Heterogeneous expectations and financial instability in a pure finance economy

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Very preliminary and incomplete

Abstract: This paper examines the role of extrapolative expectations in a model worked out to explain periodic bursts of financial instability (Sordi & Vercelli, 2006; Dieci, Sordi & Vercelli, 2006). The nexus between extrapolative expectations and financial instability was emphasised long ago by Keynes in the General Theory (1936). Enthoven and Arrow (1956) proved that extrapolative expectations do not necessarily produce dynamic instability in a general equilibrium model. However, it is possible to show that, under more realistic conditions that periodically take root in the economy, the widespread adoption of extrapolative expectations produces growing financial instability in the economy. The adoption of extrapolative expectations reflects the short-termism of economic agents and their herd behaviour. Even more sophisticated agents who rationally choose the best forecasting rule are likely to get locked into the choice of an extrapolative rule. We intend to demonstrate (1) that the widespread use of extrapolative expectations by economic agents brings about a financial bubble that may lead to a serious financial crisis and (2) that the use by economic agents of a mix of extrapolative and regressive expectation formation mechanisms reduces the dynamical instability of the model but may give rise to complex dynamics.

Keywords: Pure finance economy, financial instability, heterogeneous expectations, chaotic dynamics

JEL Classification: G01, C61, D84

1. Introduction

(…)

2. A bi-variate measure of units’ financial conditions

The decisions of economic units (banks, firms, households) are heavily affected by their current and expected financial conditions. Thus, in order to study their influence on the economy, we need a
measure of financial conditions of economic units. It appears that the two crucial dimensions are their liquidity and solvency. In what follows we express units’ financial conditions with a pair of numbers that measure exactly these two crucial dimensions.

With regard to units’ liquidity, we measure it in each period \( t \) as the surplus of financial inflows \( y_{it} \) over financial outflows \( e_{it} \) in proportion to inflows, all flows being measured in the same time unit:

\[
f_{it} = \frac{y_{it} - e_{it}}{e_{it}} \quad i = 1, 2, \ldots, n
\]

The solvency of an economic unit, on the other hand, may be measured by its net worth \( f^*_i \), defined as the capitalization of its expected surplus and deficits,

\[
f^*_i = \sum_{j=0}^{T} E_{t-1} \left[ f_{i+j} \right] (1 + r)^j \quad i = 1, 2, \ldots, n
\]

where \( E_{t-1} \) denotes the conditional expectation operator based upon information available at the end of period \( t-1 \), \( T \geq 1 \), the time horizon of the units and where, for the sake of simplicity, the nominal interest rate \( r \) is used as the discount factor. Given this definition of \( f^*_i \), we have the following condition of financial sustainability for each financial unit \( i = 1, 2, \ldots, n \):

\[
0 \leq f^*_i (< T + 1)
\]

We can understand this condition in intuitive terms by observing that when \( f^*_i < 0 \) the ‘net value’ of the financial unit is negative. In this case the unit is virtually insolvent unless it succeeds to promptly realize a radical financial restructuring or to be bailed out by other units or the state.

For the sake of simplicity we call \( f_{it} \) the “liquidity condition” and \( f^*_i \) the “solvency condition”.

The classification of financial units in terms of the two variables is shown in Fig. 1.
The derivation of the figure can be explained as follows. In principle, there are infinite financial conditions that can be represented in such a Cartesian space. However, it is possible to establish a link between our representation and the well-known classification of financial units suggested by Minsky (e.g., 2008/1986). The blue dotted vertical line drawn at $f_{it}^* = 0$ represents the solvency barrier. If we consider the space to the right of this barrier, we can easily verify that the units with a solvency/liquidity combinations above the red dotted horizontal line (representing the liquidity barrier) may be defined as hedge units in the language of Minsky, while the units underneath may be defined as speculative or Ponzi units. Minsky does not explicitly consider in his classification the units with a negative solvency condition that are virtually insolvent. We believe that this is a crucial shortcoming of Minsky’s classification and that all units with a $(f_{it}^*, f_{it})$ combination to the left of the solvency line should also be considered. The reason for this is that a financial unit that is virtually insolvent according to the expectations of the unit itself or its creditors does not necessarily go broke as it may be bailed out by the state, or another firm through merger or acquisition or it may save itself through a radical restructuring/downsizing of its activity. The destiny of such distressed financial units, as we are going to call them, is crucial to describe, explain and forecast financial crises and in order to choose the best possible policy to keep them under control. Thus, for the time being, the suggested continuous measurement of units’ financial conditions allows a ternary classification that is similar, but not identical, to Minsky’s classification:
hedge, speculative (and Ponzi), and distressed units. In order to use this Cartesian space for the study of financial fluctuations we need a further essential ingredient. We assume that each financial unit, in order to minimize its risk of bankruptcy, chooses a margin of safety, i.e. a maximum value of the intertemporal ratio sufficiently higher than zero, beyond which it does not want to go. Let’s call this safety margin \( \mu_i \). So we have to draw a further vertical line at the right of the solvency barrier (the green dotted vertical line drawn at \( f_{u}^{*} = \mu_i \) ) and this allows a refinement of the classification in six financial postures (see Fig. 1). Units in region 1 may be called hyper-hedge as they do not have problems neither from the liquidity point of view nor from the solvency point of view (\( f_{u} > 0, f_{u}^{*} > \mu_i \)). Units in region 2 are speculative as they have liquidity problems but do not perceive solvency problems (\( f_{u} < 0, f_{u}^{*} > \mu_i \)). Units in region 3 are hyper-speculative as they have both liquidity problems and solvency problems (\( f_{u} < 0, 0 < f_{u}^{*} < \mu_i \)). Units in region 4 are hedge units because they do not have liquidity problems but perceive that they may have solvency problems in the future as their safety margin is too small (\( f_{u} > 0, 0 < f_{u}^{*} < \mu_i \)). Finally we have to consider the units in financial distress. We can distinguish between units in region 5, which we define highly distressed financial units being both illiquid and virtually insolvent (\( f_{u} < 0, f_{u}^{*} < 0 \)), and units in region 6, which we define distressed units. The latter are virtually insolvent units, but that have managed in the current period to obtain financial inflows higher than the financial outflows raising hopes of survival (\( f_{u} > 0, f_{u}^{*} < 0 \)). This six-fold classification of financial conditions of economic units keeps a bridge with Minsky’s classification but eliminates some of its shortcomings.

3. Financial instability and extrapolative expectations

In Vercelli (2009a,b) the classification just described is used to suggest a reformulation of the Financial Instability Hypothesis (FIH)’s core through a very simple model of financial fluctuations in the space defined by \( k_{u} = 1 - f_{u} \) and the corresponding intertemporal variable \( k_{u}^{*} \) (see also Vercelli 2000, Sordi and Vercelli 2006 and Dieci et al. 2006). The main idea is that the basic building block of the FIH is the interaction between liquidity and solvency conditions as represented by \( k_{u} \) and \( k_{u}^{*} \) respectively. The cyclical dynamics in the space \( \{k_{u}^{*}, k_{u}\} \) is easily described once the following assumptions are introduced. First of all, each unit is assumed to prefer higher returns ceteris paribus. In addition, financial returns are assumed to be positively correlated
with the financial leverage within the desired margin of safety so that speculative units show a positive correlation between returns and risk-taking as expressed by the distance from the safety margin. Finally units are assumed to be characterized by herd behaviour due to the pressure of the market and mass psychology. Under these assumptions it turns out that there is a tendency to a clockwise cycle. It is not too difficult to adjust to the present case, expressed in terms of \( f_u \) and \( f_u^* \), the description of the resulting cyclical dynamics given in Vercelli (2009a,b). To do this, let us consider a financial unit which initially has liquidity and solvency conditions such as to be in region 1 of Fig. 1. For this unit it is possible to increase its financial outflows more than its inflows without getting into liquidity troubles; in addition, since it continues to have an excess of inflows, the unit revises its expectations in such a way to reduce further its perceived risk of insolvency. This may cause the unit to enter in region 2. When this happens the unit has an incentive to increase its leverage in order to increase its returns and this is true until it reaches its margin of safety \( \mu_i \). When this happens, and the unit enter region 3, it then tries to reduce the excessive risk of insolvency by de-leveraging; however, since it continues to have an excess of outflows over inflows, though a diminishing one, its perception of insolvency risk continues to increase. When it enters region 4 it has succeeded in rebuilding an excess of inflows and this progressively reduces the risk of insolvency. Most units follow this sequence of financial conditions describing a financial cycle. If the margin of safety is too small and the reaction to liquidity problems and/or solvency risk is too week, the financial unit may cross the solvency barrier and become virtually insolvent (region 5). After the solvency barrier is crossed, the behaviour of the unit has to change radically to avoid bankruptcy. This result may be obtained either through a restructuring that abates current and prospective outflows much more than inflows or through a bail-out by the state or another firm. If the unit is able and lucky, it may rapidly shift in region 6 and immediately after again in region 4, starting a new financial cycle. In any case there is a sudden and huge reduction of outflows that reduces the inflows of other units that are pushed to trespass the solvency barrier.

Our purpose is now to derive a dynamical system in the two variables \( f_u \) and \( f_u^* \) capable of generating a persistent dynamics of the type just described. Following the procedure of Dieci, Sordi & Vercelli (2006), where the equation for the dynamics of the intertemporal financial ratio \( k_i^* \) was derived starting from its definition and by assuming that financial units revise their expectations about future inflows and outflows according to a simple adaptive scheme, we first assume that at the end of period \( t-1 \) the structure of financial units’ expectations is flat and such that:

\[
E_{t-1}[f_{t+s}] = f_i^n, \quad s = 0,1,...,T
\]

While the economic rationale for this assumption was not discussed by Dieci et al. (2006), we can
justify it on the basis of Minsky’s idea of “periods of tranquility”. As is well known (see, for example, Minsky 2008/1986, p. 197 and also Papadimitriou and Wray 2008), the latter are periods characterized by a robust financial system and few innovations. At the beginning of a period of this type, it appears reasonable to assume that the units’ structure of expectations is flat, this reflecting the expectation that in the near future, within the time-horizon \( T \), the inflows and outflows will be more or less constant. Thus, inserting (3) into (2), we obtain:

\[
\begin{align*}
    \bar{f}^a = \sum_{s=0}^{T} \frac{E_{t+1} \left[ f_{t+s} \right]}{(1+r)^s} = \bar{f}^a (1+r)^{T+1} - 1 \quad \frac{1}{r(1+r)^T} \\
    \bar{f}^a = \frac{\bar{f}^a}{a}
\end{align*}
\]

where:

\[
a = a(r, T) = \frac{r(1+r)^T}{(1+r)^T - 1}
\]

This function is such that:

\[
\begin{align*}
    \frac{\partial a}{\partial r} &= \frac{(1+r)^T [1+r + Tr - (1+r)^T]}{[1+r)^T - 1]^2} > 0 \\
    \frac{\partial a}{\partial T} &= \frac{r(1+r)^T \ln(1+r)}{(1+r)^T - 1]^2} < 0
\end{align*}
\]

Moreover, given that \( a(r, 0) = 1 \), we can conclude that \( a \leq 1 \) for all \( T \geq 0 \) and \( \lim_{T \to \infty} a(r, T) = 0 \) (see Fig. 2).
Minsky’s FIH, however, warns us that “stability is destabilizing”. Indeed, in a situation of tranquility, net profits (i.e., profits less debt commitments) are positive and this fact leads to a state of increasing euphoria. In turns, this causes expectations to improve, a fact that we can model as a mechanism of extrapolative expectations formation. The simplest way to do this is to assume that if at time $t$ a financial unit observes a liquidity index less (greater) of what it considered to be “normal” at the end of the previous period ($\bar{f}_i^n$), then it expects its liquidity index to further decrease (increase) below (above) that level next period, i.e.:

$$E_t[f_{it+1}] = f_t + \rho \left( f_t - \bar{f}_i^n \right) = (1 + \rho) \left( f_t - \bar{f}_i^n \right) + \bar{f}_i^n$$

where $\rho > 0$. Thus, we also have:

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1Many years after the classical contributions by Goodwin (1947), Enthoven & Arrow (1956) and Negishi (1964), there has recently been a revival of interest in this type of expectations formation mechanism as testified, for example, by Vega-Redondo (1989), Pesaran (1996) and, more recently, Lansing (2006) and all contributions to the “chartists-fundamentalists” (or, more in general, to the heterogenous expectations) literature. We will come back on this in the next section of the paper. In short, the motivation behind this renewed interest in extrapolative expectations is a large empirical evidence suggesting that in reality expectations are usually less than fully rational and that agents prefer rather to rely on simple heuristic principle, the more so, the more irregular is the dynamics of the crucial variables.
\[ E_t[f_{it+2}] = (1 + \rho) \left( E_t[f_{it+1}] - \bar{f}_i^n \right) + \bar{f}_i^n \]

\[ \vdots \]

\[ E_t[f_{it+3}] = (1 + \rho) \left( E_t[f_{it+2}] - \bar{f}_i^n \right) + \bar{f}_i^n \]

Repeated substitution yields:
\[ E_t[f_{it+s}] = (1 + \rho)^t \left( f_{it} - \bar{f}_i^n \right) + \bar{f}_i^n \quad s = 0, 1, \ldots, T \]

Thus:
\[ f_{it+1}^* = \sum_{s=0}^{T} E_t[f_{it+s+1}] = \sum_{s=0}^{T} E_t[f_{it+s}] \]
\[ = \sum_{s=1}^{T} \frac{(1 + \rho)^s \left( f_{it} - \bar{f}_i^n \right) + \bar{f}_i^n}{(1 + r)^{s-1}} \]
\[ = (1 + \rho) \left( f_{it} - \bar{f}_i^n \right) \sum_{s=1}^{T} \frac{(1 + \rho)^{s-1}}{1 + r} + \bar{f}_i^n \sum_{s=0}^{T} \frac{1}{1 + r} \]
\[ = (1 + \rho) \left[ \frac{1 - \left( \frac{1 + \rho}{1 + r} \right)^T}{1 - \left( \frac{1 + \rho}{1 + r} \right)} \right] \left( f_{it} - \bar{f}_i^n \right) + f_{it}^* \]
\[ = \frac{(1 + \rho) \left[ (1 + r)^T - (1 + \rho)^T \right]}{(1 + r)^T \left( r - \rho \right)} \left( f_{it} - \bar{f}_i^n \right) + f_{it}^* \]
\[ = \beta \left( f_{it} - \bar{f}_i^n \right) + f_{it}^* \quad (8) \]

where:
\[ \beta = \beta(\rho, r, T) = \frac{(1 + \rho) \left[ (1 + r)^T - (1 + \rho)^T \right]}{(1 + r)^T \left( 1 - r - \rho \right)} > 0 \]

such that (see Fig. 3):
\[ \frac{\partial \beta}{\partial \rho} = \frac{(r + 1)^T \left[ (1 + r + Tr - T \rho)(\rho + 1)^T - (1 + r)(r + 1)^T \right]}{(r - \rho)^2} > 0 \]
\[ \frac{\partial \beta}{\partial T} = -(r + 1)^T (\rho + 1)^T \ln(\rho + 1) - \ln(r + 1) \frac{r - \rho}{r - \rho} > 0 \]
\[ \frac{\partial \beta}{\partial r} = -\frac{(\rho + 1) \left[ (1 + \rho)(r + 1)^T - (1 + \rho + Tr - T \rho)(\rho + 1)^T \right]}{(r - \rho)^2 (r + 1)^T} \leq 0 \]
Finally, given (4), equation (8) becomes:

\[ f_{i+1}^* = \beta f_i + (1-a\beta) f_i^* \]  

(9)

To close the model, in analogy with what done in Sordi & Vercelli (2006) for the variable \( k_i = 1-f_i \), we consider the following equation for the dynamics of the liquidity condition of unit \( i \):

\[ f_{i+1} = f_i - \alpha_i (f_i^* - \mu) \]  

(10)

The interpretation of this equation in the present context is the following. When a unit at time \( t \) observes a value of its liquidity index less the safety margin \( \mu_i \), it reacts in the next period by reducing outflows relatively to inflows and vice versa when it observes a value greater than \( \mu_i \).

Thus, the dynamical system of the model turns out to be the simple linear feedback mechanism between \( f_i \) and \( f_i^* \) described by equations (9) and (10), or, in matrix notation:

\[
\begin{bmatrix}
  f_{i+1}^* \\
  f_{i+1}
\end{bmatrix} = A \begin{bmatrix}
  f_i^* \\
  f_i
\end{bmatrix} + \begin{bmatrix}
  0 \\
  \alpha, \mu
\end{bmatrix}
\]

where
\[
A = \begin{bmatrix}
1 - a\alpha & \beta \\
-\alpha & 1
\end{bmatrix}
\]
such that \(\text{tr}(A) = 2 - a\beta\) and \(\det(A) = 1 - a\beta + \alpha, \beta\).

Simple passages show that the equilibrium values of the two variables are:

\[
f_i^e = \mu
\]

\[
f_i^* = a\mu < \mu
\]

while the characteristic equation is given by:

\[
\lambda^2 - (2 - a\beta)\lambda + (1 - a\beta + \alpha, \beta) = 0
\]

(11)

The dynamics of \(f_i\) and \(f_i^*\) is cyclical when the discriminant \(\Delta\) of (11) is negative, that is when:

\[
\Delta = (2 - a\beta)^2 - 4(1 - a\beta + \alpha, \beta) = \beta(a^2\beta - 4\alpha) < 0
\]

condition which is satisfied when:

\[
\alpha_i > \frac{a^2\beta}{4}
\]

(12)

In Fig. 4, the region of the \((\rho, \alpha_i)\)-plane where condition (12) holds is shown in yellow for the case in which \(r = 0.05\) and \(T = 5\). Other cases, for different combinations of \(r\) and \(T\), are shown in Figs. 5 and 6. These figures give evidence that the region of combinations of parameters such that the dynamics of the model is cyclical decreases with \(T\), whereas remains almost unaltered by changes in \(r\).

To study the dynamical stability of the solution, we consider the necessary and sufficient conditions for (11) to only have roots of absolute value less than one (see, for example, Gandolfo 1997, pp. 58-59). In terms of the trace and determinant of the matrix \(A\) these conditions can be expressed as:

\[
1 + \text{tr}(A) + \det(A) = 4 - 2a\beta + \alpha, \beta > 0 \iff \alpha_i > \frac{2a\beta - 4}{\beta}
\]

(13)

\[
1 - \text{tr}(A) + \det(A) = \alpha, \beta > 0
\]

(14)

\[
1 - \det(A) = |a - \alpha_i|\beta > 0 \iff \alpha_i < a
\]

(15)
While condition (14) is always satisfied, so that we can disregard it, the other two are satisfied or not depending on the combination of $\rho$ and $\alpha_i$. This is shown in Fig. 7 for the same case as in Fig. 4 and in Figs. 8 and 9 for other various combinations of the parameters.

Finally, as an example, the full spectrum of all possible typologies of dynamics is given in Fig. 10 for the case $r = 0.05$ and $T = 5$.

Thus, the conclusion is that the fact that financial units revise their expectations according to an extrapolative mechanism destabilises the system.
This is easily visualised by numerical simulation: the typical dynamics at the micro level generated by the solvency and liquidity conditions of the financial unit are fluctuations of increasing amplitude (yellow region) or an explosive monotonic dynamics (grey region), implying the collapse of the unit. In Fig. 11 we have chosen values of $\rho$ such that condition (13) is redundant and taken values of $r = 0.05$, $T = 1, 4, 7$ and $\alpha_i$ such that $\alpha_i = a$. In this special case the dynamical system generates persistent fluctuations of constant amplitude of the financial conditions of the unit. In all three cases we have considered, we obtain a clockwise cyclical sequence of the financial condition.
of the unit of the type we have verbally described at the beginning of this section.\textsuperscript{2} It turns out that the shorter the time horizon of the unit, the less probable is that its financial conditions cross the solvency barrier and it becomes virtually insolvent. In all cases, however, as shown in Fig. 12 it is then enough to take a slightly greater value of $\alpha$, let us say such that $\alpha = a + 0.01$, to have the system to generate explosive fluctuations.

\textbf{Fig. 8:} The same as in Fig. 7 for various combinations of $r = 0.01, 0.04$ and $T = 2, 6, 10$

\textbf{Fig. 9:} The same as in Fig. 8 for various combinations of $r = 0.07, 0.10$ and $T = 2, 6, 10$

\textsuperscript{2}It should be stressed, however, that there is a minor difference between Fig. 1, we have used to classify financial units on the basis of their solvency and liquidity conditions, and the Figures we are now discussing. Indeed, in the latter, the red dotted line – as a result of the specification of the dynamical system – is drawn at $f_a = a \mu$ rather than at $f_a = 0$. It should be reminded however (see Fig. 2) that $a$ quickly approaches zero as the time horizon of the unit increases.
$\alpha_i = \frac{(2 a \beta - 4)}{\beta}$

$\alpha_i = a$

$\alpha_i = \sqrt{4 \beta}$

$T = 0.05, \ T = 5$

Fig. 10: xxx

$T = 1, \ \alpha_i = a = 0.5122$

$T = 7, \ \alpha_i = a = 0.1474$

Fig. 11: Persistent fluctuations of constant amplitude for $\alpha_i = a$ and $T = 1,4,7$

$T = 1, \ a = 0.5122, \ \alpha_i = 0.5222$

$T = 4, \ a = 0.22, \ \alpha_i = 0.23$

$T = 7, \ a = 0.1474, \ \alpha_i = 0.1574$

Fig. 12: Explosive fluctuations for $\alpha_i = a + 0.01$ and $T = 1,4,7$
4. Heterogenous expectations and persistent dynamics

An unsatisfactory aspect of the previous analysis is the assumption that the financial unit, having started in a period of tranquillity and euphoria to form extrapolative expectations, keeps to this mechanism of expectations formation regardless of the phase of the cyclical pattern of its financial conditions. More realistic, and perhaps closer to Minsky’s view of the FIH is to assume that units’ expectations regarding their future cash outflows and inflows endogenously evolve over the financial cycle. The simplest way to do so is to assume that financial units form extrapolative expectations (due to ‘euphoria’) until they observe a liquidity index not too far from what they consider ‘normal’, but start to form regressive expectations – such that the liquidity index is expected to reverse toward its long run equilibrium $a\mu_t$ – when a certain critical distance is reached.\(^3\) Thus, in each period over the cycle, there will be financial units that form their expectations according to an extrapolative mechanism of the type:

$$E_t' [f_{t+1}] = f_t + \rho' (f_t - \bar{f}_t) = (1 + \rho')(f_t - \bar{f}_t) + \bar{f}_t$$

and others that instead use the following regressive mechanism:

$$E_t' [f_{t+1}] = f_t - \rho' (f_t - \bar{f}_t) = (1 - \rho')(f_t - \bar{f}_t) + \bar{f}_t$$

We can expect that this behaviour of individual financial units will lead to a persistent dynamics of the aggregate financial conditions of our pure finance economy.

To show this, we first assume that, in the aggregate, the mechanism of expectations formation turns out to be a mix of the two basic mechanisms of expectations formation, with weights that vary over the financial cycle:

$$E_t [f_{t+1}] = \alpha_t E_t' [f_{t+1}] + (1-\alpha_t) E_t' [f_{t+1}]$$

\(^3\)In assuming this we are closely following some recent contributions by Westerhoff (e.g., 2006a,b,c) – where the simple linear multiplier-accelerator model is revised and extended by including the role played by agents’ heterogenous expectations about future income – and, more in general, the recent, growing body of literature on financial markets (see, e.g., Day & Huang 1990, Brock & Hommes 1998 and Chiarella, Dieci and He 2007, 2009 to mention only a few) in which a crucial role is played by the consideration of different groups of agents (e.g., ‘chartists’ and ‘fundamentalists’) which revue their expectations according to different schemes.
where \( \rho^e_r = w_r \rho^e + (1 - w_r)(-\rho^e) \) turns out to be a weighted average of the two individual coefficients \( \rho^e \) and \( -\rho^e \). To make the study of the dynamics possible to handle, we assume that \( w_r \) can take only the two extreme values 0 and 1 according to the following mechanism:

\[
w_r = \begin{cases} 
0, & \text{if } |\bar{f}^n - f^e| \geq b \\
1, & \text{if } |\bar{f}^n - f^e| < b 
\end{cases}
\]

where \( b \) is a small positive magnitude. In this special case, at the aggregate level, the extrapolative mechanism prevails when the aggregate liquidity index is not too far from its normal level, whereas the regressive mechanism prevails in the opposite case, i.e.:

\[
\rho^e_r = \begin{cases} 
-\rho^e < 0, & \text{if } |\bar{f}^n - f^e| \geq b \\
\rho^e > 0, & \text{if } |\bar{f}^n - f^e| < b 
\end{cases}
\]

Thus, the first equation of the dynamical system for the whole economy becomes:

\[
f^e_{t+1} = f^e_t + \beta^e_t \left( f^e_t - \bar{f}^n \right)
\]

where:

\[
\beta^e_t = \frac{1}{(1 + r)^{T-1}} \left[ \begin{array}{c} w_r \rho^e + (1 - w_r)(-\rho^e) \\ 1 \end{array} \right]
\]

such that:

\[
\frac{\partial \beta^e_t}{\partial \rho^e} > 0, \quad \frac{\partial \beta^e_t}{\partial \rho^r} > 0, \quad \frac{\partial \beta^e_t}{\partial T} > 0, \quad \frac{\partial \beta^e_t}{\partial r} \leq 0
\]

Numerical simulation gives evidence that the system is now able to generate a wide range of dynamics, including persistent fluctuations and complex dynamics. Some examples are given in Fig. 13, 14 and 15, where we have taken \( \rho^e = 0.2 \), \( \rho^r = 0.8 \), \( b = 0.02 \), \( \alpha = 0.25 \), \( r = 0.07 \), \( \mu = 0.1 \) and different values of \( T \) (\( T = 1, 2, 5 \) respectively).

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4 This is in perfect agreement with what we read in the following passage of Goodwin’s 1947 article (p. 192):

“It is in contradiction to known facts, and inherently quite unlike that all producers would have the same expectations. Upon inspection, however, it turns out that the aggregate expectations coefficient (...) can be considered as a weighted average of the various individual coefficients”
For the case considered in Fig. 15, the following Fig. 16 clearly shows the property of sensitive dependence to initial conditions, a property which we take as an indicator that in this case the dynamics is chaotic.
5. Conclusions

(...)
References


