Are There Common Upswings and Downswings between NAFTA Countries?

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Preliminary and Incomplete Draft

Abstract

Following the characteristics of business cycle fluctuations, comovement among aggregate variables, and busines cycle asymmetry, this paper studies the simultaneous estimation of comovement and asymmetry within an international business cycles framework. The objective of this paper is to measure the relative importance of common transitory and growth components for the outputs of NAFTA countries. A multivariate state space model with Markov regime switching is applied with a twofold purpose: a decomposition of macroeconomic variables into permanent and transitory components, as well as an application of dynamic factor analysis to identify the business cycle fluctuations of common components.

JEL Classification: C32, E32
1. Introduction

Two major characteristics of business cycle fluctuations are the comovement among aggregate variables and business cycle asymmetry, which implies the division of fluctuations into separate regimes of expansion and recession. Simultaneous estimation of comovement and asymmetry was formerly examined for a single country case, yet it has not been applied to international fluctuations. The objective of this paper is to measure whether the NAFTA countries business cycle fluctuations exhibit common recession and expansion phases, as well as to determine whether those common phases arise from the common comovement in each country’s trend or cyclical components.

Highlighting the asymmetric nature of business cycles Mitchell (1927) noted that “business contractions seem to be a briefer and more violent process than business expansion.” Using a Markov process framework Neftci (1984) presented the first statistical tests of asymmetric behavior of time series, which triggered the development of nonlinear univariate models to address the problem. Since then, several different approaches have been developed to model business cycle asymmetry. Hamilton (1989) modeled asymmetry in which the growth rate of real output is governed by an unobserved Markov switching state variable. In his case, the economy faced one of two states: positive growth, expansion, or negative growth, recession. Later aiming to capture peak reversion in levels, Beaudry and Koop (1993), Sichel (1994), and Kim and Nelson (1999a) modeled asymmetry in the persistence of shocks. Kim and Nelson (1999) proposed asymmetry of the transitory component of real output.

Another no less important feature of business cycle fluctuations is the comovement among macroeconomic variables. Comovement of economic time series has played an important role in analyzing and forecasting business cycle fluctuations. In particular, Composite Indexes of Coincident and Leading Economic Indicators, initially developed by Mitchell and Burns (1938), are used to this day to summarize the current state of macroeconomic activity. In later work, Burns and Mitchell (1946) emphasized comovement between aggregate variables as a significant empirical fact on business cycle fluctuations:
“...a cycle consists of expansions occurring at about the same time in many economic activities, followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle.”

Addressing the problem of constructing an index of coincident indicators, Stock and Watson (1989, 1991, 1993) applied a linear dynamic factor model to capture the comovement of major macroeconomic variables. In the dynamic factor model, comovement among the variables is expressed through a single unobserved component common to all the series and is estimated through Kalman filter.

Though comovement and asymmetry had been recognized to be two major features of the business cycle, they had not been analyzed simultaneously until Diebold and Rudebusch (1996). Their work was followed by M.-J. Kim and Yoo (1995), Chauvet (1998), and Kim and Nelson (1998) in which the dynamic factor models of Stock and Watson (1989, 1991, 1993) are utilized along with the asymmetry in growth estimated by Hamilton (1989), so that the series common growth component is subject to two-state Markov regime switching. Similar to Hamilton’s (1989) univariate model, those multivariate two-phase models are unable to capture the peak-reversion proposed by Sichel (1994). In this regard, Kim and Murray (2002) present a more general regime-switching dynamic factor model, which allows for recessions arising from both the permanent growth component and the transitory cyclical component.

2. Model Specification

The unobserved components model is characterized by the following equations:

\[ Y_{it} = \gamma_i T_{it}^w + \alpha_i c_i^w + \tau_{it} + \epsilon_{it} \]  \hspace{1cm} (1)

\[ \phi^w(L) \Delta T_{it}^w = \mu_{it}^w + \nu_t, \quad \nu_t \sim iidN(0, \sigma_{\nu_t}^2) \]  \hspace{1cm} (2)

\[ \phi^c(L) c_i^w = \lambda_{it}^w + u_t, \quad u_t \sim iidN(0, \sigma_{u_t}^2) \]  \hspace{1cm} (3)

\[ \tau_{it} = \mu_t + \tau_{it-1} + \omega_{it}, \quad \omega_{it} \sim iidN(0, \sigma_{\omega_t}^2) \]  \hspace{1cm} (4)

\[ \psi_i(L) c_i^w = \epsilon_{it}, \quad \epsilon_{it} \sim iidN(0, \sigma_{\epsilon_i}^2) \]  \hspace{1cm} (5)
where \( Y_{it} \) is 100 times the log of the individual time series, \( i = 1, \ldots, N \), and \( N \) is the number of time series, which in this model is equal to the number of countries. \( Y_{it} \) is decomposed into \( T_{it}^w \), the common stochastic trend, \( \tau_{it} \), the idiosyncratic stochastic trend, \( c_{it}^w \), the common cyclical component, and \( c_{it}^u \), the idiosyncratic cyclical component.

Throughout the paper, \( T_{it}^w \) and \( c_{it}^w \) will be referred to as the common permanent component and the common transitory component respectively or as the common trend and cycle. Both the common cycle and the idiosyncratic cycle are assumed to follow an autoregressive process. \( \gamma_i \) and \( \alpha_i \) are factor loadings for the common trend and the common cycle respectively. \( \gamma_i \) indicates the extent to which each series is affected by the common permanent component, while \( \alpha_i \) indicates the extent to which each series is affected by the common transitory component.

\( S_{1t} \) and \( S_{2t} \) are Markov switching state variables that switch between 0 and 1 and have \( q_1, q_2 \) and \( p_1, p_2 \) transition probabilities such that:

\[
S_{1t} = \{0,1\}, \quad \Pr[S_{1t} = 0|S_{1t-1} = 0] = q_1, \quad \Pr[S_{1t} = 1|S_{1t-1} = 1] = p_1
\]

\[
S_{2t} = \{0,1\}, \quad \Pr[S_{2t} = 0|S_{2t-1} = 0] = q_2, \quad \Pr[S_{2t} = 1|S_{2t-1} = 1] = p_2
\]

The common permanent component is subject to Hamilton (1989) regime switching.

\[
\sigma_{\nu,S_{1t}}^2 = \sigma_{\nu,0}^2 (1 - S_{1t}) + \sigma_{\nu,1}^2 S_{1t}
\]

\[
\mu_{S_{1t}}^w = \mu_{0}^w (1 - S_{1t}) + \mu_{1}^w S_{1t}
\]

The common transitory component is subject to Kim and Nelson (1999a) regime switching.

\[
\sigma_{\nu,S_{2t}}^2 = \sigma_{\nu,0}^2 (1 - S_{2t}) + \sigma_{\nu,1}^2 S_{2t}
\]

\[
\lambda_{S_{2t}} = \lambda_{S_{2t}}
\]

\( \lambda_{S_{2t}} \) is an asymmetric shock to the common cycle that is equal to 0 during the expansionary periods when \( S_{2t} = 0 \). In this case, the shared economic fluctuations are near their potential output level, which is the common trend. The common permanent component is a “ceiling level” for the common transitory component. \( \lambda_{S_{2t}} \) is expected to
be negative during the recessionary periods, when \( S_{2t} = 1 \) and \( \lambda_{S_{2t}} = \lambda \), and when the common cycle is hit with a transitory shock. In Friedman’s (1964) terminology, \( \lambda \) is the size of the pluck for the common transitory component so that the transitory component is plucked down during recessionary periods.

The variances of the symmetric shock to the common transitory and permanent components are allowed to be different during recessions and expansions. Accordingly, the disturbances \( u_t \) and \( \nu_t \) are heteroskedastic and follow a Markov-switching process. The variance of \( u_t \) is normalized to 1 when \( S_{2t} = 0 \), and equal to \( \sigma^2_{u, S_{2t}} = \sigma^2_{u,1} = \sigma^2_u \) when \( S_{2t} = 1 \). The variance of the common permanent component is normalized to 1 when \( S_{1t} = 0 \), and equal to \( \sigma^2_{\nu, S_{1t}} = \sigma^2_{\nu,1} = \sigma^2_\nu \) when \( S_{1t} = 1 \). Normalizing the variances of the permanent and transitory components to 1 in one of the regimes is necessary for model identification purposes (Kim and Nelson, 1999b).

The model is estimated in differences and is written in deviations from the mean. The estimation of the model in first differences is necessary in order to ensure stationarity and to allow for the use of the unconditional mean of the unobserved state variable as a starting value for the Kalman filter.

\[
\Delta y_{it} = \gamma_i \Delta \tau_{it}^w + \alpha_i \Delta c_{it}^w + z_{it} \tag{6}
\]

\( \Delta y_{it}, \Delta \tau_{it}^w, z_{it} \) are defined as \( \Delta y_{it} = \Delta y_{it} - \Delta \bar{Y}_i \), \( \Delta \tau_{it}^w = \Delta T_{it}^w - \delta \), \( z_{it} = \Delta c_{it} + \Delta \tau_{it}^w \).

The idiosyncratic component \( z_{it} \) follows an autoregressive process such that:

\[
\psi_i(L) z_{it} = \eta_{it}, \eta_{it} \sim iidN(0, \sigma^2_{\eta, i}) \tag{8}
\]
To estimate the parameters, as well as unobserved components of the model the state space representation of the model is used to apply Kalman filtering and Kim’s (1994) approximate maximum likelihood estimation algorithm. The measurement equation of multivariate unobserved component model is: \( \Delta y_t = H \beta_t \). The transition equation of the model is: \( \beta_t = \mu + F \beta_{t-1} + V_t \), and \( E(V_t' V_t) = Q \). Where:

\[
\Delta y_t = \begin{bmatrix}
\Delta y_{1t} \\
\Delta y_{2t} \\
\Delta y_{3t}
\end{bmatrix}, \quad H = \begin{bmatrix}
\gamma_1 & a_1 & -a_1 & 1 & 0 & 0 \\
\gamma_2 & a_2 & -a_2 & 0 & 1 & 0 \\
\gamma_3 & a_3 & -a_3 & 0 & 0 & 1
\end{bmatrix},
\]

\[
\beta_t = \begin{bmatrix}
\Delta \tau^w_t \\
\zeta_t \\
\xi_t
\end{bmatrix}, \quad \mu = \begin{bmatrix}
\mu^\tau S_t \\
\lambda S_t \\
0
\end{bmatrix}, \quad F = \begin{bmatrix}
\phi^\tau & 0 & 0 & 0 & 0 & 0 \\
0 & \phi^c & 0 & 0 & 0 & 0 \\
0 & 0 & \phi^c & 0 & 0 & 0 \\
0 & 0 & 0 & \phi_1 & 0 & 0 \\
0 & 0 & 0 & 0 & \phi_2 & 0 \\
0 & 0 & 0 & 0 & 0 & \phi_3
\end{bmatrix},
\]

\[
V_t = \begin{bmatrix}
\xi_t \\
\varepsilon_t \\
0
\end{bmatrix}, \quad Q = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma^2_{\varepsilon_t} & 0 \\
0 & 0 & 0 & 0 & \sigma^2_{\varepsilon_t}
\end{bmatrix},
\]

where \( \zeta_{it} = \Delta \tau_{it} + \Delta c_{it} \) with \( \nu_{it} \sim iidN(0, \sigma^2_{\nu_t}) \) residuals. \( \xi_{it} \sim iidN(0,1) \) is the residual term of \( \Delta \tau^w_t \). \( \Delta \tau^w_t, c^w_t, \zeta_t \) are assumed to follow first order autoregressive process with \( \phi^\tau, \phi^c, \phi \) autoregressive parameters.\(^1\)

The Kalman filter equations together with the Kim (1994) method to approximate the likelihood function for state space models with Markov switching can be written as follows:

\(^1\) Construction of \( T_t \) from \( \Delta \tau_t \) is described in Kim and Murray (2002) Appendix (3).
\[ \beta_{ij,t-1}^{(i,j)} = \mu_{S_i} + F\beta_{t-1|i,t-1} \]

\[ P_{ij,t-1}^{(i,j)} = FP_{t-1|i,t-1}F' + Q \]

\[ \eta_{ij,t-1}^{(i,j)} = y_{t} - H\beta_{ij,t-1}^{(i,j)} \]

\[ f_{ij,t}^{(i,j)} = HP_{ij,t-1}^{(i,j)}H' \]

\[ \beta_{ij,t}^{(i,j)} = \beta_{ij,t-1}^{(i,j)} + P_{ij,t-1}^{(i,j)}H[ f_{ij,t-1}^{(i,j)} ]^{-1} \eta_{ij,t-1}^{(i,j)} \]

\[ P_{ij,t}^{(i,j)} = (I - P_{ij,t-1}^{(i,j)}H[ f_{ij,t-1}^{(i,j)} ]^{-1})HP_{ij,t-1}^{(i,j)} \]

were \( \beta_{ij,t-1}^{(i,j)} \) is an inference on \( \beta \) based on information up to time \((t-1)\) and \( \beta_{ij,t}^{(i,j)} \) is an inference on \( \beta \) based on information up to time \( t \), with corresponding \( P_{ij,t-1}^{(i,j)} \) and \( P_{ij,t}^{(i,j)} \) MSE matrices, conditional on \( S_t = j \) and \( S_{t-1} = i \). \( \eta_{ij,t-1}^{(i,j)} \) is the conditional error of \( y_{t} \) based on information \((t-1)\) and \( f_{ij,t-1}^{(i,j)} \) is the conditional variance of \( \eta_{ij,t-1}^{(i,j)} \). The model is estimated in GAUSS.

3. **Empirical Results**

**Data**

The paper examines quarterly real GDP of the U.S., Canada and Mexico, covering the period from first quarter of 1970 to the first quarter of 2006, with 2000 as a base year. The time series for Canada and Mexico are taken from OECD Quarterly National Accounts database and for the U.S. from the Bureau of Economic Analysis database. All the time series are in logs, multiplied by 100, so that the cycle can be interpreted as a percent deviation from the trend. DF-GLS unit root testing with modified Akaike Information Criteria (MAIC) lag selection is conducted for each of the series. It fails to reject the null of unit root for the series.

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2 For a more detailed explanation on estimation of the regime switching dynamic factor model refer to Kim and Murray (2002).
Discussion of Results

Maximum likelihood estimates of the model are presented in Table 1. The state variable of the common trend is defined as $S_{tr} = \{0,1\}$ where $q_1 = \Pr[S_{tr} = 0|S_{tr-1} = 0] = 0.9696$ and $p_1 = \Pr[S_{tr} = 1|S_{tr-1} = 1] = 0.6639$. $\mu_0$ is positive and corresponds to the $\Pr[S_{tr} = 0|S_{tr-1} = 0]$ expansion phase of common business cycles. $\mu_1$ is negative and corresponds to the $\Pr[S_{tr} = 1|S_{tr-1} = 1]$ recession phase of common business cycles. The growth rate of the common trend during state zero is $\delta + \mu_0 = 2.8683\%$. The growth rate of the common trend during state one is $\delta + \mu_1 = -0.475\%$. The transition probabilities imply that the unconditional probability of common trend being in expansion regime is $(1-p_1)/(2-p_1-q_1) = 0.9171$, 92% and in recession is $(1-q_1)/(2-p_1-q_1) = 0.0829$, 8%. The expected duration of the common expansion is $(1-q)^{-1}$ is approximately 32 years. Accordingly the expected duration of the common recession is $(1-p)^{-1}$ about 3 years. The common trend and the smoothed probability of recession for the common trend are shown in Figures 1 and 2. Figure 2 illustrates that the model estimated 3 switches when the common trend changes from expansion to recession state. The switch occurs when $p > 0.5$, which correspond to the third quarter of 1981, the second quarter of 1990, and the fourth quarter of 2004. Those switches of the common trend from expansion to recession are reflected in Figure 1. In each of these cases the common recession lasts for one year.

The state variable of the common cycle is defined as $S_{2r} = \{0,1\}$ where $q_2 = \Pr[S_{2r} = 0|S_{2r-1} = 0] = 0.9978$ corresponds to the expansion phase of the cycle and $p_2 = \Pr[S_{2r} = 1|S_{2r-1} = 1] = 0.9883$ corresponds to the recession phase of the cycle. The downward pluck, $\lambda = -0.6974$, is negative. However the common cycle exhibits a recessionary phase right away and switches from the recession in the second quarter of 1987. Figures 3 and 4 illustrate the common cyclical component and the smoothed probability of recession for the common cyclical component.

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3 For the simplification of language, state zero and state one are referred to as $\Pr[S_t = 0|S_{t-1} = 0]$ and $\Pr[S_t = 1|S_{t-1} = 1]$ respectively.
4. Conclusion

This paper applies a multivariate unobserved components model with Markov regime switching in order to estimate the common trend and cycle of the NAFTA countries’ outputs and to explore whether the common trend and cycle exhibit business cycle fluctuations. The study finds that the common trend exhibits simultaneous upswings and downswings. The economic swings in the common trend occur for three out of the five recessions that were experienced in the U.S. during the examined time period. The common recession phases according to this study are 1981 third quarter to 1982 fourth quarter, 1990 second quarter to 1991 second quarter, and 2000 fourth quarter to 2001 fourth quarter.

References


### Table 1: Maximum Likelihood Estimates: Quarterly GDP, 1970:1 to 2006:1

#### Transition Probabilities of Common Trend

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<thead>
<tr>
<th>$q_1$</th>
<th>0.9696</th>
<th>$p_1$</th>
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#### Transition Probabilities of Common Cycle

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#### Common Trend Factor Loadings

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#### Common Cycle Factor Loadings

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#### Autoregressive Parameters for the Common and Idiosyncratic Components

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#### Common and Idiosyncratic Innovation Standard Deviations

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Standard errors are in parentheses
Figure 1: Common Trend for NAFTA Countries

Figure 2: Smoothed Probability of Recession for Common Trend
Figure 3: Common Cycle for NAFTA Countries

Figure 4: Smoothed Probability of Recession for Common Cycle