A DSGE model of the term structure with regime shifts*

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Abstract

We construct and estimate the term structure implications of a small DSGE model with nominal rigidities in which the laws of motion of the structural shocks are subject to stochastic regime shifts. We demonstrate that, to a second order approximation, switching regimes generate time-varying risk premia. We then estimate the model on US data relying on information from both macroeconomic variables and the term structure. Our results support the specification with regime-switching: heteroskedasticity is a clear feature of the model’s residuals and the regimes have intuitively appealing features. The model is also capable of generating sizable time-variability in term premia. With non-expected utility preferences, the DSGE model can match yield for standard values of the parameters which affect its first order dynamics.

JEL classification:

Keywords: DSGE models, term structure of interest rates, policy rules, particle filter, Bayesian estimation.

PRELIMINARY

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1 Introduction

The yield curve plays implicitly a central role in macro (DSGE) models, because the expectations channel is a fundamental component of their monetary policy transmission mechanism. In the recent generation of models with nominal rigidities, the central bank can often afford to react little, on impact, to deviations of inflation from its target value, because at the same time it promises – and private agents believe this promise – that it will keep reacting over a long time in the future. This type of monetary policy rule – often described as "inertial," or including a concern for "interest rate smoothing" – stabilises inflation because aggregate demand is affected by the whole expected future path of policy interest rates, not just the current rate. Given this central role of the yield curve in DSGE models, it would also be desirable to include bond prices in the information set of the econometrician when the models are taken to the data.

The reason why this is not common practice is that estimated DSGE models are typically solved to a first-order approximation. Linearised models appear to be inconsistent with yield data at a basic level. They imply that the unconditional slope of the term structure should be zero, contrary to overwhelming evidence that the average term structure is positively sloped. Some recent papers, however, have argued that DSGE models, or in general models in which the endogeneity in inflation is explicitly taken into account, can perform better than expected when matching unconditional features of yields data, including slope and volatility – provided they are solved using second-order approximations or higher.\(^1\)

In this paper, we therefore explore the ability of a small microfounded model with nominal rigidities to match the conditional dynamics of both macroeconomic and term structure data, using a full-information estimation approach. Consistently with the aforementioned papers, we estimate the second-order approximate solution of the model, rather than its log-linearised version, so that the model is capable of generating non-zero term-premia.

However, models solved to a second-order approximation can only generate constant risk-premia, while the finance literature has highlighted the importance of allowing for

\(^1\)See e.g. Piazzesi and Schneider (2006), Wachter (2006), Gallmeyer, Hollifield, Palomino, and Zin (2007), Bansal and Shaliastovich (2008), Hördahl, Tristani and Vestin (2008) and Rudebusch and Swanson (2008)
time-variation in risk premia to match the conditional features of yields – see e.g. Dai and Singleton (2002). In order to allow for time-variation in risk premia, we allow for heteroskedasticity in the model’s structural shocks, i.e. time-variation in the "amount of risk" faced by bond-holders at any point in time. We assume that heteroskedasiticy takes the specific form of regime switching, because discrete regimes are potentially more amenable to an economic interpretation. Moreover, the assumption of regime switching has already been shown to help fit yields in the finance literature – see Hamilton (1988), Naik and Lee (1997), Ang and Bekaert (2002a,b), Bansal and Zhou (2002), Bansal, Tauchen and Zhou (2004), Ang, Bekaert and Wei (2008), Dai, Singleton and Yang (2008), Bikbov and Chernov (2007) – and is also increasingly used in macroeconomics following Sims and Zha (2007).

Finally, in the most general version of our theoretical model we also adopt the non-expected utility preferences proposed by Epstein and Zin (1989) and Weil (1990). These preferences are quite standard in the finance literature – see e.g. Campbell (1999) – and they have already been successfully used to model yields in a partial equilibrium model by Piazzesi and Schneider (2006) and, more recently, Bansal and Shaliastovich (2008). Gallmeyer et al. (2007), Backus, Routledge and Zin (2007) and Rudebusch and Swanson (2008) have used these preferences in calibrated models.

Our empirical results are based on US data from 1966 to 2006. We estimate different variants of our model, from the simplest case of a linearised model with power utility and heteroskedastic shocks, to the case of a quadratic model with non-expected utility and regime switching along three dimensions. The following general features of our results can be highlighted.

The first is that we find considerable support for a specification with regime switches, compared to a model with Gaussian shocks. The residuals of a model with Gaussian shocks show clear signs of heteroskedasticity and serial correlation. All model variants with regime switching are capable of fitting yields reasonably well. This result is somewhat

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2The finance literature, especially in affine term structure models, emphasises more time-variations in risk premia due to changed in the "price of risk". Time variations in the price of risk can be generated within general equilibrium models if they are solved up to a third order approximation. This approach is pursued in Ravenna and Seppala (2007a, b), Rudebusch, Sack and Swanson (2007) and Rudebusch and Swanson (2007, 2008). However, these papers are purely theoretical: the estimation of DSGE models solved using third order approximations appears to be infeasible at this point in time.
suprising for the linear model, which can generate the observed slope of the yield curve solely through expectations terms.

Smoothed regimes are in line with one’s intuition and roughly consistent across model specifications. Not surprisingly, the variance of policy shocks becomes higher at the time of the so-called "monetarist experiment" of the Fed. The variances of technology shocks and of a demand shock (either in preferences or in government spending) show clear signs of the "Great moderation" after 1984 and/or of counter-cyclical features, with switches to the high-variance regime during recessions.

The second general feature of our results is that nonlinear models are capable of generating considerable variations in risk premia, i.e. expected excess holding period returns. Premia vary over time because of changes in regime. While the exact identification of the various regimes is not identical across model-variants, variations in excess holding period returns are quite robustly identified. Premia tend to be countercyclical, namely they tend to increase during NBER recessions. For example, they increase sharply at the beginning of the new millennium. At the same time, they tend to be higher and more volatile over the whole 1980s. They then increase again at the beginning of the new millennium.

From the viewpoint of economic interpretation, the model with non-expected utility preferences has one clear advantage: the posterior mean of estimated parameters are more in line with estimates derived solely from macro-data. As a result, the propagation mechanism implicit in the model plays an active role. In contrast, parameter estimates from the model with power utility often reach extreme values. For example, the monetary policy rule is very aggressive, so that variations in inflation are always attributed to exogenous changes in the target.

Our model is related to a growing literature exploring the term structure implications of new-Keynesian models. The closest papers to ours is Doh (2006), which also estimates a quadratic DSGE model of the term structure of interest rates with heteroskedastic shocks. However, Doh (2006) allows for additional non-structural parameters to model the unconditional slope of the yield curve, while our approach is fully theoretically consistent. Another difference between the two papers is that heteroskedasticity in Doh (2006) is modelled through ARCH shocks, while it is generated by regime switching in our case. Andreasen (2008) shows that the estimation of a richer term structure model, which includes capital accumulation, is feasible to second order. However, the model cannot gen-
erate time-variation in risk premia because shocks are homoskedastic. Bekaert, Cho and Moreno (2006) and De Graeve, Emiris and Wouters (2007) estimate the loglinearised reduced form of DSGE models using both macroeconomic and term structure data. As in Doh (2006), these papers do not impose theoretical restrictions on the unconditional slope of the yield curve. In addition, they assume at the outset that risk-premia are constant.

2 The model

In order to highlight the marginal contribution of heteroskedasticity, we rely on a relatively standard model in the spirit of Woodford (2003). The central feature is the assumption of nominal rigidities.

We only deviate from the standard model in postulating that households’ preferences can be described by the non-expected utility specification proposed by Epstein and Zin (1989) and Weil (1990). This specification is quite standard in the consumption-based asset pricing literature and it has already been employed to analyse the term structure of interest rate in a partial equilibrium model by Piazzesi and Schneider (2006). Here we extend this specification to a general equilibrium model in which we also allow for habit persistence in consumption and a labour-leisure choice – see also Backus, Routledge and Zin (2004, 2005). Rudebusch and Swanson (2008) also use non-expected utility preferences in a model similar to ours, but they rely on the assumption of homoskedastic shocks.

2.1 Households

We assume that each household $i$ provides $N(i)$ hours of differentiated labor services to firms in exchange for a labour income $w_l(i)N_l(i)$. Each household owns an equal share of all firms $j$ and receives profits $\int_0^1 \Pi_l(j)dj$.

As in Erceg, Henderson and Levin (2000), an employment agency combines households’ labor hours in the same proportions as firms would choose. The agency’s demand for each household’s labour is therefore equal to the sum of firms’ demands. The labor index $L_t$ has the Dixit-Stiglitz form $L_t = \left[ \int_0^1 N_l(i) \frac{\theta_{w,t}^{-1}}{\theta_{w,t} - 1} \Pi_l(j) dj \right] \frac{\theta_{w,t}}{\theta_{w,t} - 1}$ where $\theta_{w,t} > 1$ is subject to exogenous shocks. At time $t$, the employment minimizes the cost of producing a given amount of the aggregate labor index, taking each household’s wage rate $w_l(i)$ as given and then sells units of the labor index to the production sector at the aggregate wage index.
\[ w_t = \left[ \int_0^1 w(i) \left( \frac{1}{1-\theta w_i} \right) \right]^{\frac{1}{1-\theta w_i}}. \]

The employment agency’s demand for the labor hours of household \( i \) is given by

\[ N_t(i) = L_t \left( \frac{w_t(i)}{w_t} \right)^{-\theta w_i} \]  

(1)

Each household \( i \) maximizes its intertemporal utility with respect to consumption, the wage rate and holdings of contingent claims, subject to its labor demand function (1) and the budget constraint

\[ P_tC_t(i) + E_tQ_{t,t+1}W_{t+1}(i) \leq W_t(i) + w_t(i)N_t(i) + \int_0^1 \Xi_t(j) \, dj \]

(2)

where \( C_t \) is a consumption index satisfying

\[ C_t = \left( \int_0^1 C_t(z) \frac{\pi(z)}{\pi(z)} \, dz \right)^{\frac{1}{\pi(z)}} \]

(3)

the price level \( P_t \) is defined as the minimal cost of buying one unit of \( C_t \), hence equal to

\[ P_t = \left( \int_0^1 p(z)^{1-\theta} \, dz \right)^{\frac{1}{1-\theta}} \]

(4)

\( W_t \) denotes the beginning-of-period value of a complete portfolio of state contingent assets, \( Q_{t,t+1} \) is their price, \( w_t(i) \) is the nominal wage rate and \( \Xi_t(j) \) are the profits received from investment in firm \( j \).

Equation (2) states that each household can only consume or hold assets for amounts that must be less than or equal to its salary, the profits received from holding equity in all the existing firms and the revenues from holding a portfolio of state-contingent assets.

Households’ preferences are described by the Kreps and Porteus (1978) specification proposed by Epstein and Zin (1989). In that paper, utility is defined recursively through the aggregator \( U \) such that

\[ U \left[ C_t, \left( E_tV_{t+1}^{1-\gamma} \right) \right] = \left\{ (1 - \beta) C_t^{1-\sigma} + \beta \left( E_tV_{t+1}^{1-\gamma} \right)^{\frac{1-\sigma}{1-\gamma}} \right\}^{\frac{1}{1-\sigma}}, \quad \sigma, \gamma \neq 1 \]

(5)

where \( \beta, \sigma \) and \( \gamma \) are positive constants. Using a specification equivalent to that in equation (5), Weil (1990) shows that \( \beta \) is, under certainty, the subjective discount factor, but time preference is in general endogenous under uncertainty. The parameter \( \gamma \) is the relative risk aversion coefficient for timeless gambles. The parameter \( 1/\sigma \) measures the elasticity of intertemporal substitution for deterministic consumption paths.

The distinguishing feature of these preferences, compared to the standard expected utility specification, is that the coefficient of relative risk aversion can differ from the
reciprocal of the intertemporal elasticity of substitution. In addition, Kreps and Porteus (1978) show that, again contrary to the expected utility specification, the timing of uncertainty is relevant in their class of preferences. The specification in equation (5) displays preferences for an early resolution of uncertainty when the aggregator is convex in its second argument, i.e. when $\gamma > \sigma$. Any source of risk will be reflected in asset prices not only if it makes consumption more volatile, but also if it affects the temporal distribution of consumption volatility.

We generalise the model in equation (5) by allowing for habit formation and a labour-leisure choice. More specifically, time-$t$ utility will not only depend on consumption $C_t$ but it will more generally be given by

$$ u_t = (C_t - hC_{t-1}) \cdot v(N_t) $$

where $v(N_t(i))$ is a function to be specified below and the $h$ parameter represents the force of habits in the model in the standard sense: the higher $h$, the less utility is generated by a given amount of current consumption.\(^3\)

Each households $i$ therefore maximises

$$ U[C_t(i), N_t(i), E_tV_{t+1}] = \left\{ (1 - \beta) \left[ (C_t(i) - hC_{t-1}(i)) \cdot v(N_t(i)) \right]^{1-\sigma} + \beta \left( E_tV_t^{1-\gamma} \right)^{\frac{1-\sigma}{1-\gamma}} \right\}^{\frac{1}{1-\sigma}} $$

subject to

$$ P_tC_t(i) + E_tQ_{t,t+1}W_{t+1}(i) \leq W_t(i) + w_t(i) N_t(i) + \int_0^1 \Xi_t(j) \, dj $$

and

$$ N_t(i) = L_t \left( \frac{w_t(i)}{w_t} \right)^{-\rho_{w,t}} $$

where the choice variables are $w_t(i)$ and $C_t(i)$.

To Bellman equation for this problem (abstracting from the $i$ subscript to simplify the notation) is

$$ J(W_t, C_{t-1}) = \max \left\{ (1 - \beta) \left[ (C_t - h_tC_{t-1}) v(N_t) \right]^{1-\sigma} + \beta \left[ E_tJ^{1-\gamma} (W_{t+1}, C_t) \right]^{\frac{1-\sigma}{1-\gamma}} \right\}^{\frac{1}{1-\sigma}} $$

$$ -\Lambda_t \left[ P_tC_t + E_tQ_{t,t+1}W_{t+1} - W_t - w_t N_t - \int_0^1 \Xi_t(i) \, di + T_t \right] $$

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\(^3\)Guariglia and Rossi (2002) also use expected utility preferences combined with habit formation to study precautionary savings in UK consumption. Koskievic (1999) studies an intertemporal consumption-leisure model with non-expected utility.
The appendix shows that the first order conditions for this problem can be written as

\[
\tilde{w}_t = -\frac{v'(N_t)}{v(N_t)} \left[ (C_t - hC_{t-1}) v(N_t) \right]^{1-\gamma} \Lambda_t \tag{7}
\]

\[
Q_{t,t+1} = \beta \tilde{\Lambda}_{t+1} \frac{1}{\tilde{\pi}_{t+1}} \left( \frac{E_t J_{t+1}^{1-\gamma}}{J_{t+1}} \right)^{\gamma-\sigma} \tag{8}
\]

\[
\tilde{\Lambda}_t = (C_t - hC_{t-1})^{-\sigma} [v(N_t)]^{1-\sigma} - \beta \theta J_t \tilde{\pi}_t^{-\sigma} [v(N_t)]^{1-\sigma} \left( \frac{E_t J_{t+1}^{1-\gamma}}{J_{t+1}} \right)^{\gamma-\sigma} \tag{9}
\]

where \( \tilde{\Lambda}_t \equiv \Lambda_t P_t \frac{1}{(1-\beta)} J_t^{-\sigma} \) and \( \tilde{w}_t \equiv w_t / P_t \). In equation (7), we also define \( \mu_{w,t} \equiv \theta_{w,t} / (\theta_{w,t} - 1) \); in the rest of the paper, we refer to exogenous changes in \( \mu_{w,t} \) as cost-push shocks.

The gross interest rate, \( I_t \), equals the conditional expectation of the stochastic discount factor, i.e.

\[
I_t^{-1} = E_t Q_{t,t+1} \tag{10}
\]

Note that we will focus on a symmetric equilibrium in which nominal wage rates are all allowed to change optimally at each point in time, so that individual nominal wages will equal the average \( w_t \).

Equations (8)-(9) highlight how this specification nests the standard power utility case, when \( \sigma = \gamma \) and the maximum value function \( J_t \) disappears from the first order conditions. The same equations also demonstrate that the parameter \( \gamma \) only affects the dynamics of higher order approximations. To first order, the term \( \left[ E_t J_{t+1}^{1-\gamma} \right]^{\gamma-\sigma} / J_{t+1}^{\gamma-\sigma} \) in equations (9) and (10) are constant in expected terms.

### 2.2 Firms

We assume a continuum of monopolistically competitive firms (indexed on the unit interval by \( j \)), each of which produces a differentiated good. Demand arises from households’ consumption and from government purchases \( G_t \), which is an aggregate of differentiated goods of the same form as households’ consumption. It follows that total demand for the output of firm \( i \) takes the form \( Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\theta} Y_t \). \( Y_t \) is an index of aggregate demand which satisfies \( Y_t = C_t + G_t \).
Firms have the production function

\[ Y_t(j) = A_t L_t^\alpha(j) \]

where \( L_t \) is the labour index \( L_t \) defined above.

Once aggregate demand is realised, the firm demands the labour necessary to satisfy it

\[ L_t(j) = \left( \frac{Y_t(j)}{A_t} \right)^{\frac{1}{\alpha}} \]

The total nominal cost function for the firm will therefore be given by

\[ TC_t(j) = w_t \left( \frac{Y_t(j)}{A_t} \right)^{\frac{1}{\alpha}} \]

where \( w_t \) is the wage index defined above. As a result, real marginal costs will be

\[ mc_t(j) = \frac{1}{\alpha} \frac{w_t}{P_t} \left( \frac{Y_t(j)}{A_t} \right)^{\frac{1}{\alpha} - 1} \]

where nominal costs are deflated using the aggregate price level (not the individual firm’s price).

As in Rotemberg (1982), we assume the firms face quadratic costs in adjusting their prices. This assumption is also adopted, for example, by Schmitt-Grohé and Uribe (2004) and Ireland (1997). It is well-known to yield first-order inflation dynamics equivalent to those arising from the assumption of Calvo pricing. From our viewpoint, it has the advantage of greater computational simplicity, as it allows us to avoid having to include an additional state variable in the model, i.e. the cross-sectional dispersion of price across firms.

The specific assumption we adopt is that firm \( j \) faces a quadratic cost when changing its prices in period \( t \), compared to period \( t - 1 \). Consistently with what is typically done in the Calvo literature, we modify the original Rotemberg (1982) formulation to allow for indexation of prices in part to lagged inflation, in part to the inflation objective

\[ \frac{\zeta}{2} \left( \frac{P_t^j}{P_{t-1}^j} - (\Pi_t^* |_{t-1})^{1-\varepsilon} \Pi_t^{t-1} \right)^2 Y_t \]

So, firms maximise their real profits

\[ \max_{P_t^j} \sum_{s = t}^{\infty} Q_{t,s} \left[ P_t^j Y_s^j \left( \frac{P_t^j}{P_s^j} \right) - TC_s \left( Y_s^j \frac{P_t^j}{P_s^j} \right) - \frac{\zeta}{2} \left( \frac{P_t^j}{P_{s-1}^j} - (\Pi_s^* |_{s-1})^{1-\varepsilon} \Pi_s^{s-1} \right)^2 Y_s \right] \]

\(^4\)The two pricing models, however, have in general different welfare implications – see Lombardo and Vestin (2008).
subject to
\[ Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\theta} Y_t \]
and to
\[ Y_t(j) = A_t L_t^o(j) \]

Focusing on a symmetric equilibrium in which all firms adjust their price at the same time, the first order condition for price setting can be written as
\[ (\theta - 1)Y_t + \zeta \left( \Pi_t - (\Pi_t^*)^{1-\epsilon} \Pi_{t-1}^* \right) Y_t \Pi_t = \frac{\theta}{\alpha} \bar{\epsilon}_t \left( \frac{Y_t}{A_t} \right)^{1/\gamma} + E_t Q_{t+1} + \zeta \left( \Pi_{t+1} - (\Pi_{t+1}^*)^{1-\epsilon} \Pi_t^* \right) Y_{t+1} \Pi_{t+1} \]

2.3 Monetary policy

We close the model with the simple Taylor-type policy rule
\[ I_t = \left( \frac{\Pi_t^*}{\beta} \right)^{1-\rho_t} \left( \frac{\Pi_t^*}{\Pi_t} \right)^{\psi_n} Y_t^{\psi_Y} I_{t-1}^{\rho_t} e^{\eta_{t+1}} \]
where \( Y_t \) is aggregate output, \( \Pi_t^* \) is a stochastic inflation target and \( \eta_{t+1} \) is a policy shock.

Some authors, notably Clarida, Galí and Gertler (2000) and Lubik and Schorfheide (2004), have argued that the start of the Volcker era also signed a structural change in US monetary policy, which resulted in a much stronger anti-inflation determination of the Federal Reserve. The change allegedly manifests itself in an increase of the inflation reaction coefficient (\( \psi_n \) in our notation above) in a simple Taylor rule characterisation of monetary policy. Until 1979Q2, monetary policy was allegedly such as to induce an indeterminate equilibrium.

Here, we propose a different interpretation of Federal Reserve behaviour. We maintain fixed the Taylor rule parameters, but allow for the possibility of changes in the inflation target \( \Pi_t^* \). A lower anti-inflationary determination would therefore be captured by an increase in the target. This formulation allows us to abstract from issues of equilibrium determinacy when estimating the model.

2.4 Market clearing

Market clearing in the goods market requires
\[ Y_t = C_t + G_t \]

In the labour market, labour demand will have to equal labour supply. In addition, the total demand for hours worked in the economy must equal the sum of the hours worked
by all individuals. Taking into account that at any point in time the nominal wage rate
is identical across all labor markets because all wages are allowed to change optimally,
individual wages will equal the average $w_t$. As a result, all households will chose to supply
the same amount of labour and labour market equilibrium will require that

$$L_t = \left( \frac{Y_t}{A_t} \right)^{1/\eta}$$

### 2.5 Exogenous shocks

In macroeconomic applications, exogenous shocks are almost always assumed to be (log-
)normal, partly because models are typically log-linearised and researchers are mainly
interested in characterising conditional means. However, Hamilton (2008) argues that a
correct modelling of conditional variances is always necessary, for example because infer-
ence on conditional means can be inappropriately influenced by outliers and high-variance
episodes. The need for an appropriate treatment of heteroskedasticity becomes even more
compelling when models are solved nonlinearly, because conditional variances have a direct
impact on conditional means.

In this paper, we assume that variances are subject to stochastic regime switches fall
shocks other than the inflation target. More specifically

$$A_{t+1} = A_t^\epsilon e^{A_t \epsilon_{t+1}}, \quad \epsilon_{t+1} \sim N \left(0, \sigma_{a,sY,t} \right)$$
$$\eta_{t+1} = \eta_t^\epsilon e^{\eta_t \epsilon_{t+1}}, \quad \epsilon_{t+1} \sim N \left(0, \sigma_{p,sP,t} \right)$$
$$G_{t+1} = (gY)^{1-p_G} G_t^\epsilon e^{G_t \epsilon_{t+1}}, \quad \epsilon_{t+1} \sim N \left(0, \sigma_{G,sG,t} \right)$$

where $g$ is the steady state government spending to GDP ratio and

$$\sigma_{a,sY,t} = \sigma_{a,LsY,t} + \sigma_{a,H} (1-sY,t)$$
$$\sigma_{p,sP,t} = \sigma_{p,LsP,t} + \sigma_{p,H} (1-sP,t)$$
$$\sigma_{G,sG,t} = \sigma_{G,LsG,t} + \sigma_{G,H} (1-sG,t)$$

and the variables $s_{G,t}$, $s_{P,t}$, $s_{Y,t}$ can assume the discrete values 0 and 1. For each variable
$s_{j,t}$ ($j = C, I, Y$), the probabilities of remaining in state 0 and 1 are constant and equal to
$p_{j,0}$ and $p_{j,1}$, respectively.

We assume regime switches in these particular variances for the following reasons. The
literature on the "Great moderation" (see e.g. McDonnell and Perez-Quiros, 2000) has
emphasised the reduction in the volatility of real aggregate variables starting in the second half of the 1980s. We conjecture that this phenomenon could be captured by a reduction in the volatility of technology shocks in our structural setting. The heteroskedasticity in policy shocks aims to capture the large increase in interest rate volatility in the early 1980s, the time of the so-called "monetarist experiment" of the Federal Reserve. Finally, the finance literature has found a relationship between regimes identified in term-structure models and the business cycle. In our model, this relationship could be accounted for by regime switches of the volatility of demand (government spending) shocks.

Concerning the process followed by the inflation target, we assume that

$$\Pi^*_t = \Pi^*_{p,L} s_{p,t} + \Pi^*_{p,H} (1 - s_{p,t})$$  \hspace{1cm} (12)$$

so that the inflation target is allowed to take either a high or a low value over time. To restrict the parameter space, we assume that switches in the inflation target are associated with switches in the policy shocks. A similar assumption in made in Schorfheide (2005).

2.6 Alternative specifications

In the empirical section, we also document results based on alternative specifications. These are characterised by: standard, power-utility preferences; a smoothly time-varying inflation target, modelled as an AR(1) shocks; demand shocks modelled as preference shocks (with regime switching variance), rather than government spending shocks. In addition, in these alternative models we assume Calvo pricing with partial indexation to lagged inflation (or to the target). In this cases, the $\zeta$ parameter represents the probability for firms not to be able to optimally change prices.

3 Solving and estimating the model

3.1 Solution method

To solve the model, we exploit the recursive nature of bonds in equilibrium. We first solve for all macroeconomic variables and then construct the prices of bonds of various maturities.
We start by writing the macroeconomic system in compact form as

\[ y_t = g(z_t, \sigma) \]  
\[ z_{t+1} = h(z_t, \sigma) + \zeta(z_t) \sigma \tilde{u}_{t+1} \]

where \( g(\cdot), h(\cdot), \) and \( \zeta(\cdot) \) are matrix functions and we define the vectors: \( z_t, \) including the lagged endogenous predetermined variables, the state variables with continuous support and the state variables with discrete support; \( y_t, \) collecting all jump variables (excluding bond yields); and \( \tilde{u}_t, \) containing all innovations. In order to write the law of motion of the discrete processes in the form implied in equation (14), we rely on Hamilton (1994). The law of motion of state \( s_{G,t}, \) for example, is written as

\[ s_{G,t+1} = (1 - p_{G,0}) + (-1 + p_{G,1} + p_{G,0}) s_{G,t} + \nu_{G,t+1}, \]

where \( \nu_{G,t+1} \) is an innovation with mean zero and heteroskedastic variance.

We then seek a second-order approximation to the functions \( g(z_t, \sigma) \) and \( h(z_t, \sigma) \) around the non-stochastic steady state \( z_t = \bar{z} \) and \( \sigma = 0. \) We define the non-stochastic steady-state as vectors \( \bar{y} \) and \( \bar{z} \) such that \( f(y, \bar{y}, z, \bar{z}). \)

For the continuous state variables, the non-stochastic steady state \( \bar{z} \) corresponds to the value which they would eventually attain in the absence of further shocks. For the state variables with discrete support, the non-stochastic steady state is instead the ergodic mean of the Markov chain. Formally, when we take the limit as \( \sigma = 0 \) we shrink the support of the regime-switching processes, so that their two realisations become closer and closer to each other. Eventually, the two realisations coincide on the ergodic mean of the process.

Amisano and Tristani (2007b) show that the second-order approximate solution can be represented as

\[ \hat{g}(z_t, \sigma) = F \tilde{z}_t + \frac{1}{2} \left( I_{n_y} \otimes \tilde{z}_t' \right) E \tilde{z}_t + k_{y,s} \sigma^2 \]

and

\[ \hat{h}(z_t, \sigma) = P \tilde{z}_t + \frac{1}{2} \left( I_{n_z} \otimes \tilde{z}_t' \right) G \tilde{z}_t + k_{z,s} \sigma^2 \]

for vectors \( k_{y,s}, k_{z,s} \) and matrices \( F, E, P \) and \( G \) to be determined. Note that \( k_{y,s} \) and \( k_{z,s} \) are vectors dependent on the realisation of the discrete states.

Once the solution of the macroeconomic model is available, yields can be solved for analytically.
To start with, we rewrite the stochastic discount factor more simply as

$$Q_{t,t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{1}{\pi_{t+1}}$$

for

$$\pi_t \equiv \pi_t J_t^{1-\gamma} \left[ E_t J_t^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$$

$$\Lambda_t \equiv \Lambda_t \left[ E_t J_t^{1-\gamma} \right]^{\frac{1}{1-\gamma}}$$

Since these relationships are all loglinear, the law of motion for $\pi_t$ and $\Lambda_t$ can immediately be derived from those of $\pi_t$, $\Lambda_t$, $J_t$, and $D_t \equiv E_t J_t^{1-\gamma}$. It follows that

$$\lambda_t = F_{\lambda} \hat{\lambda}_t + \frac{1}{2} \zeta \hat{E}_{\lambda} \hat{\lambda}_t + k_{\lambda,s} \sigma^2$$

$$\pi_t = F_{\pi} \hat{\pi}_t + \frac{1}{2} \zeta \hat{E}_{\pi} \hat{\pi}_t + k_{\pi,s} \sigma^2$$

and

$$\tilde{z}_{t+1} = P \tilde{z}_t + \frac{1}{2} (I_{n_z} \otimes \tilde{z}_t') G \tilde{z}_t + k_{z,s} \sigma^2 + \tilde{\zeta} (z_t) \sigma \tilde{u}_{t+1}$$

where $F_{\lambda}$ and $F_{\pi}$ are the appropriate rows of vector $F$ and $E_{\lambda}$ and $E_{\pi}$ are appropriate submatrices of matrix $E$. We can now compute bond prices using the method in Hördahl, Tristani and Vestin (2008). The appendix shows that, in log-deviation from its deterministic steady state, the approximate price of a bond of maturity $n$, $\hat{b}_{t,n}$, can be written as

$$\hat{b}_{t,n} = F_{B_n} \hat{\pi}_t + \frac{1}{2} \zeta \hat{E}_{B_n} \hat{\pi}_t + k_{B_n,s} \sigma^2$$

where $F_{B_n}$, $E_{B_n}$, and $k_{B_n,s}$ are defined through a recursion. Note that $k_{B_n,s}$ changes depending on the realisation of the discrete states, but matrices $F_{B_n}$ and $E_{B_n}$ are state-independent.

### 3.2 Estimation methodology

Looking at the system of equations (13) and (14), given that discrete state variables appear linearly and in a quadratic way, the system can be re-written as quadratic in the continuous state variables with intercept and linear terms changing according to the discrete state variables. This alternative representation is particularly convenient for describing the estimation methodology. It is straightforward to show that the model can
be rewritten as

\[ y_{t+1}^o = c_j + C_{1,j} x_{t+1} + C_{2} \text{vech}(x_{t+1} x_{t+1}') + D v_{t+1} \]  
\[ x_{t+1} = a_i + A_{1,i} x_t + A_{2} \text{vech}(x_t x_t') + B_i w_{t+1} \]  
\[ s_t \sim \text{Markov switching} \]

where the vector $y_t^o$ includes all observable variables, vector $x_t$ only includes the states with continuous support, vector $s_t$ includes the states with discrete support, and $v_{t+1}$ and $w_{t+1}$ are measurement and structural shocks, respectively. In this representation, the regime switching variables affect the system by changing the intercepts $a_i$ and $c_j$, the slope coefficients $A_{1,i}$ and $C_{1,j}$, and the loadings for the of the structural innovations $B_i.$ (we indicate here with $i$ the value of the discrete state variables at $t$ and with $j$ the value of the discrete state variables at $t + 1$).

If the approximation of the state space form is truncated to the linear terms, then the system becomes

\[ y_{t+1}^o = c_j + C_{1} x_{t+1} + D v_{t+1} \]  
\[ x_{t+1} = a_i + A_{1} x_t + B_i w_{t+1} \]  
\[ s_t \sim \text{Markov switching} \]

i.e. a linear system with (conditionally) Gaussian innovations and intercepts and loading factors which depend on the value of the discrete state variables. We describe how to obtain the likelihood of the model separately for the linear and the quadratic cases. With the likelihood in hand and a choice for prior specification, estimation is carried out by posterior simulation.

3.2.1 The linear case

In the linear case, we have a linear state space model with Markov switching. See Kim (1994), Kim and Nelson (1999) and Schorfheide (2005). The likelihood cannot be obtained by recursive methods and it is approximated using a discrete mixture approach. Things are easier when the number of continuous shocks (measurement and structural) is equal to the number of observables. In such a case the continuous latent variables can be obtained via
inversion and the system can be written as a Markov Switching VAR. The likelihood can be obtained by using Hamilton’s filter i.e. by integrating out the discrete latent variables.

3.2.2 The quadratic case

In the quadratic case, the likelihood cannot be obtained in closed form. One possible approach to compute the likelihood is to rely on sequential Monte Carlo techniques (see e.g. Amisano and Tristani, 2007a, and the reference therein). These methods, however, are computationally expensive in a case, such as the one of our model, in which both nonlinearities and non-Gaussianity of the shocks characterise the economy.

We thus adopt a simple extension of the filter we employ in the linear case. The only problem in this respect is the quadratic term in $x_t$ in the observation equation (15). For this reason, at each point in time $t$ we compute a linear approximation of this term around the estimate of $\hat{x}_{t-1}$. Relying on the assumption that the number of continuous shocks (measurement and structural) is equal to the number of observables, we then invert equation (15) to find a candidate $\hat{x}_t^c$ from the observation of $y_t^o$. We finally expand the quadratic term again around $\hat{x}_t^c$ and repeat this procedure until convergence.

4 Regime switching and the variability of risk premia

The state-dependence of $B_{n,s}$ implies that bond risk premia will also become time-varying. In order to see this, it is useful to derive expected excess holding period returns, i.e. the expected return from holding a $n$-period bond for 1 period in excess of the return on a 1-period bond. To a second order approximation, the expected excess holding period return on an $n$-period bond can be written as

$$\widehat{h}_{pr, t, n} - \hat{r}_t = \text{Cov}_t \left[ \hat{x}_{t+1}, \hat{b}_{t+1, n-1} \right] - \text{Cov}_t \left[ \Delta \hat{\lambda}_{t+1}, \hat{b}_{t+1, n-1} \right]$$

This expression can be evaluated using the model solution to obtain

$$\widehat{h}_{pr, t, n} - \hat{r}_t = \sigma^2 F_{B,n-1} \tilde{\zeta}_t \tilde{\zeta}'_t \left( F'_\pi - F'_\lambda \right)$$

(21)

where $\tilde{\zeta}_t \equiv \tilde{\zeta} (z_t)$ and $\tilde{\zeta}_t \tilde{\zeta}'_t$ is the conditional variance-covariance matrix of vector $z_t$, which depends on state $s$. In our model, therefore, risk premia change every time there is a switch in any of the discrete state variables.
Since the conditional variance of the price of a bond of maturity $n$ can be written, to a second order approximation, as $E_t \left[ \tilde{B}_{t+1,n-1} \tilde{B}_{t+1,n-1}^\prime \right] = \sigma^2 F_{n-1} \tilde{\zeta}_t^{-1} F_{n-1}^\prime$, it follows that we can define the (microfounded) price of risk for unit of volatility, or the "market prices of risk," in our model as

$$\xi_t = \sigma \tilde{\zeta}_t^{-1} (F_\pi - F_\lambda) \quad (22)$$

The market prices of risk are only affected by first-order terms in the reduced-form of the model. All terms in equation (22) would be constant in a world with a single regime. They becomes time-varying in our model due to the possibility of regime switches, because the variance-covariance matrix $\tilde{\zeta}_t^{-1}$ is regime-dependent.

In the empirical finance literature, the market prices of risk are often postulated exogenously using slightly different specifications. For example, Naik and Lee (1997), Bansal and Zhou (2002) and Ang, Bekaert and Wei (2008) assume that the market prices of risk are regime dependent, but the risk of a regime-change is not priced. On the contrary, regime-switching risk is priced in Dai, Singleton and Yang (2008).

In our model, these specifications can arise endogenously depending on how the regime-switching processes affect the model. Based on the definition $z_t' = [x_t', s_t']'$, where vector $x_t$ only includes the states with continuous support and vector $s_t$ includes the states with discrete support, we can partition the matrix $\tilde{\zeta}_t$ (recall that shocks with continuous and discrete support are all independently distributed) and the vectors $F_\pi$ and $F_\lambda$ conformably as

$$\tilde{\zeta}_t = \begin{bmatrix} \tilde{\zeta}_t^x & 0 \\ 0 & \tilde{\zeta}_t^s \end{bmatrix}, \quad F_\pi = \begin{bmatrix} F_\pi^x \\ F_\pi^s \end{bmatrix}, \quad F_\lambda = \begin{bmatrix} F_\lambda^x \\ F_\lambda^s \end{bmatrix}$$

As a result, equation (22) can be split into the vectors $\xi_t^x$ and $\xi_t^s$ such that $\xi_t' = [(\xi_t^x)', (\xi_t^s)']'$ and

$$\xi_t^x = \sigma \left( \tilde{\zeta}_t^x \right)' \left[ (F_\pi^x)' - (F_\lambda^x)' \right] \quad (23)$$

As a result, equation (22) can be split into the vectors $\xi_t^x$ and $\xi_t^s$ such that $\xi_t' = [(\xi_t^x)', (\xi_t^s)']'$ and

$$\xi_t^s = \sigma \left( \tilde{\zeta}_t^s \right)' \left[ (F_\pi^s)' - (F_\lambda^s)' \right] \quad (24)$$

Vector $\xi_t^x$ in equation (23) includes the prices of risk associated with variables with continuous support. These prices change across regimes. If, for example, technological risk were not diversifiable, then the price of risk associated with technology shocks would be higher in a high-variance regime for technology shocks (and lower in a low-variance regime). This is the regime-dependence of market prices of risk which is present in all the aforementioned finance models.
Vector $\xi_t$ in equation (24) includes instead the market prices of regime-switching risk, i.e. the price of risk associated with the possibility of regime changes. These prices of risk are also regime-dependent, because they will be affected by the conditional variance of the discrete process, which depends on the regime prevailing at each point in time.

In our set-up, the prices of risk associated with variables with continuous support, $\xi_t$, will always be non-zero. Whether the prices of regime-switching risk are zero or not depends instead on the exact way in which regime-switching affects the economy. When only the variance of exogenous shocks is allowed to change regime stochastically, the market price of regime-switching risk is zero. The reason is that, as in a model with homoskedastic shocks, variances have no effect on the first order approximation of the model. The possibility that variances may change is therefore also irrelevant, to first order.

On the contrary, the prices of regime-switching risk would be non-zero if regime-switching affected other structural elements of the model. One obvious possibility would be to replace the inflation target process in equation (12) with a specification allowing for regime switching in the target mean. In this case, a shift in the inflation target regime would have direct implications on, for example, inflation expectations. As a result, the possibility of such a regime-shift would also command a non-zero market price.

Our set-up can therefore offer a microfoundation for the different assumptions adopted in the finance literature. It should be emphasised, however, that papers in the finance literature also allow the prices of risk to be affine functions of the continuous states of the model. This would only be possible in our set-up if we solved the model to third order.

5 Empirical results

5.1 Data and prior distributions

We estimate the model on quarterly US data over the sample period from 1966Q1 to 2006Q2. Our estimation sample starts in 1966, because this is often argued to be the date after which a Taylor rule provides a reasonable characterisation of Federal Reserve policy.\footnote{According to Fuhrer (1996), "since 1966, understanding the behaviour of the short rate has been equivalent to understanding the behaviour of the Fed, which has since that time essentially set the federal Funds rate at a target level, in response to movements in inflation and real activity". Goodfriend (1991) argues that even under the period of official reserves targeting, the Federal Reserve had in mind an implicit
The data included in the information set are real per-capita GDP, the GDP deflator, the 3-month nominal interest rate and yields on 3-year and 10-year zero-coupon bonds. Prior to estimation, GDP is demeaned and detrended using a linear trend.

Prior and posterior distributions for our preferred, quadratic model with regime switches and non-expected utility are presented in Table 5. We refer to this models as $MQ^*$.

In previous estimates, we also estimated linear and quadratic versions of a similar model with homoskedastic shocks and/or standard expected utility preferences. Prior and posterior distributions for these other models are reported in Tables 1-4. We refer to the model with a single regime as $M0$ and to the regime-switching model as $M3$, where the digit refers to the number of discrete processes included in the specification. We denote the estimates of the first order (or linear) approximation of these models with an $L$; estimates of the second-order (or quadratic) approximation with a $Q$.

Prior distributions are relatively standard. We only discuss here the priors for the parameters related to the regime switching processes and for the $\gamma$ parameter of the $MQ^*$ model. More specifically, we set the prior means for the standard deviations of policy, preference and technology shocks so as to induce an ordering in which state 0 is the high-volatility state.

Concerning transition probabilities, we assume beta priors such that the probabilities of persistence in each state are symmetric and high means. For $\gamma$, we assume a very flat prior, but we centre it on a very high value because this helps to initialise the estimation process.

5.2 Posterior distributions and goodness of fit

Posterior distributions in Tables 1-4 show that all versions of the model with standard power utility must be stretched to different extents to replicate macro and yields data at the same time.

The first sign of strain arises from the marked increase, compared to the prior mean, in the posterior mean of the standard deviations of almost all shocks. For example, compared target for the Funds rate.

In preliminary estimation rounds, we have estimated our model under the simplifying assumption of absence of regime-switching and introducing incrementally regime switching in policy, demand and technology shocks. Since the linear model with three regime-switching processes dominates those with one or two in terms of marginal likelihood, we focus here on the first model.
to a prior mean around 1%, the estimated standard deviation of technology shocks increase to 3% in $M_0L$, to 10% in $M_0Q$, to between 3% and 4% in $M3L$ and to between 2% and 5% in $M3Q$. Large standard deviations tend to be necessary in order to produce movements in 10-year yields, which would otherwise tend to stay close to their long-run mean in an environment where the expectations hypothesis holds (see also Gürkaynak, Sack and Swanson, 2005).\footnote{Even in model $M0Q$ a weak version of the expectations hypothesis holds because risk-premia are constant.} In these models, the posterior mean of the standard deviation of the target shock is also particularly large. This increase must be interpreted jointly with the estimates of the posterior means of the policy rule coefficients. In all models, the policy rule becomes very aggressive against inflation deviations from target, with short-term reaction coefficients above 1.0 and a degree of interest rate smoothing which is consistent with inertial or superinertial policy – in the sense of Woodford (2003). These coefficients imply that inflation is almost always kept on target by the central bank. All models are therefore forced to explain the inflation rates observed in our sample as induced by the central bank through a sequence of target shocks. This feature also explains the low posterior mean of the inflation indexation parameter. Finally, linear models with standard preferences require a very small value of the elasticity of substitution of consumption (i.e. a high $\sigma$). Posterior estimates of this coefficient are less extreme for the quadratic models and especially in the $M3Q$ model, where $\sigma$ is approximately equal to 6.

Overall, the posterior distribution of models with power utility have some puzzling implications. This property changes for the model with non-expected utility and with regime-switching (see Figure 5). In this model, the standard deviations of all shocks are smaller and the policy rule displays standard parameter values. The $\gamma$ parameter is high, but broadly consistent with the values used e.g. in Piazzesi and Swanson (2006).

The advantage of the regime switching specifications is to permit large standard deviations only in periods when explaining movements in long-yields is particularly difficult. The posterior estimates of the transition probabilities suggest that the low-variance states are more persistent than the high-variance ones, except for the case of government spending shocks. Overall, this implies that the ergodic variance of the shocks is not necessarily higher than in the homoskedastic case, even if, at the same time, the model with regime-switching would be able to occasionally generate bursts in volatility, hence in risk premia.
Turning to goodness of fit measures, Figures 1-5 display 1-step-ahead forecasts and realised variables for each of the three models.

The most striking feature emerging from these figures is probably that all models are capable of fitting the data to a surprisingly good extent. What is particularly noticeable is that the level of yields can be matched by the linear models. In this context, Bekaert, Cho and Moreno (2006) and De Graeve, Emiris and Wouters (2007) fit yields by introducing exogenous parameters to explain their unconditional slope. In our case, however, the unconditional slope is zero. Nevertheless the models manage to replicate it in sample, thanks to the high persistence of the exogenous shocks.

A second feature which emerges from Figures 1-5 is the clear heteroskedasticity of the residuals. This is problematic for the models with Gaussian shocks, while it is explained by the model with regime-switching.

5.3 Implications of regime switching

Figures 6-8 display smoothed and filtered estimates of the discrete states in $M3L$, $M3Q$ and $M3^*$ models, respectively, together with the official NBER recession dates. In all cases, 1 denotes the low-variance state, 0 the high-variance state.

In the case of all models, the regimes associated with the policy shock tracks quite well the Fed’s monetarist experiment. This state jumps abruptly to the value 0 in 1980 and remains there until 1983; it then drifts back to the low-variance state over most of the remaining the sample (with marginal exceptions). There are only small revisions noticeable in the smoothed estimates, compared to the filtered ones. The

The various models also tend to detect the Great moderation in output volatility starting in the mid-1980s. In the linear model, the moderation is strongly associated with the technology shock, while it is interpreted as a drop in the volatility of the demand shock in model $M3^*$. In both cases, the switch to a low-variance regime occurs over the early 1980s and it is quite clearly identified also in real time. In previous years, however, only smoothed estimates confirm that the economy was in a high-variance regime. Filtered estimates are much more volatile and tend to repeatedly move away towards the low-variance regime. The association with the Great moderation is much less clear in the $M3Q$ model.

The remaining shocks display some association with the economic cycle in the $M3Q$
and $M3^*$ models. This is not the case in the $M3L$ model, in which the probability of being in a low-variance state is almost never above 0.5. This suggests that the cyclicality of the variance of preference shocks tend to reflect changes in risk-premia, which are ruled out by construction in the linear model.

The various states can be composed to define 8 possible combinations of regimes. This is done to construct Figures 9-10, which displays excess holding period returns derived from the models with power utility and non-expected utility, respectively. As discussed in relation to equation (21), these measures of risk premia vary over time only as a result of regime changes.

Two notable features emerge from Figure 9-10. The first one is that both models are capable of generating sizable risk-premia. Premia are strictly increasing in the maturity of bonds and hover around levels of several percentage points at the 10-year horizon.

The second feature emerging from these figures is that the premia are significantly variable over time, which is a desirable feature to explain observed deviations of the data from features consistent with the expectations hypothesis (see e.g. Dai and Singleton, 2002). In addition, premia dynamics are quite consistent between the models with standard, or non-expected utility, in spite of the somewhat different interpretation of the states.

A peak in risk-premia is visible at the time of the monetarist experiment in the early 1980s. This is encouraging, because deviations of yields from values consistent with the expectations hypothesis are known to be particularly marked during the Volcker tenure. For example, Rudebusch and Wu (2006) note that the performance of the expectations hypothesis improves after 1988 and until 2002.

Estimated premia increase strongly again after the recession at the beginning of the new millennium, but then they drop sharply during the phase of monetary tightening which began in the middle of 2004 up to the lowest levels in the sample in 2006. This is the period which was characterised as a conundrum by Federal Reserve Chairman Greenspan in congressional testimony on 16 February 2005, because long-term rates did not rise as policymakers raised short-term rates. Our model explains the conundrum in terms of a drop in risk-premia on long-term bonds.
5.4 Impulse responses

Figures 11-12 shows the impulse responses of our variables to shocks with continuous support in the $M3Q$ and $M3^*$ models.

Nonlinear impulse responses are defined as the difference between the expected future sample path of a variable conditional on a given initial state $x_t$, and the expected future path conditional on $x'_t$, where $x_t$ is equal to $x'_t$ except for an individual element which is perturbed by a known amount. The dependence of nonlinear impulse response functions on initial conditions is well-known (see e.g. Gallant, Rossi and Tauchen, 1993). Figures 11-12 show simulations starting from the steady state of the model. Posterior median responses and bounds corresponding to a 80% posterior coverage are reported. Both continuous and discrete states are simulated.

Compared to similar evidence based on estimates relying solely on information from macro-variables, the notable feature of Figure 11 is that the responses of the policy interest rate and, to a lesser extent, output and inflation, are much more persistent. In many cases, there is still no sign of a return to the baseline 3 years after the shock. Only after monetary policy shocks do endogenous variables go back to baseline quickly. The impulse responses also confirm that most of the movements in inflation are due to changes in the target. Technology and preference shocks affect output, but they have negligible effects on inflation.

In contrast, impulse responses are much more standard in Figure 12. Technology shocks remain very persistent, but the other shocks have effects only at business cycle frequencies. Macroeconomic variables are affected by structural shocks in a more intuitively appealing manner. Inflation arises mostly from cost-push shocks at business cycle frequencies, mostly by technology shocks over the longer run. GDP responds mostly to demand shocks over a 2-3 years horizon, while it is determined by technology shocks over the longer run.

6 Conclusions

Our results of the estimation of the second order approximation of a macro-yield curve model with regime switches and non-expected utility preferences show considerable support for this specification, compared to models with homoskedastic shocks.

Different regimes clearly help fitting macroeconomic variables, notably the heteroskedas-
ticity of the model’s residuals. Estimated regimes also bear an intuitively appealing structural interpretation. Moreover, changes in regimes generate sizable changes in risk premia.

At the same time, the additional parameter introduced by non-expected utility preferences allows the model to fit yields data without altering the basic functioning of the macro-model. All other parameter estimates are quite consistent with values obtained relying solely on macroeconomic information.
Appendix

A The household problem

Using the definitions $U_1, t = \partial U \left[ u_t, \left( E_{t+1}^{1-\gamma} \right)^{\frac{1}{1-\gamma}} \right] / \partial u_t$ and $U_2, t = \partial U \left[ u_t, \left( E_{t+1}^{1-\gamma} \right)^{\frac{1}{1-\gamma}} \right] / \partial \left( E_{t+1}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}$, we can write the first order conditions for the optimum as

$$
\Lambda_t \rho_t = U_1, t v(N_t) - h E_t U_1, t+1 v(N_{t+1}) U_{2, t} \left( \frac{E_{t} J_{t+1}^{1-\gamma}}{J_{t+1}} \right)^{\gamma},
$$

$$
U_1, t (C_t - h C_{t-1}) v' (N_t) = \Lambda_t \frac{1 - \theta_{w,t}}{\theta_{w,t}} w_t,
$$

$$
Q_{t,t+1} = \frac{\Lambda_{t+1}}{\Lambda_t} U_{2, t} \left( \frac{E_{t} J_{t+1}^{1-\gamma}}{J_{t+1}} \right)^{\gamma},
$$

plus the envelope conditions

$$
J_{W,t} = \Lambda_t,
$$

$$
J_{C,t} = -h U_1, t v(N_t)
$$

where we also defined $J_t \equiv J(W_t, C_{t-1})$, $J_{C,t} \equiv \partial J(W_t, C_{t-1}) / \partial C_{t-1}$.

Note that the two derivatives $U_1, t$ and $U_2, t$ can be rewritten as

$$
U_1, t = (1 - \beta) \left\{ (1 - \beta) (C_t - h C_{t-1})^{1-\sigma} [v(N_t)]^{1-\sigma} + \beta \left[ E_t J_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \right\}^{\frac{1-\sigma}{\sigma}} (C_t - h C_{t-1})^{-\sigma} [v(N_t)]^{-\sigma}
$$

and

$$
U_2, t \left[ E_{t} J_{t+1}^{1-\gamma} \right]^{\frac{1-\sigma}{\sigma}} = \beta \left\{ (1 - \beta) (C_t - h C_{t-1})^{1-\sigma} [v(N_t)]^{1-\sigma} + \beta \left[ E_t J_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \right\}^{\frac{1}{1-\gamma}}
$$

Moreover, at the optimum, the maximum value function will obey the recursion

$$
J_t = \left\{ (1 - \beta) [(C_t - h C_{t-1}) v(N_t)]^{1-\sigma} + \beta \left[ E_t J_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \right\}^{\frac{1}{1-\gamma}}
$$

Using these equations, we can rewrite the FOCs in the text as

$$
\tilde{w}_t = -\frac{v' (N_t)}{v(N_t)} \frac{\theta_{w,t}}{\theta_{w,t}-1} \frac{[(C_t - h C_{t-1}) v(N_t)]^{1-\sigma}}{\Lambda_t}
$$

and

$$
Q_{t,t+1} = \beta \frac{\tilde{\Lambda}_{t+1}}{\Lambda_t} \frac{1}{\pi_{t+1}} \left( \frac{E_t J_{t+1}^{1-\gamma}}{J_{t+1}} \right)^{\gamma-\sigma}
$$

25
where

\[ \tilde{\Lambda}_t \equiv [v(N_t)]^{1-\sigma} (C_t - hC_{t-1})^{-\sigma} - \beta hE_t [v(N_{t+1})]^{1-\sigma} (C_{t+1} - hC_t)^{-\sigma} \left( \frac{E_t J_{t+1}^{-\gamma} \tilde{v}_{t+1}}{J_{t+1}} \right)^{\gamma-\sigma} \]

and \( \tilde{w}_t \equiv \tilde{w}_t/P_t \).

Note that in the absence of labour-leisure choice \((v(N_t) = 1 \text{ for all } t)\), we would obtain

\[ Q_{t,t+1} = \frac{\beta}{\pi_{t+1}} \tilde{\Lambda}_{t+1} \left( \frac{E_t J_{t+1}^{-\gamma} \tilde{v}_{t+1}}{J_{t+1}} \right)^{\gamma-\sigma} \]

\[ \tilde{\Lambda}_t \equiv (C_t - hC_{t-1})^{-\sigma} - \beta hE_t (C_{t+1} - hC_t)^{-\sigma} \left( \frac{E_t J_{t+1}^{-\gamma} \tilde{v}_{t+1}}{J_{t+1}} \right)^{\gamma-\sigma} \]

If habits were also set to zero, we would go back to the standard case

\[ Q_{t,t+1} = \frac{\beta}{\pi_{t+1}} \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{E_t J_{t+1}^{-\gamma} \tilde{v}_{t+1}}{J_{t+1}} \right)^{\gamma-\sigma} \]

\[ \tilde{\Lambda}_t = C_t^{-\sigma} \]

Finally, when \( \gamma = \sigma \)

\[ \tilde{w}_t = -\frac{v'(N_t)}{v(N_t)} \frac{\theta_{w,t} - 1}{\tilde{\Lambda}_t} \]

\[ Q_{t,t+1} = \frac{\beta}{\pi_{t+1}} \tilde{\Lambda}_{t+1} \]

\[ \tilde{\Lambda}_t = [v(N_t)]^{1-\sigma} (C_t - hC_{t-1})^{-\sigma} - \beta hE_t [v(N_{t+1})]^{1-\sigma} (C_{t+1} - hC_t)^{-\sigma} \]

**B Solution for bond prices**

Recall that

\[ \tilde{z}_{t+1} = P\tilde{z}_t + \frac{1}{2} (I_{n_z} \otimes \tilde{z}_t') G\tilde{z}_t + k_{z,s} \sigma^2 + \tilde{\zeta}_t \sigma \tilde{w}_{t+1} \]

\[ \tilde{\lambda}_t = F_{\lambda} \tilde{z}_t + \frac{1}{2} \tilde{\lambda}_t' E_{\lambda} \tilde{z}_t + k_{\lambda,s} \sigma^2 \]

\[ \tilde{\pi}_t = F_{\pi} \tilde{z}_t + \frac{1}{2} \tilde{\pi}_t' E_{\pi} \tilde{z}_t + k_{\pi,s} \sigma^2 \]

where \( P \) is a \( n_z \times n_z \) matrix, \( G \) is a \( n_z^2 \times n_z \) matrix, \( k_{z,s} \) is an \( n_z \times 1 \) vector (whose elements are state dependent), \( \tilde{\zeta}_t \equiv \tilde{\zeta}(z_t) \) is a \( n_z \times n_u \) matrix, \( F_{\lambda} \) and \( F_{\pi} \) are \( 1 \times n_z \) vectors, \( E_{\lambda} \) and \( E_{\pi} \) are \( n_z \times n_z \) matrices, and finally \( k_{\lambda,s} \) and \( k_{\pi,s} \) are (state dependent) scalars.
B.1 1-period bonds

To derive the price of 1-period bonds, note first that a second order approximation to the stochastic discount factor is

\[ \hat{q}_{t,t+1} = (F_\lambda - F_\pi) \hat{z}_{t+1} + \frac{1}{2} \hat{z}_{t+1}' (E_\lambda - E_\pi) \hat{z}_{t+1} - F_\lambda \hat{z}_t - \frac{1}{2} \hat{z}_t' E_\lambda \hat{z}_t - k_{\pi,s} \sigma^2 \]

or

\[ \hat{q}_{t,t+1} = ((F_\lambda - F_\pi) P - F_\lambda) \hat{z}_t + \frac{1}{2} (F_\lambda - F_\pi) \left( I_{n_z} \otimes \hat{z}_t' \right) G \hat{z}_t \\
+ \frac{1}{2} \hat{z}_t' P' (E_\lambda - E_\pi) P \hat{z}_t - \frac{1}{2} \hat{z}_t' E_\lambda \hat{z}_t \\
+ \sigma^2 (F_\lambda - F_\pi) k_{\pi,s} - k_{\pi,s} \sigma^2 \\
+ \sigma (F_\lambda - F_\pi) \xi_t \tilde{u}_{t+1} + \frac{1}{2} \sigma \hat{z}_t' P' (E_\lambda - E_\pi) \xi_t \tilde{u}_{t+1} \\
+ \frac{1}{2} \sigma \tilde{u}_{t+1}' \xi_t (E_\lambda - E_\pi) P \hat{z}_t + \frac{1}{2} \sigma^2 \tilde{u}_{t+1}' \xi_t (E_\lambda - E_\pi) \tilde{u}_{t+1} \]

To second order, the price of a 1-period bond is

\[ \hat{b}_{t,1} = -i_t = E_t [\hat{q}_{t+1}] + \frac{1}{2} \left( E_t [\hat{q}_{t+1}^2] - (E_t [\hat{q}_{t+1}])^2 \right) \]

for which we need

\[ E_t [\hat{q}_{t+1}^2] - (E_t [\hat{q}_{t+1}])^2 = \sigma^2 (F_\lambda - F_\pi) \xi_t \tilde{z}'_t (F_\lambda - F_\pi) \xi_t \]

and

\[ E_t [\hat{q}_{t,t+1}] = (F_\lambda - F_\pi) P \hat{z}_t + \frac{1}{2} (F_\lambda - F_\pi) \left( I_{n_z} \otimes \hat{z}_t' \right) G \hat{z}_t + \sigma^2 (F_\lambda - F_\pi) k_{\pi,s} \\
+ \frac{1}{2} \hat{z}_t' P' (E_\lambda - E_\pi) P \hat{z}_t + \frac{1}{2} \sigma^2 E_t \left[ \tilde{u}_{t+1}' \xi_t (E_\lambda - E_\pi) \xi_t \tilde{u}_{t+1} \right] \\
- F_\lambda \hat{z}_t - \frac{1}{2} \hat{z}_t' E_\lambda \hat{z}_t - k_{\pi,s} \sigma^2 \]

Now note that, for any matrix \( A \) and vector \( x \),

\[ E \left[ x' Ax \right] = E \left[ \text{vec} \left( x' Ax \right) \right] \\
= E \left[ x' \otimes x' \right] \text{vec} \left( A \right) \\
= \left( \text{vec} \left( E \left[ xx' \right] \right) \right)' \text{vec} \left( A \right) \]

where the vec operator transforms a matrix into a vector by stacking its columns. It follows that

\[ E_t \left[ \tilde{u}_{t+1}' \xi_t (E_\lambda - E_\pi) \xi_t \tilde{u}_{t+1} \right] = \left( \text{vec} \left( 1 \right) \right) \text{vec} \left( \tilde{z}_t' (E_\lambda - E_\pi) \tilde{z}_t \right) \\
= \text{tr} \left( \tilde{z}_t' (E_\lambda - E_\pi) \tilde{z}_t \right) \]

27
where \( \text{tr} \) represents the trace, i.e. the sum of the diagonal elements of a matrix.

Hence,

\[
\hat{b}_{t,1} = ((F_\lambda - F_\pi) P - F_\lambda) \tilde{z}_t + \sigma^2 ((F_\lambda - F_\pi) k_{z,s} - k_{\pi,s}) + \frac{1}{2} \sigma^2 \text{tr} \left( \tilde{\zeta}_t' (E_\lambda - E_\pi) \tilde{\zeta}_t \right) + \frac{1}{2} \sigma^2 (F_\lambda - F_\pi) \tilde{\zeta}_t \zeta_t' (F_\lambda - F_\pi)' + \frac{1}{2} \zeta_t' (P' (E_\lambda - E_\pi) P - E_\lambda) \tilde{z}_t + \frac{1}{2} (F_\lambda - F_\pi) (I_{n_z} \otimes \tilde{z}_t^2) G \tilde{z}_t
\]

Finally, note that

\[
(F_\lambda - F_\pi) (I_{n_z} \otimes \tilde{z}_t^2) G \tilde{z}_t = (F_\lambda - F_\pi) \begin{pmatrix} \tilde{z}_t^2 & 0 & \cdots & 0 \\ 0 & \tilde{z}_t^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \tilde{z}_t^2 \end{pmatrix} G \tilde{z}_t = \begin{bmatrix} (F_{\lambda,1} - F_{\pi,1}) \tilde{z}_t^2 & (F_{\lambda,2} - F_{\pi,2}) \tilde{z}_t^2 & \cdots & (F_{\lambda,n_z} - F_{\pi,n_z}) \tilde{z}_t^2 \end{bmatrix} G \tilde{z}_t = \tilde{z}_t \begin{bmatrix} \sum_{j=1}^{n_z} (F_{\lambda,j} - F_{\pi,j}) G_j \end{bmatrix} \tilde{z}_t
\]

where \( F_{\lambda,i} \) and \( F_{\pi,i} \) denote the \( i \)-th elements of vectors \( F_\lambda \) and \( F_\pi \), respectively, and \( G_i \) denotes the \( i \)-th \( n_z \times n_z \) matrix which are vertically stacked to make up \( G \). We can therefore rewrite the 1-period bond as

\[
\hat{b}_{t,1} = F_{B,1} \tilde{z}_t + \frac{1}{2} \zeta_t' E_{B,1} \tilde{z}_t + k_{B,1,s} \sigma^2
\]

where

\[
F_{B,i} = (F_\lambda - F_\pi) P - F_\lambda \\
k_{B,i,s} = (F_\lambda - F_\pi) k_{z,s} - k_{\pi,s} + \text{tr} \left( \tilde{\zeta}_t' (E_\lambda - E_\pi) \tilde{\zeta}_t \right) + (F_\lambda - F_\pi) \tilde{\zeta}_t \zeta_t' (F_\lambda - F_\pi)' + \sum_{j=1}^{n_z} (F_{\lambda,j} - F_{\pi,j}) G_j
\]

Note also that, by construction, \( \check{b}_{t,1} = -\hat{b}_t \), so \( F_{B,i} = -F_i, E_{B,i} = -E_i \) and \( k_{B,i,s} = -k_{i,s} \). Note that this definition also allows us to rewrite \( \hat{q}_{t,t+1} \) as

\[
\hat{q}_{t,t+1} = F_{B,1} \tilde{z}_t + \frac{1}{2} \zeta_t' E_{B,1} \tilde{z}_t + \sigma^2 ((F_\lambda - F_\pi) k_{z,s} - k_{\pi,s}) + \sigma (F_\lambda - F_\pi) \tilde{\zeta}_t \tilde{u}_{t+1} + \sigma^2 \tilde{\zeta}_t' P' (E_\lambda - E_\pi) \tilde{\zeta}_t \tilde{u}_{t+1} + \frac{1}{2} \sigma^2 \tilde{u}_{t+1}' \tilde{\zeta}_t' (E_\lambda - E_\pi) \tilde{\zeta}_t \tilde{u}_{t+1}
\]
B.2 2-period bonds

2-period bond prices can be written as (up to a second order approximation)
\[ \hat{b}_{t,2} = \hat{b}_{t,1} + E_t \left[ \hat{b}_{t+1,1} \right] + \frac{1}{2} \text{Var}_t \left[ \hat{b}_{t+1,1} \right] + \text{Cov}_t \left[ \hat{b}_{t+1,1}, \hat{b}_{t+1,1} \right] \]

Based on 1-period prices, we can derive
\[
E_t \left[ \hat{b}_{t+1,1} \right] = F_{B_t} P_z \tilde{z}_t + \frac{1}{2} F_{B_t} \left( I_{n_z} \otimes \tilde{z}_t' \right) G \tilde{z}_t + \frac{1}{2} \tilde{z}_t' P' E_{B_t} P \tilde{z}_t + F_{B_t} k_{z,s} \sigma^2 + \sigma^2 k_{B_1,s} + \frac{1}{2} \sigma^2 \text{tr} \left[ \tilde{z}_t E_{B_1} \tilde{z}_t \right] \]
\]
and
\[
E_t \left[ \hat{b}_{t+1,1} \hat{q}_{t+1,1} \right] - E_t \left[ \hat{b}_{t+1,1} \right] E_t \left[ \hat{q}_{t+1,1} \right] = \sigma^2 F_{B_t} \tilde{z}_t \tilde{z}_t' F_{B_1} \]
\[
E_t \left[ \hat{b}_{t+1,1} \hat{q}_{t+1,1} \right] - E_t \left[ \hat{b}_{t+1,1} \right] E_t \left[ \hat{q}_{t+1} \right] = F_{B_t} \tilde{z}_t \tilde{z}_t' (F_{\lambda} - F_\pi)' \]

It follows that
\[ \hat{b}_{t,2} = F_{B_2} \tilde{z}_t + \frac{1}{2} \tilde{z}_t' F_{B_2} \tilde{z}_t + k_{B_2,s} \sigma^2 \]

where
\[
F_{B_2} = F_{B_1} (I + P) \\
E_{B_2} = E_{B_1} + P' E_{B_1} P + \sum_{j=1}^{n_z} F_{B_1,j} G_j \\
k_{B_2,s} = k_{B_1,s} + F_{B_1} k_{z,s} + \text{tr} \left( \tilde{z}_t' E_{B_1} \tilde{z}_t \right) + F_{B_1} \tilde{z}_t \tilde{z}_t' F_{B_1} + 2 F_{B_1} \tilde{z}_t \tilde{z}_t' (F_{\lambda} - F_\pi)' 
\]

B.3 n-period bonds

Using the same procedure, we find that n-period bond prices can be written as
\[ \hat{b}_{t,n} = F_{B_n} \tilde{z}_t + \frac{1}{2} \tilde{z}_t' F_{B_n} \tilde{z}_t + k_{B_n,s} \sigma^2 \]

where for \( n > 1 \)
\[
F_{B_n} = F_{B_1} + F_{B_{n-1}} P \\
E_{B_n} = E_{B_1} + P' E_{B_{n-1}} P + \sum_{j=1}^{n_z} F_{B_{n-1},j} G_j \\
k_{B_n,s} = k_{B_1,s} + k_{B_{n-1},s} + F_{B_{n-1}} k_{z,s} + \text{tr} \left( \tilde{z}_t' E_{B_{n-1}} \tilde{z}_t \right) + F_{B_{n-1}} \tilde{z}_t \tilde{z}_t' F_{B_{n-1}} + 2 F_{B_{n-1}} \tilde{z}_t \tilde{z}_t' (F_{\lambda} - F_\pi)' 
\]
References


Model with power utility for households and Calvo pricing for firms. The exogenous processes with continuous support include the inflation target, a policy shock, a preference shock and a technology shock. These results are based on 500000 simulations. Legend: "sd" denotes the standard deviation; "low q" and "up q" denote the 5th and 95th percentiles of the distribution. Acceptance rate: .78. Priors: beta distribution for $\psi_\pi$, $\psi_y$, $\psi_\sigma$, $\psi_{m}$; gamma distribution for $\psi_\pi$, $\psi_y$ and all standard deviations; shifted gamma distribution (domain from 1 to $\infty$) for $\gamma$, $\phi$, $\Xi$, $\Pi^*$; normal distribution for $\rho_\pi$.

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Table 1: Parameter estimates: $MOL$ model.
Table 2: Parameter estimates: M0Q model

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<td>0.138284</td>
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Model with power utility for households and Calvo pricing for firms. The exogenous processes with continuous support include the inflation target, a policy shock, a preference shock and a technology shock. These results are based on 200000 simulations. Legend: "sd" denotes the standard deviation; "low q" and "up q" denote the 5th and 95th percentiles of the distribution. Acceptance rate: .73. Priors: beta distribution for $\beta$, $h$, $i$, $\zeta$, $\rho_\pi$, $\rho_y$, $\rho_\nu$; gamma distribution for $\psi_{\pi}$, $\psi_y$ and all standard deviations; shifted gamma distribution (domain from 1 to $\infty$) for $\gamma$, $\phi$, $\Xi$, $\Pi^*$; normal distribution for $\rho_i$. 
Table 3: Parameter estimates: M3L model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>post mean</th>
<th>post sd</th>
<th>post low q</th>
<th>post up q</th>
<th>prior mean</th>
<th>prior sd</th>
<th>prior low q</th>
<th>prior up q</th>
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<td>0.9642</td>
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<td>0.9867</td>
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<td>0.0168</td>
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</table>

Model with power utility for households and Calvo pricing for firms. The exogenous processes with continuous support include the inflation target, a policy shock, a preference shock and a technology shock. These results are based on 1000000 simulations. Legend: "sd" denotes the standard deviation; "low q" and "up q" denote the 5th and 95th percentiles of the distribution. Acceptance rate: .44 Priors: beta distribution for $\gamma$, $\phi$, $\Xi$, $\Pi^*$; gamma distribution for $\psi_{\Pi}$, $\psi_{y}$ and all standard deviations; shifted gamma distribution (domain from 1 to 1) for $\gamma$, $\phi$, $\Xi$, $\Pi^*$; normal distribution for $\rho_{\Pi}$. 
Table 4: Parameter estimates: M3Q model

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<th>Parameter</th>
<th>Post Mean</th>
<th>Post SD</th>
<th>Post Low Q</th>
<th>Post Up Q</th>
<th>Prior Mean</th>
<th>Prior SD</th>
<th>Prior Low Q</th>
<th>Prior Up Q</th>
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<td>0.0090</td>
<td>0.9642</td>
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</table>

Model with power utility for households and Calvo pricing for firms. The exogenous processes with continuous support include the inflation target, a policy shock, a preference shock and a technology shock. These results are based on 100000 simulations. Legend: "sd" denotes the standard deviation; "low q" and "up q" denote the 5th and 95th percentiles of the distribution. Acceptance rate: .54 Priors: beta distribution for $\beta$, $h$, $\iota$, $\zeta$, $\rho_{c}$, $\rho_{y}$, $\rho_{\psi}$, $\psi_{\pi}$, $\psi_{y}$ and all standard deviations; shifted gamma distribution (domain from 1 to $\infty$) for $\gamma$, $\phi$, $\Xi$, $\Pi^*$; normal distribution for $\rho_{I}$. 
Table 5: Parameter estimates: $MQ^*$ model

<table>
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<tr>
<th>Parameter</th>
<th>Post Mean</th>
<th>Post SD</th>
<th>Post Low Q</th>
<th>Post Up Q</th>
<th>Prior Mean</th>
<th>Prior SD</th>
<th>Prior Low Q</th>
<th>Prior Up Q</th>
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Model with non-expected utility for households and Rotemberg pricing for firms. The exogenous processes with continuous support include the cost-push shock, a policy shock, a government spending shock and a technology shock. These results are based on 40000 simulations. Legend: "sd" denotes the standard deviation; "low q" and "up q" denote the 5th and 95th percentiles of the distribution. Acceptance rate: .24 Priors: beta distribution for $\rho_e, \rho_i, \zeta, \rho_g, \rho_A$; gamma distribution for $\psi_x, \psi_y$ and all standard deviations; shifted gamma distribution (domain from 1 to $\infty$) for $\gamma, \sigma, \phi, \Xi, \Pi^*$; normal distribution for $\rho_i$. 
Figure 1: Actual variables and 1-step-ahead predictions: M0L model
Figure 2: Actual variables and 1-step-ahead predictions: M0Q model
Figure 3: Actual variables and 1-step-ahead predictions: M3L model
Figure 4: Actual variables and 1-step-ahead predictions: M3Q model
Figure 5: Actual variables and 1-step-ahead predictions: $MQ^*$ model.
Model with power utility for households and Calvo pricing for firms. The exogenous processes with continuous support include the inflation target, a policy shock, a preference shock and a technology shock. Legend: "state X" corresponds to the policy shock; "state C" corresponds to the preference shock; "state \( \Lambda \)" corresponds to the technology shock.
Model with power utility for households and Calvo pricing for firms. The exogenous processes with continuous support include the inflation target, a policy shock, a preference shock and a technology shock. Legend: "state X" corresponds to the policy shock; "state C" corresponds to the preference shock; "state A" corresponds to the technology shock.
Model with non-expected utility for households and Rotemberg pricing for firms. The exogenous processes with continuous support include a cost-push shock, a policy shock, a government spending shock and a technology shock. Legend: "state X" corresponds to the policy shock; "state G" corresponds to the government spending shock; "state A" corresponds to the technology shock.
Figure 9: Term premia in the M3Q model
Figure 10: Term premia in the MQ* model
Figure 11: Impulse responses in the M3Q model
Figure 12: Impulse responses in the $MQ^*$ model