Investigating Price Level Dynamics with an Unobserved Components Model
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Introduction

Ongoing research into the persistence of inflation has been mixed, with recent evidence leaning toward a finding of a stationary inflation rate once structural breaks in the process have been accounted for and once a non-linear inflation process is allowed. For example, Beechley and Osterholm (2008) estimate a nonlinear ESTAR model for United States CPI and find the inflation process to be trend stationary. In addition, Kim, Kang and Morley (2007) demonstrate that there have been three separate regimes since 1950, each with differing degrees of persistence. Yet, a stationary inflation process does not preclude persistent shocks to inflation. Persistent, although ultimately temporary, inflation movements can occur if permanent shocks to the price level have long lasting effects.

At the same time there has been a growing body of research investigating the nature of “core” inflation, typically defined as that part of price level growth not cause by temporary price changes in price-volatile commodities such as energy and food. There have been a number of different approaches to estimating the core inflation rate including the “exclusion” approach, in which highly volatile components such as food and energy are removed from the price index (or from the inflation) calculation. Alternative statistical and parametric approaches have been proposed and estimated, but generally have not been applied by policy makers.\footnote{Silver, M., (2007) “Core Inflation: Measurement and Statistical Issues in Choosing Among Alternative Measures,” IMF Staff Papers, Vol. 54. No.1, 163-190}

What is apparent, however, is that a consensus has emerged, and, for reasons of maintaining credibility, there is a natural starting point for many countries. First is the use of the CPI as the basis of the core inflation measure as the most visible and credible measure to anchor inflation expectations. Second is the widespread adoption of exclusion-based CPIs.
In this paper, we bring together these two strands of the literature. We use an unobserved components model to represent both the core price index (as defined by the exclusion approach) and the more volatile components of the price index. An unobserved components model (UC) includes specification of both the long run movement and the short run dynamics in the three series. This is an important characteristic of the various components, because “core” inflation has often been thought of as the long run movement in all prices including food and energy. Typically prices for goods in the food and energy sectors are omitted because there short run volatility is thought to overwhelm the contribution of the food and energy components to measuring long run increases in the price level. By estimating a simultaneous unobserved components model in all three, we can estimate the degree to which these important components of the overall price level have similar or different properties than the remaining core prices. Also, because we estimate a simultaneous model in the series, the UC model permits the estimation of both the long run and short run correlations of innovations in the series.

We find that core inflation does not appear to be so different from the other components, especially food. Core and food components have similar order of magnitude volatility, and standard deviations of shocks are also of the same magnitude. There are some differences in the dynamics, but both series have highly persistent transitory components. The energy component is more volatile, and its dynamics are negatively autocorrelated and mean revert more quickly than core and food components.

Test for Stationarity in Inflation

The first step in modeling the dynamics of the components of inflation is to determine whether the inflation process is stationary. This is required for the specification of the
unobserved components model, which models the permanent component of a series as a random walk. Thus the input series must be difference stationary. If inflation is found to be non-stationary, we would apply the UC model to the inflation rate (here the log difference of the price indexes). On the other hand, if inflation is found to be stationary, so that the price level is non-stationary, then it is appropriate to specify the UC model in terms of the log of the price level.

We investigate the time period from January 1983 – December 2007. Based on the KPSS stationarity test (Kwiatkowski et al. (1992)), we reject trend-stationarity for all three price level series. However, for the first differences we cannot reject trend-stationarity for any of the three series at the 5% level. Furthermore, if we allow for structural breaks in these series, as discussed in Levin and Piger (2002), then the persistence of inflation may be reduced further, giving additional support to the finding of stationarity. Our results indicate that all three price level series are difference-trend-stationary over this time period. These results are consistent with Leybourne, Kim, Smith and Newbold (2003), who find that U.S. inflation switched from being nonstationary to being stationary in 1982.

A Multivariate Unobserved Components Model

We model the movements of the levels key price subseries through time. The US model has the following form:

\[ p_{it} = \tau_{it} + c_{it} \]

where, \( i = c \) (Excluding Food and Energy), \( f \) (Food), and \( e \) (Energy).

The trend components, \( \tau_{it} \), are stochastic (random walks) and the formulation permits testing for and including trend breaks:

\[ \tau_{it} = \mu_i + \tau_{it-1} + \eta_{it} \]
The transitory components are also stochastic:

\[ c_{it} = \phi_{i1} c_{it-1} + \phi_{i2} c_{it-2} + \epsilon_{it} \]

Note that the model not only permits estimation of the permanent and transitory portion of the two parts of inflation but also permits correlation between the innovations in the two components. Correlation between the permanent innovations in the two parts implies that permanent innovations in the volatile parts of the CPI (like food and energy) may contain information about permanent innovations in the “core” CPI, and call into question the consensus on the exclusion approach.

**Model Estimation**

We apply the model from described above to monthly US CPI data from January of 1983 through December of 2007. The model includes three series (all seasonally adjusted): 1) All Items Less Food and Energy (Core CPI), 2) Food, and 3) Energy. Seasonal adjustment factors are recalculated each January for the previous year, so our data end in December, 2007.² Our data begin in 1983 to avoid the definitional change regarding shelter in the CPI. Before 1983, mortgage interest rates which were included in the CPI as a part of homeowner’s costs, whereas since 1983 a rental equivalence measure has been used (Smith, 2005).

The final model includes three structural breaks in the drift term for each series. We determined the break dates by estimating a univariate correlated UC model for each series. Based on testing for a single drift break in each of these univariate models, we found a significant drift break for all three series. We then estimated two different models, one which

² The data were obtained from the Federal Reserve Bank of St. Louis at http://research.stlouisfed.org/fred2.
allowed for only the single break for each series, and another allowing all three series to break jointly at all the dates determined by the univariate structural break tests. Based on a likelihood ratio test, we were able to reject the restricted model in favor of the model allowing all three series to break together with a p-value of 0.03. The break dates are June of 1991 (based on univariate results for Food CPI), April 1993 (based on univariate results for Core CPI), and February 2002 (based on univariate results for Energy CPI). The parameter estimates are presented in Table 1. The estimated components are presented in Figure 1.

These results show that while the estimated models share some similarities, the models for all three price series have some important differences. First, the size of the drift term for Core CPI has been monotonically decreasing over the sample, from 0.37 (4.44% annual inflation) down to 0.18 (2.16% annual inflation). In contrast the energy CPI had a monotonically increasing drift over the sample, switching from negative to positive in 1993M04. The drift in Food CPI decreased, with a clear structural break in 1991M06 from 0.36 (4.32% annually) to 0.07 (0.84% annually), but then increased again to 0.23 in 1993, staying near there at 0.21 (2.52% annually) from 2001M02.

In terms of the transitory components, the energy series has negative AR parameters indicating its dynamics follow a damped sinusoidal pattern, possibly reflecting overshooting and undershooting. The core series has large positive AR parameters, summing to .94. Although the second AR parameter for the food series is negative, the AR parameter for Food is positive and their sum is positive and large, about .95. Thus both the core and food series demonstrate a high degree of persistence in their transitory components. Finally, the standard deviations for both the permanent and transitory shocks to the energy series are much larger than the standard deviations for either the core series or the food series, which are of about the same magnitude. This result
suggests that the volatility in the food series may no longer be sufficiently large to justify its exclusion from the “core” CPI.

Important information is also contained in the contemporaneous correlation of the shocks. First, the correlations between the permanent and transitory shocks within an individual series provide insight into the pattern of arrival of the shocks to the series. For both the core series and the energy series, permanent and transitory within-series shocks are high negatively correlated. In unobserved component models, this correlation is often interpreted to imply that the full effects of permanent shocks are partially mitigated in the short run. If the adjustment to a permanent shock is somewhat gradual, the actual value in the period of incidence will be below the permanent value, giving rise to a temporary negative shock in the opposite direction. In contrast, there is a relatively small positive correlation between permanent and transitory shocks to the food series. In this instance, the temporary shock is reinforcing the permanent shock, occasionally causing a form of overshooting in the period in which the permanent shock occurs.

Cross series correlations also yield some interesting information about the relationship among the shocks to the three series. First, permanent shocks to the core series are correlated with permanent shocks to both the food series and the energy series, despite the fact that permanent shocks to food and energy are not themselves correlated. We interpret this as suggesting that there may be underlying structural shocks to the two series that have permanent components that can affect the permanent movement in the core series. In other words, exclusion of the overall food and energy series from the calculation of “core” inflation may exclude important information about “true” long run changes in the overall price level. In addition, permanent shocks to both the energy and food series are negatively correlated with the temporary core shock, mimicking the relationship between the permanent core shock and the
temporary core shock. This also suggests that the dynamic behavior of the core series may be
influenced by innovations in food and energy prices. In the next section, we investigate a
structural interpretation of these results.

A Structural Interpretation of the Results

The empirical results presented above provide information about the trend and cyclical
movements in the three price series and the relationship among the estimated innovations.
However, these innovations can be considered to be the set of reduced form shocks derived from
the estimation of the simultaneous unobserved components model. To gather a deeper
understanding of the information contained in these shocks, it is useful to construct and simulate
a structural model for the innovations. This exercise will allow us to further interpret the
correlations among the reduced form shocks and to derive a sense of magnitude of those
relationships.

Our equations of interest can be written as nine equations, 3 identities and six estimated
equations. In principal we can analyze the response of the components of core, food, and energy
price indices to shocks to each of these components via impulse response functions.

\[
\begin{align*}
P_{ct} &= \tau_{ct} + c_{ct} \\
\tau_{ct} &= \mu_c + \tau_{ct-1} + \eta_{ct} \\
c_{ct} &= \varphi_{c1} c_{ct-1} + \varphi_{c2} c_{ct-2} + \epsilon_{ct} \\
P_{ft} &= \tau_{ft} + c_{ft} \\
\tau_{ft} &= \mu_f + \tau_{ft-1} + \eta_{ft} \\
c_{ft} &= \varphi_{f1} c_{ft-1} + \varphi_{f2} c_{ft-2} + \epsilon_{ft} \\
P_{et} &= \tau_{et} + c_{et} \\
\tau_{et} &= \mu_e + \tau_{et-1} + \eta_{et} \\
c_{et} &= \varphi_{e1} c_{et-1} + \varphi_{e2} c_{et-2} + \epsilon_{et}
\end{align*}
\]
To pursue the IRF analysis it is necessary to identify the structural shocks. We discuss our identification restrictions in what follows. Note that we need 15 restrictions to just-identify our system of six estimated equations.

First, we assume that permanent shocks to a category (e.g., food) exert a contemporaneous impact on transitory shocks to that same category, but that transitory shocks to a category do not exert a contemporaneous impact on permanent shocks to that same category. That is, within category, causality is unidirectional from permanent shocks to transitory shocks. These restrictions follow from the idea motivating the permanent/transitory decomposition in our UC model. Permanent shocks define the long run movements of a series, and the transitory shocks are deviations from this trend. In such a model permanent shocks, changes in trend, may lead to changes in the long run of the series, and may also lead to transitory shocks at the moment of impact. But transitory shocks do not impact permanent shocks, because to do so would give transitory shocks a role in causing permanent movements in trend. This assumption gives us three zero restrictions.

Along the same lines, we further assume that transitory shocks in any category $i$ (e.g., food) do not cause permanent shocks in any other category $j$. The justification is analogous to the first assumption, and follows from the idea that transitory shocks, to be transitory, cannot impact the long run trends of the variables in the model. This adds six more zero restrictions.

Finally, we use information from the estimated correlations among the shocks. The estimated correlation between transitory shocks to food and transitory shocks to core is quite

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3 The idea of using low and statistically insignificant estimated correlations to zero out causation in VAR analysis has been used previously by a number of authors including Granger and Swanson (JASA, 1997), Hoover (Econometric Theory 2005), and Bessler and Haigh (Journal of Business 2004). The later two authors explicitly set there analysis in a Directed Acyclic Graph framework in which finding insignificant estimated correlations is just the first step in an identification process.
low, at -0.07, so we impose the restriction that is no causality between these two terms, yielding two more zero restrictions. Similarly, the estimated correlation between permanent shocks to food and transitory shocks to energy is low at -0.11, so we assume there is no causality between these two terms as well. (This adds one restriction, because we had already assumed causation cannot run from transitory energy shocks to permanent food shocks.) Finally, the estimated correlation between permanent food and permanent energy is also quite low, -0.11, so we assume no causality between these two terms. Our last restriction comes from imposing that causation flows from transitory energy shocks to transitory food shocks, but not the other way. This is based upon the assumption that changes in energy prices could exert a contemporaneous impact on food prices but not vice versa.

The above assumptions provide the following restrictions:

<table>
<thead>
<tr>
<th>Reduced Form Disturbances</th>
<th>Structural Disturbances</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\nu_c$</td>
</tr>
<tr>
<td>Perm-Core</td>
<td>$\xi_c$</td>
</tr>
<tr>
<td>Trans-Core</td>
<td>$\nu_f$</td>
</tr>
<tr>
<td>Per-Food</td>
<td>$\xi_f$</td>
</tr>
<tr>
<td>Perm-Energy</td>
<td>$\nu_e$</td>
</tr>
<tr>
<td>Trans-Energy</td>
<td>$\xi_e$</td>
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</table>

Simulations

The model was simulated for two standard deviation permanent positive (structural) shocks to the core series, the food series and the energy series. A subset of those simulations is
presented below. Recall that a shock to the permanent component of a series is a permanent shock to the level of the series, but a temporary shock to the inflation rate. In fact, if the series only had the I(1) permanent component, then a shock to the permanent component would raise the price level at the time of the shock, causing an instantaneous jump in the inflation rate. Then one period later, the price level would revert to growing at its long run rate, and the inflation rate would return to its long run level. Our impulse response functions for permanent shocks will exhibit a more gradual response to a permanent shock, reflecting the impact of the transitory component responding to the permanent shock.

Figures 3 through 5 present the impact on the core series of each of those shocks. Figure 3 shows how the core inflation responds to a permanent core shock. The baseline core inflation rate is 2.14 percent (annual rate). A two standard deviation shock to the core CPI – a shock of 1.94% annualized -- causes a small but persistent impact on the core inflation rate. The permanent part of the core inflation series jumps by the size of the shock on impact, but the temporary part of the core inflation series is impacted in the opposite direction, an effect discussed above, so that the impact on the aggregated core inflation series (permanent plus temporary components) is modest. On the other hand, while the core shock increases the core inflation rate by only three-tenths of a percent, it takes nearly 36 months for the effect to disappear entirely. In effect, the permanent structural shock by also impacting the transitory component causes a slow adjustment process to the shock. Eventually the core price level will be permanently higher and the core inflation rate will revert to 2.14%, but the adjustment process is gradual.

Figure 4 shows the response in the core inflation rate from a two standard deviation permanent food shock, a shock of 1.69% annualized. Because of the negative correlation
between the transitory core shock and the permanent food shock, the first reaction in the core inflation rate is negative, but then the core inflation rate increases by about two tenths of a percent over the baseline rate. The impact then begins to dissipate, but takes nearly 24 months to virtually approach zero. Again, the slow dynamic adjustment is due to the high persistence in the transitory component of core inflation.

Finally, Figure 5 presents the response in core inflation to a positive energy shock. The variance of the underlying energy series is higher than the other series, and the standard deviation of the shock to the permanent component of energy is 29.6% annualized. After the initial negative response, the pattern in core inflation is remarkably like its response to a core shock. Core inflation is increased by about three tenths of a percent and takes about 36 months to dissipate.

In contrast to permanent shocks, the impact on core inflation of transitory shocks to the various components of inflation is in general much weaker. Energy is the exception. In fact, our identification assumptions preclude an impact of structural transitory food shocks on core inflation. The impact of transitory core shocks on core inflation is very weak, as the standard deviation of transitory core shocks is almost nill, .006% annualized.

The exception is transitory energy shocks, with a standard deviation of 5.16% annualized. These shocks have a notable impact on core inflation, as illustrated in Figure 7. At impact this shock raises core inflation to 2.22%. After that the core inflation rate falls to 2.13%, just slightly below its baseline value of 2.14%, and gradually converges back to the baseline over time.

One feature of the inflation series that this work illustrates is the importance of the transitory component of inflation, especially core inflation, to the dynamics of the process. While the transitory component of core inflation appears somewhat unimportant in the sense that
structural shocks to transitory core components are very small, the persistent dynamics of the transitory component are the crucial feature in explaining the slow and gradual adjustment of core inflation to shocks, even to shocks to core inflation itself. Further work examine the nature of the transitory – permanent relationship among the components of inflation, as well as the robustness of our results to alternative structural identification.

CONCLUSIONS AND FURTHER WORK?

To be added.
REFERENCES


Table 1: Maximum Likelihood Estimates of the UC Model (LLV: -319.410)

Table 1a: Standard Deviations, Drift Terms, and AR Parameter Estimates

<table>
<thead>
<tr>
<th>Series</th>
<th>Standard Dev. Permanent Shocks ($\sigma_\eta$)</th>
<th>Standard Dev. Transitory Shocks ($\sigma_\epsilon$)</th>
<th>Drift 1983.1-1991M06 ($\mu_1$)</th>
<th>Drift 1991.7-1993M04 ($\mu_2$)</th>
<th>Drift 1993M05-2002M02 ($\mu_3$)</th>
<th>Drift 2002M03-2007M12 ($\mu_4$)</th>
<th>AR1 Parameter ($\phi_1$)</th>
<th>AR2 Parameter ($\phi_2$)</th>
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<tbody>
<tr>
<td>Core</td>
<td>0.155</td>
<td>0.198</td>
<td>0.366</td>
<td>0.281</td>
<td>0.215</td>
<td>0.178</td>
<td>0.922</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.009)</td>
<td>(0.012)</td>
<td>(0.018)</td>
<td>(0.010)</td>
<td>(0.010)</td>
<td>(0.020)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>Food</td>
<td>0.178</td>
<td>0.128</td>
<td>0.359</td>
<td>0.067</td>
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<td>0.210</td>
<td>1.197</td>
<td>-0.244</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.013)</td>
<td>(0.022)</td>
<td>(0.045)</td>
<td>(0.016)</td>
<td>(0.021)</td>
<td>(0.034)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>Energy</td>
<td>2.485</td>
<td>0.992</td>
<td>-0.089</td>
<td>-0.028</td>
<td>0.297</td>
<td>0.862</td>
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<td>-0.353</td>
</tr>
<tr>
<td></td>
<td>(0.091)</td>
<td>(0.056)</td>
<td>(0.056)</td>
<td>(0.058)</td>
<td>(0.062)</td>
<td>(0.067)</td>
<td>(0.051)</td>
<td>(0.024)</td>
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</table>

Table 1b: Correlation Parameter Estimates

<table>
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<tr>
<th>Shock</th>
<th>Perm – Core</th>
<th>Perm – Food</th>
<th>Perm – Energy</th>
<th>Trans – Core</th>
<th>Trans – Food</th>
<th>Trans – Energy</th>
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<tbody>
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<tr>
<td>Perm – Food</td>
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<tr>
<td>Perm – Energy</td>
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<td>1</td>
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<tr>
<td>Trans – Core</td>
<td>-0.913</td>
<td>-0.618</td>
<td>-0.629</td>
<td>1</td>
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<tr>
<td>Trans – Food</td>
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<td>0.249</td>
<td>0.305</td>
<td>-0.069</td>
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<td>Trans – Energy</td>
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<td>-0.901</td>
<td>0.568</td>
<td>-0.249</td>
<td>1</td>
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</tbody>
</table>
Figure 1. Permanent Components of Core, Food, and Energy CPI
Figure 2. Transitory Component of Core, Food, and Energy CPI
Figure 3

Core Inflation Response to Permanent Core Shock

Baseline  With Core Shock
Figure 4

Core Inflation Response to Permanent Food Shock

Baseline

With Food Shock
Figure 5

Core Inflation Response to Permanent Energy Shock

Baseline

With Energy Shock
Figure 6

Core Inflation Response to Transitory Energy Shock