Cyclicality and term structure of Value-at-Risk within a threshold autoregression setup

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Abstract: This paper explores empirically the link between stocks returns Value-at-Risk (VaR) and the state of financial markets cycle. The econometric analysis is based on a self-exciting threshold autoregression setup. Using quarterly French data from 1970Q4 to 2008Q4, it turns out that the $k$-year VaR of equities is actually dependent on the cycle phase: the expected losses as measured by the VaR are smaller in bear market than in bull market, whatever the horizon. These results strongly suggest that the European rules regarding the solvency capital requirements for insurance companies should adapt to the state of the financial market’s cycle.

Keywords: Expected equities returns, Value at Risk, Financial cycle, Investment horizon, Threshold Autoregression.

JEL classification: G11.

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Introduction

After the renewal of banks regulatory framework with the Basel II agreement in 2003, the European Solvency II committee is currently working on new capital standards for insurance companies. A crucial question for life insurers and collective Defined-Contribution pension funds is to determine whether the new prudential regulation in Europe should recognize the long maturity of their liabilities and their role as intermediaries promoting intergenerational risk sharing (see Gollier [2008]). If policyholders have themselves a long-term perspective for their saving, it is important that these financial intermediaries get the right incentives to select assets portfolios that fit best the duration of the assets holding period. In this paper, we quantify the relationship between the equity risk and both the holding duration and the position in the financial cycle.

Actually, the Basel II risk-based capital requirements have been widely criticized because they could exacerbate financial cycles, or more generally business cycle fluctuations (see e.g. Kashyap and Stein [2004], Adrian and Shin [2007, 2008], Plantin, Sapra and Shin [2008], Rochet [2008]). Basically, these authors claim that solvency capital requirements (SCR hereafter) rules which do not depend on the state of the business/financial cycle may lead to large pro-cyclical leverage effects. As a result of such rules, investors demand of securities increases during financial booms, thereby reinforcing them. Conversely, investors have to sell securities during financial downturns in order to restore their solvency ratios, which exacerbates the financial recession.¹ Yet, a cyclical SCR rule allowing for smaller capital requirements during downturns could at least dampen, if not completely eliminate, this procyclical leverage effect.

Providing further support to such a cyclical SCR rule, a growing empirical literature points to predictability and mean-reversion in stocks returns (see e.g. Campbell [1991], Campbell [1996], Barberis [2000], Campbell and Viceira [2002], Bec and Gollier [2007], Campbell and Thompson [2008] or Jondeau and Rockinger [2009]). More precisely, excess stock returns risk is found to be mean reverting in the sense that the risk associated

¹See Adrian and Shin [2007] for a very clear presentation of this procyclical leverage effect.
with long holding periods is lesser than the one associated with short holding horizons as e.g. the widely scrutinized one-year horizon. Beyond this potential investment horizon effect, returns mean reversion may also imply a cyclical effect. In other words, the financial cycle’s position could help predicting future returns and future risk.

Our contribution to this literature is twofold. First, we assess empirically the importance of these cyclical and investment horizon effects for French stock price data. This question is explored by modelling the dynamics of excess return of equities from a self-exciting threshold autoregression (hereafter SETAR) model. This setup aims at disentangling bear and bull markets dynamics. The choice of this representation is basically motivated by the fact that it allows for straightforward computation of the conditional first and second-order moments matrices, namely the conditional mean and variance-covariance matrices. Hence, two crucial variables for dynamic portfolio allocation optimization are obtained easily — the time-\(t\) conditional expectation (forecast) and conditional variance (risk measure) for asset returns at horizon \(t+h\). We also propose a measure of the Value-at-Risk based on the SETAR estimates. It is in line with existing measures in that it derives from the empirical distribution of the expected \(k\)-period returns. Nevertheless, it has the advantage of not imposing any assumption regarding the law of distribution of the sample but relies on bootstrapped quantiles instead.\(^2\) Our second contribution is then to propose a VaR measure which takes the influence of the recent cycle conditions into account. Furthermore, we take advantage of this to propose a cycle-dependent measure of the Solvency Capital Requirement which accounts for the illiquidity risk.

Using quarterly French data from 1970Q4 on, it turns out that both cyclical and horizon effects do influence the Value-at-Risk: it is higher during booms than during recessions, and lower for long than for short investment horizons. Hence, beyond the fact that constant SCR rules may be destabilizing, they are not even justified by a constant VaR. By contrast, our findings support SCR rules which would be flexible enough so

\(^2\)See e.g. Feunou and Meddahi [2007] for a different approach to derive the term structure of risk.
as to take these cyclical and horizon effects into account. This modification of the methodology is countercyclical: it should induce intermediaries to be more conservative in long expansionary phases and to be more risk-taking in downturns.

The paper is organized as follows. Section 1 presents the econometric methodology. Section 2 describes the data used for the threshold autoregression presented in Section 3. In Section 4, estimated stocks returns VaR are compared across investment horizons and phases of financial cycle. Section 5 concludes.

1 SETAR modelling of VaR

1.1 The SETAR model

Let $R_{0t}$ denote the nominal short rate and $r_{0t} = \log(1 + R_{0t})$ the log (or continuously compounded) return on this asset that is used as a benchmark to compute excess returns on equities. Then, with $r_{et}$ the log stock return, let $x_{et} = r_{et} - r_{0t}$ denote the corresponding log excess returns that we wish to model as a Self-Exciting Threshold AutoRegression. In what follows, two kinds of SETAR will be considered for $x_{et}$. The first one is a two-regime SETAR model which aims at capturing expansion vs recession phases and is given by:

$$x_{et} = (\mu_\ell + \sum_{i=1}^{n} \rho_{\ell i} x_{e,t-i})s_t + (\mu_u + \sum_{i=1}^{n} \rho_{ui} x_{e,t-i})(1 - s_t) + v_t,$$

where $s_t$ is a zero-one valued transition function defined by:

$$s_t = \begin{cases} 
1 & \text{if } x_{e,t-1} \leq \lambda, \\
0 & \text{otherwise}. 
\end{cases}$$

Hence, we expect the dynamics of the excess returns to be regime-switching with $x_{e,t-1}$ as the transition variable. More precisely, for values of $x_{e,t-1}$ smaller than or equal to the threshold $\lambda$, the dynamics is governed by $\mu_\ell$ and $\rho_{\ell i}$: this corresponds to the bear market regime. In the bull market regime, where values of $x_{e,t-1}$ are greater than $\lambda$, the dynamics is governed by $\mu_u$ and $\rho_{ui}$. It is assumed that the roots of the characteristic
polynomials $\rho_j(z) = 1 - \rho_j z - \ldots - \rho_j n z^n$, $j = \ell, u$, lie strictly outside the unit circle in absolute value, a condition which rules out nonstationary or explosive behavior in $x_{e,t}$. Finally, the innovations $v_t$ are assumed to be i.i.d. distributed with mean zero and variance $\sigma_v^2$. The second SETAR specification of interest is a 3-regime SETAR of the form:

$$x_{et} = (\mu_\ell + \sum_{i=1}^{n} \rho_\ell i x_{e,t-i}) s_{\ell t} + (\mu_m + \sum_{i=1}^{n} \rho_m i x_{e,t-i}) s_{mt} + (\mu_u + \sum_{i=1}^{n} \rho_u i x_{e,t-i}) s_{ut} + v_t,$$

(2)

where $s_{\ell t} = 1$ if $x_{e,t-1} \leq \lambda_\ell$ and 0 otherwise, $s_{ut} = 1$ if $x_{e,t-1} > \lambda_u$ and 0 otherwise, and $s_{mt} = 1 - s_{\ell t} - s_{ut}$. There are now two real-valued thresholds, $\lambda_\ell$ and $\lambda_u$. In this 3-regime SETAR, we allow for a middle regime on top of the lower and upper regimes. This middle regime could correspond i.e. to a “normal” regime, in between the bear and bull markets. Contrary to the 2-regime model, the specification given in equation (2) could also capture a distinction between large absolute values of the returns (in the lower and upper regimes) and smaller ones (in the middle regime).

### 1.2 From SETAR to Value-at-Risk

Following Campbell and Viceira [2004], the one-period log excess returns are added over $k$ successive periods in order to get the cumulative $k$-period log excess returns on equities, denoted $x_{et}^k \equiv \frac{1}{k}(x_{e,t+1} + \cdots + x_{e,t+k})$. Thanks to its autoregressive nature, the SETAR model is particularly well suited for forecasting purposes. Indeed, the $k$ successive one-period returns used to define $x_{et}^k$ are obtained by forward recursion of models (1) or (2).

Hence, the value-at-risk obtains straightforwardly from this model. The VaR is basically defined as a number such that there is a probability $p$ that a worse excess (log-)return occurs over the next $k$ periods. As such, the VaR is a quantile of this return distribution. The VaR of a long position (left tail of the distribution function) over the
time horizon $k$ with probability $p$ may hence be defined from:

$$p = Pr \left[ x_{et}^k \leq VaR \right] = F_k(VaR),$$

where $F(\cdot)$ denotes the cumulative distribution function of $x_{et}^k$. The quantile function is the inverse of the cumulative distribution function from which the VaR obtains:

$$VaR_k(p) = F_k^{-1}(p).$$

Since $x_{et}^k$ is the sum of log excess returns over $k$ periods, it is also the log of the product of the excess returns (not taken in log) over $k$ periods. Hence, the VaR of the corresponding capital requirement simply obtains as:

$$VaR^cR_k(p) = \exp(VaR_k(p)) - 1$$

Since we are interested in the value-at-risk for various time horizons, it is desirable to keep an equivalent risk level over all the horizons, which means adjusting $p$ with $k$. For instance, the $1 - p = 95\%$ level retained in VaR analysis is chosen on a yearly basis. In order to maintain the same yearly probability, the corresponding probability for horizon $k$ must be adjusted accordingly, that is $1 - p = (95\%)^k$. All the computations below will retain this horizon-adjusted probability.

As can be seen from equation (4), such a VaR measure is directly affected by the distribution chosen for $F(\cdot)$. It is now well-known that the normal distribution is not suitable for most speculative assets, even at the quarterly or yearly frequency. Since there is no consensus regarding which alternative distribution to choose, we propose to retain a bootstrap approach relying on the empirical distribution. Basically, this approach consists in resampling $S$ times the residuals estimated from the SETAR model so as to re-built $S$ simulated sequences of $\frac{1}{k}(x_{e,t+1} + \cdots + x_{e,t+k})$. The method will be discussed to greater extend below and will be applied to the data described in the next section.
2 The data

The benchmark asset from which the excess returns on equities will be calculated is a short rate. The 3-month PIBOR rate obtained from Datastream is retained from 1970M11 to 1998M12. It is then continued using the 3-month EURIBOR rate from 1999M1 to 2008M12. The end-of-quarter values from this monthly series are retained to get quarterly observations, and \( r_{0t} \) denotes the log return on the 3-month rate.

French data for stock prices and returns come from Morgan Stanley Capital International (MSCI) database and are available since December 1969. More precisely, quarterly stock market data are based on the monthly MSCI National Price and Gross Return Indices in local currency. From these data, a quarterly stock total return series and a quarterly dividend series are obtained following the methodology described in Campbell [1999]\(^4\). Note that we depart from Campbell’s approach by not including the tax credits on dividends. Indeed, MSCI calculates returns from the perspective of US investors, so it excludes from its indices these tax credits which are available only to local investors. For France, Campbell chooses to add back the tax credits quite roughly, by applying the 1992 rate of 33.33\% to all the sample. Nevertheless, this rate hasn’t remained fixed over the sample considered here (1970Q1—2008Q4). On top of this, the way dividends are taxed has also changed during that period. We couldn’t find exact tax rate data for our sample and have chosen to work with data excluding tax credits. The equities excess return, \( x_{et} \), is then obtained by substracting \( r_{0t} \) from the log return on equities.

Figure 3 in appendix reports the French risk-free rate and the log excess return on equities.

\(^4\)See also Campbell’s “Data Appendix for Asset Prices, Consumption and the Business Cycle”, March 1998, downloadable from Campbell’s homepage.
3 Empirical assessment of the influence of the financial market cycle on excess equities log returns

3.1 The SETAR models results

The lag order $n$ of the SETAR models is chosen so as to eliminate residuals serial correlation, which leads to retain one lag. For both models (1) and (2), we have also considered a constrained version in which the intercept is assumed to be the same across regimes, i.e. $\mu_t = \mu_u (= \mu_m)$. The threshold estimates were obtained by grid search so as to

1) leave at least 5% of the the observations in each regime and

2) maximize the model’s likelihood. First, we performed linearity tests. Since the thresholds are unidentified nuisance parameters under the null hypothesis, we use the SupLR statistic whose asymptotic distribution obtains from Hansen [1996]. Consequently, the residual bootstrap method described in Hansen and Seo [2002] is used to compute the p-value. Based on 5000 simulations, the null of linearity is only rejected at the 14.46% and 11.35% for the unconstrained two- and three-regimes models respectively. By contrast, it is rejected at the 3.32% and 2.65% for their constrained intercept versions. The OLS estimates of the latters are reported in Table 1, see Appendix. For these SETAR(1) models, the null of no residuals serial correlation up to order 4 is not rejected according to the Portmanteau and LM test statistics. It is also worth noticing that both ARCH and White F tests do not reject the homoskedastic null hypothesis.

In the two-regime model, the threshold estimate is -18.83, which is strongly negative. As can be seen from Figure 4 in Appendix, which plots the log excess returns together with $\lambda_t$, the lower regime basically accounts for the financial crisis which happened during the period: the two oil price shocks with the financial market troughs in 1974 and 1977, the election of the left-hand President François Mitterrand in the second quarter of 1981, the stocks markets Black Monday in the fourth quarter of 1987, the Asian financial crisis at the end of 1990 and 1998, the internet bubble burst in 2001 and 2002, and finally the subprimes crisis which makes the French returns on equities visit the lower regime.
in 2008. Since only nine observations out of 152 are classified in this low regime\(^5\), it seems to capture the troughs, or crisis, rather than recession times: The average of the log excess returns in this regime is -29.03\%. The autoregressive coefficient estimate \(\rho\ell\) is -0.25 — the nullity of this coefficient is rejected at the 6\%-level — which implies strong mean reversion in this regime. In the upper regime, where most of the observations lie, \(\rho_u\) is 0.25 which suggests a slight persistence. The average of the observations lying in the upper regime is 2.47\%.

The three-regime model fails to capture a “normal” regime: While \(\lambda\ell\) is still estimated at -18.83, the second threshold is also found strongly negative with an estimate of -10.38. Looking at Figure 4, where \(\lambda_u\) is also plotted, it appears that the middle regime basically captures the returns fall which forgoes a large trough. As can be seen from Table 1, \(\rho_m = 0.65\), which means that a negative observation in this regime will have a negative impact on the next observation, whereas there is still strong mean reversion in the lower regime since \(\rho\ell\) is -0.23.

Overall, these two SETAR models yield quite similar results: There is strong mean-reversion in the lower regime and there is some persistence in the upper regime, even though rather weak. If the dynamics of the log returns is regime-dependent, so should be the dynamics of the Value-at-Risk as will be checked below.

According to the information criteria reported at the bottom of Table 1, the Schwartz and Hannan-Quinn criteria are smaller for the two-regime model than for the three-regime model. This is not the case for the AIC criterium but as shown in \([?]\) from simulation experiments, it is outperformed by the two other criteria when quarterly data are considered. Hence, we will base our subsequent VaR analysis on the two-regime model.\(^6\)

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\(^5\)Note however that the estimated threshold is not the lower boundary of the grid-search interval.

\(^6\)The conclusions below remain unchanged with the three-regime SETAR.
4 The dynamics of Value-at-Risk

4.1 The proposed empirical measures of the VaR

The bootstrap method described below belongs to the multivariate filtered historical simulation (FHS) method presented in Chistoffersen [2009]. This method consists in simulating future returns from a model using historical return innovations. It is qualified by “filtered” because it does not use simulations from the set of returns directly, but from the set of shocks, which are basically returns such as filtered here by the SETAR model.

The FHS method described in Chistoffersen [2009] would amount in our case to the following: First, using random draws from a uniform distribution, the estimated residuals of model (1) are resampled \( S \) times. Using these \( S \) series of \( v^s \) together with the estimated parameters of model (1) and the observed value of \( x_{e,t-1}^s \), \( S \) hypothetical sequences of \( x_{et}^k \) are obtained by forward recursion of the SETAR model. The VaR\(_k\) then obtains by retaining — amongst these \( S \) simulated sequences — the value of return such that there is a probability \( p \) that a worse value occurs at horizon \( k \). This method clearly accounts for the uncertainty of the shocks realization. However, by setting \( x_{e,t-1}^s = x_{e,t-1} \), it makes the VaR measure strongly dependent on the last available observations. In order to illustrate this, Figure 1 reports this date-dependent VaR measure calculated from 200,000 simulations for the one- to three-year investment horizons and for all \( t \) from 1980Q1 to 2008Q4. For each date \( t \), we have estimated model (1) from 1971Q1 until \( t \) and obtained the \( k \)-year VaRs by the bootstrap method described above. These figures also plot the ex-post observed values of \( \exp(x_{et}^k) - 1 \). For all these investment horizons, the VaR(95%) under-estimates the stock return risk during the 2001-2002 recession episode.

Table 2 in appendix reports out-of-sample tests of predictive accuracy of the models considered here for the \( \text{VaR}(95\%) \)’s up to five years. Following the lines of e.g. Guidolin and Timmermann [2006], we consider the unconditional coverage probability which is the percentage of VaRs above the corresponding ex-post observed return, as well as
Figure 1: VaR(95\%) and corresponding observed returns
the \( S_P \) test statistic given in Escanciano and Olmo [2009], eq. (5) therein. This \( S_P \) statistic refers to the so-called unconditional backtesting which tests whether or not the unconditional expectation of the ‘hits’ or ‘exceedances’ is equal to the theoretical one.\(^7\) From the unconditional coverage probabilities, it turns out that the model is slightly too liberal at the one-year horizon and slightly too conservative at longer horizons. The unconditional backtesting does not reject the null of forecasting accuracy for the two-year horizon only.

4.2 Empirical measures of VaR\(_k\) across investment horizon and financial cycle

Finally, since we aim at evaluating the impact of the financial cycle on the VaR for various investment horizons, we would rather control for the position in the cycle. This is done by initializing the value of the excess return for the forward recursion of model (1) as follows: it is set to the average of observations lying in the lower (respectively upper) regime in order to characterize the VaR in financial recession (resp. expansion).

The results reported below were obtained for \( S = 200,000 \) simulations for each \( k = 1, \cdots, 15 \) years, from which we picked up the corresponding \((1 - 95\%_k)\) quantile for each \( \text{VaR}_k \). Figure 2 plots the two measures of \( \text{VaR}_k \) described above, namely the expansion and recession ones against holding horizons up to fifteen years. The first important result emerging from this figure is that whatever the investment horizon, the VaR depends on the position in the financial cycle. For all horizons, the VaR is stronger in expansion than in recession. The VaR’s gap between recession and expansion times at the one-year horizon is around 5%. This gap widens up to about 15% at the 15-year horizon, but such long-horizons results must be cautiously interpreted since the model is estimated using thirty-seven years only. Overall, these results suggest that a rule imposing the same solvency capital requirement whatever the state of the financial system should be avoided.

\(^7\)Under the null, \((\rho(1 - \rho))^{-0.5}S_P\) has a standard normal distribution given the DGP is known. In practice, the DGP parameters are estimated but Escanciano and Olmo [2009] show that this unconditional S-test still possesses rather good finite-sample power properties even in presence of estimation risk.
The market cycle could actually be pro-cyclical.

The second important result concerns the dynamics of the VaR across investment horizons. In a previous study (see Bec and Gollier [2007]), mean-reversion was found in log returns on French equities relatively to other assets returns: their relative risk was found decreasing with the holding period. This is confirmed by the results in Figure 2. Indeed, the worst expected loss in terms of capital requirement at the \((1 - 0.95^k)\)–percent level, decreases with the investment horizon after two or three years. Starting from a recession period, it even becomes a gain after seven years according to our estimates.

As a further check, the simulations were also performed by re-estimating of the vector autoregression for each \(s \in S\) so as to take the parameters estimates uncertainty into account — which is not done in the common FHS approach. Indeed, the impact of parameter uncertainty on the conclusions regarding the horizon effect has been stressed in a recent empirical work by Pastor and Stambaugh [2009]. In this variant, we also adapt the bootstrap procedure to account for possibly neglected residuals heteroskedasticity.
following the lines described in e.g. Cavaliere, Rahbek and Taylor [2008]: instead of being resampled, the estimated SETAR residuals are multiplied by a Gaussian i.i.d. $\mathcal{N}(0, 1)$ sequence so that the resulting simulated residuals keep the same heteroskedastic features as the estimated ones. As can be seen in Figure 5 reported in appendix, the cyclical effect is robust when the parameters uncertainty and possible heteroskedasticity are taken into account. So is the horizon effect but after five years in this case.

5 Concluding remarks

The SETAR modelling of French stocks excess returns provides evidence of the regime-dependent nature of their dynamics: large downturns are strongly corrected whereas a slight persistence is found otherwise. Since the Value-at-Risk is evaluated from the expected excess returns, it is also influenced by the state of the financial cycle. The VaR evaluated from a trough is lower than the one obtained in expansion for all investment horizons. Our results provide support to the claim that fixed solvency capital requirements may have important procyclical consequences on the dynamic investment strategies of the financial intermediaries. They also suggest some predictability in French equities returns since they point to a decrease in the VaR as the holding period increases. In future work, we will check if our conclusions hold in other countries. One limit of the approach retained here is that it assumes the existence of financial markets cycles without explaining it. A better understanding of this phenomenon is a challenging question on our research agenda.

References


## Appendix

Table 1: SETAR estimates

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<th>2-regime</th>
<th>3-regime</th>
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<td></td>
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<td>[0.70]</td>
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<tr>
<td>$\rho_\ell$</td>
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<td></td>
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<td>[0.09]</td>
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<td></td>
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<tr>
<td>$\lambda_\ell$</td>
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<tr>
<td>$\lambda_u$</td>
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<tr>
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<td>7.742</td>
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<td>Q(4) p-val.</td>
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<tr>
<td>White F p-val.</td>
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<td>0.44</td>
</tr>
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$p$-values of $t$-statistics in [ ].
Table 2: Out-of-sample tests of predictive accuracy

<table>
<thead>
<tr>
<th>Expected % of violations</th>
<th>1 year</th>
<th>2 years</th>
<th>3 years</th>
<th>4 years</th>
<th>5 years</th>
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<td>0.0648</td>
<td>0.0865</td>
<td>0.0900</td>
<td>0.1354</td>
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</table>

| Unconditional backtesting | (\(p(1-p)\))^{-0.5} S_P | 4.07* | -1.55 | -2.62* | -4.38* | -4.08* |

The expected percentage of violations is given by \((1 - 0.95^k)\).

* means rejection at 5%-level.

Figure 3: The data (1970Q4—2008Q4)
Figure 4: The estimated thresholds

Figure 5: Value-at-Risk(9\sigma^k\%) when taking parameters uncertainty and heteroskedasticity into account