Financial market volatility and the business cycle: a stochastic volatility approach*

Gianluca Moretti (Banca d’Italia)

December 14, 2008

Abstract

This is an extended abstract of the paper. All the results shown are preliminary and incomplete. Do not quote without permission.

1 Introduction

Understanding the sources of financial markets volatility is of great importance. Starting with the seminal work by Engle (1982) an enormous number of papers has focused on developing time series models to forecast changes in the volatility based on lagged conditional and unconditional volatilities. However, only a few papers have directly addressed the problem of linking aggregate financial market volatility to the evolution of economic conditions. These works include Officer (1973), Schwert (1989) and Hamilton and Lin (1996). Recently, Engle and Rangel (2005) and Engle, Ghysels and Sohn (2008) made a step forward in this direction. The former proposed a modified GARCH, called the Spline-GARCH, where they model movements in the low frequency volatility by introducing a slow-moving component, represented by an exponential spline. Once this component is estimated they regress it on several macroeconomic variables and show evidence of a significant relation between aggregate financial market volatility and the state of the economy. More recently, Becker and Clements (2007) use the same

---

*This paper represents the authors’ personal opinions and does not reflect the view of the Bank of Italy.
approach for longer term forecasts of S&P 500. In this paper, we propose a stochastic volatility model that embodies the same ideas behind the spline GARCH, but that improves its applications in several ways. Firstly, while the Spline GARCH is based on a two-step procedure, our approach estimates contemporaneously both the low frequency volatility and its relation with the macroeconomy. Secondly, rather than using a cubic spline, which does not carry any economic meaning, we model the low frequency volatility using a factor which is extracted directly from a few economic variables and that represent a measure of the ‘business cycle’. This implies that our definition of low frequency volatility does not depend on the choice of the number of knots in the spline function. Finally, our model allows us to produce in real time both short and long term forecasts of the conditional variance.

2 The model and preliminary results

We start with a brief outline of the Spline-GARCH in order to introduce the main ideas behind modelling low frequency volatility. In the original specification of Engle and Rangel (2005), the Spline GARCH is defined as

$$r_t = \sqrt{\tau_t} g_t \varepsilon_t \sim N(0, 1)$$

$$g_t = (1 - \alpha - \beta) + \alpha \left( \frac{r_t^2}{\tau_{t-1}} \right) + \beta g_{t-1}$$

$$\tau_t = c \exp(w_0 t + \sum_{i=1}^{k} w_i ((t - t_i)_+)^2)$$

where \(\{t_i\}_{i=1}^{k}\) represents a partition of the time line into \(k\) equally spaced intervals. As suggested by Engle and Rangel and Becker and Clements (2007), one way to select an optimal number of knots \(k\) is to use an information criteria. The spline GARCH allows the level of unconditional volatility to be time varying and to coincide with the low frequency volatility \(\tau_t\). In Figure 1 we show the unconditional variance for the monthly log return of the S&P500 together along with the low frequency volatility \(\tau_t\). We chose a number of knots equal to 14 which corresponds to a two year interval between two consecutive knotpoints. The Spline function generates a cyclical pattern in the conditional volatility that follows the low frequency movements in the unconditional volatility. The smoothness of these cyclical pattern depends on the number on knotpoints.
Once the long term volatility $\tau_t$ is estimated, there are two ways to link macroeconomic variables to long-term volatility. The first, followed by Engle and Rangel, is to aggregate $\tau_t$ temporally to convert it to the same frequency of the macro data (yearly) and regress it on macroeconomic variables or on proxies of their volatility. The other approach, suggested by Adams and Clements, is to constrain the knotpoints $\{t_i\}_{i=1}^k$ to be a linear function of macroeconomic data. The Spline GARCH cannot easily be used to forecast future movements in the low frequency volatility. In fact, in order to make predictions of $\tau_{t+k}$, we need forecasts of the macroeconomic variables $k$ steps ahead. When the knotpoints are placed between long intervals of time (say every 1 or 2 years), forecasts of the macroeconomic variables for such horizons have a large degree of uncertainty.

An alternative approach to model volatility, proposed in this paper, is to use a stochastic volatility model. This class of models treats volatility as an unobserved variable whose logarithm is modeled as a linear autoregressive process. The simplest version of a SV model is given by

$$r_t = \exp \left( \frac{g_t}{2} \right) \varepsilon_t$$

$$g_t = \mu + \phi g_{t-1} + \eta_t \quad \varepsilon_t \sim N \left( 0, \sigma^2_\varepsilon \right)$$

where $g_t = \ln (\sigma^2_t)$. The model can easily be linearized by taking the logarithm of the square

$$\ln \left( r_t^2 \right) = E \left( \ln \left( \varepsilon_t^2 \right) \right) + g_t + \xi_t$$

$$g_t = \mu + \phi g_{t-1} + \eta_t$$

where $\xi_t = \ln (\varepsilon_t^2) - E \left( \ln \left( \varepsilon_t^2 \right) \right)$. A survey of the statistical properties of the SV model can be found in Harvey et al. (1994). What is important for our application is the fact that it can be easily casted into a state space form. This property allows us to combine it with a factor of the business cycle extracted by a few economic indicators. Specifically, if we define with $y_t$ the logarithm of a vector of economic variables, the model we consider is given by
\[ y_t = \beta f_t + u_t \]  \hspace{1cm} (1)
\[ \theta (L) f_t = v_{1,t} \]  \hspace{1cm} (2)
\[ \varphi (L) u_t = v_{2,t} \]  \hspace{1cm} (3)
\[ \ln \left( r_t^2 \right) = E \left( \ln \left( \varepsilon_t^2 \right) \right) + g_t + \xi_t \]  \hspace{1cm} (4)
\[ g_t = \mu + \phi g_{t-1} + \gamma f_t + \eta_t \]  \hspace{1cm} (5)
\[ v_{1,t} \sim N \left( 0, \sigma_{v_1}^2 \right); \quad v_{2,t} \sim N \left( 0, diag(\Sigma) \right) \]

where \( \beta \) is a factor loading vector, \( f_t \) is a scalar common factor and \( u_t \) a vector of specific factors whose components are pairwise uncorrelated. Both the common and specific factors are assumed to be stationary ARMA processes. Our approach extends a standard SV by introducing a measure of the business cycle \( f_t \) extracted from a dynamic factor model (eq.1-3) similar to those proposed by Stock and Watson (1989). This specification has several advantages. Firstly, differently from the Spline GARCH, it allows us to estimate contemporaneously a measure of the economic conditions and its impact on the volatility. In this respect, we can assess at each point of time the contribution of the economic variables to the overall aggregate volatility. Secondly, by construction, the factor \( f_t \) is smooth and slow moving, replicating somehow the cyclical pattern generated by the exponential spline in the spline GARCH model. However, differently from the latter, it has a direct economic meaning by being related to the business cycle. Finally, it can easily be used to forecast volatility in real time. The ‘Factor Augmented’ SV model can be easily estimated by Quasi maximum likelihood based on the Kalman Filter.

We perform two different exercises. In the first we compare the FASV with the standard SV model and the spline GARCH using monthly data for the S&P log returns and US coincident business cycle indicators. Specifically we use Industrial Production index (IP), rate on unemployment (UN) and the conference Board leading indicator (CI).\textsuperscript{1} The sample period is from January 1980 to August 2008 and to make the economic series stationary we take yearly growth rates. All the series are plotted in Figure 2. From a preliminary analysis we chose a AR(1) specification for the common and all

\textsuperscript{1}These variables were chosen by looking at the most influential economic indicators for U.S. financial markets. We are currently investigating the role of other indicators.
the specific factors. The parameter estimates for FASV model are displayed in Table 1.

<table>
<thead>
<tr>
<th>Factor loadings</th>
<th>$\beta_{(IP)}$</th>
<th>$\beta_{(UN)}$</th>
<th>$\beta_{(CI)}$</th>
<th>$\gamma$</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0121</td>
<td>-0.0641</td>
<td>0.086</td>
<td>-0.9085</td>
<td>-9.616</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>AR parameters</th>
<th>$\varphi_{(IP)}$</th>
<th>$\varphi_{(UN)}$</th>
<th>$\varphi_{(CI)}$</th>
<th>$\theta$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.9326</td>
<td>0.94028</td>
<td>0.9326</td>
<td>0.983</td>
<td>0.985</td>
</tr>
</tbody>
</table>

The factor loading of the conditional volatility is significant and negative. This is consistent with the findings of Hamilton and Lin, that during recessions stock market volatility increases. All the autoregressive parameters indicate strong persistence in the volatility and in the common and specific factors. In Figure 3 and we show the unconditional and the conditional variance $g_t$ respectively for the standard SV model and the FASV. The FASV follows quite accurately the cyclical pattern of low frequency volatility. Remarkably, the business cycle factor seems to explain a large part of the volatility increase during the recessions of 1983, 1990, 2001 and 2008. In particular, the increase in the volatility of the 2008, which seems to be partly caused by the deterioration of the economic activity in the US, is not captured by either the standard SV or the Spline GARCH.

In the second exercise we run a real time forecasting exercise to show how the FASV model can be used to predict current and future movements in the low frequency variance of financial markets.

to be added

References


Figure 1: Conditional and unconditional variance from the Spline GARCH model
Figure 2: Macroeconomic indicators of the business cycle
Figure 3: Standard deviation and SV model estimate of the standard deviation.
Figure 4: Conditional and unconditional variance from the FA-SV model