Real and Nominal Business Cycles: New Evidence from a Generalized Unobserved Components Model

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December 1, 2009

JEL classification: C32, E32

Keywords: Unobserved Components decomposition, Real Business Cycle, Beveridge-Nelson Decomposition

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Abstract

We show that the random walk assumption for permanent component in an Unobserved Components (UC) decomposition of U.S. output likely overstates the importance of nominal shocks in the sense of imposing a lower bound on the size of them. At the same time various economic models would imply non-trivial dynamics for equilibrium output to grow toward its steady-state level following real productivity shocks. Therefore we propose a multivariate UC model in which the permanent component corresponds to equilibrium output in absence of nominal shocks and model it as an ARIMA(2,1,0) process. We show that this UC model is identified and estimate it using U.S. real GDP and Industrial Production data. The estimation results provide supportive evidence of the ARIMA(2,1,0) model and confirms the empirical findings of Morley, Nelson, and Zivot (2003) that the nominal shocks are relatively unimportantly in driving the U.S. business cycles during the post-war period.
1. Introduction

Nelson and Plosser (1982) find evidence that US output contains a stochastic trend. Following this seminal finding, Harvey (1985) and Clark (1987) propose an Unobserved Components model (hereafter UC) to decompose real output into permanent and transitory components. In a typical UC decomposition it is usually assumed that the permanent component follows a random walk. On one hand, it is a statistical convenience to assume a random walk for the permanent component because by doing so one could easily track the steady-state level of the equilibrium output following productivity shocks. On the other hand, if one wants to track the equilibrium output and its dynamic evolution to its steady-state level following a productivity shocks, it may be too restrictive to have a random walk permanent component as various economic theories imply that equilibrium output driven by productivity shocks could display non-trivial dynamics on its path toward its steady-state level, see e.g. the Real-Business-Cycle (RBC) literature as pioneered by King, Plosser and Rebelo (1988), King, Plosser, Stock and Watson (1991), Cogley and Nason (1993, 1995), and the technology diffusion literature as in Griliches (1957), Lippi and Reichlin (1994), and Rotemberg (2003).

This paper breaks new ground in the previous literature in this area in a number of important ways. We develop and estimate an unobserved components (UC) model of aggregate output that can capture the complex dynamics implied by the diffusion of real productivity shocks and nominal demand shocks. In contrast to existing UC models, in which the trend component is driven by real productivity shocks, which are assumed to follow a random walk, the trend component in our model follows a more general unit root process. An important result is that the UC model, with a more general unit root process, can be identified, even when allowing for the correlation between transitory and permanent components. A second important
result is that the consequence of incorrectly imposing a simple random walk trend is that the importance of nominal shocks will be overstated. Third, we demonstrate that allowing for a more general unit root trend process is empirically relevant for US data on aggregate output. Finally, our paper makes an important contribution to the literature on the trend/cycle decomposition using UC models. Specifically, our proposed model is shown to be empirically superior to standard existing UC models. In particular, Morely, Nelson, and Zivot (2003) (hereafter MNZ), demonstrated the empirical importance of allowing correlation between permanent and transitory components in an UC model of aggregate output by considering a likelihood ratio ($LR$) test. Our paper demonstrates the additional empirical importance of a more general unit root trend process using a $LR$ test. We confirm the empirical findings of MNZ (2003) that nominal shocks are relatively unimportant, at least within a linear framework, and demonstrate the slow diffusion of productivity shocks implied by Real Business Cycle (RBC) models are empirically supported.

This paper is organized in the following way: in Section 2, we present a motivation of the generalized UC model based on a brief discussion of the economic models which imply richer dynamics for the equilibrium output. In Section 3 we prove that the random walk assumption for the permanent component imposes a lower bound and thereby likely overstates the importance of nominal shocks. We also further discuss the relationship between our generalized UC decomposition with the Beveridge-Nelson (hereafter BN) decomposition. In Section 4, we address identification issues of our proposed bivariate UC model and estimate this model using the real GDP and Industrial Production data. Section 5 concludes.
2. Motivation: The Equilibrium Output Dynamics

In our proposed UC model the permanent component would correspond to the equilibrium output resulting from the optimization behaviors of households and firms, given technology and resource constraints. If the technology flow is roughly a random walk a typical RBC model generates serial correlations in the growth rate of equilibrium output because of the time-to-build effect, the importance of which may vary depending on values of the underlying structural parameters. Furthermore, such an observation is reinforced if the technology shock has a richer dynamics than the random walk as suggested by the technology diffusion literature.

First, consider a basic RBC model in the spirit of King, Plosser and Rebelo (1988). In such an economy the representative agent optimizes the following objective function:

\[ E_t \left\{ \sum_{j=0}^{\infty} \beta^j u(C_{t+j}, N_{t+j}) \right\} \]  

(2.1)

Where, \( u(\cdot) \) is the utility function and is assumed to be additive and log-linear:

\[ u(C_t, L_t) = \ln C_t + b \cdot \ln(1 - N_t), b > 0 \]  

(2.2)

Where, \( \beta \) is the discount factor, \( C_t \) is consumption and \( N_t \) is working hours which grow at rate \( n \) (\( n < \beta \)).

The production function is Cobb-Douglas with labor-augmenting technology:

\[ Y_t^* = K_t^\alpha (A_t N_t)^{1-\alpha}, 0 < \alpha < 1 \]  

(2.3)

Where, \( K_t \) is the capital input, \( A_t \) is the technology factor, and \( \alpha \) is the share of capital input. We may think of \( Y^* \) here as the equilibrium output in absence of nominal shocks. The capital stock evolves as following (notice the government spending is absent for illustration convenience):
\[ K_{t+1} = (1 - \delta) \cdot K_t + Y_t^\ast - C_t \] 

Where, \( \delta \) is the depreciation rate of capital.

By following King, Plosser, Stock and Watson (1991), we assume the technology shock is difference-stationary with drift \( g \):

\[ (1 - L) \ln A_t = g + e_t \] 

Where, \( e_t \) is stationary and \( \phi_e(L)e_t = \theta_e(L)\eta_{A,t} \cdot \eta_{A,t} \sim i.i.d. N(0, \sigma^2_{\eta_A}) \). It is important to notice that the serial correlations in \( e_t \) capture potential dynamics of technology diffusions.

By assuming a 100% depreciation rate as in Long and Plosser (1983) and McCallum (1989) we may solve the model analytically to obtain:

\[ (1 - L) \ln Y_t^\ast = (1 - \alpha) \cdot (n + g) + \alpha \cdot (1 - L)\tau_{t-1} + (1 - \alpha) \cdot \left[ \phi_e(L) \right]^{-1} \theta_e(L)\eta_{A,t} \] 

Apparently even in absence of technology diffusion (i.e., \( e_t = \eta_{A,t} \)), the model implies an important AR(1) dynamics for the equilibrium output growth rate and this has been pointed out by McCallum (1989). Intuitively, such a growth momentum arises and is determined by the capital share (\( \alpha \)) because it takes time for new capital investment to gradually reach a new steady-state level in response to a productivity shock.

For general cases in which the depreciation rate is less than 100\%, the model cannot be solved analytically. However, as demonstrated by Cogley and Nason (1993), we can follow the solution algorithm described in King, Plosser and Rebelo (1988, 2002) to approximate the equilibrium result by taking a log-linearization of the first-order conditions around the steady state. The resulting system of stochastic differential equations could be solved by following Hansen and Sargent (1980) to obtain the following dynamics for output (ignoring constants):

\[ \phi_e(L) \cdot (1 - M_2L) \cdot (1 - L) \ln Y_t^\ast = M_0 \cdot (1 - M_1L) \cdot \theta_e(L)\eta_{A,t} \] 

(2.7)
Where, $M_0, M_1, M_2$ are complicated nonlinear functions of the underlying structural parameters. Cogley and Nason (1993, 1995) investigate the equilibrium output dynamics from a typical RBC model and find that the implied dynamics for the equilibrium output growth rate is likely to be small in that $M_1$ and $M_2$ are close to each other given parameter values commonly chosen in the RBC literature. Note, however, that the equilibrium output may still contain a non-trivial dynamics if there is a significant presence of technology diffusion dynamics.

Our discussions so far do not tend to argue that RBC models along with technology diffusion is able to fully explain the actual macroeconomic fluctuations but rather that these economic models suggest that the equilibrium output may follow a more general unit root process, rather than a random walk. Since the actual output could permanently deviate from the equilibrium output, due to nominal shocks, the UC model provides a nice way to decompose the real economic activity into a permanent component, driven by real productivity shock, and transitory component, due to nominal demand shocks. If one is to compare the contributions of real productivity shocks and nominal demand shocks in explaining actual macroeconomic fluctuations it may be important to explicitly allow the permanent component to have a more general unit root process.

3. A Generalized UC Model and Its Relationship with the BN Decomposition

3.1. A Generalized UC Model

We propose a generalized UC model to decompose the actual output into its equilibrium level and the deviation of the actual output from the equilibrium level due to nominal shocks:

$$y_t = \tau_t + \psi_t$$  \hspace{1cm} (3.1)
\[ \alpha(L)(1-L)\tau_i = \mu + \eta_i, \eta_i \sim i.i.d N(0, \sigma^2_\eta) \]  
(3.2)

\[ \phi(L)\psi_i = \varepsilon_i, \varepsilon_i \sim i.i.d N(0, \sigma^2_\varepsilon) \]  
(3.3)

where, the logarithm of real output \( y_t \) is the sum of a permanent component \( \tau_t \) and transitory component \( \psi_t \); \( \alpha(L) \) and \( \phi(L) \) are stationary autoregressive representations; \( \mu \) is the drift term.

Notice that if \( \alpha(L) \) is restricted to be 1 as in Harvey (1985), Clark (1987) and MNZ (2003), the permanent component \( \tau_t \) then follows a random walk with drift\(^1\) and \( \tau_t \) would track the steady-state level of the equilibrium output following each productivity shock. The transitory component \( \psi_t \) is usually assumed to follow a stationary AR(2) process following Clark (1987) and MNZ (2003), i.e., \( \phi(L) = 1 - \phi_1 L - \phi_2 L^2 \). When the permanent component is assumed to be a random walk, it would correspond to the steady-state level of the output. Therefore, the transitory component \( \psi_t \) contains not only nominal shocks due to which the output constantly deviate from its equilibrium level but also the temporary deviation of the equilibrium output from its steady-state level following productivity shocks. As illustrated in the above section, the economic models imply that the equilibrium output would not be able to jump immediately to its new steady-state level following a productivity shock. Rather it takes several periods for the equilibrium output to reach its new steady-state level. While MNZ (2003) models this general idea with non-zero correlation between the permanent and transitory shocks, we allow the permanent component in our model to explicitly correspond to the equilibrium output by departing from the random walk assumption.

\(^1\)The generalization of allowing \( \mu \) to be random as well (so-called drifting model) is often estimated but appears to be insignificant for U.S. real output data, see e.g., Harvey (1985), Clark (1987).
Given the above discussions it is straightforward to illustrate that the UC model with \( \alpha(L) = 1 \) may overstate the importance of nominal shocks in explaining output fluctuations. To see this, recall from MNZ (2003) that such an UC model implies a reduced-form ARMA structure for \( \Delta y_t \):

\[
\Delta y_t = \mu + \eta_t + (1 - L)(1 - \phi_1 L - \phi_2 L^2)^{-1} \cdot \varepsilon_t
\]

(3.4)

The first-order auto-covariance of \( \{\Delta y_t\} \) may be derived as below (see Appendix A for details):

\[
Cov(\Delta y_t, \Delta y_{t-1}) = (\phi_1 - 1) \rho_{\eta\varepsilon} \cdot \sigma_{\varepsilon} \cdot \sigma_{\eta} + \frac{\phi_1 - \phi_2 - 1}{(1 + \phi_2)(1 + \phi_1 - \phi_2)} \cdot \sigma_{\varepsilon}^2
\]

(3.5)

Where, \( \rho_{\eta\varepsilon} \) is the correlation between the permanent and transitory shocks. Notice in (3.5), we have the \( \lim_{\sigma_{\varepsilon} \to 0} Cov(\Delta y_t, \Delta y_{t-1}) = 0 \). Therefore, in estimating this UC model the transitory shock variance \( \sigma_{\varepsilon}^2 \) must be considerably above zero in order to reconcile the significantly positive first-order auto-correlation in U.S. real GDP growth rate. This literally puts a lower bound \( a \) priori on the size of nominal shocks.

This issue, however, would not be present if we allow permanent component to have a richer dynamics. For illustration purpose, consider a simple case \( \alpha(L) = 1 - \alpha L \):

\[
(1 - L) \tau_t = \mu + \alpha \cdot (1 - L) \tau_{t-1} + \eta_t, \eta_t \sim i.i.dN(0, \sigma_{\eta}^2)
\]

(3.6)

Where, \( 0 < \alpha < 1 \). The first-order auto-covariance of the output growth from such a revised UC model may be derived as below (see Appendix A for details):

\[
Cov(\Delta y_t, \Delta y_{t-1}) = \frac{\alpha}{1 - \alpha^2} \cdot \sigma_{\eta}^2 + \frac{[\alpha \cdot (1 + \phi_2 - \alpha) + (\phi_1 - 1)]}{1 - \alpha(\phi_1 + \alpha \phi_2)} \cdot \rho_{\eta\varepsilon} \cdot \sigma_{\eta} \cdot \sigma_{\varepsilon} + \frac{\phi_1 - \phi_2 - 1}{(1 + \phi_2)(1 + \phi_1 - \phi_2)} \cdot \sigma_{\varepsilon}^2
\]

(3.7)
Notice here we have \( \lim_{\sigma_\eta \to 0} \text{Cov}(\Delta y_i, \Delta y_{i-1}) \to \frac{\alpha}{1 - a^2} \cdot \sigma_\eta^2 \), the scenario in which most output variation is due to productivity shocks either because there are no nominal shocks or nominal shocks have no real impact; or we may have \( \lim_{\sigma_\eta \to 0} \text{Cov}(\Delta y_i, \Delta y_{i-1}) \to \frac{\phi_1 - \phi_2 - 1}{(1 + \phi_2)(1 + \phi_1 - \phi_2)} \cdot \sigma_\varepsilon^2 \), where all output variation is due to nominal shocks. Therefore, this generalization allows the data to reveal by itself how important the real and nominal shocks are relative to each other in explaining the actual output fluctuations without imposing any \textit{a priori} restriction on the model.

We notice that a richer dynamics for the permanent component in the UC model have been studied by Quah (1992) in a theoretical framework and Blanchard and Quah (1989) (hereafter BQ) through a bivariate structural-VAR framework. Our approach is different from BQ in that we directly modify and estimate a multivariate UC model and use the technique laid out in MNZ (2003) to identify such models. Furthermore, Lippi and Reichlin (1994) argue that the technology diffusion process may lead to an \( S \)-shape pattern in the permanent component and subsequently estimate such a univariate UC model. Our model is distinct from theirs also in that we do not impose that a particular dynamic pattern and we will consider a multivariate UC model in our estimation.

\textit{3.2. The Relationship with the BN Decomposition}

Consider the UC model given by (3.1), (3.6) and (3.3) for illustration purpose. Watson (1986) points out that the conditional expectation of a random walk permanent component for a UC model gives the BN trend, regardless of its cycle dynamics. This finding may be easily extended to our generalized UC model when the permanent component is more general than a random walk. Consider the definition of BN trend for the given UC model:
\[ \text{BN}_t = \lim_{k \to \infty} E \left[ y_{t+k} - k \cdot \mu^* \mid \Omega_t \right] \]  

(3.8)

Where, \( \mu^* = \frac{\mu}{1 - \alpha} \), \( \Omega_t \) is the information set at time \( t \); \( \text{BN}_t \) is the BN trend.

Since \( y_{t+k} = \tau_{t+k} + \psi_{t+k} \) and \( \lim_{k \to \infty} E[y_{t+k} \mid \Omega_t] = 0 \), we may have (see Morley (2002)):

\[ \text{BN}_t = \lim_{k \to \infty} E \left[ \tau_{t+k} - k \cdot \mu^* \mid \Omega_t \right] = E \left[ \tau_t + \frac{\alpha}{1 - \alpha} (\Delta \tau_t - \mu^*) \mid \Omega_t \right] \]  

(3.9)

The BN cycle is the difference between the observed output and the BN trend:

\[ \psi_t = y_t - E \left[ \tau_t + \frac{\alpha}{1 - \alpha} (\Delta \tau_t - \mu^*) \mid \Omega_t \right] \]  

(3.10)

As evident from (3.9), the BN trend is the conditional expectation of \( \tau_t \) plus additional growth momentum of the permanent component. If the permanent shock dominates output variation, i.e.,

\[ \psi_t \approx -E \left[ \frac{\alpha}{1 - \alpha} (\Delta \tau_t - \mu) \mid \Omega_t \right] \], the resulting cycle would merely reflect a temporary deviation of the equilibrium output from its new steady-state level after a productivity shock, the possibility of which was discussed by Stock and Watson (1988a).

In light of the discussions in Morley (2008) and Oh, Zivot and Creal (2009), we may rewrite the generalized UC model as below:

\[ y_t = \tilde{\tau}_t + \tilde{\psi}_t \]  

(3.11)

\[ \tilde{\tau}_t = \mu(1 - \alpha)^{-1} + \tilde{\tau}_{t-1} + (1 - \alpha)^{-1} \cdot \eta_t \]  

(3.12)

\[ (1 - \alpha L)(1 - \phi_1 L - \phi_2 L^2) \tilde{\psi}_t = -\frac{\alpha}{1 - \alpha} (1 - \phi_1 L - \phi_2 L^2) \eta_t + (1 - \alpha L) \epsilon_t \]  

(3.13)

Here \( \tilde{\tau}_t \) refers to the steady-state level of output which will ultimately be achieved following a productivity shock. If our proposed UC model truly describes the economy, the UC model with a
random walk permanent component and orthogonal shocks ($\rho_{\eta e} = 0$) would be mis-specified. MNZ show that in a particular class of UC models with a random walk permanent component, the correlation $\rho_{\eta e}$ is identifiable and the estimated value of it is approximately -0.9. From (3.13) one can see the result of a strong negative correlation in MNZ may be explained by the temporary deviation of the equilibrium output from its steady-state level as discussed so far.

4. Identification and Estimation of a Generalized Bivariate UC Model

Recently Morley (2007), Sinclair (2009a) and Basistha (2009) show that the bivariate UC models are better able to utilize more information in gauging the relative importance of different types of the shocks in explaining the economic fluctuations. Following their suggestions, we propose the following bivariate UC model for decomposing real economy activity:

$$\alpha(L)(1-L)\tau_t = \mu + \eta_t, \eta_t \sim i.i.dN(0, \sigma^2_{\eta})$$

(4.1)

$$y_{1t} = \tau_t + \psi_{1t}$$

(4.2)

$$y_{1t} = a + b \cdot \tau_t + \psi_{2t}$$

(4.3)

$$\phi_1(L)\psi_{it} = \varepsilon_{it}, \varepsilon_{it} \sim i.i.dN(0, \sigma^2_{\varepsilon})$$

(4.4)

Where, $i = 1$ corresponds to GDP and $i = 2$ corresponds to the Industrial Production. And the variance of shocks is: $\text{var}[\eta_t, \varepsilon_{1t}, \varepsilon_{2t}] = \begin{bmatrix} \sigma^2_{\eta} & \sigma_{\eta e1} & \sigma_{\eta e2} \\ \sigma_{\eta e1} & \sigma^2_{\varepsilon1} & \sigma_{\varepsilon1e2} \\ \sigma_{\eta e2} & \sigma_{\varepsilon1e2} & \sigma^2_{\varepsilon2} \end{bmatrix}$.

It is clear that we have imposed the cointegration restriction that there is only one common permanent shock. We implemented the Stock and Watson (1988b) test of common trend. By using 4 lags and including a constant and a linear trend, the Stock and Watson common
trend test statistic of testing the null of two common stochastic trends versus 1 trend is -38.88 with a 5% critical value of -30.8, indicating that we can reject 2 common trends in favor of one common stochastic trend. To achieve greater estimation efficiency we impose the cointegration relationship. Our results are qualitatively unaffected regardless of whether we impose one permanent component or not.

We follow the previous literatures (see e.g., Clark (1987), MNZ (2003) and Sinclair (2009a)) and assume a stationary AR(2) process for the transitory component, i.e., \( \phi(L) = (1 - \phi_1 L - \phi_2 L^2) \). At the same time, we propose an AR(2) process to capture potential growth momentum for the permanent component, i.e., \( \alpha(L) = (1 - \alpha_1 L - \alpha_2 L^2) \). We argue that the flexible AR(2) specification is able to capture a potentially rich dynamics in the permanent component growth\(^2\).

It is straightforward to check whether our proposed UC model is identified by following the identification strategy laid out in MNZ (2003). Employing the Granger and Newbold’s (1986) Theorem, this UC model implies a particular reduced-form VARMA process for vector variable \( \begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \end{bmatrix} \):

\[
\begin{bmatrix}
\alpha(L) \phi_1(L) & 0 \\
0 & \alpha(L) \phi_2(L)
\end{bmatrix}
\begin{bmatrix} \Delta y_{1t} \\ \Delta y_{2t} \end{bmatrix}
=
\begin{bmatrix}
\theta_{11}(L) & \theta_{12}(L) \\
\theta_{21}(L) & \theta_{22}(L)
\end{bmatrix}
\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}
\]

\[(4.5)\]

\(^2\) It may seem natural to estimate potential moving average terms given the discussion in Section 2. However, the addition may not be that beneficial, see e.g. Kascha and Mertens (2009). After all, the AR(2) dynamics has been shown to be able to capture dynamics of the transitory component and it ought to perform just as well in capturing any potential dynamics in the permanent component.
Where, the moving average coefficients are normalized such that: $\theta_{g,0} = 1$ for $i = j$ and $\theta_{g,0} = 0$ for $i \neq j$. Morley (2007) show that such a UC model without the AR(2) dynamics in the permanent component growth is identified. An intuitive reason for our proposed model to remain identified is that the additional parameters we add are introduced in the autoregressive part. More rigorously, to see our model is identified, notice that the UC model has 15 parameters while there are 22 parameters in the reduced-form VARMA representation with one restriction on the long-run matrix due to the cointegration relationship, see e.g., King, Plosser, Stock and Watson (1991).

We obtain the quarterly U.S. real GDP and Industrial Production data from the Federal Reserve Bank of St. Louis Website for the sample period from 1947:1 to 2008:4. The proposed UC model is estimated by casting it into a state-space representation and maximizing the log-likelihood function based on the Kalman filter (see Appendix B for details). The estimation results are given by Table 1.

Based on the estimation results the growth momentum in the permanent component, i.e., the AR(2) coefficients, are indeed significant. At the same time the correlation between the permanent shock and the transitory shock to GDP is nearly zero. This result, along with the discussions following equations (3.11)-(3.13) in the previous section, confirms the interpretation in MNZ (2003) that the correlation under the random walk assumption reflects effects of permanent shocks on transitory deviations rather than vice versa because here permanent shocks are identified by having different dynamics than transitory shocks. The size of the AR

\[3 \text{ See Lutkepohl (2006) for a more general discussion of the VARMA model.}\]

\[4 \text{ We obtain the monthly Industrial Production Index and convert it to quarterly observation by using the last month of each quarter.}\]
coefficients in GDP transitory component implies that there is a very small role for nominal shocks to play in explaining the real GDP fluctuations. The permanent shocks are apparently much more important than the transitory shocks.

Given so many parameters to be estimated the identification is likely to become fairly weak, and the standard Wald-type t-test may not deliver reliable results as pointed out by Nelson and Startz (2007). For this reason, we re-estimate the model by imposing the null $\alpha_1 = \alpha_2 = 0$ and calculate the resulting LR test statistic to be 9.1081 which clearly indicates a rejection of the null at the 5% significance level.

The stylized economic models as discussed in Section 2 imply more general unit root process than the random walk for the equilibrium output. How the equilibrium output responds to a productivity shock depends on the underlying structure of the economy (in particular, the preference and technology parameters). Instead of directly estimating these underlying preference parameters we adopt an AR(2) process to estimate the equilibrium output dynamics. Figure 1 gives the Impulse Response Function of the permanent output (equilibrium output) in response to a unit increase of permanent shock. Our empirical finding seems to support a hump-shaped response of the equilibrium output to the productivity shock. According to our estimation results, following a productivity shock the equilibrium output first “overshoots” and then settles down at its new steady-state level. Intuitively, this explains why our model attributes more output fluctuations to the real productivity shock. According to the IRF one unit increase of permanent shock ultimately increases the steady-state GDP level by about 1.95. Figure 2 present the Impulse Response Functions of the transitory components for both real GDP and Industrial Production in response to a unit of transitory shock. There is not much persistence in the transitory component of GDP as illustrated by the IRF. It takes only about 2.5 years for the effect
of nominal shocks to real GDP to completely die out. However, there seems to be a fair amount of persistence in the industrial production transitory component\(^5\).

With respect to the correlation, the permanent and transitory components are being interpreted as completely structural shocks and therefore should be uncorrelated with each other. Based on the estimation results we find that the correlation between the permanent and transitory shocks to the real GDP is indeed nearly zero and we cannot reject the null that they are uncorrelated.

Figure 3 plots the real GDP, filtered estimates of the common permanent component, and the real GDP transitory component along with shaded area representing the NBER-dated recessions. Figure 4 places the NBER-dated recessions along with the Industrial Production, filtered estimates of the common permanent component, and the Industrial Production transitory component. It appears that real productivity shocks have played a very significant role in explaining the GDP fluctuations while the nominal shocks are not as important.

Since it takes the equilibrium output several quarters to reach the new steady-state level following a productivity shock, the difference between the equilibrium output and its steady-state level might have interesting implications. To visualize this, we plot the permanent component (equilibrium output), its steady-state level and the difference between these two for real GDP in Figure 5. The steady-state level of the equilibrium output is obtained by applying the BN decomposition to the permanent component by following the method provided in Morley (2002)

\(^5\) Note that the transitory component of the Industrial Production becomes much less persistent if we do not impose the cointegration.
and it reflects the level that the equilibrium output will ultimately reach following a productivity shock.

It is very interesting to notice that in all recessions the difference between the steady-state level and equilibrium level remains positive. This finding seems to suggest that during recessions immediately after the negative productivity shocks hit the economy the equilibrium output would overreact in the sense that it would first drop below its steady-state level before coming up back to its steady-state level. On the other hand, during the expansionary periods the difference between the steady-state level and equilibrium level remains largely negative, which seems to suggest that the equilibrium output may go beyond its steady-state level immediately following a positive productivity shock before coming down and settling at its steady-state level.

5. Conclusion

This paper develops and estimates an unobserved components (UC) model of aggregate output that can capture the complex dynamics implied by the diffusion of real productivity shocks and nominal demand shocks. In contrast to the existing UC models, in which the trend component is driven by real productivity shocks, which are assumed to follow a random walk, the trend component in our model follows a more general unit root process. An important finding of our model is that the UC model, with a more general unit root process, can be identified, even when allowing for the correlation between transitory and permanent components. A second important point that we make is that the consequence of incorrectly imposing a single random walk trend is that the importance of nominal shocks will be overstated. Third, we demonstrate that allowing for a more general unit root trend process is empirically relevant for US data on aggregate output. Finally, our paper makes an important contribution to the literature on the
trend/cycle decomposition using UC models. Specifically, our proposed model is shown to be empirically superior to standard existing UC models.

We argue that a growth momentum in the permanent component is important when using the UC model to decompose real economic activity in gauging the relative importance of real productivity shocks and nominal demand shocks in explaining the actual output fluctuations. The estimation results in this paper offer supportive evidence of the proposed growth momentum and imply an “overshooting” pattern of the equilibrium output following the real productivity shocks. The results also tend to imply that productivity shocks accounts for most post-war recessions leaving a much smaller role for nominal shocks.

Our analysis has not considered the nonlinear literature within UC models. There could be possible asymmetry or nonlinearity that may assign a larger role to the transitory components, especially in recessions, than we ascribed to them using a linear framework, see e.g., Kim and Nelson (1999) and Sinclair (2009b). Our paper focuses on a linear decomposition in the spirit of Beveridge and Nelson (1981) and Morley, Nelson and Zivot (2003). We leave nonlinear specification as potential extension of our model which may be explored in future’s work.
References


APPENDIX A

This Appendix derives (3.7), and (3.5) follows as a special case when \( \alpha(L) = 1 \). From the UC setting-up, we have (ignoring the constant term):

\[
\Delta y_t = \Delta \tau_t + \psi_t - \psi_{t-1}
\]  

(A.1)

Therefore,

\[
\begin{align*}
\text{Cov}(\Delta y_t, \Delta y_{t-1}) &= \text{Cov}(\Delta \tau_t + \psi_t - \psi_{t-1}, \Delta \tau_{t-1} + \psi_{t-1} - \psi_{t-2}) \\
&= \text{Cov}(\Delta \tau_t, \Delta \tau_{t-1}) + \text{Cov}(\Delta \tau_t, \psi_{t-1}) - \text{Cov}(\Delta \tau_t, \psi_{t-2}) \\
&\quad + \text{Cov}(\Delta \tau_{t-1}, \psi_t) + \text{Cov}(\psi_t, \psi_{t-1}) - \text{Cov}(\psi_t, \psi_{t-2}) \\
&\quad - \text{Cov}(\Delta \tau_{t-1}, \psi_{t-1}) - \text{Var}(\psi_{t-1}) + \text{Cov}(\psi_{t-1}, \psi_{t-2})
\end{align*}
\]

(A.2)

Apparently, the random sequence \( \{\Delta \tau_t, \psi_t\}_{t=1}^{\infty} \) is jointly stationary. Therefore, define the cross covariance \( \sigma_{\Delta \tau, \psi} = \text{Cov}(\Delta \tau_j, \psi_j), \forall j \). It is easy to show that:

\[
\sigma_{\Delta \tau, \psi} = \frac{\sigma_{\psi \psi}}{1 - \alpha \phi_1 - \alpha^2 \phi_2}
\]  

(A.3)

Then it is straightforward to derive all the covariance and variance terms in (A.2):

\[
\begin{align*}
\text{Cov}(\Delta \tau_t, \Delta \tau_{t-1}) &= \alpha \cdot \frac{\sigma_{\psi \psi}}{1 - \alpha^2} \\
\text{Cov}(\Delta \tau_t, \psi_{t-1}) &= \alpha \cdot \text{Cov}(\Delta \tau_{t-1}, \psi_{t-1}) = \alpha \cdot \sigma_{\Delta \tau, \psi} \\
\text{Cov}(\Delta \tau_t, \psi_{t-2}) &= \alpha^2 \cdot \text{Cov}(\Delta \tau_{t-1}, \psi_{t-2}) = \alpha^2 \cdot \sigma_{\Delta \tau, \psi} \\
\text{Cov}(\Delta \tau_{t-1}, \psi_t) &= \phi_1 \text{Cov}(\Delta \tau_{t-1}, \psi_{t-1}) + \phi_2 \alpha \text{Cov}(\Delta \tau_{t-2}, \psi_{t-1}) = (\phi_1 + \phi_2 \alpha) \cdot \sigma_{\Delta \tau, \psi} \\
\text{Cov}(\psi_{t-1}, \psi_{t-2}) &= \phi_1 \text{Cov}(\psi_{t-1}, \psi_{t-1}) + \phi_2 \text{Cov}(\psi_{t-2}, \psi_{t-1}) - \text{Var}(\psi_{t-1}) = -\frac{(1 - \phi_1 \phi_2)}{(1 + \phi_2)(1 + \phi_1 - \phi_2)} \cdot \sigma_{\epsilon \epsilon}^2
\end{align*}
\]

(A.7)

Following Hamilton (1994, page 58), one has:

\[
\text{Cov}(\psi_{t-1}, \psi_{t-2}) - \text{Cov}(\psi_t, \psi_{t-1}) - \text{Cov}(\psi_{t-1}, \psi_{t-2}) - \text{Var}(\psi_{t-1}) = -\frac{(1 - \phi_1 \phi_2)}{(1 + \phi_2)(1 + \phi_1 - \phi_2)} \cdot \sigma_{\epsilon \epsilon}^2
\]

(A.7)

Combining all terms in (A.2), we obtain (3.7).
APPENDIX B

We cast the UC model given by (4.1) – (4.4) into a state space form:

Measurement Equation:

\[
\begin{bmatrix}
  y_{1t} \\
  y_{2t}
\end{bmatrix} = \begin{bmatrix}
  0 \\
  a
\end{bmatrix} + \begin{bmatrix}
  1 & 0 & 0 & 1 & 0 & 0 \\
  b & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \cdot Z_t \tag{B.1}
\]

Where, \( Z_t = [\tau_t \ \tau_{t-1} \ \tau_{t-2} \ \psi_{1t} \ \psi_{1t-1} \ \psi_{2t} \ \psi_{2t-1}] \).

Transition Equation:

\[
Z_t = \tilde{\mu} + F \cdot Z_{t-1} + G \cdot \tilde{v}_t \tag{B.2}
\]

Where, \( \tilde{\mu} = \begin{bmatrix}
  0 \\
  0
\end{bmatrix}, \quad F = \begin{bmatrix}
  (1 + \alpha_1) & (\alpha_2 - \alpha_1) & (-\alpha_2) & 0 & 0 & 0 & 0 \\
  1 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}, \]

\[
G = \begin{bmatrix}
  1 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, \quad \tilde{v}_t = \begin{bmatrix}
  \eta_t \\
  \epsilon_{1t} \\
  \epsilon_{2t}
\end{bmatrix}, \text{ and } E[\tilde{v}_t \tilde{v}_t'] = \begin{bmatrix}
  \sigma_\eta^2 & \sigma_{\eta\epsilon_1} & \sigma_{\eta\epsilon_2} \\
  \sigma_{\eta\epsilon_1} & \sigma_{e_1}^2 & \sigma_{e_1e_2} \\
  \sigma_{\eta\epsilon_2} & \sigma_{e_1e_2} & \sigma_{e_2}^2
\end{bmatrix}.
\]

The estimation is obtained by maximizing the log-likelihood function written with the aid of Kalman filter. After obtaining the parameter values the filtered estimates of the state variables may be easily computed using the Kalman filter. See Kim and Nelson (1999) for more details of the estimation procedure.
### Table 1: Maximum Likelihood Estimation of the Bivariate UC Model for U.S.

<table>
<thead>
<tr>
<th>Cointegrated Unobserved Components Model</th>
<th>Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Permanent Component</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Drift: $\mu$</td>
<td>0.4166</td>
<td>0.0650</td>
</tr>
<tr>
<td>Innovation: $\sigma_\eta$</td>
<td>0.6830</td>
<td>0.0676</td>
</tr>
<tr>
<td>AR Coeffs: $\alpha_1$</td>
<td>0.7164</td>
<td>0.1097</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>-0.2299</td>
<td>0.0799</td>
</tr>
<tr>
<td><strong>Transitory Components</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_{11}$</td>
<td>0.3355</td>
<td>0.0997</td>
</tr>
<tr>
<td>$\phi_{12}$</td>
<td>0.2299</td>
<td>0.0791</td>
</tr>
<tr>
<td>Innovation: $\sigma_{e1}$</td>
<td>0.3935</td>
<td>0.0452</td>
</tr>
<tr>
<td>Correlation: $\rho_{\eta e1}$</td>
<td>0.0281</td>
<td>0.1508</td>
</tr>
<tr>
<td><strong>GDP</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_{21}$</td>
<td>1.2294</td>
<td>0.0756</td>
</tr>
<tr>
<td>$\phi_{22}$</td>
<td>-0.2371</td>
<td>0.0756</td>
</tr>
<tr>
<td>$a$</td>
<td>-450.5662</td>
<td>91.6328</td>
</tr>
<tr>
<td>$b$</td>
<td>0.9753</td>
<td>0.0972</td>
</tr>
<tr>
<td>Innovation: $\sigma_{e2}$</td>
<td>1.3371</td>
<td>0.0921</td>
</tr>
<tr>
<td>Correlation: $\rho_{\eta e2}$</td>
<td>0.8650</td>
<td>0.0532</td>
</tr>
<tr>
<td>Correlation: $\rho_{e1e2}$</td>
<td>-0.4771</td>
<td>0.1567</td>
</tr>
<tr>
<td><strong>Log Likelihood Value</strong></td>
<td>-739.4218</td>
<td></td>
</tr>
</tbody>
</table>

Note: The real GDP and Industrial Production Index data are obtained from the Federal Reserve Bank of Saint Louis website. The quarterly Industrial Production data is converted from the monthly data by using the end of month series for each quarter. The sample period is from 1947:1 to 2008:4. We take natural logarithm of the level and multiply by 100. $\mu$ is the drift term in equation (4.1). $\sigma_\eta$ is the shock to the permanent component (actual size of the permanent shock is $\sigma_\eta^2/(1-\alpha_1-\alpha_2)^2$, see e.g., Cochrane (1988)). $\alpha_1$ is the first AR coefficient in equation (4.1) and $\alpha_2$ is the second AR coefficient in equation (4.1). $\phi_{11}$ is the first AR coefficient for the Real GDP transitory component. $\phi_{12}$ is the second AR coefficient for the
Real GDP transitory component. $\sigma_{e_1}$ is the innovation to the Real GDP transitory component. $\rho_{qe_1}$ is the correlation between the Real GDP permanent and transitory shocks. $\phi_{21}$ is the first AR coefficient for the industrial production transitory component. $\phi_{22}$ is the second AR coefficient for the industrial production transitory component. $\sigma_{e_2}$ is the innovation to the industrial production transitory component. $\rho_{qe_2}$ is the correlation between the permanent and the Industrial Production transitory shock. $\rho_{e_1e_2}$ is the correlation between the Real GDP and industrial production transitory shocks.
Figure 1: The Impulse Function of the Permanent Component
Figure 2: The Impulse Function of the Transitory Components of GDP and Industrial Production
Figure 3: The Bivariate UC Decomposition of Real GDP into Permanent and Transitory Components
Figure 4: The Bivariate UC Decomposition of Industrial Production into Permanent and Transitory Components
Figure 5: The Permanent Component and Its Steady-State Level from the Bivariate UC Decomposition of the Real GDP