Does the Central Bank, in reacting to macroeconomic indicators, influence the term structure of interest rates when faced with a binding constraint at the zero lower bound?

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Abstract

With U.S. interest rates constrained at the zero lower bound, conventionary monetary policy may be unable to achieve desired expansionary effects. With zero, or near zero, interest rates, the true stance of monetary policy is elusive when conventional policy actions seem ineffective. In these instances the shadow rate, which is free to fluctuate between positive and negative values and encompasses the option for investors to hold cash at zero interest, acts to proxy for monetary policy objectives. I estimate a Gaussian affine term structure model in which the observed short-term interest rate is driven by a shadow rate which is modeled as an affine function of standard latent yield curve factors, observable macroeconomic indicators and the central bank policy target.

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1 Introduction

The global recession of 2008 and 2009, stemming mainly from the financial crisis permeating throughout the U.S., triggered unprecedented responses in both fiscal and monetary policy. For the first time since the Great Depression, U.S. interest rates faced the binding constraint of the zero lower bound, thus stifling the efficacy of efforts of the Federal Reserve to stimulate economic activity and introducing the necessity of developing alternative monetary policy initiatives to combat the recession. Similar experiences have been observed in Japan. The FOMC policy rate has been pegged effectively at zero since December of 2008. Short-term nominal interest rates have traditionally been considered an indicator of the stance of monetary policy - looser policy coincides with lower interest rates. However, when interest rates face a binding constraint at zero, the true stance of monetary policy can be obscured.

When short term interest rates are essentially zero, including their values in a VAR framework does not sufficiently represent their role in financial markets if we assume that the real stance of monetary policy would result in negative interest rates. Thus, the true impact of monetary policy is muddled when the tool of conventional policy is rendered ineffective. Nominal interest rates cannot fall below zero as investors always have alternative of holding cash and earning zero interest. However, short rates can become negative in a general Gaussian affine term structure model (GATSM), especially when traditional expansionary monetary policy puts pressure on short term interest rates, forcing them to fall further and further. This issue motivates the development of term structure models imposing a binding constraint for interest rates at the zero lower bound. Black [1995] modified the typical GATSM to eliminate the occurrence of negative interest rates by introducing the idea of interest rates as options. Nominal interest rates cannot take on negative values in a world in which cash is a viable alternative to standard short-term assets. Since cash essentially pays zero interest, should nominal bonds pay a negative interest rate, investors would have an arbitrage opportunity by issuing bonds at negative interest rates and using the borrowed funds to purchase cash, earning a zero rate of interest. Black [1995] introduced the concept of a shadow instantaneous interest rate which is free to take on negative values while the nominal interest rate is the positive part of the shadow rate due to the always present option to convert an asset to currency. Krippner [2012] modifies the Black [1995] framework of modeling interest rates as options by calculating the value of a call option to hold cash. The modifications in Krippner [2012] generate closed-form solutions for bond prices and yields. The model calculates a shadow short-term interest rate which would be seen in financial markets if the cash option did not exist. The shadow rate has been considered a proxy for the stance of monetary policy in an environment in which the zero lower bound is binding. From this foundation one can develop a full model of the shadow term structure based upon the shadow short rate depicting the fundamental policy objectives.
Most Taylor-type rules for monetary policy would currently recommend setting a negative policy target, and thus suggest a negative short rate, which is infeasible. Since the central bank cannot target negative interest rates, when the observed target is at zero, unconventional policy measures and external dynamics in the economy push the shadow rate down further and may have been necessary to reach the recommended policy rate. The shadow rate, according to the specifications in Krippner [2012], dropped to more than 300 basis points below what the standard Taylor rule would prescribe for much of 2012. This suggests that actual monetary policy is much looser than this rule. The shadow rate generated from the Krippner [2012] method provides a measurement of the actual policy which can then be compared against that recommended by the policy rule. Through the shadow rate mechanism we can trace out the effects of policy and decipher the connection between the behavior of the monetary authority and term structure dynamics. Looking at both the Fed target and the shadow rate simultaneously allows for modeling the feedback between the perceived and effective stances of monetary policy and should help to reconcile the difference when looking at standard policy rules that do not account for the term structure at the ZLB.

Piazzesi [2001] estimates a term structure model including a central bank policy rule that incorporates term structure and macroeconomic data and finds that the Federal Reserve is mainly reacting to information contained in the yield curve. When the short end of the yield curve is effectively constrained at zero, do policy makers adjust their reaction to term structure dynamics? Furthermore, Ang and Piazzesi [2003] find that macroeconomic factors help explain movements in the short end and middle of the yield curve. If the central bank abides by a Taylor-type policy rule, the same macro factors will influence monetary policy and thus displaying a relationship between the yield curve, central bank behavior and overall macro indicators. While these two approaches incorporate important macroeconomic factors and monetary policy into a model of the term structure dynamics, neither address the possibility of negative nominal interest rates.

I extend the methodologies of Piazzesi [2001] and Ang and Piazzesi [2003] to describe the joint dynamics of bond yields, monetary policy and macroeconomic variables while explicitly modeling the zero lower bound (ZLB) using the Krippner [2012] framework. I estimate a GATSM in which the short-term interest rate is modeled as a function of two standard latent yield curve factors, two observable macroeconomic indicators (inflation and real output) and the central bank policy target. The observable short rate is driven by the underlying process for the shadow short rate, which can take on negative values. Whenever the shadow rate is less than zero, we observe an interest rate of zero in financial markets. Following the convention in much of the term structure literature to describe short rate dynamics, I incorporate latent state variables to model term premia effects on the level and slope of the yield curve. In the same tradition of Piazzesi [2001], I assume a Taylor-type rule for the policy target in which the central bank reacts to the current state of the macroeconomy but allow for responses to latent yield curve data as well. This makes it possible to
examine the effects of monetary policy in an environment which solves the ZLB issue. If the macro factors or central bank policy do not influence the term structure dynamics, the loadings on these factors in the short rate equation will be close to zero with little or no weight placed upon the volatilities of these factors when pricing risk and the term premia demanded by investors. Ultimately I will be able to trace out how a shock to the Fed target propagates throughout the economy, particularly in terms of the effects on bond prices and yields. When the policy target is stuck near zero, along with observed short-term interest rates, shocks to monetary policy permeate through the shadow rate to affect the term structure and the macroeconomy.

2 Empirical Methodology

Krippner [2012] extended the ideas of Black [1995] and Gorovoi and Linetsky [2004] in constructing a shadow short term interest rate to address the issue of the inability of observed nominal rates to take on negative values. These methods introduce a hypothetical shadow bond for which the interest rate is free to fluctuate between positive and negative values. Therefore investors will choose to hold the shadow bond if the interest rate is above zero or will default to holding cash should the interest rate fall below zero. As a result, anytime the shadow interest rate takes on negative values we replace it with zero and establish the binding constraint observed in financial markets.

2.1 ZLB-GATSM Interest Rates

I merge the methodology developed by Krippner [2012] and Dai and Singleton [2002] for constructing the shadow short-term interest rate. The nominal short rate can be expressed as the maximum of the shadow short rate and zero:

\[ r^* (t) = \max \{ r(t), 0 \} \]  

(1)

Based upon this approach, the ZLB-GATSM structure describes the dynamics of interest rates, forward rates and bond prices in a realistic way accounting for the impossibility of negative returns on nominal bonds of all maturities. I briefly summarize the details of the general ZLB-GATSM expressions from Krippner [2012]. See that paper for an exhaustive discussion of the motivation and intuition of this framework. The observable ZLB values are underlined while their shadow counterparts are not underlined.

According to standard term structure relationships, the observed forward rate \( f(t, \tau) \) at time \( t \) for a bond with time to maturity \( \tau \) is a function of the current price of that bond. The observed nominal interest rates for any time to maturity \( \tau \) shares the following relationship with forward rates:
\[
R(t, \tau) = \frac{1}{\tau} \int_0^\tau f(t, v) \, dv \\
= \frac{1}{\tau} \sum_{i=1}^I f\left(t, \left[i - \frac{1}{2}\right] \Delta \tau\right),
\]

(2)

where the second line comes from applying the rectangular rule for numerical integration with \(I\) constant increments. The short rate is the limit as \(\tau\) approaches zero and thus gives the instantaneous return. Through this framework we link the short rate to forward rates and forward rates to interest rates for all maturities. Observed forward rates are functions of the state variables and through this avenue these states affect observed interest rates of all maturities. Hence this establishes a direct mechanism through which macro indicators and monetary policy can influence the entire yield curve, rather than just the short end traditionally thought to be under the control of the central bank.

### 2.2 Shadow Interest Rate

Krippner [2012] constructs a GATSM adopting the specifications of Chen [1995] with closed-form expressions for bond and option prices. Following the tradition of Vasicek [1977], Krippner [2012] describes the shadow short rate as the sum of two latent yield curve factors:

\[
r^* (t) = s_1 (t) + s_2 (t).
\]

(3)

I release some of the restrictions introduced through this functional form assumption and adopt the general multi-factor GATSM specification which describes this shadow short rate as a linear function of an \(N\) dimensional state vector:

\[
r^* (t) = a + b' X (t),
\]

(4)

where \(a\) is a constant, \(b\) an \(N \times 1\) vector of coefficients and \(X (t)\) is the \(N \times 1\) vector of state variables driving the dynamics of the yield curve. The state vector includes the observable macro factors, unobservable standard yield curve factors and the monetary policy target. I adjust the derivation from Krippner [2012] to allow for the short rate to be an affine function of the state, rather than just the sum of its components. As a result, I lose some of the parsimonious nature of the Krippner model but allow for more flexibility in how interest rates respond to the factors in the model and introduce more dynamics in the relationship between macro indicators and financial market fluctuations.
2.3 Unobservable Latent Factors

The state vector includes two unobservable factors from standard yield curve model assumptions: \( s_1(t) \) and \( s_2(t) \). The two latent factor structure is often used in affine term structure literature to achieve a parsimonious model while still accounting for varied effects on different components of the yield curve. These two latent factors embody level and slope effects on the yield curve and I first estimate the model including only these two state variables to allow a chance for them to explain the term structure on their own.

In this baseline model, the intuition follows exactly as in Krippner [2012] for explaining the zero-lower bound GATSM (ZLB-GATSM): \( s_1(t) \) is the "level" component of the shadow forward rate where innovations to \( s_1(t) \) shift \( f(t, \tau) \) equally at all horizons; \( s_2(t) \exp(-\kappa \tau) \) is the "slope" component and innovations to \( s_2(t) \) shift short horizon shadow forward rates by more than long horizon shadow forward rates. Collecting these latent factors provides the unobservable block of the state vector: \( X^U(t) = [s_1(t), s_2(t)]' \) whose dynamics can be summarized as:

\[
X^U(t) = \Phi^U(L)X^U(t) + \Sigma^U \varepsilon^U(t),
\]

where \( \Phi^U(L) \) is the lag operator.

2.4 Central Bank Policy Rule and Observable Macroeconomic Factors

According the Taylor [1993] policy rule, the Fed sets the monetary policy target in response to the current state of the economy and makes adjustments according to changes in inflation and real activity. In addition to this, I allow the Fed target to depend on the two latent state variables underlying the yield curve \((s_1(t), s_2(t))\). Denote the observable block of the state vector with the macro variables as \( X^O(t) \). Therefore, the central bank target is a function of both \( X^O(t) \) and \( X^U(t) \). The policy rule takes on the form:

\[
\theta_t = \Psi_\pi \pi(t) + \Psi_y y(t) + \Psi_{s_1} s_1(t) + \Psi_{s_2} s_2(t) + \Sigma^\theta \varepsilon^\theta(t)
\]

where \( \pi(t) \) is a measurement of core inflation and \( y(t) \) a measurement of the output gap and \( \varepsilon^\theta(t) \sim N(0, 1) \) is an idiosyncratic monetary policy shock.

I model the joint dynamics of \( \pi(t) \) and \( y(t) \) over time. I allow each macro variable to be a function of it’s own first lag, the first lag of the other macro variable, as well as the lagged policy rule. Therefore, like a standard monetary policy VAR, the policy instrument does not have a contemporaneous effect on
macroeconomic dynamics. The evolution of inflation and the output gap can be defined as:

\[ \pi(t) = \rho_\pi \pi(t - 1) + \rho_y y(t - 1) + \rho_\theta (t - 1) + \Sigma_\pi \varepsilon_\pi(t), \]  
(7)

\[ y(t) = \alpha_\pi \pi(t - 1) + \alpha_y y(t - 1) + \alpha_\theta (t - 1) + \Sigma_y \varepsilon_y(t), \]  
(8)

where \( \varepsilon_\pi \) and \( \varepsilon_y \) are i.i.d. \( N(0, 1) \) with associated inflation and output volatilities \( \Sigma_\pi \) and \( \Sigma_y \). The first block of the state vector is composed of the observable macro factors: \( X^O(t) = [\pi(t), y(t)]' \). In the absence of the latent states \( s_1 \) and \( s_2 \), the macro variables and the policy target would behave as a traditional monetary policy VAR with \( \theta(t) \) listed last in the vector. I estimate this as a separate case for comparison and describe the dynamics of all observable macro components as:

\[
\begin{bmatrix}
X^O(t) \\
\theta(t)
\end{bmatrix} = \Phi^O(L) \begin{bmatrix}
\pi(t) \\
y(t) \\
\theta(t)
\end{bmatrix} + \Sigma^O \varepsilon^O(t),
\]  
(9)

where again \( \Phi^O(L) \) is the lag operator.

### 2.5 Model Specification

The processes for the state vector and the observable interest rates can be combined into a non-linear state space system. The observation equation describes the relationship between nominal interest rates observed in financial markets and the explanatory state variables. These state variables feed through forward rates which are modeled explicitly as non-linear functions of the states, their volatilities and the market prices of risk associated with each. Therefore the observation equation takes the following form:

\[
\mathcal{R}(t, \tau) = \frac{1}{\tau} \sum_{i=1}^{I} f\left(t, \left[ i - \frac{1}{2} \right] \Delta \tau \mid X^O(t), X^U(t), \theta(t) \right).
\]  
(10)

For now, I model \( X^O(t) \) and \( X^U(t) \) separately and assume that their cross-dynamics are independent. The purpose of this is mainly for ease in calculation but separability between the observable macroeconomy and standard latent yield curve factors may also be a realistic assumption. Imposing separate dynamics avoids the additional convolution from feedback between macro and latent yield curve factors. The only convolution stems from how all of these factors together feed into \( \theta(t) \). See Ang and Piazzesi [2003] for a brief discussion on this issue as they also model macro and latent factors as independent processes.
research will include introducing correlated dynamics among the observable macro factors and unobserved yield curve and policy inertia factors by allowing each to be a function of either contemporaneous or lagged values of the other states. Rewriting equation 6 in terms of variables dated $t-1$ and before, we can define the evolution of the state vector with the following transition equation:

\[
\begin{bmatrix}
X^O (t) \\
X^U (t) \\
θ (t)
\end{bmatrix}
= \Phi
\begin{bmatrix}
X^O (t-1) \\
X^U (t-1) \\
θ (t-1)
\end{bmatrix}
+ \Sigma
\begin{bmatrix}
ε^O (t) \\
ε^U (t)
\end{bmatrix}
\]

where $\Phi$ is the coefficient matrix in the lag operator $\Phi (L)$ corresponding to the first lag.

\[ (11) \]

3 Estimation of Shadow Short Rate

I estimate the system of equations defining the dynamic evolution of the state variables in conjunction with their relationship to observable interest rates, through the shadow rate mechanism, using a non-linear particle filter. I apply Bayesian state space estimation methods to derive the shadow rate as an affine function of the state vector $X (t) = [X^O (t), X^U (t)]'$. First, I estimate the model with only the latent states $s_1$ and $s_2$ to give the traditional term structure factors a chance to explain term structure dynamics on their own. Second, I estimate the model incorporating macro factors and the central bank policy target in order to glean the additional explanatory nature of these other aspects of the economy not generally addressed in the term structure literature. As discussed in Ang and Piazzesi [2003], bond prices in the model are driven by shocks to the macro and unobservable variables alike. Since macro variables seem to be correlated with yields observed in financial markets, incorporating these additional factors may improve forecasting ability of the model. Furthermore, since many economists agree that the central bank follows something similar to the Taylor-type class of policy rules, the same macro indicators that influence monetary policy may be correlated with bond yields. This would suggest a relationship between the yield curve, central bank behavior and overall macro indicators. The present model allows for exploiting this relationship when the ZLB acts as a binding constraint and introduces alternative dynamics between financial markets and monetary policy.
The basis of the model allows for establishing how the behavior of the monetary authority in setting the target policy rate affects the term structure. From here, we can address the question of how this effect changes when interest rates face a binding constraint at the ZLB and conventional monetary policy can no longer diffuse through the economy along customary channels.

Like Krippner [2012], the data for $R(t, \tau_k)$ spans the years 1986-2012 and includes the end of month 3- and 6-month U.S. Treasury bill rates (from the FRED database on the St. Louis Reserve website) and 1-, 2-, 3-, 4-, 5-, 7-, 10-, 15-, and 30-year continuously compounding zero-coupon U.S. Treasury bond rates as developed in the data set used for Gurkaynak et al. [2006] (from the Federal Reserve Board Research Data website). Therefore, the number of different maturities used to construct the term structure is $K = 11$. For the macro factors, I use data on CPI inflation and the difference real GDP and potential GDP (as projected by the Congressional Budget Office). The policy instrument data includes the exact target for the Federal Funds Rate announced by the Federal Reserve until 2008 and the midpoint of the target range from 2008-2012. Applying the particle filter allows for constructing a time-series of the shadow short rate working beneath the observable term structure.

When the policy target is positive, and thus the shadow rate is likely to be as well, a standard monetary policy VAR would describe the macroeconomy. The relationship between macro indicators and monetary policy is direct and the central bank can influence inflation and real output with conventional measures. However, when the target rate is at zero, and the shadow rate falls to negative values, we need to use the term structure dynamics to flesh out the true stance of monetary policy. In this scenario a shock to policy results in a shock to the shadow rate, through $\theta(t)$, and plays out through this path in order to affect the yield curve and the macroeconomy.

### 4 Preview of Results

As a basis for comparison, initially I estimate the shadow short rate as a function of only two latent state variables, according to the model developed in Krippner [2012]. Figure 1 depicts the the model implied shadow rate from 1986-2012. This procedure generates a time-series for the shadow rate which has been consistently negative since November 2008. In order to narrow the focus specifically to ZLB episodes, I consider the time period in which the U.S. has witnessed a prolonged episode with short-term interest rates around zero and the main instrument of monetary policy paralyzed. Figure 2 focuses in on the years from 2006-2012 in order to capture the time leading up to and during the ZLB episode.

The shadow rate becomes negative around the implementation of the QE1 program in late November of 2008 when the Federal Reserve began purchasing mortgage-backed securities and held the FFR around zero.
Figure 1: Krippner (2012) Shadow Short Rate from 1986-2012. Baseline model in which the shadow rate is a function of only two latent yield curve factors representing "level" and "slope" effects.

Figure 2: Krippner (2012) Shadow Short Rate focused on pre-ZLB and throughout the ZLB episode from 2006-2012. Baseline model in which the shadow rate is a function of only two latent yield curve factors.
I first estimate a VAR of the macroeconomy under the traditional policy scenario with the effective federal funds rate as the observed policy instrument. The constructed shadow rate series allows for estimating a VAR in the alternative scenario with the shadow rate acting as the true stance of monetary policy thus serving as a proxy for the effective policy instrument. The second VAR may better illustrate the unconventional policy tactics pursued by the Federal Reserve including QE1, QE2 and Operation Twist which won’t be captured in a VAR with the Fed Funds rate at zero.

References


