Monetary Policy Regime Shifts and Inflation Persistence∗

Troy Davig       Taeyoung Doh

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Abstract

This paper reports the results of estimating a Markov-Switching New Keynesian (MSNK) model using Bayesian methods. The broadest and best fitting MSNK model is a four-regime model allowing independent changes in the regimes governing monetary policy and the volatility of the shocks. We use the estimates to investigate the mechanisms that lead to a decline in the persistence of inflation. We show that the population moment describing the serial correlation of inflation is a weighted average of the autocorrelation parameters of the exogenous shocks. Changes in the monetary or shock volatility regimes shift weight over these serial correlation parameters. Estimation results indicate that a shift to a more active monetary regime reduces the weight on the more persistent shocks, so lowers the serial correlation of inflation. Similarly, a shift to the low volatility regime in which the volatility of the most persistent shock declines more reduces the weight on the more persistent shocks and also contributes to reducing inflation persistence. We find that the timing of regime shifts is consistent with evidence in the other literature and both a change in monetary regime and shift in shock volatility are empirically relevant in explaining changes in U.S. inflation persistence.

∗Federal Reserve Bank of Kansas City, 1 Memorial Drive, Kansas City, MO 64198. E-mail: Troy.Davig@kc.frb.org and Taeyoung.Doh@kc.frb.org. We thank Chris Otrok and Ferre DeGraeve for very helpful comments. We also thank Marco Del Negro, Jesus Fernandez-Villaverde, Alejandro Justiniano, Thomas Lubik, Jim Nason, Giorgio Primiceri, Frank Schorfheide, and Dan Waggoner, including all participants in the Workshop on Methods and Applications for DSGE Models hosted by the Federal Reserve Bank of Cleveland. The views expressed herein are those of the authors and do not necessarily represent those of the Federal Reserve Bank of Kansas City or the Federal Reserve System.
1 Introduction

An important issue for monetary policy is understanding the factors that contribute to persistence in deviations of inflation from its underlying trend. Several studies address the empirical aspects of U.S. inflation persistence and how it may have changed over time.\(^1\) Motivated by this literature, we investigate the mechanics of how inflation persistence can change in the context of a small-scale New Keynesian model. Either aggressive monetary policy or a decline in the unconditional variance of the shock processes reduce the autocorrelation of inflation. To infer the roles of these factors in explaining U.S. data, we estimate variants of a Markov-switching New Keynesian model using Bayesian methods. The broadest model encapsulates both mechanisms, allowing independent changes in the regimes governing monetary policy and the volatility of the shocks. We find both mechanisms are empirically relevant in explaining changes in U.S. inflation persistence. Our primary results indicate that monetary policy is not so easily characterized by the pre- and post-Volcker distinction. Nor is the evolution in shock volatility so easily separated into the pre- and post- Great Moderation periods. Explaining the decline in inflation persistence, as well as decline in aggregate volatility, requires a monetary policy actively responding to inflation along with a decline in the volatility of the exogenous shocks.

Each Markov-switching New Keynesian (MSNK) model we estimate allows a different aspect of the model to change regime. Specifically, we estimate three different versions of the MSNK model. First, we estimate a MSNK model with a switching monetary policy rule that allows policy to shift between active and passive stances.\(^2\) Second, we estimate a MSNK model with a constant monetary rule, but allows the volatilities of the shocks to change. We find the timing of the low volatility regime corresponds closely to the timing of the active monetary regime from the model with switching in only in the monetary policy rule. This suggest that the estimation procedure exploits switching in whatever parameters we make available to capture the high volatility of the 1970s. So the final model allows for four regimes with independent switching in both monetary policy and the volatility of the shocks. Incorporating independent regime shifts in these two factors provides a framework for untangling the source of the changes in inflation persistence and can also inform us on the source of the decline in aggregate volatility associated with the Great Moderation.

Overall, U.S. data favors the four-regime MSNK model with switching in the monetary rule and shock volatility. The results indicate that monetary policy was active in responding to inflation throughout the latter half of the 1950s and most

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\(^2\)Following the language in Leeper (1991), active (passive) monetary policy refers to a policy that adjusts the nominal interest rate more (less) than one-for-one with movements in inflation.
of the 1960s. Beginning in the late 1960s, policy reverted to passively responding to inflation that lasted throughout the 1970s. Policy began again responding aggressively to inflation after about 1980, but returned to its passive stance around the recessions in 1990-91 and 2001. In many respects, dividing monetary policy behavior between the pre- and post-Volcker periods is useful because it captures the broad contours of policy. However, it fails to capture the shifts in the systematic aspect of monetary policy around recessions and poorly describes policy in the 1960s. For the shock volatility regimes, they fluctuate between episodes of high and low volatility. The low-volatility regime is in place throughout the post-1984 period, or Great Moderation era. However, this regime is also in place for the 1960s.

Focusing on the four-regime MSNK model, we investigate how the different monetary and volatility regimes affect inflation persistence. We show that the population moment describing the serial correlation of inflation is a weighted average of the autocorrelation parameters of the exogenous shocks, which include a technology, monetary policy and markup shock. The weights are functions of the monetary policy coefficients and shock volatilities. Changes in regimes then reshuffle weights over these serial correlation parameters and alter the serial correlation properties of inflation. A shift to the hawkish monetary regime or low-volatility regime reduces the weight on the more persistent shock, so reduces inflation persistence.

The alternative to the MSNK approach is to exogenously split samples and use a more conventional fixed-regime framework. The MSNK model, however, has advantages over a fixed-regime approach with respect to letting the data speak regarding regime changes and overall logical coherence. In structural estimation of DSGE models, the informational assumption underlying exogenous sample splitting is undesirable. For example, the econometrician is assumed to know precisely when regime changes occur, yet agents behave as if the regimes in place are permanent.\(^3\) Also, an information asymmetry arises in that private agents know all the parameter values, which are objects unknown to the econometrician. This informational asymmetry is resolved using an MSNK model, where the econometrician must infer both the timing of regimes and structural parameters. Also, private agents incorporate the possibility of future regime change into their expectation formation.

The general issue of the inflation persistence has generated a large empirical literature. Recently, Cecchetti and Debelle (2006), Clark (2006) and Levin and Piger (2006) conclude that once allowing for shifts in the mean rate of inflation, persistence has not declined much over the past few decades. Pivetta and Reis (2007) report statistical measures indicating that inflation persistence is approximately unchanged in the U.S. using a sample beginning in 1947. Cogley, Primiceri, and Sargent (2007) provide evidence that the inflation-gap, measured as the difference between actual

inflation and trend inflation, has declined in recent decades. These papers focus on measurement, where only Cogley, Primiceri, and Sargent (2007) interpret their findings within a medium-scale DSGE framework. A benefit of using a DSGE model to address changes in inflation persistence is the ability to disentangle whether any shifts in inflation persistence are a result of changes in monetary policy or the processes driving the shocks. Our approach is similar to Cogley, Primiceri, and Sargent (2007), except we use a smaller-scale Markov-switching rational expectations model, but with the benefit of allowing the data to indicate when shifts occur.

2 A Markov-Switching New Keynesian Model

This section presents a New Keynesian model with a relatively standard private sector specification, following closely the setups in Ireland (2004) and An and Schorfheide (2007). The primary difference relative to these specifications is that the parameters in the monetary rule and shock volatilities are subject to regime shifts. Several previous studies explore the implications of regime changes in the policy rule, though no study has estimated a fully specified DSGE model using a monetary rule with switching reaction coefficients - a task we take up in the next section.

The basic elements of the model economy include a representative household, a representative firm that produces a final good and a continuum of monopolistically competitive firms that each produce an intermediate good indexed by \( j \in [0, 1] \).

2.1 Households

The representative household chooses \( \{C_t, N_t, B_t\}_{t=0}^\infty \) to maximize lifetime utility

\[
E_t \sum_{t=0}^\infty \beta_t \left( \frac{(C_t/A_t)^{1-\tau}}{1-\tau} - H_t \right),
\]

where \( C_t \) denotes consumption of a composite good, \( H_t \) are hours worked, \( A_t \) is a measure of technology, \( \beta \in (0, 1) \) is the discount factor and \( \tau > 0 \) is the coefficient of relative risk aversion.\(^4\) Utility maximization is subject to the intertemporal budget constraint

\[
P_t C_t + Q_t B_t = B_{t-1} + W_t H_t + P_t D_t - P_t T_t,
\]

where \( B_t \) are nominal bond holdings, \( D_t \) are real profits from ownership of firms, \( T_t \) are lump-sum taxes, \( P_t \) is the aggregate price level, \( W_t \) is the nominal wage and \( Q_t \) is the inverse of the gross nominal interest rate.

\(^4\) As we discuss below, technology follows a non-stationary process and induces a stochastic trend in consumption. Detrending \( C_t \) by \( A_t \) is convenient because the model has a well-defined steady state in terms of detrended variables.
2.2 Firms

Intermediate goods-producing firm $j$ produces output, $y_{jt}$, according to

$$y_{jt} = A_t n_{jt},$$

where $A_t$ is an exogenous measure of productivity that is the same across firms and $n_{jt}$ is the labor input hired by firm $j$. The labor market is perfectly competitive and firms are able to hire as much as demanded at the real wage.

The monopolistic intermediate goods-producing firms pay a cost of adjusting their price, given by

$$ac_{jt} = \frac{\varphi}{2} \left( \frac{p_{jt}}{\Pi p_{jt-1}} - 1 \right)^2 Y_t,$$

where $\varphi \geq 0$ determines the magnitude of the price adjustment cost, $\Pi$ denotes the central bank’s inflation target and $p_{jt}$ denotes the nominal price set by firm $j \in [0, 1]$. The price adjustment cost is in terms of the final good $Y_t$. Each intermediate goods-producing firm maximizes the expected present value of profits,

$$E_t \sum_{s=0}^{\infty} \beta^s \Delta_{t+s} \frac{d_{jt+s}}{P_{t+s}},$$

where

$$\Delta_{t+s} \equiv \left( \frac{C_{t+s}}{C_t} \right)^{-\tau} \left( \frac{A_t}{A_{t+s}} \right)^{1-\tau}$$

is the representative household’s stochastic discount factor and $d_{jt}$ are nominal profits of firm $j$ at time $t$. Real profits are

$$\frac{d_{jt}}{P_t} = \frac{p_{jt}}{P_t} y_{jt} - \psi_t y_{jt} - \frac{\varphi}{2} \left( \frac{p_{jt}}{\Pi p_{jt-1}} - 1 \right)^2 Y_t,$$

where $\psi_t$ denotes real marginal cost, where $\psi_t = (W_t/P_t)/A_t$.

There is a representative final-goods producing firm that purchases the intermediate inputs at nominal prices $p_{jt}$ and combines them into a final good using the following constant-returns-to-scale technology

$$Y_t = \left[ \int_0^1 y_t (j) \frac{\theta_t - 1}{\theta_t} dj \right] \frac{\theta_t}{\theta_t - 1},$$

where $\theta_t > 1 \ \forall \ t$ is the elasticity of substitution between goods. Variations in $\theta_t$ translate into shocks to the desired markup of the firm towards which the actual markup approaches over time. In the case of fully flexible prices (i.e. $\varphi = 0$), firms
set their nominal price equal to a constant markup over marginal cost, where the steady state markup is
\[ u = \frac{\theta}{\theta - 1}, \]  
(5)
and \( \theta \) is the steady state elasticity of substitution.

The profit-maximization problem for the final-goods producing firm yields a demand for each intermediate good given by
\[ y_{jt} = \left( \frac{p_{jt}}{P_t} \right)^{-\theta_t} Y_t, \]  
(6)
where \( p_{jt} \) is the nominal price of good \( j \). The zero-profit condition for the final goods-producing firm implies \( P_t \equiv \left[ \int_0^1 p_{jt}^{1-\theta_t} \, dj \right]^{1/(1-\theta)} \) is the aggregate price level.

### 2.3 Policy

The monetary authority sets the short-term nominal rate using the following rule
\[ R_t = \tau \Pi \left( \frac{\Pi_t}{\Pi} \right)^{\alpha(s_t)} \left( \frac{Y_t}{A_t y^*} \right)^{\gamma(s_t)} \exp(\epsilon_t), \]  
(7)
where \( R_t \) is the gross nominal interest rate, \( \Pi_t = P_t/P_{t-1} \), \( \Pi \) is the target rate of inflation, \( \tau \) is the steady state real rate, \( y^* \) is the steady state level of the detrended output and the regime, \( s_t \), is a discrete-valued random variable that follows a two-state Markov chain,
\[ P_1 = \begin{bmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{bmatrix}, \]  
(8)
where \( p_{ii} = \text{Pr} \left[ s_t = i | s_{t-1} = i \right] \). The active, or hawkish regime, corresponds to \( s_t = 1 \) and the less active regime, or ‘dovish’ regime, corresponds to \( s_t = 2 \). This labeling implies \( \alpha(2) < \alpha(1) \).

The assumption of a constant inflation target may seem at odds with the empirical literature that stresses the importance of allowing for mean shifts when measuring inflation persistence, such as Cecchetti and Debelle (2006), Clark (2006) and Levin and Piger (2006). Schorfheide (2005) also focuses on a policy rule with a shifting mean in a similar DSGE model. The rationale then for imposing a constant mean, but shifting reaction coefficients, is to give monetary policy a potential mechanism to affect inflation persistence. A shifting mean in the policy rule of this DSGE model

\[ ^5 \text{In the taxonomy of Leeper (1991), the dovish regime is less active, but may also be passive.} \]
\[ ^6 \text{The potential exists for the response coefficients on output (i.e. the } \gamma \text{s) to be sufficiently large and different to confound the simple dove vs. hawk classification. However, estimates given in the next section imply that the current classification is descriptive of the underlying monetary policy stances.} \]
\[ ^7 \text{A subsequent section demonstrates how changes in reaction coefficients affect persistence.} \]
does not affect the model implied degrees of persistence, so is simply an inadequate specification to address this issue.

The potential exists, however, that allowing for a change in trend inflation may affect estimates of the timing and magnitude of the reaction coefficients in the monetary policy rule. To capture this possibility, we consider an alternative specification that specifies trend inflation as following a driftless random walk,

\[ \Pi_t^* = \Pi_{t-1}^* + \varepsilon_{\pi t}, \]  

where \( \varepsilon_{\pi t} \sim N(0, \sigma^2_{\pi}). \)

Also, the fiscal authority passively adjusts lump-sum taxes to satisfy the government’s flow budget constraint and transversality condition on government debt.

### 2.4 Exogenous Shock Processes

Some versions of the MSNK model we estimate will allow regime change in the monetary policy rule, but other specifications will also allow regime change in the variances for shock processes driving the dynamics of output and inflation.

Aggregate productivity follows

\[ \ln A_t = \lambda + \ln A_{t-1} + \ln a_t, \]  

where \( \ln a_t = \rho_a \ln a_{t-1} + \varepsilon_{at}, \) \( \varepsilon_{at} \sim N(0, \sigma^2_a(r_t)) \) and \( |\rho_a| \leq 1 \) for \( r_t \in \{1, 2\} \). The process for productivity imposes that it grows at an average rate of \( \lambda \), but is subject to serially correlated shocks that have varying degrees of persistence depending on the regime.

Shocks to the markup and monetary policy rule follow

\[ \ln u_t = (1 - \rho_u) \ln u + \rho_u \ln u_{t-1} + \varepsilon_{ut}, \]
\[ \ln e_t = \rho_e \ln e_{t-1} + \varepsilon_{et}, \]

where \( \varepsilon_{ut} \sim N(0, \sigma^2_u(r_t)), \varepsilon_{et} \sim N(0, \sigma^2_e(r_t)) \), \( |\rho_u| \leq 1 \) and \( |\rho_e| \leq 1 \) for \( r_t \in \{1, 2\} \).

The regime governing the volatility of the shock process, \( r_t \), also follows a two-state Markov chain,

\[ P_2 = \begin{bmatrix} q_{11} & 1 - q_{11} \\ 1 - q_{22} & q_{22} \end{bmatrix}, \]

where \( q_{ii} = \Pr [r_t = i|r_{t-1} = i] \).
In the four-regime MSNK model, the shock volatility regime, $r_t$, is independent from the monetary regime, $s_t$. As a result, the shock volatility regime can change without requiring a change in the monetary regime. This approach allows the data to indicate whether a, say, high volatility period is more likely the cause of either monetary policy or higher exogenous volatility. Ideally, the volatilties for each shock could change regime according to their own independent Markov chains. However, the estimation of the four-regime MSNK model is computationally demanding, so estimation with higher-order chains is left for future research.

### 2.5 Symmetric Equilibrium

In a symmetric equilibrium, each intermediate good-producing firm facing the same marginal cost, so makes the same pricing and production decisions. In equilibrium, we can then eliminate the $j$ subscripts, yielding $y_{jt} = Y_t$, $p_{jt} = P_t$, $n_{jt} = N_t$, $ac_{jt} = AC_t$ and $d_{jt} = D_t$. In equilibrium, the first-order condition for the firm’s pricing decision is

$$0 = (1 - \theta_t) \Delta_t + \theta_t \Delta_t \psi_t - \varphi \Delta_t \left( \frac{\Pi_t}{\Pi} - 1 \right) \left( \frac{\Pi_t}{\Pi} \right) + \beta \varphi E_t \left[ \Delta_{t+1} \left( \frac{\Pi_{t+1}}{\Pi} - 1 \right) \left( \frac{\Pi_{t+1} Y_{t+1}}{\Pi Y_t} \right) \right].$$  \hfill (15)

The household’s first-order conditions are

$$1 = A_t \left( \frac{C_t}{A_t} \right)^{-\tau} \frac{W_t}{P_t^\tau}$$  \hfill (16)

$$\left( \frac{C_t}{A_t} \right)^{-\tau} = \beta E_t \left[ \frac{R_t}{\pi_{t+1}} \left( \frac{C_{t+1}}{A_{t+1}} \right)^{-\tau} \frac{A_t}{A_{t+1}} \right].$$  \hfill (17)

In addition, the aggregate resource constraint must hold in equilibrium,

$$Y_t = C_t + AC_t,$$  \hfill (18)

and also,

$$H_t = N_t,$$  \hfill (19)

where $N_t = \int_0^1 n_{jt} dj$.

In the case of fully flexible prices, (15) and (16) imply aggregate output is

$$Y_t^* = A_t u_t^{\frac{1}{\tau}},$$  \hfill (20)

where $u_t$ is the markup.
3 The Conditionally Linear MSNK Model

3.1 Equilibrium Relations and Shock Processes

The log-linearized private sector relations (15) and (17) are

\[ \hat{y}_t = E_t \hat{y}_{t+1} - \tau^{-1} \left( \hat{R}_t - E_t \hat{\pi}_{t+1} - E_t \hat{a}_{t+1} \right), \]  
\[ \hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa (\hat{y}_t + \hat{u}_t), \]  

where \( \hat{y}_t \equiv \ln \left( \frac{y_t}{y} \right) \), \( y_t = \left( \frac{Y_t}{A_t} \right) \), \( y = \psi^{-\frac{1}{\tau}} \) is the output gap, \( \hat{\pi}_t = \ln \left( \frac{\Pi_t}{\Pi} \right) \) and \( \hat{R}_t = \ln \left( \frac{R_t}{R} \right) \). Conditioning on a given regime, the monetary rule and shock processes are linear, given by

\[ \hat{R}_t = \alpha (s_t) \hat{\pi}_t + \gamma (s_t) \hat{y}_t + \varepsilon_t, \]  
\[ \hat{a}_t = \rho_a \hat{a}_{t-1} + \varepsilon_{at}, \]  
\[ \hat{u}_t = \rho_u \hat{u}_{t-1} + \varepsilon_{ut}, \]  
\[ \hat{e}_t = \rho_e \hat{e}_{t-1} + \varepsilon_{et}. \]

Equations (21) – (26) represent the full MSNK model.

As in Davig and Leeper (2007), we compute solutions using the method of undetermined coefficients on the minimum set of state variables.\(^8\) Solutions have regime-dependent coefficients as follows

\[ \begin{bmatrix} \hat{\pi}_t(s_t) \\ \hat{y}_t(s_t) \\ \hat{R}_t(s_t) \end{bmatrix} = \begin{bmatrix} A_\pi(s_t) & B_\pi(s_t) & C_\pi(s_t) \\ A_y(s_t) & B_y(s_t) & C_y(s_t) \\ A_R(s_t) & B_R(s_t) & C_R(s_t) \end{bmatrix} \begin{bmatrix} \hat{a}_t \\ \hat{u}_t \\ \hat{e}_t \end{bmatrix}. \]  

since we use a first-order approximation to the equilibrium conditions, the solution coefficients depend only on the monetary regime and not the shock volatility regime.

3.2 Determinacy Restrictions

In estimating the MSNK model, we allow one monetary regime to be passive. However, the presence of a passive regime can render the equilibrium indeterminate. In a purely forward-looking New Keynesian model with a fixed policy rule, a passive policy fails to uniquely determine the equilibrium.\(^9\) This results in the possibility

\(^8\)See Farmer, Waggoner, and Zha (2006) for an adaptation of this method that permits inclusion of lagged predetermined variables and an algorithm for checking whether the MSV solution is determinate (i.e. bounded and unique).

\(^9\)This is very often the case, but some qualifications do apply. See Bullard and Mitra (2003) for an analysis of the New Keynesian model under various monetary policy rules.
that agents will coordinate on extraneous information, or sunspots, which generate ‘non-fundamental’ macroeconomic fluctuations. The equilibrium representation under indeterminacy complicates estimation because it differs from the determinate representation.\(^{10}\) One potential approach to account for indeterminacy when estimating the MSNK model follows Lubik and Schorfheide (2004), where posterior weights apply to the determinate and indeterminate regions of the parameter space.

Since the current paper is a first pass at estimating a MSNK with switching monetary rules, we restrict parameter estimates to lie within the determinate region.\(^{11}\) To impose the determinacy conditions for the MSNK model, we use the approach of Davig and Leeper (2007) who formulate the restrictions based on a linear representation, or ‘stacked’ system, of the model given in (21) – (23). This approach permits one monetary regime to be passive, but restricts the extent of the passive behavior.

To illustrate how to derive the stacked system, first assume all shocks are i.i.d. and rewrite expectations as follows

\[
E_t\pi_{t+1} = E[\pi_{t+1} | s_t = i, \Omega_t^{-s}] = p_{i1}E[\pi_{1t+1} | \Omega_t^{-s}] + p_{i2}E[\pi_{2t+1} | \Omega_t^{-s}],
\]

\[
E_t x_{t+1} = E[x_{t+1} | s_t = i, \Omega_t^{-s}] = p_{i1}E[x_{1t+1} | \Omega_t^{-s}] + p_{i2}E[x_{2t+1} | \Omega_t^{-s}],
\]

where \(\pi_{it} = \pi_t(s_t = i, \varepsilon_t), x_{it} = x_t(s_t = i, \varepsilon_t)\), and \(\varepsilon_t = [\hat{a}_t, \hat{a}_t, \hat{e}_t, \hat{e}_t]'\) for \(i = 1, 2.\)\(^{12}\) The information set, \(\Omega_t^{-s} = \{s_{t-1}, \ldots, r_t, r_{t-1}, \ldots\}\), excludes the current regime, so \(\Omega_t = \Omega_t^{-s} \cup \{s_t\}\). Distributing probability mass across the different conditional expectations for inflation, as in (28) – (29), is the same approach as in Gordon and St-Amour (2000) and Bansal and Zhou (2002).

Next, define the forecast errors

\[
\eta_{\pi j, t+1} = \pi_{jt+1} - E[\pi_{jt+1} | \Omega_t^{-s}],
\]

\[
\eta_{x j, t+1} = x_{jt+1} - E[x_{jt+1} | \Omega_t^{-s}],
\]

for \(j = 1, 2\). Substituting expectations, (28) – (29), and the policy rule, (7), into (21) – (22) yields the stacked system

\[
AY_t = BY_{t-1} + A\eta_t + Cu_t,
\]

\(^{10}\)By ‘equilibrium representation,’ we mean the set of equations and state variables that completely describe the evolution of the economy.

\(^{11}\)Although these restrictions allow one monetary regime to be passive, they are not innocuous. Davig and Leeper (2007) highlight that using the MSNK model to interpret recent U.S. monetary history in terms of switching from passive (i.e. indeterminacy pre-Volcker) to active (i.e. determinacy post-Volcker) policy is not a viable option. Either the entire time series must be interpreted as the outcome of a unique fundamental equilibrium, or always (and still) subject to sunspot shocks.

\(^{12}\)Whether shocks are i.i.d. or serially correlated does not matter for determinacy, so is made here for convenience.
where
\[
Y_t = \begin{bmatrix}
\pi_{1t} \\
\pi_{2t} \\
x_{1t} \\
x_{2t}
\end{bmatrix}, \quad \eta_t = \begin{bmatrix}
\eta_{\pi 1t} \\
\eta_{\pi 2t} \\
\eta_{x 1t} \\
\eta_{x 2t}
\end{bmatrix}, \quad u_t = \begin{bmatrix}
\varepsilon_{at} \\
\varepsilon_{ut} \\
\varepsilon_{et}
\end{bmatrix},
\] (33)

and \(A, B\) and \(C\) are conforming matrices consisting of private sector parameters, policy parameters and the transition matrix. The stacked system has constant coefficient matrices, yet captures the impact potential regime changes in monetary policy have on expectation formation. Further, standard methods for solving linear rational expectations systems are applicable to (32), such as Blanchard and Kahn (1980) or Sims (2001).\(^{13}\)

Necessary and sufficient conditions for determinacy, which is the existence of a unique bounded solution to (32), is that all the generalized eigenvalues of \((B, A)\) lie inside the unit circle. The determinacy conditions are intuitive. First, the passive monetary regime cannot be too passive, meaning the response to inflation can be less than one, but still has to be above some minimum threshold. And second, the passive regime cannot be too persistent, meaning that the expected duration of the regime must be below a given threshold. The determinacy conditions are joint restrictions over both monetary regimes, so the parameters governing the active regime affect the determinacy restrictions over the passive regime. Therefore, the more persistent or active the active regime is, the more persistent or passive the passive regime can be.

Farmer, Waggoner, and Zha (2008) present an example of solutions to the conditionally linear system, (21) – (23), where lagged regimes affect the conditional distribution of endogenous variables. These solutions include a sunspot shock and can exist in regions of the parameter space where the stacked system has a unique equilibrium. The reason for the difference is that the stacked system, by assumption, rules out the potential for lagged regimes to affect the conditional distribution for inflation and output. Farmer, Waggoner, and Zha (2008) provide a sufficient condition to identify regions of the parameter space that yield indeterminacy in the conditionally linear system.\(^{14}\)

\(^{13}\)McCallum (2004) proves the equivalence between MSV solutions and determinate (i.e. unique and non-explosive) solutions from solving a system of linear expectations difference equations for purely forward looking models. Davig and Leeper (2007) show the equivalence between the MSV solution and determinate solution of the stacked system in regime-switching rational expectation models.

\(^{14}\)This condition requires that every eigenvalue of a conditionally linear system is greater than its corresponding transition probability. However, neither the Davig and Leeper (2007) or Farmer, Waggoner, and Zha (2008) conditions indicate whether the equilibrium in the full nonlinear model is unique. In the following estimation, we throw away parameters which do not satisfy necessary and sufficient conditions for determinacy in the stacked system.
4 Estimation

4.1 Econometric Methodology

The linear structure of the model solution conditional on the current regime makes the application of the approximate Kalman filter of Kim and Nelson (1999) feasible. Given laws of motion for the shock processes and the minimum state variable solutions of inflation, output, and nominal interest rate, we can write down the following regime-dependent state-space model representation

\[ x_t = \rho(s_t)x_{t-1} + \varepsilon_t, \quad x_t = [\hat{a}_t, \hat{u}_t, \hat{e}_t]' \]

\[ Z_t = A_z + B_z(s_t)x_t, \quad Z_t = [\ln Y_t, \pi_t, R_t]' \]

where \( x_t \) is a vector of state variables and \( Z_t \) is a vector of three observed variables consisting of per capita real GDP, inflation (log difference of GDP deflator), and 3 month Treasury bill rate. The period is from 1953:Q1 to 2006:Q4. The model has three observables and three structural shocks, so there is no need to introduce measurement errors. Constructing the likelihood for the MSNK model requires integrating out latent variables, including the history of regimes. Kim and Nelson (1999) note that collapsing some paths of regimes with very small probability is necessary to make the filtering algorithm operable. Otherwise, we have to consider \( 2^t \) different paths of regimes to evaluate the likelihood value at \( t \). Allow for 8 different paths of regimes, in the likelihood is

\[ p(Z_t|Z_{t-1}, \vartheta) = \sum_{s_t \in \{1, 2\}} p(Z_t|Z_{t-1}, \vartheta, s_t)p(s_t|Z_{t-1}, \vartheta) \]

where \( \vartheta \) is a vector of structural parameters and \( Z_{t-1} \) denotes observations up to time \( t-1 \). After constructing the likelihood, we can attempt to obtain maximum likelihood estimates of \( \theta \) via maximization. In this paper, we follow Bayesian approach and combine the likelihood with a prior distribution of \( \vartheta \). From Bayesian perspective, the posterior distribution of \( \vartheta \) which updates the prior distribution by the information from the likelihood is a key tool for inference. Incorporating prior information on \( \vartheta \) provides additional curvature for the posterior density and excludes implausible estimates of parameters which may overfit the sample data. The posterior distribution of \( \vartheta \) is also hard to characterize analytically, but repeated draws of \( \vartheta \) from a Markov chain, whose stationary distribution corresponds to the posterior distribution of \( \vartheta \), provide draws of \( \vartheta \) from the posterior distribution. We use

\[ \text{We increased the number of histories that are considered to 64 and found little difference in terms of the likelihood value.} \]

\[ \text{For further discussion of advantages of Bayesian approach in the estimation of DSGE models, see An and Schorfheide (2007).} \]
a random-walk Metropolis-Hastings algorithm to obtain the posterior draws.\textsuperscript{17} We initialize the Markov chain at the (local) mode of the posterior density by using a numerical optimization routine (\textsc{Csmi}n\textsc{wel} provided by Christopher Sims). The inverse of the negative hessian evaluated at the local model is used as the covariance matrix of the proposal density.

In constructing the likelihood, we use the filtered probability for each regime to integrate out the latent regimes. Since regimes are not directly observable to the econometrician, we are often interested in computing the estimates of the probability of different regimes conditional on all the observations available. This approach provides an indication of which history of regimes is most probable given the available observations. The smoothed probability of each regime can be obtained by applying the filtering step backwards as follows

\begin{equation}
p(s_t|Z^T, \vartheta) = \frac{p(s_t|Z^t, \vartheta)p(s_{t+1}|s_t)p(s_{t+1}|Z^T, \vartheta)}{\sum_{s_t \in \{1,2\}} p(s_t|Z^t, \vartheta)p(s_{t+1}|s_t)p(s_{t+1}|Z^T, \vartheta)}.
\end{equation}

Since $p(s^T|Z^T, \vartheta)$ and $p(s_t|Z^t, \vartheta)$ are obtained as byproducts of the likelihood evaluation, this is relatively easy to implement.

To identify the sources of the changes in inflation persistence, we need to compare different regime switching models. The marginal likelihood of each model provides a coherent framework to compare non-nested models. Conceptually, it is obtained by integrating the posterior kernel over the entire parameter space in each model $M_i$

\begin{equation}
p(Z^T|M_i) = \int p(Z^T|\vartheta, M_i)p(\vartheta|M_i)d\vartheta.
\end{equation}

The practical computation of this constant is done by the numerical approximation based on the posterior simulator as in Geweke (1999), for example.

### 4.2 Prior Distribution

Prior means of parameters are calibrated to match sample moments of observed variables, if possible. For example, the prior mean of the average technology growth rate ($\lambda$) is matched with the average growth rate of per capita real GDP. Similarly the prior mean of the steady state inflation ($\pi$) for the model with a constant inflation target is set to match the average inflation rate in the data. And the prior mean of the discount factor ($\beta$) is set to match the average nominal interest rate. The autocorrelation of technology growth ($\rho_a$) and the standard deviation of technology shock

\textsuperscript{17}We use a mixture of normal distribution and $t$ distribution as a proposal density. The relatively fat-tailed $t$ distribution makes it more likely for the proposal density to cover the tail area in the target density and, therefore, facilitates the convergence of MCMC chain.
\( (\sigma_a) \) are set to match the autocorrelation and the standard deviation of per capita real GDP. Prior distributions of other parameters are mostly set to be consistent with the existing literature on the estimation of New Keynesian models. For example, the prior distribution of the slope of the Phillips curve is from Lubik and Schorfheide (2004). For switching parameters, prior distributions are set to be roughly consistent with split sample (pre-1983, post-1983) estimation results.\(^{18}\) This induces the natural ordering of regime-dependent parameters and mitigates the potential risk of 'label switching' problem as noted in Hamilton, Waggoner, and Zha (2007). Table 1 provides detailed information on the prior distribution of parameters.

### 4.3 Posterior Distribution

We estimate three versions of the MSNK model. The first allows switching only in the monetary policy rule and the second allows switching only in the shock volatilities. The third model is the four-regime MSNK model that allows independent switching in monetary policy and the shock volatility regimes. Posterior estimates of parameters are based on 200,000 draws after throwing away the initial 800,000 (700,000) draws for two-regime models (four-regime model).\(^{19}\) Table 2 provides prior and posterior probability intervals for all the parameters for the model with a constant inflation target.\(^{20}\)

For the MSNK model with the switching monetary policy rule, the different regimes have different reaction coefficients to inflation. The mean of the reaction coefficient to inflation in the active regime is 2.13, which is much larger than .75 coefficient in the passive regime. The response to output is similar across regimes but the uncertainty for the coefficient in the active regime is much higher than that for the coefficient in the passive regime.

The timing of the different regimes are given by the posterior expected values of the smoothed probabilities in Figure 1. The timing of monetary regimes shows two major breaks during the sample period. The hawkish monetary regime was in place most of the time from 1954-1970, then again from 1982-2007.

For the four-regime MSNK model, the regime-switching parameters are broadly similar to the previous estimates with one important exception. The monetary policy

---

\(^{18}\)Additional consideration in the construction of the prior distribution is whether or not the model implied inflation persistence in each regime can capture moments in split sample data. We compute the model implied inflation persistence for 200 prior draws and check that there is a positive probability of matching actual moments in split sample data.

\(^{19}\)Plots of recursive means of parameters confirm that draws from the Markov chain stabilize after 600,000 draws.

\(^{20}\)Estimation of the MSNK models with an inflation target that follows a random walk had a substantially lower log marginal likelihood, indicating models with this feature fit the data poorly in comparison to the MSNK models with a constant inflation target.
reaction coefficient in the dovish regime is now substantially greater than unity, implying active policy. Figure 3 reports the posterior expected values of the smoothed probabilities and indicate that monetary policy was active in responding to inflation throughout the latter half of the 1950s and most of the 1960s. Beginning in the late 1960s, policy reverted to passively responding to inflation that lasted throughout the 1970s. Policy began again responding aggressively to inflation after about 1980, but returned to its passive stance around the recessions in 1990-91 and 2001. For the shock volatility regimes, they fluctuate between episodes of high and low volatility. The low-volatility regime is in place throughout the post-1984 period, or Great Moderation era. However, this regime is also in place for the 1960s.

The timing and nature of monetary regimes is roughly consistent with estimates from Clarida, Galí, and Gertler (2000) and Lubik and Schorfheide (2004), where both find substantial differences in the reaction of the nominal interest rate to inflation before and after approximately 1980. The key difference to these papers is our finding that policy was largely active in the latter half of the 1950s and throughout the 1960s.

Table 3 reports the log marginal likelihood values for each model and indicates that the data prefers the model with switching shock volatility over the model with switching monetary policy. However, the model that best fits the data is the four-regime MSNK model.

5 Changes in Inflation Persistence

An important issue for monetary policymakers is detecting changes in the persistence of inflation. In the MSNK model, monetary policy affects the contemporaneous effect of shocks - that is, the $A_\pi (i)$, $B_\pi (i)$, and $C_\pi (i)$ terms - but not the rate of decay (i.e. persistence) of a given shock. The persistence generated by a particular shock is driven by the serial correlation properties of the shock, which inflation inherits. From this standpoint, monetary policy has nothing to do with inflation persistence, since the only source of propagation in the model is exogenous. However, this does not imply that monetary policy is unable to affect the serial correlation properties of inflation. This section shows that the serial correlation of inflation is a weighted average of the autocorrelation parameters of the exogenous shocks. Changes in either the monetary or volatility regime induce changes in the serial correlation of inflation by shifting weights over these parameters.

21 There are, of course, alternative interpretations of monetary policy before 1980. Schorfheide (2005) reports regime switching in the inflation target, while reaction coefficients are left to be time invariant. Sims and Zha (2006) place emphasis on changes in the volatility of innovations to the monetary policy rule, instead of changes to the reaction coefficients.
Using several metrics, Pivetta and Reis (2007) argue the persistence of inflation in the U.S. is approximately unchanged. Focusing on the inflation gap, which is the difference between actual inflation and a potentially time-varying trend, Cogley, Primiceri, and Sargent (2007) find the persistence of the inflation gap has declined in recent decades.\footnote{In the context of the MSNK model, the trend, or steady state, rate of inflation is constant, so there is no distinction between persistence of the inflation gap and the inflation rate.} To more closely link inflation persistence in the MSNK model to this empirical literature, we compute the model implied inflation persistence as the population moment for the autocorrelation of inflation. For the four-regime MSNK model, this statistic conditioning on a given regime is

\[
AR(\pi_t|s_t = i, r_t = j, \forall t) = w_a(i, j)\rho_a + w_u(i, j)\rho_u + (1 - w_a(i, j) - w_u(i, j))\rho_e, \tag{39}
\]

where

\[
\begin{align*}
w_a(i, j) &= A\pi(i)^2W(i, j)\left(\frac{\sigma^2_a(j)}{1 - \rho_a^2}\right), \tag{40} \\
w_u(i, j) &= B\pi(i)^2W(i, j)\left(\frac{\sigma^2_u(j)}{1 - \rho_u^2}\right), \tag{41}
\end{align*}
\]

and

\[
W(i, j) = \left[A\pi(i)^2\frac{\sigma^2_a(j)}{1 - \rho_a^2} + B\pi(i)^2\frac{\sigma^2_u(j)}{1 - \rho_u^2} + C\pi(i)^2\frac{\sigma^2_e(j)}{1 - \rho_e^2}\right]^{-1},
\]

for \(i, j = 1, 2\). Equation (39) shows how inflation persistence is a weighted average of the persistence parameters of the underlying shocks. A change in monetary or shock volatility regime that shifts the relative weights from more persistent to less persistent shocks will decrease this measure of inflation persistence.\footnote{This result is consistent with the argument in Taylor (2000), which connects the conduct of monetary policy to inflation persistence.}

Table 4 reports the model implied inflation persistence statistic for each MSNK model. Focusing on the four-regime MSNK model (i.e. specification P3 in Table 4), the lowest degree of inflation persistence is in the regime with hawkish monetary policy and low shock volatility (i.e. \(s_t = 1\) and \(r_t = 1\)). According to Figure 3, this regime was in place throughout most of the 1960s and after 1980, excluding the periods around the 1990-91 and 2001 recessions. This timing of the active monetary and low shock volatility regimes in the four-regime MSNK model is similar to the timing of the regimes producing low inflation persistence in Evans and Wachtel (1993). They estimate a Markov-switching model for inflation and report estimates that imply a decline in inflation persistence from 1953-1967, then again from 1983 to the end of their sample in 1993. Thus, our estimates of the timing of regimes and implications for inflation persistence accord with their reduced-form empirical evidence.

As an additional rough check as to the plausibility of these statistics, Table 4 also reports autocorrelation coefficients for U.S. inflation, measured as the percentage

\[
\text{16}
\]
change in the GDP deflator, over two subsamples - 1953:Q1-1979:Q2 and 1984:Q1-2006:Q4. The measures use an AR(1) specification and are slightly different than Stock and Watson (2007), who report the largest autoregressive root across two similar subsamples. The estimates in Table 4 show that serial correlation in U.S. data is lower in the later sample, dropping from .89 to .57. The drop in persistence in the MSNK model from the switch in monetary policy is quantitatively relevant, but does not appear to generate a large enough drop to fully account for the decline in persistence. For example, Table 4 indicates that in the model with switching in only the monetary rule, persistence in the passive regime is around .95, then falls to around .82 in the active regime. Based on this evidence, a reasonable conclusion is that active monetary policy played an important role in reducing inflation persistence, but that shifts in shock volatility are also needed to match the observed moderation in inflation persistence.

6 Conclusion

This paper reports the results of Bayesian estimation of MSNK models with regime switching in monetary policy and shock volatility. Overall, U.S. data favors the model with independent switching in both the monetary policy and shock volatility regimes. We show that the population moment describing the serial correlation of inflation is a weighted average of the autocorrelation parameters of the exogenous shocks. The weights depend on the different monetary and shock volatility regimes. Consequently, changes in either of the regimes reshuffles the weights over these serial correlation parameters. Estimation indicates that a shift to the hawkish monetary regime reduces the weight on the more persistent shocks, so lowers the serial correlation of inflation. Similarly, a shift to the low volatility regime reduces the weight on the more persistent shocks and also contributes to reducing inflation persistence. We conclude for the MSNK model to generate a reduction in inflation persistence similar to the observed decline in U.S. data, both an active monetary policy and low shock volatility regime is necessary.

24 Using the percentage change in the GDP deflator, Stock and Watson (2007) show that the largest autoregressive roots differs little across the two periods. However, they conclude that inflation persistence has still fallen in the post-1984 sample, since the proportion of mass of the spectrum close to frequency zero is considerably larger in the first subsample.
References


Table 1: PRIOR DISTRIBUTION

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Notes: Para (1) and Para (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions; $s$ and $\nu$ for the Inverse Gamma distribution, where $p_{\text{IG}}(\sigma|\nu,s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$, $a$ and $b$ for the Uniform distribution from $a$ to $b$. P1 allows switching only in monetary policy coefficients while P2 allows switching coefficients only in variance parameters of shocks. P3 allows switching for both policy coefficients and variances.
Table 2: Posterior Distribution

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Table 3: Log Marginal Data Densities

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Table 4: Autocorrelation of Inflation

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<tr>
<td>AR(π</td>
<td>(s_t = 1, r_t = 1\forall t)</td>
<td>[.78,.86]</td>
<td>[.83,.88]</td>
</tr>
<tr>
<td>AR(π</td>
<td>(s_t = 1, r_t = 2\forall t)</td>
<td>-</td>
<td>[.78,.83]</td>
</tr>
<tr>
<td>AR(π</td>
<td>(s_t = 2, r_t = 1\forall t)</td>
<td>[.94,.96]</td>
<td>-</td>
</tr>
<tr>
<td>AR(π</td>
<td>(s_t = 2, r_t = 2\forall t)</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>


AR(π) 0.8856 (0.0656) 0.5734 (0.1156)

Notes: The posterior mean of the model implied inflation autocorrelation is reported with posterior standard deviation in [ ]. For data, numbers in ( ) are standard errors for sample autocorrelation coefficients. All the standard-errors are Newey-West (1987) adjusted with 4 lags.

Figure 1: Posterior Expected Values of the Active Monetary Policy Regime Probability
Figure 2: Posterior Expected Values of the Low Volatility Regime Probability
Figure 3: Posterior Expected Values of Monetary and Volatility Regimes

Active Policy, High Volatility

Active Policy, Low Volatility

Passive Policy, High Volatility

Passive Policy, Low Volatility