Heterogeneous Behavioral Expectations, Exchange Rate Dynamics and Monetary Policy Rules

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September 21, 2009

– Preliminary Draft –
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Abstract

In this paper the performance of alternative monetary policy rules in a macroeconomic environment characterized by different behavioral expectations formation is analyzed. We assume two behavioral forecasting rules concerning the nominal exchange rate based on fundamentalism and chartism, as it is usual in the literature. Based on stochastic simulation, we can identify the flexible PPI inflation targeting as the dominant monetary policy rule, since it delivers the lowest volatility not only of output and inflation, but also of the nominal exchange rate.

Keywords: Behavioral Expectations, Foreign Exchange Dynamics, Monetary Policy Rules

JEL CLASSIFICATION SYSTEM: E52, F31

*E-mail: christian.proano@web.de. I would like to thank the IMK research team, and especially Gustav Horn, Silke Tober and Rudolf Zwiener, Peter Flaschel, Roy Nitze and Tassios Malliaris, as well as participants at the WEAI 2009 Conference in Vancouver, the Money Macro and Finance Research Group 41th Annual Conference in Bradford, UK, and the 2009 Annual Conference of the German Economic Association in Magdeburg, Germany, for valuable comments on previous drafts of this paper.
1 Introduction

Despite of the increasing empirical evidence supporting the importance of “behavioral rules” and other heuristics for the decision making of economic agents (see e.g. Akerlof (2002)), the rational expectations paradigm still represents a main building block in most mainstream macroeconometric models.

This is also the case in the NOEM (New Open Economy Macroeconomics) DSGE framework, where the dynamics of the nominal exchange rate are also explained as the result of the forward-looking behavior of economic agents with rational expectations. However, even though theoretically appealing, the empirical implications of the rational expectations assumption seem to be at odds with empirical data of the dynamics of nominal exchange rates: Indeed, as pointed out e.g. by De Grauwe and Grimaldi (2005b), Efficient Markets Rational Expectations (EMRE) models seem incompatible with important stylized facts on foreign exchange (hereafter FX) rate fluctuations as well as the occurrence of speculative bubbles, herding behavior and currency runs. Indeed, as shown for example in Ehrmann and Fratzscher (2005), the volatility of fundamentals (modeled in that study through an index of interest rate and output growth differentials and current account deficits) is by far not as large as the dynamics of the corresponding nominal exchange rates that would be predicted by rational expectations models.

On the contrary, “non-rational” models, that is, models which feature economic agents with heterogenous beliefs, attitudes or trading schemes (heuristics), seem much more successful in this task, see e.g. Frankel and Froot (1987), Allen and Taylor (1992), Cheung and Chinn (2001) and Manzan and Westerhoff (2007). Indeed, the inclusion of such heterogeneity, and therefore of a somewhat “non-rational” behavior by the economic agents has proven quite valuable in providing insights and explanations concerning some of the “puzzles” which arise when “rationality” is assumed (see De Grauwe and Grimaldi (2006, ch.1) for an extensive discussion of the advantages of the heterogenous agents-approach with respect to the rational-expectations approach in the explanation of empirical financial market data).

In the majority of such non-rational, heterogenous expectations models, however, the analysis is often constrain to the FX markets (assuming an exogenous stochastic process for the fundamental nominal exchange rate); The effects of a non-rational behavior by the FX market participants for the dynamic stability at the macroeconomic level have still remained widely uninvestigated.

1See Engel and West (2005) for an alternative view on this respect.
In this paper an attempt is made to fill in this gap by setting up in a standard stylized macroeconomic model with a FX market where traders choose between two behavioral forecasting rules concerning the future development of the nominal exchange rate: fundamentalism and chartism, also known as “technical analysis” in the literature. The main contribution of this paper to the literature is thus the focus on the one hand on the role of behavioral FX trading not only for the stability of that single market but for the whole macroeconomic system, and on the other hand, the analysis of the effectiveness of alternative monetary policy rules concerning macroeconomic stabilization in such an environment.\(^2\)

The remainder of the paper is organized as following: In section 2 the theoretical framework is described, the local stability properties of a continuous-time representation of the model are analyzed. The basic dynamics of the model are discussed in section 3. Section describes a variety of stochastic simulations of the model under different monetary policy rule specifications. Finally, section 5 draws some concluding remarks from this study.

2 The Model

The International FX Market

An international FX market is assumed, where traders – no matter their nationality – can freely trade domestic in foreign currency (and vice versa) and then invest in both domestic and foreign bonds given the perfect capital mobility between the two economies. This international FX market is characterized by “boundedly rational” traders which, due to informational, time and/or cognitive constraints, do not/cannot calculate “mathematically rational” expectations with respect to the future dynamics of the nominal exchange rate (as it is assumed in the NOEM/DSGE framework) but use behavioral forecasting rules for this task instead.

Now assume that the following sequence of events holds: At the beginning of a period \(t\), only variables determined in \(t - 1\) are known by the FX market participants, which form their forecasts of the nominal exchange rate at \(t + 1\) on the basis of that information. Independently, the monetary authorities in both countries set the nominal interest rates on the basis of the same information. Then, given the perfect capital mobility between

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\(^2\)Recent research within the DSGE framework also has focused on the interplay of macroeconomic fundamentals, nominal and real exchange rate dynamics and monetary policy rules: see e.g. Mark (2007) and Benigno and Benigno (2008).
the two countries, the nominal exchange rate level in \( t \) is determined via the Uncovered Interest Rate Parity (UIP). Finally, the real variables output and inflation are determined in both countries.

As it is done in the majority of heterogeneous expectations models, see e.g. De Grauwe and Grimaldi (2005a) and Manzan and Westerhoff (2007), the FX traders are assumed to choose between two types of behavioral forecasting rules: one which only takes into account certain macroeconomic fundamentals (the “fundamentalist” rule), and one which is based only on the past developments of the nominal exchange rate (the “chartist” or “technical analysis” rule).

Let \( s_t \) represents the logarithm of nominal exchange \( S_t \) and \( E^f_t \) denote the expectations operator of a particular behavioral forecasting rule \( j \) based on the information set available at the beginning of period \( t \). Then, according to the “fundamentalist” forecasting rule, the expected log nominal exchange rate at \( t+1 \) is given by

\[
E^f_t s_{t+1} = s_{t-1} + \beta^f_s (f_{t-1} - s_{t-1}),
\]

where \( f_{t-1} \) represents the (log) fundamental nominal exchange rate at time \( t-1 \) and \( \beta^f_s > 0 \) a scaling factor linked with the speed of adjustment of the log nominal exchange rate towards its long-run equilibrium level \( f \) assumed by the fundamentalists.\(^3\) As it is usually done in the literature (see e.g. Froot and Rogoff (1995), Taylor and Peel (2000) and Taylor, Peel and Sarno (2001)), the PPP postulate (in its absolute form) is assumed to represent the long-run point of reference for the nominal (and real) exchange rate, that is

\[
f_t = p_t - p_t^*,
\]

with \( p_t = \ln(P_t) \) and \( p_t^* = \ln(P_t^*) \) denoting the log price levels in the domestic and foreign economies, respectively.\(^4\) Inserting this expression in eq.(1) delivers

\[
E^f_t s_{t+1} = s_{t-1} + \beta^f_s (p_{t-1} - p_{t-1}^* - s_{t-1}) \quad \Rightarrow \quad s_{t-1} - \beta^f_s (\eta_{t-1})
\]

where \( \eta_t \) is the log of the real exchange rate \( N = S P^*/P \) at time \( t \) and \( \eta_0 = 0 \) its PPP-consistent level.

\(^3\)This scaling factor could depend on the past absolute deviations of the expected nominal exchange rate values from their actual level, as assumed for example in De Grauwe and Grimaldi (2005b).

\(^4\)Note that this is not the only possible specification for the fundamentalist rule: The “fundamentalist” expected nominal exchange rate depreciation could also be determined by the PPP in its relative form, that is \( E^f_t (\Delta s_{t+1}) = \Delta p_t - \Delta p_t^* \) (\( \Delta p_t \) and \( \Delta p_t^* \) being the domestic and foreign price inflation rates).
By contrast, according to the “chartist” forecasting rule, it is assumed that the respecting expected log nominal exchange rate at $t + 1$ is given by

$$E^c_t s_{t+1} = s_{t-1} + \beta^c_s \Delta s_{t-1},$$

(4)

where $\Delta s_{t-1} = s_{t-1} - s_{t-2}$ and $\beta^c_s > 0$ is a scaling factor representing the degree of “persistence” or trend-chasing by the nominal exchange rate expected by the “chartist” forecasting rule.

With (eventually) different expectations concerning the future development of the nominal exchange rate resulting from the two behavioral forecasting rules just described, the last-period earnings of investing one unit of domestic currency in the foreign currency depend of course on whether a nominal appreciation ($\Delta s_{t-1} < 0$) or a depreciation ($\Delta s_{t-1} > 0$) took actually place between the periods $t - 2$ and $t - 1$ (see De Grauwe and Grimaldi (2006)), that is

$$\psi^j_{t-1} = [S_{t-1}(1 + i^*_t) - (1 + i_{t-1})S_{t-2}] \text{ sgn}[E^j_{t-2} \Delta s_{t-1}] \quad j = c, f$$

(5)

with

$$\text{sgn}[E^j_{t-2} \Delta s_{t-1}] = \begin{cases} 1 & \text{for } E^j_{t-2} \Delta s_{t-1} > 0 \\ 0 & \text{for } E^j_{t-2} \Delta s_{t-1} = 0 \\ -1 & \text{for } E^j_{t-2} \Delta s_{t-1} < 0 \end{cases}$$

According to eq.(5), if for example a domestic currency depreciation from $t - 2$ to $t - 1$ was implied by the chartism rule ($E^c_{t-2} \Delta s_{t-1} > 0$) and the nominal exchange rate indeed depreciated ($\Delta s_{t-1} > 0$), the profit associated with the use of the technical analysis rule is equal to $S_{t-1}(1 + i^*_t) - (1 + i_{t-1})S_{t-2}$. If in contrast the FX market traders use the fundamentalist rule according to which $E^f_{t-2} \Delta s_{t-1} < 0$, but $\Delta s_{t-1} > 0$ actually occurs, they make an analogous associated loss of the same amount.

At every $t$, the share of FX traders using the fundamentalist forecasting rule (the so-called “market mood” in Dieci, Foroni, Gardini and He (2005)) is given by the variable $\omega_t$, which, in the spirit of Brock and Hommes (1997, 1998) – see also De Grauwe and Grimaldi (2006) –, is determined by

$$\omega_t = \frac{\exp[\gamma(\psi^f_{t-1} - \sigma^2_{j,t-1})]}{\exp[\gamma(\psi^f_{t-1} - \sigma^2_{j,t-1})] + \exp[\gamma(\psi^c_{t-1} - \sigma^2_{j,t-1})]}$$

(6)

with

$$\lim_{\psi^f_{t-1} \to \infty} \omega_t = 1 \quad \text{and} \quad \lim_{\psi^f_{t-1} \to -\infty} \omega_t = 0.$$
and

\[ \sigma_{j,t-1}^2 = (E_{j,t-2}S_{t-1} - S_{t-1})^2 \quad j = c, f, \]

being the last period’s squared forecast error of the behavioral forecasting rule \( j \), and \( \gamma \) measuring the sensitivity with which traders revise their choice of the forecasting rules (a higher \( \gamma \) implying a stronger reaction to the profitabilities differentials between the two rules). The evolution of the market mood variable \( \omega_t \) is thus assumed to be determined by the relative profitability resulting from the fundamentalist and the chartist forecast rules, as well as by their accuracy. The FX market traders are thus assumed to choose between the two forecasting rules according to their relative profitability in the previous period.

Figure 2 illustrates the \( \omega \) function for different values of \( \psi_f \) and \( \psi_c \) (assuming \( \omega_f \) and \( \omega_c \) to be zero for simplicity). As it can be clearly observed, for similar values of \( \psi_f \) and \( \psi_c \)

(i.e. for similar returns associated with the use of the fundamentalist and chartist rules), \( \omega \) is approximately 0.5. But as the differential between \( \psi_f \) and \( \psi_c \) grows, the share of traders using the fundamentalist rule \( \omega \) either increases (for \( \psi_f > \psi_c \)) or decreases (for \( \psi_c > \psi_f \)), moving towards 1 in the first case and towards 0 in the second case.

On the basis of the expressions for \( E_{t+1}^f \) and \( E_{t+1}^c \) given by eqs. (1), (3), and (6), respectively, the market expectation of the log nominal exchange rate at \( t + 1 \) is simply the weighted average of the two expected nominal exchange rates, that is

\[ E_{t+1} = \omega E_{t+1}^f + (1 - \omega)E_{t+1}^c \]

with \( \omega \) given by eq.(6).
Given the perfect capital mobility assumed in the model, the expected rates of return of domestic and foreign bonds are equated by the adjustment of the log nominal exchange rate in \( t \) to the interest rate differentials and the market expectations on the future nominal exchange rate, that is

\[ s_t = i_t^* - i_t + E_t^m s_{t+1}. \]  

(8)

The UIP condition is thus assumed to hold in this framework – though under a behaviorally determined \( E_t^m s_{t+1} \) instead of the “mathematically rational” \( E_t s_{t+1} \) assumed in the standard international finance literature –. This turns clear after the insertion of \( E_t^m s_{t+1} \) in eq.(8), namely

\[ s_t = i_t^* - i_t + s_{t-1} - \omega_t \beta^f \eta_{t-1} + (1 - \omega_t) \Delta s_{t-1}. \]  

(9)

By subtracting \( s_{t-1} \) from both sides, we obtain the following behaviorally founded law of motion for the log nominal exchange rate

\[ \Delta s_t = i_t^* - i_t + E_t^m s_{t+1} = i_t^* - i_t - \omega_t \beta^f \eta_{t-1} + (1 - \omega_t) \Delta s_{t-1}. \]  

(10)

It should be noted that since the relative weight of the two forecasting rules \( \omega_t \) is still determined by eq.(6), the dynamics of the log nominal exchange rate described by eq.(9) or rather eq.(10) are determined not only by the nominal interest rate differentials as it is standard in the literature, but – innovatively – also by the relative importance of the “fundamentalist” and “chartist” forecasting rules in the FX market (the “market mood”), which depends in turn in a nonlinear manner (see Figure 2) on the relative profitability of both rules and therefore, indirectly, also on the nominal interest rate differentials.

Eqs.(9) or rather eq.(10) indeed open up the possibility for a regime switching behavior of the log nominal exchange (determined by the relative profitability and interplay of the two forecasting rules), with periods of large persistence in the nominal exchange rate (and of deviations of the real exchange rate from the PPP level) as well as nonlinear adjustments of the nominal exchange rate as assumed theoretically e.g. by De Grauwe and Grimaldi (2005b) and documented empirically by Taylor and Peel (2000) Taylor et al. (2001), among others. We will address some of these issues below.

The Macroeconomy

In order to keep this exposition as transparent as possible, the real side of the economy is modeled in a quite parsimonious manner. Accordingly, the output dynamics are repre-
sented by the following standard open-economy IS-relationship

\[ y_t = \alpha_y y_{t-1} - \alpha_y (i_t - \pi_t - (i_o - \pi_o)) + \alpha_y \eta_{t-1} \]  

(11)

where \( y_t \) denotes the output gap (defined as log deviations of actual output from its potential level), \( i_{t-1} \) the short-term nominal interest rate (\( i_o \) being the steady state nominal interest rate), \( \pi_t \) the price inflation rate (\( \pi_o \) being the steady state inflation rate) and \( \eta_t \) the log real exchange rate, with \( \eta_t = \eta_o = 0 \).

With respect to the domestic price inflation dynamics, we assume a standard backward-looking Phillips Curve equation of the form

\[ \pi_t = \alpha_{\pi y} y_{t-1} + \alpha_{\pi} \pi_{t-1}, \]  

(12)

where \( \alpha_{\pi y} \) represents the slope of the Phillips curve and \( \alpha_{\pi} \) the degree of inflation persistence present in the economy.\(^5\)

Together with the law of motion for the log nominal exchange rate given by eq.(10), the price inflation adjustment equation for the domestic economy (assuming \( \pi^f_t = \bar{\pi}^f = \text{const.} \)) delivers the following equation for the evolution of the log real exchange rate:

\[
\Delta \eta_t = \Delta s_t + \bar{\pi}^f - \pi_t = -\exp(\psi^f_{t-1}) \beta_f \eta_{t-1} + \exp(\psi^c_{t-1}) \beta^c (\Delta s_{t-1}) + \bar{\pi}^f - \pi_t.
\]  

(13)

**Monetary Policy**

Concerning monetary policy, as it is usual in standard modern macroeconometric models, we assume that the short-term nominal interest rate is determined by a classical Taylor rule:

\[ i^T_t = i_o + \phi_\pi (\pi_t - \pi_o) + \phi_y (y_t - y_o). \]  

(14)

where the target nominal interest rate of the central bank \( i^T_t \) is assumed to depend on the steady state nominal rate of interest \( i_o \), on the inflation gap \( \pi_t - \pi_o \) (with a reaction strength \( \phi_\pi \)) – for now assumed to be defined in terms of producer prices inflation – and on the output gap (with a reaction strength \( \phi_y \)).

\(^5\)The use of a forward-looking Phillips Curve as usually done in the literature would imply additional assumptions concerning the expectations of future inflation which would detract from the focus of this paper on the FX markets. In this sense, the use of a New Keynesian Phillips curve of the form \( \pi_t = E_t(\pi_{t+1}) + \kappa y_t \), with \( E_t \) as the mathematical expectations operator, would imply a rational inflation expectations formation concerning price inflation which would stand at odds with the behavioral expectations formation in the FX market assumed in this paper.
It should be pointed out that for now we prescind from the inclusion of an interest rate smoothing term in the determining equation for the nominal interest rate, assuming implicitly that it represent at every point the target level of the domestic monetary authorities. In the stochastic simulations of the next section we however will investigate the role of interest rate smoothing for the effectiveness of monetary policy concerning output and inflation stabilization with this behavioral framework.

Local Stability Analysis

In order to obtain first insights on the role of heterogenous expectations and the main international transmission channels for the stability of the FX market and the whole macroeconomy, in this section the model’s stability conditions are investigated in an analytical manner. Based on the notion that the qualitative dynamics and stability properties of a macroeconomic model should not depend on whether it is formulated in continuous- or discrete time, we use a continuous-time representation of the model for the following local stability analysis. For this the underlying period length is defined in general terms as $\Delta t$, so that for $\Delta t \to 0$ the following continuous time approximation for the output dynamics equation can be formulated

$$\dot{y} = (\alpha_y - 1)y - \alpha_{yr}(i - \pi - (i_o - \pi_o)) + \alpha_{y\eta}(\eta),$$

with $\alpha_y \leq 1$. Concerning the Phillips Curve relationship, the straightforward continuous time approximation of eq.(12) is

$$\dot{\pi} = \alpha_{\pi y}y + (\alpha_{\pi} - 1)\pi,$$

with $\alpha_{\pi} \leq 1$.

With respect to the nominal exchange rate dynamics expressed by eq.(10), due to the non-differentiability of eq.(5) and the significant nonlinearity comprised in eq.(6), see De Grauwe and Grimaldi (2006, p.25ff), as well as to the fact that this will be investigated extensively in the next section by means of stochastic simulations, for now we prescind

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6In the academic literature there is an ongoing and still unsolved debate about whether there is an interest smoothing parameter in the monetary policy reaction rule of the central banks or whether the observed high autocorrelation in the nominal interest rate is simply the result of highly correlated shocks or only slowly available information, see e.g. Rudebusch (2002) and Rudebusch (2006) for a throughout discussion of this issue.

7As pointed out by Foley (1975), “No substantive prediction or explanation in a well-defined macroeconomic period model should depend on the real time length of the period”, see also Flaschel, Franke and Proaño (2008).
from the dynamics of $\omega$, the market mood variable, and assume it to remain constant at its steady state level $\omega = \omega_o$.

Under this simplifying assumption, it follows for the dynamics of the log nominal exchange rate

$$\dot{s} = -\omega_o \beta_s^f \eta + (1 - \omega_o) \beta_s^c \dot{s},$$

where $\dot{s} \approx \Delta s_t = \Delta s_{t+\Delta t}$ is assumed to hold. Reordering delivers

$$\dot{s} = -\omega_o \beta_s^f \eta \frac{1}{1 - (1 - \omega_o) \beta_s^c}.$$  \hfill (17)

For the continuous-time approximation of the log real exchange rate dynamics described by eq.(13), we obtain

$$\dot{\eta} = \dot{s} + \bar{\pi}_f - \pi = -\omega_o \beta_s^f \eta \frac{1}{1 - (1 - \omega_o) \beta_s^c} + \bar{\pi}_f - \pi$$ \hfill (18)

Together, eqs. (15)--(18) represent the following autonomous 3D nonlinear dynamical system:

$$\dot{y} = (\alpha y - 1) y - \alpha y_r (i_o + \phi_\pi (\pi - \pi_o) + \phi_\eta (y - y_o) - \pi - (i_o - \pi_o)) + \alpha y \eta$$
$$\dot{\pi} = \alpha \pi y (y - y_o) + (\alpha \pi - 1) \pi.$$  
$$\dot{\eta} = -\omega_o \beta_s^f \eta \frac{1}{1 - (1 - \omega_o) \beta_s^c} + \bar{\pi}_f - \pi$$

It can be easily corroborated that the unique steady state of this nonlinear 3D system is given by

$$y = y_o, \quad \pi = \pi_o = \bar{\pi}_f = 0, \quad i = i_o = i_o^f, \quad \eta = 0, \text{ and } \omega_o = 1/2.$$

The corresponding Jacobian of the system evaluated at the steady state is

$$J_{2D} = \begin{bmatrix}
\partial \dot{y} / \partial y & \partial \dot{y} / \partial \pi & \partial \dot{y} / \partial \eta \\
\partial \dot{\pi} / \partial y & \partial \dot{\pi} / \partial \pi & \partial \dot{\pi} / \partial \eta \\
\partial \dot{\eta} / \partial y & \partial \dot{\eta} / \partial \pi & \partial \dot{\eta} / \partial \eta
\end{bmatrix} = \begin{bmatrix}
J_{11} & J_{12} & J_{13} \\
J_{21} & J_{22} & J_{23} \\
J_{31} & J_{32} & J_{33}
\end{bmatrix}$$

with

$$J_{11} = (\alpha y - 1) - \alpha y_r \phi_y, \quad J_{12} = -\alpha y_r (\phi_\pi - 1), \quad J_{13} = \alpha y \eta,$$
$$J_{21} = \alpha \pi y, \quad J_{22} = \alpha \pi - 1, \quad J_{23} = 0,$$
$$J_{31} = 0, \quad J_{32} = 0, \quad J_{33} = -\omega_o \beta_s^f \frac{1}{1 - (1 - \omega_o) \beta_s^c}.$$

This reduced 3D dynamical system can be proofed to be stable around its interior steady state, if the conditions described by the following proposition are fulfilled:
Proposition

Under Assume that \( \alpha_y \leq 1 \) and \( \alpha_\pi \leq 1 \), as well as the validity of the conditions:

1. \[ \beta_s^c < \beta_s^{c,\text{max}} = 1 + \frac{\beta_f^c \omega_o}{(2-\alpha_y-\alpha_\pi+\phi \alpha \pi)(1-\omega_o)}, \]
2. \( \phi_y \geq 0 \) and \( \phi_\pi > 1 \).

Then: The Routh-Hurwitz conditions are fulfilled and the unique steady state of the reduced 3D dynamical system is locally asymptotic stable.

Proof:

According to the Routh-Hurwitz stability conditions for a 3D dynamical system, asymptotic local stability of a steady state is fulfilled when

\[ a_i > 0, \quad i = 1, 2, 3 \quad \text{and} \quad a_1 a_2 - a_3 > 0, \]

where \( a_1 = -\text{tr}(J) \), \( a_2 = \sum_{k=1}^{3} J_k \) with

\[
J_1 = \begin{vmatrix}
J_{22} & J_{23} \\
J_{32} & J_{33}
\end{vmatrix},
J_2 = \begin{vmatrix}
J_{11} & J_{13} \\
J_{31} & J_{33}
\end{vmatrix},
J_3 = \begin{vmatrix}
J_{11} & J_{12} \\
J_{21} & J_{22}
\end{vmatrix}.
\]

and \( a_3 = -\det(J) \).

It can be easily confirmed that under the validity of the stability conditions enlisted in the above proposition, \( a_1 = -\text{tr}(J) = -(J_{11} + J_{22} + J_{33}) > 0 \) holds unambiguously, since

\[ a_1 = 2 - \alpha_y - \alpha_\pi + \alpha_{yr} \phi_y + \frac{\beta_f^c \omega_o}{1 - \beta_s^c (1-\omega_o)}. \] \hspace{1cm} (19)

Concerning the second Routh-Hurwitz stability condition \( a_2 = \sum_{k=1}^{3} J_k > 0 \), it can be easily observed that

\[
a_2 = \frac{\beta_f^c \omega_o (2-\alpha_\pi - \alpha_y + \alpha_{yr} \phi_y)}{1 - \beta_s^c (1-\omega_o)} + (\alpha_\pi - 1)(\alpha_y - 1 - \alpha_{yr} \phi_y) + \alpha_{yr} \alpha_{\pi y} (\phi_\pi - 1),
\]

is indeed unambiguously positive under conditions 1 and 2, as well as

\[
a_3 = -\det(J) = \alpha_{yr} \alpha_{\pi y} + \frac{\beta_f^c \omega_o [(\alpha_\pi - 1)(\alpha_y - 1 - \alpha_{yr} \phi_y) + \alpha_{yr} \alpha_{\pi y} (\phi_\pi - 1)]}{1 - \beta_s^c (1-\omega_o)}.
\]
and the critical condition

\[ a_1a_2 - a_3 = -\alpha y \pi y + (2 - \alpha y - \alpha \pi + \alpha y \phi y)(\alpha y \pi y(\phi \pi - 1)) + \left(2 - \alpha y - \alpha \pi + \alpha y \phi y + \frac{\beta f \omega_o}{1 - \beta f(1 - \omega_o)}\right) \cdot \\
(\alpha \pi - 1)(\alpha y - 1 - \alpha y \phi y) + \left(\frac{\beta f (1 - \alpha y + \alpha y \phi y)\omega_o}{1 - \beta f(1 - \omega_o)}\right) + \\
\left(1 - \alpha \pi + \frac{\beta f \omega_o}{1 - \beta f(1 - \omega_o)}\right)\left(\frac{(1 - \alpha \pi)\beta f \omega_o}{1 - \beta f(1 - \omega_o)}\right) \]  

(20)

for reasonable parameter values of \( \alpha y \pi \) and \( \alpha \pi y \).

Summing up, the steady state of the 3D dynamic system given by eqs. (15)–(18) can thus be proven to be asymptotically stable if, on the one hand, the intrinsic dynamics of the nominal exchange rate are not all-too explosive (\( \beta c_s < \beta c_{s,\text{max}} \)) and, on the other hand, if the conduction of monetary policy is active enough to bring about price inflation and output stability (\( \phi y \geq 0 \) and \( \phi \pi > 1 \)). Note that these two conditions must jointly hold, being by no means substitutes from each other: If \( \beta c_s > \beta c_{s,\text{max}} \), not only the FX market but also the real economy is subject to explosive forces which would make the steady state unstable, and this for irrespective values of \( \phi \pi \) and \( \phi y \). This result thus implies that an exclusive focus on price (and output) stability might not be sufficient to achieve macroeconomic stability if the FX (and in more general terms, the financial markets) are highly unstable.

It should be pointed out, however, that these results (and in general the local stability analysis just performed) are based on the (simplifying) assumption that the market mood (represented by \( \omega \)) remained constant and at its steady state level \( \omega = \omega_o = 1/2 \) due to the highly nonlinear nature of the FX market mood dynamics given by eq.(6), so that the interaction between the macroeconomic dynamics of a small open economy and the profitability of the assumed behavioral forecasting rules has been still not investigated. This is done by means of stochastic simulations in sections 3 and 4.

3 Model Dynamics

3.1 Dynamic Adjustment Analysis

Before analyzing in detail the performance of alternative monetary policy rules in this behavioral macroeconomic framework for the small open economy case, in this section the
model’s dynamic responses to one-time shocks are discussed.\textsuperscript{8}

There is a large empirical literature concerning the transmission of monetary policy shocks in small open economies: In a nutshell, the two main stylized facts concerning the monetary policy transmission mechanisms in the U.S. are

- an unexpected increase in the U.S. nominal interest rate (a contractionary monetary policy shock) leads to a slowdown of economy activity, which reaches its peak after five quarters, approximately, and
- Price inflation initially increases (the price puzzle discussed, e.g., by Sims (1992)), but, after some quarters, an unambiguously negative effect can be observed.

In contrast, the empirical evidence on the nominal exchange rates reaction to monetary policy shocks is not as undisputed: While Eichenbaum and Evans (1995, p.976) for example find that “the maximal effect of a contractionary monetary policy shock on U.S. exchange rates is not contemporaneous; instead the dollar continues to appreciate for a substantial period of time [, something] inconsistent with simple rational expectations overshooting models of the sort considered by Dornbusch (1976)”, Kim and Roubini (2000), Kalyvitis and Michaelides (2001) and Bluedorn and Bowdler (2006) find little evidence on such a behavior for the G-7 nominal exchange rates after the inclusion of alternative measures of monetary policy shocks as well as of relative output and prices in their specifications.

<table>
<thead>
<tr>
<th>Table 1: Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Gap</td>
</tr>
<tr>
<td>$\alpha_y = 0.9$</td>
</tr>
<tr>
<td>$\alpha_{\eta y} = 0.1$</td>
</tr>
<tr>
<td>$\alpha_{\eta \eta} = 0.01$</td>
</tr>
</tbody>
</table>

In our model, as it can be observed in Figure 2, a one-time increase in the domestic nominal interest rate (under the parameter values of Table 1) leads to a differentiated performance of the unexpected earnings of both chartist and fundamentalist forecasting strategies, which in turn leads to a shift in the market sentiment towards chartism. The initial appreciation of the nominal and real exchange rate – together with the nominal interest rate increase – leads to a downturn of economic activity and of domestic price inflation, which in turn lead to a decrease in the nominal interest rate beyond its initial

\textsuperscript{8}Due to the fact that we do not assume rational expectations formation here, the differentiation between anticipated and unanticipated shocks does not make sense.
value due to its endogenization via the monetary policy rule given by eq.(14) (not shown in Figure 2). The domestic interest rate reaction, in turn, feeds back again in the performance of the chartists and fundamentalists in the FX markets, therefore influencing again the path of the nominal exchange rate, which experiences a depreciation beyond its initial level. It is worth stressing again that the dynamics depicted in Figure 2 (amplitude and persistence) are determined by the relative predominance of the chartist forecasting strategy in the FX market and its relative profitability following the nominal interest rate shock.

![Figure 2: Dynamic responses of FX markets and real economy to a one-time domestic monetary policy shock ($\beta_f = 1, \beta_s = 0.5$)](image)

It should be clear that the relative profitability of both forecasting strategies depends of the actual values of $\beta_f$ and $\beta_s$, which in reality are quite likely to be state-dependent and even time-varying (indeed, as stated before, one of the main stylized facts concerning the dynamics of nominal exchange rates is its apparent nonlinear adjustment with respect to macroeconomic fundamentals, as discussed in Taylor et al. (2001)).

Figure 3 makes this point clear: There the reaction of all macroeconomic variables to a one-time domestic nominal interest rate shock for different values of the trend-chasing parameter $\beta_s \in (0, 2)$ and $\beta_f = 0.5$ (for the sake of better graphical exposition) is illustrated.

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9Ehrmann and Fratzscher (2005, p.16) state in this respect: “[…] the reaction of exchange rates to monetary policy decisions also depends on the market’s interpretation of the underlying reason for the decisions and the expected effect on the economy.”

---

13
As it can be observed, for increasing values of $\beta^c_s$ and a lower value of $\beta^f_s$ than in Figure 2 (which therefore implies a lower adjustment speed of the log nominal exchange rate with respect to its PPP-consistent level expected by the fundamentalists), the response of the macroeconomic variables to the initial monetary policy shock becomes more persistent and with a larger amplitude. As it can be clearly observed, higher values of $\beta^c_s$ act clearly destabilizing not only for the dynamics in the FX markets, but also for the dynamics of the real economy. This destabilizing influence of the technical analysis parameter $\beta^c_s$ becomes even clearer in the case where the economy is subject to a pure FX market shock.

As Figure 4 clearly illustrates, the effect of a nominal exchange rate shock for the real economy increases the larger the $\beta^c_s$ parameter and therefore the larger the impact of this shock is in the FX market. So we can observe that while for small values of $\beta^c_s$ no particular effect on output and inflation besides the usual depreciation effect can be observable, for $\beta^c_s \to 2$ the monotonic convergence of all macroeconomic variables to its baseline levels turns into oscillating dynamics with an increasing amplitude.\(^{11}\)

\(^{10}\)It should be clear that for $\beta^c_s = 0$, the dynamics of the nominal exchange rate are driven solely by the deviation of the log real exchange rate from PPP, see eq.(10).

\(^{11}\)This result corroborates the analytical results of the (partial) local stability analysis of the previous section, where $\beta^c_s < 0$ was identified as a necessary condition for local stability.
Figure 4: Dynamic responses of FX markets and real economy to a one-time nominal exchange rate shock for varying values of $\beta^c_s \in (0, 2)$ (with $\beta^f_s = 0.5$)

4 The Performance of Alternative Monetary Policy Rules in a Heterogeneous Behavioral Expectations Framework

In this section we investigate the performance of a variety of alternative monetary policy rules concerning macroeconomic stabilization again by means of stochastic simulations of the model. For this we simulate again the economy over 1000 periods under the different monetary policy rules specifications summarized in Table 2.

On the one hand, we use different nominal target variables, namely PPI inflation (the case investigated in the theoretical framework of the previous section), CPI inflation – defined as

$$\pi^c = \gamma\pi + (1 - \gamma)\pi^m = \gamma\pi + (1 - \gamma)(\pi^f + \Delta s_t), \quad \gamma = 0.15$$

with $\gamma$ being the share of imported goods in the CPI basket (the value $\gamma = 0.15$ is taken from Rabanal and Tuesta (2006)) and $\pi^m = \pi^f + \Delta s_t$ the domestic-currency inflation of foreign goods –, and the nominal exchange rate depreciation $\Delta s_t$. Additionally, we analyze the effect of an inclusion of an interest rate smoothing term in the actual nominal interest rate, as well as the difference between a strict targeting of the nominal target variable and a flexible targeting case (with the output gap is also targeted). For instance, the flexible...
PPI inflation targeting with interest rate smoothing (rule Ia) is represented by

\[ i_t = \phi_i i_{t-1} + (1 - \phi_i) i^T_t \]

\[ = \phi_i i_{t-1} + (1 - \phi_i) \left[ i_o + \phi_\pi (\pi_t - \pi_o) + \phi_y (y_t - y_o) \right], \]

the flexible CPI inflation targeting without interest rate smoothing (rule IIb) by

\[ i_t = i^T_t = i_o + \phi_\pi (\pi^C_t - \pi_o) + \phi_y (y_t - y_o) \]

and the strict nominal exchange rate targeting without interest rate smoothing (rule IIIc) is given by

\[ i_t = i^T_t = i_o + \phi_s (\Delta s_t). \]

Table 2 shows the alternative monetary policy rules parameter values used in the following simulations.

<table>
<thead>
<tr>
<th>Table 2: Alternative Monetary Policy Rules Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. PPI Inflation Target II. CPI Inflation Target III. Nominal FX Target</td>
</tr>
<tr>
<td>a</td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>( \phi_i )</td>
</tr>
<tr>
<td>( \phi_\pi )</td>
</tr>
<tr>
<td>( \phi_y )</td>
</tr>
<tr>
<td>( \phi_s )</td>
</tr>
</tbody>
</table>

Table 3 illustrates the computed standard deviations of the main simulated time series under the alternative monetary policy rules. The analyzed monetary policy rules are grouped in three different categories: Flexible Nominal Targeting with and without Interest Rate Smoothing, Flexible Nominal Targeting without Interest Rate Smoothing, and Strict Nominal Targeting without Interest Rate Smoothing.

<table>
<thead>
<tr>
<th>Table 3: Standard Deviations (in percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexible Nominal Targeting with Interest Rate Smoothing</td>
</tr>
<tr>
<td>I. a</td>
</tr>
<tr>
<td>( y )</td>
</tr>
<tr>
<td>0.429</td>
</tr>
</tbody>
</table>

Flexible Nominal Targeting without Interest Rate Smoothing

| I. b | II. b | III. b |
| \( y \) | \( \pi \) | \( i \) | \( \Delta s \) | \( \eta \) | \( y \) | \( \pi \) | \( i \) | \( \Delta s \) | \( \eta \) | \( y \) | \( \pi \) | \( i \) | \( \Delta s \) | \( \eta \) |
| 0.225 | 0.180 | 1.111 | 0.208 | 0.298 | 0.355 | 0.238 | 1.533 | 0.259 | 0.379 | 0.816 | 0.463 | 2.897 | 0.472 | 0.715 |

Strict Nominal Targeting without Interest Rate Smoothing

| I. c | II. c | III. c |
| \( y \) | \( \pi \) | \( i \) | \( \Delta s \) | \( \eta \) | \( y \) | \( \pi \) | \( i \) | \( \Delta s \) | \( \eta \) | \( y \) | \( \pi \) | \( i \) | \( \Delta s \) | \( \eta \) |
| 0.338 | 0.201 | 1.611 | 0.226 | 0.327 | 1.403 | 0.666 | 5.337 | 0.678 | 1.022 | – | – | – | – | – |
Rate Smoothing (rules I.a, II.a and III.a, and I.b, II.b and III.b, respectively), as well as Strict Nominal Targeting without Interest Rate Smoothing (rules I.c, II.c and III.c).

First of all it should be pointed out that under the rules II.a (Flexible CPI Inflation Targeting with Interest Rate Smoothing), III.a (Flexible Nominal Exchange Rate Targeting with Interest Rate Smoothing) and III.c (Strict Nominal Exchange Rate Targeting without Interest Rate Smoothing), the simulated economy featured an explosive behavior after ca. 200 periods given the parameter values of Tables 1 and 2. This result leads us to the first finding of our analysis:

- In a macroeconomic environment characterized by potentially destabilizing heterogeneous behavioral expectations in the FX market, a sluggish behavior of the target nominal interest rate under a Flexible CPI Inflation (rule II.a) and Nominal Exchange Rate Targeting (rule III.a), as well as a Strict Nominal Exchange Rate Targeting regime (rule III.c), are not capable to induce stability.

The reasons for this result seem quite obvious: If the dynamics of the nominal exchange rate are erratic due to the influence and interaction of different expectations schemes in the FX market, an interest rate monetary policy rule targeting solely the stabilization of the nominal exchange rate (rule III.c) – or alternatively, a rule targeting but nominal exchange rate variations and the output gap but under interest rate smoothing – might simply not be able to bring about stability into the economy. This seems also to be the case for rule II.a, due to the direct influence of the nominal exchange rate variability in CPI inflation.

Respecting the remaining cases, the direct comparison between the simulated standard deviations of the different target variables shown in Table 3 lead to the following conclusions:

- Rule II.c has the poorest performance respecting overall macroeconomic stabilization.

- Among the remaining rules (besides of rule II.c), under rule III.b the target variables feature the highest volatility.

- Rule I.b seems to be the rule which generates the lowest volatility in all macroeconomic target variables.

Indeed, as Table 3 shows, rule I.b (Flexible PPI Inflation Targeting without I.R.S.) seems to be the more appropriate rule to follow in a macroeconomic framework such as the one discussed here, since under it monetary policy is capable to attain not only the
lowest volatility respecting the explicit target variables of that rule, but also of the other possible target variables.

This result is not surprising: If monetary policy focuses on PPI inflation and output (without any interest rate smoothing) and it reacts to nominal (and real) exchange rate movements only to the extent up to which these affect the output dynamics, it can reduce the overall volatility not only in the real side of the economy, but also in the financial (FX) markets. This is due to the fact that, due to the behavioral expectations determination of the nominal exchange rate, an (unnecessarily) sluggish as well as a relatively too volatile nominal interest rate behavior (the latter being an unavoidable consequence of CPI and nominal exchange rate targeting) induce an unnecessary variability in the FX market.

5 Concluding Remarks

In this paper the effectiveness of alternative monetary policy rules concerning macroeconomic stabilization in a behavioral heterogenous expectations framework was analyzed. Though mainly theoretic, this study delivered a variety of important implications not only for the better understanding of FX-market/macroeconomy interactions, but also for the conduction of monetary policy. Indeed, given the importance that, according to empirical evidence, different expectations and behavioral trading schemes have for the dynamics of the nominal exchange rate, the analysis of the performance of economic policy in macroeconomic environments not driven by “rational” economic agents is not only an interesting academic exercise, but in fact an important task which has been left aside in the academic literature in recent years due to the almost exclusive focusing on the modeling of DSGE models.

But, as the actual global financial crisis shows, financial markets might a) not function as perfect or b) economic agents might not be as well informed or act as rational as economists like to assume. Against this background, one of the main results of the analytical stability analysis of the model discussed here was that a standard monetary policy rule with an inflation and output targets is not likely to bring about macroeconomic stability if the financial markets are subject to explosive trend-chasing forces and large nominal exchange rate shocks. Alternative strategies – as the reaction of the Federal Reserve Bank during the actual crisis show – might be necessary to bring about stability in the financial markets. Given the fact that DSGE models do not (cannot, by construction) deliver any recommendations in this respect, we believe that the research direction pursued in this paper will experience a revival in the years to come, when economist realize that the economics is more about psychology than rationality.
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