Testing for Group-Wise Convergence with an Application to Euro Area Inflation

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Abstract
While panel unit root tests have been used to investigate a wide range of macroeconomic issues, the tests suffer from low power to reject the unit root null in panels of stationary series if the panels consist of highly persistent series, contain a small number of series, and/or have series with a limited length. We propose a new procedure to increase the power of panel unit root tests when used to study convergence by testing for stationarity between a group of series and their cross-sectional means. Although each differential has non-zero mean, the group of differentials has a cross-sectional average of zero for each time period by construction, and we incorporate this constraint for estimation and when generating finite sample critical values. This procedure leads to significant power gains for the panel unit root test. We apply our new approach to study inflation convergence within the Euro Area countries for the post 1979 period. The results show strong evidence of convergence soon after the implementation of the Maastricht treaty. Furthermore, median unbiased estimates of the half life for the period before and after the Euro show a dramatic decrease in the persistence of the differential after the occurrence of the single currency.

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1. Introduction

Time series investigation of the convergence hypothesis often relies on unit root tests as the rejection of the null hypothesis is commonly interpreted as evidence that the series have converged. The extension of these tests to the panel framework has significantly influenced the literature on how to measure convergence of macroeconomic variables. The combination of time-series and cross-sectional information leads to tests with improved performance, especially for the data lengths usually encountered in macroeconomic analysis.

Initial work on panel unit root tests adapted univariate unit root tests to the panel setting. Levin, Lin and Chu (2002) consider a homogenous speed of mean reversion across the series while others, such as Im, Pesaran and Shin (2003) and Maddala and Wu (1999) allow for heterogeneous speeds of reversion. These tests, and improved versions of them, are widely used to investigate the stationary behavior of macroeconomic time series such as real GDP (Fleissig and Strauss (1999)), inflation (Culver and Papell (1997)), interest rates (Wu and Zang (1996)) and real exchange rates (Papell (1997)).


Panel unit root tests for convergence utilize Bernard and Durlauf’s (1995, 1996) definition of time series convergence for long-run output movements, where two (or more) countries converge when long-run forecasts of per capita output differences tend to zero as the forecasting horizon tends to infinity. In the bivariate context, tests for time series convergence require cross-country per capita output differences to be stationary. In the multivariate, or panel, context, a group of countries converge if the null hypothesis

1 An alternative is to use a factor structure approach as in Bai and Ng (2004). Breitung and Pesaran (2005) provide a survey of the existing literature.
that the difference between each country’s output and the cross-sectional mean has a unit root can be rejected in favor of the alternative hypothesis that each difference is stationary. Panel methods have been used to investigate output convergence by Ben-David (1993, 1996), Islam (1995), Evans and Karras (1996), Evans (1998), and Fleissig and Strauss (2001), among others.

Convergence of inflation rates, especially within the European Union, is another topic for which panel unit root tests have been fruitfully employed. For these studies, inflation rates are replaced by their differentials with respect to the cross-sectional mean as variables of interest. Hence, evidence of stationarity for this group of series can be interpreted as support that the series have converged. Several works such as Kočenda and Papell (1997), Lee and Wu (2001), and Weber and Beck (2005) have used panel methods to investigate inflation convergence.

While the first and second generations of panel unit root tests have significantly enhanced finite sample performance, these tests can still have low power to reject the unit root null in a panel of stationary series if the panels consist of highly persistent series, contain a small number of series, and/or have series with a limited length. This paper proposes a new procedure that improves the power of panel unit root tests when testing for convergence of a group of time series.

Panel unit root tests for convergence involve stationarity between a group of series and their cross-sectional means. As the series may not be characterized by absolute convergence toward the cross-sectional average, each differential can have a non-zero mean. By construction, however, the group of differentials has a cross-sectional average of zero for each time period. In order to improve the panel unit root test’s performance, we exploit this extra knowledge of the data by incorporating the appropriate restriction when estimating the model and generating finite sample critical values. To our knowledge, this constraint has not been utilized for previous tests of convergence using panel unit root tests. It should be emphasized that our proposed method is only applicable for tests of convergence. The power of panel unit root tests that examine the Purchasing Power Parity hypothesis by investigating the stationarity of real exchange rates, for
example, cannot be improved by our method as this is not a groupwise convergence related issues.\(^2\)

Monte Carlo simulations confirm the enhanced size-adjusted power of the test when using the constraint. Since imposing a valid constraint will increase power, and the constraint is valid by construction, this result is not surprising. What is more interesting is that the increase in power is generally larger for more persistent data, lower numbers of series, and smaller data spans. Since these are the characteristics that make it difficult for panel tests to reject the unit root null when it is false, imposition of the constraint has the potential to increase findings of convergence in practice.

We also use Monte Carlo simulations to investigate the performance of the test with mixed panels of stationary and unit root series. While the frequency of rejection is, as expected, always higher when the constraint is imposed, this is a mixed blessing. With a large number of stationary series relative to the size of the panel, imposition of the constraint will increase the probability that the unit root null will be rejected, which is generally considered desirable. But, with a small number of stationary series relative to the size of the panel, imposition of the constraint will also increase the probability that the unit root null will be rejected, which is generally considered undesirable.

The enhanced performance of the testing procedure enables a reduction of the data length while preserving good power of the test. Hence, it allows us to analyze data sets for the post-Euro, 1999-2006 period. More specifically, we focus on inflation convergence among the Euro countries. While our main concern is to investigate whether the rates convergence after the Euro, we are also interested in any potential impact of the Euro on the rate of convergence. As a result, the study analyzes the 1979:1 to 2006:12 period using a rolling window of seven years, starting with 1979:1-1986:12 and ending with 1999:1-2006:12, which isolates the impact of the Euro. Furthermore, this approach deals with any potential time break in the data due to events such as German reunification.

Our results show that the inflation rates have converged as early as just after the implementation of Maastricht treaty. However, the results weaken during the two years before the advent of the Euro before reverting back to a strong evidence of convergence.

\(^2\) In this case, the series are individually converging to their own mean but not to a common target.
Furthermore, the generated median unbiased estimates of the half-life for before and after the advent of the Euro shows the dramatic decrease in the persistence of the differentials, after the occurrence of the single currency.

2. Panel Unit Root Tests for Convergence

We modify standard panel unit root tests to account for the restriction on the intercepts imposed by testing for convergence, focusing on the second generation of panel unit root tests that account for contemporaneous correlation by estimating the residual covariance matrix. More specifically, the test considered is an extension of the Levin, Lin, and Chu (2002) application of the ADF test to the panel framework that investigates a homogeneous rate of convergence across the series. Let consider the following system of equations:

\[ \Delta y_{it} = \epsilon_{it} + \rho_i y_{i,t-1} + \sum_{j=1}^{k_i} \phi_{ij} \Delta y_{i,t-j} + \epsilon_{it} \]  

with \( i=1,\ldots,N \) \( t=1,\ldots,T \) and \( \epsilon_{it} \sim WN \)  

The hypotheses tested investigate the evidence for a common unit root process versus a homogeneous rate of convergence under the alternative, that is:

\[ H_0 : \rho_i = \rho = 1 \text{ versus } H_1 : \rho_i = \rho < 1 , \text{ for all } i \]

While it would be desirable, following Im, Peseran, and Shin (2003), to allow for heterogeneous rates of convergence, the alternative hypothesis for these tests is that \( \rho_i < 1 \) for at least one \( i \), which is not economically relevant for investigating convergence among a group of countries. Since the focus of the paper is on panel of macroeconomic variables where the time series dimension is large compared to the cross-section dimension, it is assumed that \( T > N \). For each series, the lag length is first selected, then the system of equations (1) is estimated using the seemingly unrelated regression (SUR) method, hence the name ADF-SUR test. 3

3 Breuer, McNown and Wallace (2002) develop a procedure where heterogenous rates of convergence are allowed in Equation (1) and each series is tested separately for stationarity, but their methodology suffers from low power. We tried their method with the imposition of the convergence restriction, but failed to produce a significant improvement in power.
2.1 The new testing procedure

Our testing procedure benefits from the extra knowledge available on the data used. More specifically, this non-sample information is included in the estimation and when generating the finite sample critical values. The restriction being true by construction, it is directly used to design a model that accounts for all information available prior to the estimation. The restricted estimator ends up with a smaller variance than the unrestricted one.4 Greene (2008, p89) suggests that “one way to interpret this reduction in variance is as the value of the information contained in the restriction”.

The procedure relies on the knowledge that, once transformed, the data may have a non-zero mean for each differential \( i \) (i.e. \( c_{it} \neq 0 \) for some \( t \) in (1)) but a cross-sectional mean equal to 0 at every period (i.e. \( \sum_{i=1}^{N} c_{it} = 0 \) for all \( t \) in (1)). If \( y_{it}^{\text{diff}} \) is the differential for country \( i \) at time \( t \) with respect to the cross-sectional mean such that \( y_{it}^{\text{diff}} = y_{it} - \frac{\sum_{i=1}^{N} y_{it}}{N} \); then by construction, for each period of time \( t \), the sum of the differentials is equal to 0, that is \( \sum_{i=1}^{N} y_{it}^{\text{diff}} = 0 \). Let \( y_{it}^{\text{diff}} \) replaces \( y_{it} \) in (1), then the intercepts \( c_{jt} \) are on average equal to 0. Hence, the estimation uses the restriction \( \sum_{i=1}^{N} c_{it} = 0 \). Note that, since each regression allows for an intercept, we are not testing for absolute convergence.

The procedure follows three steps:
- **Data transformation:** the differentials with respect to the cross-sectional mean are calculated for all series
- **Lag selection:** the number of lagged first difference terms allowing for serial correlation, \( k_j \) in (1), is selected using the recursive procedure for each series

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4Judge et al. (1988, p812) explains that” if nonsample information is correct, then using it in conjunction with the sample information will lead to an unbiased estimator that has a precision matrix superior to the unrestricted least squares estimator”. 
- **Estimation**: The residual covariance matrix is estimated. This matrix, along with the pre-selected $k_j$, is used in the estimation of (1) with the SUR method while two restrictions are imposed:

a. $\sum_{i=1}^{N} c_{it} = 0$, that is the non-sample information

b. $\rho_i = \rho$, that is an homogeneous rate of convergence

The estimated $\rho$ and its corresponding standard deviation are obtained, and the t-statistic is calculated for $H_0 : \rho = 1$.

As suggested by O’Connell (1998), Maddala and Wu (1999) and Chang (2004), tests using the SUR estimation rely on simulated critical values. The bootstrap critical values are generated using a non-parametric resampling method with replacement. First, the coefficient estimates $(\hat{\phi}_j)$ and the fitted residuals $(\hat{u}_i)$ are estimated from $\Delta y_{it} = \sum_{j=1}^{k_j} \hat{\phi}_j \Delta y_{i,t-j} + u_{it}$ \(^5\). Then, the bootstrap samples $(u_{it}^*)$ are drawn from the centered fitted-residuals. More specifically, to preserve the contemporaneous correlation, the $(u_{it}^*)$ are resampled as a vector $u_{i}^* = (u_{it_1}^*, u_{it_2}^*, \ldots, u_{it_N}^*)$ from the empirical distribution of $(\bar{u}_i - \frac{1}{T} \sum_{t=1}^{T} u_{it})^T$. Next, the bootstrap samples $(\varepsilon_{it}^*)$ are recursively generated using the estimated parameters $(\hat{\phi}_j)$ and the bootstrap samples $(u_{it}^*)$ as $\varepsilon_{it}^* = \sum_{j=1}^{k_j} \hat{\phi}_j \varepsilon_{i,t-j}^* + u_{it}^*$, starting from $u_{i0}, \ldots, u_{i,-k_j+1}$. Finally, the pseudo-data $y_{it}^*$ are obtained by taking the partial sum of $(\varepsilon_{it}^*)$ as $y_{it}^* = y_{i0}^* + \sum_{j=1}^{k_j} \varepsilon_{ij}^*$. \(^6\) The system of equations is then estimated using SUR with two restrictions: $\sum_{i=1}^{N} c_{it} = 0$ and $\rho_i = \rho$. For each generated set of series, the estimation procedure previously explained is applied.

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5 The initial lag selection uses BIC.

6 Each pseudo-data $(y_{it}^*)$ is generated with $T+50$ observations, then the first 50 observations are discarded, hence each $(y_{i0}^*)$ is random.
Davidson and G. MacKinnon (2006) explain that “imposing the restriction […] yields more efficient estimates of the nuisance parameters upon which the distribution of the test statistics may depend. This generally makes bootstrap test more reliable, because the parameters of the bootstrap DGP are estimated more precisely”. Since the restriction is true by construction, we expect the restricted test to perform better in small samples than the unrestricted one.

2.2 Impact of the constraint in small samples

In order to analyze the impact of the restriction $\sum_{i=1}^{N} c_{it} = 0$, a set of simulations investigates the size-adjusted power of the ADF-SUR test with and without the restriction. Let consider the following data generating processes:

$$y_{it} = \rho_i y_{i,t-1} + u_{i,t} \quad \text{with } i=1, \ldots, N \text{ and } t=1, \ldots, T$$

The innovations $\{u_{i,t}\}$ are drawn from iid normal distributions with mean zero and variance one. The panel dimensions are $N = 5, 10, \text{ and } 20$ and $T = 25, 50, 100, \text{ and } 200$. For each experiment, the critical values and the empirical rejection probabilities calculated at a 5% nominal level are based on 2000 iterations.\(^7,8\) Since we are using randomly generated data, each experiment is repeated 20 times, hence Tables 1 and 2 report the average rejection probabilities.

Table 1 reports the 5% size-adjusted power of both restricted and unrestricted ADF-SUR tests when the data sets observe a homogeneous rate of convergence, $\rho_i = \rho$, equal to 0.99, 0.97, 0.95 and 0.90.\(^9\) The results show a significant enhancement of the restricted-test performance for most of the cases. For example, for highly persistent data such that $(N, T, \rho) = (10, 100, 0.97)$, the restriction increases the size-adjusted power of the ADF-SUR test from 0.384 to 0.595. Similarly, for moderately persistent data such that $(N, T, \rho) = (20, 50, 0.95)$, the restriction increases the power from 0.337 to 0.539. As expected, these improvements disappear as $N$ and $T$ increase and the data is less persistent, that is in the cases where the ADF-SUR test performs well. In addition, the restriction has only a moderate impact when the panel has a small time dimension, $T = 25$

\(^7\) Davidson and McKinnon (1999) advise a minimum of 1500 bootstraps when analyzing the performance of the test at 1%.

\(^8\) Davidson and McKinnon (2006) define and discuss this probability for the power of bootstrap tests.

\(^9\) The case $\rho = 0.8$ is not reported as it does not provide any new insights on the test’s behavior.
and 50, and the data is extremely persistent, $\rho = 0.99$. In sum, the restriction significantly enhances the test performance for persistent data ($\rho > 0.9$) and small to medium data spans ($T < 200$).

Table 2 focuses on the size-adjusted power of both restricted and unrestricted ADF-SUR tests when the series estimated have heterogeneous rate of convergence $\rho$. More specifically, the experiment considers a mix of stationary ($\rho_i = 0.97, 0.95, 0.90$ and 0.8 for $i = 1, \ldots, k$) and non-stationary processes ($\rho_i = 1.0$ for $i = k+1, \ldots, N$).$^{10}$ The data length $T$ is equal to 100 for $N = 5, 10$ and $20$. The ideal test, of course, would have high power to reject the unit root null if all of the series were stationary and low power if one or more of the series were nonstationary. In that case, one could reasonably conclude that rejection of the unit root null provided evidence that all of the series were stationary.

We start by considering the interpretation of the results of panel unit root tests with homogeneous rates of convergence. Taylor and Sarno (1998) and Breuer, McNowan, and Wallace (2001) have provided evidence that, in the general case where the sum of the intercepts is not constrained to equal zero, the unit root null can be rejected by panel methods with homogeneous rates of convergence even when the panels contain only a few stationary series. Breuer, McNowan, and Wallace (2002), Sarno and Taylor (2002), and Taylor and Taylor (2004) go further, arguing that the unit root null can be rejected even if only one of the series is stationary. We assess the validity of these claims in Table 2, where the columns labeled (2) present the results without the constraint. The bottom row of the three panels, for $N = 5, 10,$ and $20$, presents the (correctly sized 0.05) rejection frequencies when all series have a unit root. Going up one row, the rejection frequencies are depicted when one of the series is stationary. For $N = 5$, they range from 0.07 ($\rho = 0.97$) to 0.11 ($\rho = 0.8$), for $N = 10$, they range from 0.06 ($\rho = 0.97$) to 0.08 ($\rho = 0.8$) and, for $N = 20$, they range from 0.06 ($\rho = 0.97$) to 0.07 ($\rho = 0.8$). With the possible exception of very small panels with a mix of unit root and low persistence processes, it

$^{10}$ The case $\rho = 0.99$ is not reported as it does not provide any new insights on the test’s behavior.
seems very unlikely that the inclusion of one stationary series will produce a rejection of the unit root null with these tests.\(^{11}\)

While the argument that inclusion of one stationary series will produce rejections using panel unit root tests with homogeneous rates of convergence seems overstated, the results confirm that one needs to be careful about interpreting rejections as evidence that all of the series are stationary. For example, with \(N = 10\), a rejection frequency of about 0.50 is found with 8 stationary series if \(\rho = 0.95\), 6 stationary series if \(\rho = 0.90\), and 5 stationary series if \(\rho = 0.80\). Since the result of rejection or non-rejection would be analogous to the outcome of a coin flip, one would not want to conclude that either all of the series were stationary or non-stationary in these cases.

The results for the restricted tests are presented in the columns labeled (1) of Table 2. For all three panels with a mix of unit root and less persistent \((\rho = 0.9\) and 0.8) stationary series, the rejection frequencies for the restricted tests are smaller than those for the unrestricted tests, so that one would be less likely to falsely conclude that all of the series were stationary, for most of the cases, becoming equal as the number of unit root series approaches the number of series. For the panels with a mix of unit root and more persistent \((\rho = 0.95\) and 0.97) stationary series when \(N = 5\) or \(N = 10\), the rejection frequencies for the restricted tests are larger than or equal to those for the unrestricted tests and, when \(N = 20\), the results are mixed.

The apparent symmetry of these results obscures the fact that, in practice, one is much less likely to falsely conclude that all series are stationary with restricted than with unrestricted ADF-SUR tests. This is because, with highly persistent processes and \(N = 5\) or \(N = 10\), the tests do not have much power to reject the unit root null even when all of the series are stationary. Taking the most extreme example \((N = 5\) and \(\rho = 0.97\)) for emphasis, the 5\% size adjusted power is only 0.41 for the restricted test and 0.23 for the unrestricted test when all of the series are stationary. With one stationary series, the fact that the rejection frequency is larger for the restricted (0.22) than the unrestricted (0.16) test is unlikely to cause an inappropriate conclusion that all of the series are stationary. In

\(^{11}\) Some of our rejection frequencies without the constraint are lower than in Breuer, McNowan, and Wallace (2001) for identical panels. The differences appear to be due to their use of Levin, Lin, and Chu (2002) critical values which do not account for serial correlation. Papell (1997) discusses this issue.
the one case \((N = 10\) and \(\rho = 0.95\)) where the tests have good power when all of the series are stationary, there is no difference in the rejection frequencies for the restricted and unrestricted tests with between 1 and 10 non-stationary series.

A very different picture emerges with less persistent processes where the tests have good power to reject the unit root null when all of the series are stationary. We will focus on a comparison of the rejection frequencies between the two tests for the smallest number of stationary series for which the rejection frequency of the unrestricted test is 0.50 or higher. For \(N = 5\), the rejection frequency is 0.58 for the restricted test and 0.65 for the unrestricted test with 4 stationary series and \(\rho = 0.9\) and is 0.40 for the restricted test and 0.51 for the unrestricted test with 3 stationary series and \(\rho = 0.8\). With \(N = 10\), the rejection frequency is 0.57 for the restricted test and 0.66 for the unrestricted test with 7 stationary series and \(\rho = 0.9\) and is 0.54 for the restricted test and 0.64 for the unrestricted test with 6 stationary series and \(\rho = 0.8\). When \(N = 20\), the rejection frequency is 0.46 for the restricted test and 0.56 for the unrestricted test with 11 stationary series and \(\rho = 0.9\) and is 0.42 for the restricted test and 0.53 for the unrestricted test with 9 stationary series and \(\rho = 0.8\). In the above examples, both tests have high power to reject the unit root null when all of the series are stationary, so they represent cases where it is plausible that the unit root null might be rejected with a mixture of stationary and non-stationary series. While other examples could be chosen, the pattern is clear. For mixed panels that contain less persistent stationary series with \(\rho = 0.8\) or \(\rho = 0.9\), one is less likely to mistakenly conclude that all of the series are stationary with the restricted than with the unrestricted tests.

When the data is, by construction, restricted so that the sum of the intercepts is equal to zero for each period, the gain in efficiency obtained by imposing the restriction in the estimation has two main impacts on the ADF-SUR test. First, the more precise estimation and resulting bootstrap procedure leads to a more powerful size-adjusted test for the most commonly encountered panel dimensions in macroeconomics. Second, the rejection frequencies are generally smaller for mixed panels of stationary and non-stationary processes. Combining the results, the restriction improves the overall behavior of the test, enhancing its ability to correctly reject the unit root null hypothesis when all
series are stationary and to fail to reject the unit root null when a subset of the series are non-stationary.

3. Inflation convergence within the Euro-zone, 1979-2006

The Maastricht treaty, signed in 1992, states that "the achievement of the high degree of price stability...will be apparent from a rate of inflation which is close to that of, at most, the three best performing member States in terms of price stability." In practice, the inflation rate of a given country is measured by the CPI and must not be greater than 1.5 percentage points above of the three EU countries with the lowest inflation. While this criterion imposes a form of convergence, it is not identical to the definition of convergence used above.

In light of the achievement of the Maastricht criteria, the fixing of Euro Area exchange rates in mid-1998, and the establishment of the Euro in January 1999, one would expect Euro Area inflation rates to have converged during the period immediately preceding the advent of the Euro. This expectation is confirmed by numerous studies, including Rogers, Hufbauer and Wada (2001), Engel and Rogers (2004), Weber and Beck (2005), Busetti, Forni, Harvey and Venditti (2007) and Rogers (2007), which agree that prices were less dispersed and inflation rates among Euro Area countries converged in the mid-1990s. In contrast, research investigating the post-1998 period, including ECB (2003), Honohan and Lane (2003), Engel and Rogers (2004), Weber and Beck, (2005), Rogers (2007), and Fritsche and Kuzin (2008), concludes that the advent of the single currency resulted in the weakening of inflation convergence among the Euro Area countries and in an increase in their price dispersion. An exception is Honohan and Lane (2004), who report sharp convergence in inflation rates since 2002.

Our study focuses on time series measurement of inflation convergence. We examine the behavior of inflation differentials with respect their cross-sectional means, and then derive the implication for the inflation rates themselves. First, we investigate the evolution of convergence over time, starting with a period prior to the European Monetary System and ending with the post Euro period. Next, we highlight the impact of

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12 The text of the Maastricht Treaty can be found at www.europa.eu.
the Euro by comparing the estimated speeds of convergence before and after the adoption of the single currency.

3.1 Data and estimation results

This study aims to describe the evolution of Euro Area inflation rates from the first stage of the European Monetary System up to 2006, while isolating the post Euro period. Annual inflation rates with monthly data $\pi_t$ for the $i^{th}$ country at time $t$ are calculated such that: $\pi_t = \ln(CPI_t) - \ln(CPI_{t-12})$.\(^{13}\) The differentials $y_{it}$ are generated so that: $y_{it} = \pi_t - \overline{\pi}_t$ where $\overline{\pi}_t$ is the cross-sectional average inflation rate.$^{14}$

The monthly CPI data are from International Financial Statistics (CD June 2007) from 1979:1 to 2006:12. Euro 11 (E11) countries are Austria, Belgium, Finland, France, Germany, Greece, Italy, Luxembourg, Netherlands, Portugal and Spain, and Euro 10 (E10) countries are E11 without Greece.$^{15}$

The descriptive statistics of the data provides us with some useful insights. The cross-country means and standard deviations of the inflation rates, reported in Figure 1, show an overall decrease in the cross-sectional mean and variance throughout the entire period for both the E10 and E11 panels. More specifically, this decrease occurs in three phases: 1982-1987 has the steepest slope, followed by 1990-1999 with a flatter slope and 2000-2006 with no visible change in the slope.$^{16}$

The enhanced performance of the new estimation procedure enables us to consider relatively short periods while retaining good size and power of the test. We isolate the Euro period by considering an eight-year estimation window with 96 monthly observations, which corresponds to the data for 1999:1-2006:12. The window is then rolled from 1979:1-1986:12 to 1999:1-2006:12, one month at a time. This approach limits the impact of potential changes in the parameters on the estimation results while depicting the evolution of the results through time. In contrast to studies which use a recursive (expanding) estimation window to study convergence, our results are not

\(^{13}\) The data is seasonally adjusted
\(^{14}\) Yearly inflation with monthly data and annualized monthly average inflations yield to similar results.
\(^{15}\) The monthly data for Ireland is available only starting 1998:1, hence we do not consider E12 as we could not compare the periods before and after the advent of the Euro.
\(^{16}\) Lopez (2008b) shows that the Euro-zone inflation rates are regime-wise stationary.
affected by the fact that the power of panel unit root tests increases with the number of
observations as well as the size of the panel.

Figure 2 reports p-values for both the E10 and E11 panels using restricted and
unrestricted ADF-SUR tests. Both groups of countries lead to similar conclusions. A
comparison between the restricted and the unrestricted estimations emphasizes the impact
of the previously discussed gain in precision with the new estimation procedure. While
the results observe a similar pattern, the restricted approach consistently leads to lower p-
values. The findings based on restricted estimation show rejections of the unit root
hypothesis for pre-Euro windows ending in 1990-1994 and 1997-1998. The evidence of
convergence is stronger for the E11 panel than for the E10 panel for the windows ending
in the early 1990s. For E11, the unit root null is rejected (at the 10 percent level) for all
only found for 1990:3 - 1991:7. The opposite holds for the windows ending in the late
1990s. For E10, the unit root null is rejected for all windows ending in 1997:4 - 1998:6
while, for E11, convergence is only found for windows ending in 1997:12 - 1998:4.

The strongest evidence of convergence comes from windows ending in 2000 –
2006, after the advent of the Euro. Again focusing on the restricted estimation, the unit
root null is rejected (at the 10 percent level) with E11 for all windows ending in 2002:3 -
2006:12 and with E10 for all windows ending in 2000:9 - 2006:12 except for 2001:9-
2002:1 and for 2005:7-2005:11. The impact of imposing the restrictions is very clear for
the Euro period. For the estimates that do not impose the restrictions, evidence of
convergence is sporadic after 2004 for E11 and almost disappears for E10. It should
perhaps be emphasized that, for the particular case of testing for convergence, there is no
question that imposing the restriction that the sum of the intercepts in Equation (1) is
equal to zero is the correct procedure. Unlike the usual case of imposing restrictions,
which may or may not be correct, this restriction is correct by construction.

Figure 3 plots the values of $\rho$ for the restricted model for E10 and E11 from
variations of $\rho$ can be interpreted as a measurement of the strength of inflation
convergence. A more persistent differential (higher value of $\rho$) would correspond to
weaker inflation convergence as any shock would have a longer lasting impact, and a less
persistent differential (lower value of $\rho$) would correspond to stronger inflation convergence. In contrast to the p-values, the rate of convergence remains relatively stable up to the window ending in 2002:1. Starting in 2002:2, the rate increases (lower value of $\rho$) and decreases (higher value of $\rho$) sharply before returning to more stable behavior near the end of the sample. The rate of convergence at the end of the sample is faster than the rate that characterizes the sample for the windows ending before 2002. The lower values of $\rho$ for the windows starting in 2002:2 are consistent with Honohan and Lane’s (2004) evidence of convergence in Euro Area inflation rates since 2002.\(^\text{17}\)

The initial phase, ending in 1997:3, reports a rate of convergence close to 0.96 for both panels. The E10 panel then shows a slow decrease in $\rho$ from 0.951 to 0.939, between the windows ending in 1997:4 and 2002:1, while the E11 panel remains highly persistent. Both panels report drastic changes in the rate of convergence for the windows ending from 2002:2 to 2005:1. The windows ending between 2002:2 and 2003:12, first, report a significant reduction in persistence ($\rho$ decreases from 0.939 to 0.839 for E10 and from 0.945 to 0.866 for E11), which is then partially compensated by a strengthening of the persistence ($\rho$ increases from 0.842 to 0.898 for E10 and from 0.866 to 0.906 for E11) for the windows ending in 2004:1-2004:12. Following this period of transition, a period of stability concludes the sample: the windows ending between 2005:1 and 2006:12 report an average value for $\rho$ of 0.908 for E10 and 0.904 for E11.

The behavior of the rates of convergence closely follows the European Monetary Union (EMU) timetable. The mechanism that led to the single currency included three major steps: from 1990:7 to 1993:12 (windows ending in 1997:7-2000:12), capital was allowed to move freely within the European Economic Community, from 1994:4 to 1998:12 (windows ending in 2001:4-2005:12) the Treaty of Maastricht was implemented and in 1999:1 (window ending in 2006:12), the single currency was introduced.

It is also worth noting that evidence of stationarity, rejection of the unit root null, occurs several years before the processes reach a steady level of persistence. While inflation rates start converging with the 1995-2002 window, they do not attain a stable

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\(^{17}\) While the value of $\rho$ is biased downward, the focus in the section is on a comparison across time periods which would not be affected by bias correction. In the next section, we conduct median-unbiased estimation for several windows.
level of convergence until the 1997-2004 window. This final degree of convergence is significantly higher ($\rho$ is significantly lower) than the one estimated for the first two phases of the EMU.

3.2 Measuring persistence

A closer investigation of the impact of the Euro requires a rigorous assessment of the speed of convergence for the inflation differentials. In order to provide an accurate measure of persistence, we apply median unbiased corrections to the restricted and the unrestricted estimates. We focus on three windows: 1982:7-1990:6, which covers the pre-Maastricht era, 1990:7-1998:6, which covers the pre-Euro period and ends six months before the exchange rates were fixed, and 1999:1-2006:12, which covers the Euro period.

Following Murray and Papell (2005), we use an extension of the Andrew and Chen (1994) method to the panel framework. The originality of our approach, however, consists of generating median unbiased estimates of the homogeneous rate of convergence for the restricted model. The iterative procedure used to generate the approximately median unbiased estimate, $\rho_{AMU}$, of $\rho$ in (1) starts with the estimation of $\phi_y$ in (1) via the new procedure. Then, assuming the estimates of $\phi_y$’s are true, the first median unbiased estimate $\rho_{1,AMU}$ is obtained by finding the median-unbiased estimator that corresponds to the value of $\rho_{SUR\text{-}restricted}$. We then assume $\rho_{1,AMU}$ to be the true value of $\rho$ and obtain a new set of estimates for the $\phi_y$’s. Conditional on these new estimates, we obtain the new median unbiased estimates $\rho_{2,AMU}$. The iterative process continues until convergence occurs and median unbiased estimates of $\rho_{SUR\text{-}restricted}$ and the $\phi_y$’s are obtained.

Table 3 reports the rates of inflation convergence, the median unbiased estimates (point estimates and 95% confidence intervals of $\rho$), and the corresponding half-lives. The median unbiased point estimates are (as expected) higher than the GLS estimates. The Euro period is characterized by the fastest rates of convergence, followed by the pre-Maastricht period, with the pre-Euro period displaying the slowest convergence rates. This pattern holds for the E10 and E11 panels and the restricted and unrestricted
estimates. For example, using the restricted model, E10 demonstrates a strengthening in inflation convergence as $\rho_{MU}$ decreases from 0.970 for the pre-Maastricht period and 0.975 for the pre-Euro to 0.950 for the Euro period.

While a comparison between the restricted and unrestricted GLS estimates does not lead to any clear differences, the restricted median-unbiased estimates are lower than the unrestricted median-unbiased estimates across all periods. In all cases, the confidence intervals when the restriction is imposed are narrower than the unrestricted confidence intervals, confirming the gain in precision from the restrictions discussed above.

The 95 percent confidence intervals for the Euro period confirm the stronger evidence of inflation convergence from the point estimates. The confidence intervals for the E10 panel with the restricted model widen between the pre-Maastricht (0.950 to 0.988) and the pre-Euro (0.946 to 0.996) periods. In contrast, the confidence interval for the post-Euro period (0.914 to 0.975) has a smaller upper bound and a much smaller lower bound than the confidence intervals for the two earlier periods.

The most common measure of persistence of an economic time series is the half-life, the number of periods it takes for a shock on the inflation differential to dissipate by 50 percent. The half-life is approximated by the ratio $(\ln(0.5)/\ln(\rho_{MU}))$. The median unbiased estimates and corresponding confidence intervals for the half-lives provide a more explicit illustration of the speed of convergence. A larger half-life would imply slower decay and weaker inflation convergence.

Our results once again illustrate the gain in information due to the use of the restriction, with the restricted HL$_{MU}$ point estimates consistently lower than unrestricted estimates. More importantly, the gain in precision leads to narrower restricted confidence intervals, with a noticeable difference for the upper boundaries. For the half-lives, every

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18 The only exception is for the unrestricted E10 panel, for which the value of $\rho$ is slightly lower for the pre-Euro than for the pre-Maastricht period.
19 While the E11 panel for the pre-Maastricht period appears to be an exception, with the width of the confidence interval equal to 0.68 for the restricted and 0.53 for the unrestricted estimates, that interpretation is not correct. The upper point of the confidence interval for the unrestricted model is 1.00. Since the confidence intervals are constrained not to exceed unity, no comparison can be made in this case.
20 While it is generally preferable to compute half-lives from impulse response functions, the panel model used allows for different serial correlation across series, hence there is no common impulse response function on which the half life could be based.
restricted confidence interval is narrower than the corresponding unrestricted confidence interval.

Since the restrictions are valid by construction, we will focus on the median-unbiased estimates of the restricted model. The half-lives of the point estimates of the differentials decrease by more than 40 percent between the pre-Maastricht and Euro periods and by more that 50 percent between the pre-Euro and Euro periods. For the E10 panel, the half-lives rose from 22.76 months in the pre-Maastricht period to 27.38 months in the pre-Euro period, followed by a decline to 13.51 months in the Euro period. For the E11 panel, the half-lives rose from 23.55 months in the pre-Maastricht period to 98.67 months in the pre-Euro period, followed by a decline to 12.98 months in the Euro period. The half-lives for the E10 and E11 panels are very similar for the pre-Maastricht and Euro periods. They are, however, very different for the pre-Euro period. The E11 panel, but not the E10 panel, displays a drastic slowdown of the speed of convergence after the Maastricht treaty, which highlights the impact of Greece and its difficulties in meeting the convergence criteria.

The differences between the Euro and earlier periods are highlighted by the median-unbiased estimates of the confidence intervals of the half-lives for the restricted model, which are both smaller and narrower in the later period. This confirms that not only are inflation differentials less persistent in the Euro period, but that we are more confident about the precision of our estimates of persistence. Going from the pre-Maastricht to the pre-Euro periods, the confidence intervals of the half-lives widen for the E10 panel, which seems coherent with the numerous changes Europe had in the early 1990s (German reunification, differing economic policies) and its evolution toward the more rigorous structure defined by the Maastricht treaty. Similarly, for the same periods, the confidence intervals for the E11 panel increase and widen, again reflecting the influence of the inclusion of Greece. For the Euro period, the confidence intervals of the half-lives are very close for the E10 and E11 panels. The robustness of the results for the Euro period to the panel composition is coherent with the convergence criterion as it sets an identical inflation target for all countries.
4. Conclusion

This paper proposes a new estimation procedure that can be used when investigating convergence of a group of series. Time-series convergence is commonly measured using panel unit root tests on differentials generated as the difference between each series and the cross-sectional average. Hence, each resulting differential has a non-zero mean, but the cross-sectional mean of the group of differentials is equal to zero for each period. Our method uses that information in order to increase the size adjusted power of the test. Monte Carlo simulations report noticeable improvements of the test’s power, especially when the data is persistent data ($\rho > 0.9$) or when the data has a limited length ($T < 200$). Both of these characteristics are commonly featured in macroeconomic time series.

Using the new approach, we analyze inflation convergence within the Euro Area countries. More specifically we investigate when the rates start converging and if the Euro has had an impact on inflation persistence. The increase in size adjusted power from the imposition of the restriction that the cross-sectional mean of the differentials is equal to zero, which is true by construction, allows us to estimate the model for all eight-year rolling windows and separately for the pre-Maastricht, per-Euro, and Euro periods.

We conduct panel unit root tests for rolling windows from 1979:1-1986:12 to 1999:1-2006:12, and report evidence of convergence if the unit root null can be rejected for the inflation differentials. While sporadic evidence of inflation convergence begins with the period starting shortly after the implementation of the Maastricht treaty, consistent evidence of convergence only occurs with windows ending during the Euro period. We also calculate the rate of convergence for each window, and find that the speed of convergence increases for windows ending during the Euro period.

In order to sharpen our focus on the speed of convergence, we calculate median-unbiased point estimates, half-lives, and confidence intervals for the pre-Maastricht, per-Euro, and Euro periods. The rate of convergence is much faster and the confidence intervals are considerably narrower for the Euro period than for the two earlier periods. The half-lives of the point estimates of the differentials, the number of periods that it takes for a shock to the inflation differentials to decrease by one-half, falls by more than
40 percent between the pre-Maastricht and Euro periods and by more that 50 percent between the pre-Euro and Euro periods.

It is commonly accepted that inflation convergence in the Euro Area weakened after the advent of the Euro. We have presented compelling evidence that this view is not correct, based on estimates that, to our knowledge, are the first to solely isolate the Euro period. The statistical evidence of convergence is much stronger and the rate of convergence much faster for the Euro period than for the earlier periods.
References


Breitung, J. and M. H., Peseran, 2005, Unit Roots and Cointegration in Panels, L. Matyas and P. Sevestre (eds), *The Econometrics of Panel Data: Fundamentals and Recent Developments in Theory and Practice*.


European Central Bank, 2003, Inflation differentials in the euro area: potential causes and policy implications


Kappler, M., 2006, Panel Tests for Unit Roots in Hours Worked, ZEW Discussion WP 06-022


Table 1: Finite Sample Power of the Restricted and Unrestricted ADF-SUR test

DGP: $y_{it} = \rho y_{i,t-1} + u_{i,t}$ with $i = 1, \ldots, N$ $t = 1, \ldots, T$ and $u_{it} \sim WN$

Estimated model: $\Delta y_{it} = c_{it} + \rho y_{i,t-1} + \sum_{j=1}^{k} \phi_j \Delta y_{i,t-j} + \varepsilon_{it}$

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(1) corresponds to the restricted model that uses $\sum_{i=1}^{N} c_{it} = 0$, while (2) stands for the unrestricted case.

Reading illustration: if $(N, T, \rho) = (5, 100, 0.97)$, the size adjusted power of the restricted ADF-SUR test is 0.410 compared to 0.225 for the unrestricted case.
Table 2: Finite Sample Power of the Restricted and Unrestricted ADF-SUR test

Mixed processes, T=100

DGP: \( y_{it} = \rho_i y_{i,t-1} + u_{it} \) with \( u_{it} \sim WN \)

Where \( \rho_i < 1 \) for \( i = 1, \ldots, N - k \) and \( \rho_i = 1 \) for \( i = k, \ldots, N \).

Estimated model: \( \Delta y_{it} = c_{it} + \rho_j y_{i,t-1} + \sum_{j=1}^{k_j} \phi_j \Delta y_{i,t-j} + \epsilon_{it} \)

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(1) corresponds to the restricted model that uses \( \sum_{i=1}^{N} c_{it} = 0 \), while (2) stands for the unrestricted case.

Reading illustration: if \( k=2 \) then the panel is a mix of 2 unit roots and 3 stationary processes. If then \( \rho_i = 0.97 \), the size adjusted power of the restricted ADF-SUR test is of 0.15 compared to 0.12 for the unrestricted case.
Table 2 (continue): Finite Sample Power of the Restricted and Unrestricted ADF-SUR test
Mixed processes, T=100

DGP: $y_{it} = \rho_i y_{i,t-1} + \epsilon_{it}$ with $u_{it} \sim WN$
Where $\rho_i < 1$ for $i = 1, \ldots, N-k$ and $\rho_i = 1$ for $i = k, \ldots, N$.

Estimated model: $\Delta y_{it} = c_{it} + \rho_j y_{i,t-j} + \sum_{j=1}^{k_j} \phi_{ij} \Delta y_{i,t-j} + \epsilon_{it}$

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(1) corresponds to the restricted model that uses $\sum_{i=1}^{N} c_i = 0$, while (2) stands for the unrestricted case.

Reading illustration: if $k=4$ then the panel is a mix of 4 unit roots and 6 stationary processes. If then $\rho_i = 0.90$, the size adjusted power of the restricted ADF-SUR test is of 0.42 compared to 0.49 for the unrestricted case.
Table 2 (continue): Finite Sample Power of the Restricted and Unrestricted ADF-SUR test

DGP: \( y_t = \rho_t y_{t-1} + u_{t}, \) with \( u_t \sim WN \)
Where \( i = 1,...,N-k \) for \( \rho_i < 1 \) and \( i = k,...,N \) for \( \rho_i = 1. \)

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(1) corresponds to the restricted model that uses \( \sum_{i=1}^{N} c_i = 0 \), while (2) stands for the unrestricted case.

Reading illustration: if \( k=8 \) then the panel is a mix of 8 unit roots and 12 stationary processes. If then \( \rho_i = 0.90 \), the size adjusted power of the restricted ADF-SUR test is of 0.53 compared to 0.65 for the unrestricted case.
Table 3: Persistence Measurement: Median Unbiased Estimator ($\rho_{MU}$) and Half-Life ($HL_{MU} = \ln(0.5)/\ln(\rho_{MU})$)

\[
\Delta y_t = c_i + \rho v_{t,i-1} + \sum_{j=1}^{k_j} \phi_j \Delta y_{t,j} + \varepsilon_t, \text{ with } i = 1,...,N \quad t = 1,...,T \text{ and } \varepsilon_t \sim WN
\]

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<td>$\rho_{MU}$</td>
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<td>0.958</td>
<td>0.970</td>
</tr>
<tr>
<td>1990:7-1998:6</td>
<td>0.943</td>
<td>0.975</td>
</tr>
<tr>
<td>1999:1-2006:12</td>
<td>0.897</td>
<td>0.950</td>
</tr>
<tr>
<td><strong>Unrestricted ADF-SUR estimation</strong></td>
<td></td>
<td></td>
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<tr>
<td>1982:7-1990:6</td>
<td>0.957</td>
<td>0.979</td>
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<tr>
<td>1990:7-1998:6</td>
<td>0.942</td>
<td>0.977</td>
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<tr>
<td>1999:1-2006:12</td>
<td>0.897</td>
<td>0.957</td>
</tr>
</tbody>
</table>
Figure 1: Cross-sectional Mean and Standard Deviation

E10

E11
Figure 2: P-values, rolling window from 1979:1-1986:12 to 1999:1-2006:12

![Figure 2](image-url)

Figure 3: Homogeneous Rate of Convergence, rolling window from 1979:1-1986:12 to 1999:1-2006:12

![Figure 3](image-url)

The x-axes report the end of the period estimated.