Threshold Effect, Financial Intermediation and Macroeconomic Performance

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ABSTRACT

This paper analyzes the theoretical finance-growth nexus in the case of developing countries. Using the Neoclassical growth framework, we raise a new issue where the finance-growth nexus has multiple stationary states with threshold effect. Threshold effect prevents the economy to reach long-run steady state equilibrium of capital stock. We show that financial intermediary is better than financial market in order to reduce threshold effect, and to ensure the existence and uniqueness of a higher long-run steady state equilibrium of capital stock by promoting long-term investment.

Keywords: Threshold Effect, Financial Intermediation, Economic Growth, Developing Countries
JEL Classification: C61, C62, O16

1. Introduction

During the last two decades, the literatures on the nexus between financial development and economic growth emerge, but the findings are still subject to relevant debate until nowadays2. In developing countries, particularly, financial development is associated with banking sector development, since financial market is underdeveloped. However, the more recent literature suggests that financial market should be also taken into account to spur economic growth, even in developing countries. Using a very large

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2 In empirical study, see King and Levine (1993a, 1993b), Levine (1998); Rajan and Zingales (1998) at the country level study, and Fisman and Love (2002) at the industry level; or recently Demirgüç-Kunt and Maksimovic (2002) at the firm level. In theoretical study, see Bencivenga and Smith (1991), or recently Hung and Cothren (2002). Levine (2005) provides a comprehensive literature review.
cross-country sample incorporating both developed and developing countries, Levine and Servos (1998) show that stock market liquidity leads to faster rate of growth, productivity improvement, and capital accumulation\(^3\). Their paper supports Levine (1991) and Bencivenga et al (1995), where stock market liquidity facilitates long-term investment, since investors can easily sell their stake in the project if they need liquidity before their project matures. Enhanced liquidity and long-term investment, therefore, increase higher-return projects that boost productivity growth.

Meanwhile, it is also well accepted that financial market tends to be more prone to asymmetric information problems and thus, financial liberalization fostering stock market liquidity is often blamed for macroeconomic downturn, as well as banking vulnerability and crisis (Bihde, 1993; Demirgüç-Kunt and Detagriache, 1999). Thus, the adverse effect of financial market occurs. This is why according to Diamond (1984) the presence of bank as financial intermediation is necessary, since banks have technology to gain information from investors which enhance investor’s rational decision based on their consumption profile.

Building on the previous literatures on the importance of financial intermediation on economic growth, Bencivenga and Smith (1991) show that financial intermediation is better than financial autarky (financial market) in order to spur economic growth. In their contribution, there are basic lists of bank activities such as loans funded deposits, holding liquid reserves against predictable withdrawal demands, issuing liabilities that are more liquid than their primary asset, and reducing the need of self-investment. The main result of their model is that financial intermediation promotes the productive long-term (illiquid) investment rather than short-term (liquid) ventures.

However, the optimal proportion of long-term investment is decreasing in the income of long-term investment itself, although it is increasing in the fraction of entrepreneurs. It is also surprising that the optimal proportion of long-term investment is increasing in the income of short-term ventures and the fraction of non-entrepreneurs. Hence, although the income of long-term investment is higher than the income of short-term ventures, it does not always incitate agents to be entrepreneur. This implies that entrepreneurship is not always a growth-enhancing factor.

\(^3\) Stock market liquidity refers to the less expensive cost of equities trading.
Recently, both theoretical and empirical studies have questioned the positive link between financial intermediation and economic growth. Deidda and Fattouh (2002) theoretically show a non-linear relationship between financial intermediation and endogenous growth. The effect of financial intermediation on economic growth remains ambiguous at low initial levels of banking sector development and the existence of risk-averse agents. This is because risk-averse agents always prefer to incur financial transaction costs even though the expected return on their savings is lower than under financial autarky. Such a situation occurs because financial intermediation can fully perform in risk diversification process. As a result, economic growth rate under banking sector is lower than under financial autarky. At high levels of the banking sector development, the relationship between banking sector development and economic growth is always positive, where the level of banking sector development depends on the initial level of real per capita income.

In the empirical examination, Deidda and Fattouh (2002) also find that there is no significant effect of financial development on economic growth in low-income countries, whereas in high-income countries, there is a positive link between financial development and economic growth. Mihci (2006) highlights that the relationship between finance and growth does not necessarily positive when substantial variations across different periods and country groups are taken into account. Meanwhile, Crouzille et al (2007) indicate the presence of threshold effect on the link between rural bank development and regional growth in the Philippines.

The aim of this paper is therefore to reevaluate a theoretical finance-growth nexus. We modify several hypothesis used by Bencivenga and Smith (1991). First, since our motivation is to model the most suitable condition for developing countries, we consider that externalities changes due to technological innovation may be less important, so that they may not much play a pivotal role in boosting economic growth. Hence, we use the Neo-classical growth hypothesis without externalities in an overlapping generation (OLG) model with three periods instead of drawing endogenous growth model as developed by Bencivenga and Smith (1991), or Deidda and Fattouh (2002). Using the Neo-classical growth framework allows us to obtain more reasonable growth rate in developing countries, where the growth rate in consecutive years is not necessarily
positive. Second, we distinguish the behaviour vis-à-vis of risk between non-entrepreneur and entrepreneur. More precisely, the entrepreneurs are supposed to be risk neutral⁴. This hypothesis allows us to consider that entrepreneurs’ risk-taking behaviour may be the source of costly overinvestment which reduces long-term economic growth⁵.

Using these features, our contributions are threefold. First, we show that entrepreneurship is always growth-enhancing factor in both financial intermediary equilibrium and financial market equilibrium, since the optimal proportion of long-term investment is increasing in the fraction of entrepreneurs, the income of long-term investment and short-term ventures, as well as the agent’s savings rate. Second, we show that agent’s saving is a main determinant of the optimal proportion of long-term investment, where in Bencivenga and Smith (1991), financial intermediary is not incitated to raise agents’ savings as input. Therefore, we characterize the traditional role of bank as financial intermediary (deposits and investments). Third, our model is characterized by the existence of multiple steady states equilibrium with the threshold effect of capital stock, as development trap problem, which impedes the economy to reach the higher long-run steady state equilibrium. In this case, financial intermediation is better than financial autarky, since the threshold level of the financial intermediation model is lower than that of the financial autarky model, and financial intermediation yields a higher transition of capital stock than financial autarky.

Our results differ from that of Deidda and Fattouh (2002) for several reasons. First, we use the Neoclassical growth framework, while they use endogenous growth. Second, we emphasize that banking sector development is always better than financial autarky to decrease threshold level and increase long-run capital stock, while in Deidda and Fattouh (2002) the opposite is true at low levels of the financial development. Third, our threshold effect is due to the initial level of capital stock, while in their model, threshold effect is due to the initial level of real per capita income. Since the real per capita income depends on the initial level of capital stock for production, their model may suffer from major reverse causality problems on the finance-growth nexus.

⁴ Azariadis and Smith (1998) also use this hypothesis for a different framework of model.
⁵ Baumol (1990) analyzes the riskiness of entrepreneurship activity which may be unproductive or even destructive. This fact should not be neglected by financial sectors whose role is to provide financial supports for entrepreneurship activity.
The rest of this paper is organized as follows. Section 2 describes the model set-up. Section 3 lays out the financial market model. Section 4 shows the financial intermediary equilibrium. Section 5 examines the study of capital stock dynamic and threshold effect. Section 6 concludes.

2. The Set-Up

The framework we use is one of overlapping generations (OLG) model with three periods and a unique good. We draw the set-up of Bencivenga and Smith (1991) which is originated from Diamond and Dybvig (1983), where all agents need liquidity and banks play a role to provide liquidity for them. We assume that there is no population growth in the economy and each generation consists of a continuum of agents with size \( N_t = N = 1 \).

Each agent may live for two or three periods. Let \( t \) be the time index, where the young and middle-age generations are endowed with an initial per firm capital stock of \( k_0 \) units at \( t = 0 \) and \( k_1 \) units at \( t = 1 \), respectively. Moreover, each young agent supplies inelastically one unit of labour in the first period.

At the first period, all agents of a generation are identical. At the beginning of the second period, the agents learn whether they will be either non-entrepreneurs (two-period-lived agents) or entrepreneurs (three-period-lived agents) with probability \( (1 - \pi) \) and \( \pi \), respectively. Thus, there are \( (1 - \pi)N \) agents who will be non-entrepreneur at the second period and \( \pi N \) agents who will be entrepreneur at the third period. All young agents save entirely their labour income in the first period.

If agents are non-entrepreneur, they consume their second period incomes, \( c_{1t} \). If the agent is entrepreneur, he consumes the profit of production in the third period, \( c_{2t} \). Thus, agents have different liquidity needs and the non-entrepreneurs have higher liquidity need than entrepreneurs, since non-entrepreneurs only live for two periods. Meanwhile, the young agents have incentive to be entrepreneur because the profit of long-term investment is relatively higher than the return of non-entrepreneur’s saving. We also assume that entrepreneurs are risk-neutral following Azariadis and Smith (1998).

Finally, whatever the type of agents, we can define the agent’s preferences by the following expected utility function.
We define \( c_{it} \) as the period \( i \) consumption of an agent who is born at \( t \). The constant relative risk aversion is denoted by \( \gamma > -1 \). The variable \( \phi \) stands for the individual specific random variable realized at the beginning of period 2. Thus, the value of \( \phi \) is equal to 0 with probability \( 1 - \pi \), or 1 with probability \( \pi \).

In order to complete this model, we characterize the production function and the entrepreneur’s behaviour. The entrepreneur’s production \( y_t \) is realized by physical capital \( k_t \) and units of labour \( L_t \). We follow the Cobb-Douglas production function as follows

\[
y_t = A k_t^\theta L_t^{1-\theta}
\]

where \( \theta \in [0,1] \) is the part of production that uses \( k_t \) and \( A \) is an arbitrary coefficient. For simplification, we assume that capital depreciates completely at the end of period. Furthermore, there is no endowment of capital at period \( t > 0 \) except for the initial old generation and middle-age generation. In order to complete the entrepreneur program, the profit function must be established. The entrepreneur’s profit \( \Pi_t \) is the difference between the production and the cost of quantity units of labour defined

\[
\Pi_t(k_t, L_t) = A k_t^\theta L_t^{1-\theta} - w_t L_t.
\]

At the equilibrium of labour market, labour demand \( L_t \) is equal to labour supply, \( N_t = N \), which is obtained by maximizing the entrepreneur’s profit subject to \( L_t \). Thus, we have \( w_t = A(1-\theta)k_t^\theta \pi^{\theta-1} \) and the maximized profit function at each period \( t \) as much as

\[
\Pi_t = A \psi k_t^\theta, \text{ with } \psi = L_t^{1-\theta} = \pi^{\theta-1}
\]

3. The Financial Market Equilibrium

This system refers to an economy without the presence of bank as financial intermediary. In the first period, the agents divide their savings \( s_t \) between liquid and illiquid assets. Liquid assets are considered as inventory of consumption goods. One unit invested in liquid asset at \( t \) directly yields \( n > 0 \) units of consumption goods at both \( t + 1 \) and \( t + 2 \). On the other hand, one unit invested in the illiquid asset yields \( R \) units of
capital goods at $t+2$. If illiquid asset is liquidated at $t+1$, then the agents receive the “scrap value” of $x$ units of consumption goods, where $0 < x < n$.

In order to establish the agents’ budget constraint, we define $z_i^m$ and $q_i^m$ as the proportion of liquid asset and illiquid asset invested at $t$, respectively. The superscript $m$ stands for the financial market. Hence, we have

$$z_i^m + q_i^m = 1, \text{ where } z_i^m \geq 0, \ q_i^m \geq 0 \tag{4}$$

At the first period, the agents’ saving is equal to labour income, $s_t = w_i$, and is divided into $z_i^m s_t$ units of liquid asset and $q_i^m s_t$ units of illiquid asset. Let $i_L, i_R, i_S$ be the interest rate of the liquid asset, illiquid asset, and “scrap” value, respectively. At the second period, let $\omega_{1t}$ be the income of non-entrepreneur after one period, then

$$\omega_{1t} = (nz_i^m + xq_i^m) w_i, \text{ where } n = 1 + i_L \text{ and } x = (1 + i_r) \tag{5}$$

By the hypothesis, if the agents are entrepreneur, then their consumption at the second period is equal to zero. At the beginning of the third period, the entrepreneur sells his illiquid assets and reinvests them in the physical capital, so that $(1 + i_L)q_i^m s_t = k_{r+2}$. This situation corresponds to the financial autarky case. At the third period, let $\omega_{2t}$ be the income received by entrepreneur before the production, then

$$\omega_{2t} = nz_i^m w_i + Rq_i^m w_i, \text{ where } R = 1 + i_L, 0 < x < n < R \tag{6.a}$$

We use (6.a) to construct the dynamics of capital stock as follows

$$k_{r+2} = Rq_i^m w_i = \varphi_m (k_r) \tag{6.b}$$

Using the profit function (3) and the budget constraints in the equation (4), (5) and (6.b), we now define the agent’s expected utility function when investment is self-financed.

$$U(q_i^n) = \left( -\frac{(1-\gamma)}{\gamma} (xq_i^m w_i + n (1-q_i^m) w_i) \right)^{-\gamma} + \pi \left( A0\psi (Rq_i^m w_i) + (1-q_i^m)nw_i \right) \tag{7}$$

Meanwhile, the agents’ optimization program is defined as $\arg\max_{0 \leq q_i^m \leq 1} \{U(q_i^n)\}$. From the first order condition, we obtain the optimal proportion of illiquid asset ($\overline{q_i^m}$) as follows.

$$\overline{q_i^m} = \overline{q_i^m} (w_i) = \frac{n}{(n-x)} \left( \frac{B(w_i)}{w_i (n-x)} \right)^{\frac{1}{\gamma}} \tag{8}$$
where \( B(w_t) = \frac{\pi}{\pi - 1} \left( \frac{nw_t - AR^bw_t ^2\psi}{w_t(n - x)} \right) \)

The optimal proportion of illiquid investment \( \bar{q}_t^m \) depends on the labour income \( w_t \). Moreover, the existence of \( \bar{q}_t^m \) in which \( 0 \leq \bar{q}_t^m \leq 1 \) can be examined by the limit value of \( \bar{q}_t^m \) when \( w_t \to 0^+ \) and \( w_t \to \infty^+ \). From (8), it is straightforward to obtain \( \lim_{w_t \to 0^+} q_t^m = -\infty \) and \( \lim_{w_t \to \infty^+} q_t^m = 1 \), if \( AR^bw_t ^2\psi > nw_t \). Hence, there is a value of \( w_t \) which implies that \( \bar{q}_t^m = 0 \).

4. The Financial Intermediary Equilibrium

We assume that agent’s financial decisions are intermediated through the banking system. Therefore, we can directly define the program of financial intermediaries realized by an institution called as “bank”. We assume that bank is a coalition of young agents who can be either non-entrepreneur or entrepreneur. Let \( z_t \) and \( q_t^b \) be the proportion of liquid and illiquid investment realized by banks, respectively. Thus, we have

\[
z_t^b + q_t^b = 1
\]

Banks ensure non-entrepreneur to receive \( R_t^b \) units of consumption goods at \( t+1 \) from each unit invested at \( t \) as following

\[
(1 - \pi) R_t^b = \alpha_{1t} z_t^b n + \alpha_{2t} q_t^b x
\]

where \( \alpha_{1t} \) and \( \alpha_{2t} \) are the part of liquid and illiquid asset liquidated at the second period, respectively. The bank chooses the values of \( \alpha_{1t} \) and \( \alpha_{2t} \). Moreover, banks also ensure entrepreneurs to receive \( R_t^b \) units of capital goods at \( t+2 \) from each unit of time \( t \) illiquid investment and \( \tilde{R}_t^b \) units of time \( t+1 \) consumption goods from each unit liquid asset invested at \( t \). For the withdrawal after two periods, there are \( \pi \) entrepreneurs who must receive \( R_t^b \) units of capital goods from each unit of illiquid investment. Thus, \( \pi R_t^b \) factor

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6 In Bencivenga and Smith (1991), the optimal proportion of illiquid investment is constant.
7 The index \( b \) refers the banking interest factor \( R^b \), where \( -1 \leq R^b \leq \infty \).
must be equal to the rest of illiquid asset \((1 - \alpha_{t2})\) multiplied by the income of investment \(R q_i^b\). Thus, the bank must provide capital goods for entrepreneurs as much as

\[
\pi R_{t2}^b = (1 - \alpha_{t2}) R q_i^b
\]  

(11)

In addition, entrepreneurs must also receive \(\tilde{R}_{t2}^b\) units of consumption goods for each unit of liquid investment at \(t\). The constraint \(\pi \tilde{R}_{t2}^b\) must be equal to the rest of consumption goods \((1 - \alpha_{t1})\) multiplied by \(z_t^b n\). Thus, banks must provide consumption goods for entrepreneurs as much as

\[
\pi \tilde{R}_{t2}^b = (1 - \alpha_{t1}) z_t^b n
\]  

(12)

In the next step, we define the program of financial intermediation for two types of agent. Firstly, there are \((1 - \pi)\) non-entrepreneurs who will liquidate their investment at \(t+1\). Thus, the bank must ensure the non-entrepreneur by holding \(R_{t1}^b w_t\) units of consumption goods to be distributed at \(t+1\). Secondly, there are also \(\pi\) entrepreneurs who will liquidate their investment at the beginning of \(t+2\). Thus, the bank must ensure entrepreneurs by holding \(R_{t2}^b w_t\) units of capital goods and \(\tilde{R}_{t2}^b w_t\) units of consumption goods to be distributed at \(t+2\). Using budget constraints in the equation (10), (11), and (12) we define the financial intermediary program in the following relation

\[
U(c_t, c_{t1}) = -\frac{(1 - \pi)}{\gamma} (R_{t1}^b w_t)^{-\gamma} + \pi (A0\psi (R_{t2}^b w_t)^0 + \tilde{R}_{t2}^b w_t)
\]  

(13)

Note that in the third period \((t+2)\), entrepreneurs will use their income of investment to finance physical capital and use it in the production. Hence, we have \(R_{t2}^b w_t = k_{t+2}\). From (11), we have the relationship between the current and the future capital stock as follows

\[
k_{t+2} = \frac{1}{\pi} (R q_i^b w_t) = \varphi_b(k_t)
\]  

(14)

In order to simplify condition in the equation (13), we assume that the bank should provide the liquidity at \(t+1\), since none of the capital assets is liquidated “prematurely”. Thus, the bank should fulfil the following liquidity constraint

\[
A0\psi R > n
\]  

(15.a)
By this assumption, we can reduce some variables as follows. In the third period \((t+2)\), the bank will only consider the existence of \(\pi\) entrepreneur.

Since the entrepreneur runs the production to get the profit, then their profit should be superior to all income of liquid investment. Such condition provides incentive for agents to become entrepreneur. In other words, \(A\theta\psi R > n\), and

\[
A\theta\psi \left(1 - \alpha_{2t}\right) \left(\frac{R}{\pi}\right) q_i^b w_i > \left(\frac{n}{\pi}\right) q_i^b w_i
\]

Equation (15.b) is fulfilled if and only if the bank set

\[
\alpha_{2t} = 0
\]

Meanwhile, the bank also maximizes the expected utility of non-entrepreneur. It means that the bank will reallocate the non-entrepreneur’s illiquid assets into liquid assets at the beginning of \(t+1\). For realizing this strategy, the bank will therefore set

\[
\alpha_{u_t} = 1
\]

Using (15.c) and (15.d), we simplify (10), (11) and (12) respectively become

\[
R_{it}^b = \frac{z_i^b}{1-\pi} n
\]

\[
R_{2t}^b = \frac{R}{\pi} q_i^b
\]

\[
\bar{R}_{2t}^b = 0
\]

Using (16), (17), and (18), and the budget constraint (9) we establish the program of financial intermediaries as follows

\[
U(q_i^b) = \frac{-(1-\pi)}{\gamma} \left(1 - \frac{q_i^b}{1-\pi} n w_i \right)^{-\gamma} + \pi \left(A\theta\psi \left(\frac{R q_i^b w_i}{\pi}\right)^{\theta}\right)
\]

Hence, banks will choose \(q_i^b\) to maximize \(U(q_i^b)\). From the first-order condition, we obtain the optimal proportion of illiquid asset \((\bar{q}_i^b)\) as follows

\[
\bar{q}_i^b = \bar{q}_i^b(w_i) = 1 - \frac{(1-\pi)(B_i)^{1-\gamma}}{n w_i}
\]
where \( B_i = \frac{A\pi \left( \frac{R}{\pi} \right)^\theta w_i^\theta \psi}{nw_i} \).

5. Capital Stock Accumulation and Threshold Effect

In comparing the level of steady state equilibrium of capital stock under the financial market and financial intermediary model, we establish Proposition 1 and 2 as follows.

**Proposition 1**

*For \( x = 0 \) we show that the optimal value of illiquid investment under financial intermediary is higher than the optimal value of illiquid investment under financial market. In other words, we have \( \overline{\alpha}_t^b > \overline{\alpha}_t^w \).*

**Proof:**

From (8) and (20), we show that \( (1-\pi)(B_i)^{\frac{1}{\gamma}}/nw_i < (B)^{\frac{1}{\gamma}}/nw_i \). Thus, we examine whether \( B_i < B \). From \( B_i \) and \( B \), we only examine if

\[
(1-\pi) \left( A\pi \left( \frac{R}{\pi} \right)^\theta w_i^\theta \psi \right)^{\frac{1}{\gamma}} < \left( \frac{\pi}{1-\pi} \left( AR^\theta w_i^\theta \psi - nw_i \right) \right)^{\frac{1}{\gamma}}
\]

Equation (21.a) can be rewritten as

\[
(1-\pi)^{\frac{\gamma}{1-\gamma}} < \left( \pi^\theta - \frac{n w_i\pi}{AR^\theta w_i^\theta \psi^2} \right)^{\frac{1}{\gamma}}
\]

For \( \gamma > -1 \) the inequality is verified if the left hand side is less than one, while the right hand side is greater than one. By definition the value of the left hand side is less than one. For the right hand side, we proceed as follows
\[
\left(\pi^0 - \frac{n w_t \pi}{A R^0 w^0_0 \theta^2}\right)^{\frac{1}{\mu_r}} < 1
\]
\[\Leftrightarrow \pi^0 - \frac{n w_t \pi}{A R^0 w^0_0 \theta^2} < 1\]
\[\Leftrightarrow A R^0 w^0_0 \theta^2 \pi^0 < A R^0 w^0_0 \theta^2 + n w_t \pi \]

Since \(\pi^0 < 1\), we verify that \(A R^0 w^0_0 \theta^2 \pi^0 < A R^0 w^0_0 \theta^2\).

As discussed above, Proposition 1 is laid down for \(x = 0\). This condition can be interpreted as the best case in which financial market is efficient, since there is no premature liquidation to fulfill the liquidity of two-period-lived agents. Proposition 1 explicitly shows that although the financial market is at the best condition, the illiquid investment of the financial market equilibrium is always lower than that of the financial intermediary equilibrium. From Proposition 1, we lay out Proposition 2 as a consequence of Proposition 1.

**Proposition 2**

*The existence of banks in an economy enhances economic growth more significantly than the absence of banks.*

**Proof:**

In the financial intermediary equilibrium, economic growth is determined by the value of \(k_{r+2} = \varphi_b(k_t)\). Meanwhile, in the financial market equilibrium, economic growth is determined by the value of \(k_{r+2} = \varphi_m(k_t)\). From Proposition 1, it is straightforward to find \(\mu_b > \mu_m\), where \(\mu_b = \frac{\varphi_b(k_t)}{k_t}\) and \(\mu_m = \frac{\varphi_m(k_t)}{k_t}\) are the change of capital stock in the financial intermediary and financial market equilibrium, respectively. Proposition 2 is thus proved.
From (6.b) and (14), we illustrate the dynamics of capital accumulation in each case as follows\(^8\).

From Figure (1), we observe that there are three stationary states in both the financial intermediary and the financial market equilibrium: (i) the trivial steady state at \( k = 0 \), (ii) the low level equilibrium trap \( (k^*) \), and (iii) the high level steady state equilibrium \( (k'^*) \).

Moreover, we observe that the financial intermediary model is more accurate than the financial market model to reduce the threshold effect. We verify this property in Proposition 3.

\(^8\) Numerical examples are available on request.
Proposition 3

(i) In the financial intermediary and financial market equilibrium, the economy converges to the high long-term steady state equilibrium if initial capital stock exceeds a threshold level, and (ii) the threshold level is lower for a financial intermediation equilibrium than for a financial market equilibrium.

Proof:

To prove Proposition 3, we verify the existence of threshold effect in both the financial intermediary and financial market equilibriums. Then, we compare both of them.

(i) The financial intermediary equilibrium

At the stationary states, we have \( k = \varphi_b(k) \). Unfortunately it is difficult to solve the stationary capital stocks \( (k) \) algebraically. From Figure (1) we observe that \( k = \varphi_b(k) \) has two roots \( k_b^* \) and \( k_b^{**} \). Alternatively, we derive \( \varphi_b(k_i) \) in order to obtain the first-order condition as follows

\[
\frac{d \varphi_b(k_i)}{d k_i} = -\frac{R(-1+\theta)\theta \left( \frac{\Omega}{n} \right)^{\frac{1}{1+\gamma}}}{k_i n \pi (1+\gamma)} \left( -1 + \pi + Ak_i^0 n \pi^0 \left( 1+\gamma \right) \left( \frac{\Omega}{n} \right)^{\frac{1}{1+\gamma}} \right) \tag{22.a}
\]

where \( \Omega \equiv \left( \frac{A(\frac{R}{n} \pi)}{\pi} (-Ak_i^0 \pi^0 (-1+\theta))^{-1+0} \right)^{\frac{1}{1+\gamma}} \)

To show the existence of threshold effect \( k_b^* \) exists, we examine if there is \( k_i \) in which

\[
\frac{d \varphi_b(k_i)}{d k_i} > 1.
\]

In other words, \( \frac{d \varphi_b(k_i)}{d k_i} - 1 > 0 \) and \( \varphi_b(k_i) \) intersects \( k_{i,2} = k_i \) at \( k_b^* \) as shown at Figure 1. In order to simplify (22.a), we assume that \( \pi \to 1 \) and hence, \( \psi \to 1 \).

Under this condition, we simply obtain

\[
\lim_{\pi \to 1} \frac{d \varphi_b(k_i)}{d k_i} - 1 = -\frac{k_i + Ak_i^0 R(\theta - 1)\theta}{k_i} \tag{22.b}
\]

Despite assuming that \( \pi \to 1 \), we do not change the properties of the financial intermediary model. Since our purpose is to formalize the role of financial intermediation
in enhancing entrepreneurship through long-term investment, the absence of non-entrepreneurs does not affect the change of capital stock. This is because economic growth should not be relied on non-entrepreneurs but entrepreneurs. From (22.6), we examine if there is $k_t$ in which the right hand side becomes positive. In other words,

$$- \frac{k_t + Ak_t^2 R(\theta - 1)\theta}{k_t} > 0$$  \hspace{1cm} (23)$$

$$\Leftrightarrow S = \left\{ k_t | k_t < \infty \land k_t > \left( \frac{1}{AR(1-\theta)\theta} \right)^\frac{1}{\theta - 1} \right\}$$

Since $A, R > 0$ and $0 < \theta < 1$, then $\left( \frac{1}{AR(1-\theta)\theta} \right)^\frac{1}{\theta - 1} > 0$ and we obtain

$$k^*_b = \left( \frac{1}{AR(1-\theta)\theta} \right)^\frac{1}{\theta - 1}$$  \hspace{1cm} (24)$$

Equation (24) is simply defined as the threshold level of the financial intermediary equilibrium, since for each $k_o$ where $k^*_b < k_o < +\infty$, we have $\frac{d\phi_b(k_t)}{dk_t} > 1$. The existence of threshold effect in the financial intermediary equilibrium is therefore confirmed.

(ii) The financial market equilibrium

To prove the existence of threshold effect under the financial market equilibrium, we use the same characterization of the bank-based economy. Assume that $\pi \to 1$ and $\psi \to 1$. This means that financial market only exists for responding the entrepreneur’s needs. By solving the first-order condition for $\phi_m(k_t)$ and its limit for $\pi \to 1$, we obtain

$$\lim_{\pi \to 1} \frac{d\phi_m(k_t)}{dk_t} - 1 = \frac{Ak_t^{\theta - 1} n R(1-\theta)\theta}{n-x} - 1$$  \hspace{1cm} (25)$$

The threshold effect $k^*_m$ exists, if and only if there is $k_t > 0$ in which $\frac{d\phi_m(k_t)}{dk_t} > 1$ or $\frac{d\phi_m(k_t)}{dk_t} - 1 > 0$. From (25), we have
Since $0 < \theta < 1$, then it is straightforward to denote that $k_m^* > 0$. Hence, the existence of threshold effect in the financial market equilibrium is confirmed.

(iii). Financial intermediary vs. financial market

From (24) and (26), we verify that the threshold level of the financial intermediary equilibrium is lower than that of the financial market equilibrium by proving that

$$\frac{AnR\theta}{n-x} \left(1 - \theta\right) < \frac{1}{AR(1-\theta)\theta}$$

(27)

For $\theta \to 1$, we have that the left hand side tends to 0, but the right hand side tends to infinity. Instead, for $\theta \to 0$, we denote the left hand side tends to 0, and the right hand side tends to 1. By these results, Proposition 3 is proved.

Threshold effects in the finance-growth nexus is one of main features of our model. This finding is particularly important in developing countries, where banking sector development should be taken into very close consideration rather than financial market development.

For instance, let $k_0$ be an initial capital stock that lies below the threshold level of the financial market equilibrium ($k_m^*$) as shown in Figure 1. In order to reach the long-run steady state equilibrium of capital stock, $k_0$ should be iterated by the $\varphi_o(k_i)$ curve. Such a situation can drive the economy to converge to $k_h^*$. Conversely, if $k_0$ is iterated by the $\varphi_m(k_i)$ curve, the economy may disappear because the steady state equilibrium of capital stock tends to zero. In this case, we denote that the financial intermediary equilibrium is better than the financial market equilibrium in order to ensure the existence and uniqueness of long-run steady state capital stock, and to reduce the threshold level. In turn, a long-term economic growth can be realized due to an increase in long-term productive investments and a decline in short-term ventures. By extension, the potential source of speculations from short-term ventures can be therefore reduced.
Nevertheless, if the initial capital stock lies below the threshold level of the financial intermediary equilibrium ($k_0 < k_0^*$) as shown in Figure 1, the steady state equilibrium of capital stock can approach to zero, even if there is a financial intermediary in the economy. In such a case, we observe that there is no positive link between financial development and economic growth in developing country, when the initial capital stock is very low. On the other hand, if developing country has sufficiently a high capital stock, then the introduction of banking system ensures the economy to converge to the higher long-run steady state equilibrium.

6. Conclusion

In providing further issue on the finance-growth nexus, we have reevaluated the model of financial intermediation à la Bencivenga and Smith (1991). Our originality is twofold. First, in modelling the finance-growth nexus, we use the Neo-classical growth framework instead of drawing endogenous growth as developed by Bencivenga and Smith (1991). Second, we distinguish the behaviour vis-à-vis of risk between non-entrepreneur and entrepreneur.

Using these features, we find that the financial intermediary equilibrium is better than the financial market equilibrium in order to ensure the existence and uniqueness of long-run steady state equilibrium of capital accumulation, as a necessary condition to achieve long-run economic growth. Moreover, we found that any type of financial development (either through financial intermediary or financial market) has a threshold effect. But the presence of banks as financial intermediaries clearly reduces the threshold level of the financial market equilibrium. Threshold effect is a new finding in the finance-growth nexus, since it captures the difficulty of raising initial capital stocks and reaching long-run economic growth. Thus, threshold effect should be acknowledged in the next empirical research on the finance-growth nexus, notably in developing countries, where externalities due to human capital and technological innovations are not yet well-developed.
References


