.014
No s.v. d	0.27	0.051	0.98	0.26	0.014
No s.v. m	0.27	0.058	1.06	0.27	0.015
No s.v. z	0.27	0.058	1.06	0.27	0.015
No s.v. ζ	0.27	0.058	1.07	0.27	0.015
No s.v. g	0.26	0.054	1.06	0.27	0.015
No s.v. φ	0.23	0.057	1.04	0.24	0.015

The results of the simulated variance decomposition can be seen in Table 2. Based on the variance decomposition, the inclusion of risk shocks does not seem to be important in explaining the standard deviations of most variables since only the standard deviation of the credit spread falls when risk shocks are removed. The inclusion of stochastic volatility is important in explaining the variation of the variables in the model, but not all forms of stochastic volatility are important. The most important stochastic volatility shocks are associated with the wealth, intertemporal, and intratemporal shocks. The removal of these

²⁰The model is simulated for 50,000 periods with the first 20,000 periods thrown out.

²¹It is important to note that agents within the model still are under the assumption that innovations are normally distributed with unit variances. Therefore, it is not the ideal variance decomposition, but does give a good, general idea of the importance of shocks in explaining the variance of simulated data within the model.

shocks reduces the variances of output, inflation, investment, consumption, and the credit spread. While the inclusion of stochastic volatility is important in explaining the variance of variables, it is important to note that traditional shocks still explain the majority of the variation.

9 Conclusion

In this paper, I estimate a New Keynesian model featuring financial frictions and stochastic volatility. The model is estimated using Bayesian techniques and the particle filter. I find that uncertainty shocks can play an important role in business cycle fluctuations, especially when uncertainty about wealth shocks and intertemporal shocks change. When uncertainty increases, both of these shocks lead to a reduction in consumption, investment, and output. The magnitude of effect of most uncertainty shocks is smaller than traditional shocks, but are still economically significant. I also find that financial frictions play an important role in business cycles as can be seen by the impact of wealth shocks on output. In the variance decomposition study, I find that the inclusion of stochastic volatility with wealth and intertemporal shocks plays a key role in explaining the variance of several variables.

The findings in this paper suggest that policy makers need to pay attention to uncertainty, especially uncertainty facing households and uncertainty about wealth. Measuring the level of uncertainty for households is difficult. I am not aware of any way to measure intertemporal uncertainty. One way to measure overall uncertainty for households may be to look at survey responses. One example of this is Binder (2015), which looks at household expectations of inflation to measure uncertainty using rounding. Fortunately, there is a good proxy for wealth uncertainty: the VIX. This data is available in real time and easily attainable. This makes wealth uncertainty a reasonable variable for policy makers to study, unlike other uncertainty and risk shocks, which are all difficult to measure or find proxies for.

This paper shows the importance of uncertainty within a standard New Keynesian model

with financial frictions à la BGG. In future research, I plan to study financial shocks and uncertainty in a model with housing. I also plan to study uncertainty and financial shocks in a model featuring Kiyotaki and Moore (1997) style financial frictions.

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10 Appendix

10.1 Data

Data is from NIPA-BEA, CPS-BLS, the FRED database, and the Flow of Funds accounts from the Federal Reserve Board for the period 1954.Q4 to 2015.Q2. Data is chosen to conform with the literature and is similar to the data used in Fuentes-Albero (2013). The data used is:

- Real output: data from NIPA table 1.3.5 on nominal gross value added by the nonfarm business sector is deflated the implicit price deflator from table 1.3.4. This is divided by the civilian noninstitutional population aged 16 and over from the BLS.
- Real investment: the sum of personal consumption expenditures of durables and gross private domestic investment from NIPA table 1.1.5 is deflated using the GDP deflator from table 1.1.4. This is weighted by the relative significance in total GDP and divided by the civilian noninstitutional population aged 16 and over from the BLS.
- Real net worth: this is defined as tangible assets minus credit market instruments at market value. Tangible assets are defined as nonfinancial assets for nonfinancial corporate businesses (FL102010005.Q) from the Federal Reserve's Flow of Funds Accounts data. Liabilities are defined as credit market instruments (FL104104005.Q), which is from the Flow of Funds account. The data is converted to real and per capita data the same way output is converted.
- Real consumption: the sum of personal consumption expenditures of nondurables and services from NIPA table 1.1.5 deflated by the GDP deflator from table 1.14. This weighted and corrected to per capita terms the same way as investment.
- Real wages: real compensation per hour in the nonfarm business sector (COMPRNFB) provided by the BLS
- Inflation: the log difference of the price index for gross value added by the nonfarm

business sector from NIPA table (1.3.4).

- Federal funds rate: this is from the FRED database and is converted to quarterly rates (the rate used is the average rate for the quarter).
- Credit spread: the difference between the gross Moody's Seasoned Baa Corporate Bond Yield found on FRED and the gross 10 year treasury rate found on FRED (both rates are converted to quarterly rates and taken as the average for the quarter).

10.2 First Order Conditions

$$\tilde{\lambda}_t = \frac{d_t}{\tilde{c}_t - h\tilde{c}_{t-1}\frac{z_{t-1}}{z_t}} - \beta h \mathbb{E}_t \frac{d_{t+1}}{\tilde{c}_{t+1}\frac{z_{t+1}}{z_t} - h\tilde{c}_t} \quad (40)$$

$$\tilde{\lambda}_t = \beta \mathbb{E}_t \frac{R_t \tilde{\lambda}_{t+1} \frac{z_t}{z_{t+1}}}{\Pi_{t+1}} \quad (41)$$

$$mc_t = (1 - \alpha)^{\alpha-1} \alpha^{-\alpha} \tilde{w}_t^{1-\alpha} \tilde{r}_t^\alpha \quad (42)$$

$$f_t = \frac{\eta - 1}{\eta} (\tilde{w}_t^*)^{1-\eta} \tilde{\lambda}_t \tilde{w}_t^\eta l_t^d + \beta \theta_w \mathbb{E}_t \left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}} \right)^{-\eta} \left(\frac{\tilde{w}_{t+1}^*}{\tilde{w}_t^*} \frac{z_{t+1}}{z_t} \right)^{\eta-1} f_{t+1} \quad (43)$$

$$f_t = \psi d_t \varphi_t \left(\frac{\tilde{w}_t}{\tilde{w}_t^*} \right)^{\eta(1+\vartheta)} (l_t^d)^{1+\vartheta} + \beta \theta_w \mathbb{E}_t \left(\frac{\Pi_t^{\chi_w}}{\Pi_{t+1}} \right)^{-\eta(1+\vartheta)} \left(\frac{\tilde{w}_{t+1}^*}{\tilde{w}_t^*} \frac{z_{t+1}}{z_t} \right)^{\eta(1+\vartheta)} f_{t+1} \quad (44)$$

$$\tilde{w}_t^{1-\eta} = \theta_w \left(\frac{\Pi_{t-1}^{\chi_w}}{\Pi_t} \right)^{1-\eta} \left(\tilde{w}_{t-1} \frac{z_{t-1}}{z_t} \right)^{1-\eta} + (1 - \theta_w) (\tilde{w}_t^*)^{1-\eta} \quad (45)$$

$$\epsilon g_t^1 = (\epsilon - 1) g_t^2 \quad (46)$$

$$g_t^1 = \tilde{\lambda}_t mc_t y_t^d + \beta \theta_p \mathbb{E}_t \left(\frac{\Pi_t^\chi}{\Pi_{t+1}} \right)^{-\epsilon} g_{t+1}^1 \quad (47)$$

$$g_t^2 = \tilde{\lambda}_t \Pi_t^* \tilde{y}_t^d + \beta \theta_p \mathbb{E}_t \left(\frac{\Pi_t^\chi}{\Pi_{t+1}} \right)^{1-\epsilon} \frac{\Pi_t^*}{\Pi_{t+1}^*} g_{t+1}^2 \quad (48)$$

$$1 = \theta_p \left(\frac{\Pi_{t-1}^\chi}{\Pi_t} \right)^{1-\epsilon} + (1 - \theta_p) (\Pi_t^*)^{1-\epsilon} \quad (49)$$

$$v_t^p = \theta_p \left(\frac{\Pi_{t-1}^\chi}{\Pi_t} \right)^{-\epsilon} v_{t-1}^p + (1 - \theta_p) (\Pi_t^*)^{-\epsilon} \quad (50)$$

$$\tilde{y}_t^d = \frac{\tilde{k}_t^\alpha (\tilde{l}_t^d)^{1-\alpha}}{v_t^p} \quad (51)$$

$$\frac{\tilde{k}_t}{\tilde{l}_t^d} = \frac{\alpha}{1-\alpha} \frac{\tilde{w}_t}{\tilde{r}_t^k} \quad (52)$$

$$\tilde{r}_t^k = \gamma_1 + \gamma_2 (u_t - 1) \quad (53)$$

$$\begin{aligned} 1 = & \tilde{Q}_t \left(1 - V \left[\frac{\tilde{I}_t}{\tilde{I}_{t-1}} \frac{z_t}{z_{t-1}} \right] - V' \left[\frac{\tilde{I}_t}{\tilde{I}_{t-1}} \frac{z_t}{z_{t-1}} \right] \frac{\tilde{I}_t}{\tilde{I}_{t-1}} \frac{z_t}{z_{t-1}} \right) + \\ & \beta \mathbb{E}_t \frac{\tilde{\lambda}_{t+1}}{\tilde{\lambda}_t} \frac{z_{t+1}}{z_t} \tilde{Q}_{t+1} V' \left[\frac{\tilde{I}_t}{\tilde{I}_{t-1}} \frac{z_{t+1}}{z_t} \right] \left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t} \frac{z_{t+1}}{z_t} \right)^2 \end{aligned} \quad (54)$$

$$R_{t+1}^k = \left[\frac{(u_{t+1} \tilde{r}_{t+1}^k - a(u_{t+1})) + (1-\delta) \tilde{Q}_{t+1}}{\tilde{Q}_t \frac{\zeta_{t+1}}{\zeta_t}} \right] \Pi_{t+1} \quad (55)$$

$$\frac{R_{t+1}^k}{R_t} [\Gamma_t(\bar{\omega}_{t+1}) - \mu_{t+1} G_t(\bar{\omega}_{t+1})] = \frac{\tilde{Q}_t \tilde{K}_{t+1} \frac{\zeta_{t+1}}{\zeta_t} - \tilde{N}_{t+1} \frac{z_t}{z_{t+1}}}{\tilde{Q}_t \tilde{K}_{t+1} \frac{\zeta_{t+1}}{\zeta_t}} \quad (56)$$

$$\mathbb{E}_t \left\{ [1 - \Gamma_t(\bar{\omega}_{t+1})] \frac{R_{t+1}^k}{R_t} + \frac{\Gamma'_t(\bar{\omega}_{t+1})}{\Gamma'_t(\bar{\omega}_{t+1}) - \mu_{t+1} G'_t(\bar{\omega}_{t+1})} \left[\frac{R_{t+1}^k}{R_t} (\Gamma(\bar{\omega}_{t+1}) - \mu_{t+1} G_t(\bar{\omega}_{t+1})) - 1 \right] \right\} = 0 \quad (57)$$

$$Z_t = R_t + \frac{\mu_t G_t(\bar{\omega}_t) \tilde{K}_t \tilde{Q}_t \frac{\zeta_t}{\zeta_{t-1}}}{\tilde{K}_t \tilde{Q}_{t-1} \frac{\zeta_t}{\zeta_{t-1}} - \tilde{N}_t \frac{z_{t-1}}{z_t}} \quad (58)$$

$$\begin{aligned}
P_t N_{t+1} = & x_t \nu \left(P_{t-1} R_t^k \tilde{Q}_{t-1} \tilde{K}_t \frac{\zeta_t}{\zeta_{t-1}} - P_{t-1} R_{t-1} \left(\tilde{Q}_{t-1} \tilde{K}_t \frac{\zeta_t}{\zeta_{t-1}} - \tilde{N}_t \frac{z_t}{z_{t-1}} \right) \right. \\
& \left. - \mu_t G(\bar{\omega}_t) P_{t-1} R_t^k Q_{t-1} K_t \right) + P_t W_t^e
\end{aligned} \tag{59}$$

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{1-\gamma_R} \left(\left(\frac{\Pi_t}{\Pi} \right)^{\psi_{1,t}} \left(\frac{\tilde{Y}_t \frac{z_t}{z_{t-1}}}{\tilde{Y}_{t-1} z_{t-1}} \right)^{\psi_{2,t}} \right)^{1-\gamma_R} m_t. \tag{60}$$

$$\tilde{K}_{t+1} = (1 - \delta) \tilde{K}_t \frac{z_t \zeta_t}{z_{t+1} \zeta_{t+1}} + \frac{z_t \zeta_t}{z_{t+1} \zeta_{t+1}} \left(1 - V \left[\frac{\tilde{I}_t}{\tilde{I}_{t-1}} \frac{z_t}{z_{t-1}} \right] \right) \tilde{I}_t \tag{61}$$

10.3 Priors

Table 3: Priors

	a	b		a	b
Υ_A	0	0.03	ρ_{σ_g}	0	1
Υ_ζ	0	0.03	ρ_{σ_φ}	0	1
Π	1	1.008	ρ_μ	0	1
χ	0	1	ρ_φ	0	1
χ_w	0	1	$\log(\sigma_d)$	-250	0
γ_R	0	1	$\log(\sigma_\varphi)$	-250	0
ψ_2	0	0.5	$\log(\sigma_g)$	-250	0
ξ	0	5	$\log(\sigma_m)$	-250	0
μ	0	1	$\log(\sigma_x)$	-250	0
ψ_1	1	3	$\log(\sigma_\zeta)$	-250	0
θ_p	0	1	$\log(\sigma_A)$	-250	0
θ_w	0	1	$\log(\eta_d)$	-250	0
ρ_d	0	1	$\log(\eta_\varphi)$	-250	0
ρ_φ	0	1	$\log(\eta_g)$	-250	0
ρ_g	0	1	$\log(\eta_m)$	-250	0
ρ_x	0	1	$\log(\eta_x)$	-250	0
ρ_{σ_m}	0	1	$\log(\eta_A)$	-250	0
ρ_{σ_d}	0	1	$\log(\eta_\zeta)$	-250	0
ρ_{σ_x}	0	1	$\log(\eta_\mu)$	-250	0
ρ_{σ_ζ}	0	1	$\log(\eta_{\psi_1})$	-250	0
ρ_{σ_A}	0	1	σ_{me}	0.003	5

Note: All distributions are uniform(a,b), except prior of σ_{me} is Weibull(a,b)

10.4 Estimation Results

Table 4: Posterior Distributions

	Mean	Std. Dev.		Mean	Std. Dev.
Υ_A	0.001	(0.001)	ρ_{σ_m}	0.55	(0.19)
Υ_ζ	0.002	(0.001)	ρ_{σ_d}	0.85	(0.07)
Π	1.006	(0.003)	ρ_{σ_x}	0.18	(0.12)
χ	0.85	(0.09)	ρ_{σ_ζ}	0.79	(0.16)
χ_w	0.37	(0.25)	ρ_{σ_A}	0.76	(0.16)
γ_R	0.69	(0.13)	ρ_{σ_g}	0.62	(0.12)
ψ_2	0.36	(0.06)	ρ_{σ_φ}	0.36	(0.05)
ξ	0.74	(0.67)	$\log(\sigma_d)$	-1.34	(0.25)
μ	0.51	(0.07)	$\log(\sigma_\varphi)$	-2.19	(0.13)
h	0.83	(0.09)	$\log(\sigma_g)$	-0.95	(0.17)
γ_2	0.54	(0.32)	$\log(\sigma_m)$	-4.40	(0.12)
ψ_1	2.01	(0.24)	$\log(\sigma_x)$	-4.26	(0.15)
θ_p	0.41	(0.10)	$\log(\sigma_\zeta)$	-4.55	(0.07)
θ_w	0.39	(0.13)	$\log(\sigma_A)$	-5.05	(0.14)
σ_ω	0.65	(0.04)	$\log(\eta_d)$	-1.20	(0.35)
ρ_ω	0.55	(0.25)	$\log(\eta_\varphi)$	-0.58	(0.15)
$\log(\eta_\omega)$	-2.96	(0.34)	$\log(\eta_g)$	-0.77	(0.16)
σ_{me}	0.01	(0.002)	$\log(\eta_m)$	-1.27	(0.32)
ρ_d	0.83	(0.09)	$\log(\eta_x)$	-0.58	(0.14)
ρ_φ	0.96	(0.03)	$\log(\eta_A)$	-1.28	(0.13)
ρ_g	0.37	(0.25)	$\log(\eta_\zeta)$	-0.69	(0.38)
ρ_x	0.72	(0.16)	$\log(\eta_{\psi_1})$	-1.28	(0.06)
Acceptance Rate		18%			

10.5 IRF of Traditional Shocks

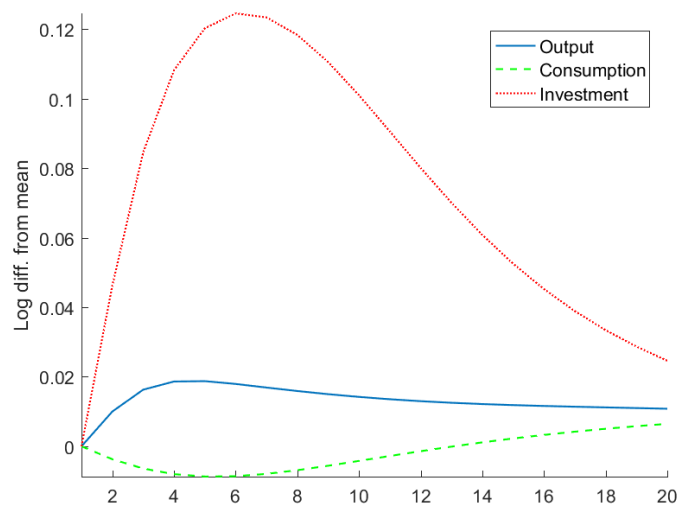


Figure 7: IRFs to productivity shock (ε_A)

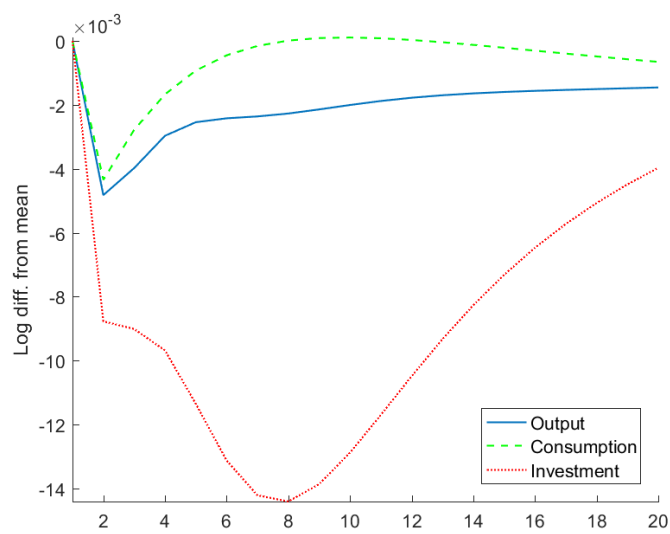


Figure 8: IRFs to capital productivity shock (ε_ζ)

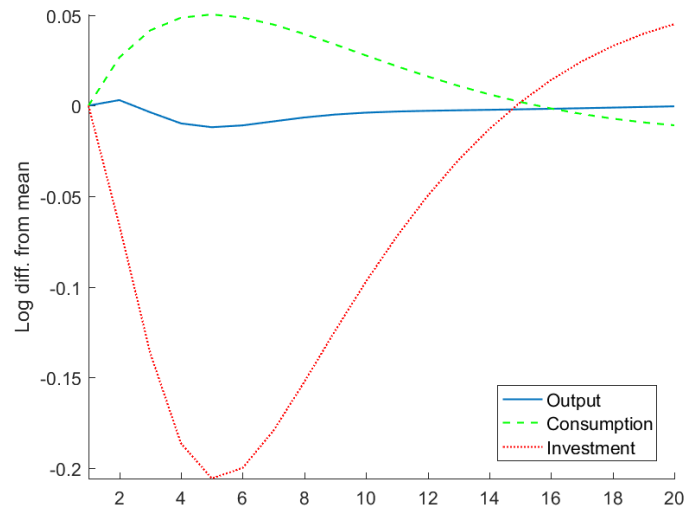


Figure 9: IRFs to intertemporal shock (ε_d)

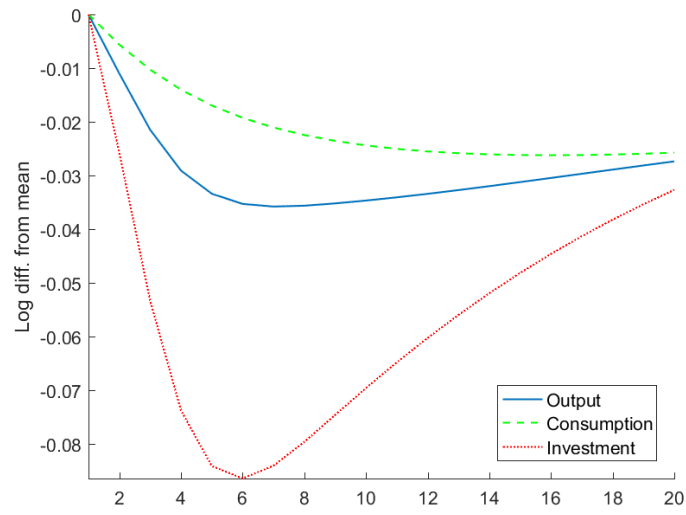


Figure 10: IRFs to intratemporal shock (ε_φ)

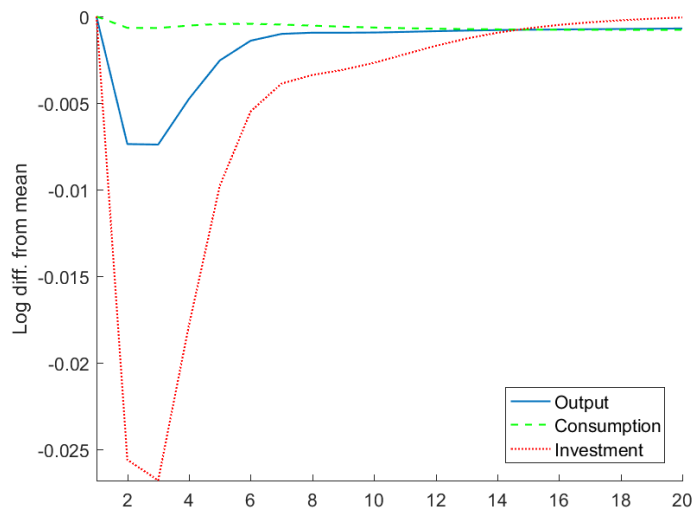


Figure 11: IRFs to monetary policy shock (ε_m)